Mathematical Description of the Regensburg Model Scenario Types RM 1 – 6

Authors:

Andreas Wolfsteiner, Master of Economics
Günter Wittmann, Master of Mathematics

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save-the-climate@online.ms (mail to) Version: 14.02.2021

1 Introduction

The Regensburg Model Scenario Types RM 1 – 6 are used to derive plausible emission paths that meet a certain budget. The emission paths are essentially determined indirectly by an assumption about the property of the annual changes. This is the innovative core of the RM Scenario Types. We pursue two approaches: determination of the course of the annual reduction rates (RM 1 - 5) and determination of a constant annual reduction amount (RM-6).

In the indirect determination of emission paths using annual reduction rates with a monotonic trajectory, the following four basic types can be distinguished:

(1) Initial less than proportional increase[1] in annual reduction rates (RM-2, RM-4) ► concave

(2) Initial over-proportional increase in annual reduction rates (RM-5) ► convex

(3) Linear increase in annual reduction rates (RM-3) ► linear

(4) Constant annual reduction rate (RM-1) ► constant

The RM Scenario Types are used in our tools to derive plausible global or national paths. The Excel tools can be downloaded from our website. At eu.climate-calculator.info you will find a web app for calculating Paris-compatible paths for the EU, which also uses the RM Scenario Types.

[1] "Increase" refers to the absolute amount of the reduction rates.

2 Constraints to be specified

B	budget for a certain period (budget period); here: 2020 - 2100
E_{BY}	emissions in the base year (*BY*); here: *BY* = 2019
E_{min}	minimum of emissions in the budget period; a negative value represents the potential for net negative emissions
RR_{BY+1}	rate of change in the first year of the budget period in RM 2 - 5; first year here: 2020; in scenario type RM-2, only a negative value is possible
TV	threshold from which the method is changed in order to map net negative emissions in a pragmatic way (from this value a constant annual reduction amount is used)

3 Formulae Regensburg Model Scenario Types

3.1 Determination of paths via annual rates of change (scenario types RM 1 – 5)

$$E_t = \begin{cases} \max\,(E_{min};\ E_{t-1} * (1 + \mathbf{RR_t})) & for\ E_{t-1} > TV\ ^2 \\ \max\,\left(E_{min};\ E_{t-1} + (E_{t-1} - E_{t-2})\right) & for\ E_{t-1} \le TV\ ^3 \end{cases}$$

where:

E_t emissions in the year *t*; here: 2020 – 2100

The **reduction rates** in the individual scenario types are based on the following formulae:

name scenario type	formula	basic function type	con-straint	course of the reduction rates
RM-2-exp[4]	$\mathbf{RR_t} = RR_{t-1} * (1 + \boldsymbol{a})$	e^x	$\boldsymbol{a} \ge 0$	► concave
RM-4-quadr[5]	$\mathbf{RR_t} = \boldsymbol{a} * (t - (BY + 1))^2 + RR_{BY+1}$	$y = ax^2 + b$	$\boldsymbol{a} \le 0$	
RM-5-rad[6]	$\mathbf{RR_t} = \boldsymbol{a} * \sqrt{t - (BY + 1) - 0.5} + RR_{BY+1}$	$y = a\sqrt{x} + b$	$\boldsymbol{a} \le 0$	► convex
RM-3-lin	$\mathbf{RR_t} = \boldsymbol{a} * \left(t - (BY + 1)\right) + RR_{BY+1} = RR_{t-1} + \boldsymbol{a}$	$y = ax + b$	$\boldsymbol{a} \le 0$	► linear
RM-1-const	$\mathbf{RR_t} = \boldsymbol{a}$	$y = a$	$\boldsymbol{a} \le 0$	► constant

Table 1: RM Scenario Types formulae[7]

The free parameter $\boldsymbol{a}$ is determined for each scenario type using an iterative solution method so that the budget (*B*) is adhered to. In the Excel tools, the integrated target value search (“goal seek”) is

[2] "Max" means here, take the larger value. Either E_{min} or E_t (which results from the application of $\mathbf{RR_t}$).

