IPT #10 - Hail

University of Ljubljana - Slovenia

Official problem statement

Extract as much information as possible about the shape and dimensions of a metal container from the sound produced when dropping small objects (such as peas) into it.

Introduction

-“Can we hear the shape of a drum?”

-Eigenfrequencies give the information

-Eigenvalues of the Laplacian with appropriate boundary conditions

-Deviations from the mathematical question

Theoretical description

To find the eigenfrequencies solve the wave equation of the volume oscillations of the air

$$\nabla^2 \Psi(\vec{r}, t) - \frac{1}{c^2} \frac{\partial \Psi(\vec{r}, t)}{\partial t} = 0$$

with the stationary wave guess $\Psi(\vec{r}, t) = \psi(\vec{r}) f(t)$, giving

$$\nabla^2 \psi(\vec{r}) = -k^2 \psi(\vec{r}) \qquad \nu = \frac{c}{2\pi} k$$

Theoretical description

-Special cases:

Rectangular geometry: $$\nu = \frac{c}{2}\sqrt{\left(\frac{n}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2 + \left(\frac{l}{L_z}\right)^2}$$

Cylindrical geometry: $$\nu = \frac{c}{2}\sqrt{\left(\frac{\rho_{m,n}}{\pi a}\right)^2 + \left(\frac{l}{H}\right)^2}$$

End corrections

The Neumann condition for open ends is only approximate

If the perpendicular dimensions are small compared to the open one we have the empirical formula

