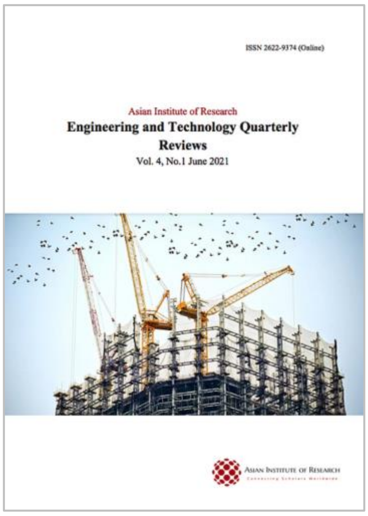

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Uniformly Valid First Approximation Shell Theory of Hybrid Anisotropic Materials

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Abstract
The theories in this article implies unique physical characteristics and formulated the governing equations. A uniformly valid shell theory which includes all the terms present in each of the asymptotic shell theories. The first approximation theories derived in this article represent the simplest possible shell theories for the corresponding length scales considered. Although twenty-one elastic coefficients are present in the original formulation of the problem, only six are appear in the first approximation theories.

Keywords: Shell Theory, Hybrid Anistropic Materials

CONSTITUTIVE EQUATIONS

The elasticity equations in terms of the dimensionless coordinate y are given by:

Stress-Displacement Relations

$$u_{r,y} = \lambda a[S_{11}\sigma_z + S_{12}\sigma_\theta + S_{13}\sigma_r + S_{14}\tau_{r\theta} + S_{15}\tau_{rz} + S_{16}\tau_{\theta z}]$$

$$u_{z,y} + \lambda a u_{r,z} = \lambda a\,[S_{51}\sigma_z + S_{52}\sigma_\theta + S_{53}\sigma_r + S_{54}\tau_{r\theta} + S_{55}\tau_{rz} + S_{56}\tau_{\theta z}]$$

$$\lambda u_{r,\theta} + (1+\lambda y)u_{\theta,y} - \lambda u_\theta = \lambda a(1+\lambda y)[S_{41}\sigma_z + S_{42}\sigma_\theta + S_{43}\sigma_r + S_{44}\tau_{r\theta} + S_{45}\tau_{rz} + S_{46}\tau_{\theta z}]$$

Equation (1)

Equilibrium Equations

$$[(1+\lambda y)\tau_{r\theta}]_{,y} + \lambda\sigma_{\theta,\theta} + \lambda a\tau_{\theta z,z} + \lambda\tau_{r\theta} = 0$$

$$[(1+\lambda y)\tau_{rz}]_{,y} + \lambda\tau_{\theta z,\theta} + \lambda a[(1+\lambda y)\sigma_z]_{,z} = 0$$

$$[(1+\lambda y)\sigma_r]_{,y} + \lambda\tau_{r\theta,\theta} + \lambda a\tau_{rz,z} - \lambda\sigma_\theta = 0$$

Equation (2)

The uniformly valid first approximation are determined by keeping only those terms found necessary in the various previous first approximation theories.

$$u_{r,y} = 0$$

$$u_{z,y} + \lambda a u_{r,z} = 0$$

$$\lambda u_{r,\theta} + (1+\lambda y)u_{\theta,y} - \lambda u_\theta = 0$$

$$u_{z,z} = S_{11}\sigma_z + S_{12}\sigma_\theta + S_{16}\tau_{\theta z}$$

$$(1/a)(u_{\theta,\theta} + u_r) = S_{21}\sigma_z + S_{22}\sigma_\theta + S_{26}\tau_{\theta z}$$

$$(1+\lambda y)u_{\theta,z} + (1/a)u_{z,\theta} = S_{61}\sigma_z + S_{62}\sigma_\theta + S_{66}\tau_{\theta z}$$

Equation (3)

Integration with respect to y can now be carried out in the same fashion as was done in the previous chapters. Integration of the first three equations yields the displacements.

