A hierarchical control scheme for optimal secondary frequency regulation with on-off loads in power networks

Andreas Kasis, Stelios Timotheou and Marios Polycarpou

***Abstract*— Load side participation can provide support to the power network by appropriately adapting the demand when required. In addition, it may allow for an economically improved power allocation. In this study, we consider the problem of providing an optimal power allocation among generation and on-off loads within the secondary frequency control timeframe. In particular, we consider a mixed integer optimization problem which ensures that secondary frequency control objectives, i.e. generation-demand balance and frequency attaining its nominal value at steady state, are satisfied. We present analytic conditions on generation and on-off load profiles such that an ϵ-optimality interpretation of the steady state power allocation is obtained, providing a non-conservative bound for ϵ. Moreover, we develop a hierarchical control scheme that provides on-off load values that satisfy the proposed conditions. Furthermore, we study the interaction of the proposed control scheme with the physical dynamics of the power network and provide analytic stability guarantees. Our results are verified with numerical simulations on the Northeast Power Coordinating Council (NPCC) 140-bus system, where it is demonstrated that the proposed algorithm enables an optimality interpretation of the steady state power allocation.**

I. Introduction

Motivation and literature review: The penetration of renewable sources of generation in power networks is expected to grow over the next years, driven by technological advances and environmental concerns [1], [2]. This will make generation more intermittent, resulting in more frequent generation-demand imbalances that may harm power quality and even cause blackouts. Hence, additional challenges are introduced to enable the safe operation of power networks, motivating the analytical study of their stability properties.

Demand side participation is considered by many to be a key way to address the above problem, due to the ability of loads to provide a fast response when required. This motivated many studies to consider controllable loads as a means of supporting existing primary [3], [4], [5], [6], [7], and secondary [8], [9], [10], frequency control mechanisms. Moreover, an issue of fairness in the power allocation, also interpreted as a problem of economic optimality, is raised if loads are used for frequency control. Attempts to address this issue [5], [9], [10], [11], [12], [13] involved constructing appropriate optimization problems that enable an economically optimal power allocation and ensuring that systems equilibria coincide with the solutions to these problems. In addition to reduced costs, such allocations may prevent the power system from reaching its safety limits, thus enhancing its stability properties.

On many occasions, loads are described by on and off states and hence a continuous representation cannot accurately characterize their behavior. The possible switching of loads has been studied in [14], [15], which consider the problem of using on-off loads to provide ancillary services in power networks and demonstrate stability for arbitrary network topologies within the primary and secondary frequency control timeframes respectively. In addition, [14] provides an optimality interpretation of the resulting equilibria by means of a centralized information structure. Moreover, [16] considers loads that switch into different modes of operation, corresponding to nominal and urgent circumstances respectively. Furthermore, several studies [17], [18], [19], consider the temperature-dependent, on-off behavior of loads, proposing various control schemes for improved performance. Hence, considering on-off load behavior is of high importance in the study of power networks.

Contribution: This study considers the stability and optimality properties of the power network when controllable on-off loads are incorporated within the secondary frequency control timeframe. The discontinuous nature of on-off loads introduces several challenges, requiring tools from switching system analysis, and makes the problem of obtaining an optimal power allocation combinatorial.

In particular, we consider a mixed integer optimization problem that associates the cost of generation and on-off controllable loads such that the secondary frequency control objectives are attained, i.e. generation-demand balance is achieved and frequency takes its nominal value at steady state. We propose equilibrium conditions that, when satisfied, ensure that the steady state power allocation cost is no greater than ϵ to the global minimum and provide a non-conservative value for ϵ. Moreover, we propose a hierarchical control policy that enables an equilibrium allocation that satisfies these conditions, by determining the on-off load values. The interaction of the proposed hierarchical scheme with the power network renders their combination a switching system. For the combined system, we explain that no chattering behavior should be expected and provide asymptotic stability guarantees. In particular, our results ensure the convergence of solutions to an equilibrium that satisfies the secondary frequency control objectives and is ϵ-optimal to the considered optimization problem.

Our analytic results are verified with numerical simulations on the NPCC 140-bus system, that demonstrate the stability and optimality properties of the proposed algorithm in a realistic setting. In particular, the simulation results demonstrate that the proposed scheme enables an ϵ-optimality interpretation of the steady state power allocation.

Paper structure: In Section II we present the considered

This work was funded by the European Union's Horizon 2020 research and innovation program under grant agreement 739551 (KIOS CoE) and from the Republic of Cyprus through the Directorate General for European Programs, Coordination, and Development.

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model for the power network and the problem formulation. In Section III we present our optimality analysis and the proposed hierarchical control scheme to solve the considered optimization problem. In Section IV we study the interaction of the proposed scheme with the physical network and present our main stability and optimality results. Our analytical results are demonstrated with numerical simulations in Section V. Finally, conclusions are drawn in Section VI. The proofs of the results have been omitted due to page constraints and will be included in an extended version of this work.