$$\delta L \approx 0.34\sqrt{A}$$

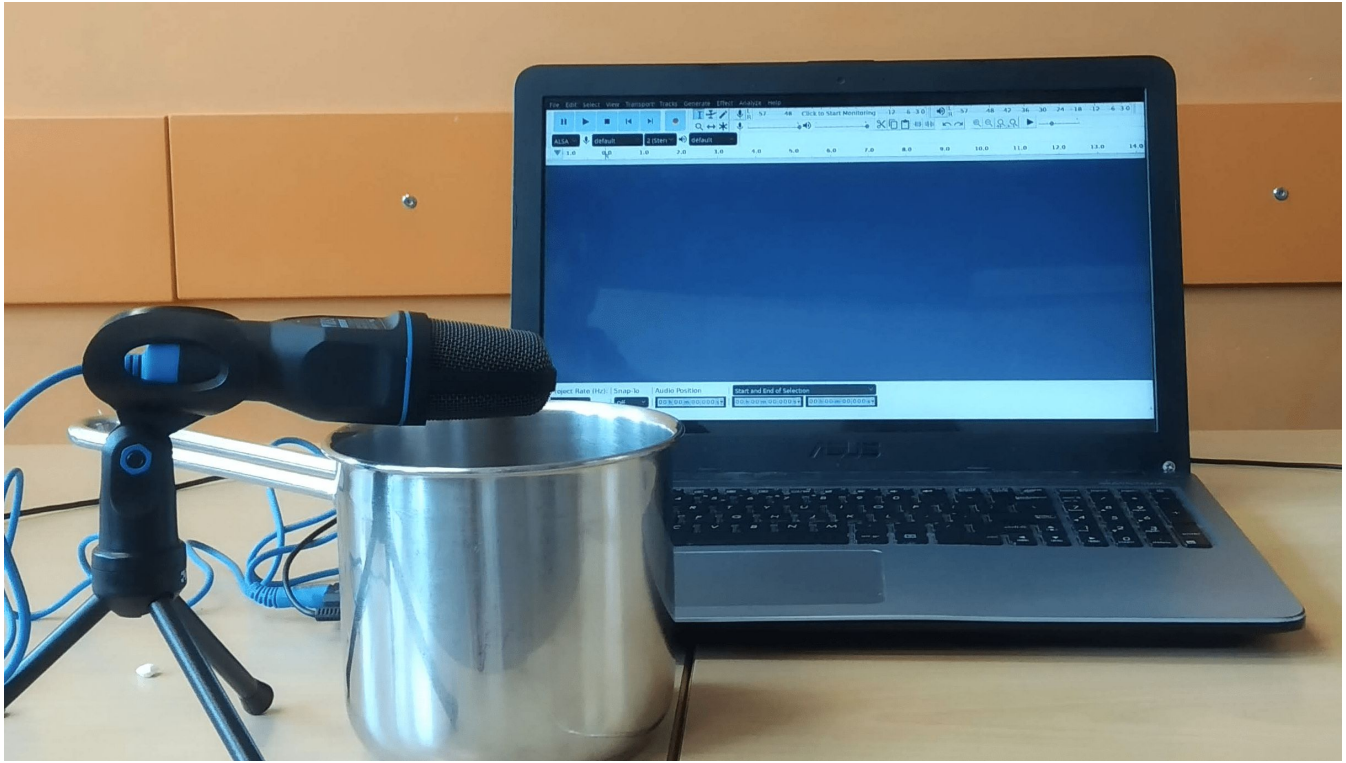
Complications

-Analytic solutions impossible for general geometry

-Can’t use side and bottom oscillations (Kirchhoff-Love plates)

-Microphone can only pick up first few peaks

Experimental setup

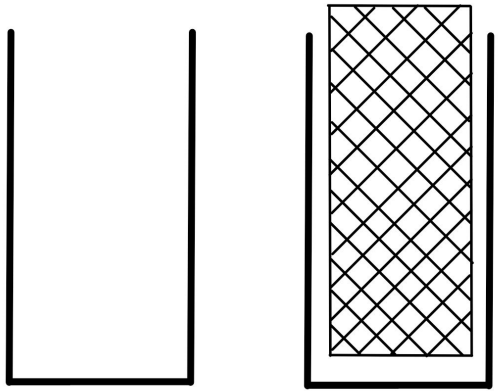


Measurements

-Dropping small stones into the container

-Two ways of measuring: volume free and volume blocked

-Microphone: Trust All-round USB Mico microphone [<7000 Hz]



-Spectrum analyzed with Audacity [ver. 2.2.1]
{Algorithm: Spectrum, Function: Hamming window, Size: 8192}

Measurements

-Containers used:

1) Cylindrical pot (H = 11.0cm, R = 6.0cm)
2) Cylindrical tea box (H = 8.0cm, R=3.0cm)
3) Rectangular cake pan (L_x= 16cm, L_y= 8.0cm, L_z=7.0cm)
4) Two blind test containers

