



Big Data to Enable Global Disruption of the Grapevine-powered Industries

D5.2 – Uncertainty-aware Visual Analytic Components

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ACRONYMS LIST

WP	Work package
LSTM	Long short-term memory
CO ₂	Carbon dioxide
SIS	Sequential Indicator Simulation

EXECUTIVE SUMMARY

In this deliverable (D5.2), we demonstrate uncertainty-aware visual analytic components that highlight uncertainties in prediction outcomes. Previous research has shown the importance of showing uncertainty as they could improve awareness and trust of the readers, particularly non-technical experts. With a growing awareness of requirements to convey uncertainty to non-expert users, researchers have proposed to extend traditional visualisation techniques to represent the uncertainty information associated with data. Nonetheless, the application of uncertainty visualisations is limited in the agriculture domain.

To demonstrate our components, in the previous version of this deliverable, we used a dataset from one of our pilot partners, INRA. The dataset contained alcoholic fermentation kinetics measured over time. The main attributes present in the dataset were the level of carbon dioxide (CO₂), temperature and time at measure. We demonstrated the possibilities of various uncertainty representation techniques.

In the current version of deliverable 5.2, we have refined the visualisations and updated the dataset to be in line with end-user evaluation requirements. Specifically, we use the price prediction scenario of agricultural products, proposed by Agroknow, to demonstrate the updated uncertainty visualisations. End-user evaluation, as part of WP8, is planned for July 2020 where our updated version of uncertainty-aware visual analytic components, along with others, will be evaluated by potential users from the industry. In this document, we present our updated version of uncertainty-aware visual analytic components and the development framework used to implement the components.

This document is structured as follows. Chapter 1 lays out an introduction to the deliverable describing the existing work and motivations. In Chapter 2, the visualisation components together with their development framework are described in detail. This document concludes with Chapter 3 where a summary of the deliverable is underlined.

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1 INTRODUCTION

Visualisation is considered as a powerful medium to help explore large datasets and to present meanings emerging from data. However, it is often impossible to encapsulate everything from real life in a dataset and to assume that the data being visualised are exact. When a data point is illustrated in a graph, we tend to interpret it as a precise representation of the true data value [1]. It is difficult to assume that a data point could actually lie somewhere it has not been drawn. Nevertheless, this scenario is abundant in data visualisation. Nearly every data set has some uncertainty, and whether and how we choose to represent this uncertainty can make a major difference in how accurately one perceives the meaning of the data. Visualisation mechanisms for communicating uncertainty have proven to be successful gaining trust particularly for non-technical experts [2]. Besides, incorporation of uncertainty into the decision-making process is crucial for making decisions and maximising benefits [3].

There is a growing awareness of the uncertainty problem within the visualisation community. Thus, researchers have been extending many traditional techniques to represent not just the data, but also the uncertainty information associated with the data [4]. Two commonly used approaches to indicate uncertainty are error bars and confidence bands [1] (see Figure 1 for example). As we can see in the figure, the two approaches are derived from the classic box-and-whisker plot. The two approaches have been widely used to indicate uncertainty because they are precise, space efficient and can show the uncertainties of many different parameter estimates in a single graph. A data point on the line may represent a mean or median whereas a bar or band may show coverage (e.g. confidence intervals or standard error).

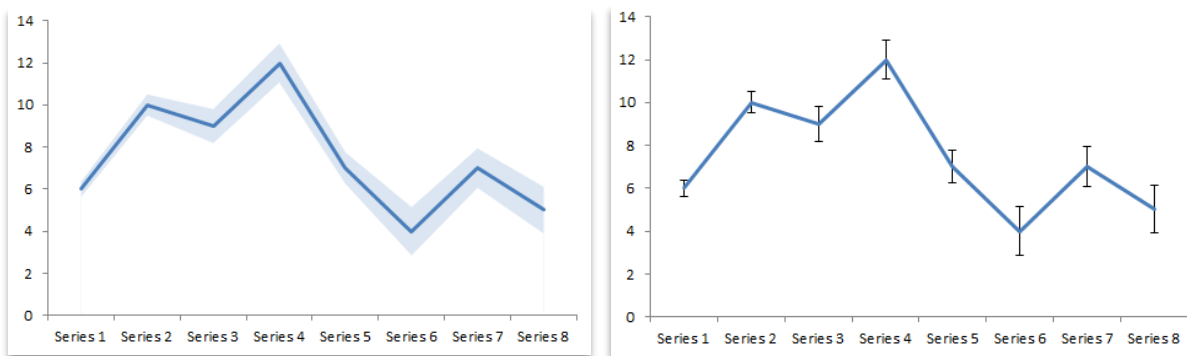


Figure 1. Example charts showing a confidence band (left) and error bars (right)¹