$$u_r = U_r(z,\theta)$$

$$u_\theta = (1+\lambda y)U_\theta(z,\theta) - \lambda U_{r,\theta}\, y$$

$$u_z = U_z(z,\theta) - a\lambda U_{r,z}\, y$$

Equation (4)

Where **Ur , U$_\theta$, Uz** are the **y=0** surface displacement components. Note here that the radial displacement is independent of the thickness coordinate while the circumferential and longitudinal displacements are of linear dependence. Therefor the theory incorporates the hypothesis of the preservation of the normal. Substitution of results into the next three equations yields the in-plane stress-strain relations:

$$\begin{Bmatrix} \sigma_z \\ \sigma_\theta \\ \tau_{\theta z} \end{Bmatrix} = [\,C\,] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} + [C] \begin{Bmatrix} K_1 \\ K_2 \\ K_{12} \end{Bmatrix} \lambda a y$$

Equation (5)

where [C] is the symmetric matrix given by

$$[\,C\,] = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix}^{-1}$$

Equation (6)

The **ε'**s are defined by

$$\varepsilon_1 = U_{z,z}$$

$$\varepsilon_2 = (1/a)(U_r + U_{\theta,\theta})$$

$$\varepsilon_{12} = U_{\theta,z} + (1/a)U_{z,\theta}$$

Equation (7)

K's. are the changes of curvature by

$$K_1 = -U_{r,zz}$$

$$K_2 = -(1/a^2)(U_{r,\theta\theta} - U_{\theta,\theta})$$

$$K_{12} = -(2/a)(U_{r,\theta z} - U_{\theta,z})$$

Equation (8)

The equations (5) and (8) play the role of correction to the y=0 strains for points away from the inner surface of the shell. The strains of the inner surface are given by (7).

This theory which contains all the terms existing in the previous theories are the equilibrium equations

$$[(1+\lambda y)\tau_{r\theta}]_{,y} + \lambda\sigma_{\theta,\theta} + a\lambda\tau_{\theta z,z} + \lambda\tau_{r\theta} = 0$$

$$\tau_{rz,y} + \lambda\tau_{\theta z,\theta} + a\lambda\sigma_{z,z} = 0$$

$$[(1+\lambda y)\sigma_r]_{,y} + \lambda\tau_{r\theta,\theta} + a\lambda\tau_{rz,z} - \lambda\sigma_\theta = 0$$

Equation (9)

Substitution of equation (5) into the equilibrium equations and carrying out the integration with respect to y:

$$
\begin{aligned}
\tau_{rz} = T_{rz} &- \lambda[(A_{13}\varepsilon_{1,\theta} + A_{23}\varepsilon_{2,\theta} + A_{33}\varepsilon_{12,\theta}) \\
&+ a\lambda(B_{13}K_{1,\theta} + B_{23}K_{2,\theta} + B_{33}K_{12,\theta})] \\
&- a\lambda[(A_{11}\varepsilon_{1,z} + A_{12}\varepsilon_{2,z} + A_{13}\varepsilon_{12,z}) \\
&+ a\lambda(B_{11}K_{1,z} + B_{12}K_{2,z} + B_{13}K_{12,z})] \\
\tau_{r\theta} = T_{r\theta} &- \lambda[(A_{12}\varepsilon_{1,\theta} + A_{22}\varepsilon_{2,\theta} + A_{23}\varepsilon_{12,\theta}) \\
&+ a\lambda(B_{12}K_{1,\theta} + B_{22}K_{2,\theta} + B_{23}K_{12,\theta}) \\
&+ a\lambda^2(F_{22}K_{1,\theta} + F_{23}K_{12,\theta})] \\
&- a\lambda[(A_{13}\varepsilon_{1,z} + A_{23}\varepsilon_{2,z} + A_{33}\varepsilon_{12,z}) \\
&+ a\lambda[(B_{13}K_{1,z} + B_{23}K_{2,z} + B_{33}K_{12,z})] \\
\sigma_r = T_r &+ [(A_{12}\varepsilon_1 + A_{22}\varepsilon_2 + A_{23}\varepsilon_{12}) + a\lambda(B_{12}K_1 + B_{22}K_2 + B_{13}K_3)] \\
&- a\lambda[T_{rz,z}y - a\lambda^2(E_{13}K_{1,\theta z} + E_{23}K_{2,\theta z} + E_{33}K_{12,\theta z}) \\
&- a^2\lambda^2(E_{11}K_{1,zz} + E_{12}K_{2,zz} + E_{12,zz})] \\
&- \lambda[T_{r\theta,\theta}y - a\lambda^2(E_{12}K_{1,\theta\theta} + E_{22}K_{2,\theta\theta} + E_{23}K_{12,\theta\theta}) \\
&- a^2\lambda^2(E_{13}K_{1,\theta z} + E_{23}K_{2,\theta z} + E_{33}K_{12,\theta z})]
\end{aligned}
$$

