Plane Partitions in Batch Track-Track Associations

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Synopsis

The most difficult multiple target tracking problem includes multiple sensors with different viewing angles, measurement geometries, fields of view, accuracies, resolutions and scan rates. Such variations in sensor output characteristics as well as channel delays, countermeasures, inherent target features and maneuvers have solidified the consensus that an effective fusion system must handle several levels of "tracklets" from distributed sources in order to produce the desired long tracks as described in Waltz and Llinas (1990). In view of the increased attention given to hypersonics as well as the increased need for low-level signal processing, the computational complexity of track association is a vital factor in determining an autonomous vehicles' ability to complete its objectives quickly. We are given a set of tracklets where the particular methods used to make the detections are taken for granted. Following joint probability density association filters, we assume short tracklets are completed (i.e, detections are correctly correlated with state estimates) and take a computational geometric approach to associating tracklets. If N is the number of short term tracklets, this method fuses them in $O(N^2)$. Using covariance as a distance, this report suggests the applicability of a class of sweep-line algorithms developed in computational geometry in data fusion.

Keywords: Multiple target tracking, data fusion, track association, radar

1 Introduction

Many classical problems in control theory such as regulation and stabilization have been solved to adequate satisfaction using well established techniques from optimization theory. Nowadays, due to the proliferation of autonomous systems, rather than optimizing an objective function, predicting relevant constraints for control systems appears to be the million dollar question. For example, the problem of path planning involves not just reference following routines but also intelligent processing of data from sensor networks to design the constraints too.

The multiple object tracking problem arises in situations whenever sensor data is available from one or more wide angle of view sensors like radar. The reports from these sensors are obtained at fixed intervals and contain noisy reflections from all the objects in their field of view. The basic tracking problem is to estimate the position and velocity of all the objects detected in the sectors surveilled using the reports gathered over time. For single objects the Kalman filter provides a suitable solution. Multiple object tracking poses a problem because correlating measurements with targets is compounded as the number of detections increase. An important algorithm that enables low-level track formation is the global nearest neighbor. This algorithm makes complete decisions for each measurement regarding their source and validity.

In joint probability density association filtering, all measurements within a vicinity are assigned to a target but the weights on these measurements are selected by proximity. We shall begin with a brief explanation of this method for the sake of completeness but the objective of this note is to address a difficulty in the subsequent track-to-track association steps.

2 Tracking Filters and Low-Level Data Association

The kinematics of a single object can be described by

$$x_{k+1} = \Phi_k x_k + G_k u_k$$

where x_k is an $n \times 1$ state vector at the k^{th} time sample and Φ_k is an apprpriate $n \times n$ transition matrix derived from Newton's equations of motion. u_k is an $m \times 1$ input vector to account for modelling errors and maneuvers which is assumed to follow a Gaussian distribution with zero mean and covariance Q_k. We take T to be the scan interval of the sensor so that $t_k = kT$. Each report y_k contains $m \times 1$ measurements that are related to the state vector x_k by

$$y_k = H_k x_k + v_k$$

where $v_k \sim \mathcal{N}(0, R_k)$. The extraneous sensor reports due to noise, false alarms and clutter satisfy

$$y_k = H\hat{x}_{k|k-1} + w_k$$

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where w_k is white and uniform over a volume V of the measurement space centered around the prediction $\hat{y}_{k|k-1} = H_k \hat{x}_{k|k-1}$. If the unnormalized extraneous report density is λ then the number of such reports in V follows a Poisson distribution with mean λV. The prediction given by the state transition matrix is

$$\begin{aligned} \hat{x}_{k+1|k} &= \Phi_k \hat{x}_{k|k} \\ P_{k+1|k} &= \Phi_k P_{k|k} \Phi_k^T + G_k Q_k G_k^T . \end{aligned}$$