Nowadays, understanding uncertainty especially in prediction tasks is perhaps one of the greatest scientific challenges [4]. It impacts on many crucial issues facing the world today—from climate studies [5], [6], to meteorology [7], [8], to interpretation of medical data [9]–[11]. By conveying the possibility that a point estimate may vary, uncertainty visualisations allow users to make more informed decisions based on prediction models [12]. One example scenario that makes use of uncertainty visualisations may be the prediction of hurricane paths. Figure 2 shows contrasting hurricane trackers from 2011 made by New York Times² (left) and Stamen³ (right), both indicating the predicted path and strength of hurricane Irene. Here, the uncertainty visualisation was extremely crucial in planning evacuations based on the predicted hurricane path.

¹ <https://alesandrab.wordpress.com/2014/09/17/create-line-charts-with-confidence-bands/>

² <http://www.nytimes.com/projects/hurricanes/index.html#!/2011/Irene>

³ <https://stamen.com/>

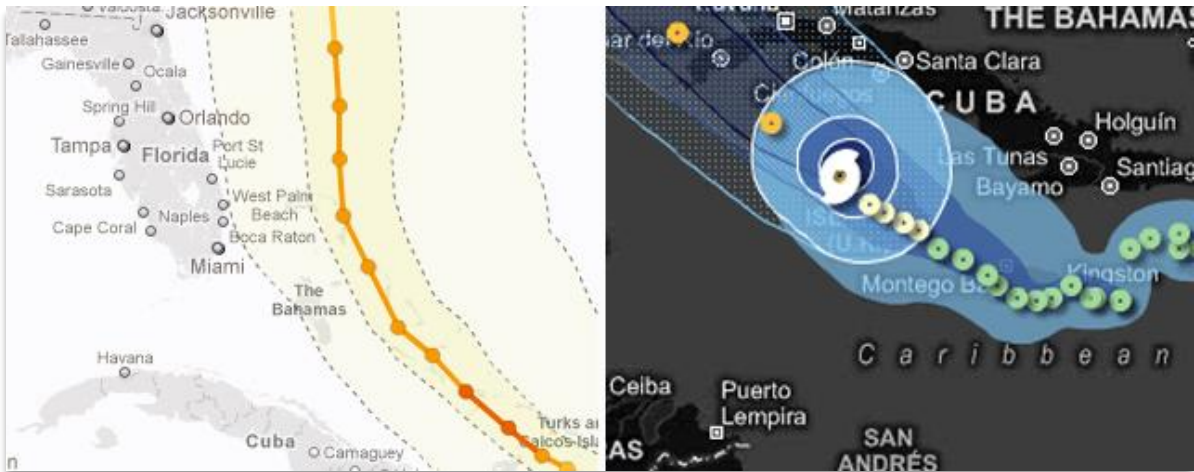


Figure 2. Hurricane trackers from 2011 made by New York Times (left) & Stamen (right) used to track the uncertainty over the strength and direction of Hurricane Irene.

Another example would be the visualisation of future economic growth in the U.K. by the Bank of England [13] as illustrated in Figure 3. The figure shows projections from November 2007 onward for changes in gross domestic product, with different shades indicating probability intervals. The black line indicates actual economic growth (according to the Office for National Statistics assessments) up to November 2007. The central interval (i.e. darker green) represents 10% probability, and the largest interval 90% probability.