Notation: Real and integer numbers are denoted by $\mathbb{R}$ and $\mathbb{Z}$ respectively. The set of n-dimensional vectors with real and integer entries are denoted by $\mathbb{R}^n$ and $\mathbb{Z}^n$ respectively. The sets of positive and non-negative real numbers are denoted by $\mathbb{R}_{>0}$ and $\mathbb{R}_{\geq 0}$ respectively. We use $\mathbf{0}_n$ and $\mathbf{1}_n$ to denote n-dimensional vectors with all elements equal to 0 and 1 respectively. The image of a vector x is denoted by $\mathrm{Im}(x)$. The cardinality of a discrete set Σ is denoted by $|\Sigma|$. The convex closure of a set A is denoted by $\bar{A}$. Finally, a function $f : \mathbb{R} \to \mathbb{R}$ is said to be monotonically increasing (respectively decreasing) if for all x and y such that $x \leq y$ it holds that $f(x) \leq f(y)$ (respectively $f(x) \geq f(y)$).

II. Problem Formulation

A. Network model

We describe the power network by a connected graph $(\mathcal{N}, \mathcal{E})$ where $\mathcal{N} = \{1, 2, \dots, |\mathcal{N}|\}$ is the set of buses and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ the set of transmission lines connecting the buses. Furthermore, we use (i, j) to denote the link connecting buses i and j and assume that the graph $(\mathcal{N}, \mathcal{E})$ is directed with an arbitrary orientation, so that if $(i, j) \in \mathcal{E}$ then $(j, i) \notin \mathcal{E}$. For each $j \in \mathcal{N}$, we use $i : i \to j$ and $k : j \to k$ to denote the sets of buses that are predecessors and successors of bus j respectively. It is important to note that the form of the dynamics in (1)–(2) below is unaltered by any change in the graph ordering, and all of our results are independent of the choice of direction. The following assumptions are made for the network:

1) Bus voltage magnitudes are $|V_j| = 1$ p.u. for all $j \in \mathcal{N}$.
2) Lines $(i, j) \in \mathcal{E}$ are lossless and characterized by the magnitudes of their susceptances $B_{ij} = B_{ji} > 0$.
3) Reactive power flows do not affect bus voltage phase angles and frequencies.
4) Relative phase angles are sufficiently small such that the approximation $\sin \eta_{ij} = \eta_{ij}$ is valid.

The first three conditions have been widely used in the literature [5], [10], [12], in studies associated with frequency regulation. The fourth condition is justified from the fact that the relative phase angles among buses are small in nominal operating conditions.

We use swing equations to describe the rate of change of frequency at each bus. This motivates the following system dynamics (e.g. [20]),

$$\dot{\eta}_{ij} = \omega_i - \omega_j,\ (i, j) \in \mathcal{E}, \tag{1a}$$

$$M_j \dot{\omega}_j = -p^L_j + p^M_j - d^u_j - \sum_{i \in \mathcal{N}_j} d^c_{i,j} - \sum_{k: j \to k} p_{jk} + \sum_{i: i \to j} p_{ij}, j \in \mathcal{N}, \tag{1b}$$

$$p_{ij} = B_{ij} \eta_{ij},\ (i, j) \in \mathcal{E}. \tag{1c}$$

In system (1) the time-dependent variables p^M_j and ω_j represent, respectively, the mechanical power injection and the deviation from the nominal value[1] of the frequency at bus j. The time-dependent variable $d^c_{i,j}$ represents the demand of controllable load i at bus j. The set of controllable loads at bus j is denoted by $\mathcal{N}_j$. Furthermore, we define the set $\tilde{\mathcal{N}} := \{(i, j) : i \in \mathcal{N}_j, j \in \mathcal{N}\}$, such that all pairs $i \in \mathcal{N}_j, j \in \mathcal{N}$ satisfy $(i, j) \in \tilde{\mathcal{N}}$. The quantity d^u_j is also a time-dependent variable that represents the uncontrollable frequency-dependent load and generation damping present at bus j. Furthermore, the quantities η_{ij} and p_{ij} are time-dependent variables that represent, respectively, the power angle difference, and the power transmitted from bus i to bus j. The constant $M_j > 0$ denotes the generator inertia. Moreover, the constant p^L_j denotes the frequency-independent load at bus j, and $\ell = \mathbf{1}^T_{|\mathcal{N}|} p^L$ its aggregate value throughout the network. Within the rest of the manuscript, we let x^* denote the equilibrium value of state x.

