# Naval Vessel Mission Fuel Expenditure Optimisation

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**Synopsis**

The requirement to maintain and uplift fuel in naval vessels is a necessary operating constraint and, with projections forecasting that fuel oil prices will continue to rise, uplifts need to be scheduled to deconflict with military tasking whilst being financially efficient. This paper presents mission fuel management as an optimisation problem, where analytical techniques are used to explore the impact of intelligent uplifts, intelligent leg speeds and the impact of minimum fuel holding restrictions and hull bio-fouling. Using a representative transit, we demonstrate that relative fuel price differences between ports may be exploited to achieve mission fuel cost savings of 15% to 25%, without impacting mission dates. For time constrained transits, being those with leg speeds limited by the minimum fuel holding restriction, a saving of 4% to 5% is achievable by varying leg speeds. Finally, we conclude that challenging minimum fuel holding requirements can yield up to 5% saving, whilst hull bio-fouling has an almost negligible effect in our model (due to the short time at sea). Extrapolation indicates that whilst fuel consumption will invariably increase for a given speed, it does not affect the fuel uplift decision making.

*Keywords:* Fuel consumption; Fuel efficiency; Fuel savings; Operational effectiveness

## 1 Introduction

Fuel consumption for seagoing vessels attracts significant operating expenditure (OPEX) and may typically account for 50% of total ship operating costs (Stopford, 2008). Naval vessels do not typically operate on a routine pattern like many other vessels and as such refuelling patterns and scheduling may vary with the nature of operations. Furthermore, there is an additional dynamic whereby the requirement to refuel either alongside or by Replenishment At Sea (RAS) may be considered to be both an operational necessity and constraint that requires careful (and sometimes reactive) scheduling to deconflict with military tasking.

The proliferation of automated data recording in vessels through integrated platform management systems presents an opportunity to exploit data analytics and machine learning algorithm techniques to better interrogate and forecast seagoing fuel consumption. Such an approach may improve accuracy and yield returns for fleet operators in their management of budgets, mission planners in their scheduling of operations, and mariners in the planning of fuel uplifts and transits.

Naval vessels operate mission profiles that are typically very different to the vast majority of other ships' profiles and these profiles are frequently affected by a number of both strategic and tactical variables as shown at Table 1. Military mission planning must therefore take consideration of such variables at both the planning stage and during the mission itself. Such considerations invariably result in a number of mission constraints (and indeed opportunities) such as restrictions on port visit locations (security), timings of visits (political and logistics chains), fuel uplift limitations (fuel management), and vessel performance (speed profiles). The importance and influence of such considerations are significant.

Vessel speed and bunkering optimisation problems have received attention in recent years and Wang et al. (2013) provide a literature review of bunkering optimisation problems in particular. Kontovas (2014) conceptualise the formulation of the so-called Green Ship Routing and Scheduling Problem as they consider the relationship between vessel speed and fuel consumption, and between fuel consumption and air emissions. Psaraftis and Kontovas (2014) study ship speed optimisation for a range of scenarios and include cost rates and factors such as fuel prices, freight rates, cargo inventory costs and the dependency of fuel consumption on payload. Their study confirms that optimal environmental performance is not necessarily the same as the optimal economic performance. Yao et al. (2012) provide a study on bunker fuel management for a single shipping liner service.

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Table 1: Examples of operating and operational constraints on naval vessels.

| **Strategic Constraints** | **Tactical Constraints** |
|---|---|
| Political | Fuel management policies |
| Security | Contingent tasking |
| Environmental | Task Group rendevous windows |
| Mission | Speed profiles |
| Time | Training requirements |
| | Deployed maintenance |

Studies of the impact of bio-fouling on fuel consumption have demonstrated increases of 5% to 20%, depending on the nature and severity of the bio-fouling, and on the vessel operating parameters (Hakim et al., 2018, 2019; Turan et al., 2016; Schultz, 2007). Historically, the British Admiralty made an allowance of a 0.25% per day increase in frictional resistance for a vessel in temperate waters and a 0.5% per day increase in tropical waters, which corresponded to a 35-50% increase in fuel consumption after 6 months in temperate waters (Woods Hole Oceanographic Institution, 1952). Recent experiments with modern anti-fouling hull coatings demonstrate similar rates of increase in frictional resistance (Schultz, 2004, 2007). Molland et al. (2011) describe typical total increases in 'roughness' (including bio-fouling) leading to increases in frictional resistance of 2-4% per month but note that initial growth is often higher, and later growth is slower.