Equation (10)

The conditions at the inner and outer surface of the shell yields

$$T_{rz} = T_{r\theta} = T_r = 0$$

Equation (11)

The in-plane stresses given by (5) into the original constitutive equations, we obtain the following expressions for the stress resultants:

$$
\begin{aligned}
&\underline{A}_{11}aU_{z,zz} + 2\underline{A}_{13}U_{z,\theta z} + \underline{A}_{33}(1/a)U_{z,\theta\theta} + \underline{A}_{13}aU_{\theta,zz} + (\underline{A}_{12} + \underline{A}_{33})U_{\theta,\theta z} + \underline{A}_{23}(1/a)U_{\theta,\theta\theta} \\
&+ \underline{A}_{12}U_{r,z} + \underline{A}_{23}(1/a)U_{r,\theta} - a\lambda[\underline{B}_{11}aU_{r,zzz} + 3\underline{B}_{13}U_{r,zz\theta} + (1/a)(\underline{B}_{12} + 2\underline{B}_{33})U_{r,z\theta\theta} \\
&+ \underline{B}_{23}(1/a^2)U_{r,\theta\theta\theta} + \underline{B}_{23}(1/a^2)U_{r,\theta} + (1/a)(\underline{B}_{12} + 2\underline{B}_{33})U_{r,z} \\
&- 2\underline{B}_{13}U_{\theta,zz}] = 0
\end{aligned}
$$

$$\underline{A}_{13}aU_{z,zz}+(\underline{A}_{12}+\underline{A}_{33})U_{z,\theta z}+\underline{A}_{23}(1/a)U_{z,\theta\theta}+\underline{A}_{22}(1/a)U_{\theta,\theta\theta}+\underline{A}_{33}aU_{\theta,zz}+$$

$$2\underline{A}_{23}U_{\theta,\theta z}+\underline{A}_{23}U_{r,z}+\underline{A}_{22}(1/a)U_{r,\theta}\quad -$$

$$a\lambda[\underline{B}_{13}aU_{r,zzz}+(\underline{B}_{12}+2\underline{B}_{33})U_{r,\theta zz}+3\underline{B}_{23}(1/a)U_{r,\theta\theta z}+\underline{B}_{22}(1/a^2)U_{r,\theta\theta\theta}+$$

$$2\underline{B}_{23}(1/a)U_{r,z}+\underline{B}_{22}(1/a^2)U_{r,\theta}]\ -a\lambda^2\left[(2\underline{F}_{23}(1/a)U_{r,\theta\theta z}+\underline{F}_{22}(1/a^2)U_{r,\theta\theta\theta}-\right.$$

$$\left.-\ 2\underline{F}_{23}(1/a)U_{\theta,\theta z}-\underline{F}_{22}(1/a^2)U_{\theta,\theta\theta})\right]\quad = 0$$

$$\underline{A}_{12}U_{z,z}+\underline{A}_{22}(U_{r}+U_{\theta,\theta})/a+\underline{A}_{23}(U_{\theta,z}+U_{z,\theta}/a)$$

$$-a\lambda[\underline{B}_{12}U_{r,zz}+\underline{B}_{22}(U_{r,\theta\theta}+U_{r})/(a^2)+2\underline{B}_{13}(U_{r,\theta z}-U_{\theta,z})/a]$$