We consider generation and frequency dependent-uncontrollable demand and frequency damping dynamics described by

$$\tau_j \dot{p}^M_j = -(p^M_j + \kappa_j \omega_j - \kappa_j p^c_j),\ j \in \mathcal{N}, \tag{2a}$$

$$d^u_j = A_j \omega_j,\ j \in \mathcal{N}, \tag{2b}$$

where $\tau_j > 0, j \in \mathcal{N}$, are time constants, p^c_j is a local power command variable available for design (see Section III-A), and $A_j > 0$ and $\kappa_j > 0$, $j \in \mathcal{N}$, are damping and droop coefficients respectively. Note that the analysis carried in this paper is valid for more general generation and demand dynamics, including cases of nonlinear and higher order dynamics, provided certain input-output conditions hold, as shown in [4], [5], [10], [15]. In this paper, we consider the simple first order generation and static uncontrollable demand dynamics for simplicity and to avoid a shift in the focus of the paper from on-off loads.

B. On-off controllable loads

Controllable on-off loads may enable an improved power allocation in power networks. Their behavior is described by

$$d^c_{i,j} = \overline{d}_{i,j} \sigma_{i,j}, (i, j) \in \tilde{\mathcal{N}}, \tag{3}$$

where $\overline{d}_{i,j} \in \mathbb{R}_+$ denotes the magnitude of load $(i, j) \in \tilde{\mathcal{N}}$. The time dependent variable $\sigma_{i,j} \in P = \{0, 1\}$ denotes the switching state of the ith load at bus j. The dynamics of σ are discussed in Sections III and IV. Furthermore, we define the constants $\rho_{i,j} \in P$, which denote the desired switching state set for each load $(i, j) \in \tilde{\mathcal{N}}$, selected by its user.

C. Optimal generation and on-off load control

In this section we consider how generation and on-off controllable loads should be adjusted such that their joint cost is minimized while at the same time the generation and demand are balanced. In particular, we let $\frac{1}{2} q_j (p^M_j)^2$ be the cost incurred when the generation is p^M_j. Furthermore, we let a cost $c_{i,j}$ be incurred when the switching state $\sigma_{i,j}$ is different than the desired state $\rho_{i,j}$ at some on-off load $(i, j) \in \tilde{\mathcal{N}}$. The cost function for on-off loads is given by

$$C^d_{i,j}(\sigma_{i,j}, \rho_{i,j}) = \begin{cases} c_{i,j}, & \text{when } \sigma_{i,j} \neq \rho_{i,j}, \\ 0, & \text{when } \sigma_{i,j} = \rho_{i,j}. \end{cases} \quad (i, j) \in \tilde{\mathcal{N}}.$$

[1] We define the nominal value as an equilibrium of (1) with frequency equal to 50Hz (or 60Hz).

We then consider the following optimization problem, called the hybrid optimal supply control problem (H-OSC),

$$\begin{aligned} &\underline{\text{H-OSC:}} \min_{p^M,\sigma} \sum_{j\in\mathcal{N}} [\frac{1}{2}q_j(p_j^M)^2 + \sum_{i\in\mathcal{N}_j} C_{i,j}^d(\sigma_{i,j},\rho_{i,j})], \\ &\text{subject to } \sum_{j\in\mathcal{N}} p_j^M = \sum_{j\in\mathcal{N}} (p_j^L + \sum_{i\in\mathcal{N}_j} \overline{d}_{i,j}\sigma_{i,j}), \qquad (4) \\ &\sigma_{i,j} \in \{0,1\}, (i,j) \in \tilde{\mathcal{N}}. \end{aligned}$$

The equality constraint in (4) requires all the frequency-independent loads to be matched by the total generation and on-off controllable demand. This ensures that when system (1) is at equilibrium, the frequency will be at its nominal value. The latter follows by summing (1b) at steady state over all $j \in \mathcal{N}$ and noting the equality constraint in (4), which using (2b) results to $\sum_{j\in\mathcal{N}} d_j^{u,*} = \sum_{j\in\mathcal{N}} A_j\omega_j^* = 0$ and hence to $\omega^* = \mathbf{0}_{|N|}$ from (1a) at steady state. The second constraint reflects that controllable loads take discrete values. The latter makes (4) a mixed integer optimization problem.

D. Problem statement

Below we state the main problem we aim to solve.

Problem 1: Design a control scheme for generation and on-off loads, described by (2a) and (3), that:

(i) Requires no recalibration when on-off loads are added or removed from the network.
(ii) Enables decentralized stability guarantees.
(iii) Is independent of (connected) network topology.
(iv) Ensures that the frequency attains its nominal value at steady state.
(v) Provides an optimality interpretation of the steady state power allocation.

The first condition requires a control scheme that does not need to be modified when on-off loads are added or removed from the network, something that is expected to frequently occur due to their large numbers. The second and third conditions require that the control scheme enables locally verifiable and network independent stability guarantees. Objective (iv) is the main objective of secondary frequency control, i.e. to ensure that the frequency takes its nominal value at equilibrium. The last condition requires an optimality interpretation of the steady state power allocation, ensuring that the incurred cost is close to the global minimum of the H-OSC problem (4).

