PROBLEM N°14
ERRATIC RAINDROPS

Ecole Polytechnique

Erratic Raindrops

"When a car moves with high speed in rain sometimes the drops on its side window walk up but not down.
***Explain the phenomenon** and find the **conditions** for it to occur (size of the drops and the car speed for example).*

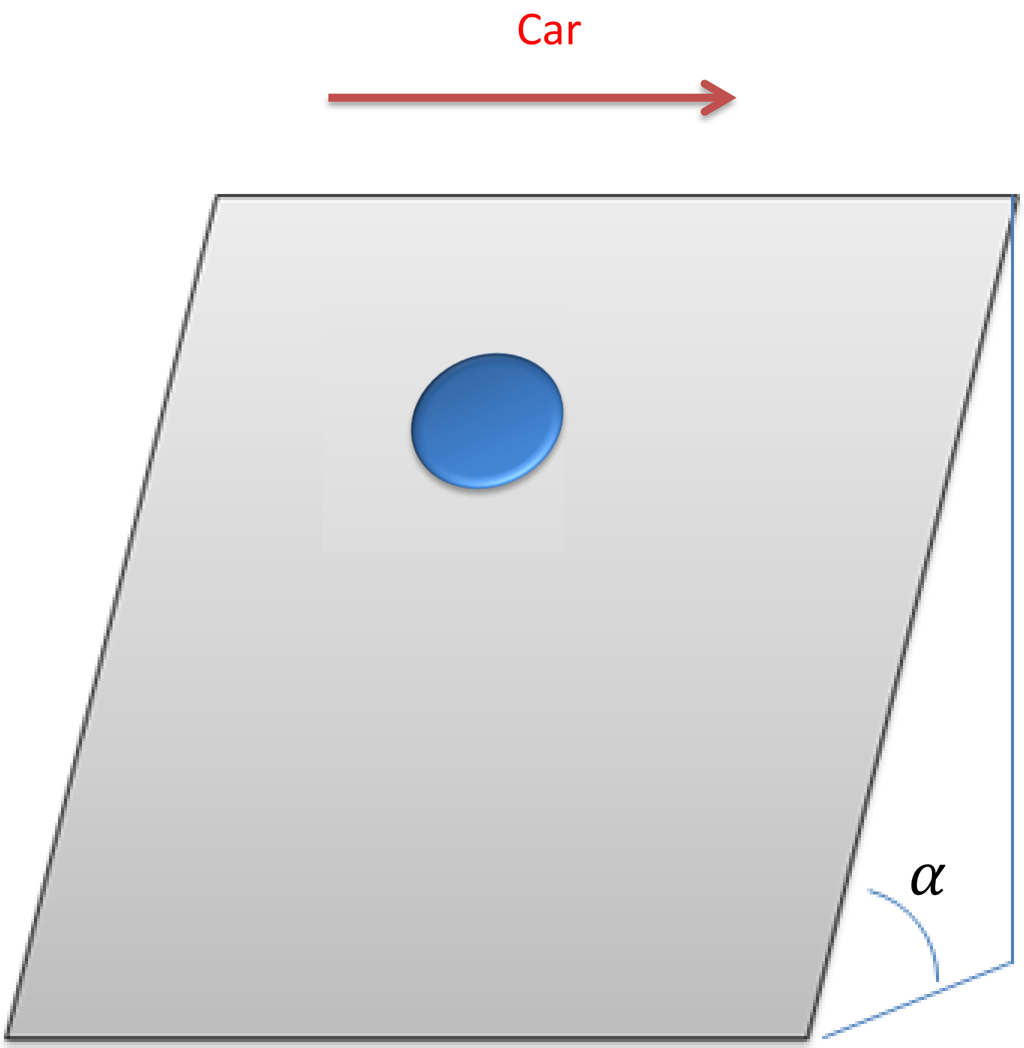
*What determines the **drop trajectory** and how does it depend on the important parameters? "*

Main idea

- **Wind going up**

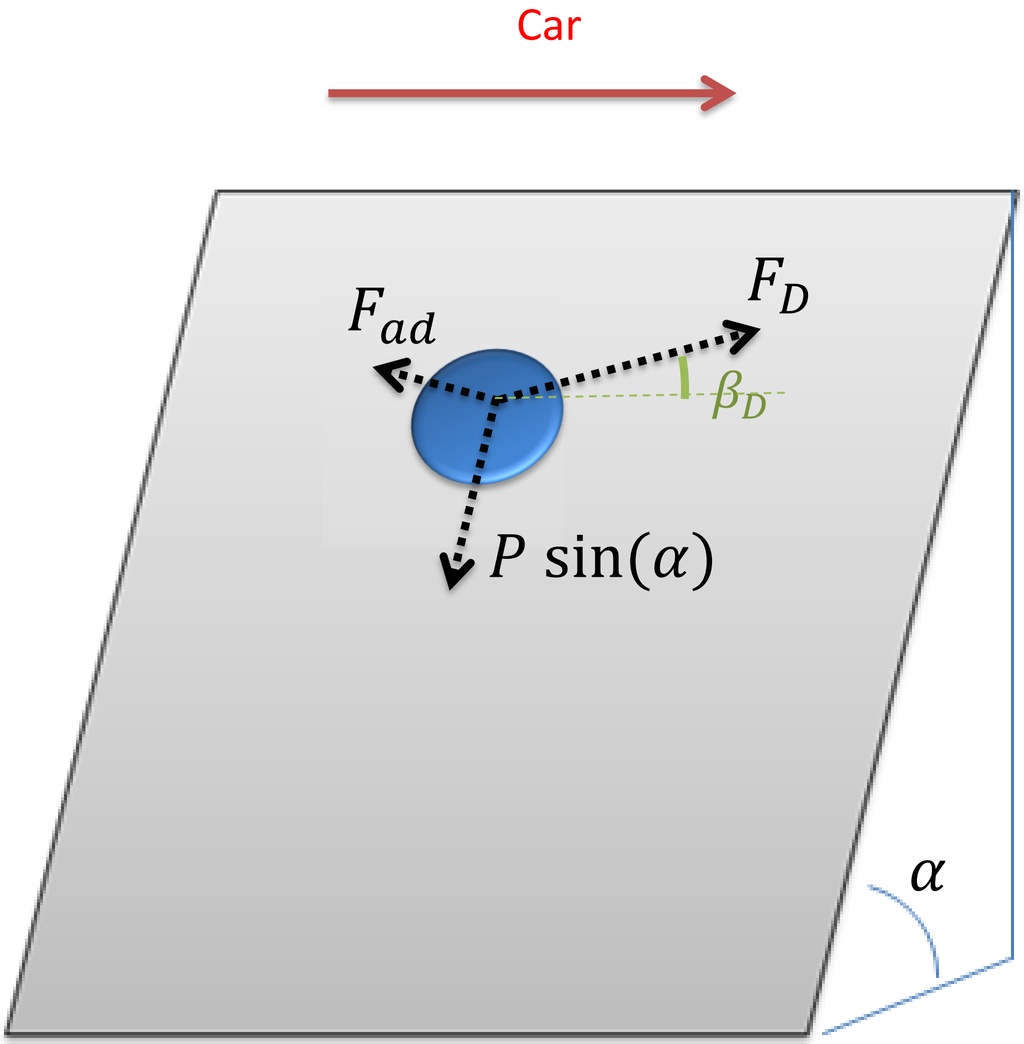
- **The drops will partly follow the wind**

One droplet



Will it stay ?

One droplet



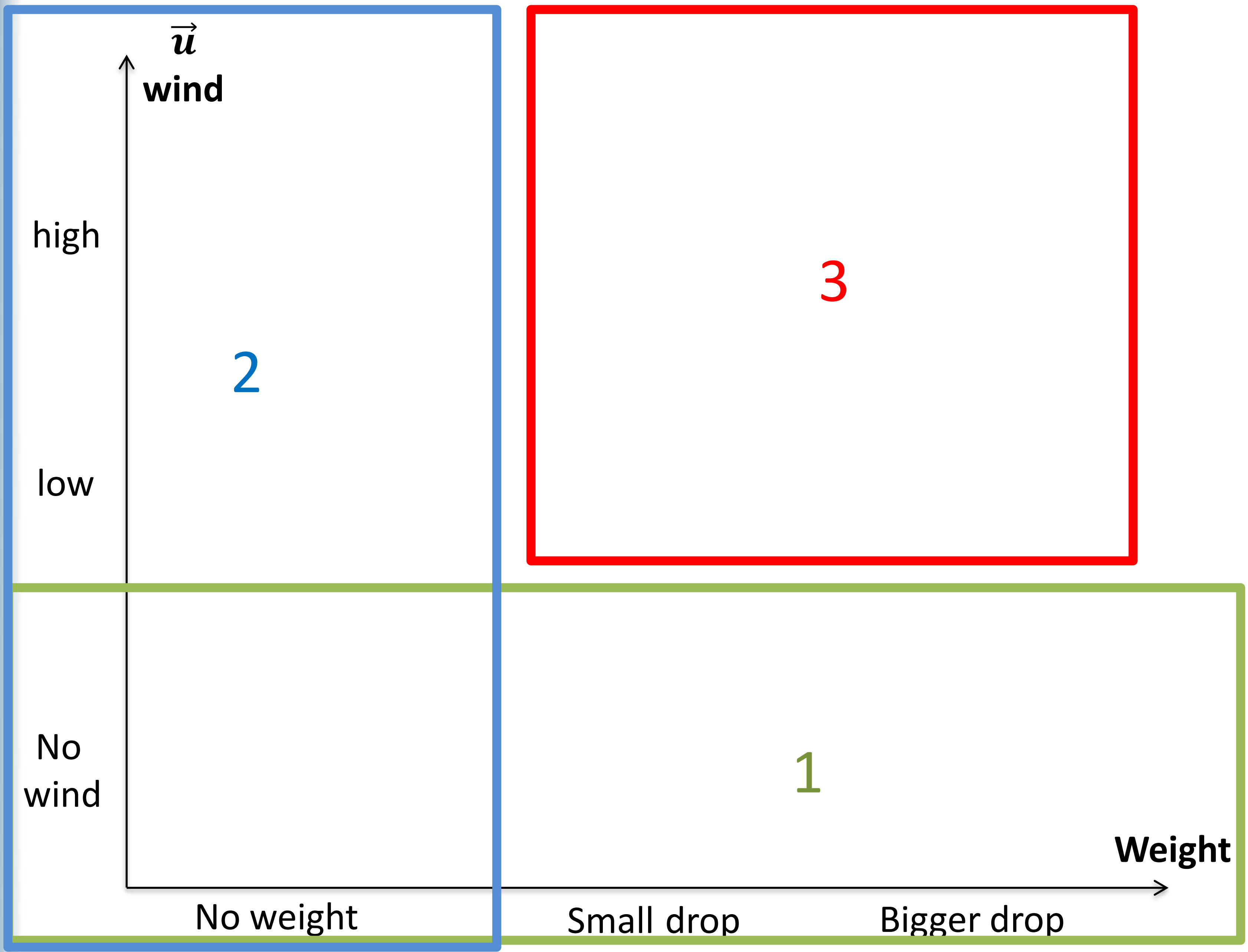
Will it stay ?

3 forces :

- Drag force F_D
- Weight P
- Adhesion F_{ad}

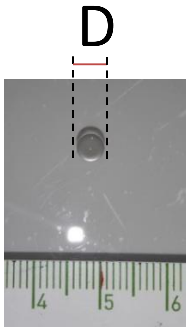
Many parameters !