[3] "Max" means here, take the larger value. Either E_{min} or E_t (which results from the application from the last absolute reduction amount; the emission path is then a straight line).

[4] In this scenario type, the free parameter $\boldsymbol{a}$ can be called the escalation rate applied to the reduction rate of the previous year. This scenario type can also be represented using the following formula: $RR_{BY+1} * e^{(t-(BY+1))*\ln(1+a)}$.

[5] Basic function type: $y = ax^2 + b$. The term [*t* - (*BY* +1)] is set for *x* in a variable transformation in order to be able to calculate with years. For *t* = 2020 the value of the term is 0. The term thus takes the values 0, 1, …, 80 for the period 2020 - 2100 considered here.

[6] Basic function type: $y = a\sqrt{x} + b$. The term [*t* - (*BY* +1) - 0.5] is set for *x*. 0.5 serves to smooth the course at the beginning (see attachment). The term [*t* - (*BY* +1)] represents a variable transformation in order to be able to calculate with years. *x* thus takes the values 0.5, 1.5, …, 79.5 in the period 2021 - 2100 considered here.

[7] In the scenario types RM 2, 5 and 3 for *t* = *BY+1* the predefined RR_{BY+1} (see chapter 2) must be used. Thus, the equations above hold for *t* > *BY*+1 (here: *t* > 2020).

used for this purpose, which is embedded in a macro that ensures that the constraint for ***a*** is also met.[8]

3.2 Determination of paths via annual change amount (scenario type RM-6)

RM-6-abs: $E_t = \max\,(E_{min};\ E_{t-1} + \boldsymbol{RA})$ [9]

The free parameter ***RA*** (constant annual reduction amount) is determined using an iterative solution method so that the budget (B) is adhered to.

3.3 Phases for determining the paths

This usually leads to the following **three phases** for determining the paths:

1. Application of the annual **reduction rates** (RM 1 - 5) or **reduction amount** (RM-6).
2. RM 1 – 5 if $E_{t-1} \leq TV$: The last reduction amount from phase 1 is used as the constant reduction amount until *E_min* is reached. In this phase, the emission path is a straight line.
3. Minimum for the annual emissions (*E_min*) is used until 2100.

4 Overview of RM Scenario Types

basic type[10]	Scenario Type	course of the annual reduction rates	basic function type	course of the annual reduction amounts	course of the emission paths
(4)	RM-1-const	constant	-	concave	convex
(3)	RM-3-lin	linear	$y = ax + b$	u-shaped	s-shaped (first concave then convex)
(1)	RM-2-exp	concave	e^x	u-shaped	
	RM-4-quadr		$y = ax^2 + b$	u-shaped	
(2)	RM-5-rad	convex	$y = a\sqrt{x} + b$	u-shaped	
-	RM-6-abs	concave	-	constant	linear

Table 2: RM Scenario Types overview

In principle, there are several options for mapping the basic types (1) and (2) using a specific function. However, as the scenario types RM-2 and RM-4 (see Figure 1) and Figure 4 in chapter 6.2 show, the results usually do not differ significantly with a tight budget and a plausible course of the reduction rates.

RM-1 with constant annual reduction rate and RM-6 with constant annual reduction amount primarily provide good indicators for the size of the challenge. In both scenario types, however, RR_{BY+1} results endogenously. In the other scenario types, RR_{BY+1} can be specified freely resp. at a realistic level.

[8] If no solution can be found with the given framework data, RR_{BY+1} is varied slightly in the Excel tools (see chapter 6.3) and B in the web app.

[9] "Max" means here, take the larger value. Either E_{min} or E_t (which results from the application of the constant annual reduction amount: ***RA***).

[10] "Basic type" here refers to scenario types in which the emission path is determined via the annual reduction rates. With RM-6, on the other hand, the emissions path is determined via the annual constant reduction amount.