III. Optimal allocation among on-off loads

A. Power Command Dynamics

In this section we consider the design of the dynamics for power command variables, used as inputs to (2a). We adopt a suitably adapted version of a scheme that has been widely used in the literature for distributed optimal secondary frequency regulation [8], [10], [13], known as the Primal-Dual scheme. In particular, we consider a communication network described by a connected graph $(\mathcal{N}, \tilde{\mathcal{E}})$, where $\tilde{\mathcal{E}}$ represents the set of communication lines among the buses. We consider the following dynamics for the power command signal p_j^c,

$$\gamma_{ij}\dot{\psi}_{ij} = p_i^c - p_j^c,\ (i,j) \in \tilde{\mathcal{E}}, \tag{5a}$$

$$\gamma_j\dot{p}_j^c = -p_j^M + p_j^L + \sum_{i\in\mathcal{N}_j} d_{i,j}^c - \sum_{k:j\to k}\psi_{jk} + \sum_{i:i\to j}\psi_{ij}, j \in \mathcal{N}, \tag{5b}$$

where γ_j and γ_{ij} are positive constants, p_i^c and p_j^c are variables shared between communicating buses i and j, and the variable ψ_{ij} is a state of the controller that integrates the power command difference of communicating buses i and j. We note that the set of communication lines $\tilde{\mathcal{E}}$ and power lines $\mathcal{E}$ can be the same or different.

The dynamics in (5) are frequently used in the literature as they achieve both the synchronization of the communicated variable p^c, something that can be exploited to guarantee optimality of the equilibrium point reached, and also that frequency attains its nominal value at steady state. These are analytically shown in Lemma 1 below.

Lemma 1: All equilibria of (1), (2), (3), (5) satisfy $\omega^* = \mathbf{0}_{|\mathcal{N}|}$ and $p^{c,*} \in \mathrm{Im}(\mathbf{1}_{|\mathcal{N}|})$.

B. Optimality analysis

In this section we consider the H-OSC problem (4) which aims to optimize the power allocation between generation and on-off loads. The considered problem is combinatorial and hence difficult to analytically solve when a large number of on-off loads is considered. Hence, an aim of this paper is to enable a steady state power allocation cost that is close to the global minimum of the H-OSC problem (4). Below, we define the notion of an ϵ-optimal point that is used throughout the rest of this manuscript.

Definition 1: Given a cost function $C_f : \mathbb{R}^n \times \mathbb{Z}^m \to \mathbb{R}$ where $n, m > 0$, a vector $\bar{x} \in \mathbb{R}^n \times \mathbb{Z}^m$ is called ϵ-optimal for C_f, for some $\epsilon \in \mathbb{R}_{>0}$, if it holds that

$$C_f(\bar{x}) \leq \min_{x\in\mathbb{R}^n\times\mathbb{Z}^m} C_f(x) + \epsilon.$$

Below, we provide conditions that, when satisfied, enable an optimality interpretation of the steady state power allocation among generation and on-off loads. Within the following proposition we let $\beta = \max_{(i,j)\in\tilde{\mathcal{N}}} \overline{d}_{i,j}$ be the largest on-off load and $K = \sum_{j\in\mathcal{N}} k_j$ the aggregate power command droop respectively within the power network.

Proposition 1: Consider an equilibrium of (1), (2), (3), (5). If $k_j = q_j^{-1}, j \in \mathcal{N}$ holds and there exists $\zeta \in \mathbb{R}$ such that

$$p^{c,*} - \frac{\beta}{K} \leq \zeta \leq p^{c,*} \tag{6a}$$

$$\sigma_{i,j}^* \in \begin{cases} \{0\}, \text{ if } \zeta > \frac{c_{i,j}}{\overline{d}_{i,j}}, \\ \{0,\rho_{i,j}\}, \text{ if } \zeta = \frac{c_{i,j}}{\overline{d}_{i,j}}, \\ \{\rho_{i,j}\}, \text{ if } |\zeta| < \frac{c_{i,j}}{\overline{d}_{i,j}}, \\ \{\rho_{i,j},1\}, \text{ if } \zeta = -\frac{c_{i,j}}{\overline{d}_{i,j}}, \\ \{1\}, \text{ if } \zeta < -\frac{c_{i,j}}{\overline{d}_{i,j}}, \end{cases} \quad (i,j) \in \tilde{\mathcal{N}}, \tag{6b}$$

then the considered equilibrium is ϵ-optimal to (4), with $\epsilon = 3\beta^2/2K$.

Proposition 1 provides a condition on the droop gains and equilibrium values which guarantees that the cost at steady state is ϵ-close to the global minimum of (4), providing a non-conservative value for ϵ. The parameter ζ associates the equilibrium values of power command variables, $p^{c,*}$, and the load costs per unit demand $c_{i,j}/\overline{d}_{i,j}$. The structure of (6a) ensures that an equilibrium that satisfies (6) is always feasible, as shown in the following lemma.

Lemma 2: There exists an equilibrium to (1), (2), (3), (5) such that (6) is satisfied.