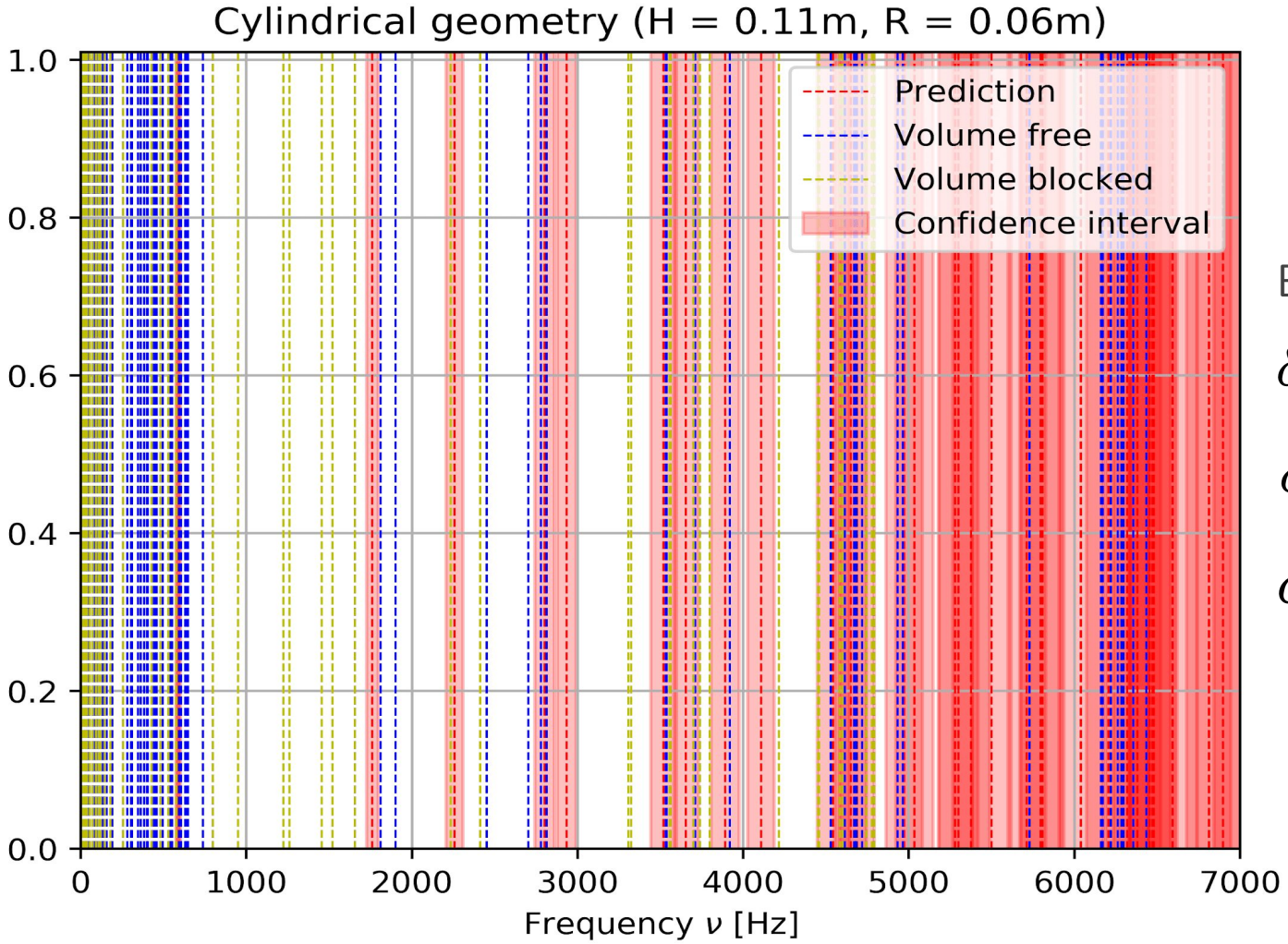

(1)


(2)


(3)

(4)

(4)

