PROBLEM N°17

Balancing Pebble

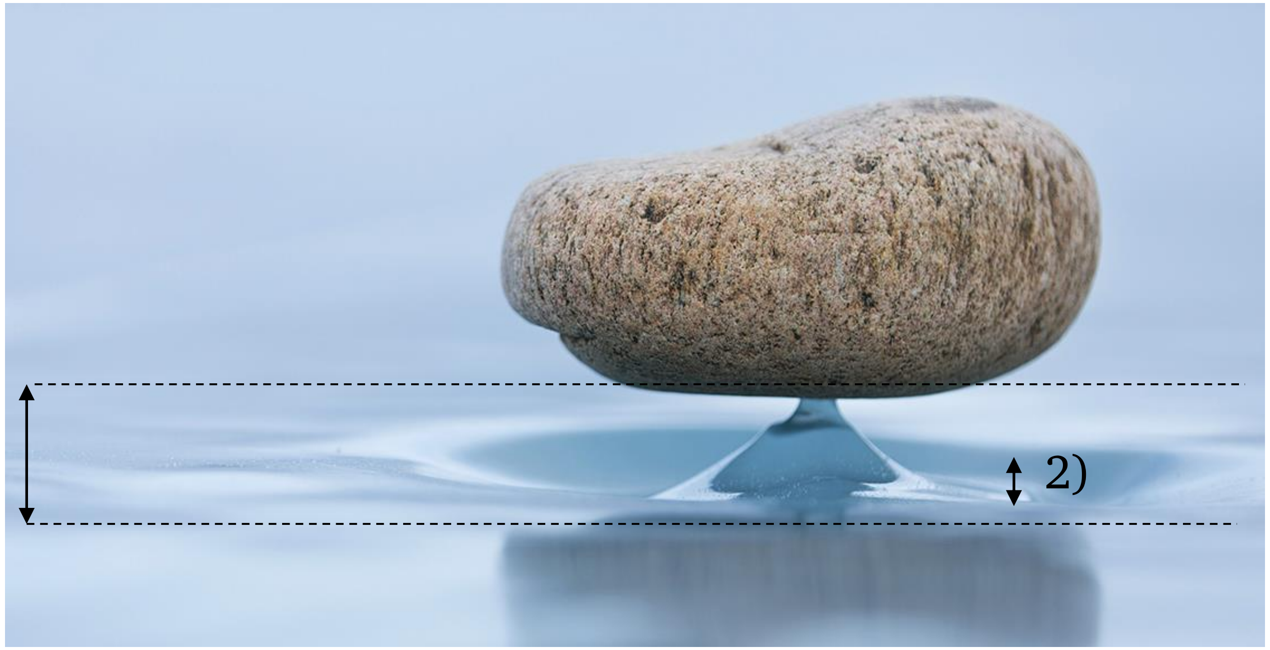
Ecole polytechnique

Balancing Pebble

Stones which are taken by wind on the ice of Baikal Lake can be found after some time staying on a thin «stand».

- **Reproduce** and **explain** this «stand» phenomenon
- Estimate **the curve** of the stand depending on the important parameters.

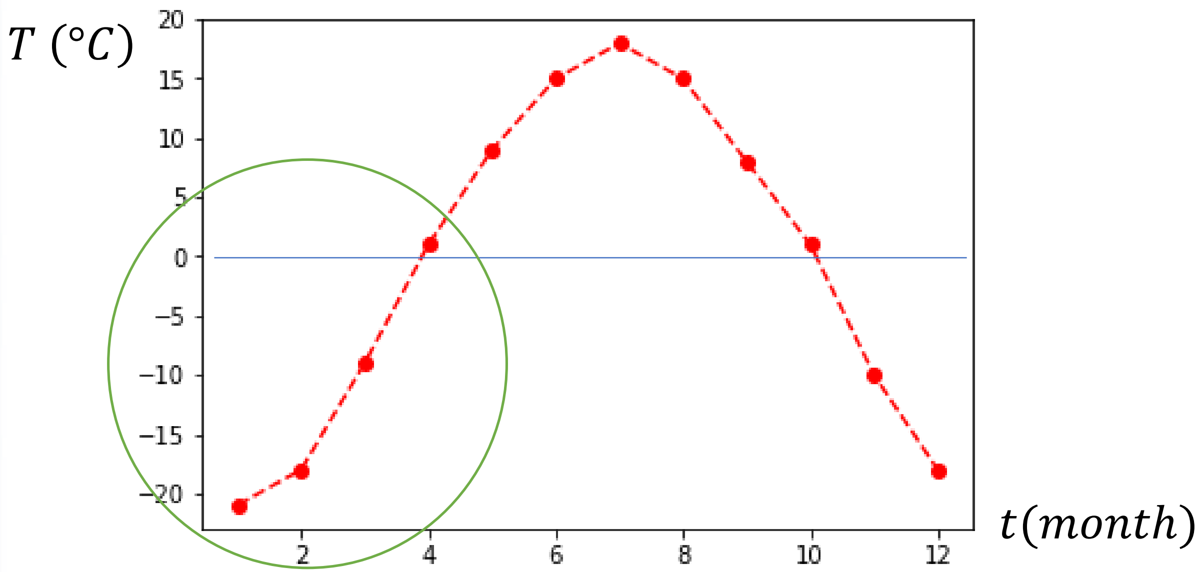
What characterizes a “Balancing Pebble” ?



1) Stone stands above the lake
2) Ice lower just around the stone
3) Asymmetry possible

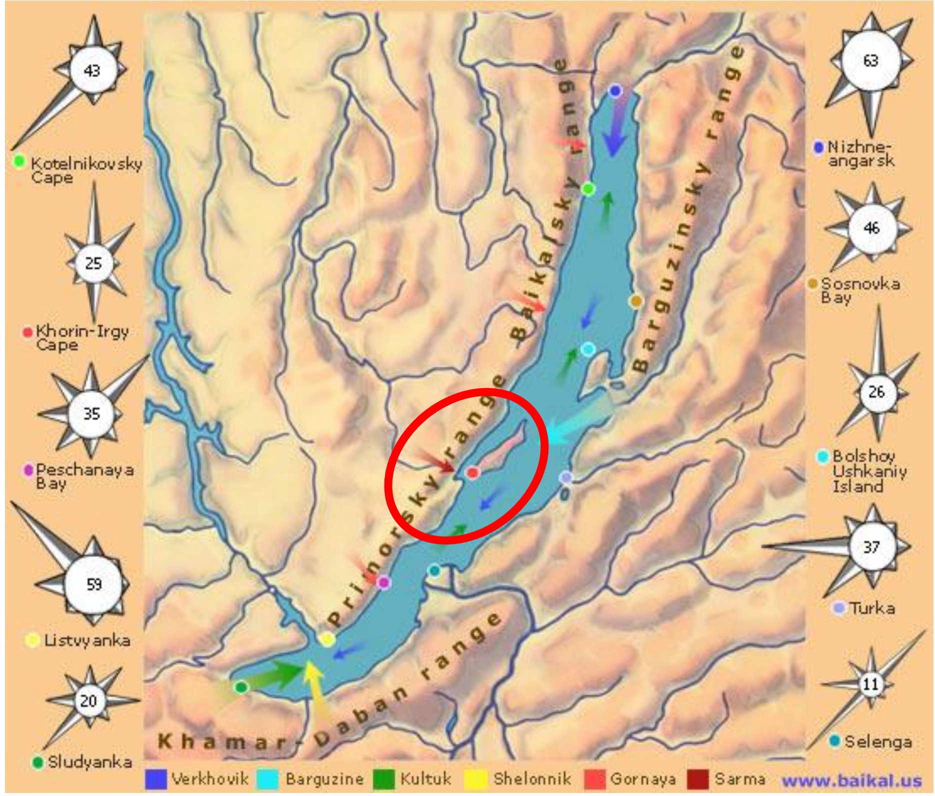
Context on Lake Baikal, Siberia

Temperature



January April December

Wind



Strong winds : 5 to 40 m/s

Sources of energy

Energetic budget of the lake ice

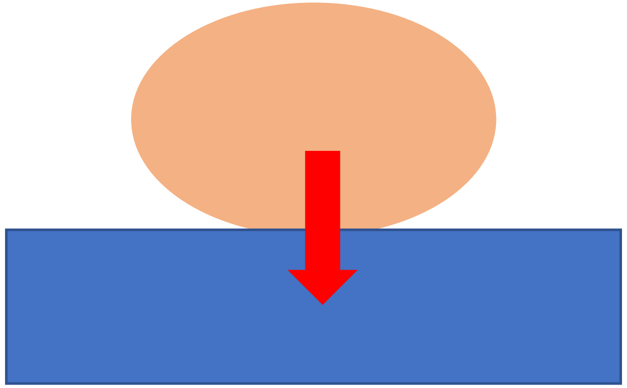
Conduction (stone)

Radiation

Wind

Formation of a stand ?

1st experiment : Conduction

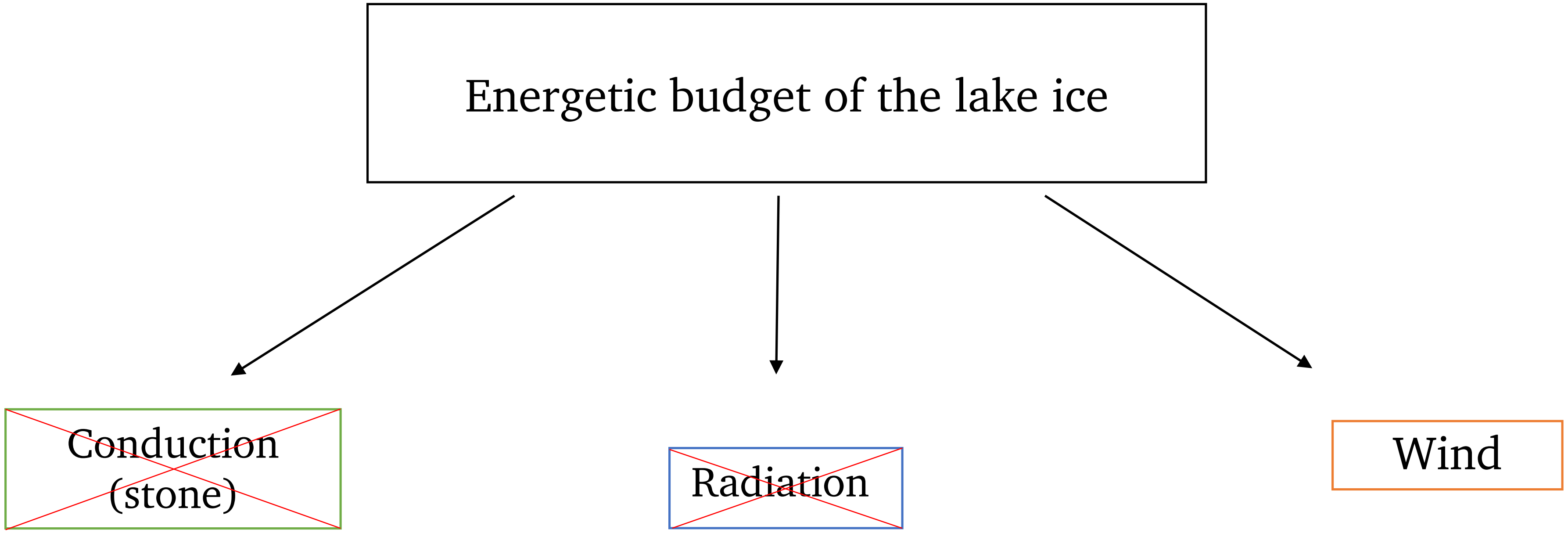


1) Stone stands above the lake X
2) Ice lower just around the stone X
3) Asymmetry possible X

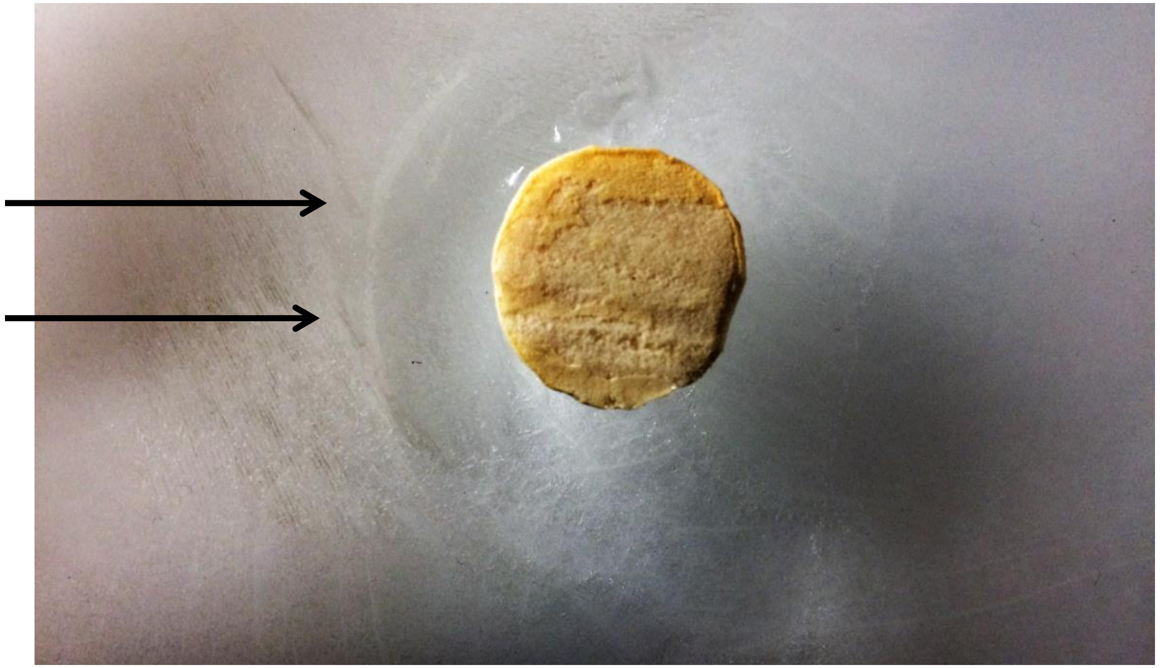
Radiation ?

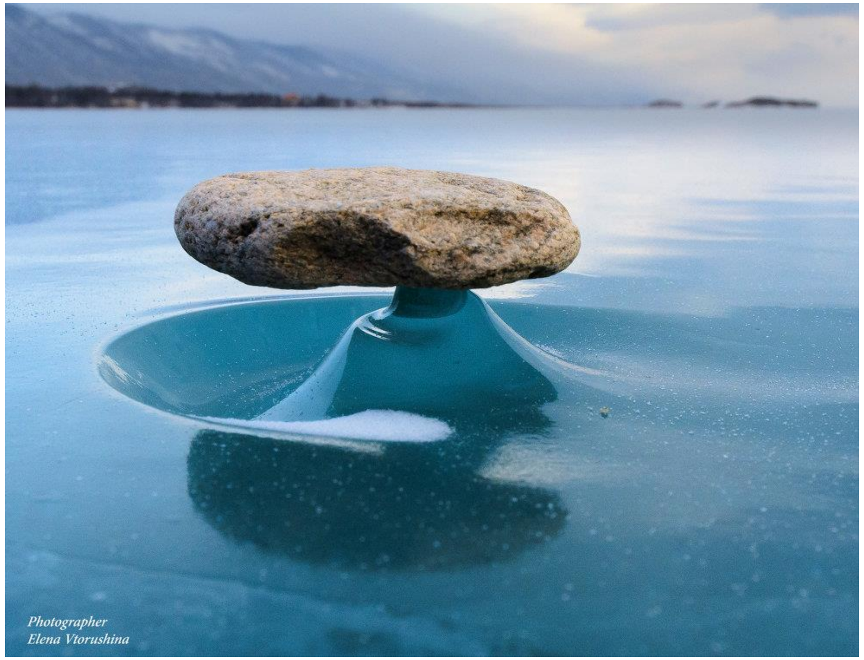
Not needed! Balancing pebbles found in a cave

Why does ice melt ?