C. Hierarchical control scheme for on-off loads

In this section we present a hierarchical control scheme which aims to obtain a vector σ^* such that (6) is satisfied and hence an ϵ-optimality interpretation of the resulting equilibria is allowed, in accordance to Proposition 1. The scheme is described by Algorithm 1 and a schematic representation of its information flow is depicted in Figure 1. Below, we provide additional explanations and intuition on its implementation.

Algorithm 1: Hierarchical optimality scheme.

Inputs: $p^L, K, \overline{d}, c.$
Output: $\overline{\sigma}(k), k \geq 1.$
Initialization: $p^c_{min}(0) = \frac{\ell}{K}$,
$p^c_{max}(0) = \frac{1}{K}(\ell + \mathbf{1}^T_{|\tilde{\mathcal{N}}|}\overline{d})$, $\mu \in (0,1), k = 0$,
$\hat{\sigma}(0) = \rho, \hat{p}^c(0) = 0, p^c_{set}(0) = 0, \phi(0) = 0.$
while $\phi(k) = 0$, **do**

$k = k + 1,$

$$p^c_{set}(k) = \mu p^c_{max}(k-1) + (1-\mu) p^c_{min}(k-1), \tag{A.1}$$

$$\hat{\sigma}_{i,j}(k) = \begin{cases} 0, \text{ if } p^c_{set}(k) > \frac{c_{i,j}}{\overline{d}_{i,j}}, \\ 1, \text{ if } p^c_{set}(k) < -\frac{c_{i,j}}{\overline{d}_{i,j}}, \ (i,j) \in \tilde{\mathcal{N}}, \\ \rho_{i,j}, \text{ otherwise}, \end{cases} \tag{A.2}$$

$$\hat{p}^c(k) = \frac{1}{K}(\ell + \overline{d}^T \hat{\sigma}(k)), \tag{A.3}$$

$$p^c_{min}(k) = \begin{cases} p^c_{set}(k), \text{ if } p^c_{set}(k) < \hat{p}^c(k) - \frac{\beta}{K}, \\ p^c_{min}(k-1), \text{ otherwise}, \end{cases} \tag{A.4}$$

$$p^c_{max}(k) = \begin{cases} p^c_{set}(k), \text{ if } p^c_{set}(k) > \hat{p}^c(k), \\ p^c_{max}(k-1), \text{ otherwise}, \end{cases} \tag{A.5}$$

$$\phi(k) = \begin{cases} 1, \text{ if } p^c_{max}(k) - p^c_{min}(k) = \\ \qquad p^c_{max}(k-1) - p^c_{min}(k-1), \\ 0, \text{ otherwise}, \end{cases} \tag{A.6}$$

$$\overline{\sigma}_{i,j}(k) = \begin{cases} \hat{\sigma}_{i,k}(k), \text{ if } \phi(k) = 1, \\ 1, \text{ if } p^c_{max}(k) < -\frac{c_{i,j}}{\overline{d}_{i,j}} - \frac{\beta}{K}, \phi(k) = 0, \\ 0, \text{ if } p^c_{min}(k) > \frac{c_{i,j}}{\overline{d}_{i,j}} + \frac{\beta}{K}, \phi(k) = 0, \\ \rho_{i,j}, \text{ otherwise}, \end{cases} \tag{A.7}$$

$(i,j) \in \tilde{\mathcal{N}}.$

Inputs, states, output and information flow: The proposed scheme uses knowledge of parameters K at the central controller, p^L at each bus and $\overline{d}_{i,j}, c_{i,j}$ at each load $(i,j) \in \tilde{\mathcal{N}}$, which can be regarded as its inputs. Furthermore, it is assumed that the value β/K is globally known[2]. It produces local values of $\overline{\sigma}_{i,j}, (i,j) \in \tilde{\mathcal{N}}$ at each iteration, used to locally update σ as explained in Section IV. The aim of Algorithm 1 is to obtain a vector $\overline{\sigma} \in P^{|\tilde{\mathcal{N}}|}$, such that (6) is satisfied when $\sigma^* = \overline{\sigma}$. To achieve this, it introduces global auxiliary variables $p^c_{min}, p^c_{max}, \hat{p}^c, p^c_{set} \in \mathbb{R}$ and $\phi \in P$ and local auxiliary variables $\hat{\sigma}_{i,j} \in P, (i,j) \in \tilde{\mathcal{N}}$. Note that, as also follows from Fig. 1, (A.2), (A.7) are implemented at each load while the rest at the central controller.

[2]The results in this paper are extendable to the case where β/K is replaced by a known upper bound, i.e. knowledge of its exact value is not necessary. This extension is omitted for brevity in presentation.