$$+a\lambda[a\underline{D}_{11}U_{z,zzz}+3\underline{D}_{13}U_{z,zz\theta}+(1/a)(\underline{D}_{12}+2\underline{D}_{33})U_{z,\theta\theta z}+(1/a\ )\underline{D}_{23}U_{z,\theta\theta\theta}+$$

$$+a\underline{D}_{13}U_{\theta,zzz}+(\underline{D}_{12}+2\underline{D}_{33})U_{\theta,zz\theta}+(3/a)\underline{D}_{23}U_{\theta,\theta\theta z}+(1/a^2)\underline{D}_{22}U_{\theta,\theta\theta\theta}+$$

$$+\underline{D}_{12}U_{r,zz}\ +2\underline{D}_{23}(1/a)U_{r,\theta z}+(\underline{D}_{22}/a^2)U_{r,\theta\theta}]$$

$$-a^2\lambda^2[\underline{E}_{13}U_{r,\theta zzz}+(\underline{E}_{23}/a^2)U_{r,\theta\theta\theta z}+2(\underline{E}_{33}/a)U_{r,\theta\theta zz}]$$

$$-a^3\lambda^2[\underline{E}_{11}U_{r,zzzz}+(\underline{E}_{12}/a^2)U_{r,\theta\theta zz}+\underline{E}_{13}(2/a)U_{r,\theta zzz}]$$

$$-a\lambda^2[\underline{E}_{12}U_{r,\theta\theta zz}+(\underline{E}_{22}/a^2)(U_{r,\theta\theta\theta\theta}+U_{r,\theta\theta})+2(\underline{E}_{23}/a)U_{r,\theta\theta\theta z}]$$

$$-a^2\lambda^2[\underline{E}_{13}U_{r,zzz\theta}+(\underline{E}_{23}/a^2)U_{r,\theta\theta\theta z}+2(\underline{E}_{33}/a)U_{r,\theta\theta zz}]\quad = \quad p$$

Equation (12)

It is convenient for the readers to compute in terms of the stress resultants as follows.

$$\begin{Bmatrix} N_z \\ N_\theta \\ N_{z\theta} \\ N_{\theta z} \\ M_z \\ M_\theta \\ M_{z\theta} \\ M_{\theta z} \end{Bmatrix} = \left[\begin{array}{c|c} \overline{A} & \overline{B} \\ \hline \overline{B} & \overline{D} \end{array}\right] \begin{Bmatrix} \varepsilon_{1d} \\ \varepsilon_{2d} \\ \varepsilon_{12d} \\ K_1 \\ K_2 \\ K_{12} \end{Bmatrix}$$

Equation (13)

Where

$$\varepsilon_{1d} = \varepsilon_1 + dK_1$$

$$\varepsilon_{2d} = \varepsilon_2 + dK_2$$

$$\varepsilon_{12d} = \varepsilon_{12} + dK_{12}$$

And

$$[\ \bar{A}\ ] = \begin{bmatrix} [\lambda a/(1+d/a)][\underline{A}_{11}, \underline{A}_{12}, \underline{A}_{13}] \\ (\lambda a)\ [\underline{A}_{21}, \underline{A}_{22}, \underline{A}_{23}] \\ [\lambda a/(1+d/a)][\underline{A}_{31}, \underline{A}_{32}, \underline{A}_{33}] \\ (\lambda a)\ [\underline{A}_{31}, \underline{A}_{32}, \underline{A}_{33}] \end{bmatrix}$$