In this paper, we offer an optimisation modelling approach to minimising mission fuel expenditure within the constraints of military mission planning whilst maintaining mission outputs. Mission fuel is fuel used at the level of the vessel during its mission profile. Such a capability offers fleet operators, mission planners and mariners cost-conscious decision support information. The remainder of this paper is presented as follows: Section 2 details the problem definition and establishes a baseline solution against which later optimisation solutions are compared. In Section 3, we present a mathematical model of the problem and detail how it is solved. In Section 4, we present the results and interpretation of a suite of computational experiments, followed by our concluding remarks in Section 5.

## 2 Problem description

In this paper, we consider the fuel cost minimisation problem for a vessel completing a multi-leg transit with a constant speed for each leg. There are multiple inter-dependent decision points along the transit at which to decide how much fuel to purchase at each port, and how fast to travel, and therefore consume fuel, on each leg. Key decision parameters are the fuel consumption rate, the volume of useable fuel on-board, the relative fuel price differences between ports, and the distances between ports.

We are interested, first, in only optimising the volume of fuel uplifted at each port, and then simultaneously optimising fuel uplifts and leg speeds. As we are interested in assessing the potential for such optimisations to minimise fuel expenditure, we make some simplifying assumptions:

- All costs associated with the transit other than fuel expenditure are assumed to be constant;
- The vessel must visit each port irrespective of any fuel uplift requirements;
- Port turnaround times are assumed to be instantaneous such that the only times considered are leg transit times.

Furthermore, we opt to study a simplified physical scenario in which the effect of bio-fouling is modelled as a linear increase in the frictional bunker fuel consumption coefficient, and that this coefficient is the only variable of the total bunker fuel consumption coefficient during a simulation. We do not consider effects such as variation in vessel displacement, trim, weather, sea state etc.

While being cognisant of the constraints experienced by the military as detailed at Table 1, we developed a baseline model from historical contexts that is indicative of a naval vessel transit. For the purpose of this paper, these allow the model to be bounded realistically in addition, they allow the model to be injected with planning assumptions and constraints for forecasting purposes. We explore how this baseline model can be optimised to minimise expenditure on fuel throughout the transit. The selected baseline model is that of a multi-leg return transit from Portsmouth→Gibraltar→Malta→Cyprus. This is represented geographically at Figure 1. The model is summarised as follows:


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Figure 1: Indicative baseline model mission profile.

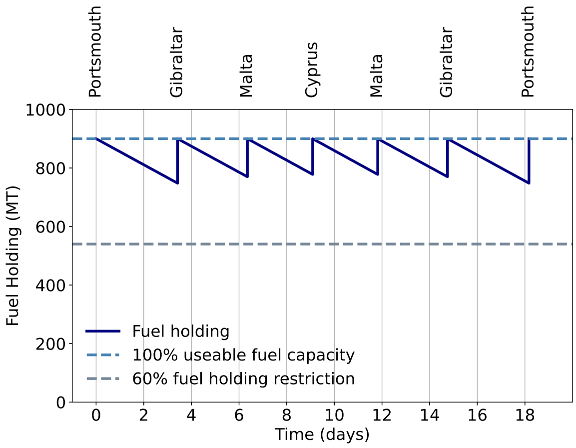


Figure 2: Baseline model fuel uplifts.

Table 2: Baseline fuel profile. Note that fuel price at Portsmouth is a parameter.

| Leg | Distance (nm) | Speed (knots) | Duration (days) | Departure (MT) | Consumed (MT) | Arrival (MT) | Uplift (MT) | Price (USD/MT) | Spend (USD) |
|---|---|---|---|---|---|---|---|---|---|
| 1 Portsmouth → Gibraltar | 1149 | 14 | 3.42 | 900 | 152 | 748 | 152 | 323 | 49,096 |
| 2 Gibraltar → Malta | 984 | 14 | 2.93 | 900 | 130 | 770 | 130 | 330 | 42,900 |
| 3 Malta → Cyprus | 920 | 14 | 2.74 | 900 | 122 | 778 | 122 | 701 | 85,522 |
| 4 Cyprus → Malta | 920 | 14 | 2.74 | 900 | 122 | 778 | 122 | 330 | 40,260 |
| 5 Malta → Gibraltar | 984 | 14 | 2.93 | 900 | 130 | 770 | 130 | 323 | 41,990 |
| 6 Gibraltar → Portsmouth | 1149 | 14 | 3.42 | 900 | 152 | 748 | 152 | 300* | 45,600 |

- The vessel completes a multi-leg return transit to the operating area within a pre-defined time limit set by a constant transit speed of 14 knots;
- The vessel is initially filled to 100% of the useable fuel capacity and re-filled to 100% of the useable fuel capacity at each port along its transit. The maximum useable fuel capacity is set to 900 Metric Tonnes (MT);
- The vessel has a 60% liquid loading restriction for minimum fuel bunkering;
- The propulsion fuel consumption rate is constant at 50 cz/day as a reasonable estimate of a 7000-8000 MT naval vessel travelling at an average speed of 14 knots. A fuel density of 890 kg/cz is assumed in accordance with ISO8217:2017 Petroleum Products – Fuels (class F) — Specifications of marine fuels.
- The vessel is ultimately filled to 100% of the useable fuel capacity at the final destination (Portsmouth) in preparation for a subsequent transit (not modelled).

