Collective and Stochastic Motion in the Time-Dependent Schrodinger Equation

Francesco R. Ruggeri Hanwell, N.B. Feb. 1, 2021

The "time-independent" Schrodinger equation appears to be directly related to stochastic motion, we argue. If there is no potential, the wavefunction exp(ipx) is statistical, indicating periodic motion with a norm of one for all x. If a time-independent potential is added, it may be decomposed as V(x)=Sum over k Vk exp(ikx). Thus, time is present not just in exp(-iEt), but in the stochastic behaviour of the potential which is only V(x) on average. At each time, one does not know which Vk acts. Thus, the time-independent Schrodinger equation could perhaps be called the purely stochastic Schrodinger equation.

It is possible to have a time dependent Schrodinger equation with a time dependent potential. Sometimes the time in the potential is hidden by a variable change as the equation involves partial derivatives. We try to argue that this potential contributes to collective motion. Time is already present in V(x), but in a stochastic manner. Explicit time (or a time related variable change) means a lack of stochasticity as one follows the potential in a predictable manner in time. It is possible that collective motion may be associated with a reference frame. For example, one may remove center-of-mass motion and consider only internal motion i.e. the quantum stochastic internal behaviour.

In this note, we briefly examine two examples. The first is a free quantum particle viewed from a constantly accelerating frame as discussed in (1). In such a frame, the free particle appears as a particle in a gravitational potential +mgx1, but x1= x-X(t) so X(t) suggests collective motion. One may argue that x represents stochastic motion, but this is not necessarily the case because one may have collective potential energy linked to collective kinetic energy. In other words, id/dt partial W may yield a term which cancels mgx indicating that it does not act as Sum Vk exp(ikx).

The second example involves a quantum oscillator with an extra xf(t) potential. We argue that this leads to collective motion as the problem may be recast in terms of a variable y=x- b(t) and solved as a "time-independent oscillator" in y as discussed in (2). Again, there exists the notion of a kind of collective motion or extra frame which leads to a phase factor exp(iy d/dt partial b(t)) as the stochastic quantum behaviour defines the physical density and the phase term exp(i phase) disappears from the density).

In such a case, W(y) the time-independent oscillator solution may be written as: Sum over p a(p) exp(ipy), but y contains t so the statistical nature of exp(ipy) is somewhat altered frome exp(ipx) i.e. there is an extra time related phase exp(i p f(t)) which has the appearance of a kind of collective motion as all p values are in synch with the same f(t) which may be sin(wt) for example.

Time Independent Schrodinger Equation and Stochasticity

It seems the time-independent Schrodinger equation is based on stochastic motion as W(x,t)=exp(-iEt)W(x) and id/dt W = EW. Thus, id/dt W may not be used to cancel time related terms on the RHS of the Schrodinger equation as in the time-dependent case. It is perhaps somewhat unusual that the time-independent equation is named in terms of an “absence of time” because we argue that time is present at least in two ways. First, there is exp(-iEt) indicating a statistical cyclic motion in time. Secondly, we argue that V(x)=Sum over k V(k) exp(ikx) and that only one Vk acts at a time. Thus, there is stochastic potential behaviour. At a given t, one does not know which virtual photon Vk yields a hit. On average, Kinetic energy average (at x) + V(x) = E, but otherwise W(x)=Sum over k a(k) exp(ikx) indicating a momentum distribution of a statistical object exp(ikx).

The time-dependent Schrodinger equation is more complicated because it may contain both stochastic and collective pieces. The collective pieces may involve both kinetic and potential energy and so potential energy is not automatically stochastic as in the time-independent case. On the other hand, the time-dependent Schrodinger equation may be purely statistical as in the case of a free quantum particle. There is a constant velocity frame associated with exp(ipx), but there is still pure statistical motion contained in exp(ipx) i.e. a kind of periodic behavioiur.