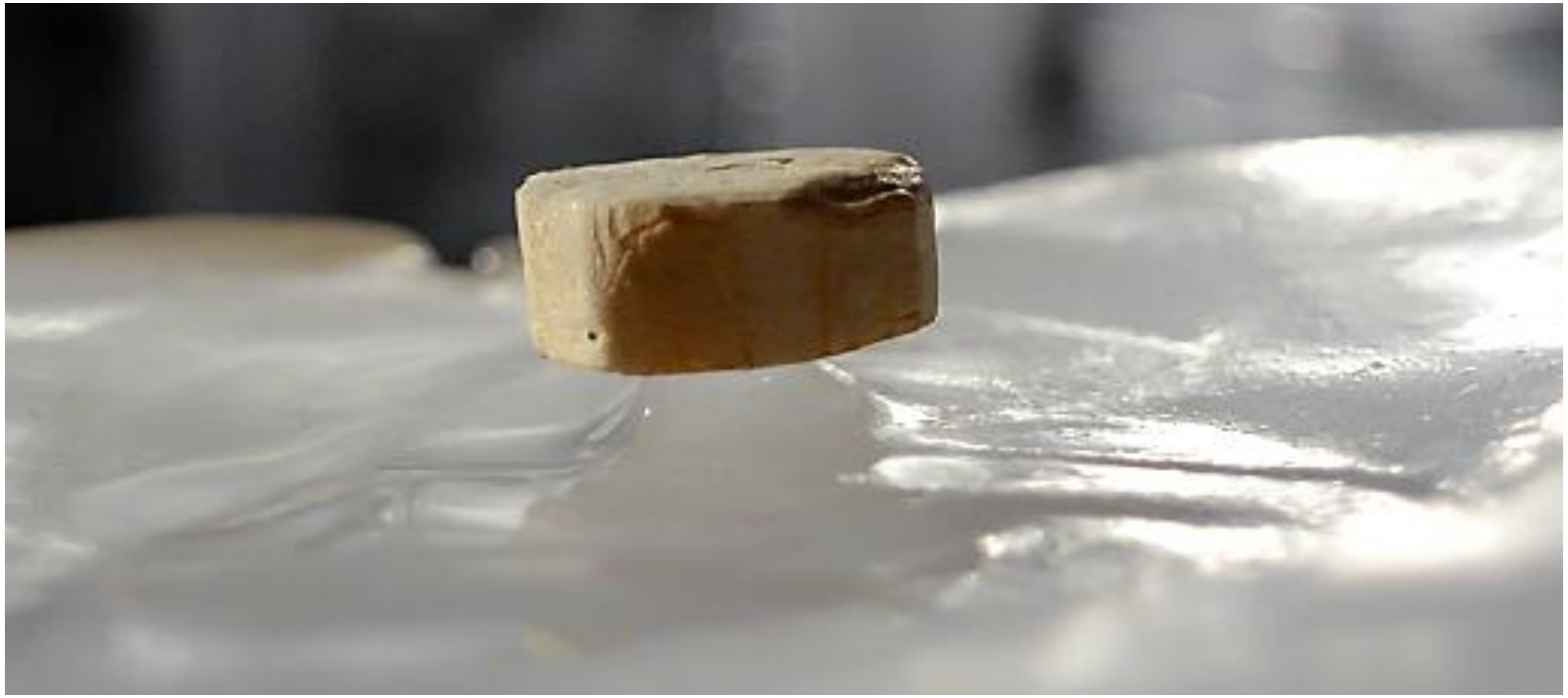
Experiment : blowing hot wind





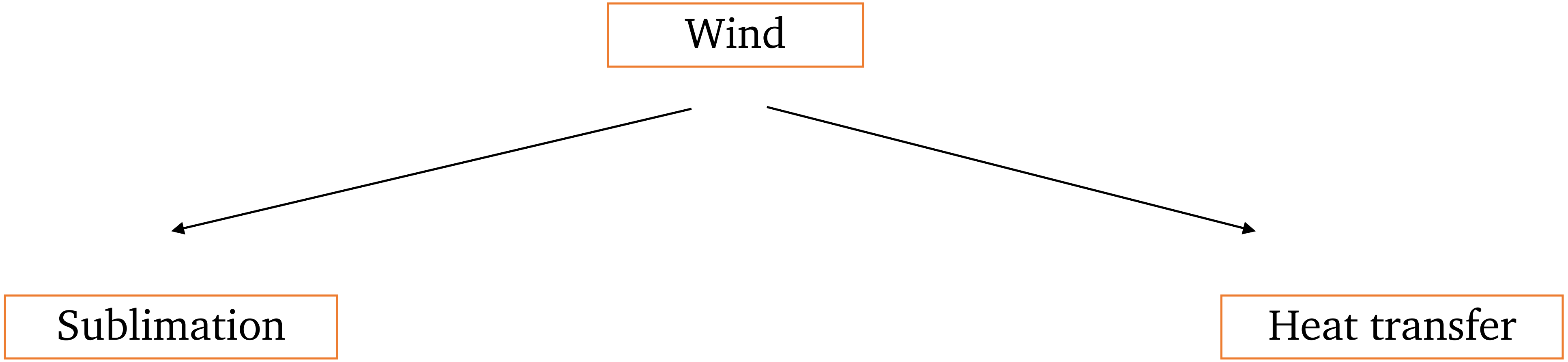
A stand
Asymmetric pattern

Experiment : with turning wind !



1) Stone stands above the lake ✓
2) Ice lower just around the stone ✓
3) Asymmetry possible ✓

Why does ice melt ?



Why does ice melt ?

Wind

Sublimation

Origin: Vapor pressure gradient

$$\Phi_{sublimation} = \boldsymbol{h_m(u)\ (n_{sat}(T) - n_{wind})}$$

n_{sat}, n_{wind} : *number of water molecules per* m^3 *of air*

h_m: mass transfer coefficient

$\sim$

Heat transfer

Origin: Temperature gradient

$$\Phi_{heat} = \boldsymbol{h(u)\ (T_{wind} - T_{ice})}$$

h: heat transfer coefficient

Conditions of temperature and humidity for Baikal lake:

Ablation rate of $\mathbf{2.10^{-5}}$ $\boldsymbol{kg/s}$ for sublimation $>>$ Ablation rate of $\mathbf{2.10^{-8}}$ $\boldsymbol{kg/s}$ for melting through heat transfer

How does ice melt ?

Modelling transfer phenomena

Newtons law of convective heating:

$$\phi_{heat} = \boldsymbol{h(u)(T_{wind} - T)}$$

Heat transfer equation:

$$\boldsymbol{\rho c \partial_t T = h(u)(T_{wind} - T)}$$

$$T = T_i + (T_{wind} - T_i)(1 - e^{-\frac{h(u)}{\rho c}t})$$

Convective mass transfer:

$$\phi_{sub} = \boldsymbol{h_m(u)(n_{wind} - n)}$$

Convective mass transfer equation:

$$\boldsymbol{\partial_t n = h_m(u)(n_{wind} - n)}$$

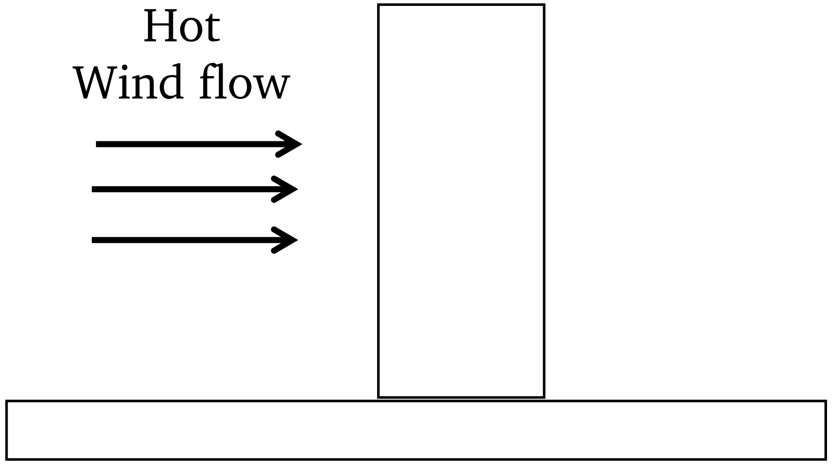
$$n = n_i + (n_{wind} - n_i)(1 - e^{-h_m(u)t})$$

Same exponential laws

How does ice melt ?

Modelling the heat transfer $h(u)$

$$T = T_i + (T_f - T_i)(1 - e^{-\frac{\boldsymbol{h(u)}}{\rho c}t})$$



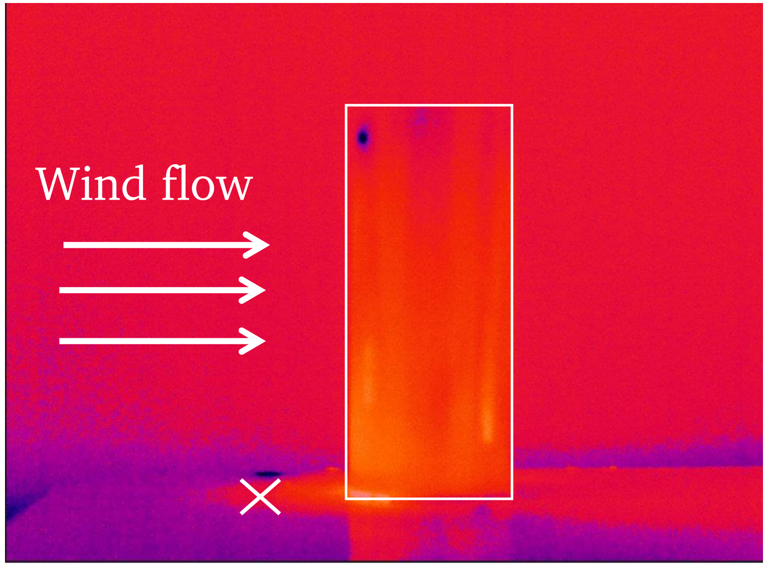
How does ice melt ?

Modelling the heat transfer $h(u)$

Newton's law gives :

$$T = T_i + (T_f - T_i)(1 - e^{-\frac{h(u)}{\rho c}t})$$

Thermal camera from 20°C to 35°C



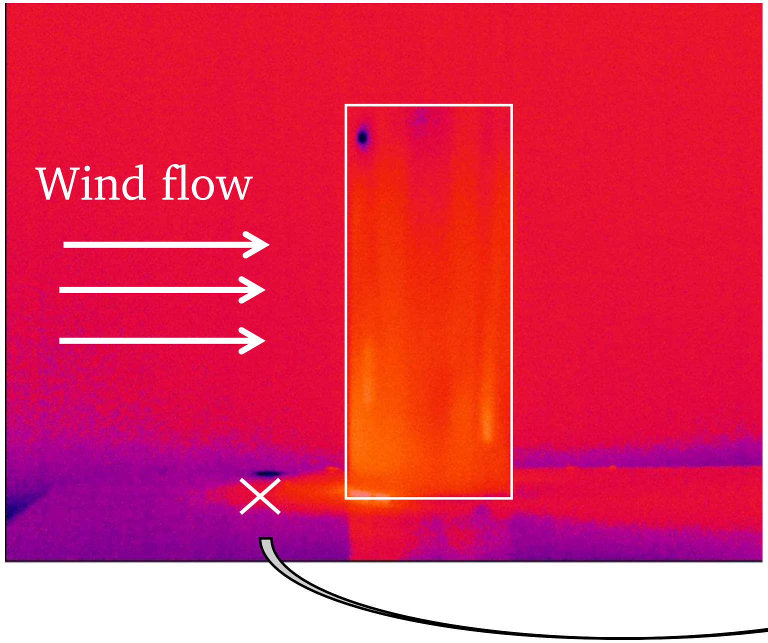
How does ice melt ?

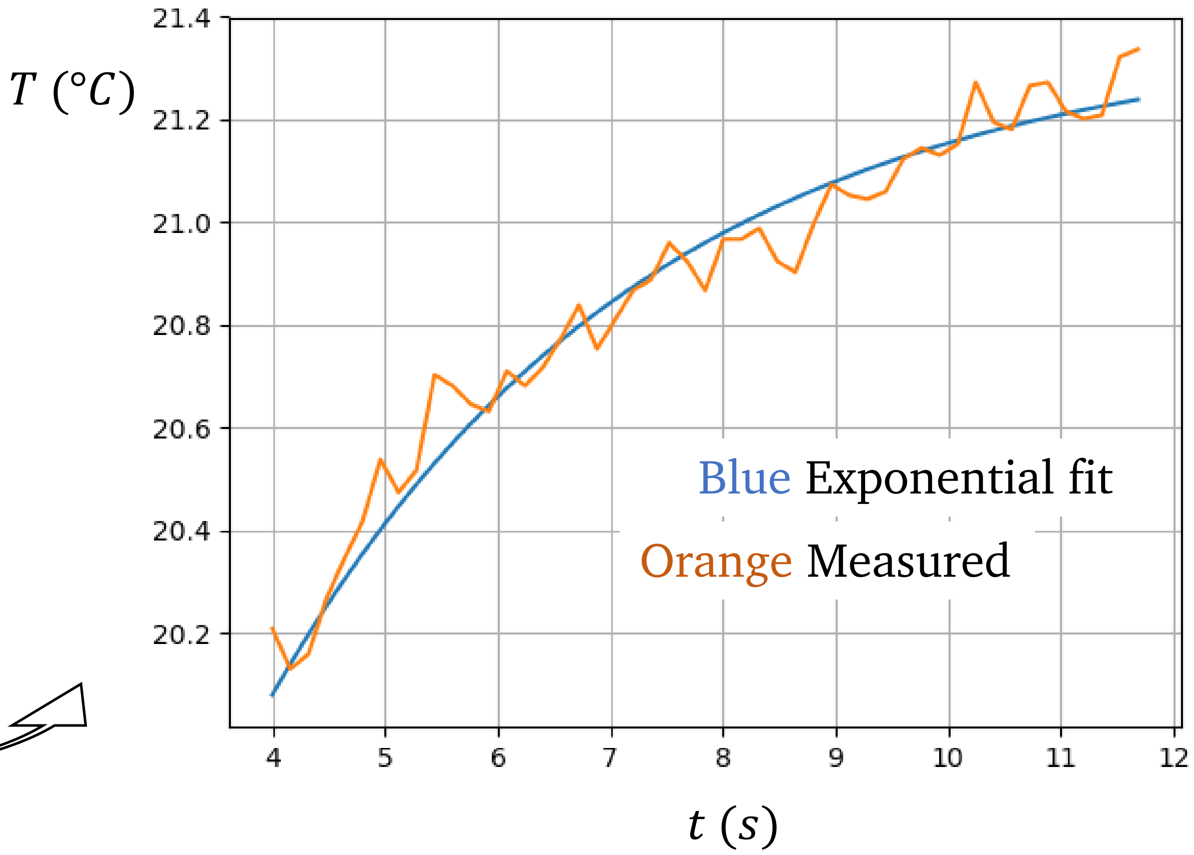
Modelling the heat transfer $h(u)$

Newton's law gives :

$$T = T_i + (T_f - T_i)\left(1 - e^{-\frac{h(u)}{\rho c}t}\right)$$

Thermal camera from 18°C to 25°C



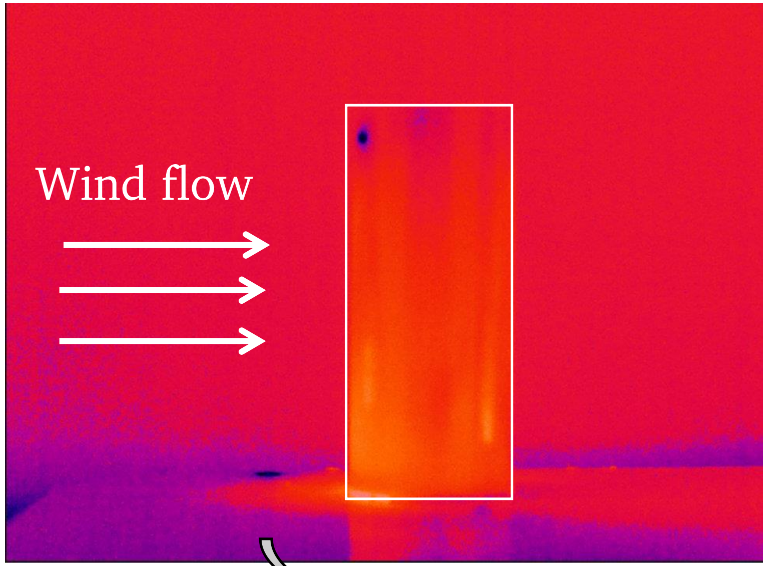


How does ice melt ?

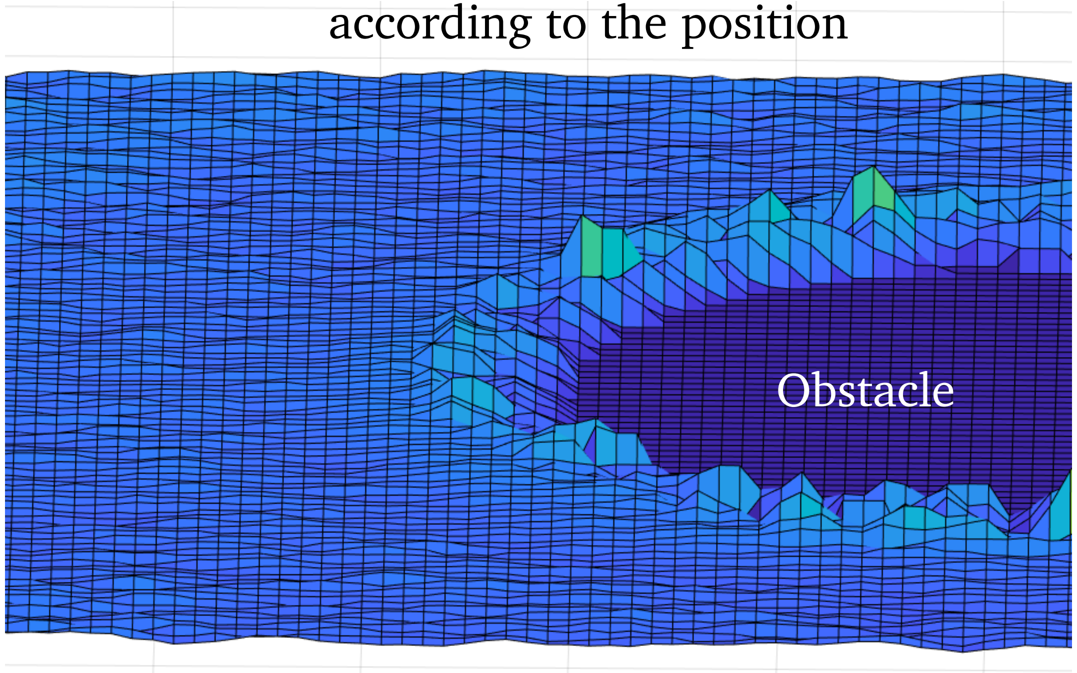
Modelling the heat transfer $h(u)$

$$T = T_i + (T_f - T_i)\left(1 - e^{-\frac{h(u)}{\rho c}t}\right)$$

Thermal camera from 20°C to 35°C



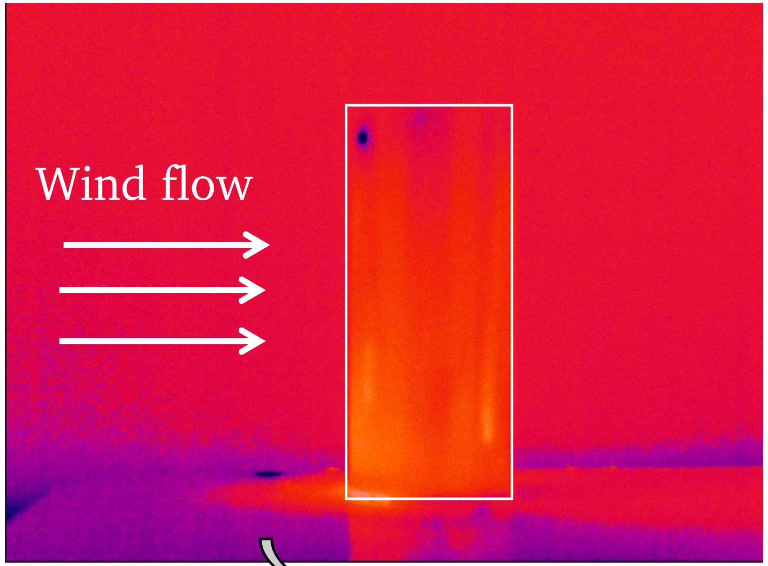
Experimental data for $h(u)$ in W m$^{-2}K^{-1}$ according to the position



How does ice melt ?