The baseline model is shown at Figure 2 and Table 2. The fuel uplifts are all set as Marine Gas Oil (MGO) and costed in US Dollars from open source fuel bunkering data, and distances are set in nautical miles (nm) via dead reckoning through open source software. The total mission fuel expenditure is calculated as the sum of the expenditures at each destination port, and totals ∼$305,000.

## 3 Model Description

In this section, we present the mathematical model and describe the solution method employed. Definitions of the nomenclature used may be found at the end of this paper.

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### 3.1 Model

The fuel consumption rate $f(v)$ is approximated by a well-known cubic function of vessel speed $v$ as (Ronen, 1982; Molland et al., 2011)

$$f(v) = k_T \cdot v^3. \tag{1}$$

We adopt the simplification such that the total bunker fuel consumption coefficient $k_T = k_F + k_R$, where $k_F$ and $k_R$ are the frictional bunker fuel consumption coefficient and residual bunker fuel consumption coefficient, respectively. We include the effect of incremental bio-fouling during a transit as a linear rate of increase $\phi$ in the $k_F$ term such that after time $t$ the fuel consumption rate is given by

$$f(v,t) = \left[1 + \frac{k_F}{k_T}\phi\,(t + t_{\text{hist}})\right] \cdot k_T \cdot v^3. \tag{2}$$

The parameter $t_{\text{hist}}$ represents the time during which bio-fouling has accumulated prior to the modelled transit. The baseline model assumes a propulsion fuel consumption rate of 44.5 MT per day at 14 knots. This determines that $k_T = 6.8 \times 10^{-4}$ and we assume that $k_F = 0.3k_T$ (Barrass, 2004).

The bunker fuel $F_i$ required by a vessel to transit a distance $d_i$ at speed $v_i$ for $i = 1$ is given by

$$F_1 = \left[1 + \frac{k_F}{k_T}\phi\left(\frac{1}{2}\frac{d_1}{v_1} + t_{\text{hist}}\right)\right] \cdot k_T \cdot d_1 \cdot v_1^2, \tag{3a}$$

and for $i > 1$

$$F_i = \left[1 + \frac{k_F}{k_T}\phi\left(\sum_{i'=1}^{i-1}\frac{d_{i'}}{v_{i'}} + \frac{1}{2}\frac{d_i}{v_i} + t_{\text{hist}}\right)\right] \cdot k_T \cdot d_i \cdot v_i^2, \tag{3b}$$

where the average effect of bio-fouling on leg $i$ is included in the term $d_i/2v_i$.

The model is formulated as the minimisation of total expenditure on bunker fuel $E$ incurred by undertaking a transit of $n$ legs and uplifting a volume of fuel $u_i$ at price per unit volume $p_i$ on leg $i$, plus the required final uplift $u_F$ at price per unit volume $p_F$ to bring the vessel up to 100% useable fuel capacity:

$$\text{Min } E = \sum_{i=1}^{n} p_i \cdot u_i + u_F \cdot p_F. \tag{4}$$

The objective function (Equation 4) is subject to a number of constraints and bounds:

1. The maximum transit time constraint ensures the vessel completes the transit within an acceptable time-frame $t_{\max}$,
$$\sum_{i=1}^{n}\frac{d_i}{v_i} \leq t_{\max}; \tag{5}$$

2. The minimum bunker fuel uplift constraint $u_{\min,i}$ ensures that the minimum fuel holding restriction $F_{\min,i}$ is not breached throughout leg $i$ given an initial bunker fuel holding $F_0$. For $i = 1$
$$u_{\min,1} = F_{\min,1} + F_1 - F_0, \tag{6a}$$
and for $i > 1$
$$u_{\min,i} = F_{\min,i} + F_i - \left(F_0 + \sum_{i'=1}^{i-1} u_{i'} - \sum_{i'=1}^{i-1} F_{i'}\right); \tag{6b}$$

3. The maximum bunker fuel uplift constraint $u_{\max,i}$ ensures that the maximum bunker fuel capacity $F_{\max,i}$ is not breached throughout leg $i$. For $i = 1$
$$u_{\max,1} = F_{\max,1} - F_0, \tag{7a}$$
and for $i > 1$
$$u_{\max,i} = F_{\max,i} - \left(F_0 + \sum_{i'=1}^{i-1} u_{i'} - \sum_{i'=1}^{i-1} F_{i'}\right); \tag{7b}$$