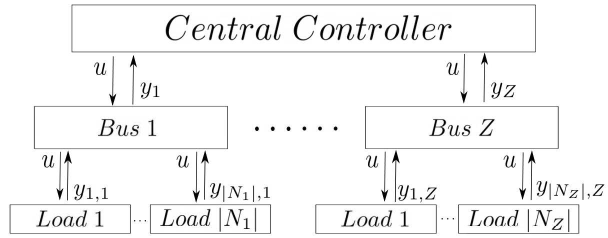

Fig. 1. Schematic representation of the information flows in Algorithm 1, where $Z = |N|$, $u(k) = [p^c_{min}(k), p^c_{max}(k), p^c_{set}(k), \phi(k)]$, $y_i(k) = p^L_i + \sum_{j \in N_i} \overline{d}_{j,i}\hat{\sigma}_{j,i}(k)$ and $y_{i,j}(k) = \overline{d}_{i,j}\hat{\sigma}_{i,j}(k)$.

Intuition: Algorithm 1 aims to obtain values of p^c_{set} and $\hat{p}^c$ such that the termination condition $\hat{p}^c(k) - \beta/K \leq p^c_{set}(k) \leq \hat{p}^c(k)$, as follows by (A.4)–(A.6), is satisfied at some finite iteration k. We show in Theorem 1 below that the termination condition suffices for (6) to hold at steady state when $\sigma^* = \overline{\sigma}$. Variable $p^c_{set}(k)$ corresponds to some price used to update $\hat{\sigma}_{i,j}(k)$. Its terminal value corresponds to ζ in (6). The update rule for $\hat{\sigma}$ is intuitive, allowing $\hat{\sigma}$ to be different than ρ only if the magnitude of its local cost per unit demand, given by $c_{i,j}/\overline{d}_{i,j}$, is less than p^c_{set}. Moreover, $\hat{p}^c(k)$ provides an estimate of the equilibrium value for $p^{c,*}$ when $\sigma^* = \hat{\sigma}(k)$. The variables $p^c_{min}(k)$ and $p^c_{max}(k)$ respectively provide a lower and an upper bound to the value of $p^c_{set}(l), \forall l \geq k$. Their values update based on the values of p^c_{set} and $\hat{p}^c$ according to (A.4)–(A.5). Their update rules are intuitive, since when p^c_{set} is increasing then $\hat{p}^c$ is non-increasing and vice-verca. Finally, the value of $\overline{\sigma}$ when $\phi = 1$ from (A.7), is such that when $\sigma^* = \overline{\sigma} = \hat{\sigma}$ then (6) holds with $\zeta = p^c_{set}$, as demonstrated in Theorem 1 below. The intermediate values of $\overline{\sigma}$ are also important, since they provide an estimate of its terminal value, e.g. since $p^c_{min}(k)$ is non-decreasing then if $\hat{\sigma}_{i,j}(k) = 0$ due to the third case in (A.7), then $\hat{\sigma}_{i,j}(l) = 0, \forall l \geq k$. The latter enables a smoother response compared to the case where all on-off loads simultaneously switch when $\phi = 1$.

Initialization: The initialization of p^c_{min} and p^c_{max} is important since their initial range needs to be sufficiently broad to include any possible terminal value of p^c_{set}. Note that, although their initialization requires knowledge of $\ell, \overline{d}$ and K, any lower and upper bounds to $p^c_{min}(0)$ and $p^c_{max}(0)$ are sufficient for all properties of Algorithm 1 to hold. Hence, the initialization of Algorithm 1 is robust to parametric uncertainty. The initialization of variables $\hat{\sigma}, \hat{p}^c$ and p^c_{set} is only made for completeness in presentation. Furthermore, $\phi(0) = 0$ is required to initiate the algorithm. Finally, $\mu \in (0,1)$ is a parameter of Algorithm 1, used in (A.1).

Implementation: In Algorithm 1, the intermediate variable p^c_{set}, updated according to (A.1), takes a value that lies strictly within $[p^c_{min}, p^c_{max}]$. The local updates of the variables $\hat{\sigma}$ make use of p^c_{set} as well as the local costs and magnitudes of on-off loads, following (A.2). Furthermore, $\hat{p}^c$ updates based on the transmitted values of $p^L_j + \sum_{i \in N_j} \overline{d}_{i,j}\hat{\sigma}_{i,j}$

from each bus j, as well as knowledge of parameter K, as demonstrated in (A.3). As already mentioned, $\hat{p}^c$ provides an estimate for $p^{c,*}$ when $\sigma^* = \hat{\sigma}$. The values of $p^c_{min}(k)$ and $p^c_{max}(k)$, which provide lower and upper bounds for $p^c_{set}(l), l \geq k$, are respectively updated according to (A.4)–(A.5), i.e. when p^c_{set} is below the considered threshold then p^c_{min} is increased and vice verca. Furthermore, the stopping condition in (A.6) ensures that $\hat{p}^c$ and p^c_{set} satisfy $\hat{p}^c(k) - \beta/K \leq p^c_{set}(k) \leq \hat{p}^c(k)$, as follows from (A.4)–(A.5). The latter is important for (6) to be satisfied, as explained above and shown in Theorem 1 below.