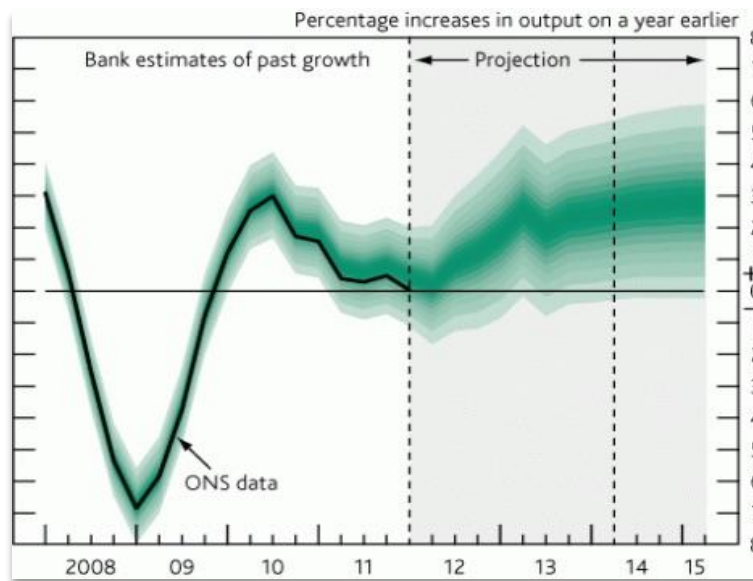


Figure 3. Fan chart for future economic growth in the U.K. as recorded in November 2007 by the Bank of England

To demonstrate potential usefulness of uncertainty visualisations, we previously presented a number of visual analytic components using a stock price data obtained from Quandl⁴. These components are illustrated in Figure 4 which shows a combination of historical stock price and a projection for the next 12 months where the uncertainty visualisation is prominent.

⁴ <https://www.quandl.com/tools/api>

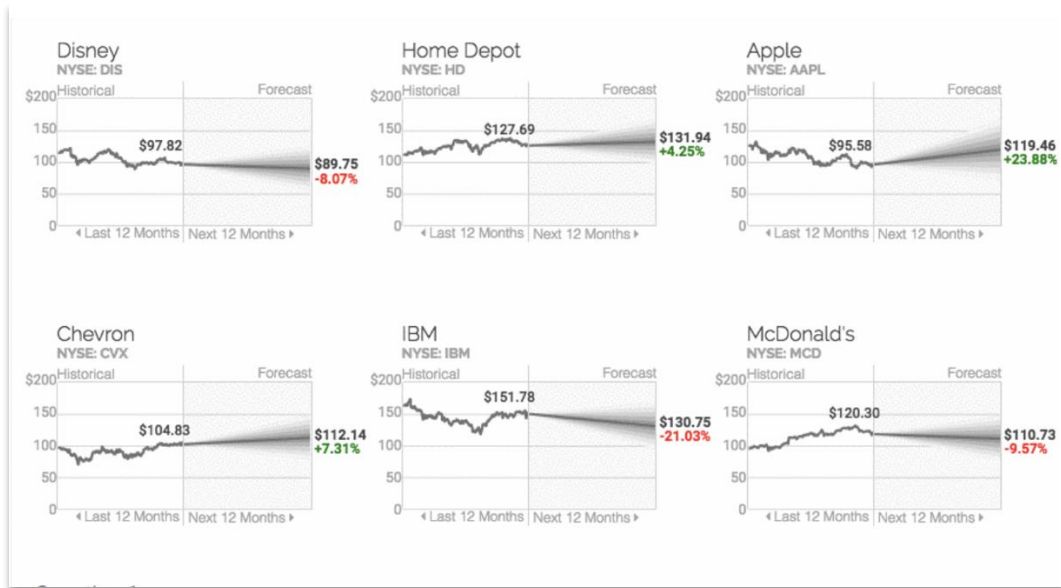


Figure 4. Uncertainty visualisation with stock price data

In the agriculture domain, the application of uncertainty visualisations is limited. One prominent example that conveyed uncertainty information is CropGIS [14] which uses a time-series to show information about biomass development of maize with a range of uncertainty describing various meteorological scenarios (see Figure 5). Weather is perhaps one of the most unpredictable variables that greatly affects agriculture. The uncertainty representations, therefore, better reflect the reality compared to static forecasts, making the prediction more reliable for farmers. Juang et al. [15] investigated the use of sequential indicator simulation (SIS) to model the uncertainty of mapping heavy-metal concentrations in soil. The result, for example, can be used to measure the reliability of delineating the contaminated area. This information is critical for decision-makers to determine which areas are contaminated and require clean-up actions.