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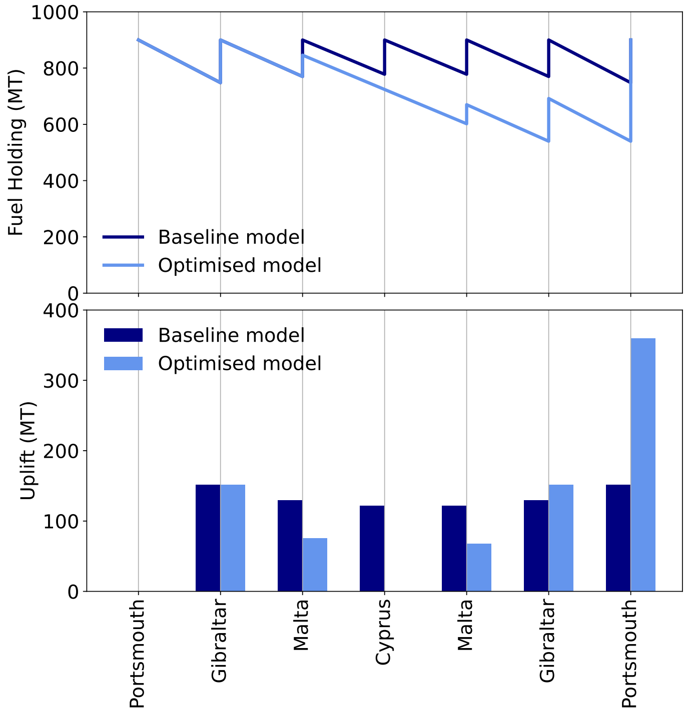


Figure 3: Comparing fuel uplift profiles from the baseline model with the optimised uplift model with $t_{\max} = 436$ hours and a Portsmouth price $p_P = \$300$.

4. The minimum bunker fuel uplift bound ensures uplifts are non-negative,

$$u_i \geq 0; \tag{8}$$

5. Minimum leg speed bound ensures vessel speeds are positive,

$$v_i > 0. \tag{9}$$

The model was programmed using `Python3` and the optimisation problem was solved using the `SciPy` library `optimize.minimize` function and the Sequential Least Square Programming (SLSQP) method.

When optimising uplifts only, the optimiser is initialised with a constant speed for all legs given by the equation $v_i = \sum_{i'=1}^{n} d_{i'}/t_{\max}$ for $i = 1 \ldots n$, and with initial uplifts $u_i$ equal to the minimum fuel required to complete leg $i$. When simultaneously optimising uplifts and speeds, the optimiser is initialised with leg speeds $v_i$ randomly sampled from a uniform distribution between 90% and 99% of the maximum speed permitted by the maximum available leg fuel $F_{\text{avail,i}}$. This is determined by setting $F_i = F_{\text{avail,i}}$ in Equations 3a and 3b, and taking the positive solution of Equations 10a and 10b. This reduces the potential for boundary condition issues. The uplifts $u_i$ are initialised such that the vessel is at 100% useable fuel capacity at the beginning of each leg. For $i = 1$

$$\left(1 + \frac{k_F}{k_T}\phi t_{\text{hist}}\right) \cdot v_1^2 + \frac{1}{2}\frac{k_F}{k_T}\phi d_1 \cdot v_1 - \frac{F_{\text{avail},1}}{k_T d_1} = 0, \tag{10a}$$

and for $i > 1$

$$\left[1 + \frac{k_F}{k_T}\phi\left(\sum_{i'=1}^{i-1}\frac{d_{i'}}{v_{i'}} + t_{\text{hist}}\right)\right] \cdot v_i^2 + \frac{1}{2}\frac{k_F}{k_T}\phi d_i \cdot v_i - \frac{F_{\text{avail},i}}{k_T d_i} = 0. \tag{10b}$$

For completeness, parameters that are held constant throughout this study are: $n = 6$, $d_i$ and $p_i$ are as per Table 2, $k_T = 6.8 \times 10^{-4}$, $k_F = 0.3k_T$, $k_R = 0.7k_T$, $F_0 = 900$ MT, $F_{\max,i} = 900$ MT for $i = 1 \ldots n$. All other parameters are subject to variation as the parameter space is explored.

## 4 Results and interpretation

In this section, we present the results and interpretation of four studies: More Intelligent Fuel Uplifts, More Intelligent Leg Speeds, A Consideration of Policy, and An Exploration of Bio-fouling.