Remark 1: As shown in Theorem 1 below, Algorithm 1 ensures that (6) is satisfied at steady state, which enables an optimality interpretation to be obtained. Note that (6) is based on a continuous relaxation of (4) and the corresponding KKT conditions. This approach is inspired from [14], which is a study on primary frequency regulation. Compared to the scheme in [14], Algorithm 1 adopts a different information structure that is not transiently coupled with the frequency control dynamics and hence provides support at slower timescales associated with secondary frequency control. Its control design enables to overcome the challenges associated with Problem 1 (see Section II-D).

Below, we demonstrate that Algorithm 1 terminates (i.e. $\phi = 1$) after a finite number of iterations. Furthermore, we show that if $\sigma^* = \overline{\sigma}^*$, where $\overline{\sigma}^*$ is the value obtained for $\overline{\sigma}$ when Algorithm 1 terminates, then (6) is satisfied at steady state. The latter allows an optimality interpretation of the steady state power allocation, in accordance to Proposition 1. The above are demonstrated in the following theorem.

Theorem 1: Algorithm 1 terminates after a finite number of iterations. Furthermore, if $\sigma^* = \overline{\sigma}^*$, then (6) is satisfied at steady state.

IV. Convergence analysis

In this section we provide a detailed description of the overall dynamical system as a switching system (see e.g. [21]) and use corresponding tools for its analysis.

In particular, system (1), (2), (3), (5), with on-off controllable loads that switch according to Algorithm 1, can be described by the states $z = (x, \sigma)$, where $x = (\eta, \omega, p^M, p^c, \psi) \in \mathbb{R}^n, n = 3|\mathcal{N}| + |\mathcal{E}| + |\tilde{\mathcal{E}}|$ is the continuous state, and $\sigma \in P^{|\tilde{\mathcal{N}}|}$ the discrete state. We also denote by $\Lambda = \mathbb{R}^n \times P^{|\tilde{\mathcal{N}}|}$ the domain where the state z takes values. For convenience, we use the following compact representation to describe (1), (2), (3), (5),

$$\dot{x} = f_\sigma(x), \sigma \in P^{|\tilde{\mathcal{N}}|}, \tag{7}$$

where $f_\sigma : \mathbb{R}^n \to \mathbb{R}^n$ is given by (1), (2), (3), (5).

Furthermore, we let t_k be the time instant where the kth iteration of Algorithm 1 terminates, satisfying $t_{k+1} \geq t_k, k \geq 1$, assuming that the time for each iteration of Algorithm 1 is finite. The switching state σ is given by

$$\sigma(t) = \sigma(0), t \in [0, t_1), \tag{8a}$$

$$\sigma(t) = \overline{\sigma}(k), t \in [t_k, t_{k+1}), \tag{8b}$$

where $\overline{\sigma}(k)$ is the output of the kth iteration of Algorithm 1 and $\sigma(0)$ the initial value of σ. Note that, as shown in Theorem 1, Algorithm 1 terminates after a finite number of iterations, which we denote by $\hat{k}$, and hence $t_{\hat{k}+1}$ is not well-defined. To resolve this, we let $t_{\hat{k}+1} = \infty$.

Remark 2: Note that although we have not explicitly set a lower bound in the time between switches, chattering behavior can be excluded for (7)–(8). The latter follows from Theorem 1 which guarantees a finite number of iterations of Algorithm 1 and hence a finite number of switches of variable σ.

A. Stability analysis

In this section we provide our main convergence result for system (7)-(8). In particular, we demonstrate that solutions to (7)-(8) globally converge to the set of its equilibria. Note that we call a point $z^* = (x^*, \sigma)$ an equilibrium of (7) if $f_\sigma(x^*) = 0$. Furthermore, we consider the Caratheodory solutions to (7)-(8) that are frequently used for the analysis of switching systems [21]. The existence of equilibria and solutions will be analytically demonstrated in an extended version of this work.

The following theorem demonstrates the stability of (7)-(8). In addition, it shows that when the condition on the local droop gains from Proposition 1 holds, then solutions to (7)-(8) globally converge to an ϵ-optimal point of the H-OSC problem (4).

Theorem 2: Solutions to (7)-(8) globally converge to the set of its equilibria. In addition, if $k_j = q_j^{-1}, j \in \mathcal{N}$, then solutions to (7)-(8) globally converge to a subset of its equilibria, satisfying $\omega^* = \mathbf{0}_{|\mathcal{N}|}$ and $p^{c,*} \in \mathrm{Im}(\mathbf{1}_{|\mathcal{N}|})$, that are ϵ-optimal to the H-OSC problem (4) with $\epsilon = 3\beta^2/2K$.

Theorem 2 demonstrates that the implementation of Algorithm 1 does not compromise the stability of the power network and allows an optimality interpretation of the steady state power allocation when on-off loads are present.