$$[\ \bar{B}\ ] = \begin{bmatrix} [\lambda a/(1+d/a)][(\lambda a\underline{B}_{11}-d\underline{A}_{11}), (\lambda a\underline{B}_{12}-d\underline{A}_{12}), (\lambda a\underline{B}_{13}-d\underline{A}_{13})] \\ (\lambda a)\ [\ (\lambda a\underline{B}_{21}-d\underline{A}_{21}), (\lambda a\underline{B}_{22}-d\underline{A}_{22}), (\lambda a\underline{B}_{23}-d\underline{A}_{23})] \\ [\lambda a/(1+d/a)][(\lambda a\underline{B}_{31}-d\underline{A}_{31}), (\lambda a\underline{B}_{32}-d\underline{A}_{32}), (\lambda a\underline{B}_{33}-d\underline{A}_{33})] \\ (\lambda a)\ [\ (\lambda a\underline{B}_{31}-d\underline{A}_{31}), (\lambda a\underline{B}_{32}-d\underline{A}_{32}), (\lambda a\underline{B}_{33}-d\underline{A}_{33})] \end{bmatrix}$$

$$[D] = \begin{bmatrix} [\lambda^2a^2/(1+d/a)][(\lambda a\underline{F}_{11}-2d\underline{B}_{11}+\frac{d^2}{\lambda a}\underline{A}_{11}),\ (\lambda a\underline{F}_{12}-2d\underline{B}_{12}+\frac{d^2}{\lambda a}\underline{A}_{12}), \\ \qquad (\lambda a\underline{F}_{13}-2d\underline{B}_{13}+\frac{d^2}{\lambda a}\underline{A}_{13}) \\ (\lambda^2a^2)\ [(\lambda a\underline{F}_{21}-2d\underline{B}_{21}+\frac{d^2}{\lambda a}\underline{A}_{21}),\ (\ldots),\ (\ldots) \\ [\lambda^2a^2/(1+d/a)][(\lambda a\underline{F}_{31}-2d\underline{B}_{31}+\frac{d^2}{\lambda a}\underline{A}_{31}),\ (\ldots),\ (\ldots) \\ (\lambda^2a^2)[\ (\lambda a\underline{F}_{31}-2d\underline{B}_{31}+\frac{d^2}{\lambda a}\underline{A}_{31}),\ (\ldots),\ (\ldots) \end{bmatrix}$$

Equation (14)

Equations (4), (5) and (10 – 14) indicate the equations of the uniformly valid first approximation theory.