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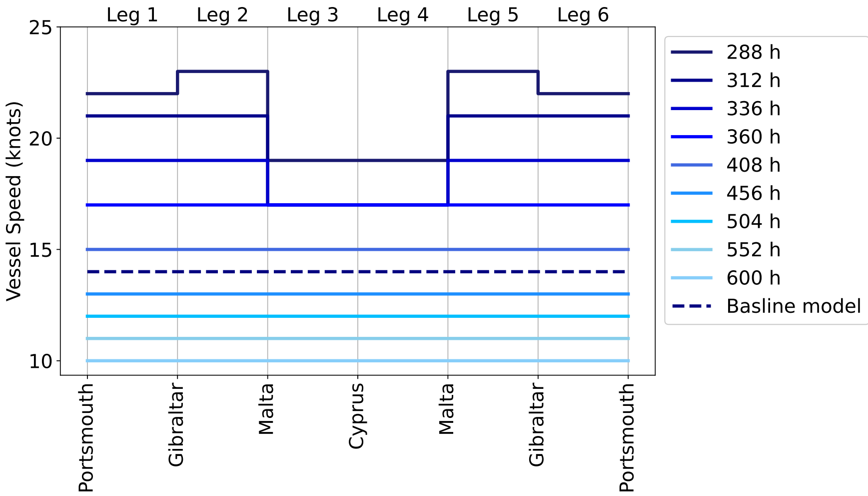


Figure 4: Comparing leg speed profiles from the baseline model with the optimised uplift & leg speed model for different $t_{\max}$ constraints and a Portsmouth price $p_P = \$300$.

### *4.1 More Intelligent Fuel Uplifts*

We first consider the case of optimising the baseline model by optimising the uplifted fuel quantities only and demonstrate that it is possible to reduce fuel expenditure without affecting mission outputs. Figure 3 compares the fuel uplift profiles between the baseline model and the optimised model with $t_{\max} = 436$ hours. While the vessel is required to visit every port en-route, it is not required to uplift any fuel. Given this flexibility, we find that the optimum solution is to avoid uplifting fuel at Cyprus where, in our model, fuel is the most expensive. This is possible because there is sufficient fuel on board to complete a round trip Malta $\rightarrow$ Cyprus $\rightarrow$ Malta without breaching the 60% minimum fuel holding restriction.

Unlike at Cyprus, the vessel is required to refuel to maximum useable fuel capacity in Portsmouth. The optimum solution is therefore dependent on the fuel price at Portsmouth $p_P$ relative to the fuel price at Gibraltar $p_G$. For $p_P$ in the range of \$200 to \$323, we find that savings between 25% to 15% are possible. Figure 3 shows the \$300 case. When $p_P > p_G$, the uplift at Portsmouth is minimised to equal the fuel required for the final leg Gibraltar $\rightarrow$ Portsmouth, as in the baseline model. The uplift at Gibraltar is commensurately increased. In all such cases, the absolute value of the saving remains constant while the total expenditure $E$ is greater for higher $p_P$. We therefore find the percentage saving declines from 15% to 12% as $p_P$ increases from \$323 to \$900. The constant absolute saving is due to the avoided uplift at Cyprus.

This study demonstrates that savings are achievable but they are dependent upon the sequencing of fuel cost differences between ports along a transit and on the minimum fuel holding constraint. By optimising uplifts only, the optimised model consumes the same volume of fuel as the baseline model. This is not expected to hold true when the weight of fuel is factored into the fuel consumption rate since the optimised model has a lower average fuel holding compared with the baseline model.

### *4.2 More Intelligent Leg Speeds*

The second case we consider is that of optimising the baseline model by simultaneously optimising the uplifted fuel quantities and the average speed travelled on each leg. In Section 4.1, we demonstrated that optimising uplifts alone was sufficient to avoid uplifting fuel at Cyprus. We find that additional savings can be made by also optimising leg speeds when $p_P > p_G$. Under these conditions, the vessel travels at higher speeds on earlier legs in order to permit a reduced speed on the final leg and complete the transit within $t_{\max} = 436$ hours. This reduces the fuel consumption on the final leg and therefore minimises the fuel uplift at Portsmouth, however, the total fuel consumption for the transit is increased. We find that up to 6% of additional savings are achievable for $p_P$ in the range \$400 to \$900 compared with optimising uplifts alone. It is therefore the case that the vessel is saving money by travelling faster earlier.

The ability to vary leg speeds permits an exploration of the impact of $t_{\max}$. With $p_P = \$300$, Figure 4 shows that when the maximum transit time $t_{\max}$ is increased, the optimal vessel speed is reduced and is constant along the entire transit. The total fuel consumption for the transit is reduced, correspondingly. At lower values of $t_{\max}$, the vessel is forced to vary its speed during the transit in order to avoid breaching the 60% liquid loading limit, and to minimise uplifts at expensive ports. This is most noticeably achieved by travelling at slower speeds on legs 3 and 4. We see a commensurate increase in speeds on other legs so that the transit is completed within the time $t_{\max}$.