As shown in Gelb (1974) given R_k and H_k, the tracking filter updates $\hat{x}_{k|k}$ and covariance $P_{k|k}$ using y_k and $\hat{x}_{k|k-1}$ as

$$\begin{aligned} \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k \left(y_k - H_k \hat{x}_{k|k-1} \right) \\ S_k &= H_k P_{k|k-1} H_k^T + R_k \\ K_k &= P_{k|k-1} H_k^T \left(H_k P_{k|k-1} H_k^T + R_k \right)^{-1} \\ P_{k|k} &= \left(I - K_k H_k P_{k|k-1} \right) . \end{aligned}$$

2.1 A Suboptimal Bayes Algorithm: JPDAF

Using the standard measurement mode (unbiased, Gaussian), the mean of the state at time k is

$$X_s = \sum_{i=0}^{m_k} X_{s_i} \beta_i$$

where β_i is the probability that a measurement originates from the track. The estimate assuming i is correct is

$$\begin{aligned} X_{s_i} &= X_p + k v_i, \quad i = 1, \ldots, m_k \\ v_i &= Z_i - z_p \end{aligned}$$

and for $i = 0$ (no update) $X_{S_0} = X_p$. Low-level data association or measurement-to-track association is performed next.

2.2 Low Level Data Association

An assignment matrix is constructed which consists of elements proportional to the distance from each observation to each track. Munkres (1957) describes an optimal assignment algorithm to correlate observations to tracks. To produce tracks, the tracking filter continues to perform the update and predict steps using optimal assignment to select measurements. In a global nearest neighbor algorithm given hit miss probabilities, tracks are considered independently. The statistical distance of each measurement z_m from track prediction z_p is given by $D(V) = [z_m - z_p]' S^{-1} = V' S^{-1} V$ where S is the residual covariance associated with that object. The report which minimizes the value of $D(V)$ is selected. This approach does not prevent two tracks from using the same measurement for the update, the main advantage being immediate decision on a single hypothesis. Disadvantages are that there is no ability to correct a bad decision and assuming the probability of detection is one, there is no mechanism to consider false alarms.

2.2.1 Track Initiation

This step involves using sensor reports that do not correlate with existing tracks to assign a new ID to a tentative track. The newborn track is confirmed shortly thereafter when there is reasonable degree of certainty that it is a real track.

2.2.2 Track Elongation

For each track, measurements at a time step is validated using a gating test where the measurement is considered valid if

$$\left(y_k - \hat{y}_{k|k-1} \right)^T S_k^{-1} \left(y_k - \hat{y}_{k|k-1} \right) \leq g^2$$

for a suitable g as described in Bar-Shalom and Fortman (1988).

2.2.3 Track Termination

Tracks are terminated to prevent them from interfering with real target tracks. Valid reasons to terminate a track are either the target is outside the coverage area or it is destroyed. When tracks are terminated prematurely, a broken set of tracklets appears for each valid measurement. The next steps suggest a way to fuse these tracklets using a technique borrowed from computational geometry.

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2.3 *Track-to-Track Association*

In some situations, a single object generates several tracks due to the limitations in either the sensor or the digital processors. It is necessary to stitch these tracklets together to obtain a single long track for each target. There may be a total of N such tracklets for r targets where $N \gg r$. This can be challenging in situations when for instance, several targets with unknown dynamics are moving at velocities that strain the update rate of a monostatic radar. This is because although the tracklets within the scan are correctly associated, the distance between related tracklets from a single target may not be close enough to make the correct association. Define a track data structure for a track T_i with the following fields: the tentative ID, a set of associated detections (valid measurements of either range or elevation grouped together using low-level optimal assignment), maximum timespan between detections L_i and the current estimates $\hat{x}_{\underline{k},i}$, $P_{\underline{k},i}$. For each tracklet, T_i where i ranges from 1 to N, a representative point c_i is generated using L_i and $\hat{x}_{\underline{k},i}$.