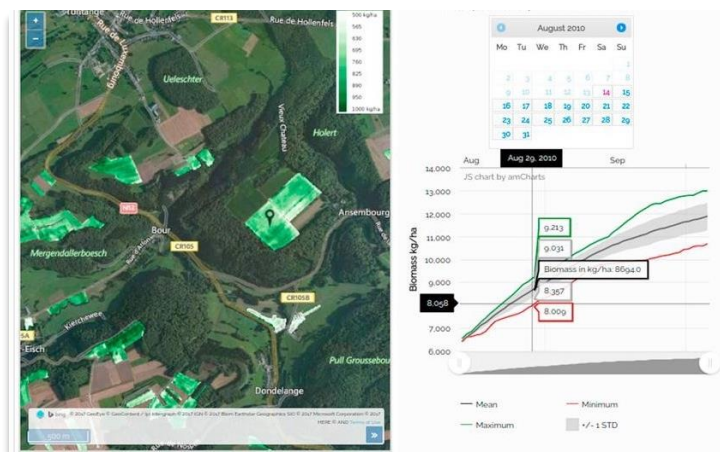


Figure 5. CropGIS showing biomass development of maize with a range of uncertainty.

In the previous version⁵ of deliverable 5.2, we demonstrated the possibilities of various uncertainty representation techniques using an alcohol fermentation kinetic dataset provided by one of the pilot partners, INRA. Figure 6 shows an overview of the visualisations demonstrated in the previous version; namely: spaghetti, uncertainty bands, gradient and hypothetical outcome plots.

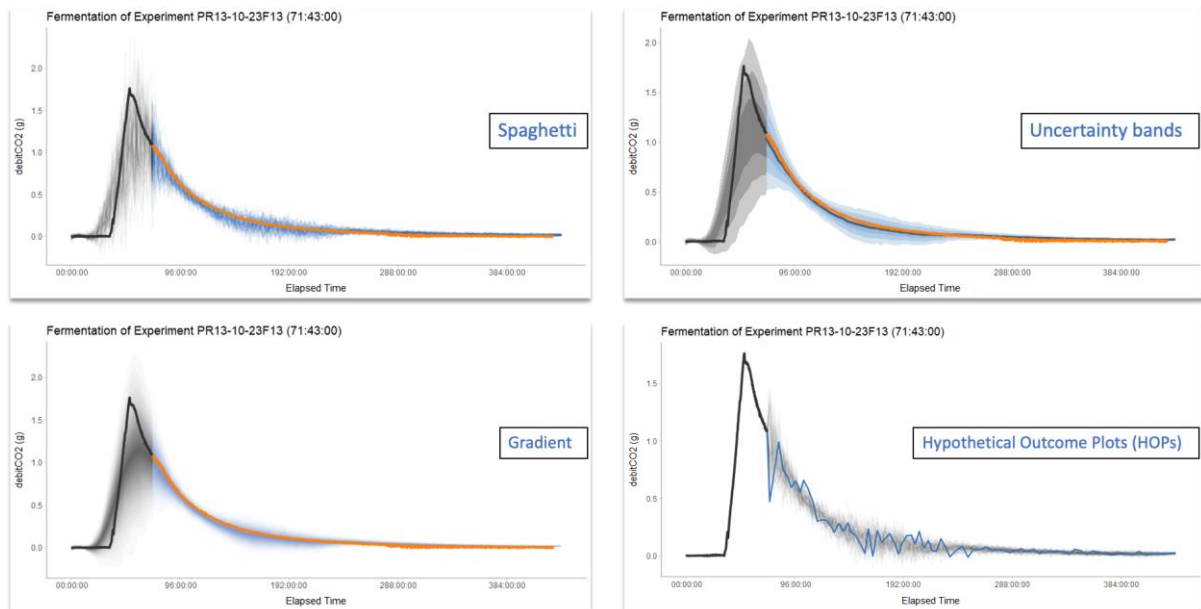


Figure 6. Uncertainty representations demonstrated in previous version of deliverable 5.2. An alcohol fermentation kinetic dataset was used.