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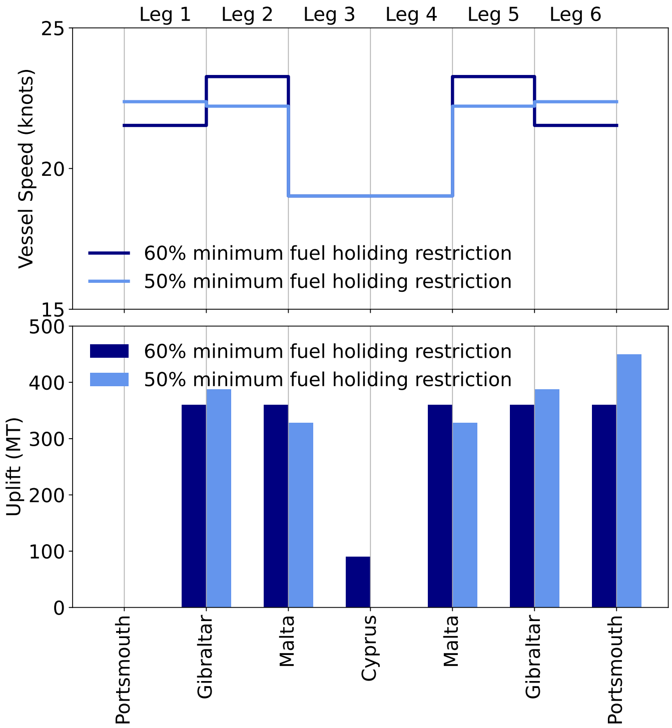


Figure 5: Comparing fuel uplift and leg speed profiles from the optimised 60% minimum fuel requirement scenario with the optimised 50% minimum fuel requirement scenario with $t_{\max}$ = 288 hours and a Portsmouth price $p_P =$ \$300.

We find that optimising uplifts and speeds can still yield a saving compared with the baseline model (where $t_{\max} = 436$ hours and the speed is a constant 14 knots) as $t_{\max}$ is reduced to approxiamtely 400 hours. We do not find savings against the baseline for smaller values of $t_{\max}$, however, we do find that optimising uplifts and leg speeds results in a lower total expenditure compared with the case of optimising only uplifts with a constant transit speed given by $\sum_{i=1}^{n} d_i/t_{\max}$. For example, with $t_{\max} = 336$ hours we find a 4% reduction in expenditure and with $t_{\max} = 312$ hours we find a 5% reduction in expenditure. When $t_{\max} = 288$ hours the reduction falls back to 4% because the fuel consumption rate has increased such that the vessel must now uplift expensive fuel at Cyprus in order to avoid breaching the 60% minimum fuel holding restriction.

As in Section 4.1, this study demonstrates that savings are achievable but that they are dependent upon the sequencing of fuel cost differences between ports along a transit and on minimum fuel holding constraints. It also demonstrates that total expenditure can be reduced by optimising uplifts and leg speeds simultaneously compared with optimising uplifts alone. It should be noted that the total fuel consumption is increased when the average transit speed is increased.

### *4.3 A Consideration of Policy*

In this third study, we explore the effect of reducing the minimum bunker fuel from 60% of maximum capacity to 50% while optimising uplifts and speeds. This offers more flexibility by allowing lower $t_{\max}$ solutions by permitting access to more of the bunker fuel and therefore higher speeds on individual legs, and by providing further scope to avoid uplifting expensive fuel. This is particularly apparent in time constrained scenarios such as when $t_{\max} = 288$ hours which, as discussed previously, necessitates an uplift at Cyprus when the minimum fuel holding restriction is 60%. By reducing this to 50%, we find the uplift at Cyprus is no longer necessary, as shown in Figure 5. This is achieved by travelling more quickly on the first leg to permit reduced speeds on later legs with the corresponding reduced fuel consumption. We have discussed this behaviour in Section 4.2 and, although this results in a larger uplift in Gibraltar, we find the total expenditure is reduced by approximately 5%. This decreases to a 1% reduction when $t_{\max} = 600$ hours because, with this limit and a 60% minimum fuel holding restriction, the uplift at Cyprus is not necessary.