V. Simulation on the NPCC 140-bus system

In this section we use the Power System Toolbox [22] to perform numerical simulations on the Northeast Power Coordinating Council (NPCC) 140-bus system, to numerically validate our analytic results. The model used by the toolbox is more detailed than our analytic one, including voltage dynamics, line resistances, a transient reactance generator model, and high order turbine governor models[3].

The NPCC network consists of 47 generation and 93 load buses and has a total real power of 28.55GW. For our simulation, we considered a step increase in demand of magnitude 2 p.u. (base 100MVA) at load buses 2 and 3 at $t = 1$ second. Furthermore, we considered 500 on-off loads at each of buses $1 - 20$ with magnitudes $\overline{d}$ and local costs c selected from uniform distributions with ranges $[0, 0.008]$p.u. and $[0, 0.1]$p.u. respectively. The value of μ, used in the implementation of Algorithm 1 was also selected from a uniform distribution with range $[0.005, 0.995]$. The initial value of σ was selected such that (6) was satisfied for some randomly selected value for $\rho \in P^{|\tilde{\mathcal{N}}|}$. The system contains 24 buses with turbine governor generation units of which 22 were assumed to contribute in secondary frequency regulation. Their power command droop gains were selected such that the power allocation among generators and on-off controllable loads was comparable. In addition, the generator cost coefficients were selected such that $q_j = k_j^{-1}$ at all units.

[3]The simulation details can be found in the data file datanp48 and the Power System Toolbox manual [22].

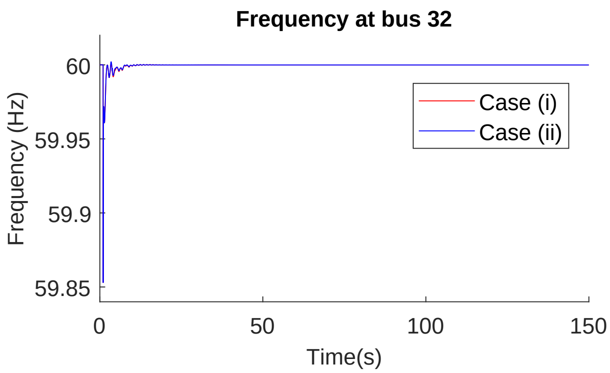

Fig. 2. Frequency response at bus 32 for the following cases: (i) No controllable on-off loads, (ii) On-off loads implementing Algorithm 1.

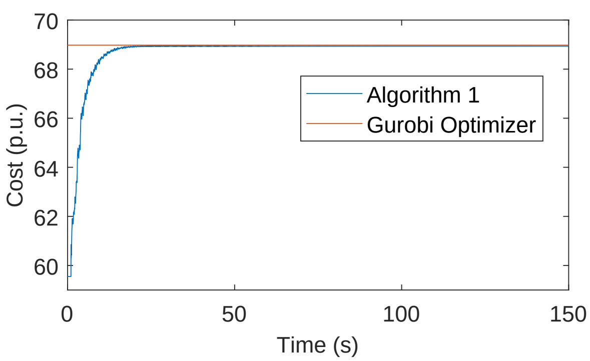

Fig. 3. Cost associated with (4) when Algorithm 1 is implemented compared to the global minimum cost obtained using the Gurobi optimizer.

The system was tested under two different cases. In case (i) the system did not include any controllable loads and in case (ii) on-off loads implementing Algorithm 1 were included. Each iteration of Algorithm 1 was assumed to require 0.3 seconds. The frequency response for these two cases at a randomly selected bus is shown on Figure 2. Figure 2 depicts a smooth frequency response for case (ii), similar to case (i), despite the additional control layer associated with Algorithm 1. Moreover, it demonstrates that the frequency converges to its nominal value, in agreement with the presented analysis.

To demonstrate the validity of the optimality analysis, the obtained results where compared with the optimal solution to (4) obtained using the Gurobi optimizer [23]. As shown in Figure 3, the proposed algorithm provides a solution that converges to the value obtained from the Gurobi optimizer, which corresponds to the global minimum of (4). In particular, it was seen that the vector $\bar{\sigma}$ provided from Algorithm 1 and the corresponding provided by the Gurobi optimizer were identical. The latter are in agreement with Theorem 2.

VI. Conclusion

We have considered the problem of controlling generation and on-off loads in power networks such that stability is guaranteed and a close to optimal power allocation is attained within the secondary frequency control timeframe. A mixed integer optimization problem has been considered which ensures that the secondary frequency control objectives are satisfied at steady state. For the considered problem, analytic conditions are derived for generation and on-off loads such that an ϵ-optimal allocation is enabled at equilibrium, providing a non-conservative bound for ϵ. Furthermore, a hierarchical control scheme has been proposed such that the aforementioned conditions are satisfied. The combined dynamics of the physical system and the controller are jointly studied as a switching system and analytic convergence and ϵ-optimality guarantees are provided. Our results are verified with numerical simulations on the NPCC 140-bus network, where a large number of on-off loads has been considered and an optimality interpretation of the steady state power allocation was obtained.

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