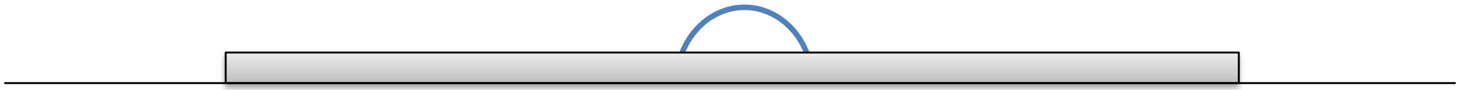
Let's begin by simple experiments



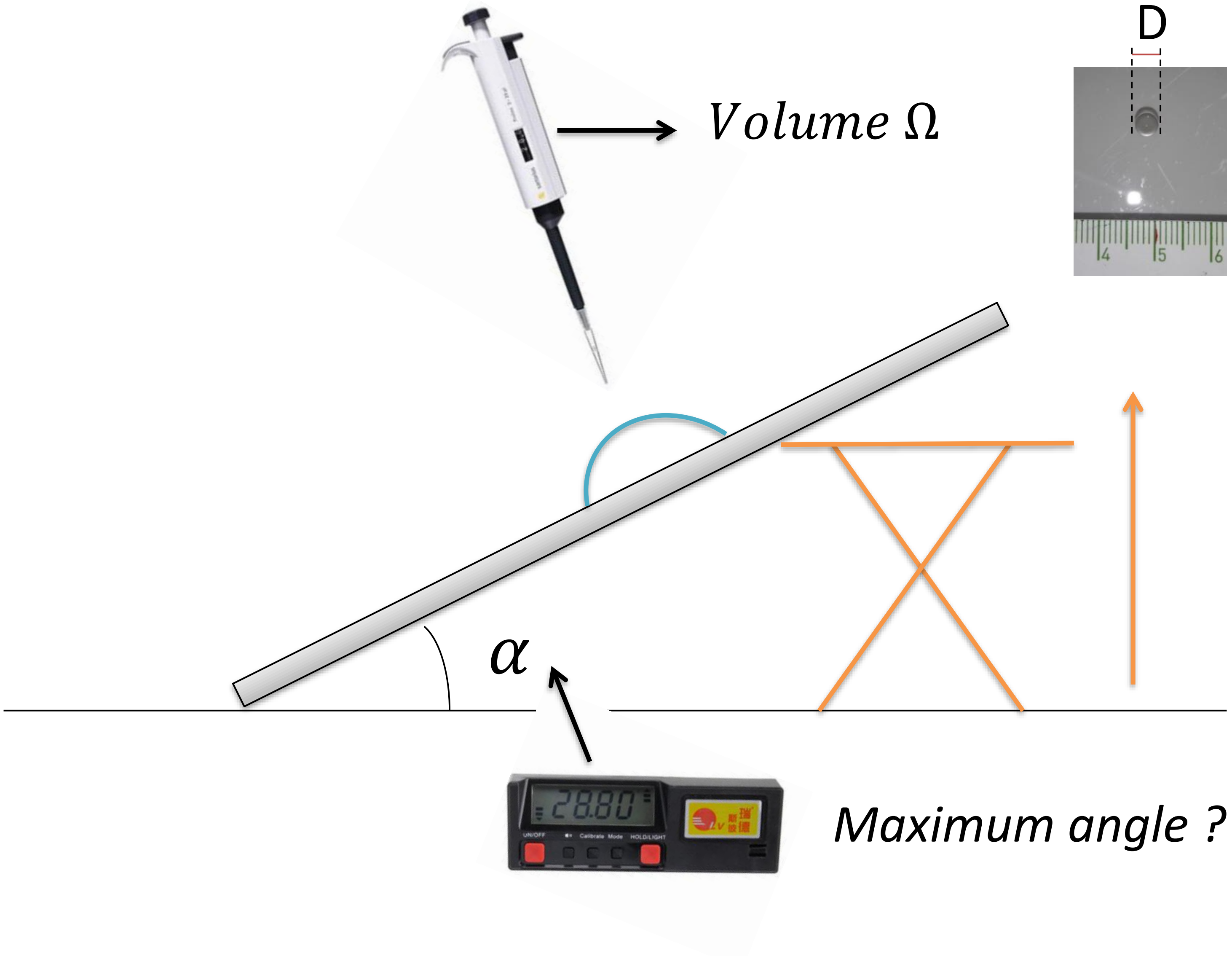
Will a drop stay ? Fighting gravity …

$Volume\ \Omega$

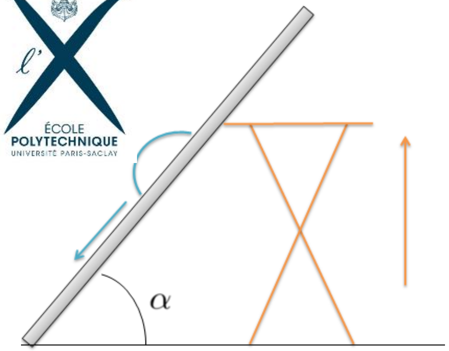


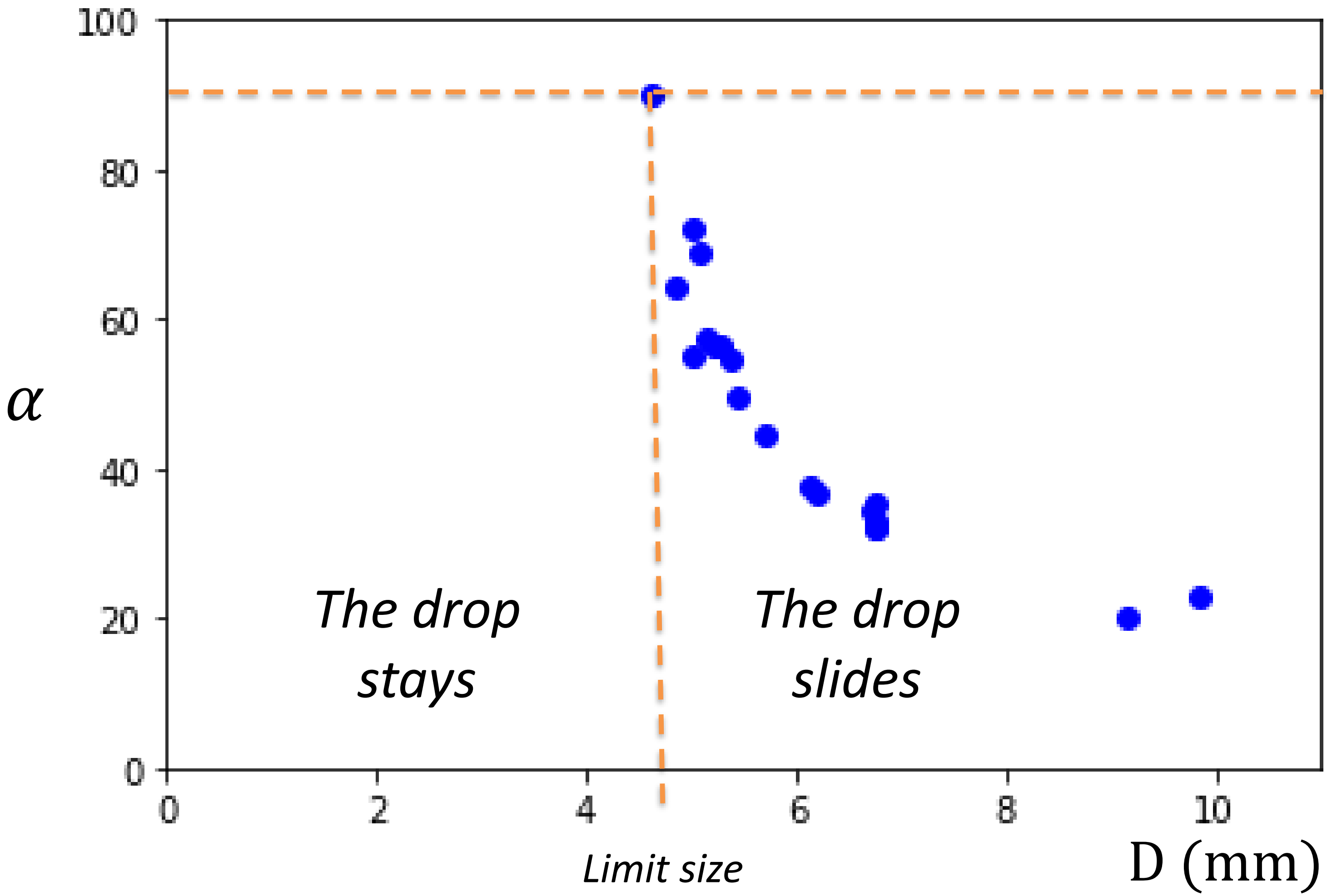


Will a drop stay ? Fighting gravity ...



Angle needed to dislodge a drop



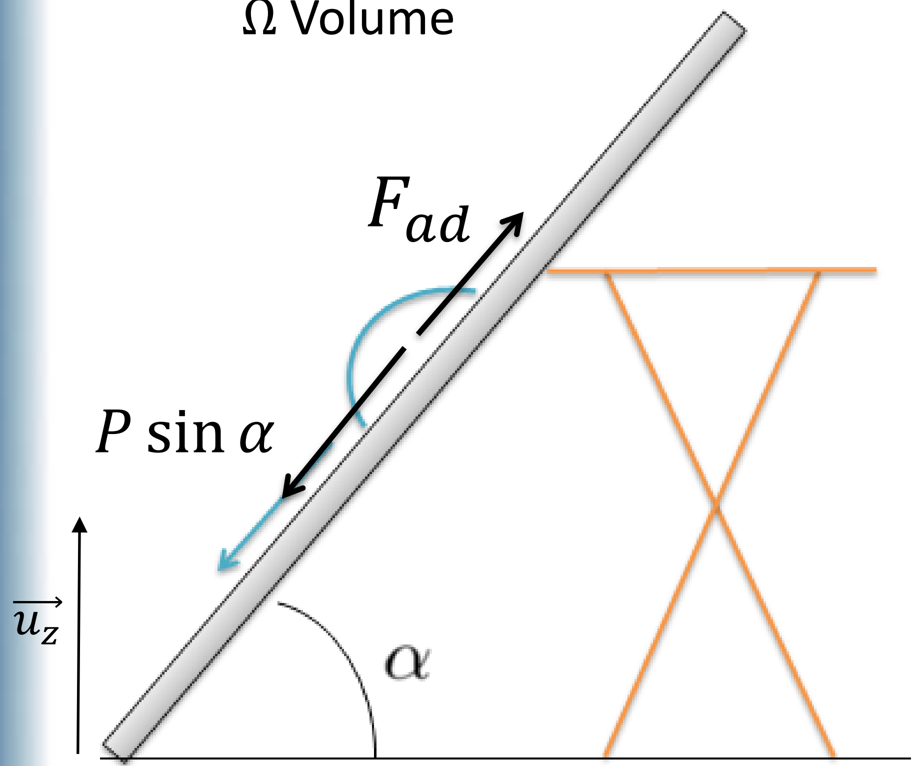


Modelisation – Maximal angle

A drop …

D Diametre

Ω Volume



Adhesion force $F_{ad} = ?$

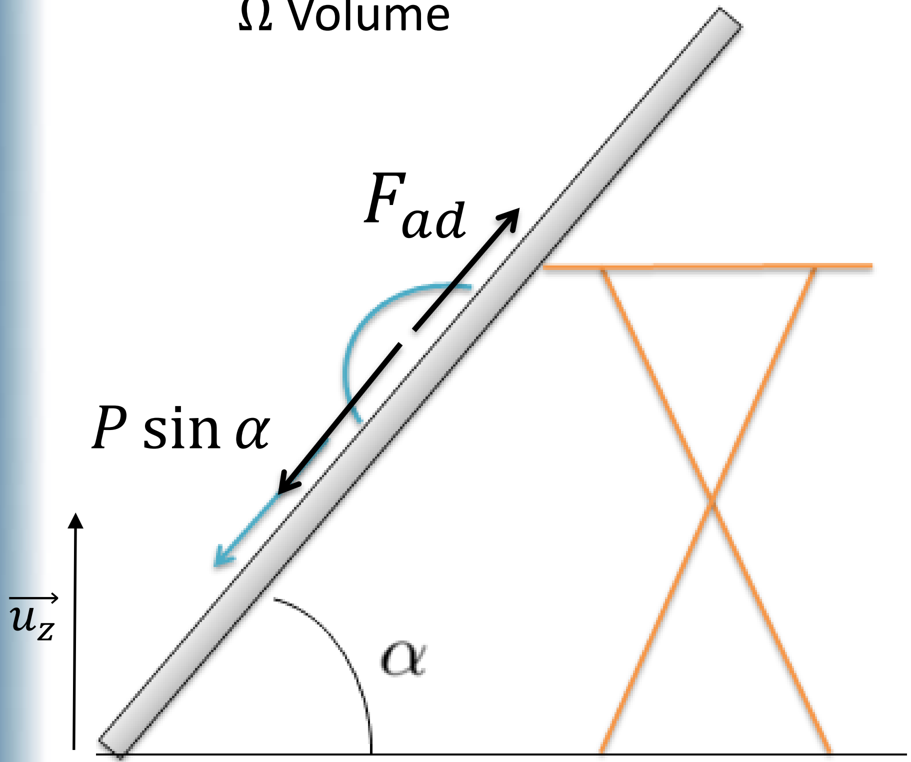
$$\vec{P} = \rho_{water} g \Omega \overrightarrow{u_z}$$

Modelisation – Maximal angle

A drop …

D Diametre

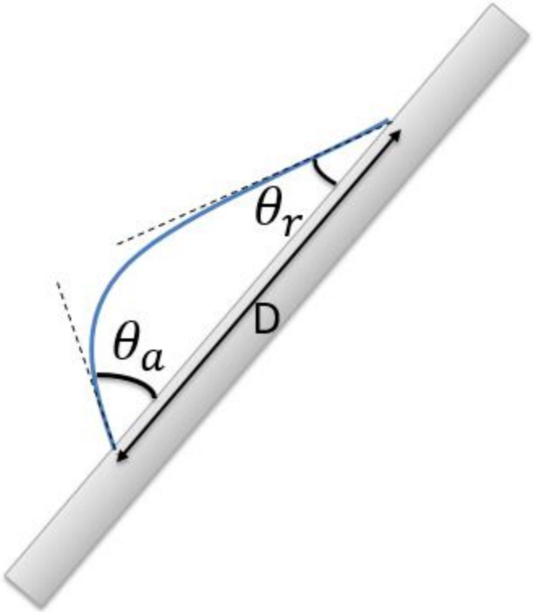
Ω Volume



$$\vec{P} = \rho_{water} g \Omega \overrightarrow{u_z}$$

Model of Furmidge

Journal of colloid science, 1962

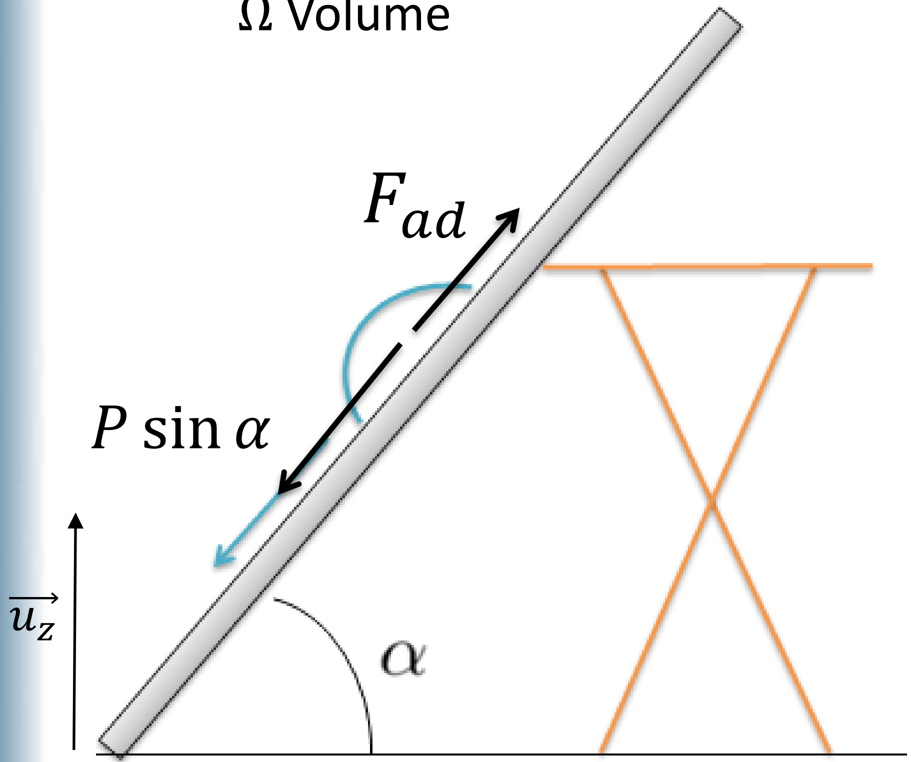


Modelisation – Maximal angle

A drop …

D Diametre

Ω Volume



$$\vec{P} = \rho_{water} g \Omega \overrightarrow{u_z}$$

Model of Furmidge

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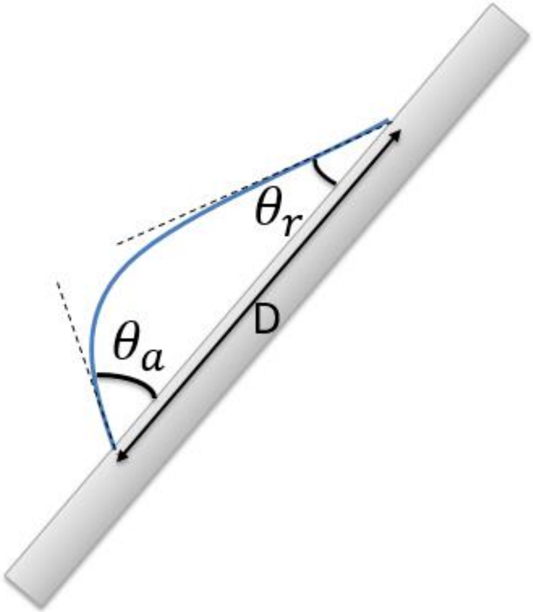
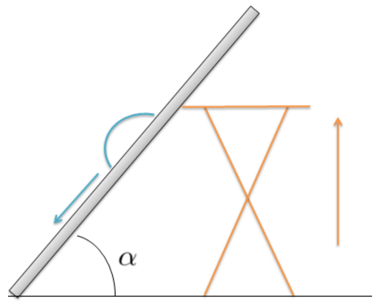


- Maximum before sliding
- Direction = axis of the deformation

$$\boldsymbol{F_{ad}} \leq \gamma(\cos\theta_r - \cos\theta_a)\mathbf{D}$$

γ = superficial tension of water

Back to results : Maximum angle



$$P \sin\alpha = F_{ad} \implies \rho g \sin\alpha = \gamma(\cos\theta_r - \cos\theta_a)\frac{D}{\Omega}$$

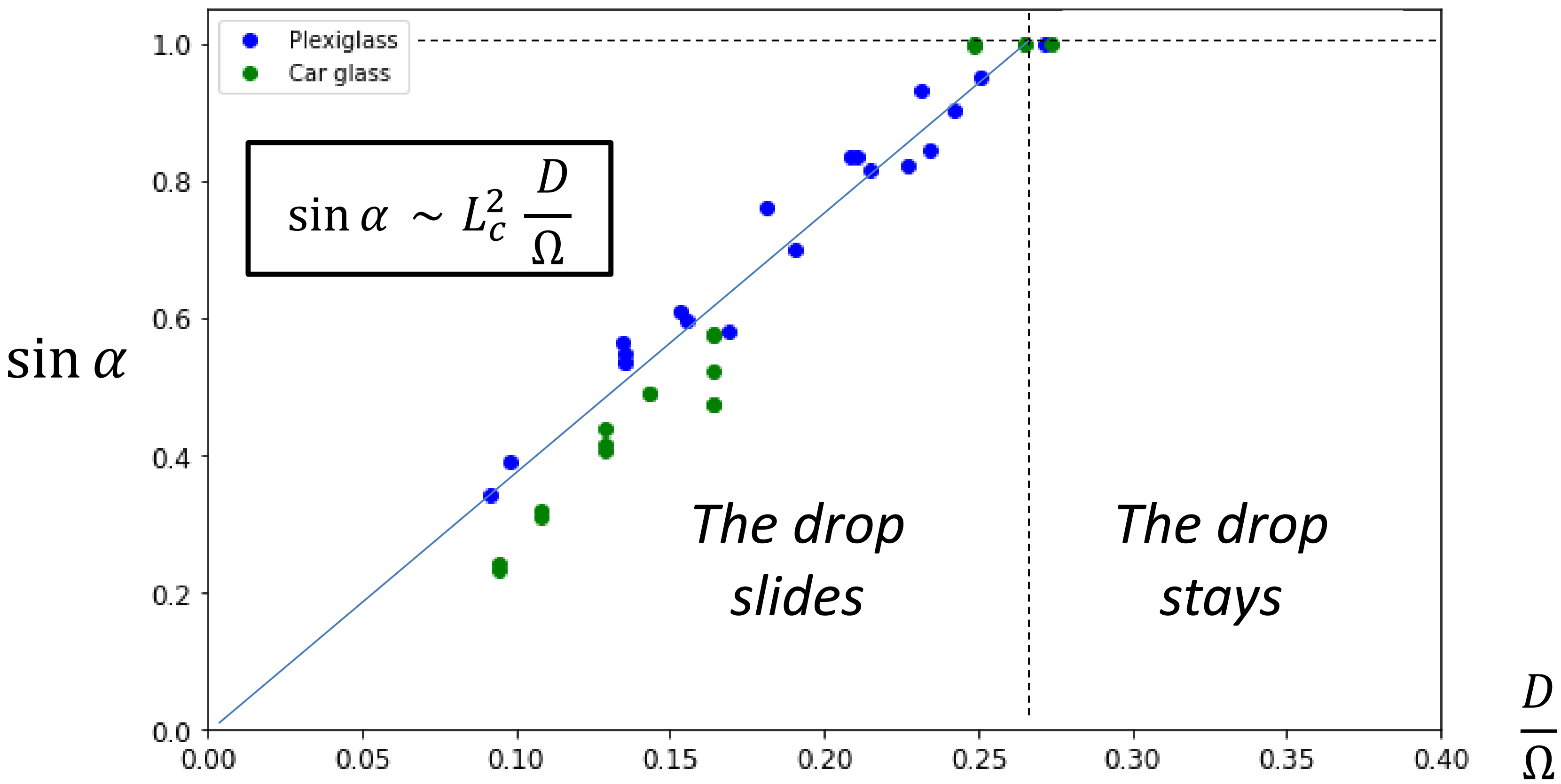
$$\implies \sin\alpha = \underbrace{(\cos\theta_r - \cos\theta_a)}_{\text{Plexiglass \& Car glass}}\ L_c^2\frac{D}{\Omega}$$

$$\implies \boldsymbol{\sin\alpha \sim L_c^2\,\frac{D}{\Omega}}$$

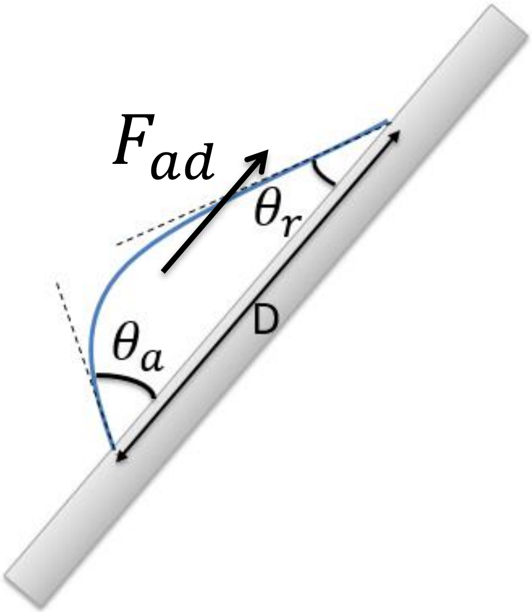
(with $L_c = \sqrt{\frac{\gamma}{\rho g}}$)