Modelling the heat transfer $h(u)$

Thermal camera from 20°C to 35°C



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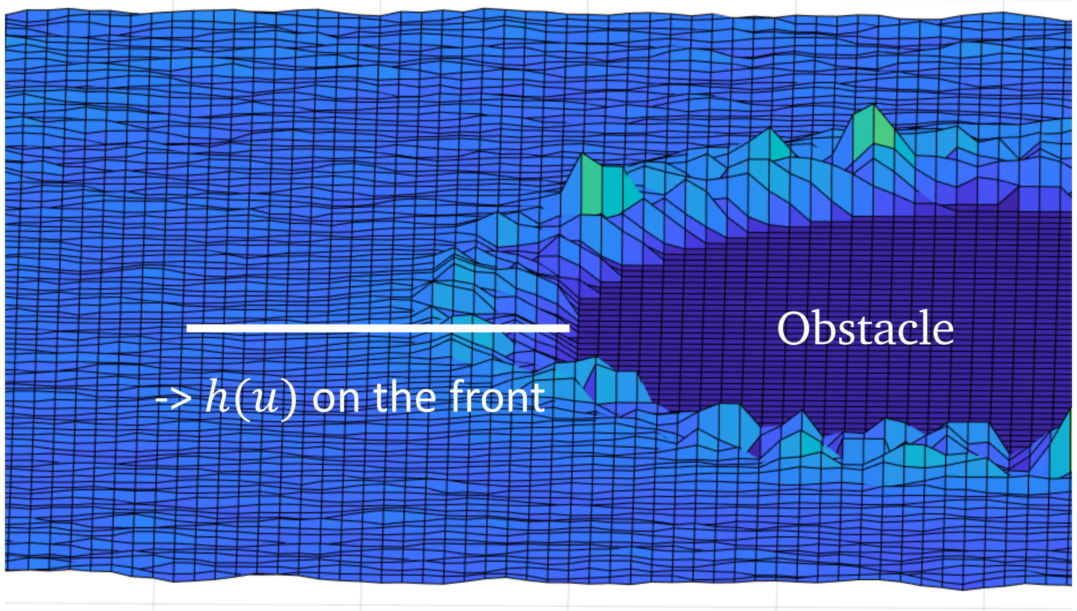
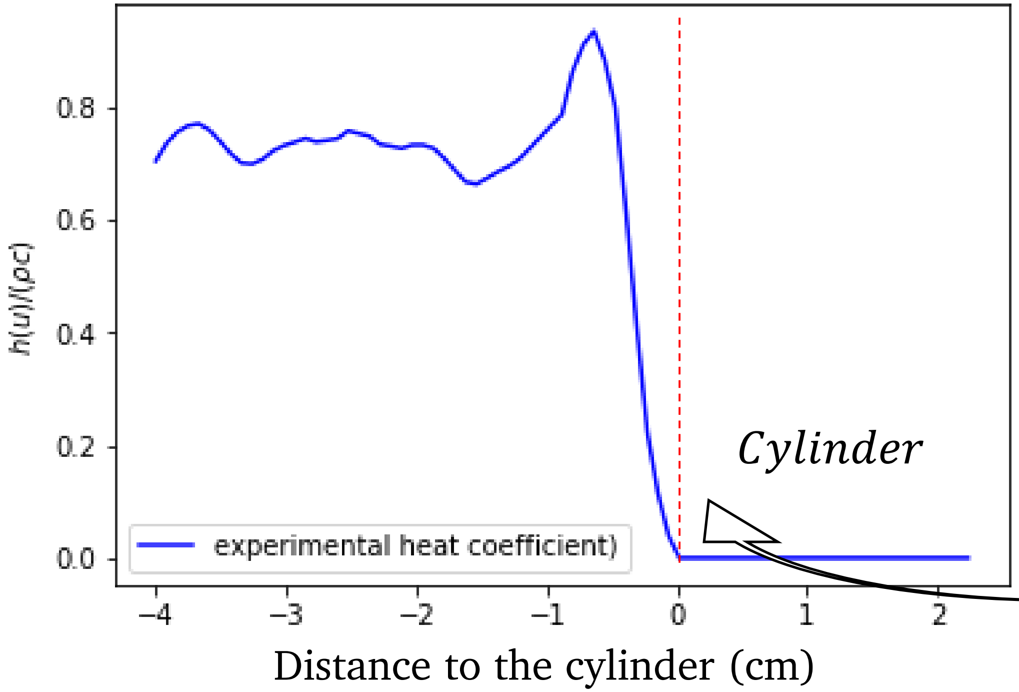


How does ice melt ?

Modelling the heat transfer $h(u)$

$h(u)$ on the front part



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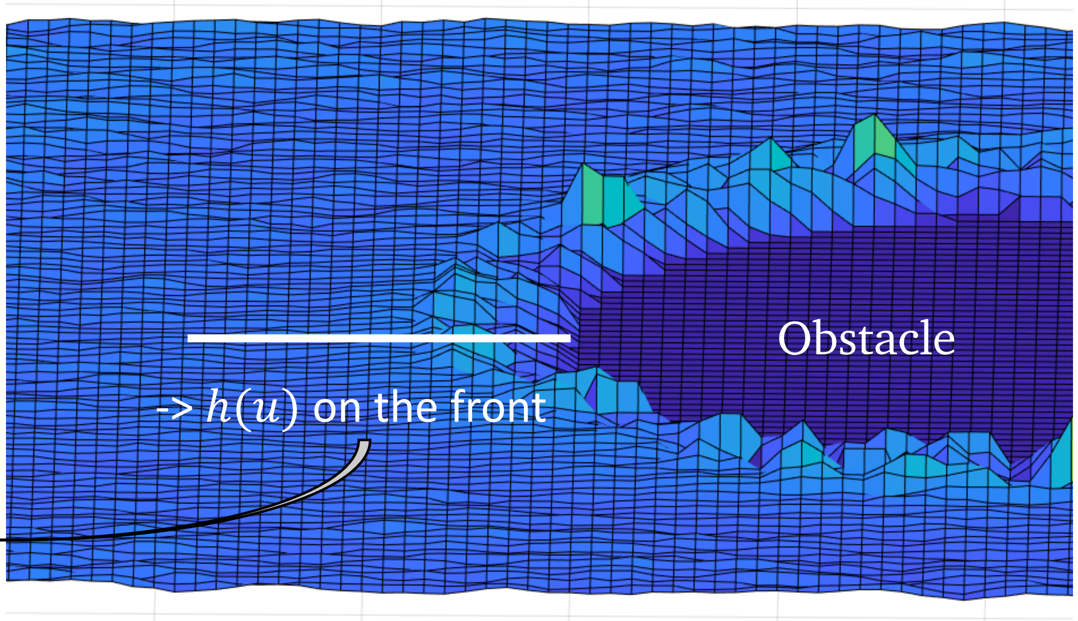
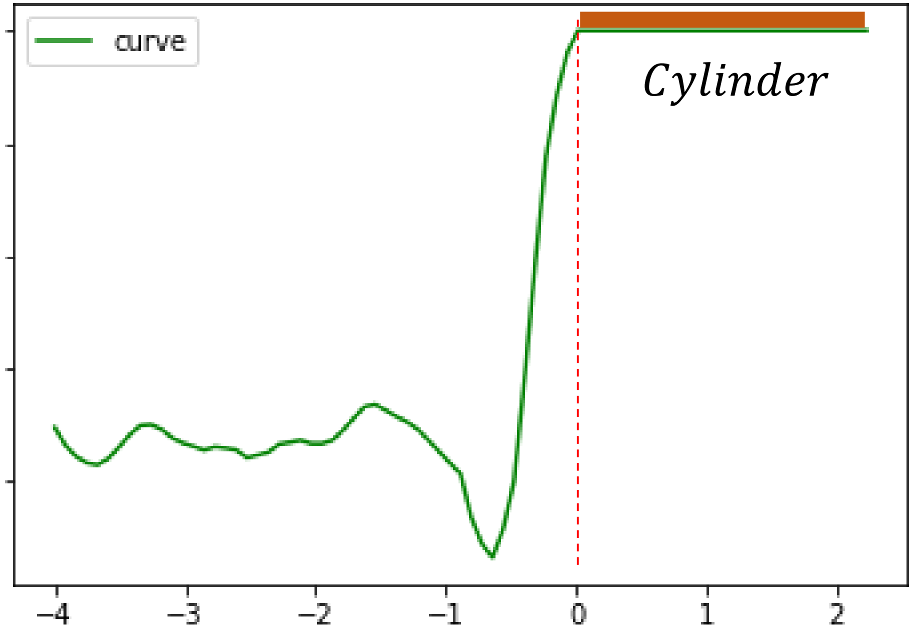


How does ice melt ?

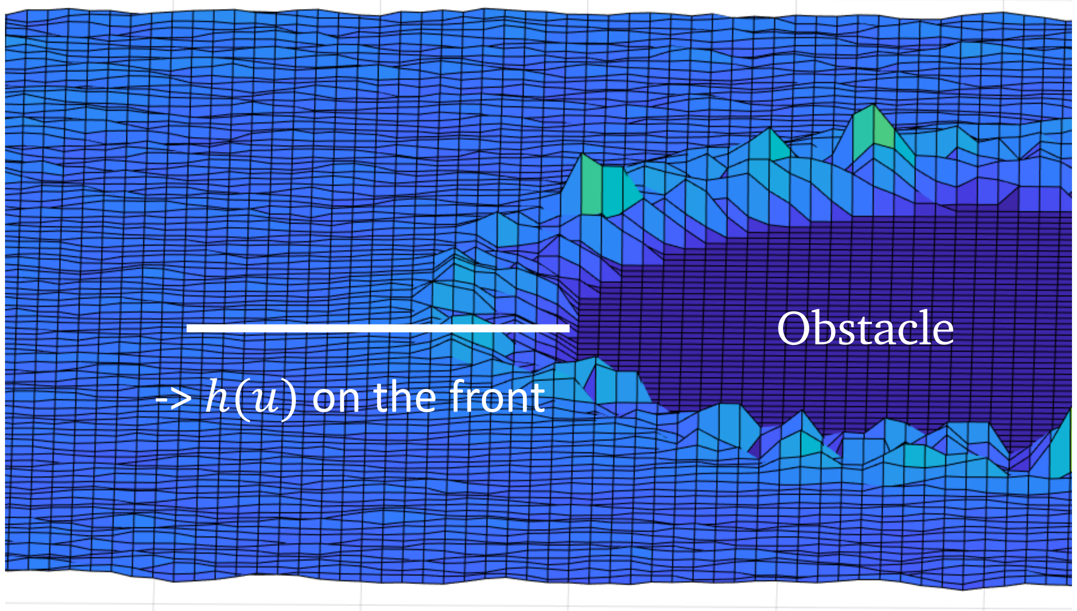
Modelling the heat transfer $h(u)$

Estimated shape of the melting ice
At t = 0+



Distance to the cylinder (cm)

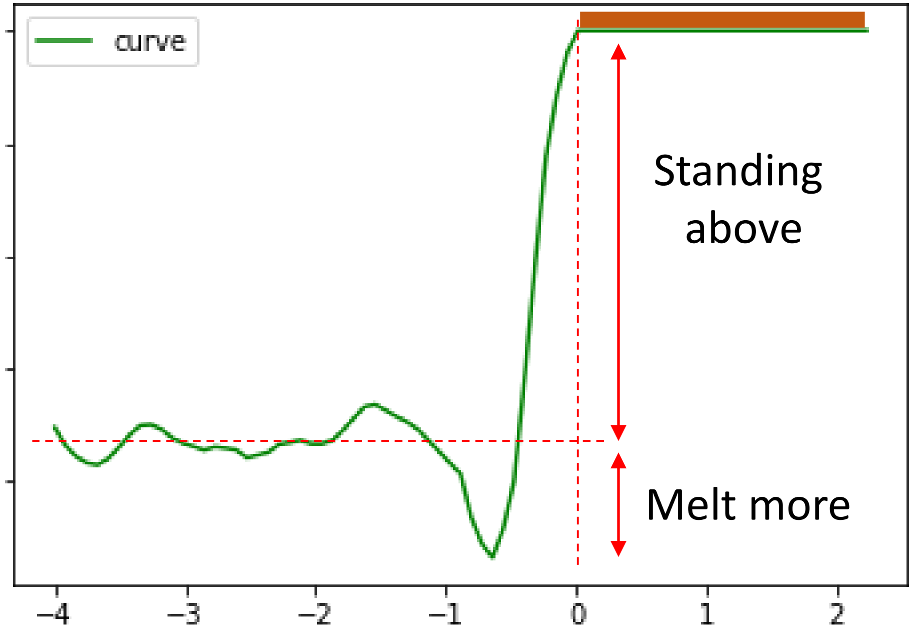
Experimental data for $h(u)$ in $\mathrm{W\,m^{-2}}K^{-1}$ according to the position



Explaining the balancing peeble

Wind creates the stand

Estimated shape of the melting ice
At t = 0+

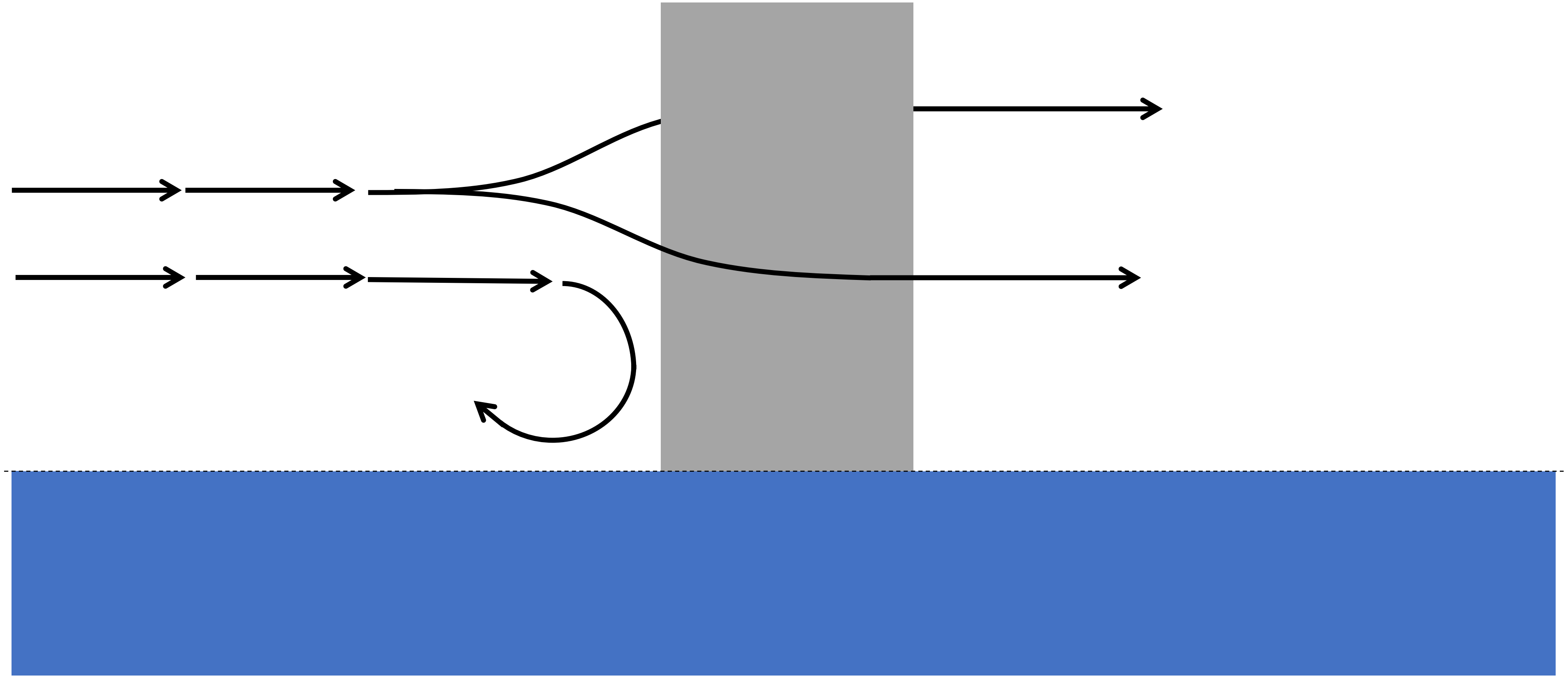


Distance to the cylinder (cm)

Curve at t = 10 min

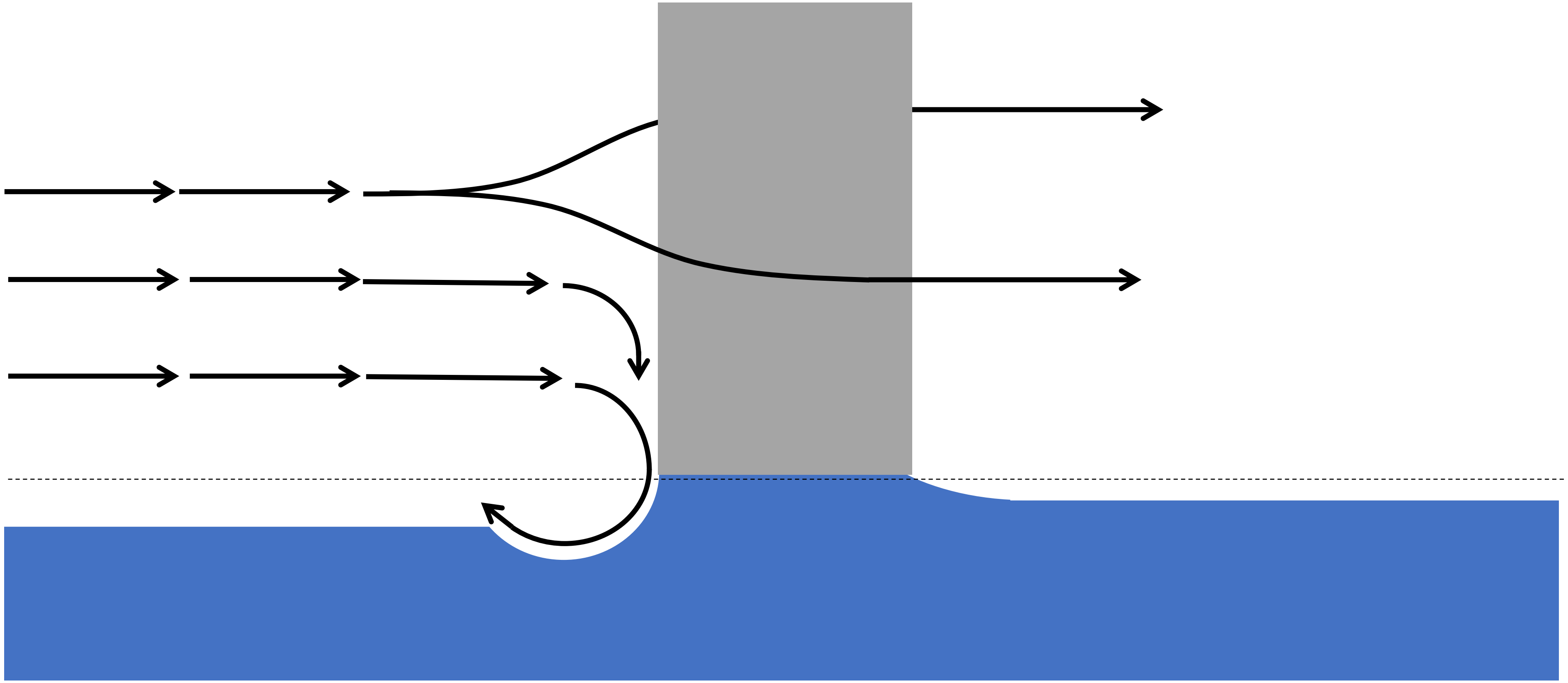
Influence of an obstacle

Air flow ?