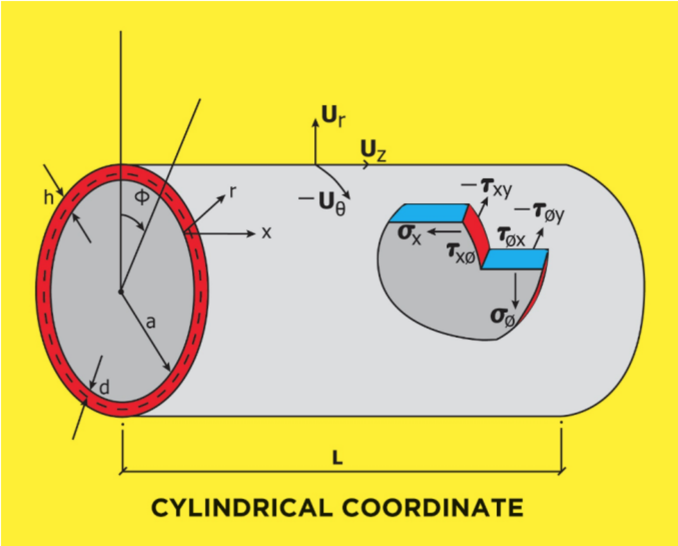

Figure 1: A Cylindrical Shell Showing Dimensions, Deformations, and Stresses

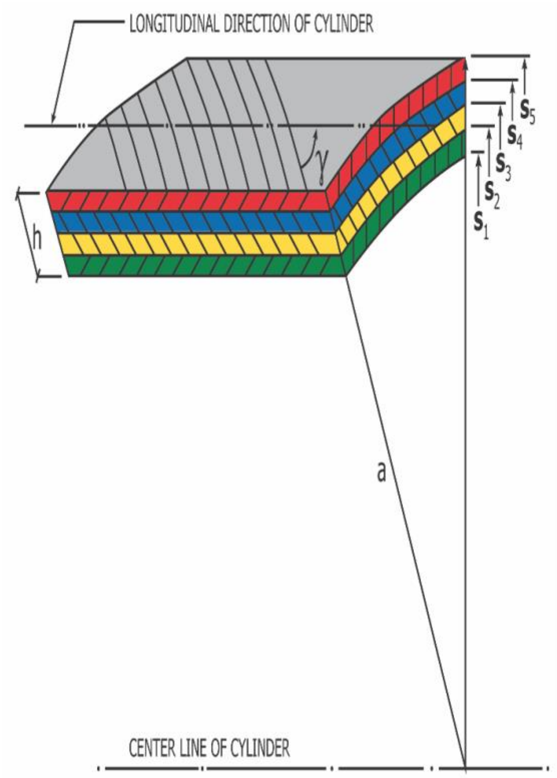

Figure 2: Laminated Cylinder and Angle of Orientation

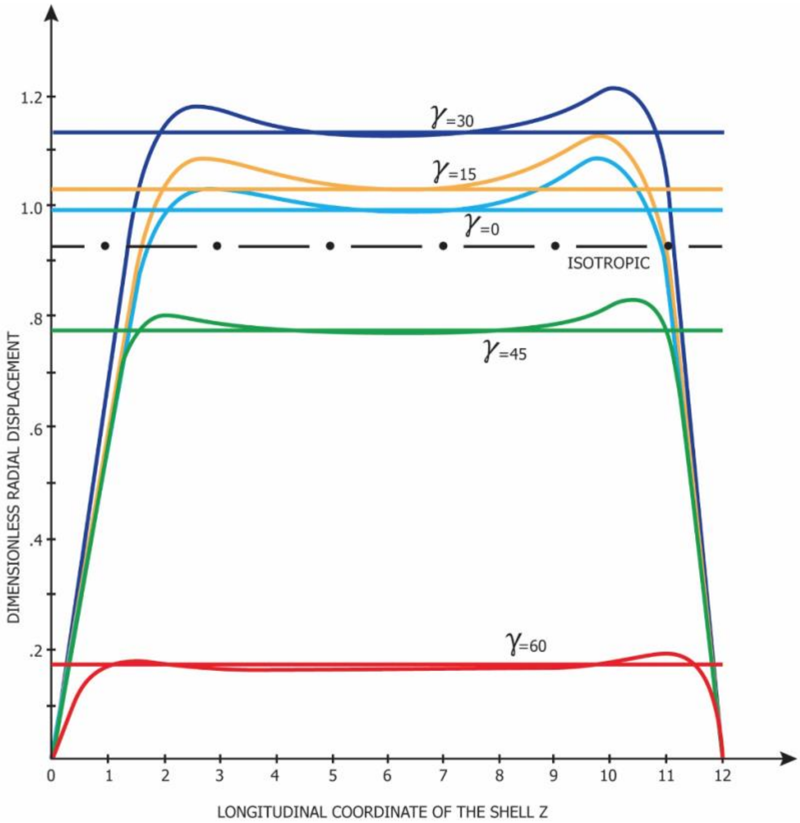

Figure 3: Radial Displacement of the Theory Associated with Longitudinal Length Scale $(ah)^{1/2}$ and Circumferential Lengths

CONCLUSION

The first approximation theories derived in this article represent the simplest possible shell theories for the corresponding length scales considered. Although twenty-one elastic coefficients are present in the original formulation of the problem, only six are appear in the first approximation theories. The fact that these expressions can be determined is extremely useful when discussing the possible failure of composite shells. It was seen that various shell theories are obtained by using different combinations of the length scales introduced in the non-dimensionalization of the coordinates and that each theory possess unique properties such as the orders of magnitudes of the stress and displacement components and edge effect penetration.

The two theories based on the axial length scale $(ah)^{1/2}$ show that a significant edge effect exists and that the penetration of the edge effect changes with the angle in similar fashion as for the radial displacement, being deepest at 30 degree. In this thesis the different theories were applied separately to the solution of a layered shell problem. The solution of a general non-homogeneous anisotropic shell problem can be obtained by a superposition of the theories derived here.

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