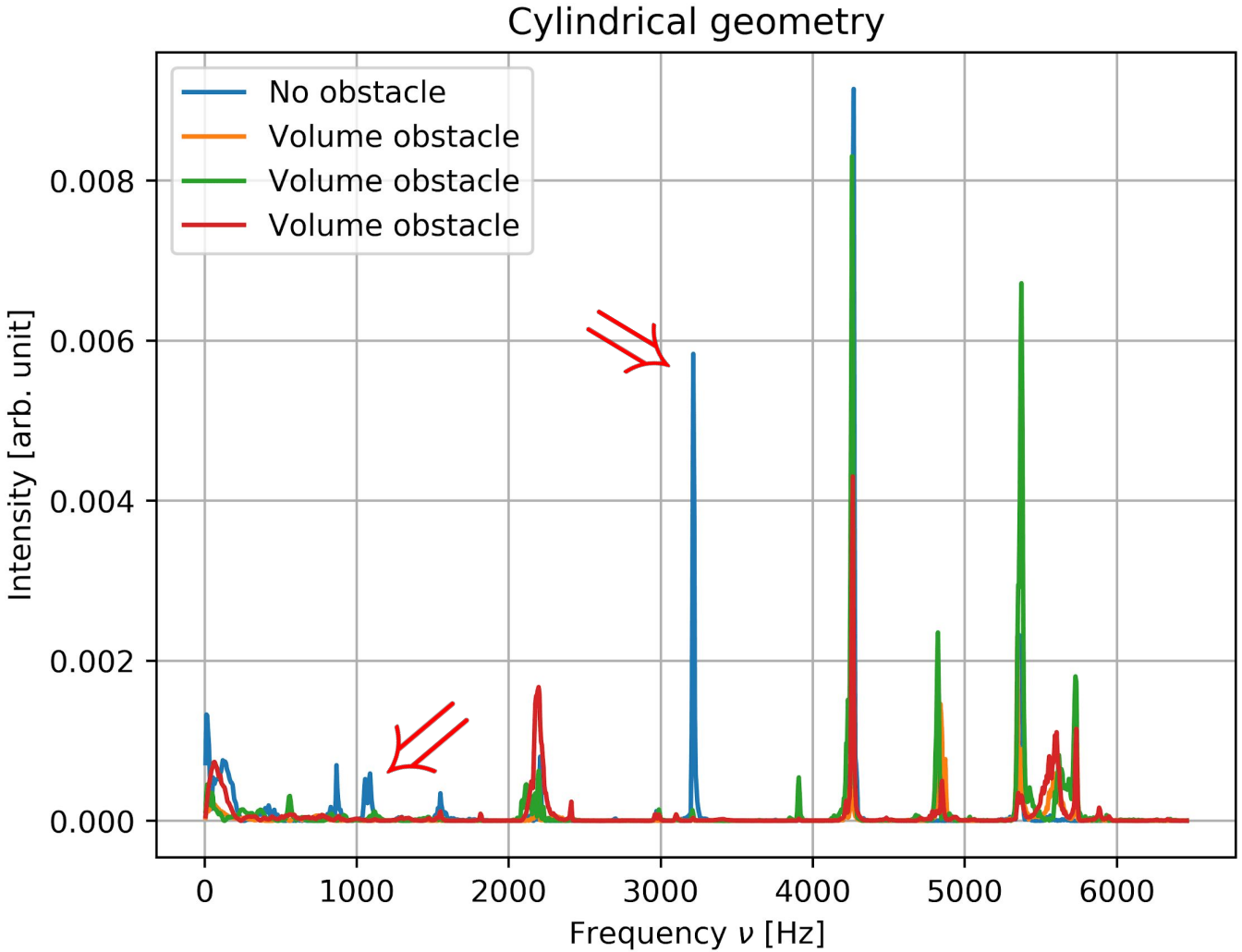


Error estimates:

$\delta H = 0.1\,\mathrm{cm}$