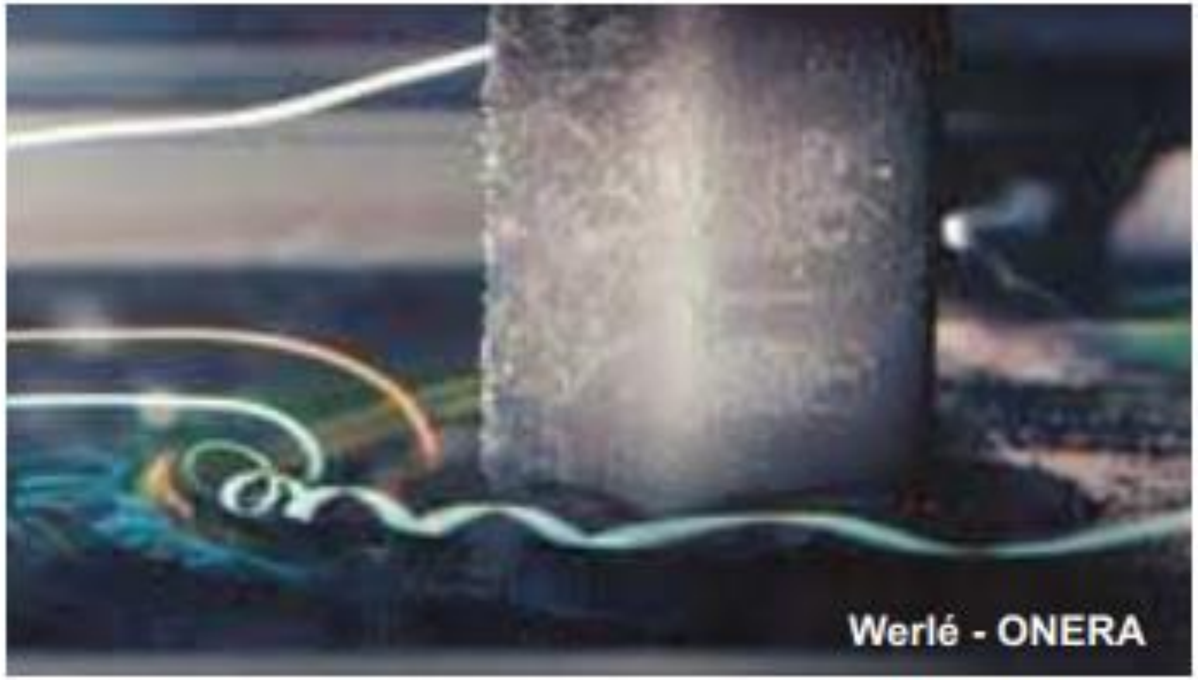
Influence of an obstacle

Turbulent vortex



Influence of an obstacle

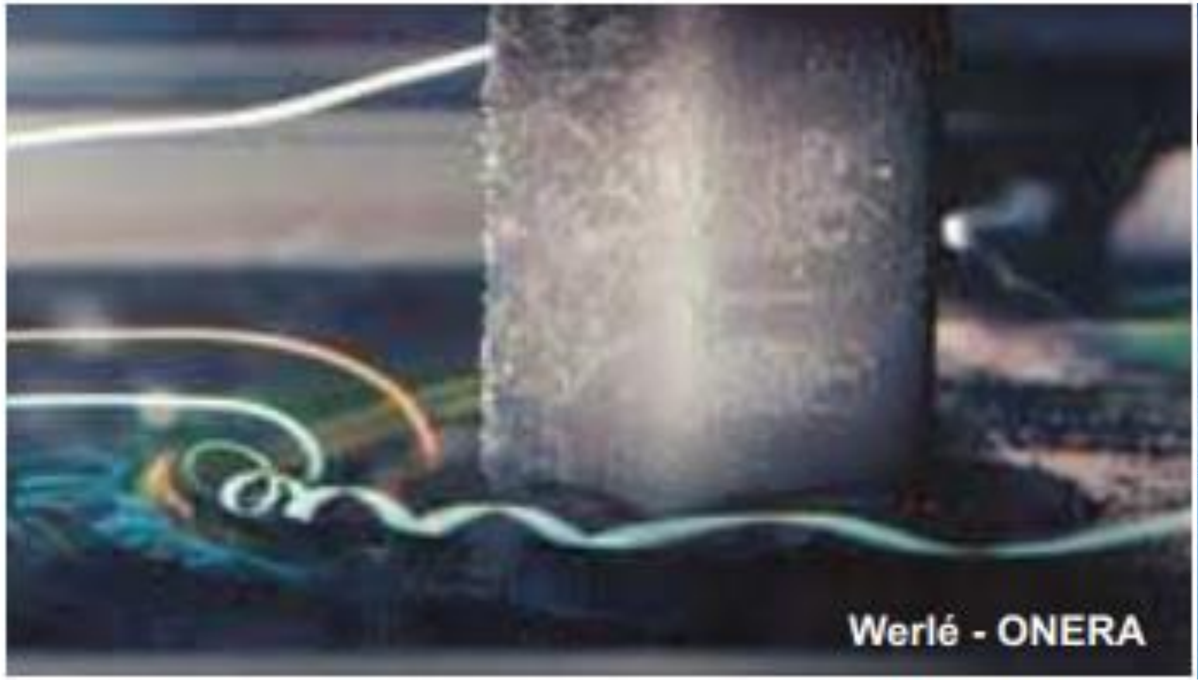
Turbulent vortex



$$Re = \frac{\rho_{air} u\, D}{\eta_{air}} \approx 10^4$$

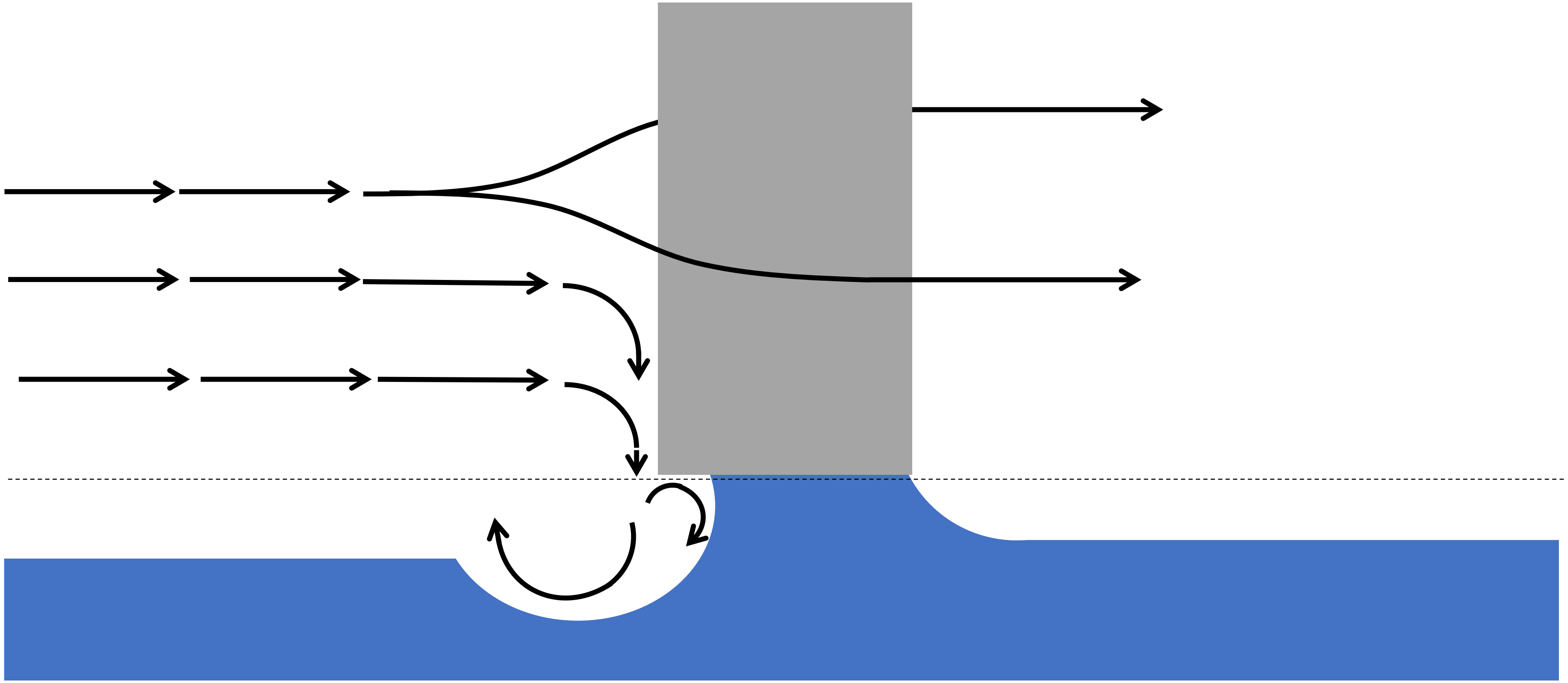
Influence of an obstacle

Turbulent vortex



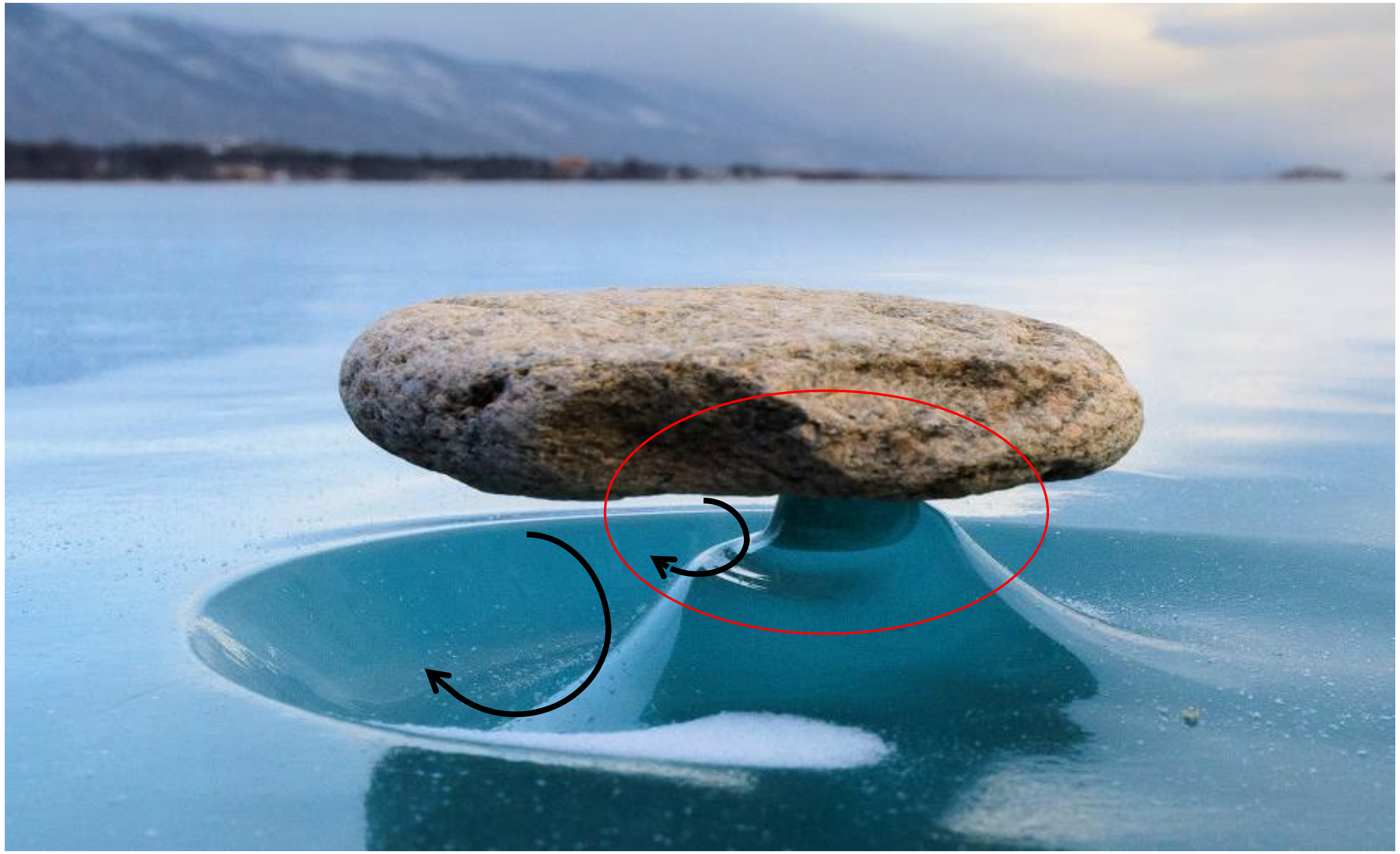
Influence of an obstacle

Turbulent vortex



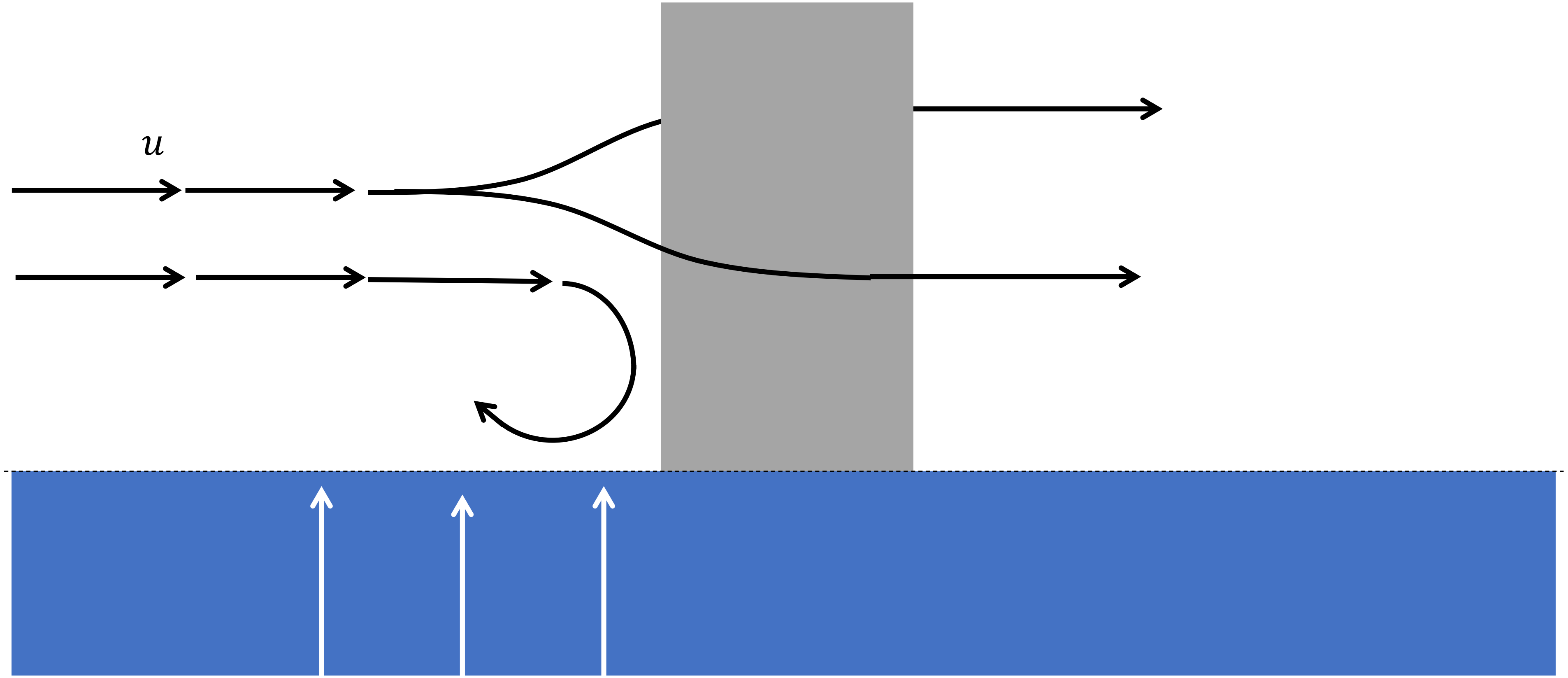
A double stand

Turbulent vortex : 2nd order effect ?