Back to results : Maximum angle

$$P \sin\alpha = F_{ad} \quad \Longrightarrow \quad \sin\alpha = \underbrace{(\cos\theta_r - \cos\theta_a)}_{\text{Plexiglass \& Car glass}} \; L_c^2 \frac{D}{\Omega}$$



Advantage of this model

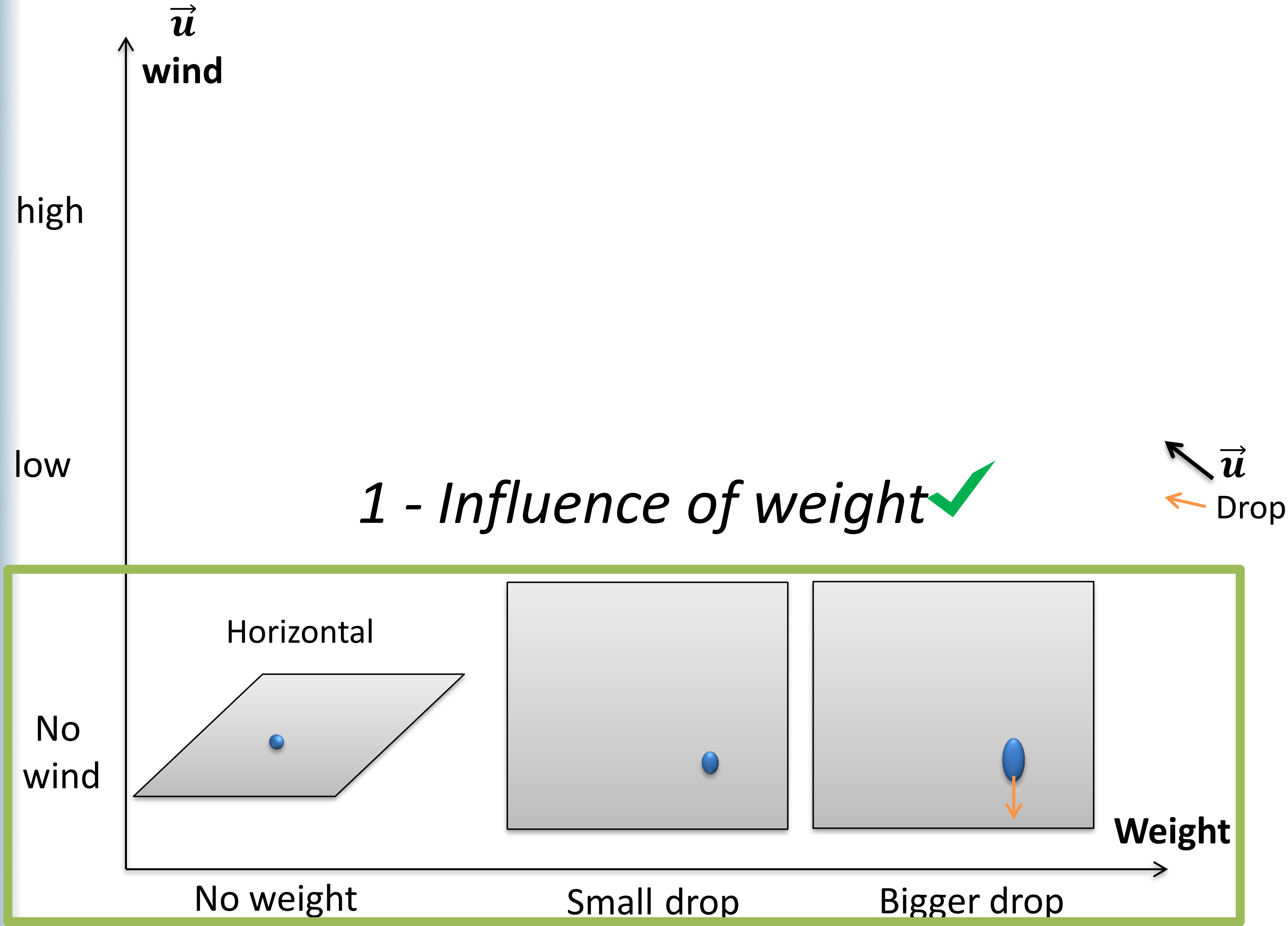


A model for the maximum adhesion :

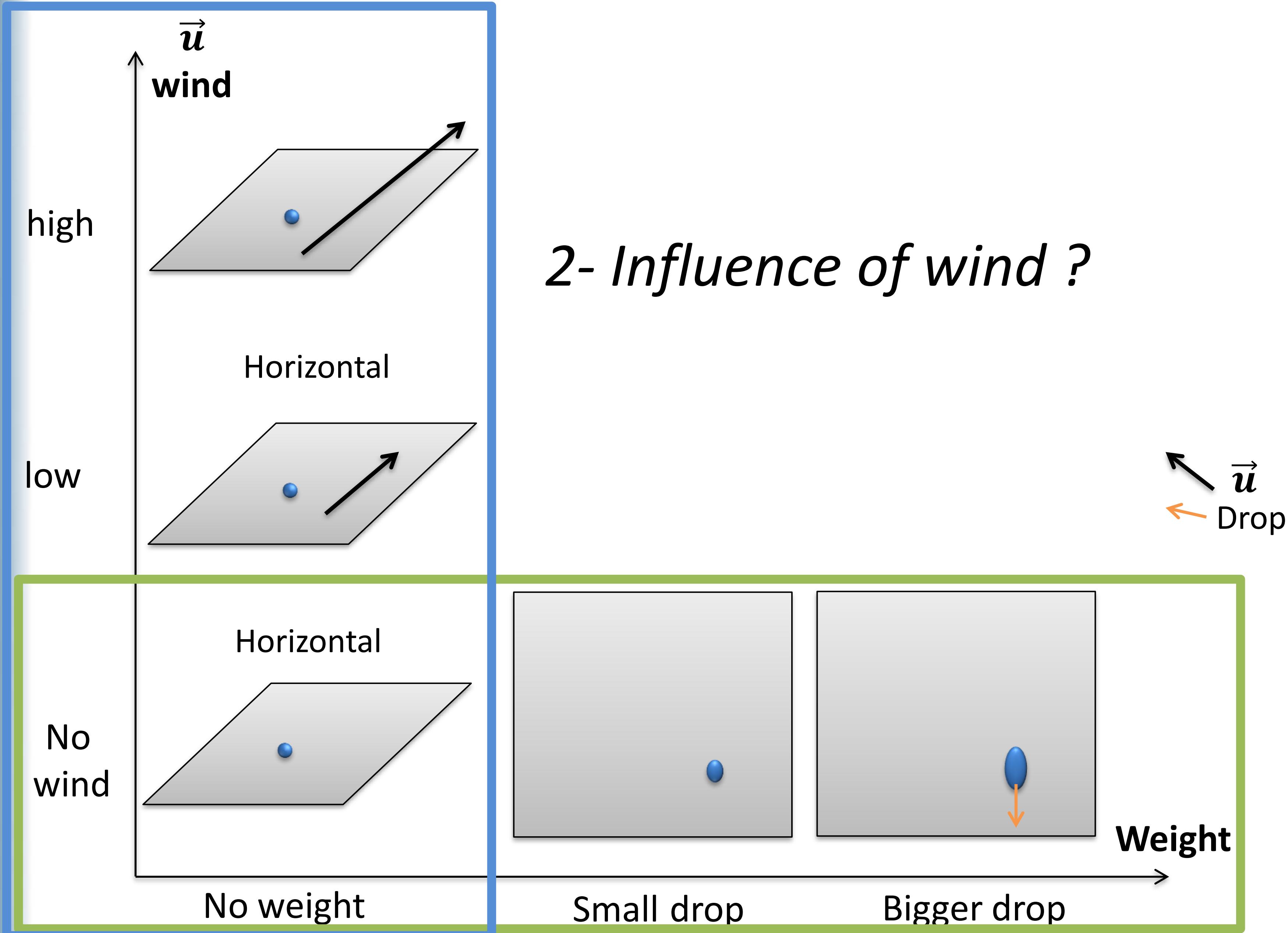
$$F_{ad} = \gamma(\cos\theta_r - \cos\theta_a)D$$

proportional to diameter.

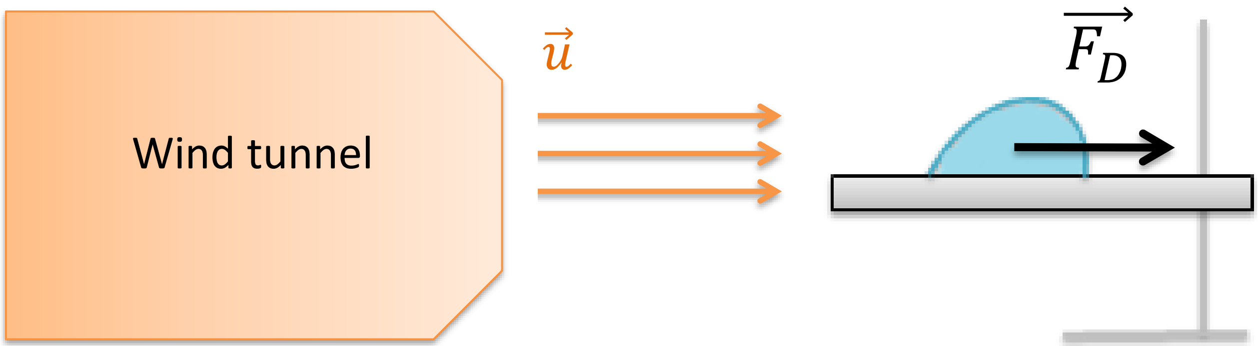
1 - Influence of weight



2- Influence of wind ?

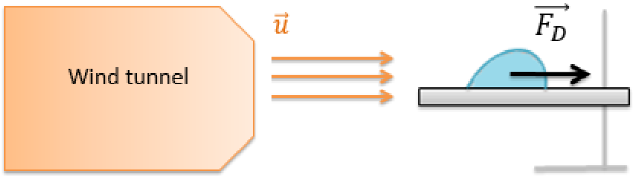


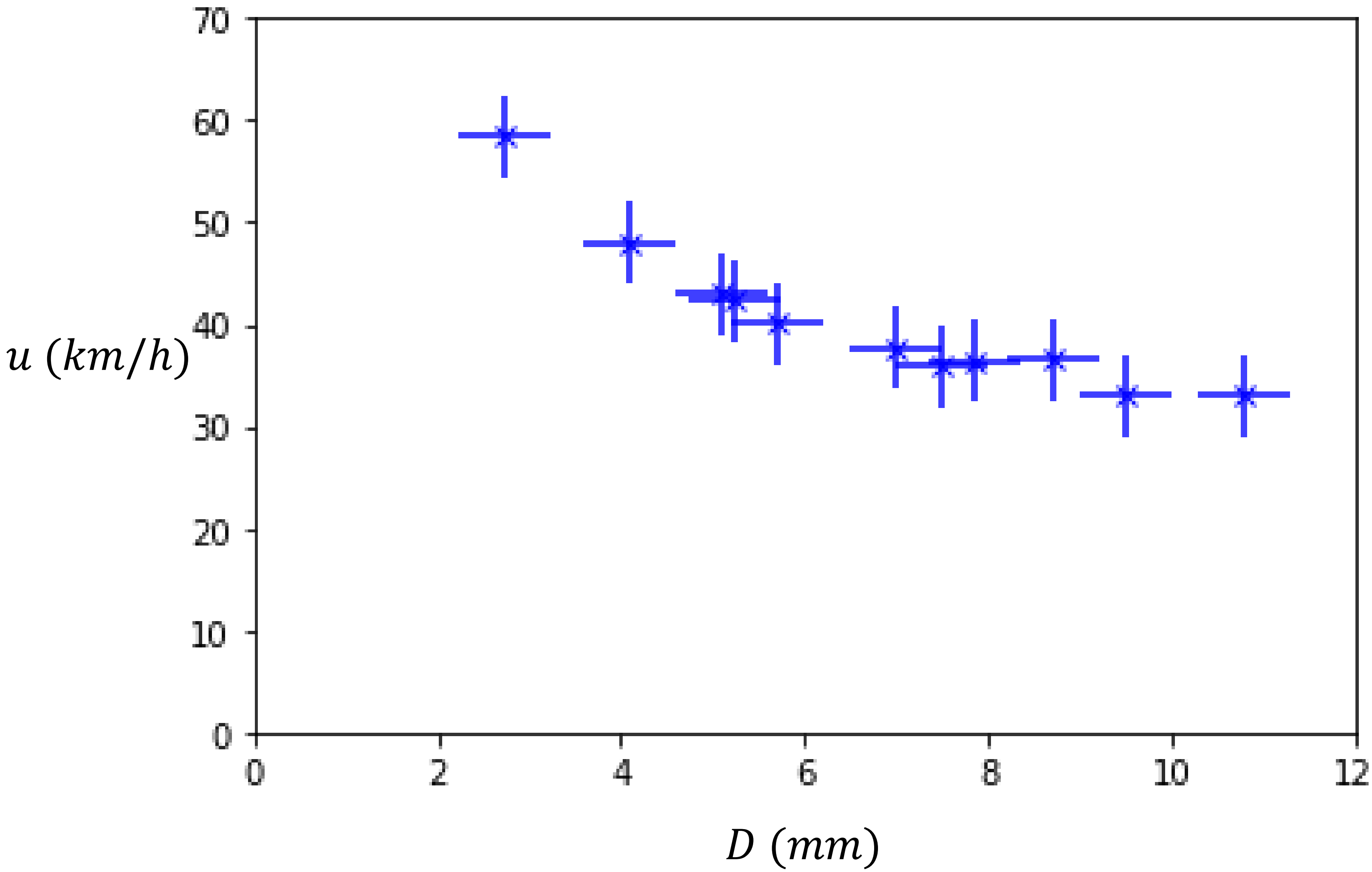
Wind needed to dislodge a drop ?



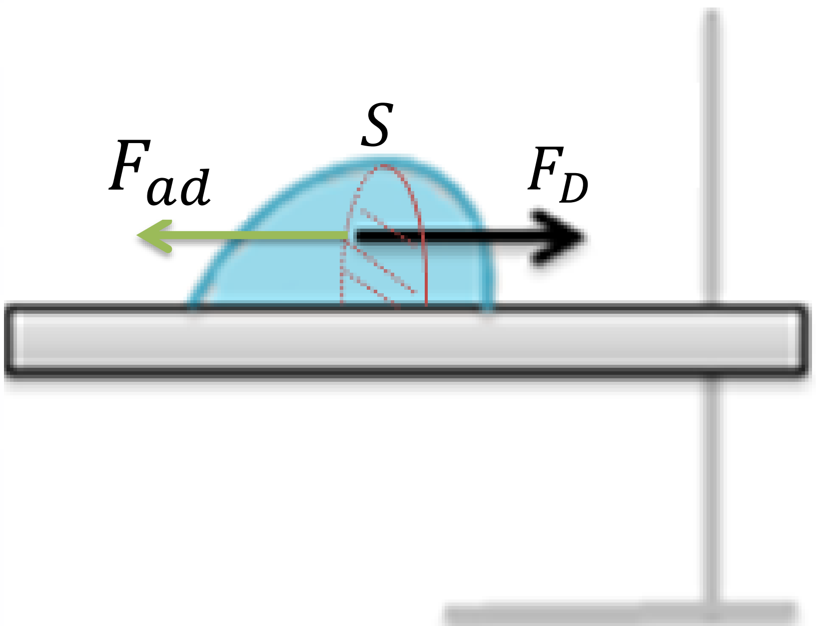
$0 \leq u \leq 25\, m/s$

Wind needed to dislodge a drop ?