Regardless, uncertainty visualisations, like many systems designed to meet end-user requirements, need to be iteratively improved using a human centred design approach. Following the 4th PMB meeting in Athens in October 2019, requirements for uncertainty visualisations arose once again from one of the pilot partners, Agroknow, when the task, price prediction of agricultural products, was presented. We discovered that selling price of agricultural products can be affected by various factors and can change over time depending on the country of origin. However, even when advanced machine learning models are employed, price prediction tasks can introduce a fair amount of uncertainty in prediction outputs. This makes it difficult to visualise the output as a precise representation of the true projection. Therefore, over the past months, we collaborated with Agroknow to refine the requirements and our existing uncertainty visualisation components. This refined version of uncertainty-aware visual analytic components will be assessed in the end-user evaluation which is planned in July 2020.

In the next section, we describe the system components, including the dataset, visualisations, and development framework we employed for the updated version.

⁵ <https://doi.org/10.5281/zenodo.2657348>

2 SYSTEM DESCRIPTION

2.1 DATASET

We used the price history dataset provided by Agroknow. The dataset contains a collection of product prices from various European countries over the past two decades. In total, the dataset contains 352,590 entries for over 400 products in various European countries. The variables include:

- *country* (the country in which the given product price was obtained),
- *dataSource* (the source from which the price data was obtained),
- *price* (the price of the product),
- *priceDate* (contains a date object from the moment the price data was collected),
 - contents of *priceDate* object are: {*millisSinceEpoch*, *daysSinceEpoch*, *month*, *year*, *day*}
- *priceStringDate* (contains a string with the date of data collection),
- *price_id* (a unique id for the data entry)
- *product* (the name of the product),
- *url* (the web address from which the data point was obtained)

2.1.1 Data Pre-processing

Given a large amount of entries, the dataset needs to be pre-processed to speed up interactions with the interface. Pre-processing was done by the following steps:

- Adding extra columns: *fit*, *lwrXX*, *uprXX*
 - *fit* is the result of multiple polynomial regression. Predictions are made with multiple linear regression of degree 3 using R's built-in *lm()* function. Linear regression was chosen as it was simple to compute. Currently, another tech partner, CNR, is working with the Long short-term memory (LSTM) neural network architecture which is more complex to compute but accurate at prediction. During the integration work, planned in the future, we will migrate from linear regression to LSTM.
 - *lwrXX* and *uprXX* are the lower and upper values of the prediction interval in percentage. They are calculated in increments of 5 starting from 50 and ending at 99, such as 50, 55, 60, ..., 95, 99.
- Extracting *millisSinceEpoch* from the *priceDate* object and removing the redundant variables: *daysSinceEpoch*, *day*, *month*, *year*, and *priceStringDate*. This is because all such variables could be obtained from a single *millisSinceEpoch* variable.
- Creating a separate dataset with all distinct products. This was used to speed up the product search function in the interface.

With the steps mentioned above, we created two separate datasets: a full pre-processed database with new variables, and a database with unique products.

During the evaluation, we will focus on agricultural products. However, it is important to note that we designed the visualisations to be quite generic. Therefore, other timeseries datasets can be visualised if they have the same structure and column names, i.e. *millisSinceEpoch*, *product* (or another category), *country* (or another subcategory), *price* (or another numeric value), *fit*, *uprXX* and *lwrXX* (XX may be any prediction level and as many levels as required can be added).

2.2 VISUALISATION COMPONENTS

The dashboard is built using the React framework⁶ and the D3 visualisation library⁷. It contains a number of independent components (see Figure 7); these are:

- Header with two search fields for products and related countries (designed with a React UI library called Ant Design⁸)
- Price evolution chart to visualise historical data, predictions and uncertainty representations (designed with D3)
- Radio buttons to show different uncertainty representations, each presenting end-user with a differing complexity. These variations are presented in the following subsections.