$\delta R = 0.1\,\mathrm{cm}$

$\delta c = 2\,\mathrm{m/s}$



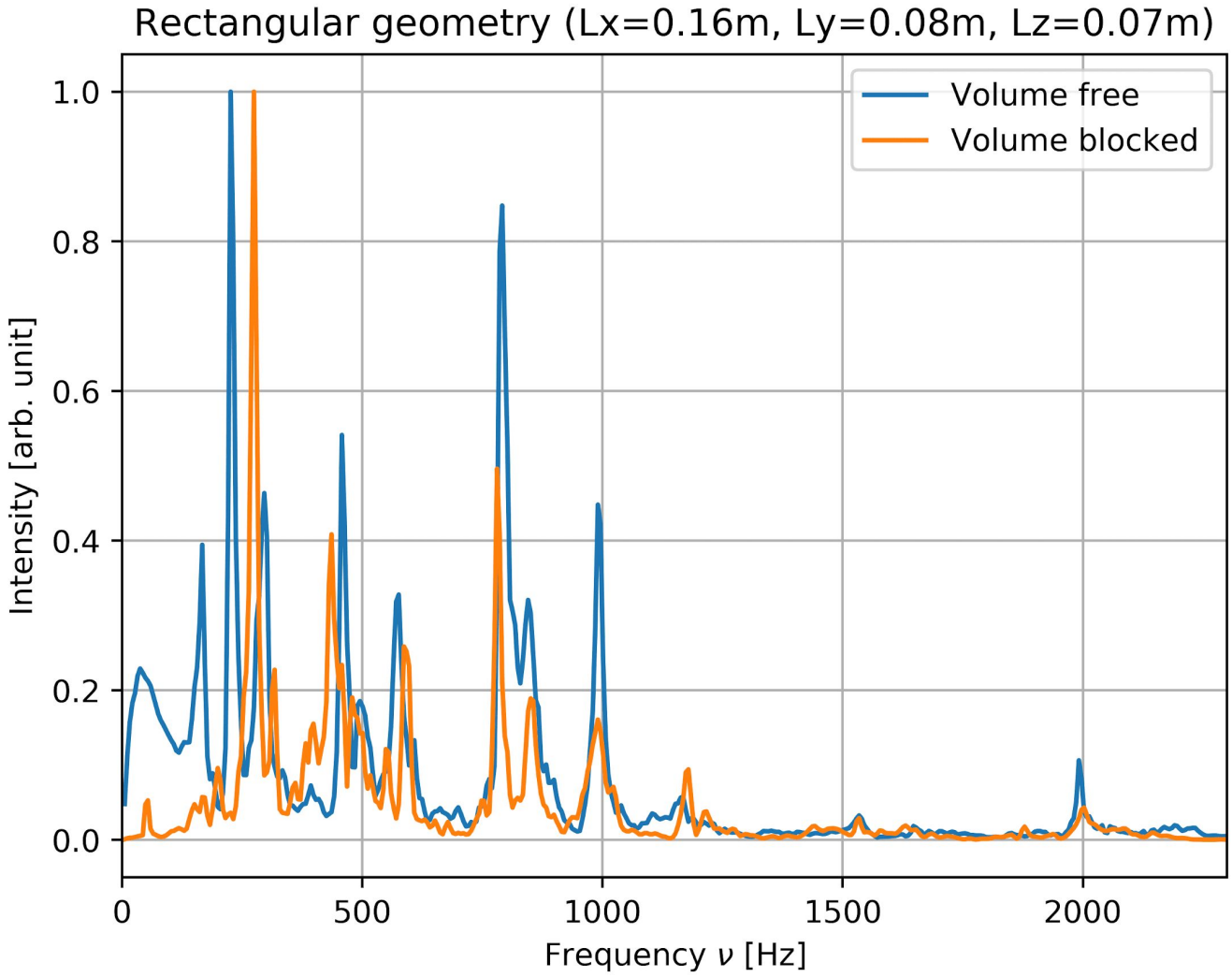
Peaks at:

$$\nu_1 = 1055\,\mathrm{Hz}$$

$$\nu_2 = 3216\,\mathrm{Hz}$$

$$\Downarrow$$

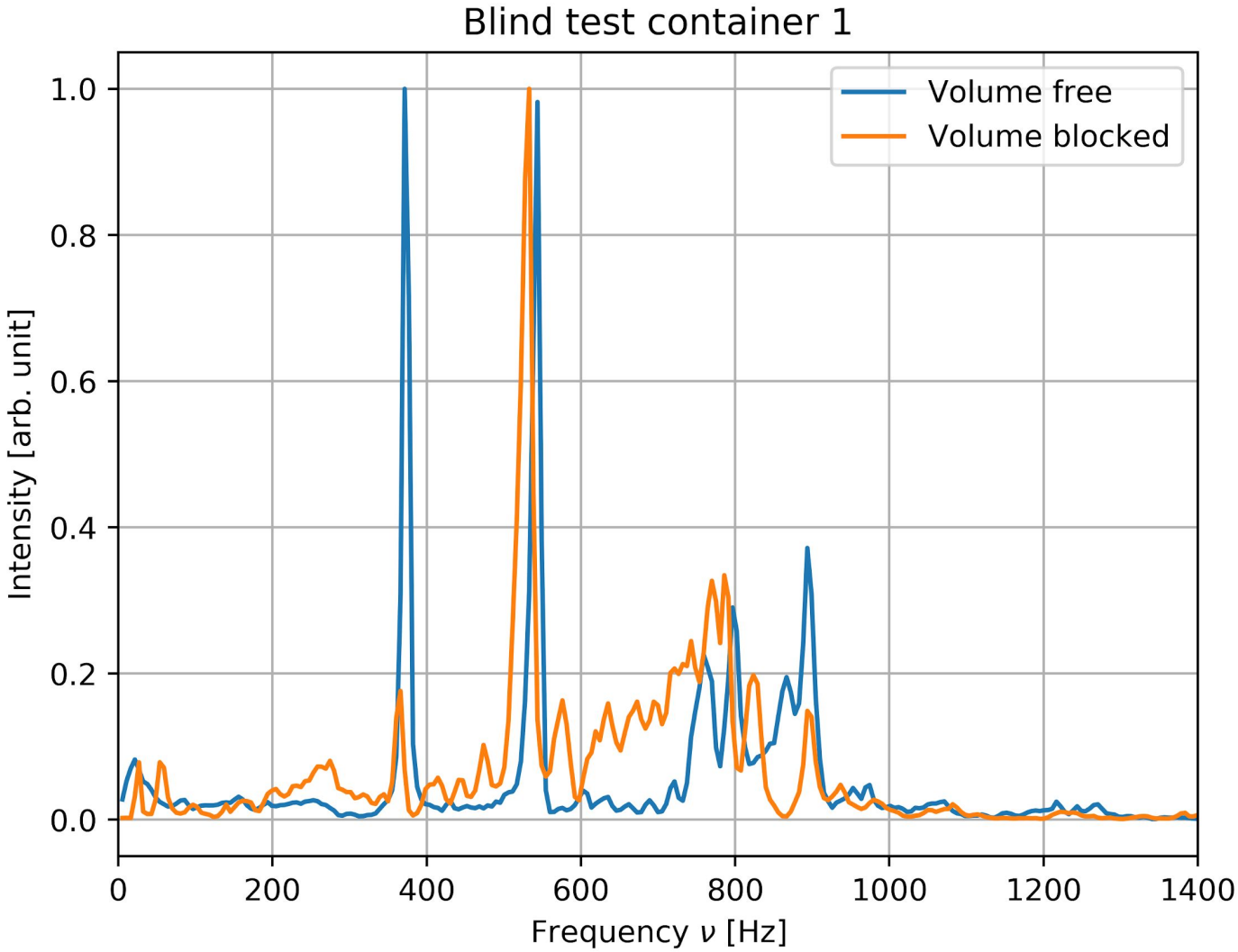
$$H = 8.04\,\mathrm{cm}$$



-Signal was noisier

-Blocking wasn't successful

-Opening was too large



First guess at peak:

$$\nu = 371\,\mathrm{Hz}$$

gave

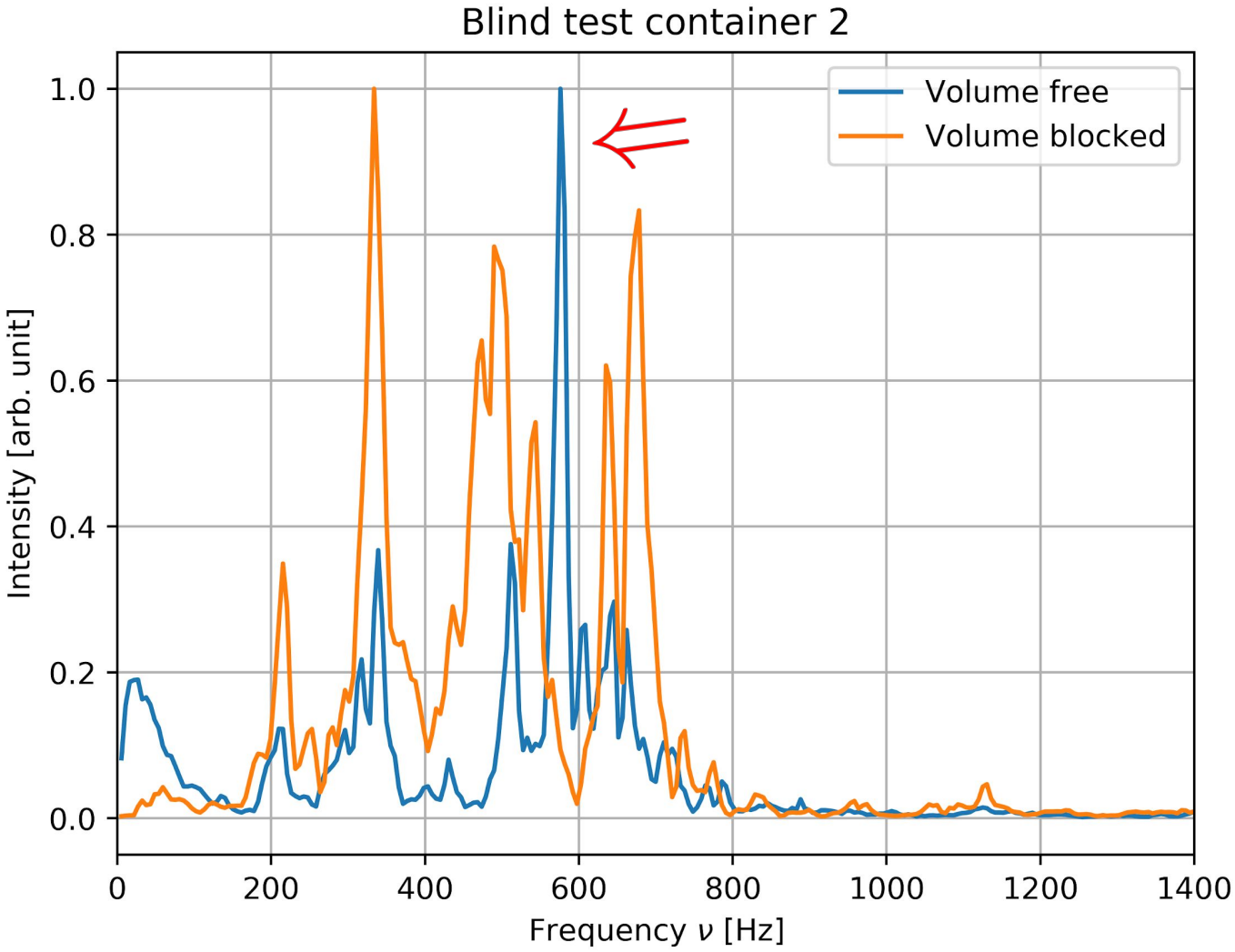
$$L \approx 23.1\,\mathrm{cm}$$

At second glance:

$$\nu = 866\,\mathrm{Hz}$$

$$\Downarrow$$

$$L \approx 9.9\,\mathrm{cm}$$



Peak:

$$\nu = 576\,\mathrm{Hz}$$

$$\Downarrow$$

$$L \approx 14.9\,\mathrm{cm}$$

Blind test 1:

Actual size:

$$L_x = 6.2\,\mathrm{cm}$$
$$L_y = 8.1\,\mathrm{cm}$$
$$L_z = 8.3\,\mathrm{cm}$$

From spectra:

$$L_z = 9.9\,\mathrm{cm}$$

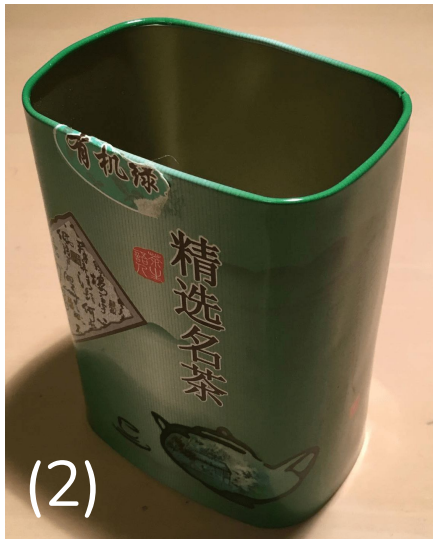
Blind test 2:

Actual size:

$$L_x = 8.7\,\mathrm{cm}$$
$$L_y = 6.0\,\mathrm{cm}$$
$$L_z = 11.6\,\mathrm{cm}$$

From spectra:

$$L_z = 14.9\,\mathrm{cm}$$



Conclusions and Discussion

-Mathematical solution has been known
-Physically, there are important complications
-Usual outcome: **can’t find geometry, can approximate biggest side**

-Better formula for end corrections
-Larger span of frequencies
-Better way to compare peaks

Bibliography

[1] Scott Wolpert Carolyn Gordon David Webb. "Isospectral plane domains and surfaces via Riemannian orbifolds". In: *Invent Math* 110 (1992). URL: https://doi.org/10.1007/BF01231320.

[2] Chad D. Balch Charles R. Steele. "Introduction to the Theory of Plates". In: (2013). URL: http://www.docdatabase.net/details-introduction-to-the-theory-of-plates-stanford-university-1122464.html.

[3] Mark Kac. "Can One Hear the Shape of a Drum?" In: *The American Mathematical Monthly* 73.4 (1966), pp. 1–23. ISSN: 00029890, 19300972. URL: http://www.jstor.org/stable/2313748.

[4] *Kirchhoff–Love Plate Theory*. [Last accessed: 11.03.2020]. URL: https://en.wikipedia.org/wiki/Kirchhoff%E2%80%93Love_plate_theory.

[5] Johan Liljencrants. *End Correction at a Flue Pipe Mouth*. [Last accessed: 11.03.2020]. 2006. URL: http://www.fonema.se/mouthcorr/mouthcorr.htm.

[6] Jan Mohring. "Helmholtz Resonators with Large Aperture". In: *Acta Acustica united with Acustica* 85 (1999), pp. 751–763.