Wind needed to dislodge a drop ?



$Re \gg 1$

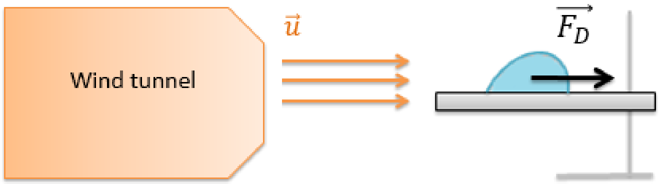
Drag force $F_D = \frac{1}{2} C_x S u^2$

$F_{ad} = F_D$

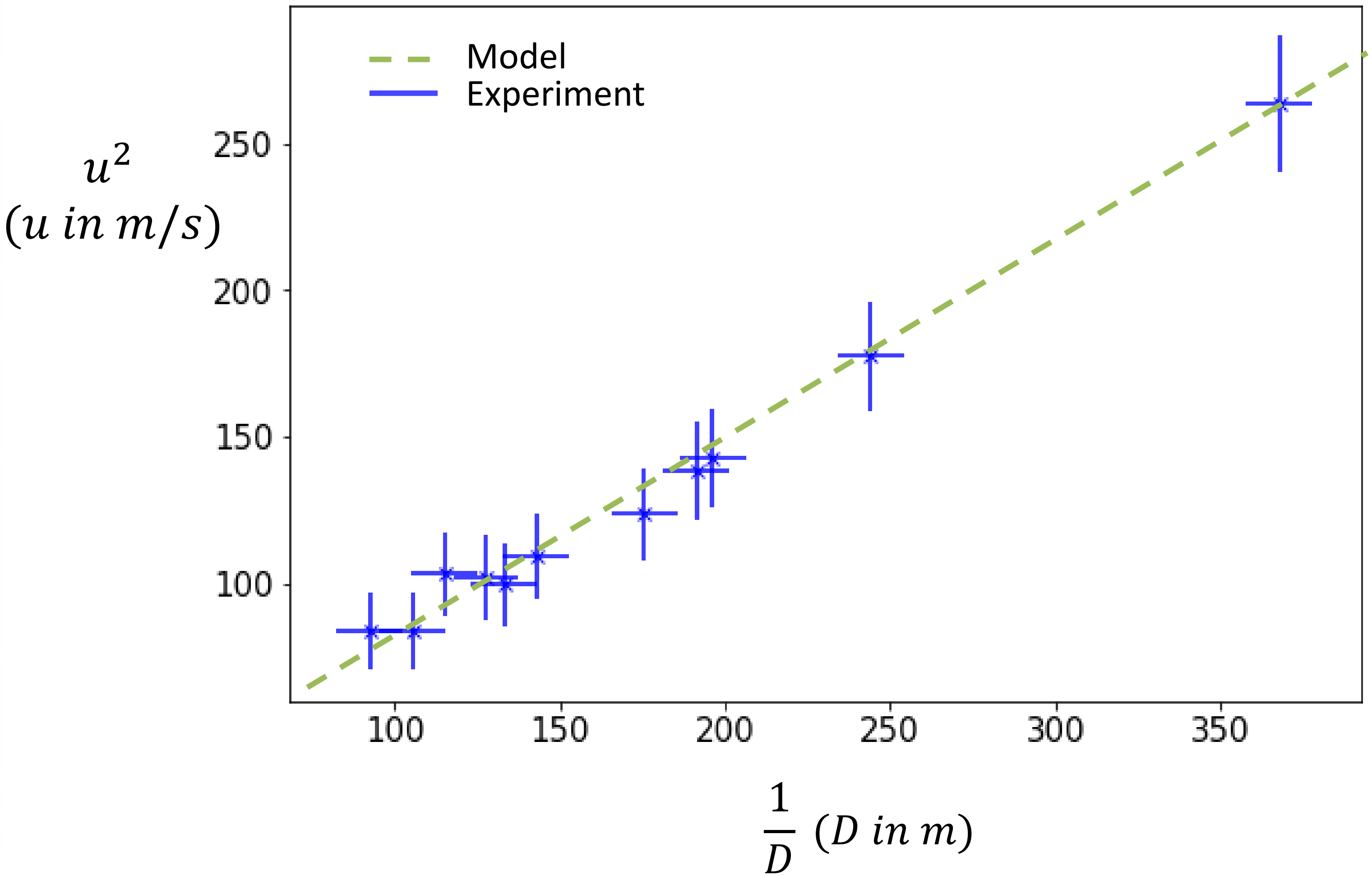
$$u^2 = \frac{2\gamma(\cos\theta_r - \cos\theta_a)}{C_x} \frac{D}{S}$$

u^2 scales in $\frac{1}{D}$

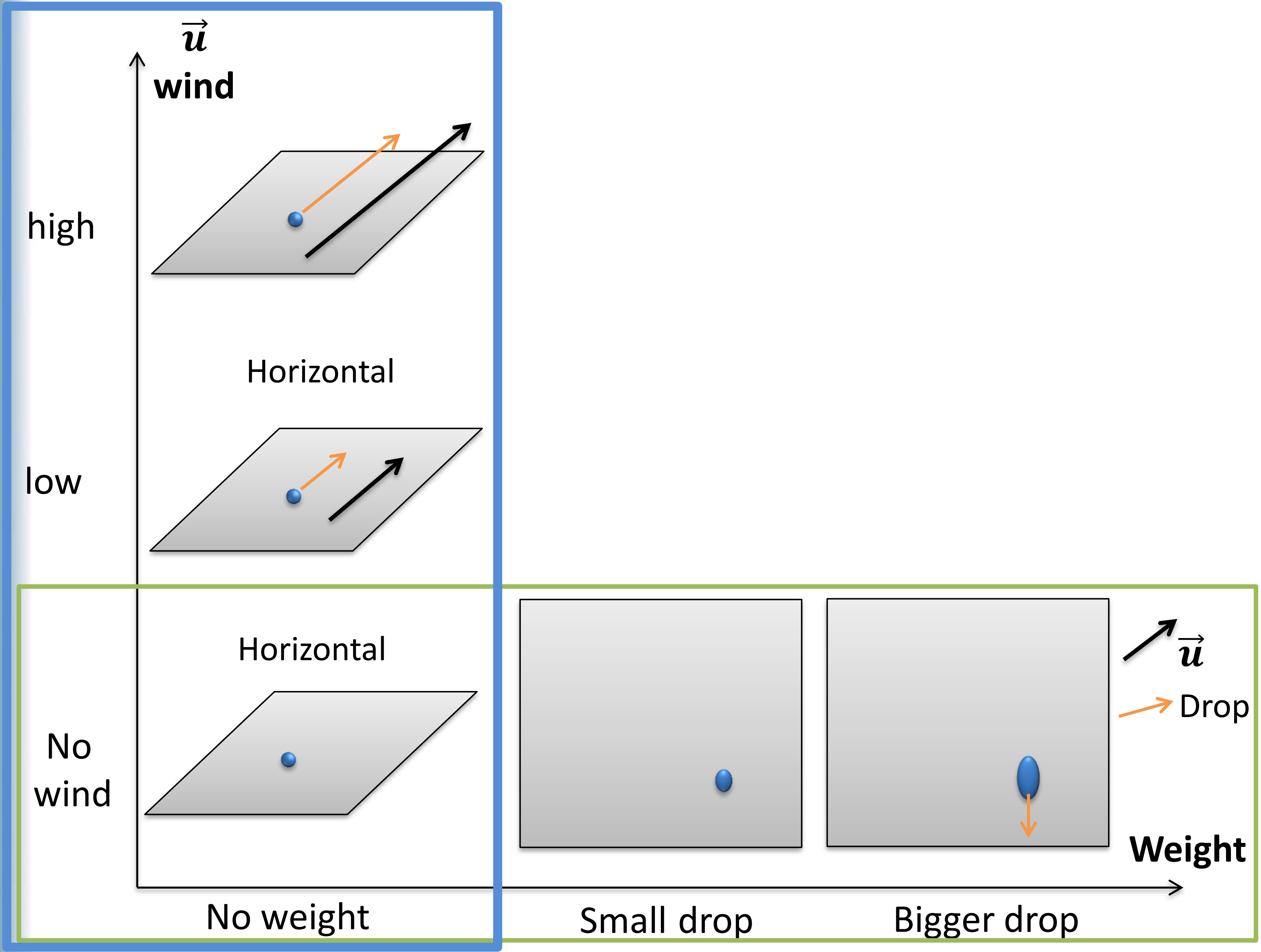
Wind needed to dislodge a drop ?