5 Exemplary global reduction rates and paths RM 1 – 6

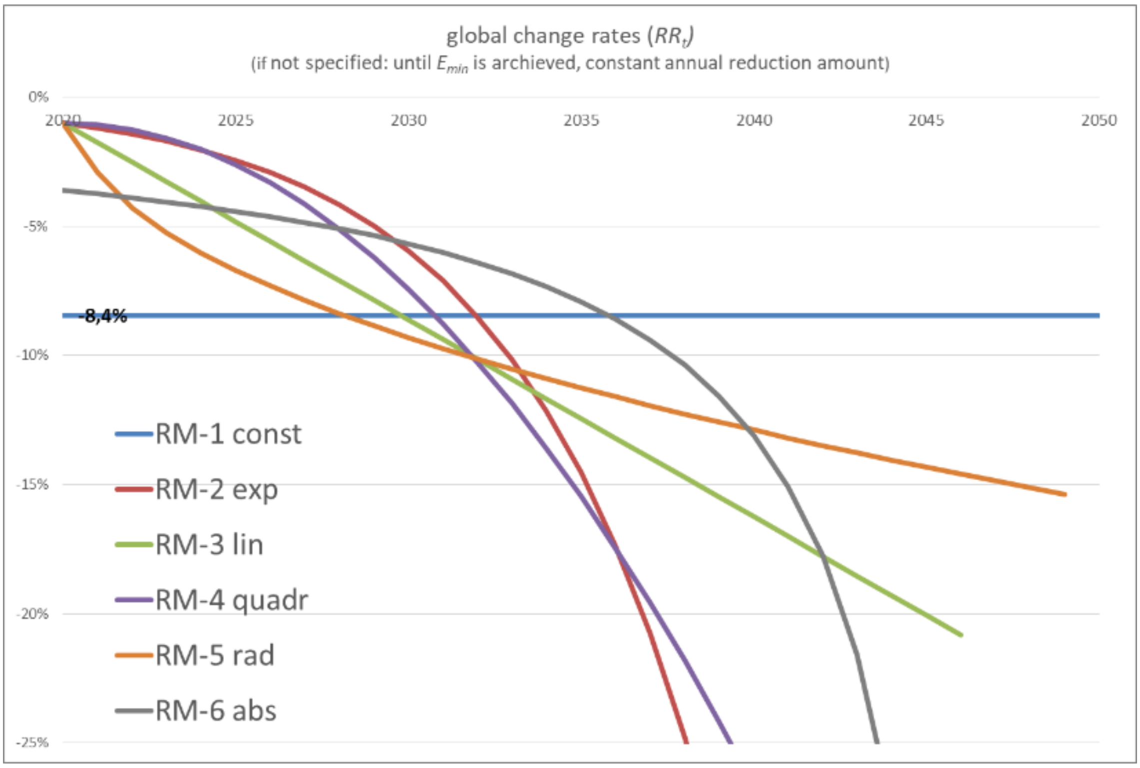

Figure 1: RM Scenario Types annual change rates

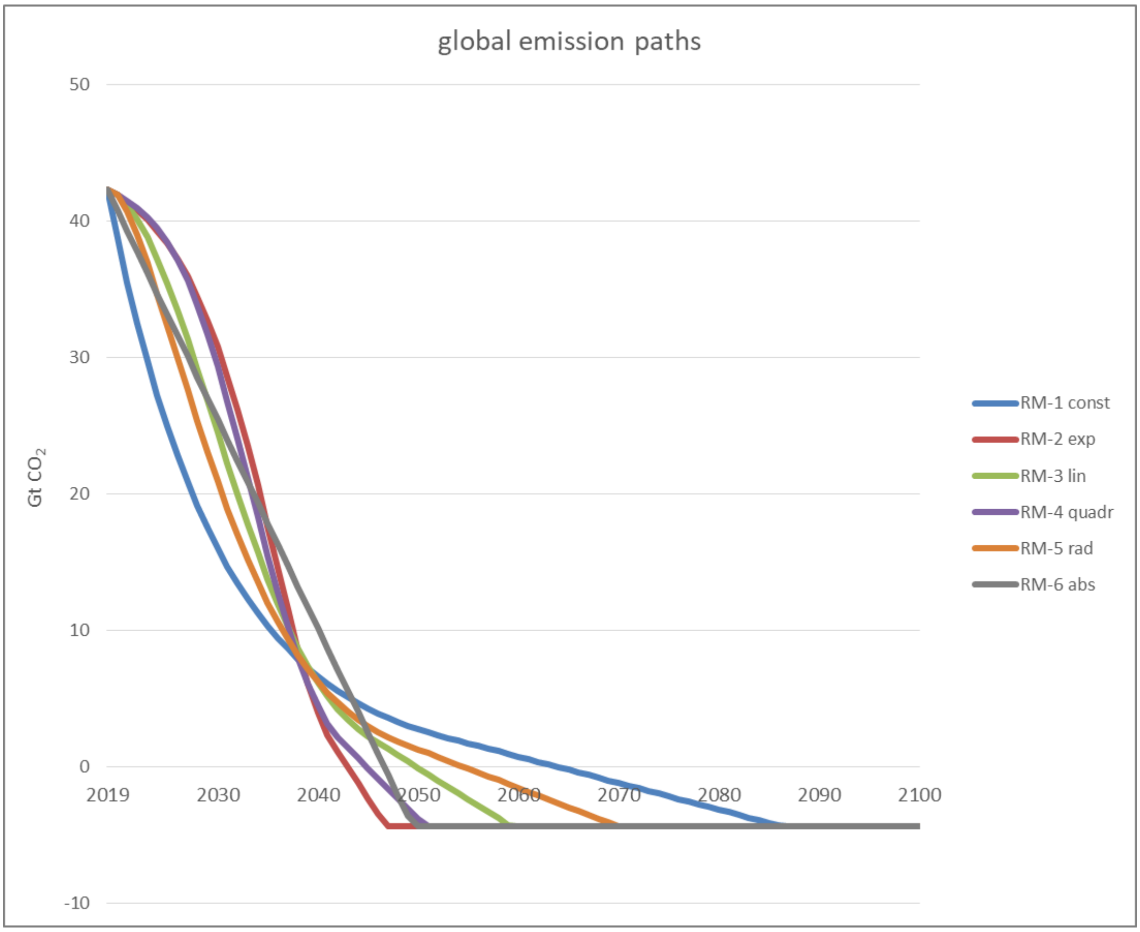

Figure 2: RM Scenario Types emission paths

6 Attachment

6.1 Correction term RM-5

$RR_t = a * \sqrt{t - (BY + 1) - 0.5} + RR_{BY+1}$

As shown in the chart below the interaction of the weighting factor ***a*** and the root without the correction term **0.5** in RM-5-rad would result in a relatively large step in the reduction rates from the first year in the budget period (2020; *BY*+1) to the second year (2021). With the correction term of **0.5**, this curve is "smoothed".