### *4.4 An Exploration of Bio-fouling*

Finally, we explore the effect of bio-fouling on our optimised solutions. We find that the accumulation of bio-fouling at rates between 0.0% and 0.5% per day do not have a significant impact on the achievable savings

(0.0% to approximately 0.6% over this range, with $p_P = \$300$ and $t_{\max} = 436$ hours). It is expected that the accumulation of bio-fouling over a longer deployment such as 6 to 9 months will be more impactful and is likely to affect both uplift and speed optimisation. The accumulation of bio-fouling prior to the task will increase the total fuel expenditure during the transit. For example, at an accumulation rate of 0.25% per day prior to and during the transit, $p_P = \$300$ and $t_{\max} = 436$ hours, total fuel expenditure increased by 2%, 7% and 14% for the accumulation of 1, 3 and 6 months of bio-fouling, respectively. However, this does not affect the optimisation decision making process during the transit.

Pescetto (2019) presented the use of machine learning (ML) techniques to analyse the effect of trim, displacement, engine performance and bio-fouling. Although we have kept some variables constant and adopted a linear bio-fouling growth rate for our mission fuel model, the use of ML and digital data from on-board platform management systems offers an error margin of just 1.5% rather than the 13% observed using the ISO15016:2015 methodology. Whilst we observe that bio-fouling has a negligible effect on vessel performance over the duration of our mission, Pescetto observes that a 17% performance improvement after re-blading a fouled blade is achievable and the reduced fuel consumption rate yields a payback period for the intervention costs on the order of 90 days.

## 5 Concluding remarks and future research

Literature reviews demonstrate that significant work has been undertaken to-date to quantify and analyse the numerous vessel and environmental variables that affect ship performance and fuel consumption. Typically, such variables are modelled across the speed range, whereas we identified an opportunity to consider vessel performance over time, thus the development of a mission fuel model to inform mission planning decision making. We developed a model with realistic naval vessel constraints and considered fuel expenditure as an optimisation problem. We applied this model, using `Python3` and the SLSQP method, to the representative case study of a return transit from Portsmouth→Gibraltar→Malta→Cyprus.

In our representative case, we demonstrate that the relative fuel price differences between ports can be exploited by intelligently uplifting fuel based on price to yield a saving of 15% to 25% compared with the baseline model. These savings are achieved by avoiding an expensive uplift at Cyprus, and are dependent on the price of fuel at Portsmouth, where an uplift is enforced. The percentage saving declines as the price at Portsmouth exceeds the price at Gibraltar, but the underlying absolute savings made by avoiding an uplift at Cyprus remains. We find additional savings of up to 6% can be achieved when Portsmouth is comparatively expensive by varying leg speeds to minimise fuel consumption on approach to Portsmouth. For transits where high speeds are required, we find that variations of the leg speeds may achieve additional savings compared with constant leg speeds. The magnitude of the savings is dependent on the price at Portsmouth with savings of 4% to 5% achieved over the parameter range of our study.

Whilst the counter-intuitive "go faster to save money" effect was observed; interrogation of the data identifies that this is to buy time in order to allow slower speeds (and the associated reduction in fuel consumption) on legs where the cost of fuel is higher. We also quantified the ability to make further savings by challenging normalised liquid loading restrictions. We demonstrate that a further 5% saving is achievable for some time constrained transits by reducing this restriction from 60% to 50%. Finally, we identified that hull fouling has a negligible effect over our short transit, but even when extrapolated, it does not change the fuel uplift decision making process.

Optimised fuel expenditure does not necessarily equate to optimised fuel consumption. Optimised fuel expenditure solutions that require increased average transit speeds, such as some of the solutions outlined above, will consume greater volumes of fuel during the transit. The environmental impact of such decisions should be considered hand-in-hand with the financial impact. However, we have demonstrated that savings may be achieved by optimising uplifts alone.

There are a number of avenues that may be explored with this model. Whilst a short mission was developed for this paper, a longer mission with greater distances and an increased number of port visits is expected to offer greater opportunity to exploit the effects of intelligent uplifts, intelligent speed, liquid loading restrictions and hull fouling. The magnitude of the savings will be dependent on the fuel consumption rate, the relative fuel price differences between ports, and the distances between ports. Alternatively, integration of a data capture capability into a platform (or interrogation of a ship's legacy dataset) would bring clarity over the $f(v) = k_T \cdot v^3$ relationship to further improve accuracy. Finally, there is an opportunity to fully optimise mission fuel management by developing a more intelligent optimisation algorithm that would select its port visits based on a number of real-time constraints, including fuel costs and security status.

## References

Barrass, C.B., 2004. Ship Design and Performance for Masters and Mates. Elsevier.