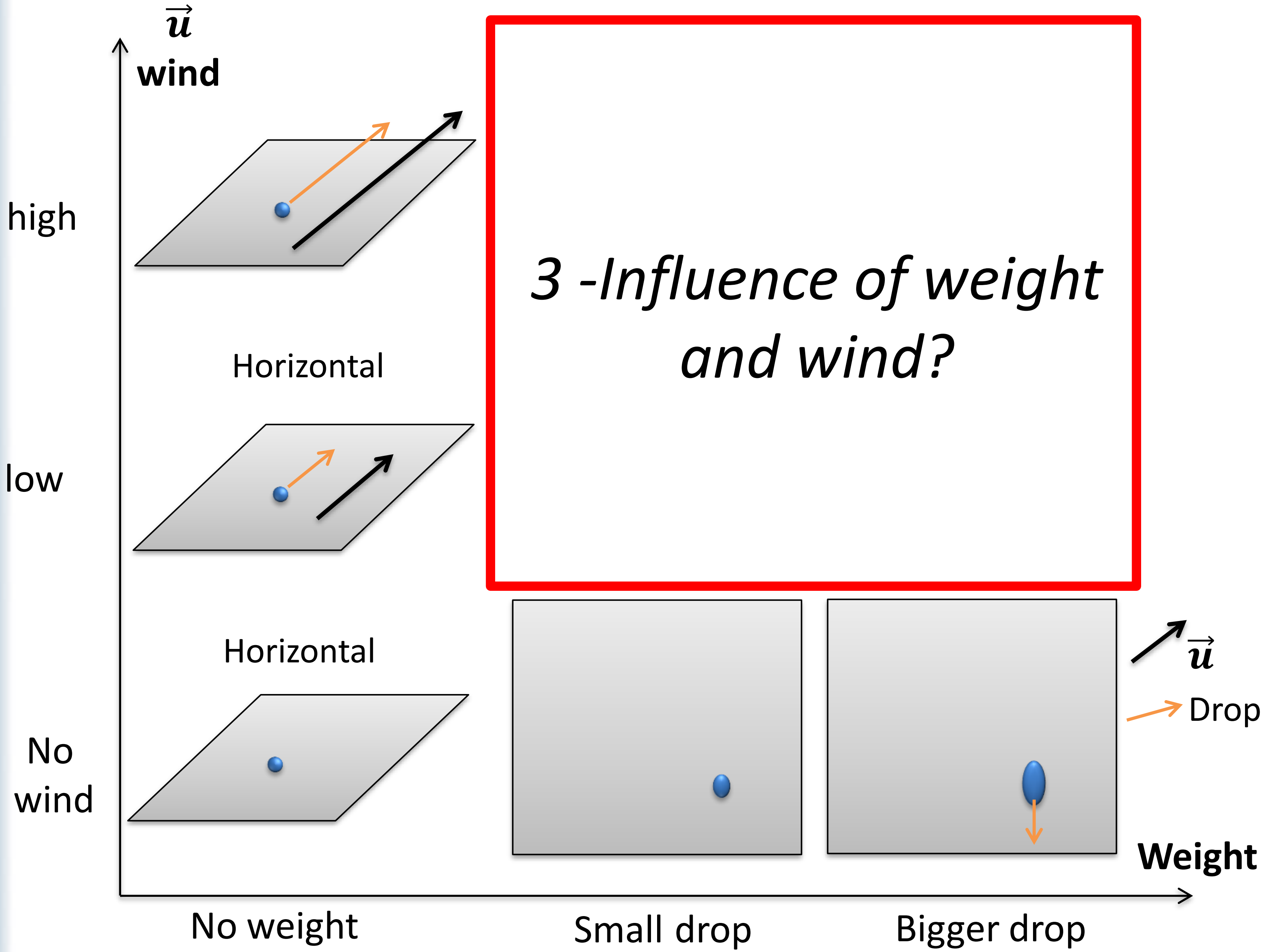
u^2 scales in $\frac{1}{D}$ ✓



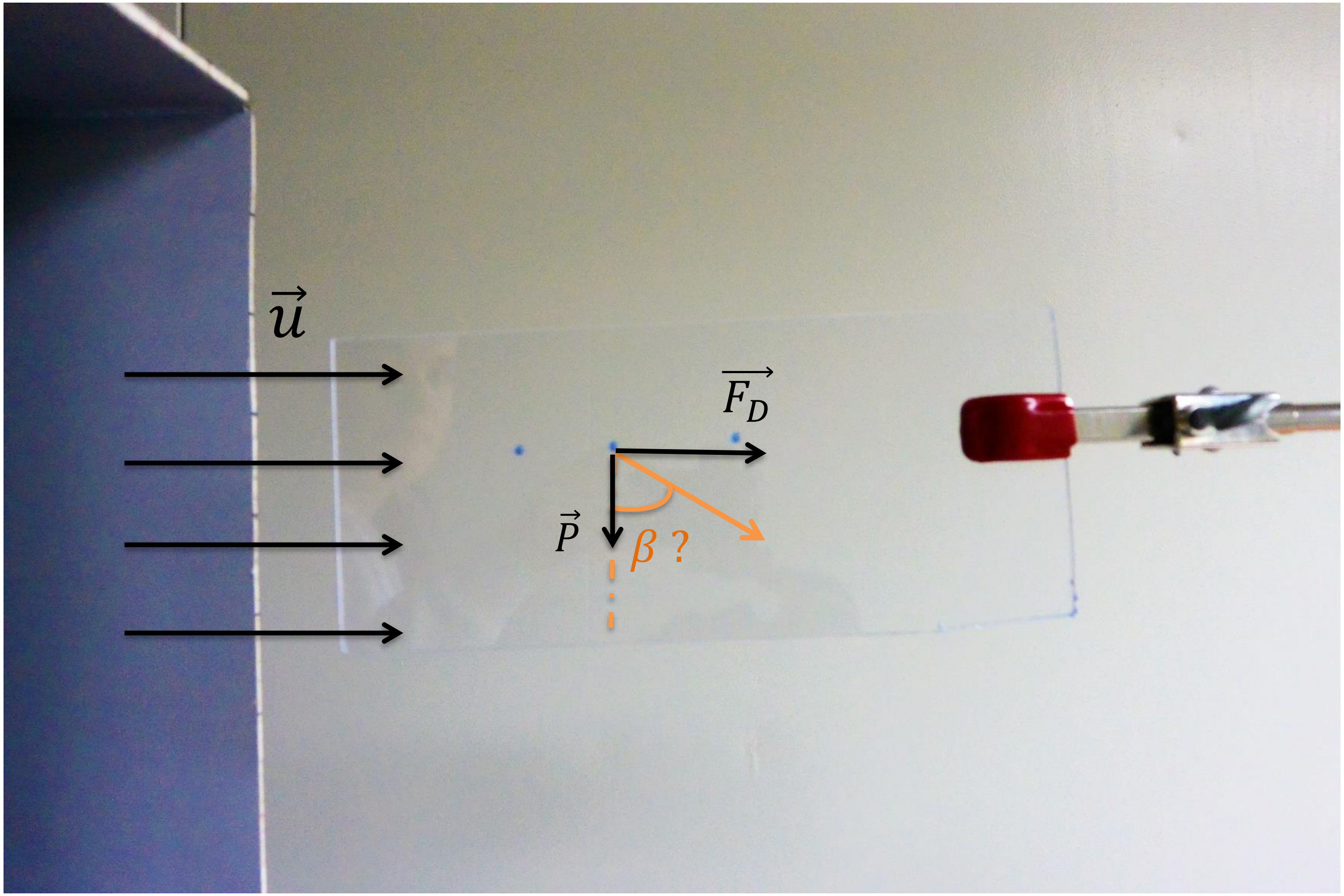
2- Influence of wind



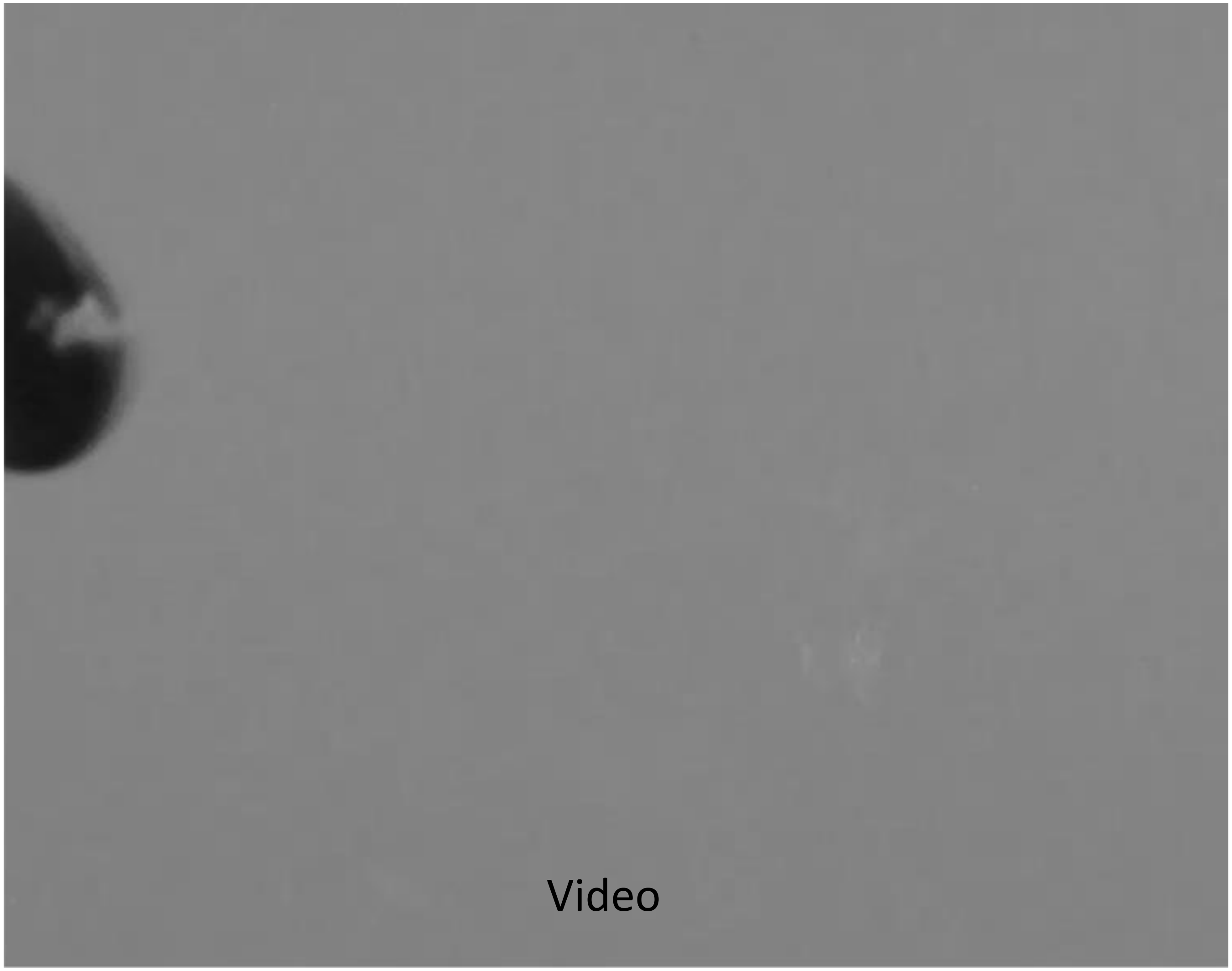
Going back to the main problem



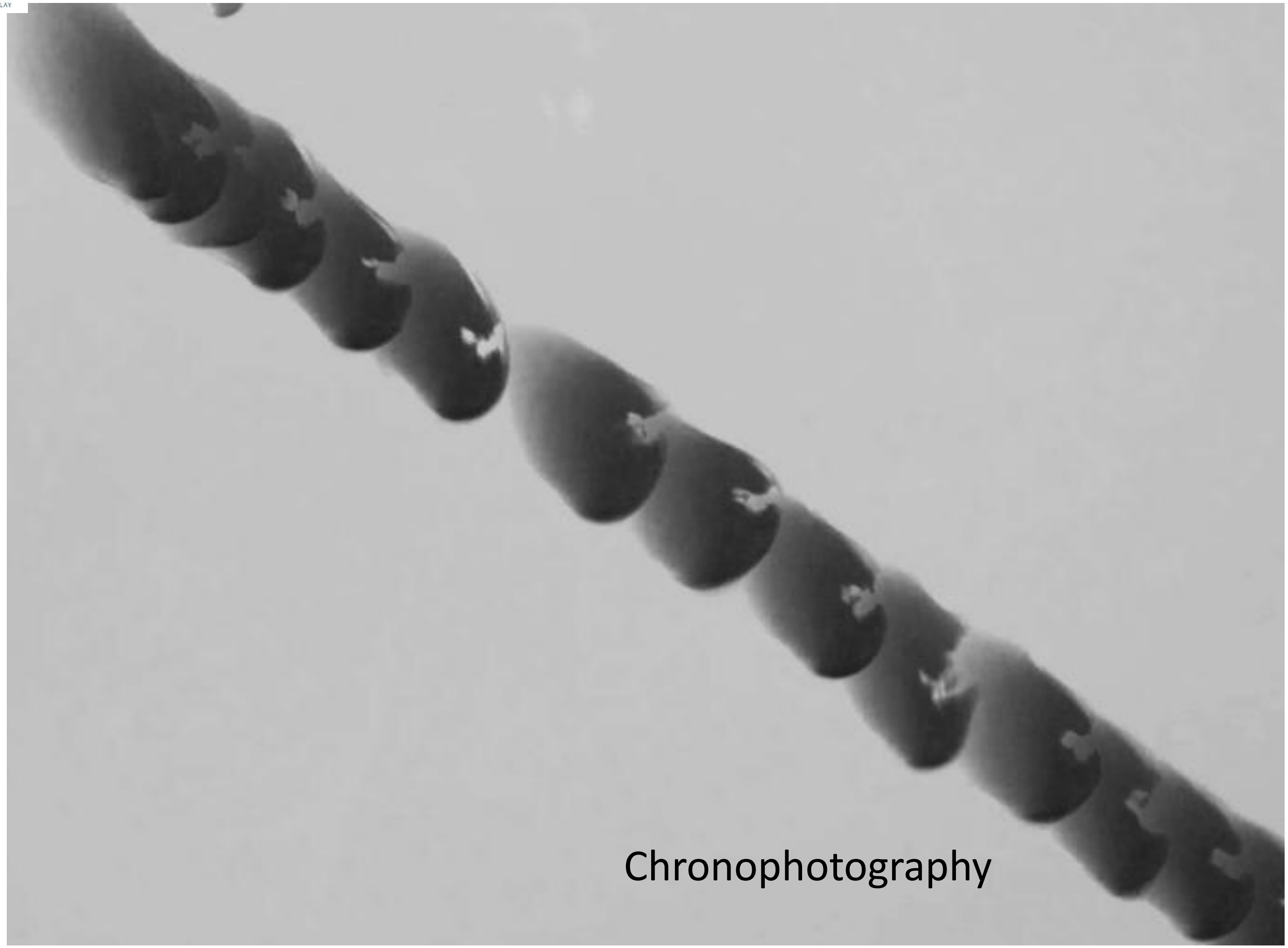
Direction of the drop ? Horizontal wind



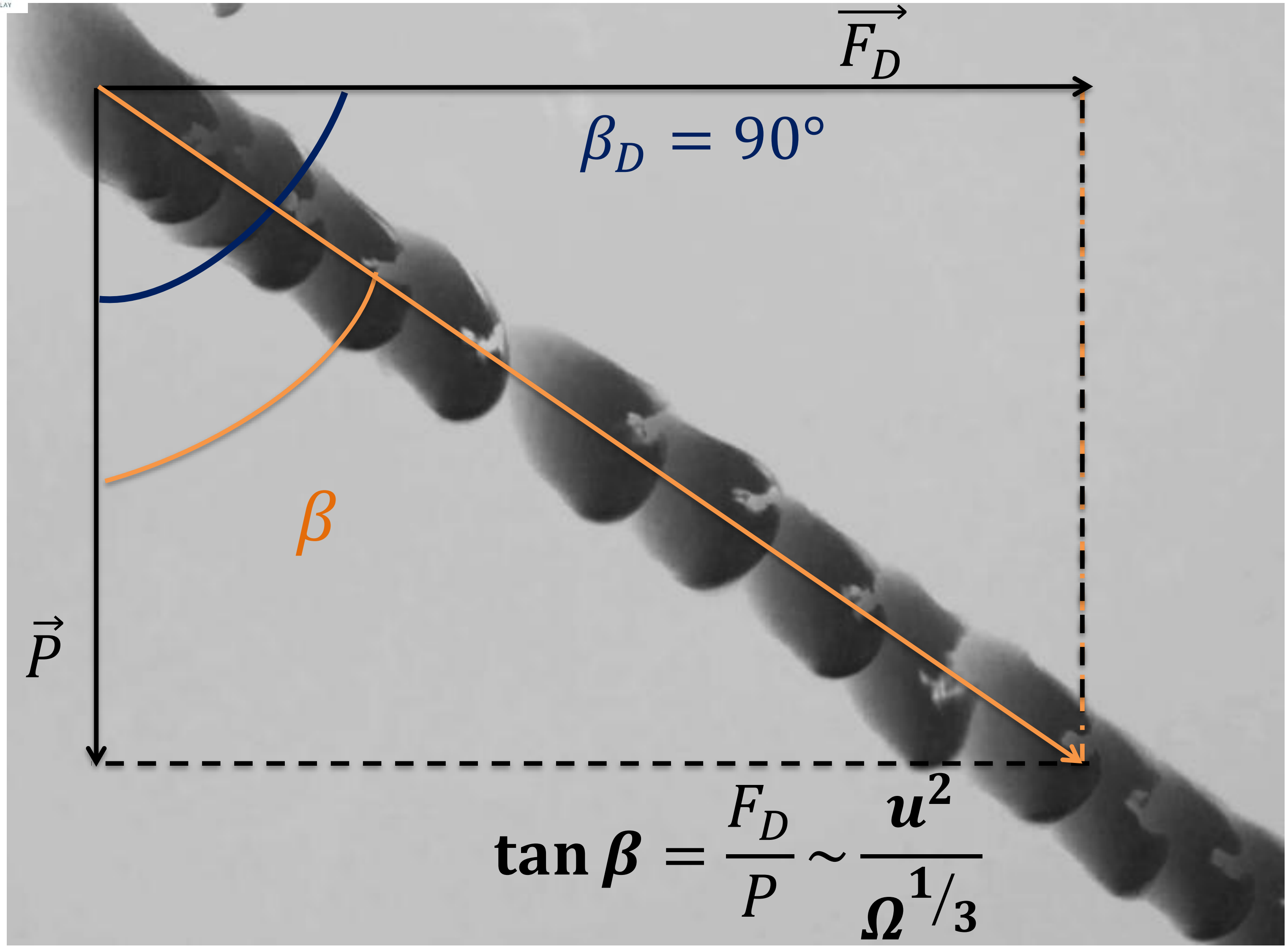
Direction of the drop ? Horizontal wind



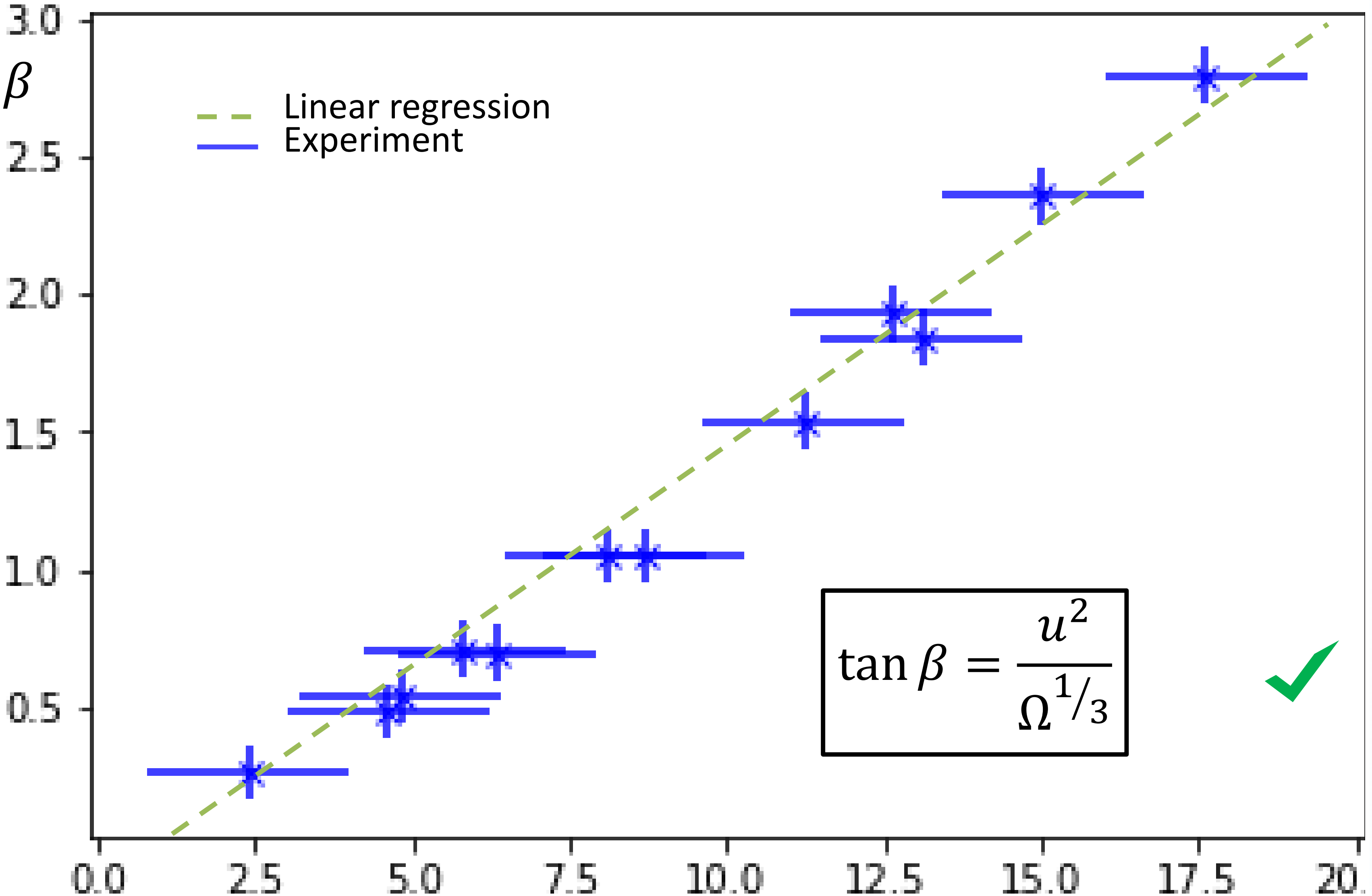
Direction of the drop ? Horizontal wind



Direction of the drop ? Horizontal wind



Direction of the drop ?