Estimation of the curve

Size of the vortex

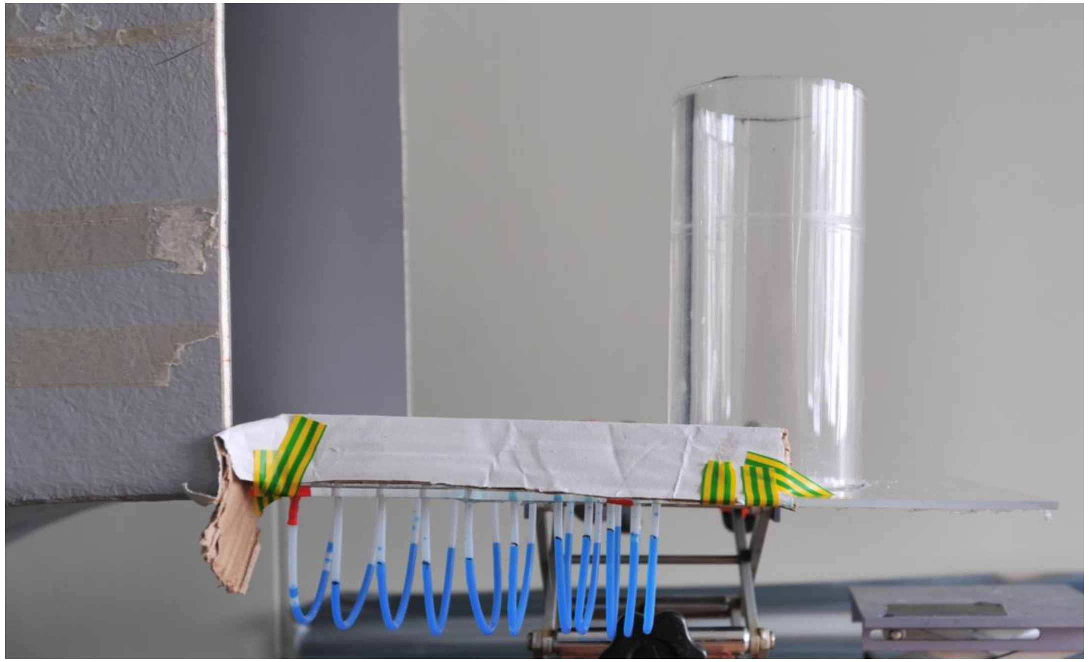


Measuring kinetic pressure

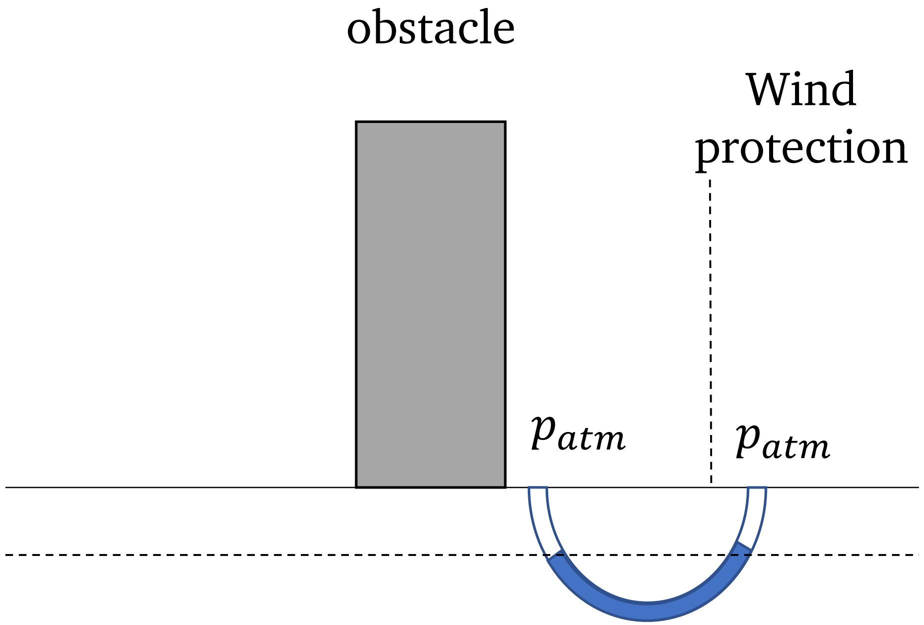
Estimation of the curve

Size of the vortex

Estimation of the characteristic size of the vortex



Set up side view

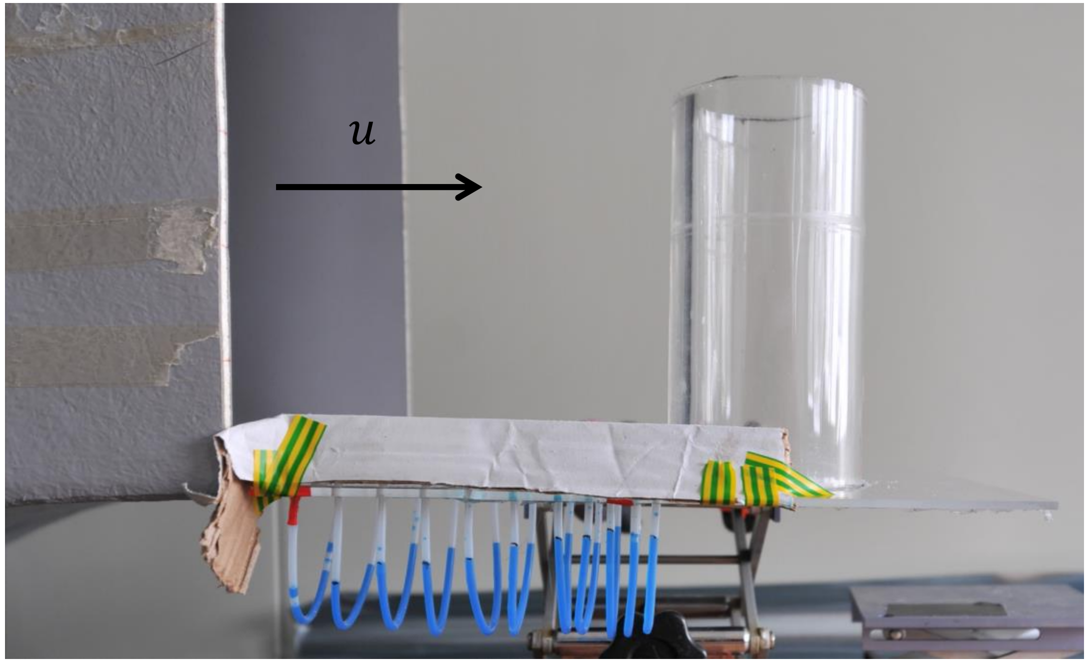


Set up front view

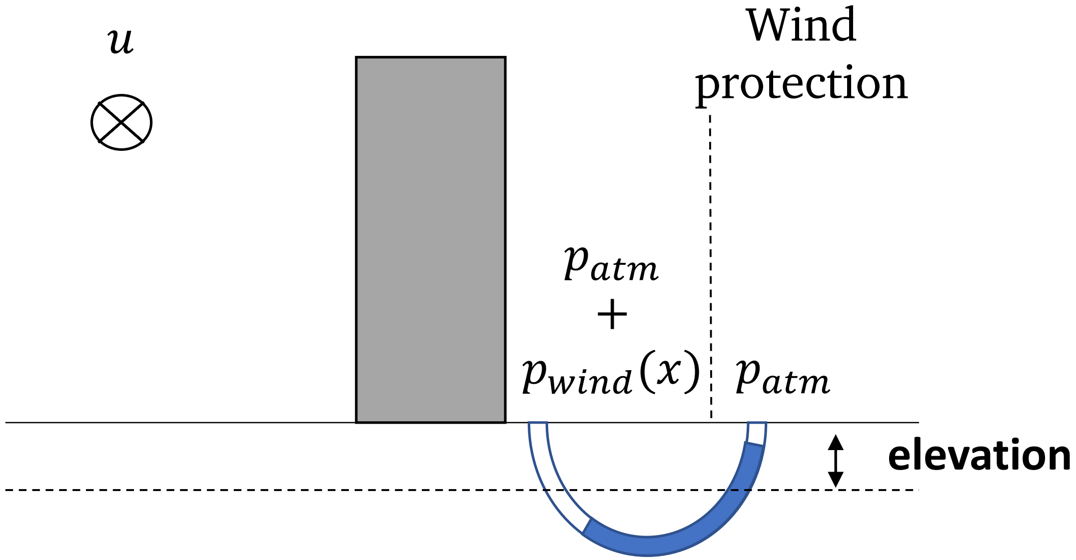
Estimation of the curve

Size of the vortex

Estimation of the characteristic size of the vortex



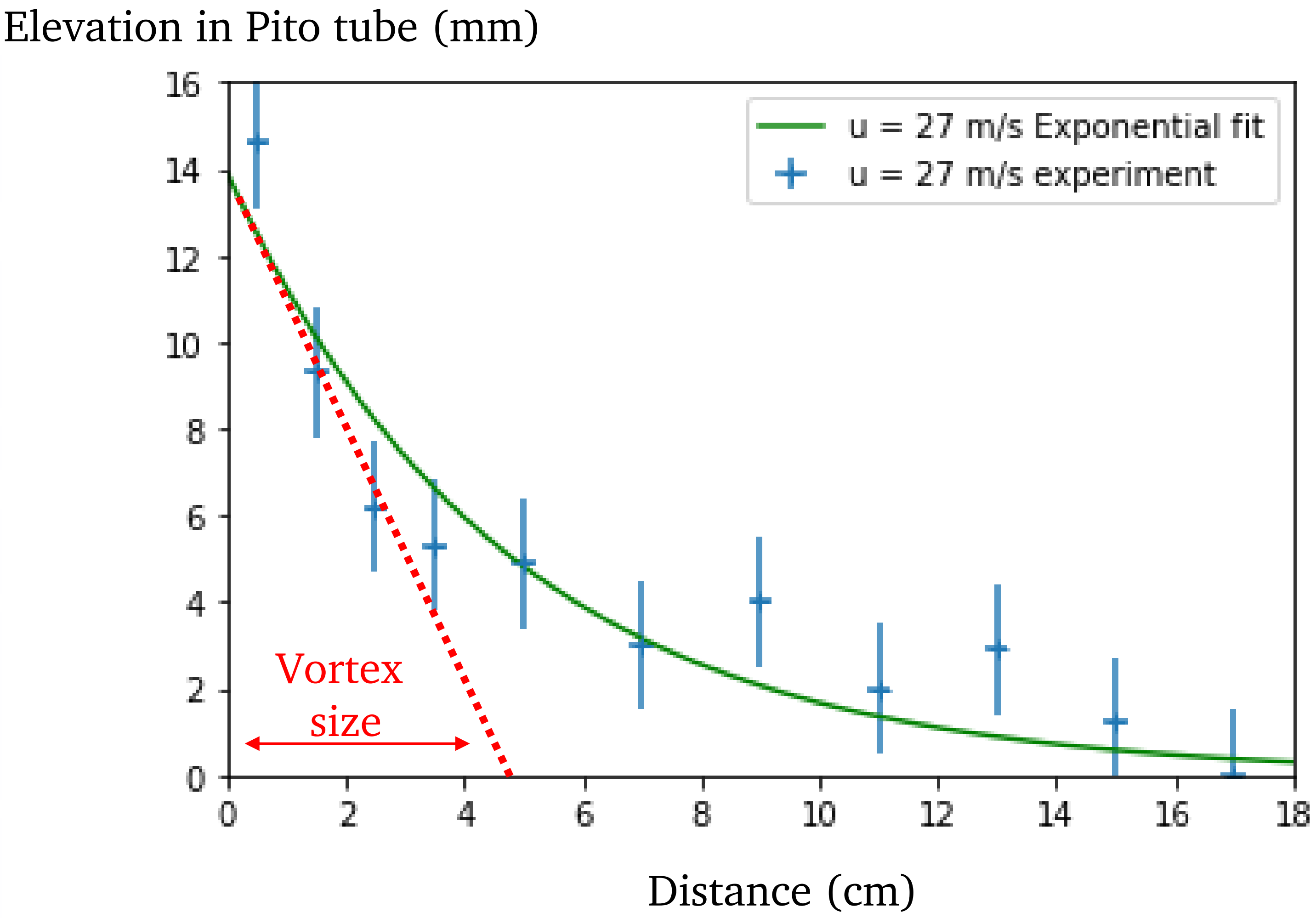
Set up side view

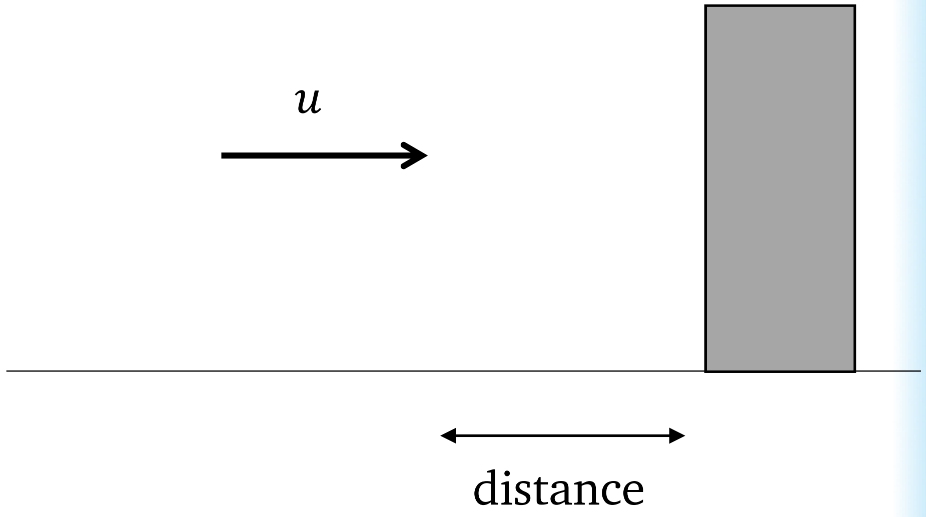


Set up front view

Estimation of the curve

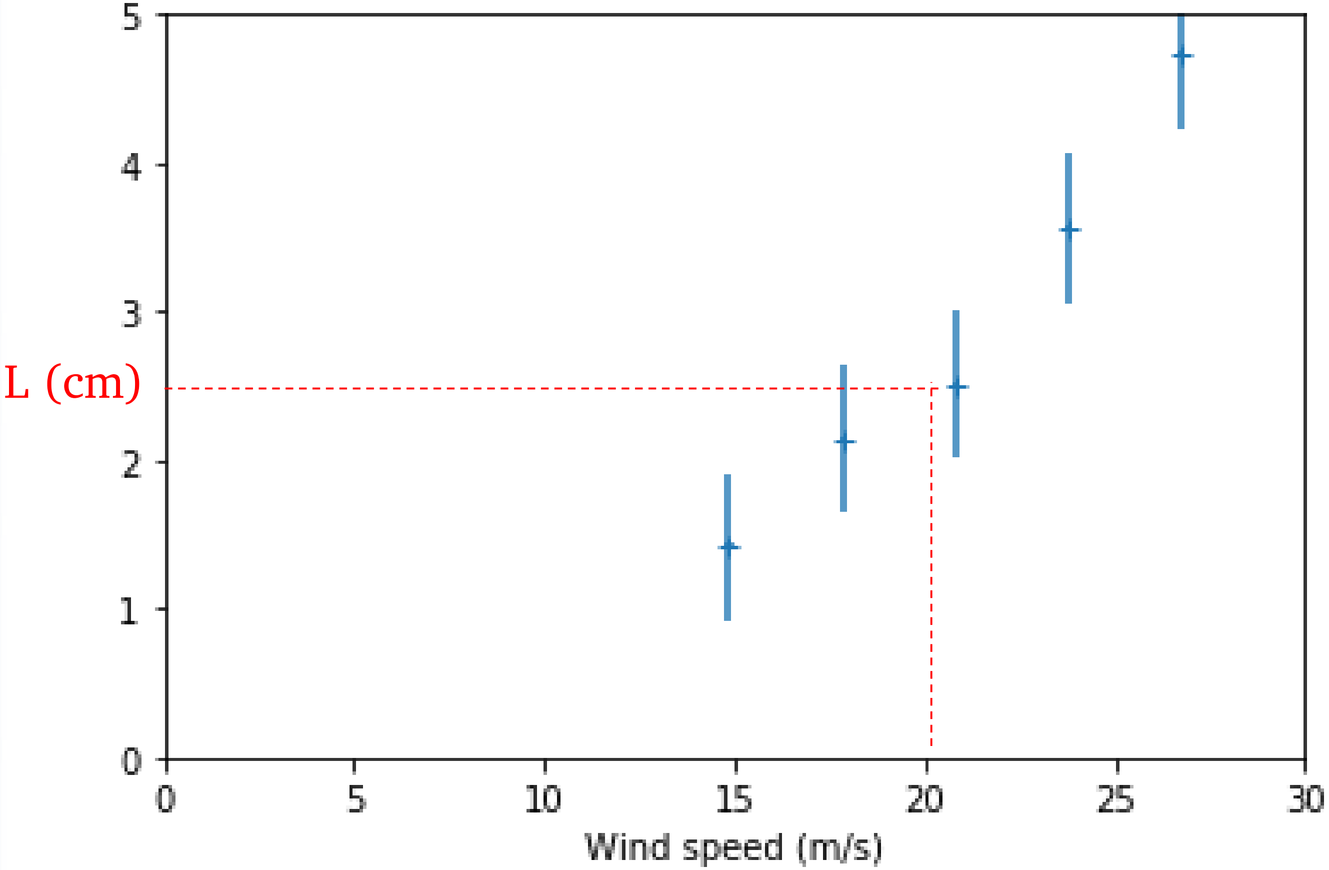
Size of the vortex

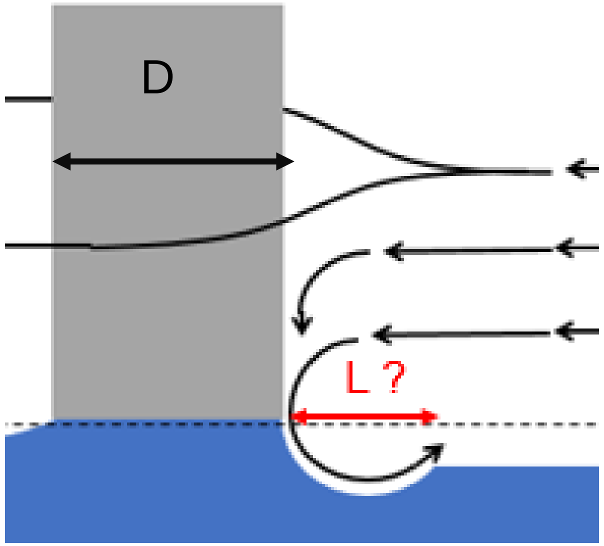




Estimation of the curve

Size of the vortex

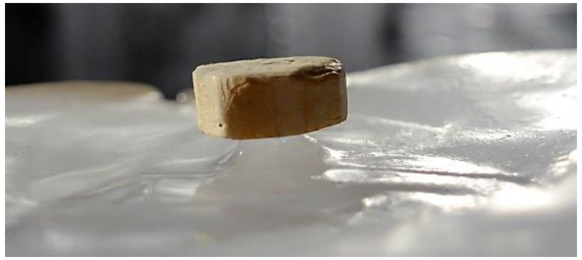




For $u = 20$ m/s, D = 7cm :
$\Rightarrow L \approx 2.5\ cm$

Curve $=$ a circle or radius $\approx \boldsymbol{L}$

Conclusion



- ✓ **Reproduce** this «stand» phenomenon

- ✓ **Explain** this «stand» phenomenon

Sublimation – Heat transfer
Same equation
Sublimation quicker on Baikal Lake

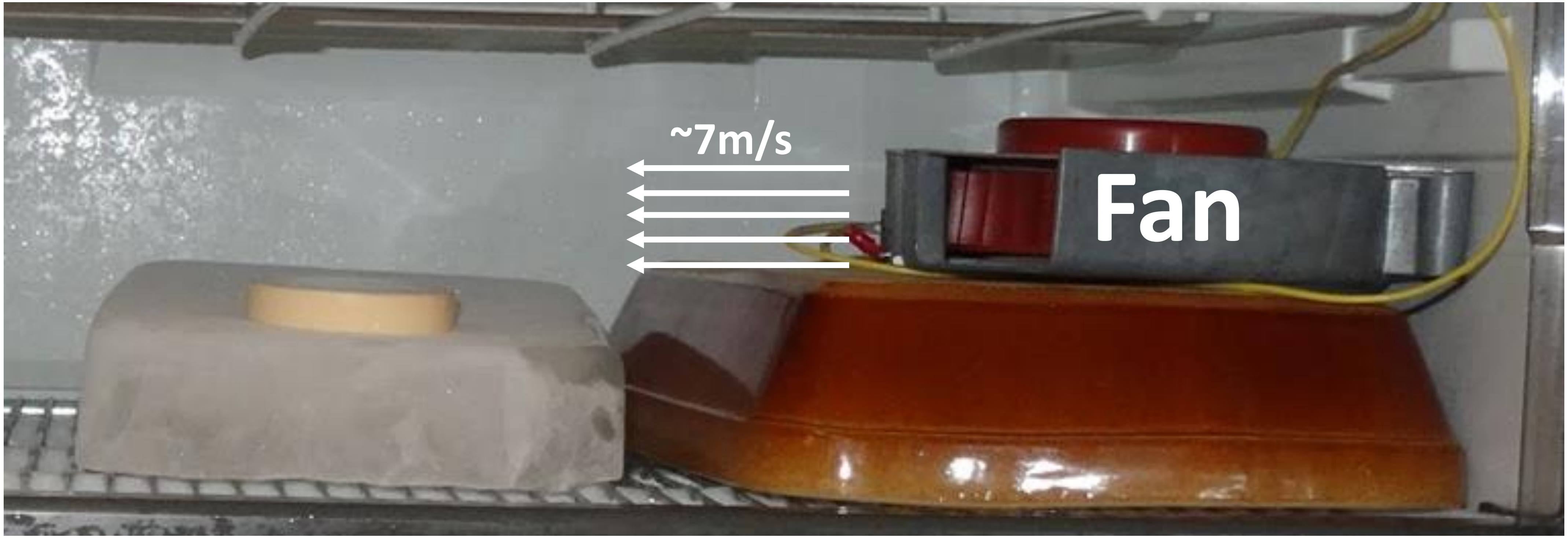
- ✓ Estimate **the curve** of the stand depending on the important parameters.

