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> # Biot_Dispersion_relations: this script calculates the
  dispersion relations of Biot's equations of poroelasticity
> # Input: non-dimensional parameters
> # Output: Matlab generated coefficients of the polynomial and
  latex generated matrix
> # Copyright (C) 2021 Yury Alkhimenkov, Ludovic Raess, Lyudmila
  Khakimova, Beatriz Quintal, Yury Podladchikov.

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  # (at your option) any later version.

> # Please cite us if you use our routine:
  # Alkhimenkov Y., Raess L., Khakimova L., Quintal B.,
  Podladchikov Y.Y., 2021.
  # Resolving wave propagation in anisotropic poroelastic media
  using graphical processing units (GPUs)

> restart;
> assume(omega>0,k>0):
> assume(del>0,rho_f>0,rho_s>0,K_dry>0,por>0,por<1,alpha>0,alpha<1,
  MM>0,Tor_fi>0,kd>0,B>0):
> fun   := exp(I*k*x+Aalpha*t):
> Aalpha:= -I*omega:
> Stress:= Q[1]*fun:
> Prf   := Q[2]*fun:
> Vx    := Q[3]*fun:
> Qxf   := Q[4]*fun:
> eq[1] := -S*diff(Stress,t) - S*a*diff(Prf,t)+diff(Vx,x);
```

$$eq_1 := \mathrm{I}\,S\,Q_1\,\omega\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}+\mathrm{I}\,S\,a\,Q_2\,\omega\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}+\mathrm{I}\,Q_3\,k\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t} \quad (1)$$

```
> eq[2] := S*a*diff(Stress,t) + S*aA*diff(Prf,t)+ diff(Qxf,x);
```

$$eq_2 := -\mathrm{I}\,S\,a\,Q_1\,\omega\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}-\mathrm{I}\,S\,aA\,Q_2\,\omega\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}+\mathrm{I}\,Q_4\,k\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t} \quad (2)$$

```
> eq[3] :=  I1*diff(Vx,t)+ I1*R11*diff(Qxf,t)-diff(Stress,x);
```

$$eq_3 := -\mathrm{I}\,I1\,Q_3\,\omega\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}-\mathrm{I}\,I1\,R11\,Q_4\,\omega\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}-\mathrm{I}\,Q_1\,k\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t} \quad (3)$$

```
> eq[4] := -I1*R11*diff(Vx,t)-I1*R22*diff(Qxf,t)-I2*Qxf- diff(Prf,
  x);
```

$$eq_4 := \mathrm{I}\,I1\,R11\,Q_3\,\omega\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}+\mathrm{I}\,I1\,R22\,Q_4\,\omega\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}-I2\,Q_4\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t}$$
$$-\mathrm{I}\,Q_2\,k\sim\,\mathrm{e}^{\mathrm{I}k\sim x-\mathrm{I}\omega\sim t} \quad (4)$$

```
> A := matrix(4,4):
  for i from 1 to 4 do
  for j from 1 to 4 do
  A[i,j] := coeff(simplify(eq[i]/fun),Q[j]): od;od;print(A);
```

$$\begin{bmatrix} \mathrm{I}S\omega\sim & \mathrm{I}aS\omega\sim & \mathrm{I}k\sim & 0 \\ -\mathrm{I}Sa\omega\sim & -\mathrm{I}SaA\omega\sim & 0 & \mathrm{I}k\sim \\ -\mathrm{I}k\sim & 0 & -\mathrm{I}I1\omega\sim & -\mathrm{I}I1R11\omega\sim \\ 0 & -\mathrm{I}k\sim & \mathrm{I}I1R11\omega\sim & \mathrm{I}I1R22\omega\sim - I2 \end{bmatrix} \tag{5}$$

```
> with(linalg):
> detA  := simplify(det(A));
```

$$detA := S^2 I1^2 (R11^2 - R22)(a^2 - aA)\omega\sim^4 - \mathrm{I}I1(a^2 - aA)S^2 I2\omega\sim^3 + 2\left(aR11 - \frac{aAR22}{2} - \frac{1}{2}\right) I1 S k\sim^2 \omega\sim^2 - \mathrm{I}I2 SaA k\sim^2 \omega\sim + k\sim^4 \tag{6}$$

```
> CallA  := (collect(detA,k));#simplify
```

$$CallA := k\sim^4 + \left(2\left(aR11 - \frac{aAR22}{2} - \frac{1}{2}\right) I1 S\omega\sim^2 - \mathrm{I}I2 SaA\omega\sim\right) k\sim^2 + S^2 I1^2 (R11^2 - R22)(a^2 - aA)\omega\sim^4 - \mathrm{I}I1(a^2 - aA)S^2 I2\omega\sim^3 \tag{7}$$

```
> Coef  := coeffs(CallA, k, 't');
```

$$Coef := S^2 I1^2 (R11^2 - R22)(a^2 - aA)\omega\sim^4 - \mathrm{I}I1(a^2 - aA)S^2 I2\omega\sim^3, 1, 2\left(aR11 - \frac{aAR22}{2} - \frac{1}{2}\right) I1 S\omega\sim^2 - \mathrm{I}I2 SaA\omega\sim \tag{8}$$

```
> with(LinearAlgebra):
> with(CodeGeneration):
> Matlab(Coef[1]);Matlab(Coef[2]);Matlab(Coef[3]);
Warning, the following variable name replacements were made:
omega~ -> cg0
cg = S ^ 2 * I1 ^ 2 * (R11 ^ 2 - R22) * (a ^ 2 - aA) * cg0 ^ 4
+ -i * I1 * (a ^ 2 - aA) * S ^ 2 * I2 * cg0 ^ 3;
cg0 = 1;
Warning, the following variable name replacements were made:
omega~ -> cg
cg1 = (0.2e1 * (a * R11 - aA * R22 / 0.2e1 - 0.1e1 / 0.2e1) *
I1 * S * cg ^ 2) + -i * I2 * S * aA * cg;
> latex(A);
 \left[ \begin {array}{cccc} iS{\it omega}&iaS{\it omega}&ik&0
\\ \noalign{\medskip}-iSa{\it omega}&-iS{\it aA}\,{\it omega}
&0&ik
\\ \noalign{\medskip}-ik&0&-i{\it I1}\,{\it omega}&-i{\it I1}\,
{\it
```

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R11}\,{\it omega}\\ \noalign{\medskip}0&-ik&i{\it I1}\,{\it
R11}\,{
\it omega}&i{\it I1}\,{\it R22}\,{\it omega}-{\it I2}\end
{array}
 \right]
>
```