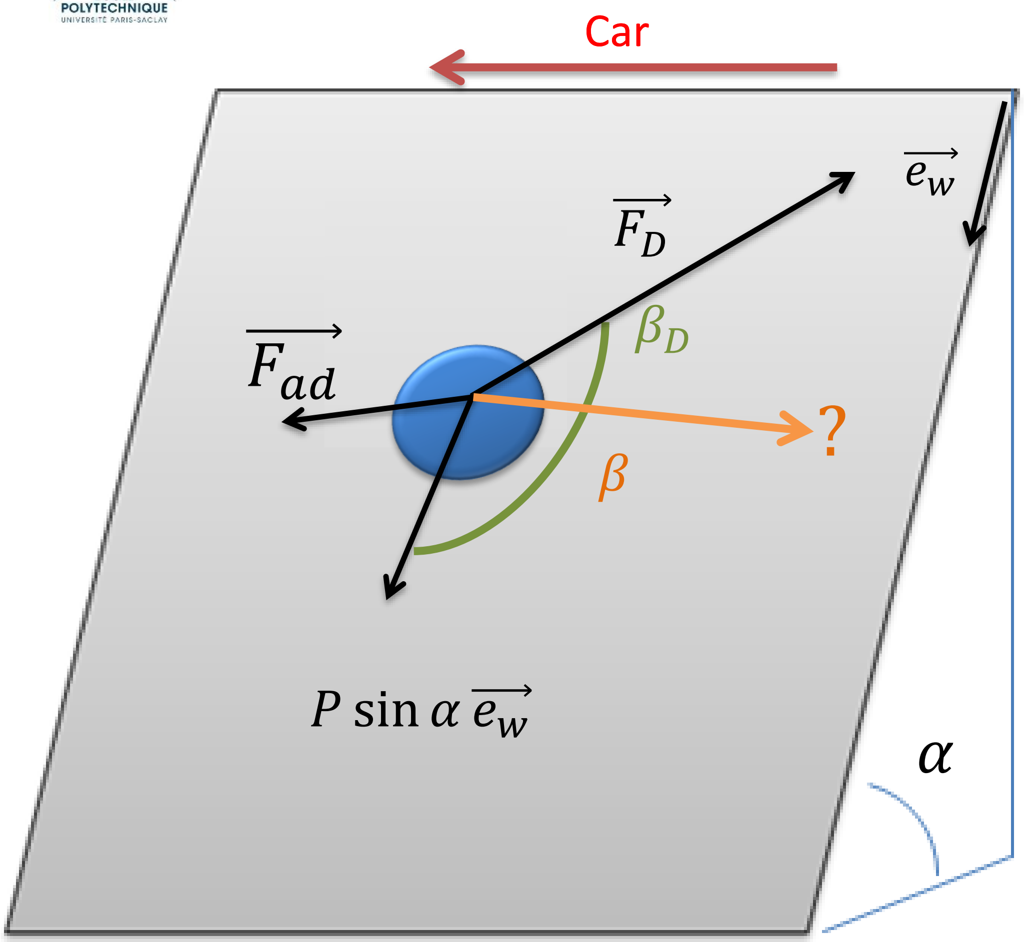


$\tan\beta$

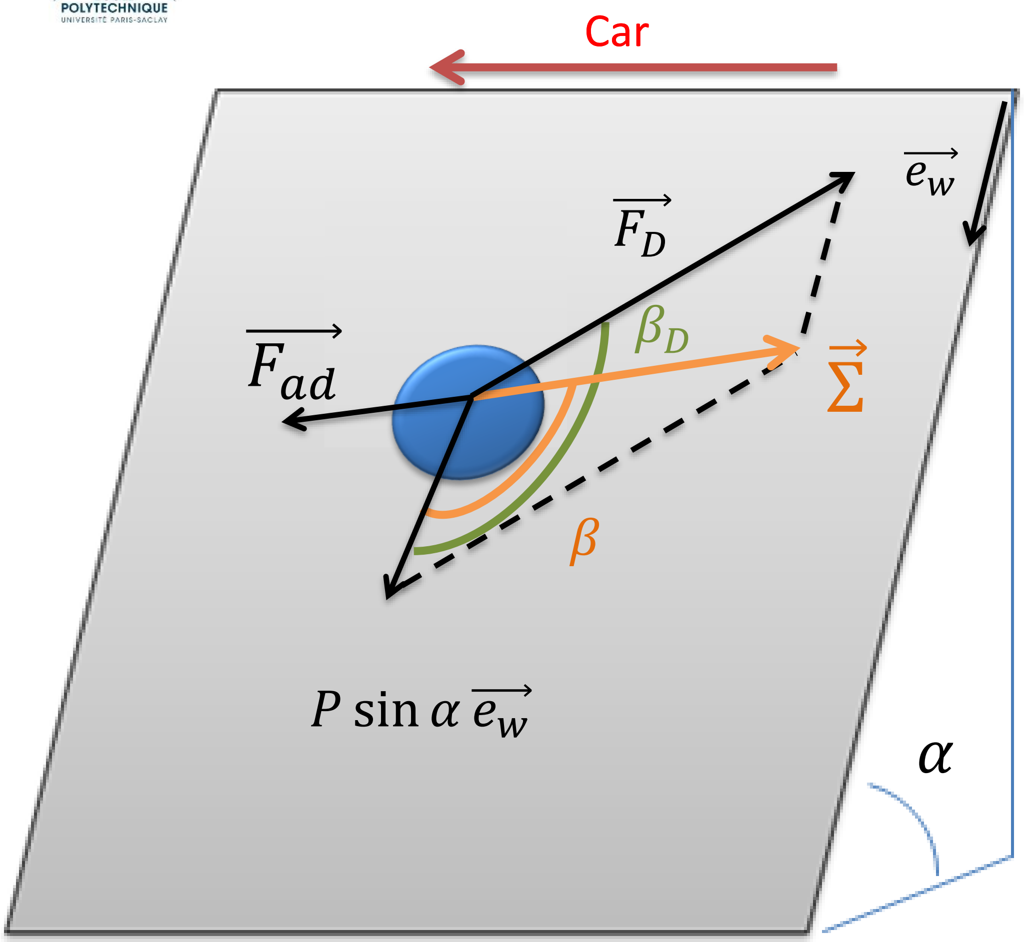
$$\tan\beta = \frac{u^2}{\Omega^{1/3}}$$

$$\frac{u^2}{\Omega^{1/3}}$$

Direction of the drop ? Any wind



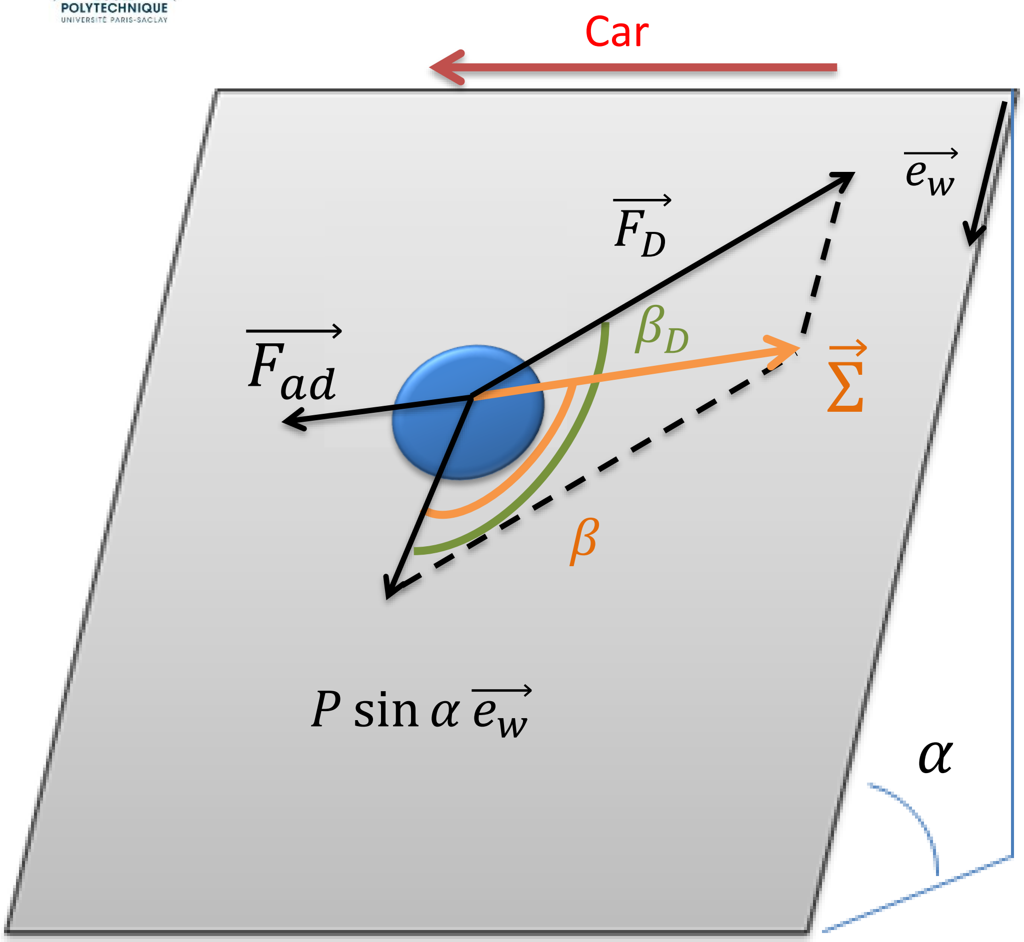
Direction of the drop ? Any wind



If a drop slides : $F_{ad} \leq \Sigma$

$$\vec{\Sigma} = \overrightarrow{F_D} + P \sin\alpha\, \overrightarrow{e_w}$$

Direction of the drop ? Any wind



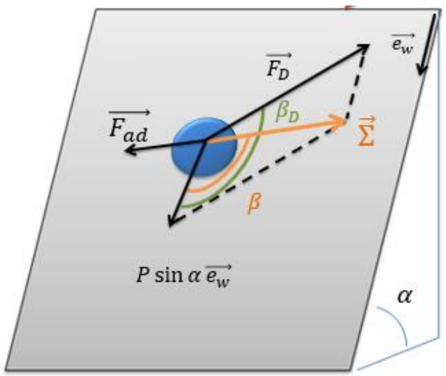
If a drop slides : $F_{ad} \leq \Sigma$

$$\vec{\Sigma} = \overrightarrow{F_D} + P \sin\alpha \overrightarrow{e_w}$$

$$\cos\beta = \frac{\vec{\Sigma}.\overrightarrow{e_w}}{\|\vec{\Sigma}\|}$$

$$\cos\beta = \frac{P\sin\alpha + F_D \cos\beta_D}{\sqrt{(P\sin\alpha)^2 + 2P\sin\alpha\, F_D \cos\beta_D + (F_D)^2}}$$

Direction of the drop



$$\cos\beta = \frac{P\sin\alpha + F_D\cos\beta_D}{\sqrt{(P\sin\alpha)^2 + 2P\sin\alpha\, F_D\cos\beta_D + {F_D}^2}}$$

Checking ✔

Set up 1 : $F_D = 0$

$\cos\beta = 1$

It follows the **weight…**

Set up 2 : $weight = 0$

$\cos\beta = \cos\beta_D$

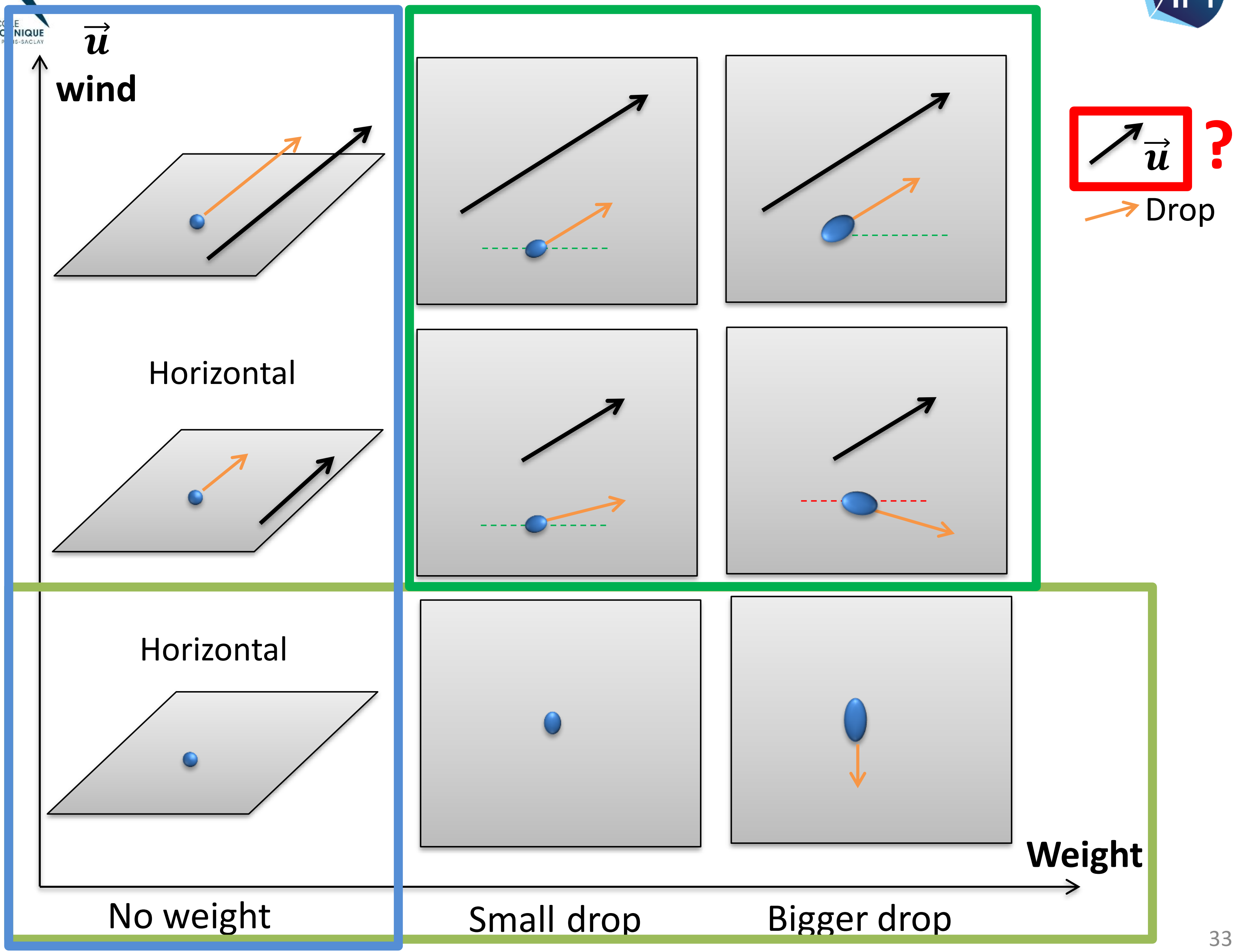
… the **wind**

Set up 3 : $\alpha = 90°$, $\beta_D = 90°$

$\tan\beta = \frac{F_D}{P}$

… the **weight** and the **wind**

Direction of the drop – Sum up



Wind on the car window ?

Experiment

Wind on the car window ?

Experiment with little threads

There IS wind going up !

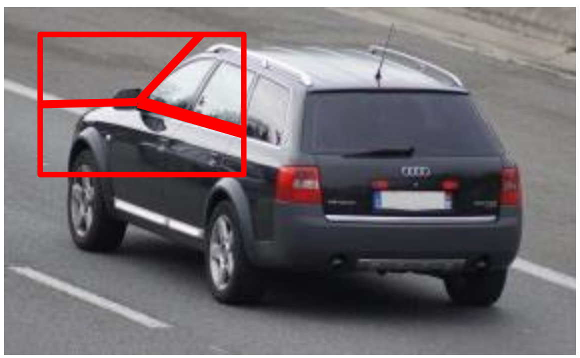
Why is there wind going up the car window ?

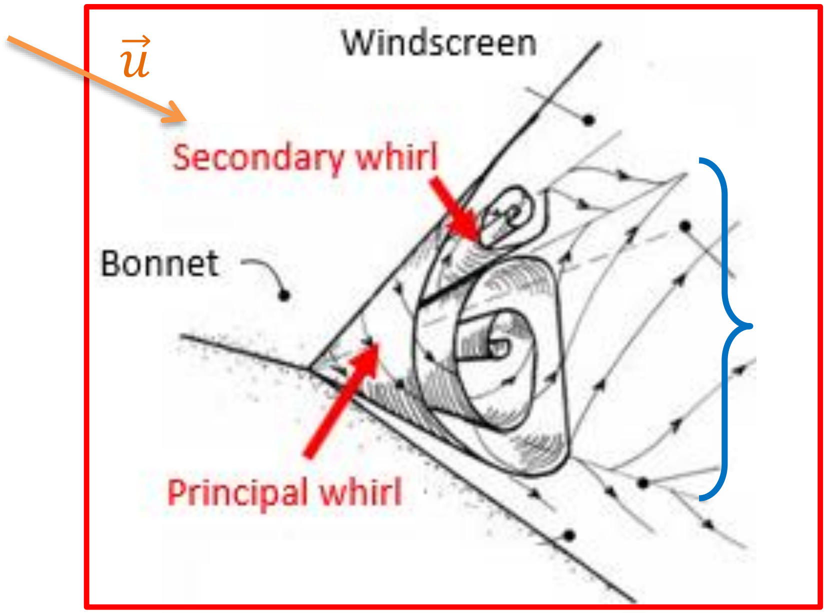
Photo : Steve Morrins

Why is there wind going up the car window ?

A cornet like structure !