Hakim, M.L., A P Utama, I.K., Nugroho, B., Yusim, A.K., Baithal, M.S., Suastika, I.K., 2018. Review of Correlation Between Marine Fouling and Fuel Consumption on a Ship, in: SENTA 2017: 17th Conference on Marine Technology, Surabaya.

 https://doi.org/10.24868/issn.2515-818X.2020.057

Hakim, M.L., Nugroho, B., Nurrohman, M.N., Suastika, I.K., Utama, I.K., 2019. Investigation of fuel consumption on an operating ship due to biofouling growth and quality of anti-fouling coating. IOP Conference Series: Earth and Environmental Science 339.

Kontovas, C.A., 2014. The Green Ship Routing and Scheduling Problem (GSRSP): A conceptual approach. Transportation Research Part D: Transport and Environment 31, 61–69.

Molland, A.F., Turnock, S.R., Hudson, D.A., 2011. Ship resistance and propulsion: Practical estimation of ship propulsive power. volume 9780521760. Elsevier Ltd.

Pescetto, A., 2019. Real cases of data driven Performance Management. Vessel Performance Optimisation, Oslo.

Psaraftis, H.N., Kontovas, C.A., 2014. Ship speed optimization: Concepts, models and combined speed-routing scenarios. Transportation Research Part C: Emerging Technologies 44, 52–69.

Ronen, D., 1982. Effect of oil price on the optimal speed of ships. Journal of the Operational Research Society 33, 1035–1040.

Schultz, M.P., 2004. Frictional resistance of antifouling coating systems. Journal of Fluids Engineering, Transactions of the ASME 126, 1039–1047.

Schultz, M.P., 2007. Effects of coating roughness and biofouling on ship resistance and powering. Biofouling 23, 331–341.

Stopford, M., 2008. Maritime Economics. Third ed., Routledge.

Turan, O., Demirel, Y.K., Day, S., Tezdogan, T., 2016. Experimental Determination of Added Hydrodynamic Resistance Caused by Marine Biofouling on Ships. Transportation Research Procedia 14, 1649–1658.

Wang, S., Meng, Q., Liu, Z., 2013. Bunker consumption optimization methods in shipping: A critical review and extensions. Transportation Research Part E: Logistics and Transportation Review 53, 49–62.

Woods Hole Oceanographic Institution, 1952. Marine Fouling and its Prevention. Technical Report. United States Naval Institute.

Yao, Z., Ng, S.H., Lee, L.H., 2012. A study on bunker fuel management for the shipping liner services. Computers and Operations Research 39, 1160–1172.

## Nomenclature

We use the index $i$ to identify a leg in a multi-leg transit. One leg comprises the origin and destination ports, and a transit between the two ports. Any fuel uplift on leg $i$ occurs before departure. In addition, fuel is uplifted at the final destination to return the vessel to maximum fuel capacity.

Parameters

| | |
|---|---|
| $n$ | Number of legs in transit; |
| $d_i$ | Distance between ports of each leg $i$ (nm); |
| $p_i$ | Fuel price per tonne at origin port of leg $i$ (USD/MT); |
| $p_F$ | Fuel price per tonne at final destination port of transit (USD/MT); |
| $F_0$ | Initial bunker fuel on board vessel (MT); |
| $F_{\min,i}$ | Minimum bunker fuel holding of vessel when arriving at destination port of leg $i$ (MT); |
| $F_{\max,i}$ | Maximum bunker fuel capacity of vessel throughout leg $i$ (MT); |
| $F_{\text{avail},i}$ | Maximum bunker fuel available for leg $i$, given by $F_{\max,i} - F_{\min,i}$ (MT); |
| $t_{\max}$ | Maximum time to complete the transit of $n$ legs (hour); |
| $t_{\text{hist}}$ | Time during which bio-fouling has accumulated prior to the modelled transit (hour); |
| $k_T$ | Total bunker fuel consumption coefficient; |
| $k_F$ | Frictional bunker fuel consumption coefficient; |
| $k_R$ | Residual bunker fuel consumption coefficient; |
| $\phi$ | Rate of increase of $k_F$ due to bio-fouling (hour$^{-1}$). |

Independent variables

| | |
|---|---|
| $u_i$ | Bunker fuel uplift at origin port of leg $i$ (MT); |
| $v_i$ | Vessel speed throughout leg $i$ (nm/h). Note that when optimising fuel uplifts only, $v_i$ are parameters. |

Dependent variables

| | |
|---|---|
| $E$ | Total expenditure on bunker fuel (USD); |
| $F_i$ | Bunker fuel required to complete leg $i$ (MT); |
| $u_F$ | Bunker fuel uplift required to return vessel to 100% useable fuel capacity at final destination (MT); |
| $u_{\min,i}$ | Minimum bunker fuel uplift at origin port of leg $i$ (MT); |
| $u_{\max,i}$ | Maximum bunker fuel uplift at origin port of leg $i$ (MT). |

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