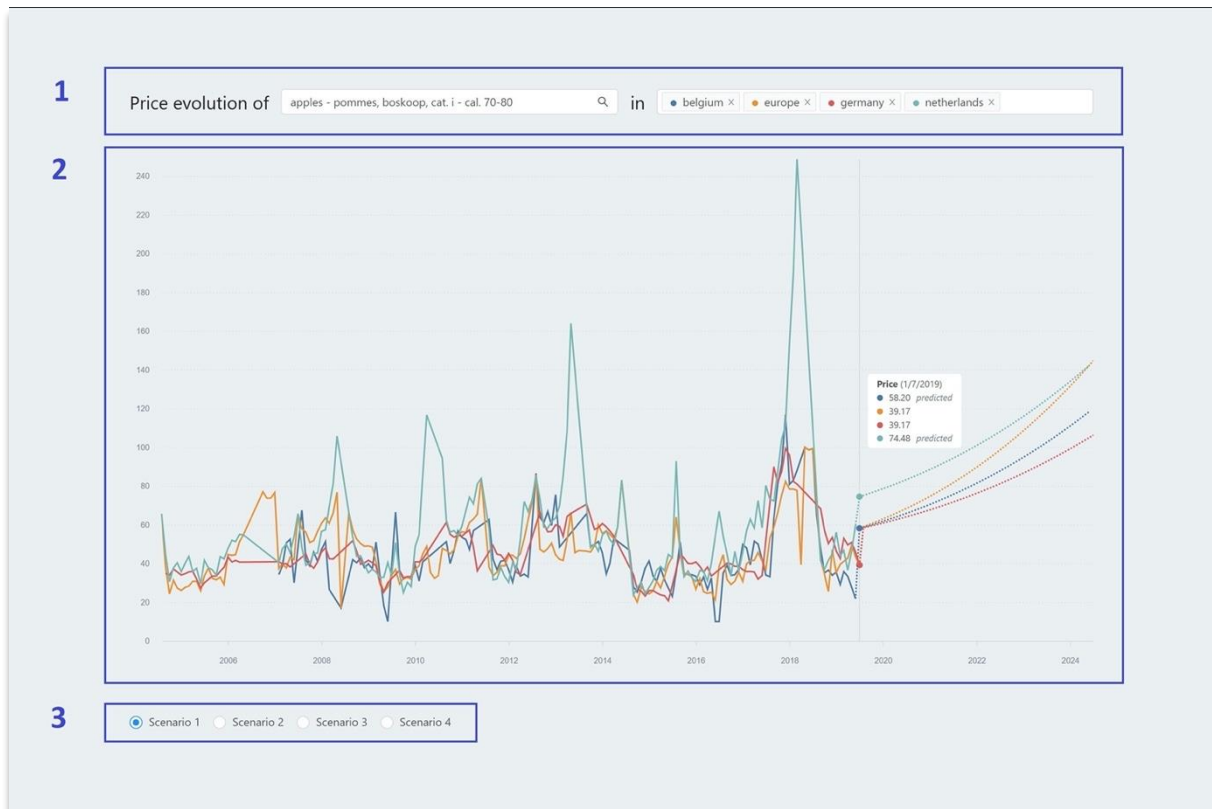


Figure 7. Price prediction dashboard with 3 distinct components: 1) header, 2) fan chart and 3) radio buttons

⁶ <https://reactjs.org/>

⁷ <https://d3js.org/>

⁸ <https://ant.design/>

2.2.1 Variation 1

Figure 8 shows a variation of the price prediction dashboard in its simplest form. This variation in fact does not contain any uncertainty representations and will be used as a baseline during the evaluation. The simple line graphs show the price evolution for a specific product in different countries. Users can compare several countries and discover mutual trends or differences. The dotted lines represent the predictions for the next 60 months (=5 years), from the last known data point. A tooltip shows the exact price and also indicates whether the value is predicted.