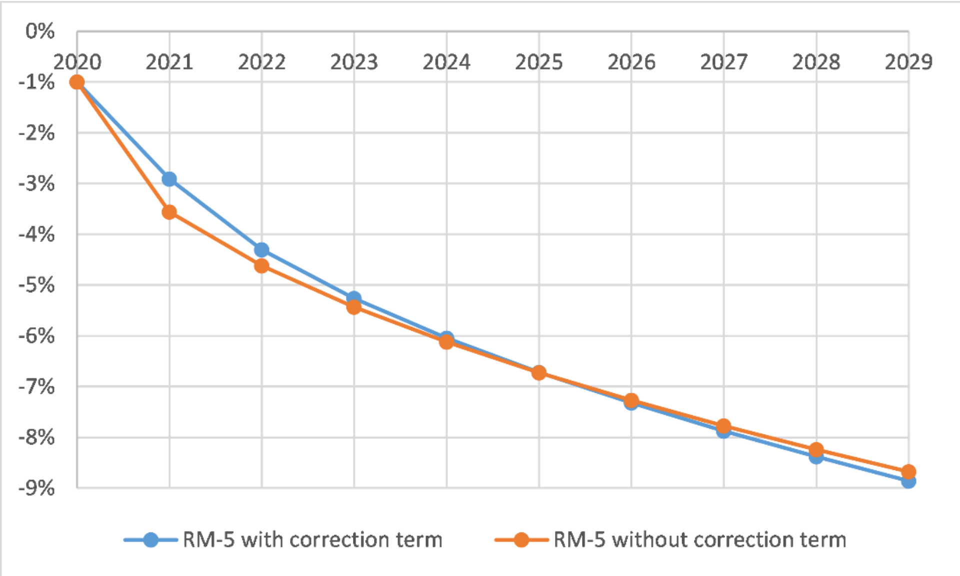

Figure 3: RM-5 correction term

6.2 Further possible scenario types

- Concave: $RR_t = RR_{BY+1} * e^{a*(t-(BY+1))}$
  This variant is almost congruent with RM-2-exp in the area used here.
- Convex: $RR_t = a * \ln(t - BY) + RR_{BY+1}$
  The following graphics show the difference to RM-5-rad:

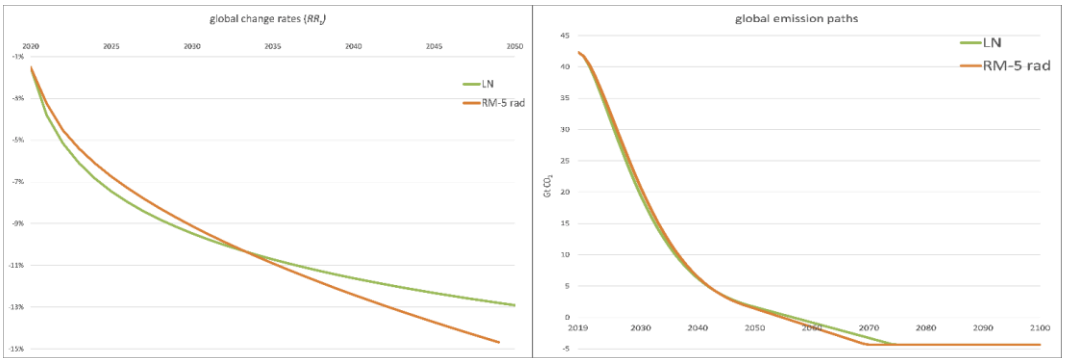

Figure 4: Further scenario type LN

- In addition to monotonous courses, a u-shaped course would also be plausible for the reduction rates. This could be based on the assessment that after “harvesting the low-hanging fruit”, the reduction rates will have to fall again. However, the lead time to the reductions, e.g., in the 2040s, can be seen as an opposing effect. This enables - with a credible climate protection policy - corresponding early long-term investments, which should make a continuous increase in the reduction rates possible. Therefor a credible climate protection policy should encourage early long-term investments, which should enable a continuous increase in the reduction rates.

- A function for the emission path can also be specified directly. See for an example: Wittmann, G.: Resource Sharing Models - A mathematical description, Chapter 3.2, published on Zenodo. In the RM Scenario Types, however, the focus is on the property of the annual changes. The focus on the necessary annual reduction rates makes clearer the challenge and makes it easier to choose a meaningful emission path.

6.3 Extracts from the program code of the macros in the Excel tools

Cell references:

- row 9: change rate 2020 (RR_{BY+1})
- row 12: free parameter ***a***
- row 15: sum of emissions 2020 - 2100 in the scenario using the formulae shown above
- row 16: given budget (*B*)

Extracts from the code (you can request the complete code from us as a bas file):

```
'target value search: determination of the free parameter
Range("E15").GoalSeek goal:=Range("E16"), ChangingCell:=Range("E12")

'try to find a solution with a slightly changed rate of change for 2020 (RR_BY+1; E9) if the free parameter is not ≤ 0
adaption = -0.0055
step = 0.0005
number = 70

initial_value = Range("E9")

Do While Range("E12") > 0
    i = i + 1
    Range("E12") = -0.01
    adaption = adaption + step
    Range("E9") = initial_value + adaption
    Range("E15").GoalSeek goal:=Range("E16"), ChangingCell:=Range("E12")
    If i > number And Range("E12") > 0 Then
        MsgBox "The macro did not find a solution in scenario RM-3. Please change the framework data."
        Range("E9") = initial_value
        i = 0
        GoTo b
    End If
Loop
```