Time-Dependent Schrodinger Equation

At first, one might consider the time-dependent Schrodinger equation to be “just as stochastic” as the time independent one, with the exception that one does not have a bound state. For example, one may have a free particle moving in one direction exp(ipx-iEt). For the bound state, one combines exp(ipx-iEt) and exp(-ipx-iEt). (Actually, for a particle in a box with infinite walls, exp(ipx) really represents exp(i pave x). Many exp(ipx) terms are stirred up to create exp(i pave x).)

The time-dependent Schrodinger equation, however, may describe stochastic motion as viewed from an accelerating frame. In such a case, a free particle Schrodinger equation transforms into one with a potential. One may expect that such a potential represents physical stochastic motion, but that is not the case. It is rather a fictitious potential associated with the accelerating frame and not a physical stochastic potential. To see this, consider the example (1):

id/dt(partial) W = -1/2m d/dx d/dx W   t1=t and x1= x- X(t)   where d/dt d/dt X = g ((1))

Transforming to t,x1 yields:

id/dt partial W1(x1,t) = -1/2m d/dx1 d/dx1 W1 + mg x1 W1   ((2))

((2)) has the form of a free particle in a gravitationao potential and one may find a solution such that W(x,t)=exp(-iEt)W(x) with W(x) having a changing density (W*W) with x. That is not, however, the solution appropriate for transformation of ((1)). X(t) depends on time which suggests a frame of reference. One might think that given x1=x-X(t), one may retain mgx which would then represent stochastic motion, but that is also not the case as time-dependent terms are affected by id/dt W which may cancel potential terms. In such a case, the potential is not related to stochastic motion we argue. For example, a solution to ((2)) appropriate for the accelerating frame is:

W1(x1,t)= exp(-iEt) exp[ipx1] exp(imx1 dX/dt) exp[i .5mIntegral (0,t) dt1 dX(t1)/dt1 dX(t1)/dt1] ((3))

In other words, one wants the original W(x,t)=exp(-ipx-iEt) multiplied by exp(-i phase(x,t)) because the physical density at x= W*(x,t)W(x,t) cannot change. The free particle density is very different from the Airy function solution of a quantum particle in a gravitational potential.

id/dt (partial) exp(imx dX/dt) = imx d/dt dX/dt which cancels the potential term mgx. Thus, the gravitational potential is an artifact of the accelerating frame used to view the free quantum particle in the rest frame. Thus, for a time-dependent Schrodinger equation, one must be careful in interpreting the physical effects of a potential as it may be linked to a moving frame.

A second example is that of a time dependent potential added to a V(x). V(x) as we have argued usually represents stochastic motion (although part or all of it may be associated with a moving frame i..e collective motion). The time dependent piece xf(t) is not fully stochastic in that it is predictable in time whereas V(x) is not. Thus, it represents a kind of collective motion or frame. It may thus be possible to change variables to incorporate xf(t) in y=x-b(t) in order to analyse the purely stochastic quantum motion, albeit in terms of a new variable y which includes time or in other words a kind of collective motion. In such a case, one may expect a phase in the solution of W(x,t) which does not affect spatial density. Spatial density should be a physical observable and thus a result of the stochastic features of quantum mechanics which create a hump behaviour. This is, however, affected by the variable y=x-b(t) which explicitly contains time (as if one were viewing the stochastic quantum system from a different frame i.e. one connected with some kind of collective motion).

The Schrodinger equation for the problem (2) is:

id/dt (partial) W = -1/2m d/dx d/dx W + .5kxx W - xf(t) W ((3))

The time dependent potential portion already looks like it could be associated with a new “spatial” variable y=x-b(t) which suggests a different frame or some kind of collective motion. Following (2), we suggest:

W= Wa(y,t)exp(iEt) exp(iydb(t)/dt)   ((4))

This is of the form of a product wavefunction: W=Wo W1 where Wo = Wa(y,t)exp(-iEt) and W1=exp(iydb/dt). Thus, one may write two equations as in the case where W=WoW1 and Wo is a ground state energy. Note: .5kxx = .5k (y-b(t))(y-b(t)) = .5kyy + b(t)b(t).5k -kb(t)y. W1 may be associated with the potential portions -kb(t)y and xf(t).