$\vec{u}$

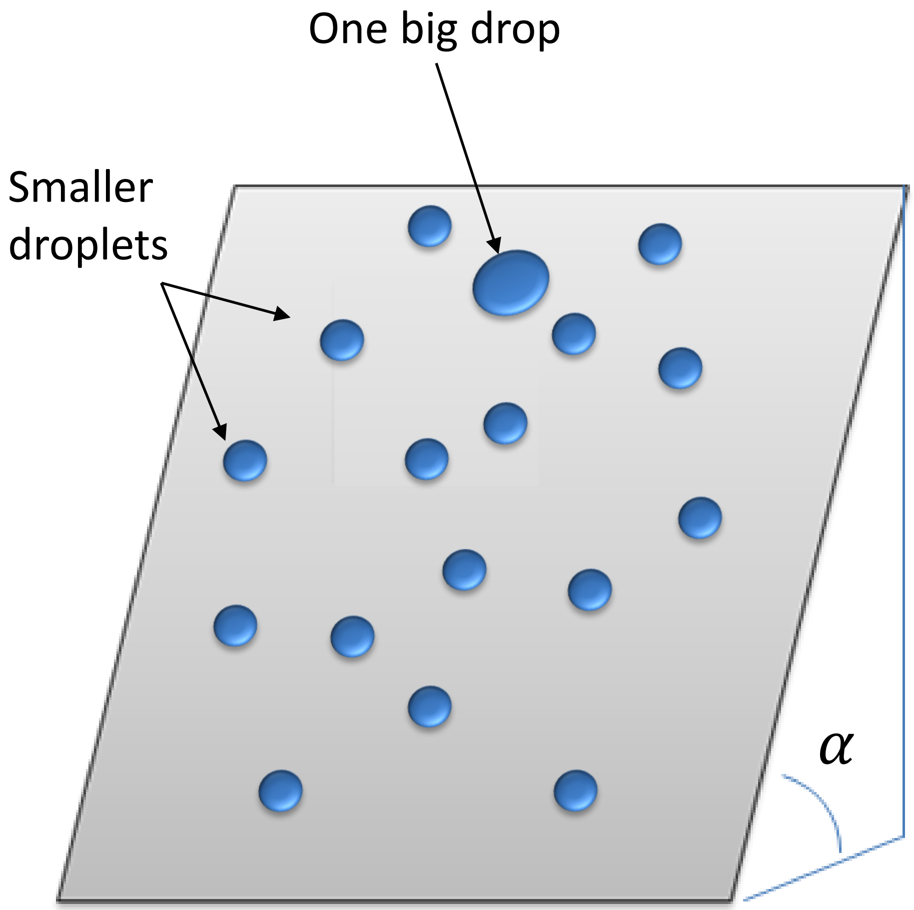




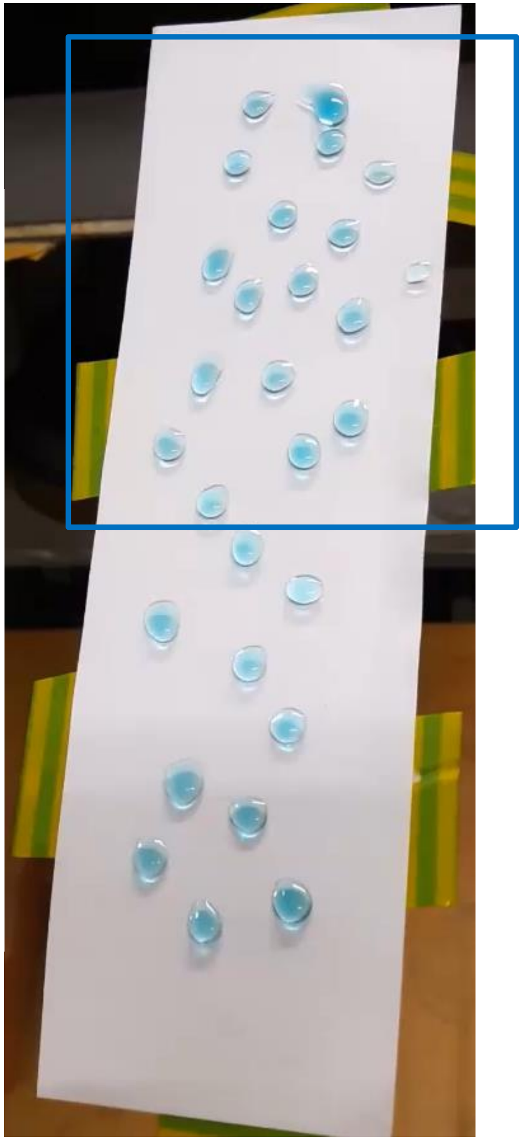
Wind locally going up

Patrick Guilleron, Aérodynamique Automobile pour l'environnement, le Design et la sécurité, édition Cépaduès, 2014, ISBN 978.2.36493.091.9.

Real conditions
Many droplets

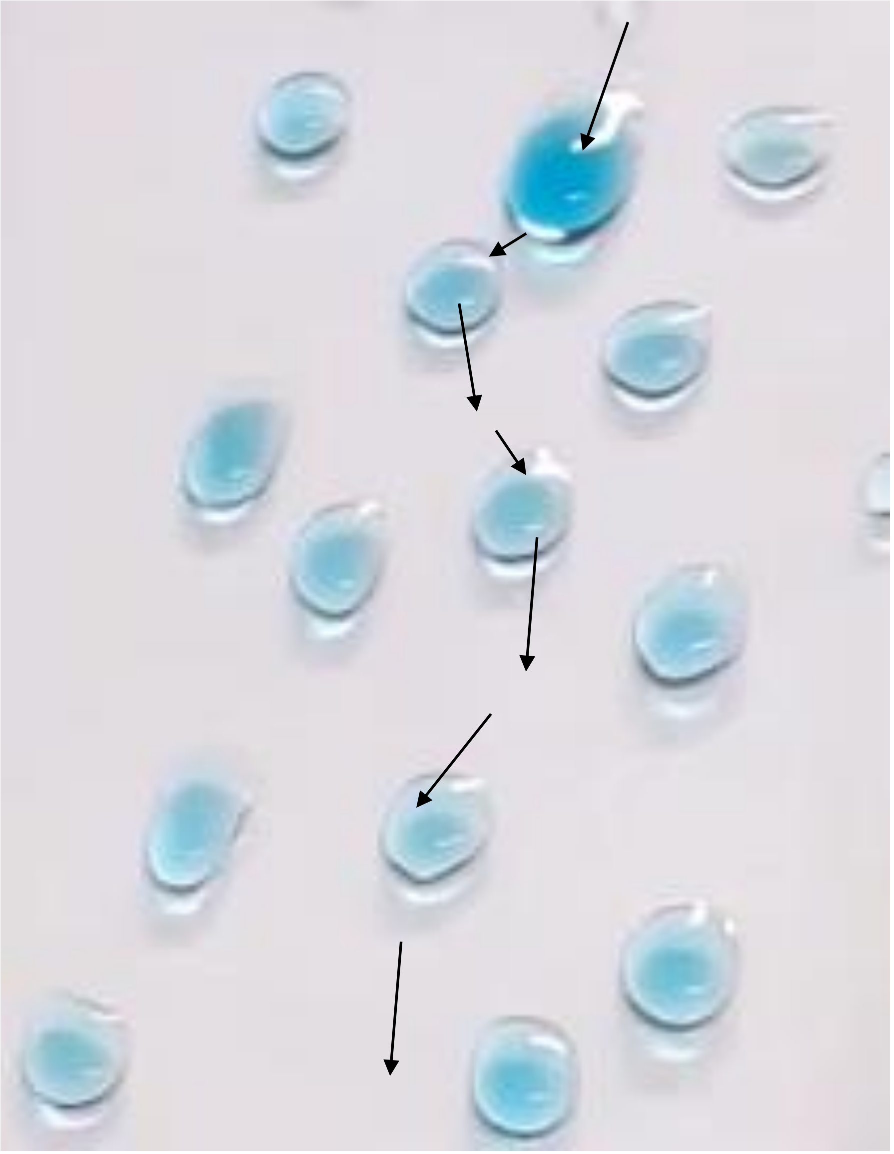


Incline the plan until the big drop slides



Real conditions
Many droplets

Zoom
on merging



It affects
the direction.

Conclusion - Erratic Raindrops

- There is wind is going up the window :
- Physical arguments : whirls on the window

- **Adhesion force :** $\boldsymbol{F_{ad}} \leq \gamma(\cos\theta_r - \cos\theta_a)\mathrm{D}$
- **Maximal inclinaison** for a given drop : $\boldsymbol{\sin\alpha} \sim L_c^2 \frac{\boldsymbol{D}}{\boldsymbol{\Omega}}$
- **Wind speed** needed to dislodge a drop : $\boldsymbol{u^2}$ scales in $\frac{\boldsymbol{1}}{\boldsymbol{D}}$
- Trajectory perfectly determined by weight $\boldsymbol{P}$, drag force $\boldsymbol{F_D}$, and the direction of the wind $\boldsymbol{\beta_D}$:

$$\boldsymbol{\cos\beta} = \frac{\boldsymbol{P}\sin\alpha + \boldsymbol{F_D}\cos\boldsymbol{\beta_D}}{\sqrt{(P\sin\alpha)^2 + 2P\sin\alpha\, F_D\cos\beta_D + (F_D)^2}}$$

- In real conditions : merging, influence of wetting, not perfect surface … erratic !

Thank you for listening !

Bibliography

- CGL. Furmidge, “Studies at phase interphases. I. The sliding of liquid drops on solid surfaces and a theory for spray retention”, Journal of colloid science, 1962
- David Quéré, “Surfaces molles”
- Kruss, “Caractérisation des interfaces”
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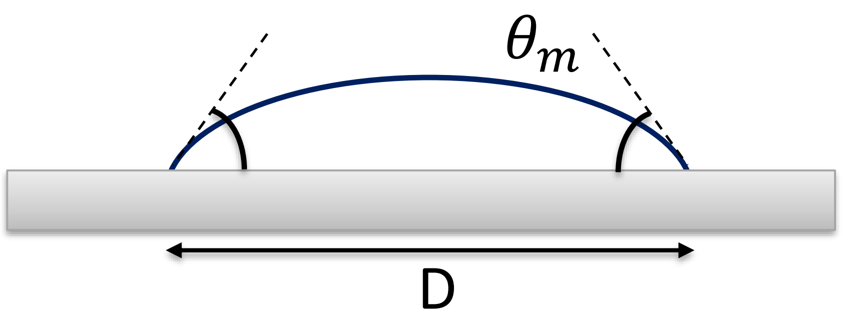
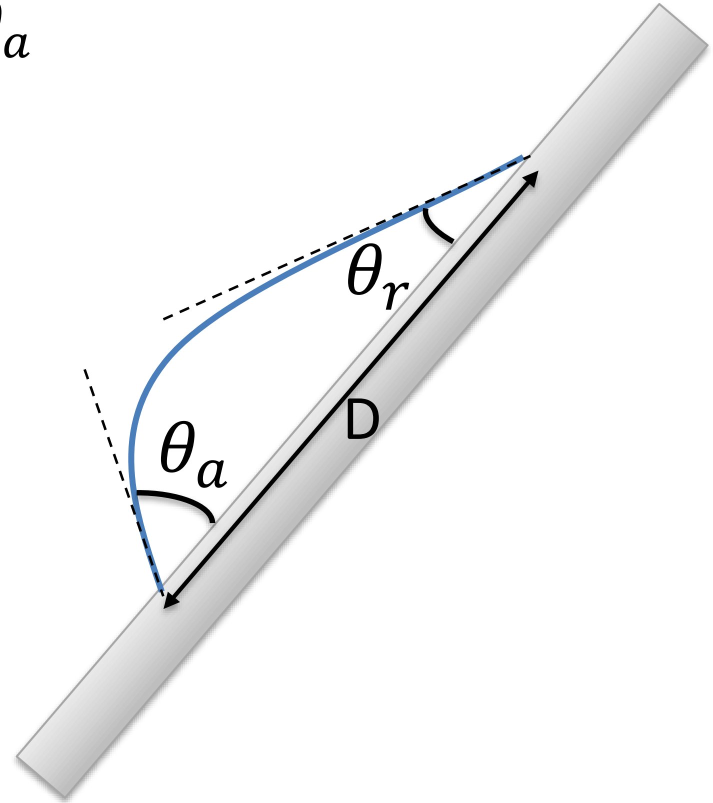
- Nolween Legrand, Adrien Daeer, laurent Limat “ Shape and motion of drops sliding down an inclined plane “ 2005 Fluid Mech.
- J. Fan a, M.C.T. Wilson b, N. Kapur, “Displacement of liquid droplets on a surface by a shearing air flow”, 2011, Journal of Colloid and Interface Science

Hysteresis

$$\theta_r < \theta_m < \theta_a$$





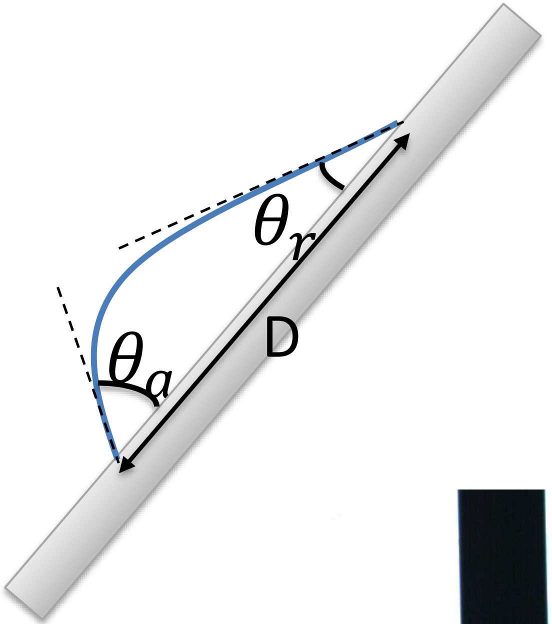
Measured :

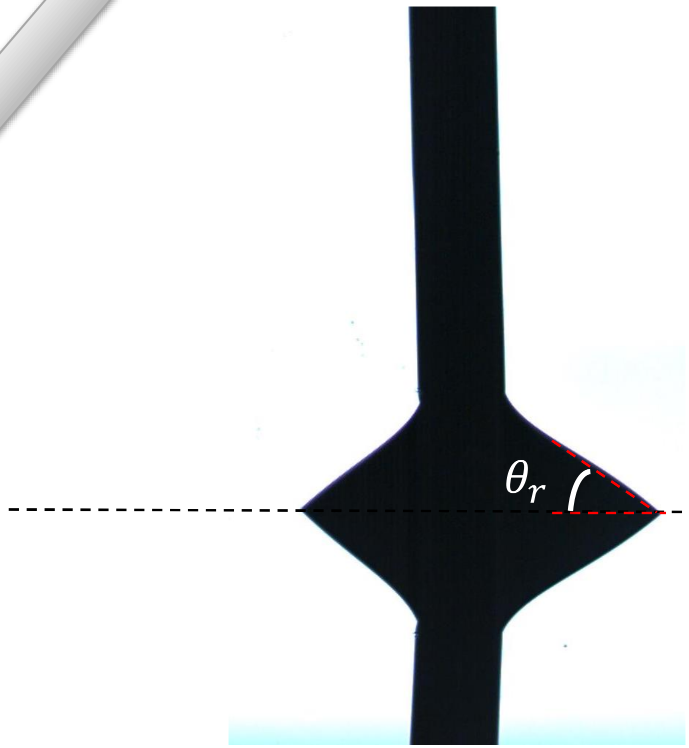
Water/plexiglas : $\theta_r = 28° \pm 2°$, $\theta_a = 69° \pm 2°$ => $\Delta\theta \approx 34°$

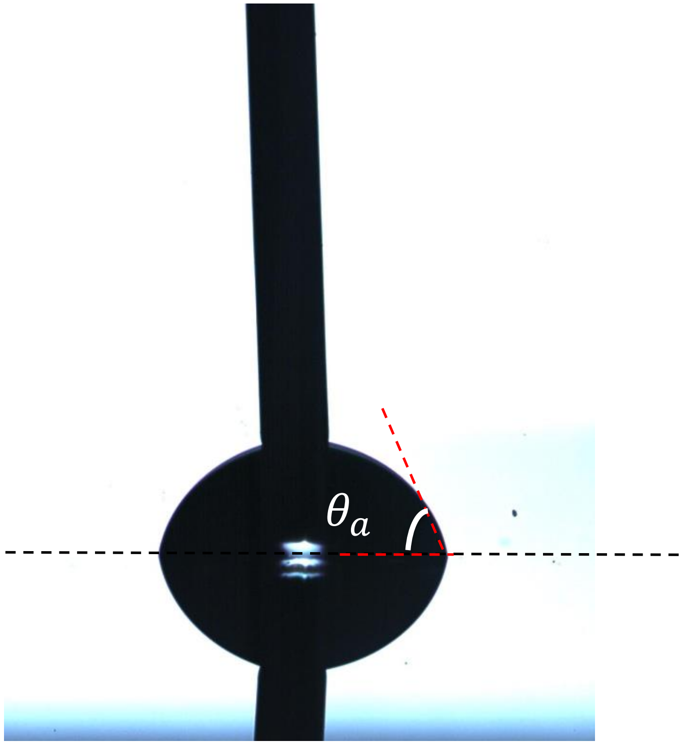
Glycerine/plexiglas : $\theta_r = 39° \pm 2°$, $\theta_a = 73° \pm 2°$ => $\Delta\theta \approx 41°$