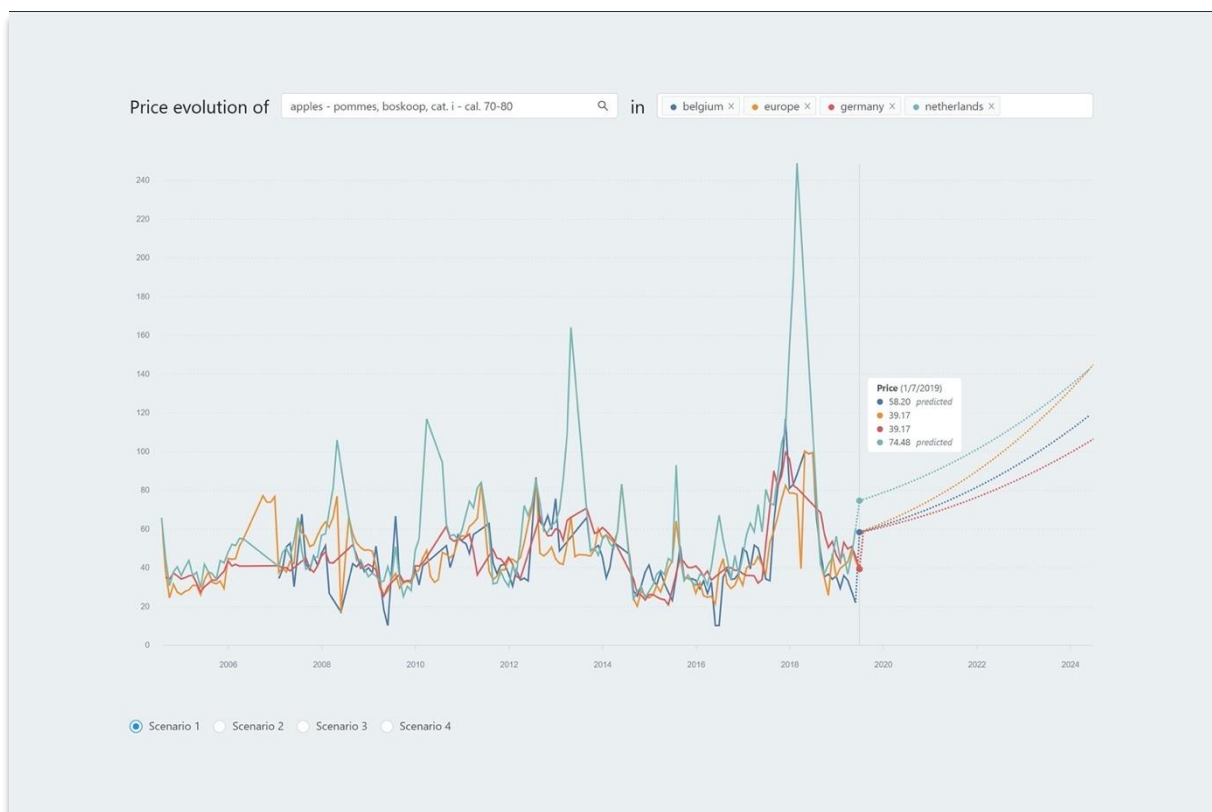


Figure 8. Variation 1 of the price prediction visualisation

2.2.2 Variation 2

Figure 9 shows the second variation of the price prediction dashboard where a simple uncertainty representation has been added. The uncertainty of future predictions is visualised by the *uncertainty fans*. At any particular point in time, the cross-section of the *uncertainty fan* corresponds to the prediction intervals according to multiple linear regression of degree 3 (as explained Section 2.1.1). A tooltip explains the meaning of the fans in layman's term and explicitly shows the upper and lower possible future price.



Figure 9. Variation 2 of the price prediction visualisation.

2.2.3 Variation 3

Figure 10 shows the third variation of the price prediction dashboard. An added component in this variation, a dotted line for historical data, is a subtle change from variation 2. The dotted line shows the fit for the historical data, according to the underlying prediction model. This gives insight in how well the model fits the known data.



Figure 10. Variation 3 of the price prediction visualisation.

2.2.4 Variation 4

Figure 11 show the fourth and final variation of the price prediction dashboard. In this variation, fans visualise the uncertainty for both predicted and historical data, according to the prediction model. From this visualisation, one could hypothesise that the timeseries with a lot of points outside the widest prediction intervals (e.g. 90, 95 and 99) are probably not very well modelled and may produce less accurate predictions.



Figure 11. Variation 4 of the price prediction visualisation.

2.3 DEVELOPMENT FRAMEWORK

The development has been carried out using the JavaScript language within the React framework⁶ with Meteor⁹ integration. Ant Design⁸, a React UI library, was used to design certain UI elements. The visualisations were designed using the D3 library⁷. Specifically, the following libraries and framework were used to design this dashboard:

- Meteor framework⁹ v1.10.2
- React framework⁶ v16.12.0
- D3 library⁷ v5.15.0
- Ant Design⁸ v3.26.9

For exploration and pre-processing of the dataset, the following libraries were used in R:

- dplyr¹⁰
- tidyr¹¹
- purrr¹²
- jsonlite¹³
- lm¹⁴

⁹ <https://www.meteor.com/>

¹⁰ <https://cran.r-project.org/web/packages/dplyr/index.html>

¹¹ <https://cran.r-project.org/web/packages/tidyr/index.html>

¹² <https://cran.r-project.org/web/packages/purrr/index.html>

¹³ <https://cran.r-project.org/web/packages/jsonlite/index.html>

¹⁴ <https://www.rdocumentation.org/packages/stats/versions/3.6.2/topics/lm>

The code of this price prediction dashboard has been published at the Github repository of BigDataGrapes project (<https://github.com/BigDataGrapes-EU/product-prices/tree/user-study>). **Please note** that this repository has been marked as private since it contains sensitive price data. All partners within the project who have access to the Github repository of BigDataGrapes project can also access this repository.