The first equation is:

-1/2m d/dy d/dy Wa(y,t) + .5kyy Wa(y,t) + c(t)Wa(y,t) = id/dt[exp(-iEt)Wa(y,t)] ((5))

Let Wa(y,t)=Wa(y)exp(-id(t)) such that: d/dt d(t) = c(t)
Then, ((5)) becomes a time-independent oscillator equation in the variable y=x-b(t) which is like a view from a different frame or collective motion.

The second equation is of the form:

-1/2m d/dx d/dx W1 -1/m dW1/dx dWo/dx + extra potential terms = id/dt W1(x) ((6)) but with W1=exp(iydb/dt) or:

-m[y d/dt db/dt] - i db/dt dWo/dy = -½ 2i db/dt dW/dy - f(t)y + .5kb(2y)

The LHS represents id/dt partial W1.

my d/dt db/t = yf(t) - kb(t)y ((7)) with y cancelling

Let id/dt exp(ig(t)) = c(t) where g(t) = Integral (0,t) dt1 c(t1)

Thus, one may solve for b(t) as a differential equation containing the original f(t) i.e. the original time dependent piece of the potential.

The point we wish to make is that the stochastic quantum behaviour seems to be associated with an oscillator as the density humps at a given t=ta are entirely due to the quantum oscillator solution. The time dependent potential xf(t) is related, we argue, to a frame or collective type of motion (the two are equivalent as the frame may accelerate). This leads to exp(i phase(x,t)) terms appearing which do not affect the density and hence are not part of the underlying stochastic nature of quantum mechanics. The problem is a little more complicated as the one uses y=x-b(t) so the origin of the oscillator changes in time (i.e. collective motion).

Conclusion

In conclusion, we argue that the time-independent Schrodinger equation describes stochastic and statistical behaviour in a quantum system. exp(ipx) which is part of W(x)=Sum over p a(p)exp(ipx) is already statistical we argue (with the norm being 1 everywhere). Furthermore, the potential V(x) =Sum over k Vk exp(ikx) and so is stochastic and also time dependent except for the fact that one cannot predict which Vk acts at a particular time. Thus, the time independent Schrodinger equation seems to be more of a “stochastic time equation”. Time does not appear because one does not predict any behaviour in time except exp(-iEt) a kind of periodic cycling.

The time -dependent Schrodinger equation may incorporate both stochastic and collective behaviours related to a different frame. For example, one may transform a free particle Hamiltonian using x1=x-X(t) t1=t into a time dependent Schrodinger equation with the appearance of a particle in a gravitational potential. It, however, does not represent the same physics. id/dt W = -1/2m d/dx d/dx W + mgx W may be solved for a physical gravitational problem using W=exp(-iEt)W(x) where the density of W(x) (related to Airy functions) changes with x. For the frame transformation, the stochastic physics of the free quantum particle i.e. its density profile cannot change. Thus, even though the transformed equation has the appearance of a particle in a gravitational potential, its solution should be of the form: exp(-iEt)exp(-ipx) exp(-iphase(x,t)). Thus, a potential in a time dependent Schrodinger equation which contains time may represent collective-new frame motion.

It is possible to have an x dependent potential added to a time dependent one, e.g. .5kxx -xf(t). The second term contains time explicitly and suggests a lack of stochasticity. Rather, it is predictable in time and is more related to collective motion or a new frame. We show (using arguments from (2)), that one may transform to y=x-b(t) (i.e. a new frame or collective motion) and solve to find a quantum oscillator solution at each t (although y keeps changing origin). Thus, the stochastic quantum behaviour is that of an oscillator while f(t)x seems to bring in collective motion (which may involve acceleration) and is similar to viewing the problem in a new frame. Thus, one has exp(i phase(x,t)) as a main part of the solution, although this contributes nothing to the spatial density which is based on quantum stochastic (statistical behaviour) it seems.

References

1. Ruggeri, Francesco R. The Free Particle Schrodinger Equation and Constant Accelerating Frames (preprint, zenodo, 2021)
2. Ruggeri, Francesco R. Time Dependent Harmonic Oscillator with a Time Dependent Force and Flux Balance (preprint, zenodo, 2019)