Ablation Parameters :
Vapor pressure, Temperature
Absolute wind speed

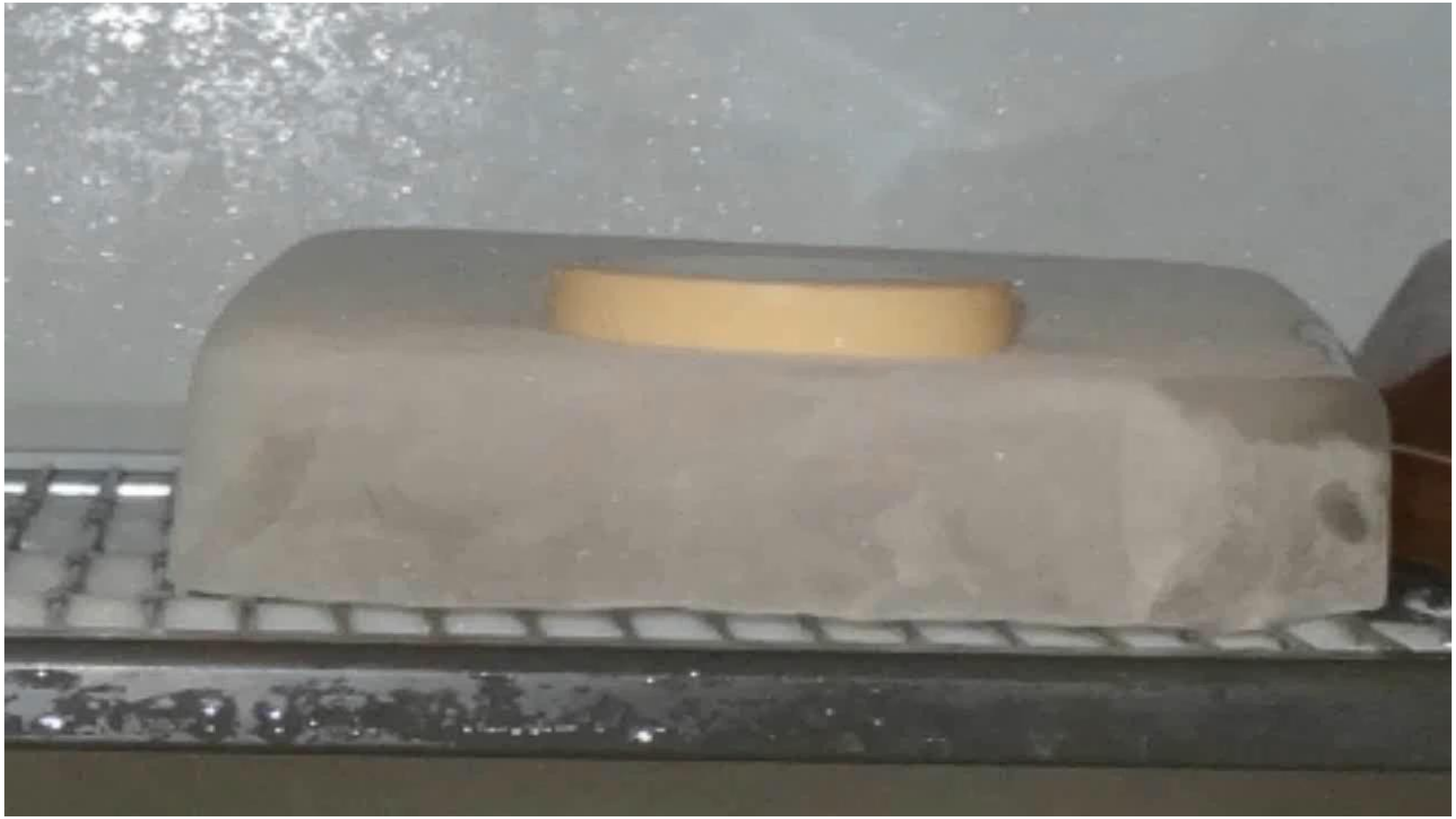
Shape parameters :
Turbulent flow
Radius of curvature $\sim L$, typical size of the vortex depends on the wind speed

Sublimation

T = -22°C

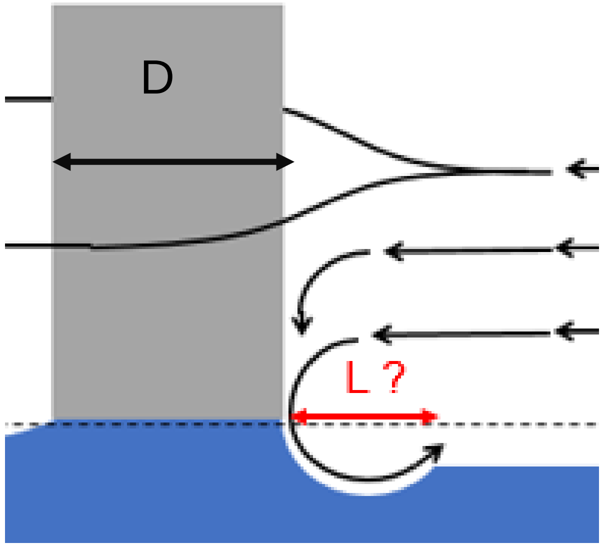


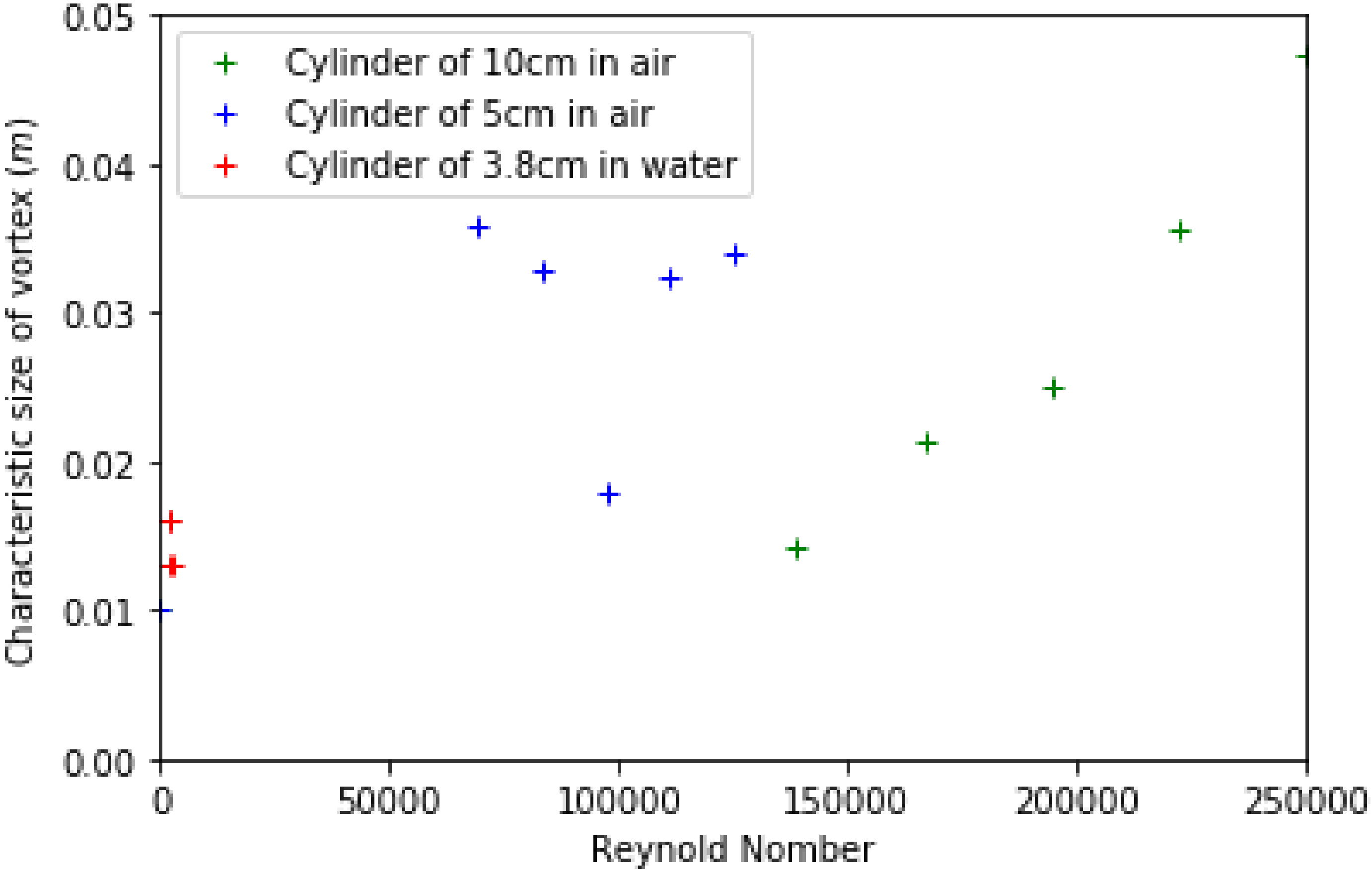
Sublimation



In 20 days !

Size of vortex VS Reynolds Number





How does ice melt ?

Heat and mass transfer

Analogous causes:

- ✓ Mass: concentration gradient
- ✓ Heat: temperature gradient

Chilton-Colburn J-analogy:

Similar formulations of basic equations:

Conduction:

- ✓ Mass: $\partial_t C = \mathrm{D_m}\, \Delta C$
- ✓ Heat: $\partial_t T = \mathrm{D_h}\, \Delta \mathrm{T}$

D_m =Mass diffusivity ; D_h = Heat diffusivity

Convection:

Mass: Sherwood number

$$Sh = h_m$$

$$Sh = Ct. \times Sc^{\frac{1}{3}} \times Re$$

Why does ice melt ?

Wind

Sublimation

Vapor pressure

$$\Phi_{sublimation} = \rho_a C_E L_{sub}\, \boldsymbol{u}\, (\boldsymbol{q_{sat}}(\boldsymbol{T}) - \boldsymbol{q_{wind}})$$

$\sim$

Heat transfer

From ice to water then vapor

$$\Phi_{heat} = \rho_a c_a C_H\, \boldsymbol{u}\, (\boldsymbol{T_{wind}} - \boldsymbol{T_{ice}})$$

$(q_{sat}, q_{wind} : mass\ of\ water\ per\ m^3 of\ air\ )$

Why does ice melt ?

Sublimation

Bulk aerodynamic approach :

$$\Phi_{humidity} = \rho_a C_E L_{sub}\, u\, (q_{sat}(T) - q_{wind})$$

$$\Phi_{humidity} \approx 25\, W/m^2 \text{ for } u = 5\text{m/s}$$

with

- ρ_a: *density of air*
- L_{sub}: *sublimation latent heat*
- q_{sat}, q_{wind} : *mass of water in* $1m^3$
- C_E: *a constant*

(Ref : The Physics of glaciers, by K.M Cuffier and W.S.B Paterson)

Characteristic time :

Δe = 10 cm

$q_{wind} = 0.8\, q_{sat}$

Δe

$$\boldsymbol{\Delta t \approx 4\ months}$$

On Lake : possible

In lab : long and difficult to reproduce

Why does ice melt ?

Heat transfer

Bulk aerodynamic approach :

$$\Phi_{heat} = \rho_a c_a C_H \, \boldsymbol{u} \, (\boldsymbol{T_{wind}} - \boldsymbol{T_{ice}})$$

$$\Phi_{heat} \approx 50 \, W/m^2 \text{ for } u = 5\text{m/s}$$

with $\begin{cases} \rho_a : density\ of\ air \\ c_a : thermal\ capacity \\ C_H : a\ constant \end{cases}$

(Ref : The Physics of glaciers, by K.M Cuffier and W.S.B Paterson)

<u>Characteristic time</u> :

Δe = 10 cm

$T_{wind} - T_{ice} = 5°C$

Δe

$\boldsymbol{\Delta t \approx 2\ months}$

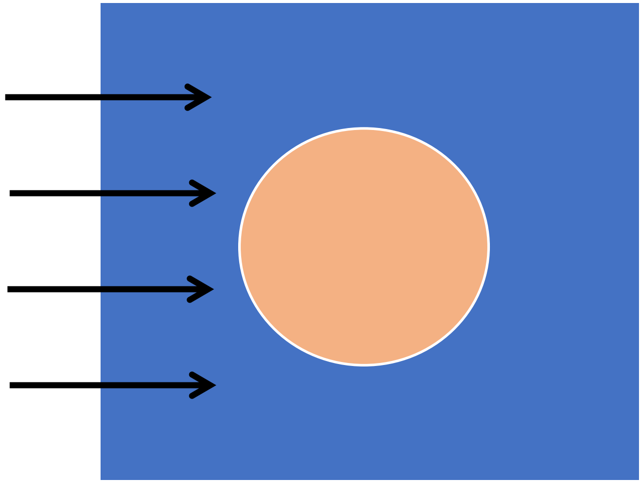
On Lake : possible

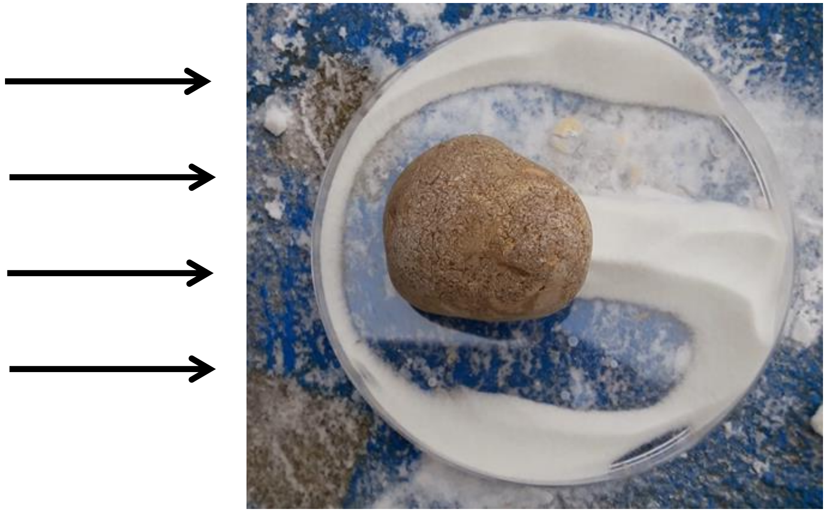
In lab : possible by increasing $T_{wind} - T_{ice}$

