Using statistical methods to investigate the kinematic and spatial history of star clusters.

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1. Introduction

In recent years the quantity and quality of kinematic data (as opposed to just spatial data) the field has for young star forming regions has increased massively. This can largely be attributed to the advent of *Gaia* among other data sources.

The kinematic and spatial evolution of young star forming regions is a topic of great scientific interest, both for its own sake and also for it relevance to other matters of scientific interest, such as the process of star formation and the question of how star forming regions originate. However, kinematic and spatial substructure is rapidly dynamically erased as young star-forming regions evolve. The exact timescale of this will depend upon the size and density of the region, amongst other factors, but typically occurs within a few Myr. To further complicate this, young star forming regions are embedded within the molecular clouds they form from. As such the younger a star-forming region is the more of its initial structure it retains but the harder it is to observe. By the time the region emerges from its parent cloud considerable dynamical evolution has had time to occur. In order to make the most of the kinematic data available to us we require statistical tools that can use it to diagnose region's kinematic state and previous evolution.

We investigate the application of the Moran's I statistic (Moran 1950) to this problem. This statistic has hitherto been mostly used by geographers to investigate the spatial autocorrelation of different ecological phenomena. Here we use it to investigate the kinematic evolution in star clusters by evaluating the spatial autocorrelation of velocity vectors in simulated star-forming regions. Specifically we employ it to try to differentiate between simulated regions that were generated with different initial conditions but have been evolved to the point that their initial spatial and kinematic substructure has been erased. In particular we focus on using this statistic as a tool to determine whether the hierarchical or monolithic models of star formation best reflect reality.

2. Methods

We perform two sets of N-body simulations of young star-forming regions. In one set the initial conditions are generated with kinematic substructure. In the other set the algorithm to generate the initial conditions is the same, except the velocity vectors are randomly shuffled between stars, so there is no kinematic substructure.

In both sets a variety of initial condition types are generated. The degree of initial spatial substructure is varied from high (H), to moderate (M), to smooth (S) substructure, and the initial virial ratio is varied between 0.3 (cool virial state, C), 0.5 (virialised, V), and 1.5 (warm, W). These three spatial structure levels and 3 initial virial ratios give a total of nine different kinds of initial conditions. Twenty simulations are performed of each type of initial conditions. Each simulated region contains 1500 stars and has a half-mass radius of 2 pc.

The simulations are run using the STARLAB software package (Portegies Zwart et al. 1999; Portegies Zwart et al. 2001). Snapshots are outputted every 0.1 Myr. Once the simulations are complete Moran's I for the components of the velocity vectors are calculated at every snapshot. In doing this calculation only stars within 2 half mass radii of the centre of mass at that snapshot are considered. This is because stars further away than this would be much less likely to be identified as cluster members if these regions were real and being observed.

3. Results

For each type of initial conditions the mean Moran's I vs time for the twenty simulations is plotted. This is done separately for the sets of simulations with and without initial kinematic substructure, and the results are presented in blue and orange respectively in Fig. 1. The standard deviation of the results of each set of 20 simulations is shown by the grey shaded area. In the top row of this figure the results for simulations which were initially highly spatially substructured are shown, the results for the initially moderately substructured simulations are on the middle row, and those with smooth initial structure on the bottom row. The results for virialy cool simulations are on the left, virialised in the centre, and warm on the right.

We note that the highly substructured & virially cool initial conditions roughly correspond to the expected initial conditions in the hierarchical model of star formation, and the initially smoothly spatially substructured and virialised initial conditions roughly correspond to the expected conditions in the monolithic model of star formation.