2.3.1 Uncertainty Fan

The *uncertainty fan* consists of:

- A tooltip that provides details on demand when user interact with lines and fans.
- An svg with all components of the fan chart: axes, dots that indicate the previous data points relative to the X coordinate of the mouse position, lines and fan bundles

The following sample of code is extracted from FanChart.jsx. It shows an example of how uncertainty fan was rendered.

```
tooltip = this.state.tooltipVisible && this.drawTooltip(scaleX);
svg = <svg height={this.state.height} width={this.state.width}
  onMouseMove={this.handleMouseMove}
  onMouseLeave={this.handleMouseLeave}>
  <g transform={`translate(${this.props.margin.left}, ${this.props.margin.top})`} >
    <XYAxis scaleX={scaleX} scaleY={scaleY} height={graphHeight} width={graphWidth} />
    {this.drawDots(scaleX, scaleY, graphHeight)}
    {this.drawLinesAndFans(scaleX, scaleY)}
  </g>
</svg>
```

```
return (
  <div className="fan-chart" ref={this.refFanChart}>
    {tooltip}
    {svg}
  </div>
);
```

Every fan bundle is a collection of fans (one for each prediction level).

- The higher the prediction level, the opaquer the fan is
- Opacity is computed automatically after rescaling all available levels to [0, 1]
- Min and max opacities are manually chosen such that the fans are visually pleasing and do not obscure underlying lines/points too much

The following sample of code is extracted from FanBundle.jsx. It shows an example of how uncertainty fan bundles were rendered.

```

getFans() {
const levels = Object.keys(this.props.data[0])
  .filter(n => n.indexOf("lwr") === 0)
  .map(n => n.substr(3, 4))
  .sort(function(a, b){return b-a}),
  maxLevel = d3.max(levels),
  minLevel = d3.min(levels);
let fans = [];
levels.forEach(function(level) {
  const l = (maxLevel - level) / (maxLevel - minLevel),
    opacityMin = 0.05,
    opacityMax = 0.2,
    opacity = opacityMin + (opacityMax - opacityMin) * l;
  let fanData = [];
  this.props.data.forEach(d => fanData.push({
    valueX: d.date,
    lower: parseFloat(d["lwr".concat(level)]),
    upper: parseFloat(d["upr".concat(level)]),
  }));
  fans.push(
    <Fan scaleX={this.props.scaleX} scaleY={this.props.scaleY} data={fanData} key={level}
country={this.props.country} colour={this.props.colour} opacity={opacity} level={level}
onFanMouseEnterOut={this.handleFanMouseEnterOut} />
  );
}, this);
return fans;
}

```

```

render() {
return (
  <g className="fans">
    {(this.props.data.length > 0) && this.getFans()}
  </g>
);
}

```

3 CONCLUSIONS

In this document, we presented an updated version of uncertainty-aware visual analytic components, delivered under the work package 5. Building upon the existing visual analytic components, we refined the way uncertainty is represented. The aim is to be able to accommodate newly arising end-user requirements and data from the pilot partners. In this version of the deliverable, we described four variations of a price prediction dashboard together with a development framework used to implement the visualisations. The variations will be assessed by end-users during the evaluation planned in July 2020. A number of metrics will be assessed, including usability, acceptance, understandability and trust.

We built the components using the JavaScript language with the help of a few open source visualisation libraries and frameworks. Detailed explanations of the components and development framework have been presented in Chapter 2. We also provided sample codes showing how *uncertainty fans* were rendered. The source code has been published under the Github repository of BigDataGrapes project (<https://github.com/BigDataGrapes-EU/product-prices>).

During the integration task that is planned in the future, we will migrate from simple linear regression to a more advanced LSTM neural network architecture which is being developed by CNR. In the meantime, we will conduct evaluations of this visualisation with end-users and iteratively improve it according to their feedback.

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