Hoodoos



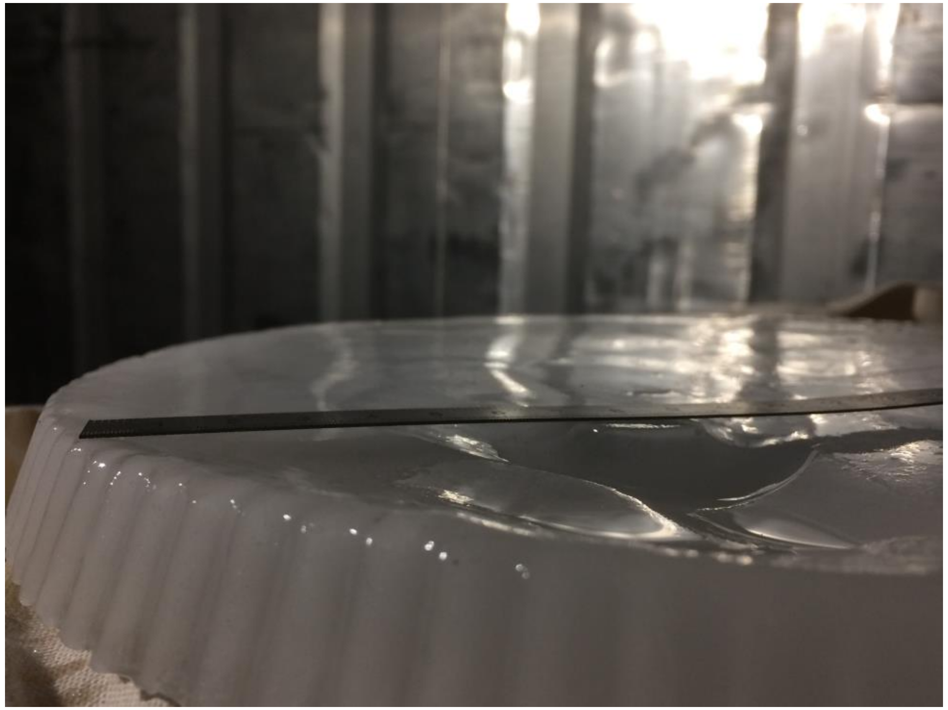
4^{th} experiment : Erosion on sand

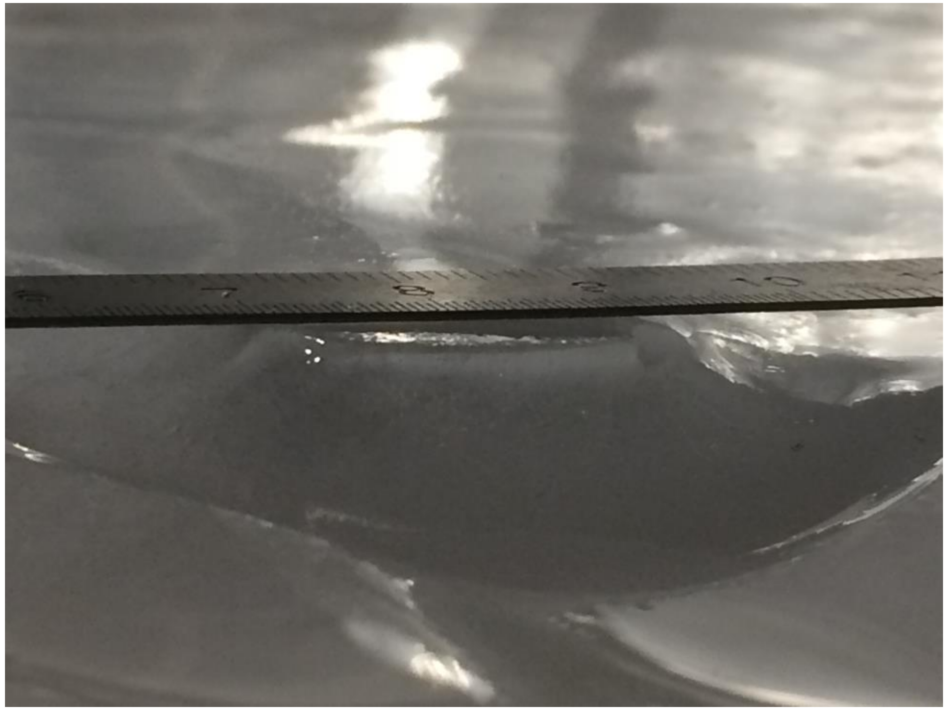




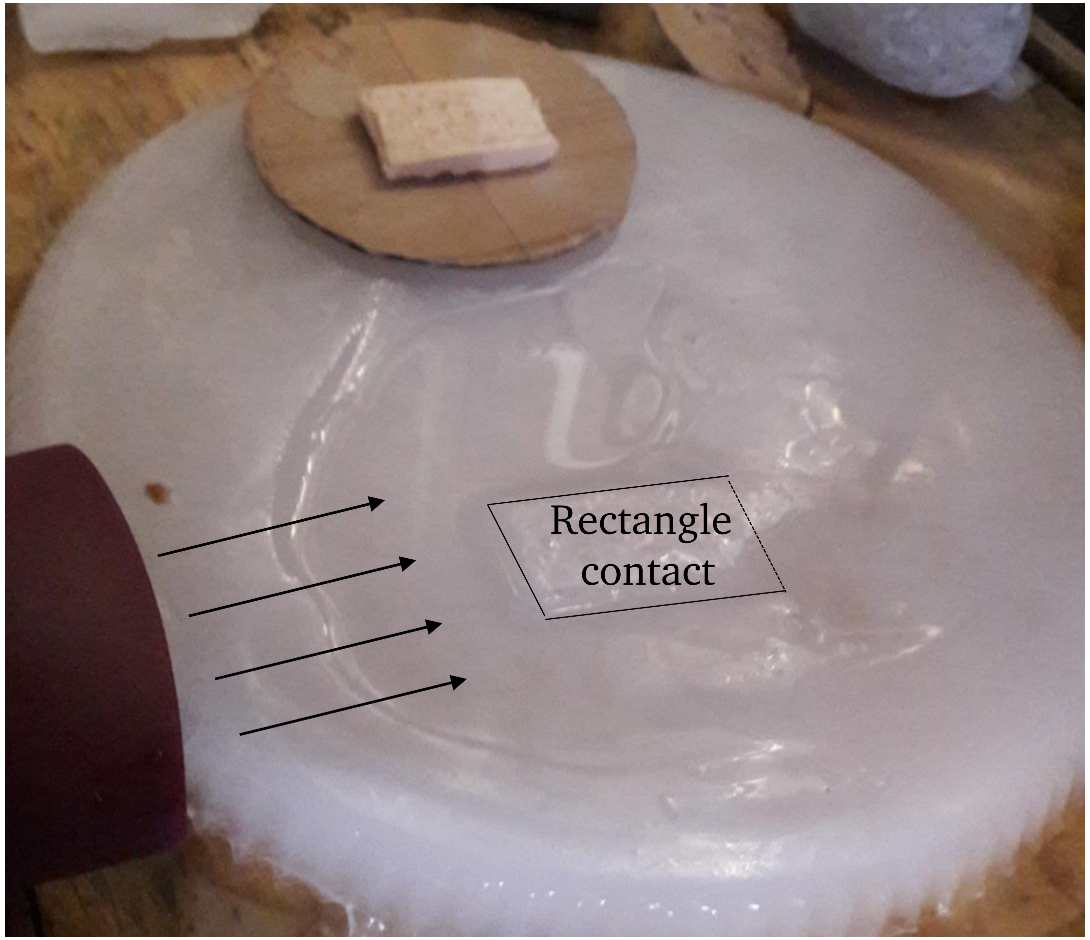
1) Stone stands above the lake ✓
2) Ice lower just around the stone ✓
3) Symmetry or Axisymmetry ✓

No elevation : only melting !

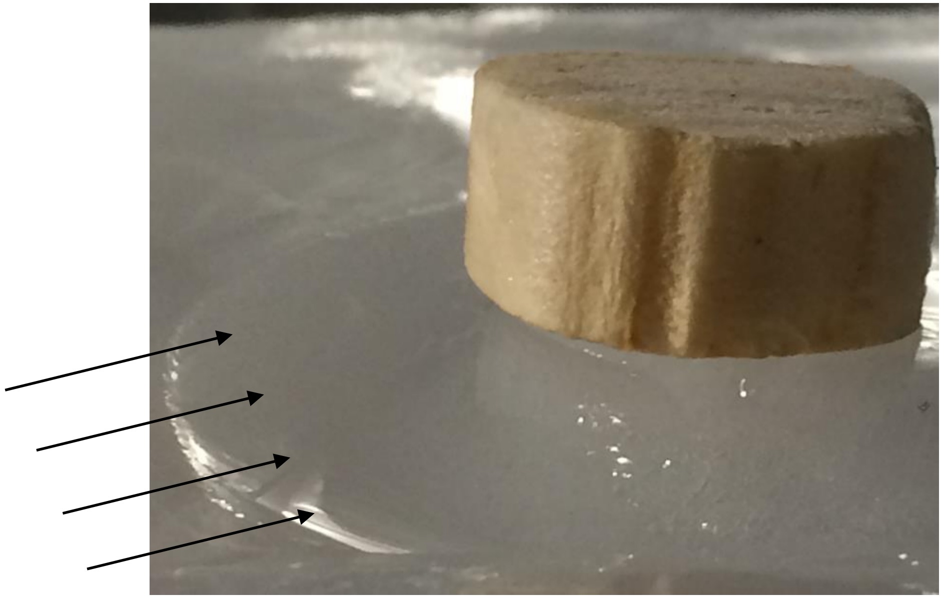


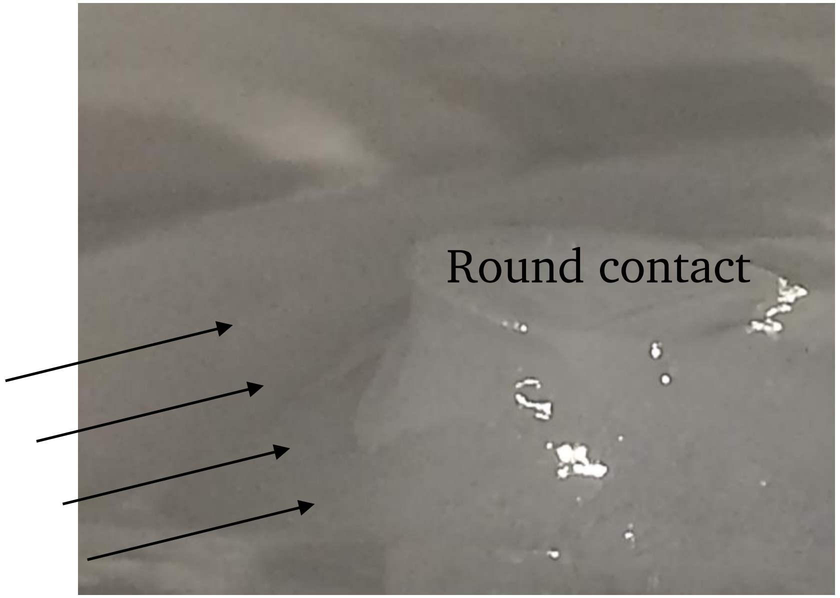


Dependence of the form of contact



Dependence of the form of contact





If no contact : the wind can blow on ice, more and more.
If contact with a solid : the ice is protected from the wind.

3rd experiment : Wind

Understanding the phenomenon

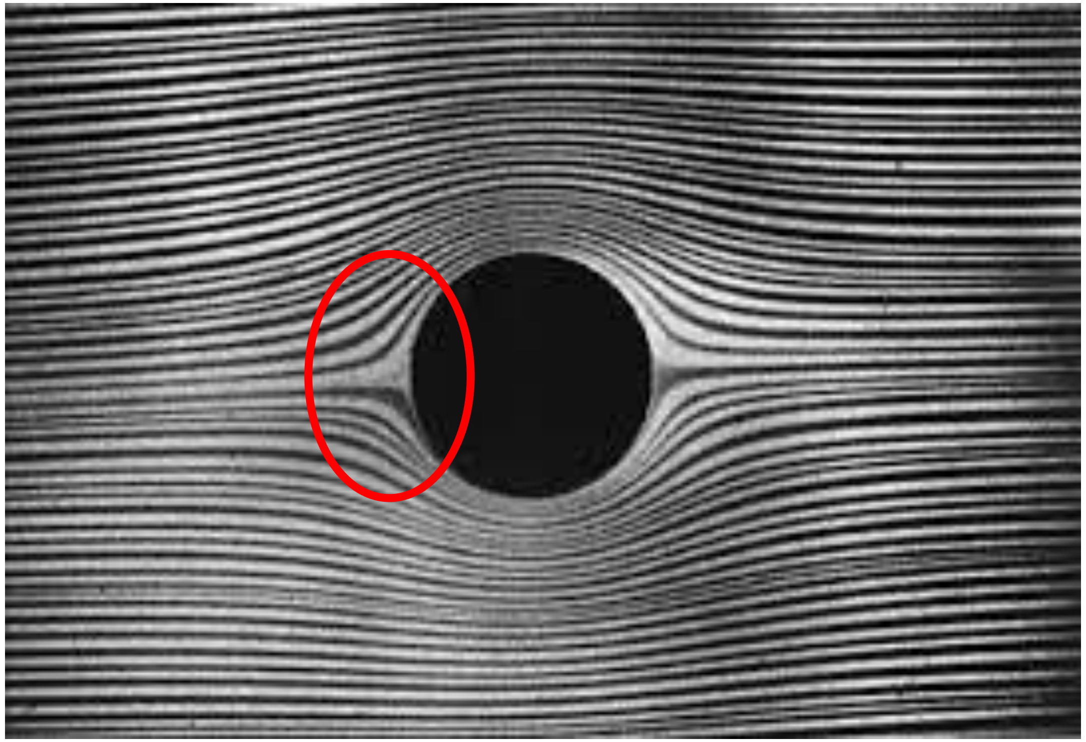
Aerodynamic flow

A simple model : Potential flow

$$\Phi_{heat} = h(u)\,(T_{wind} - T)$$

Should melt less
at the front

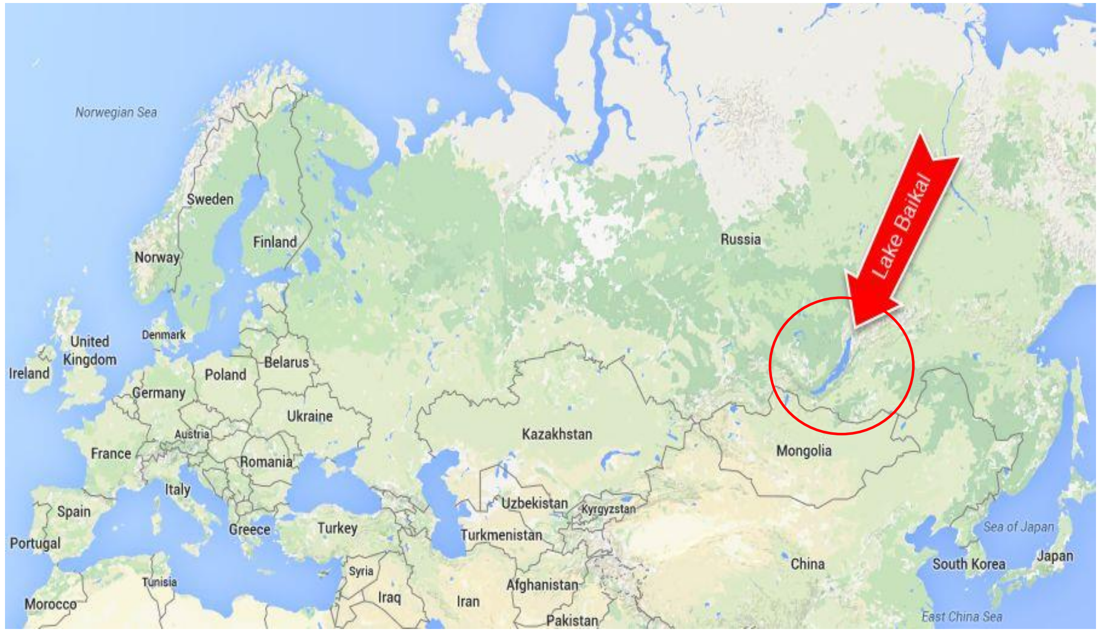
$$Re = \frac{\rho_{air} u\, D}{\eta_{air}} \approx 10^4$$



There must be something else… Turbulence !

Context of the picture

Lake Baikal, Siberia

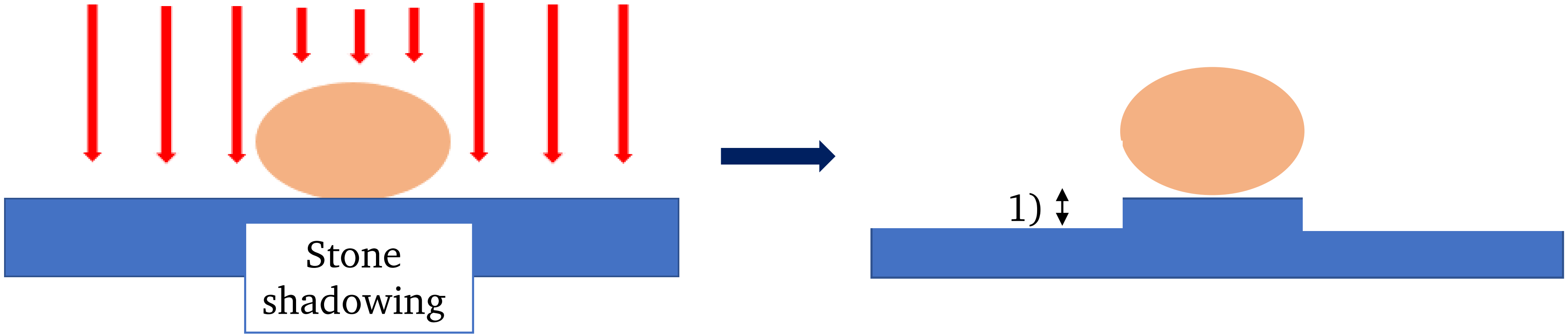


Location of Lake Baikal

Picture of Lake Baikal

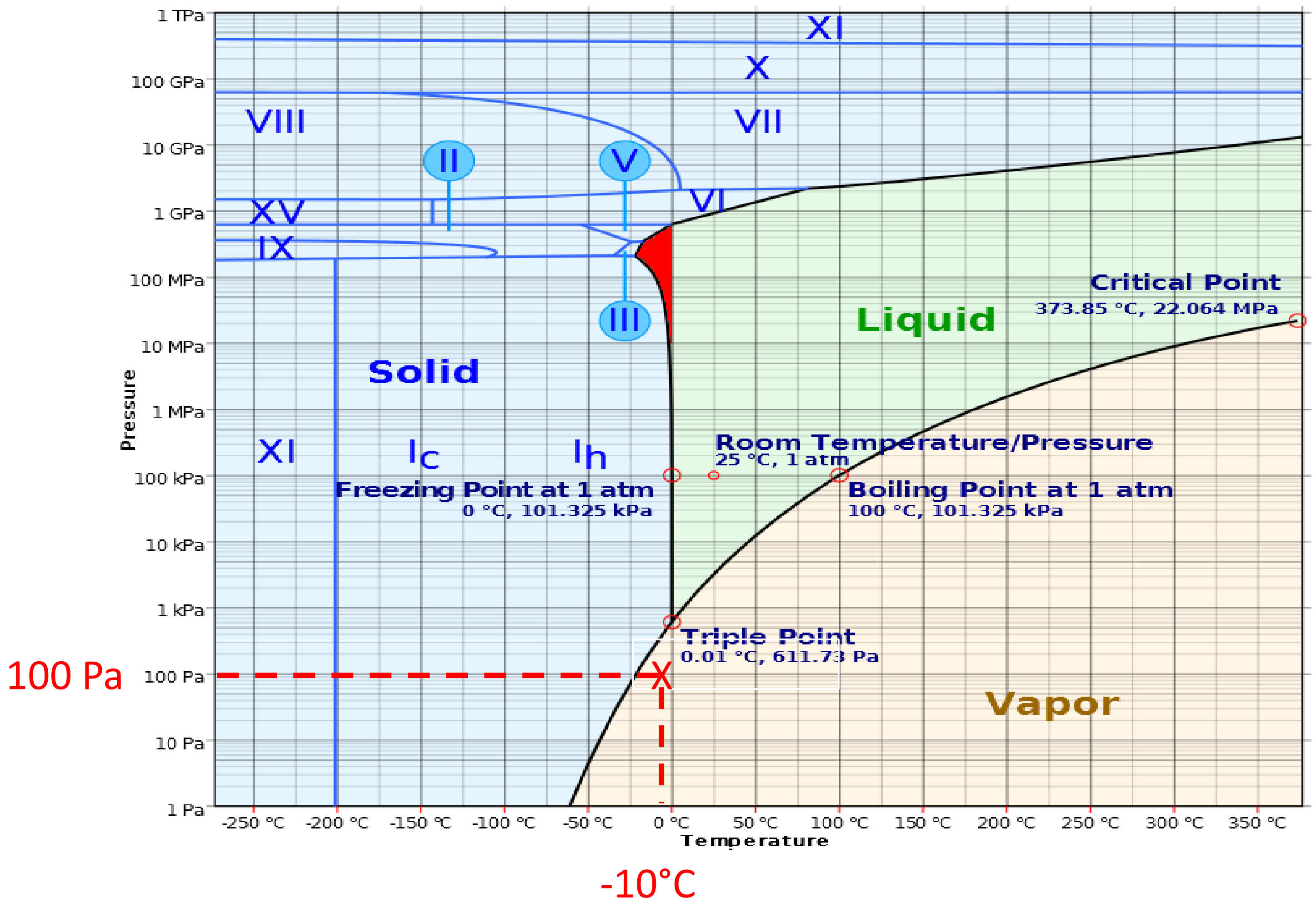
Structure formation : from December to April

2^{nd} experiment : Radiation



1) Stone stands above the lake ✓
2) Ice lower just around the stone X
3) Asymmetry possible X

Why does ice melt ?