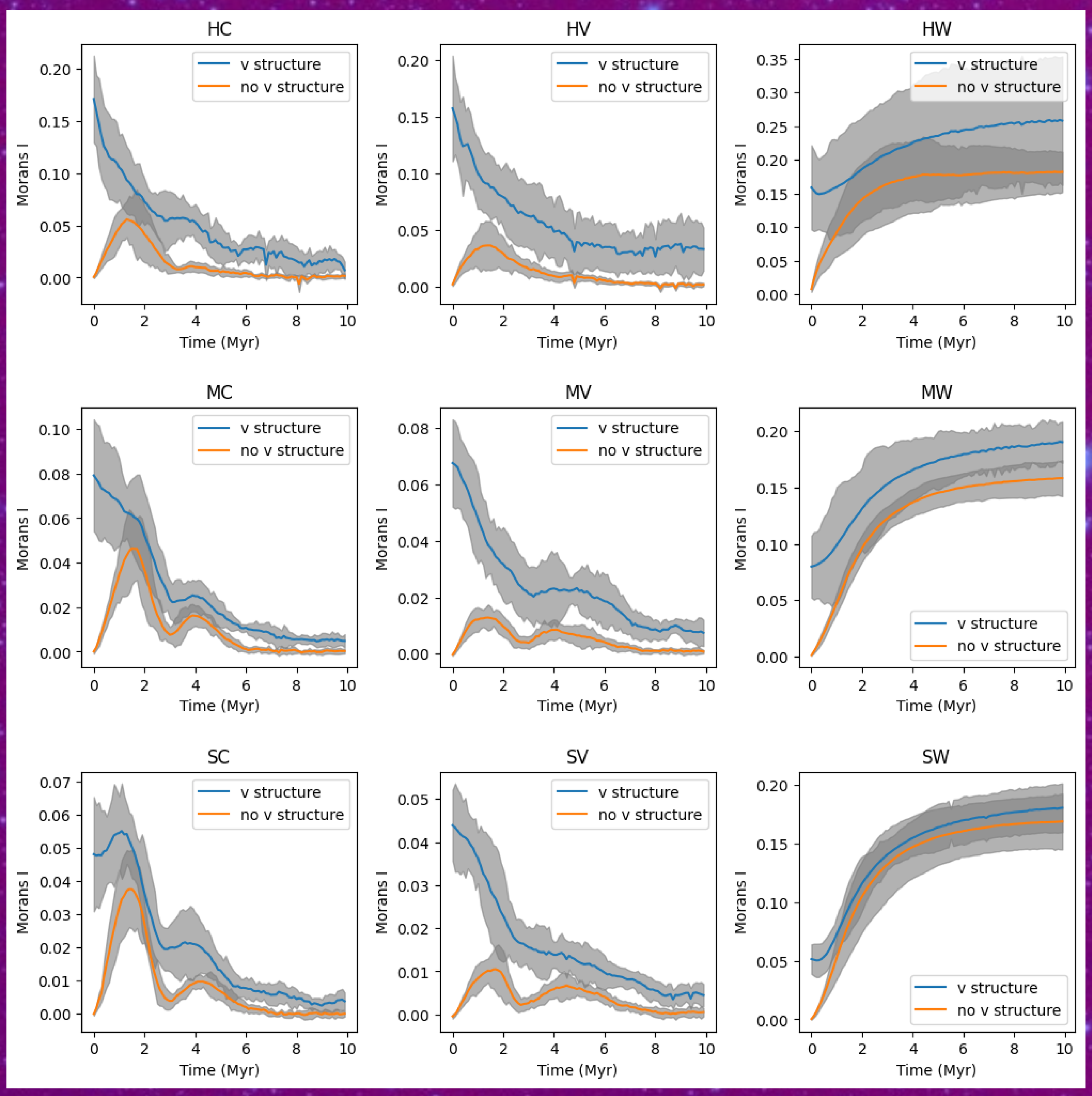

Fig 1. Plot of Moran's I vs time for nine sets of initial conditions for simulations without initial kinematic substructure.

4. Discussion

In this section we will discuss Fig. 1 in detail. In all cases the simulations without initial kinematic structure have Moran's I of ~0 at 0 Myr., confirming that there is no spatial autocorrelation between the velocity vectors. In contrast simulations initialised with kinematic substructure start with much higher values of Moran's I, demonstrating that substructure was successfully engendered by the algorithm used to generate the initial conditions.

First we will discuss the cool and virialised simulations (left and middle column) which are initialised with velocity substructure (blue lines). Moran's I trends downwards with time as the regions dynamically evolve and the initial structure is erased. However at almost all times the Moran's I remains significantly above that of regions that where initialised without kinematic structure. This is true even beyond 10 Myr. For reference, spatial structure is erased in 2-4 Myr in these simulations.

In contrast the simulations without initial kinematic substructure (orange lines) show an increase in Moran's I in the form a bump (or double bump). These can be explained as follows. The only directional force at play in these simulations is gravity, which pulls all stars towards the centre of mass of the system on average. If there is no initial velocity structure then this is uncontested and the mean velocities of the stars will be radially inwards, i.e. there will be a dependence between a star's position and its velocity vector, so the spatial autocorrelation of the velocities as measured by Moran's I increases. After 2-3 Myr the region has contracted, and a dense core has been produced. The high density means dynamical evolution is fast, kinematic structure is quickly erased, and Moran's I drops back to 0. However, this dense core is far from virial equilibrium; there is a significant excess of potential energy. This causes a "bounce back" and the core re-expands. As this occurs, on average the stellar velocities are radially outwards, so Moran's I increases, producing a 2nd peak at ~4 Myr. The region then settles down and remaining structure is erased.

Lastly we will discuss the warm (W, virial ratio 1.5, right hand column in Fig. 1) region results. In all cases Moran's I increases over time before plateauing. This is because the high virial ratio causes the regions to expand rapidly before significant dynamical evolution can occur, and they rapidly expand to a scale where such evolution *cannot* readily occur because the distances involved increase and as such the gravitational forces decrease. As such the initial kinematic structure is "frozen in", and as the region expands the signal in kinematic structure becomes clearer, so Moran's I increases. It plateaus because over time the fraction the region expands per unit time decreases as the instantaneous size is already large, so the rate of increase in Moran's I also decreases.

5. Conclusions

From inspection of the y-axes for differing levels of initial spatial substructure Fig. 1 we can see that Moran's I tends to be higher the larger the initial degree of substructure, even at times significantly past when that structure has been erased (2-4 Myrs). As such this could provide a powerful diagnostic of the initial degree of spatial substructure in regions that are now dynamically evolved.

Additionally Moran's I is substantially lower for regions without initial kinematic substructure compared to those with it. Putting these factors together, this method will provide very different results when applied to regions that formed with significant spatial and kinematic structure (as expected by the hierarchical model of star formation) compared to regions that formed without either (as expected by the monolithic model). As such this method could provide valuable evidence for determining which of these models best reflects reality.

Background credit: NASA