For $C = \{c_1, \ldots, c_n\}$ and $W = \{P_{\underline{k},1}, \ldots, P_{\underline{k},n}\}$, partition V into N pieces where each partition V_i is defined by

$$V_i = \left\{x \in C \mid (x - c_i)^2 + P_{\underline{k},i} \leq (x - c_j)^2 + P_{\underline{k},j} \quad \forall j \neq i\right\}. \tag{1}$$

Imai, Iri and Murota (1985) extended the partition defined by (1) to one in Laguerre geometry for N circles in the plane where the distance between a circle and a point is defined by the length of the tangent line. The Voronoi diagram in the Laguerre geometry may be applied to finding the connected components of N circles and finding the contours of the union of N circles. deBerg (2000) shows that this partition may be obtained using sweep line algorithms. Next, we may compute the list of edges E_i for each partition V_i and construct the Delaunay triangulation D using the elements of C. The associations for a track T_i is given by $E_i \cap D$. The association of two tracklets is controlled by the length of the edge shared by their representative points. Placing weights W on the representatives using the covariance of the tracklet enables us to fuse tracklets which are from the same track since they will have similar covariances.

2.3.1 *Selecting tracklet representatives*

Given a set of measurements in a tracklet T_i, a motion model is used to best explain the dynamics of the target within that set. For example, if the tracklet is linear with each measurement evenly spaced, a constant velocity model can be chosen. For such models, a simple representative point c is the midpoint of the best fit line. For more complex motion models, the mean of the first and last detection in the tracklet could be taken as a representative point. Another way to choose the representative is to take the centroid of the polygon whose vertices are the given by the measurements. Alternatively, the middle most measurement in the tracklet can be taken as a representative.

2.3.2 *Splitting fused tracklets*

Let us restrict ourselves to tracklets containing only range measurements. The outcome of $E \cap D$ is a $2 \times p$ matrix, say U where each column contains the association between the two range representatives that are believed to have originated from the same target. To split U into long fused tracks, we may simply reshape U into a single column matrix with each column of U stacked on top of the other beginning with the first column. Then taking the derivative of that column matrix and inserting a zero at the top of the results in a $2p \times 1$ column matrix. Reshaping the $2p \times 1$ column matrix back into a $2 \times p$ matrix with mostly zeros. If we find all the locations where the first row is non-zero, we can find long tracks because the presence of a non-zero value indicates a change in track identity. The number of locations found thus is the estimated number of targets $\hat{r}$. Create $\hat{r}$ new IDs which will serve as the final IDs for the long tracks. We can then break up the $2 \times p$ matrix into $\hat{r}$ smaller matrices by using those locations where there is a non-zero value to indicate the start of the new batch. Each smaller matrix contains all the tracklets which are associated with the same target.

3 Conclusions and Further Work

Modern sensor fusion platforms deliver long tracks through several intelligent solutions. The most common architecture involves three components: a data processing module, a correlator and an identity manager. Fusion centers must handle delayed measurements from ground based, shipborne and airborne platforms and also be able to exploit overlapping sensor networks. They must provide operators with long tracks because the length of a track provides crucial information such as range and motion patters for use in high level decision support. Detection of anomalies, conflicts and other situations of interest can be identified more effectively if machine learning is fed with better tracks. Rich and meaningful interaction is possible between automatic control theorists and the air surveillance domain as path planning becomes more reliant on single or multisensor trackers and correlators. Some of the industries most benefited from this research will be those involved in air sovereignty, surveillance, marine vessel traffic, and commercial space vehicle re-entry services.

A sweep line algorithm produces weighted plane partitions in $O(N \log N)$. The weights are given by $W_i = P_{\underline{k},i}$ and the sweep line is the vertical line that represents the $\underline{k}^{\text{th}}$ batch. Each tracklet representative c_i is treated as

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the focus of a parabola and the sweep line is the directrix. As the sweep line moves in time, it partitions the space V containing the track representative using rules described in deBerg (2000). Constructing the dual of these partitions using the same vertices produces Delaunay triangulations. Intersections of the partition edges E with the triangulations produces the associations in $O(N^2)$. In the presence of asynchronous sensor reports, it is unclear how the weights W should be modified. Also, the selection of tracklet representatives deserves more rigorous analysis.

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