- Most stable phase: **vapor**
  ⇒ sublimation

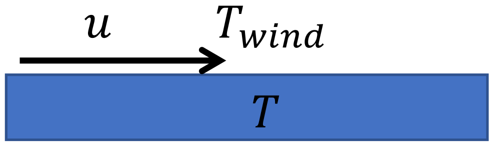
- **Endothermic** reaction
  ⇒ energy needed

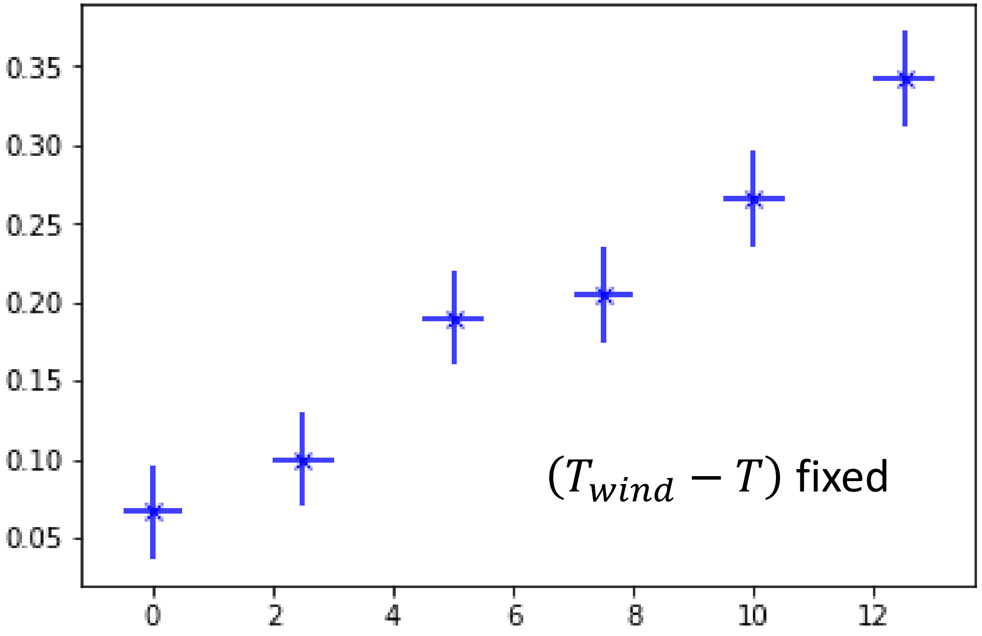
How does ice melt ?

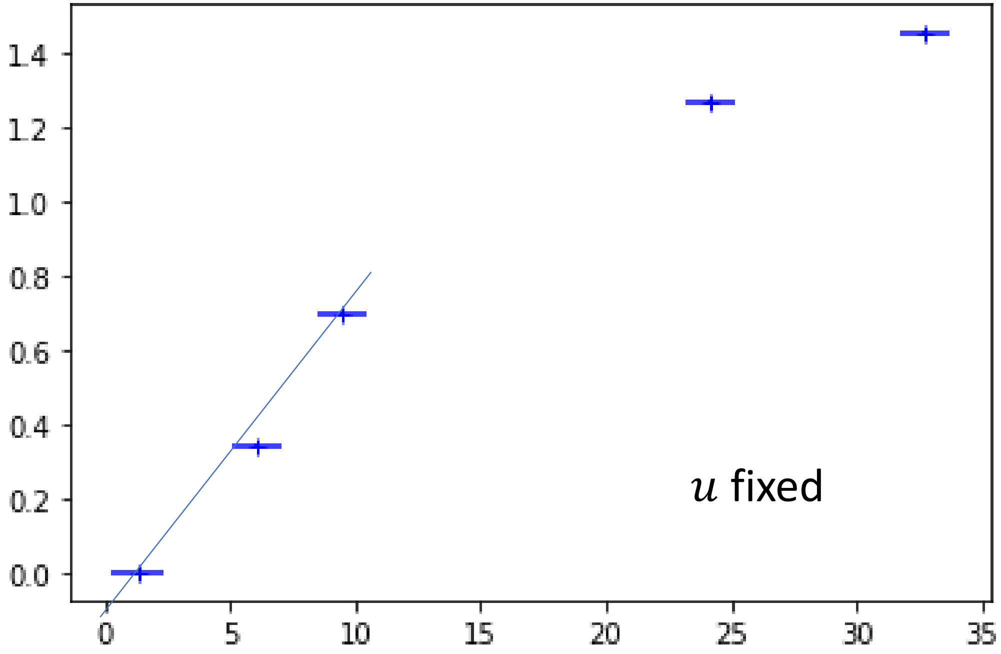
Modelling the heat transfer

Newton's law $\Phi_{heat} = h(u)\,(T_{wind} - T)$

On a simple case :

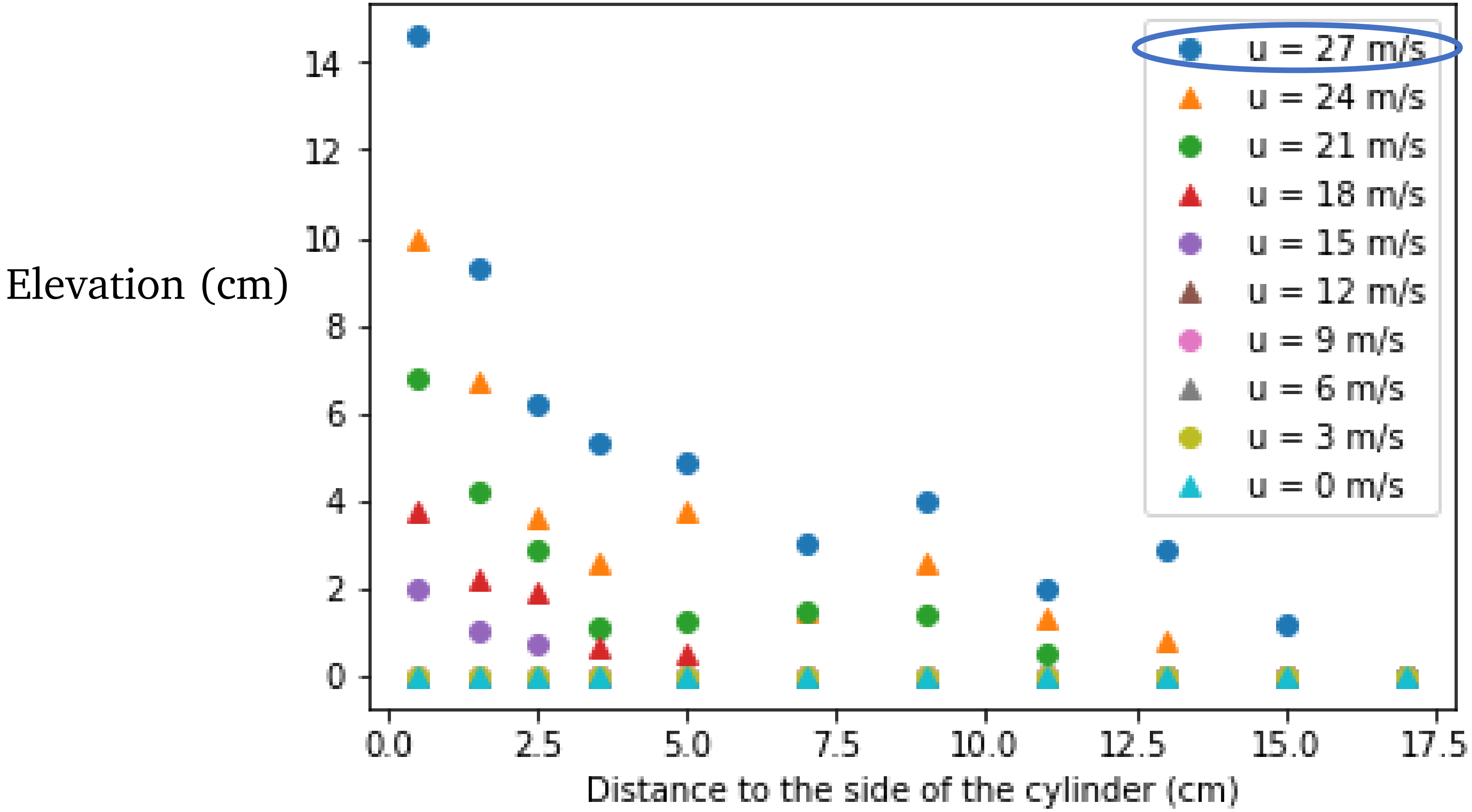






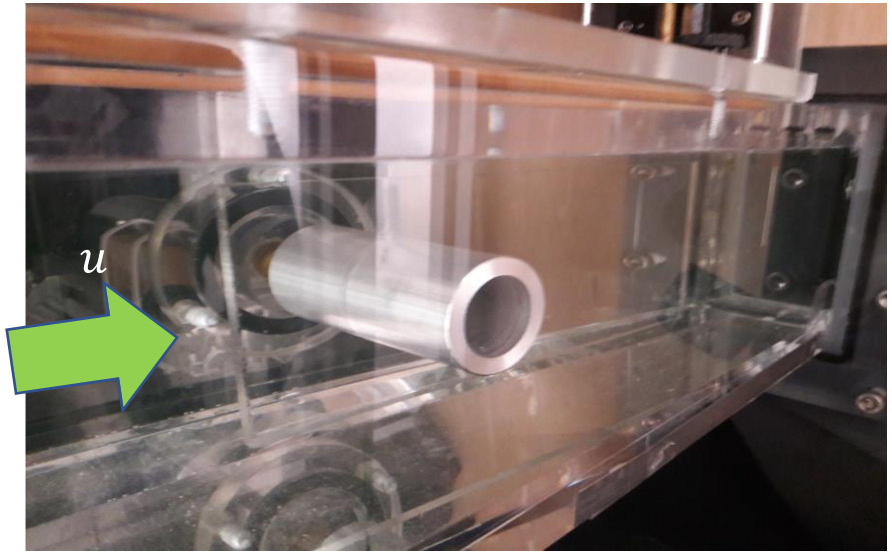
Estimation of the curve

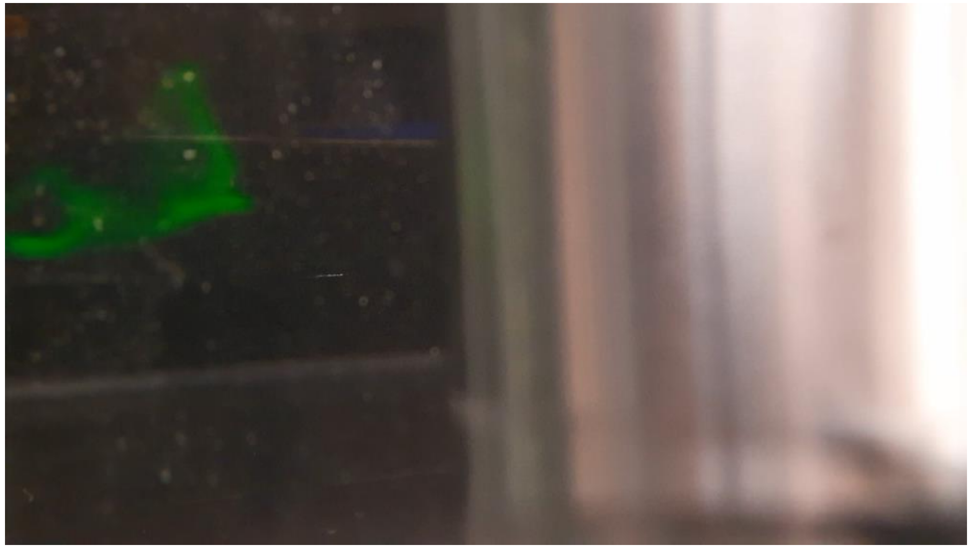
Size of the vortex



Estimation of the curve

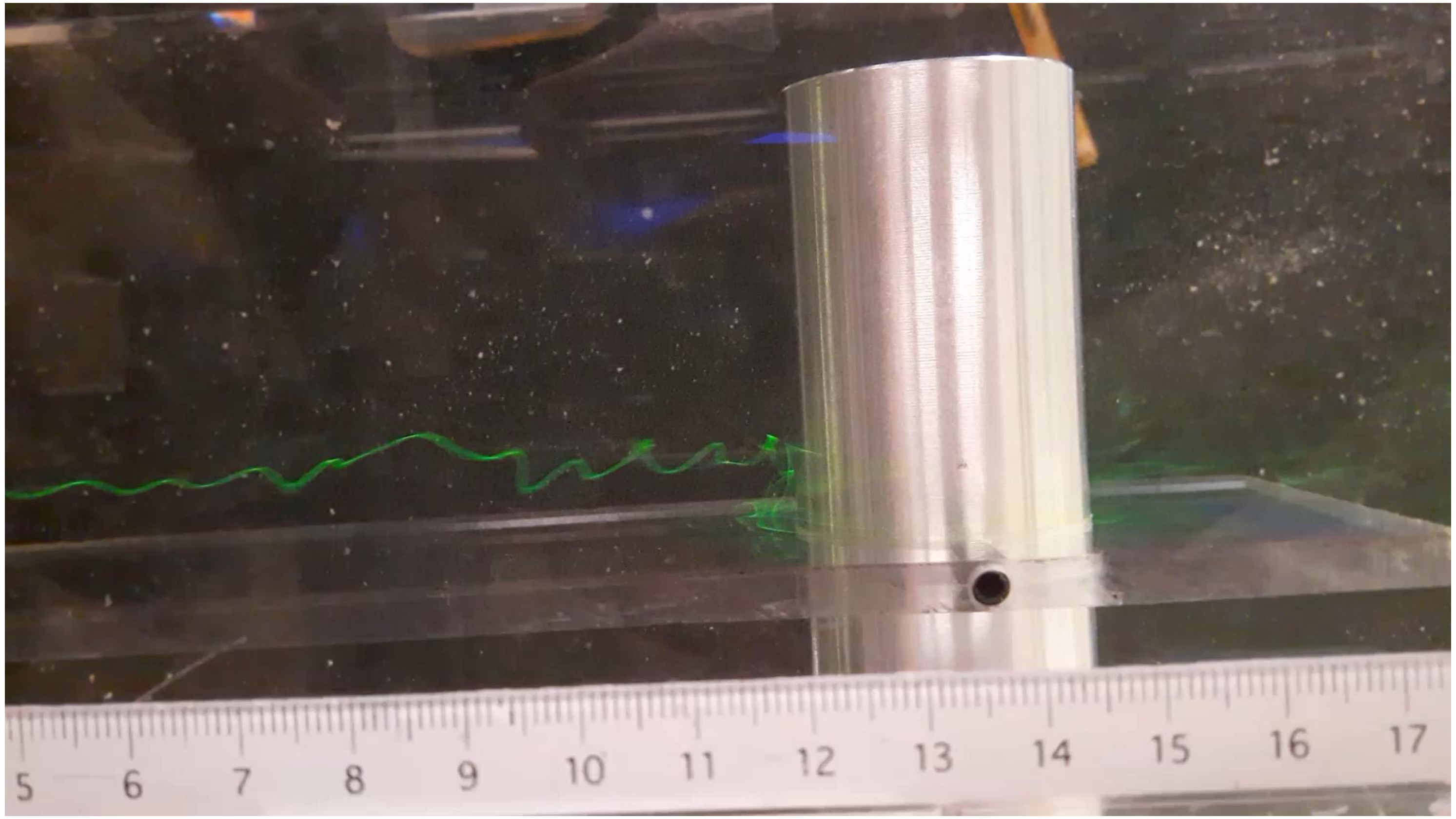
Visualizing the vortex





Estimation of the curve

Visualizing the vortex



Why does ice melt ?

Wind

Sublimation

Origin: Vapor pressure gradient

$$\Phi_{sublimation} = \boldsymbol{h_m}(\boldsymbol{u})\ (\boldsymbol{q_{sat}}(\boldsymbol{T}) - \boldsymbol{q_{wind}})$$

$(q_{sat}, q_{wind} : mass\ of\ water\ per\ m^3 of\ air\ )$

Sherwood number:

$$Sh = \frac{h_m L}{D_m} = 0.03. \times Re^{4/5} \times Sc^{1/3}$$

h_m: mass transfer coefficient

Heat transfer

Origin: Temperature gradient

$$\Phi_{heat} = \boldsymbol{h}(\boldsymbol{u})\ (\boldsymbol{T_{wind}} - \boldsymbol{T_{ice}})$$

Nusselt number:

$$Nu = \frac{hL}{D} = 0.002. \times Re^{4/5} \times Sc^{1/3}$$

h: heat transfer coefficient

$\sim$

$\sim$

$$\frac{\Phi_{sublimation}}{\Phi_{heat}} \gg 1$$