Measure Hysteresis

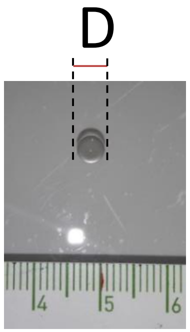
$$\theta_r < \theta_m < \theta_a$$



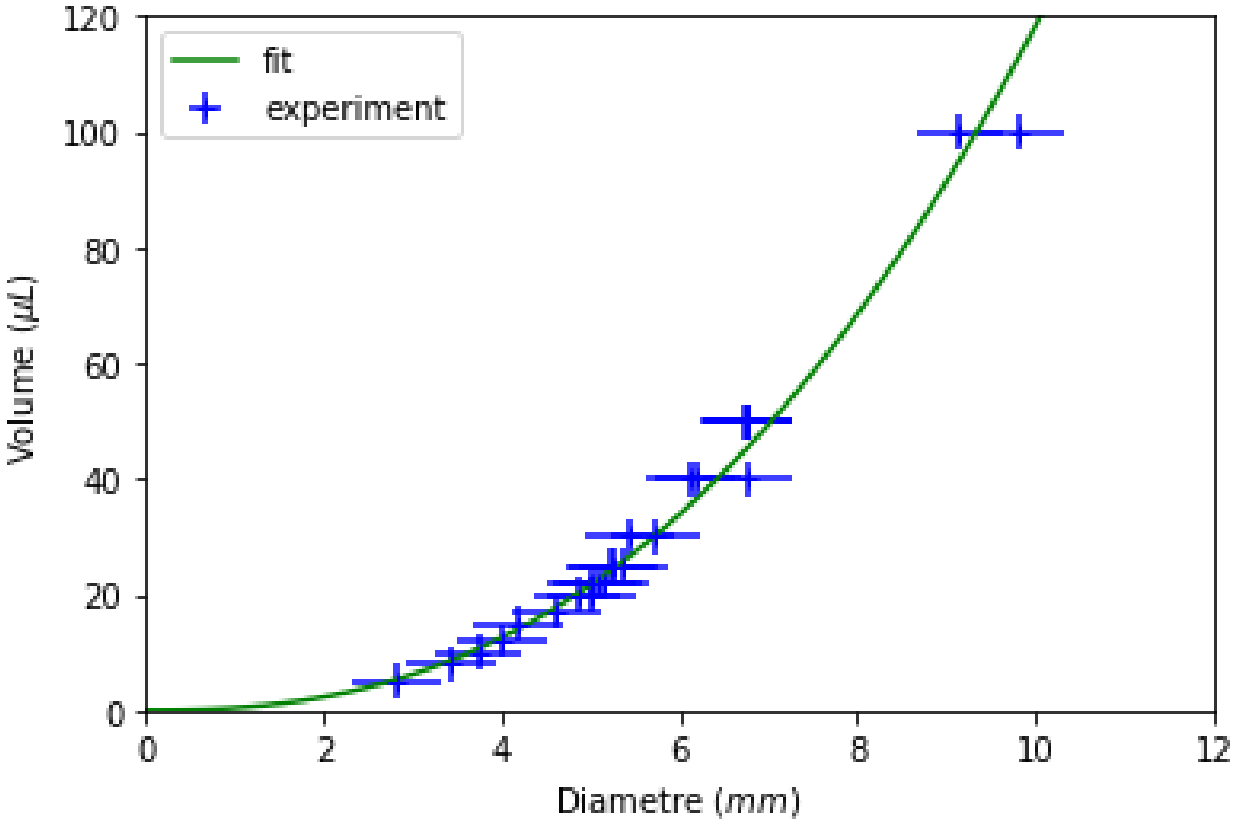




Geometry of a droplet

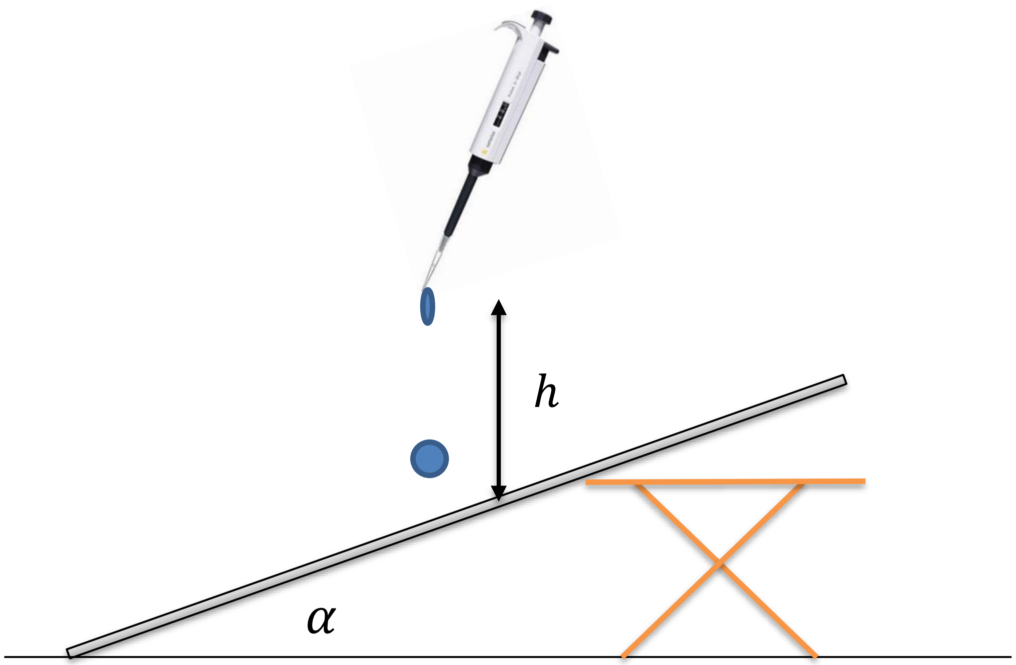


Experimental law between
Volume and Diameter :



$$\boldsymbol{\Omega} = \boldsymbol{a} * \boldsymbol{D}^{\boldsymbol{b}}\ , with\ a = 0.4;\ b = 2.45\ \neq 3$$

*What if a drop **falls** on the car window ?*



Size rain drop $\approx 5\ to\ 10\ \mu L$

Falling ≠ resting (kinetic energy)

Modification : slides more easily

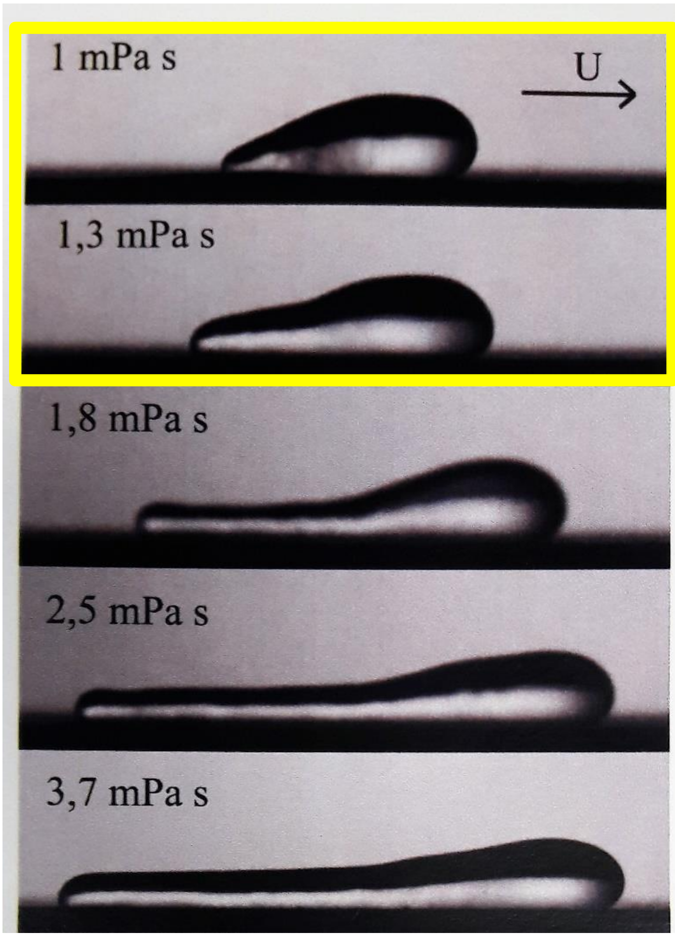
A droplet creates micro daughter drops => stay on the window

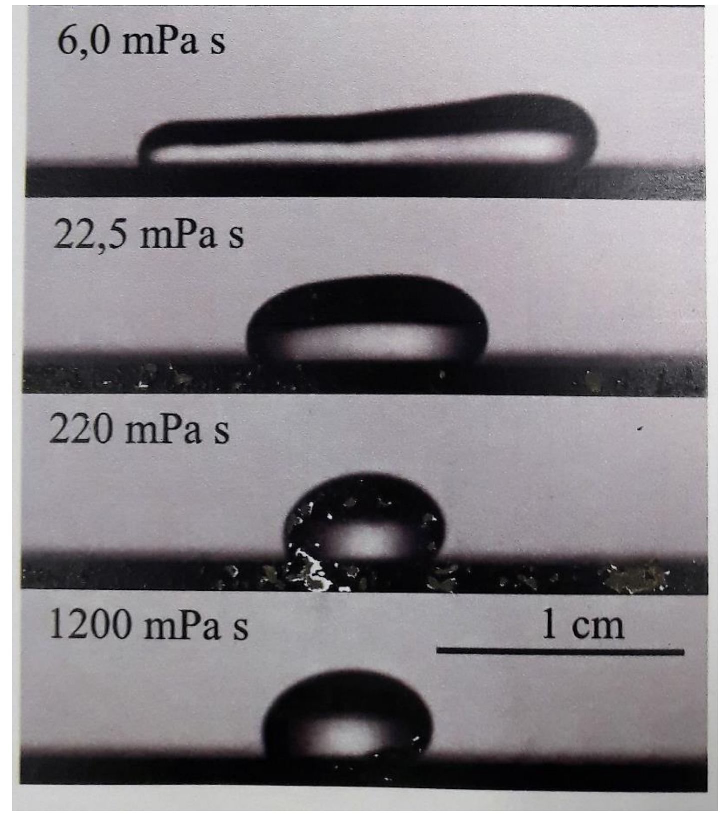
Deformation of a droplet

Influence of Viscosity (in $mPa.s$)

$Water$:
$\eta \approx 1\ mPa.s$

$Milk$





$Olive$ Oil

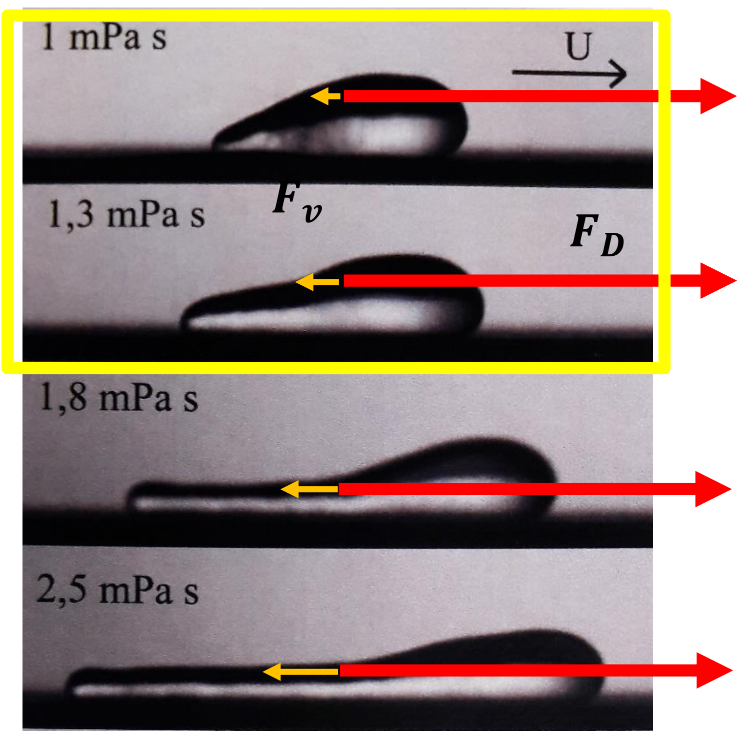
$Honey$

Laboratoire d'hydrodynamique de l'X

Viscous force within the droplet

$$F_v = \eta_{water} D\, U^2 \approx 1\ \mu N$$
$$F_D = \frac{1}{2}\, C_x S\, U^2 \approx 1\ mN$$

$Water$:



$F_v \ll F_D$: we neglict the viscous forces.

And on a train ?

Quicker !

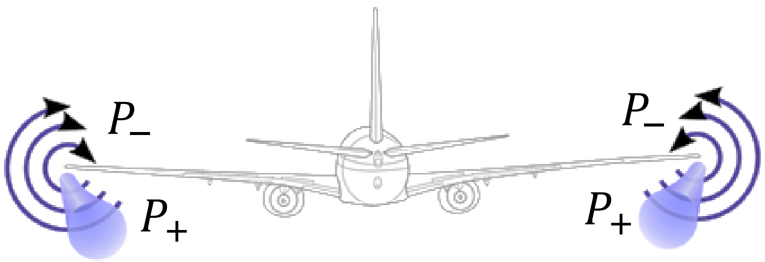
Smaller droplets,

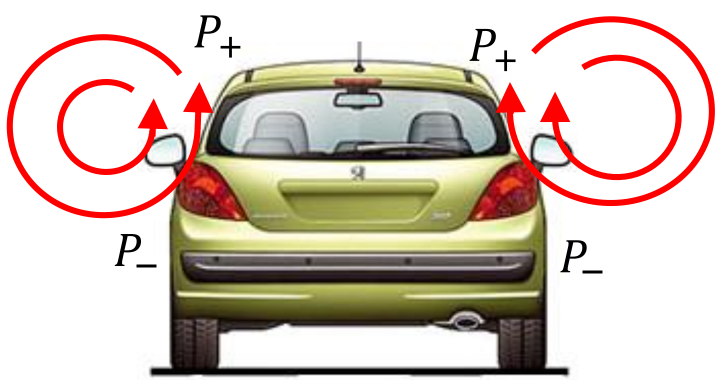
Motion mainly horizontal
(except near the very front)

Why is there wind going up the car window ?

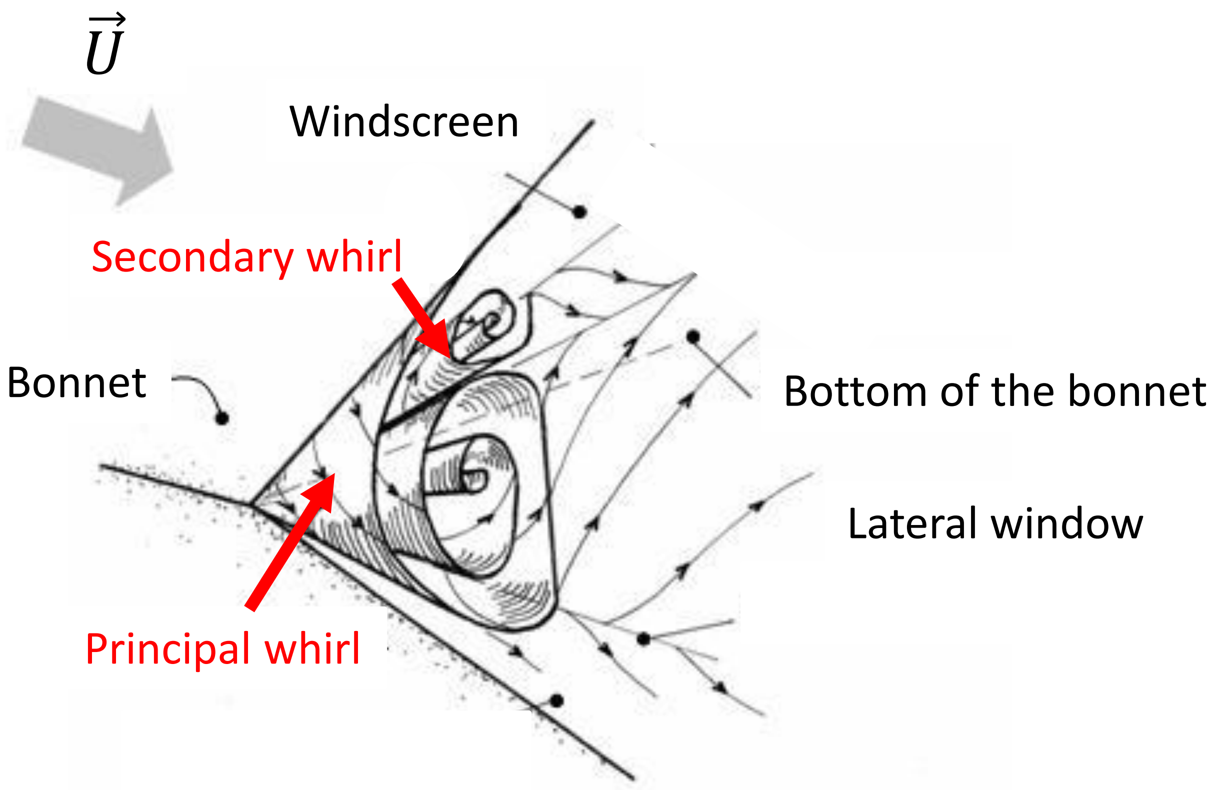
Analogy

Marginal whirls - well known on plane

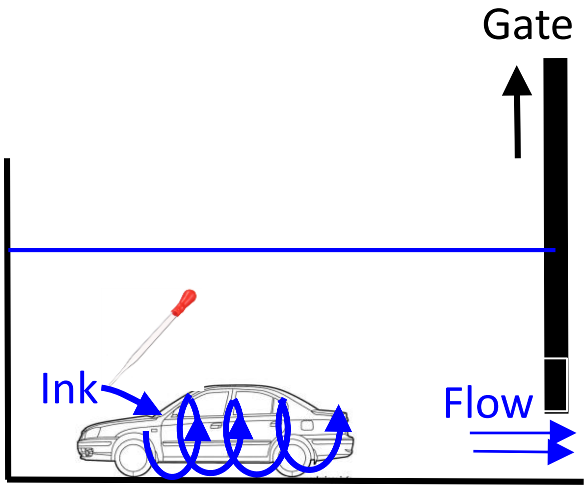




P_- pressure of extrados
P_+ pressure of intrados
$P_- < P_+$


$\vec{U}$
Windscreen
Secondary whirl
Bonnet
Bottom of the bonnet
Lateral window
Principal whirl

Why is there wind going up the car window ?

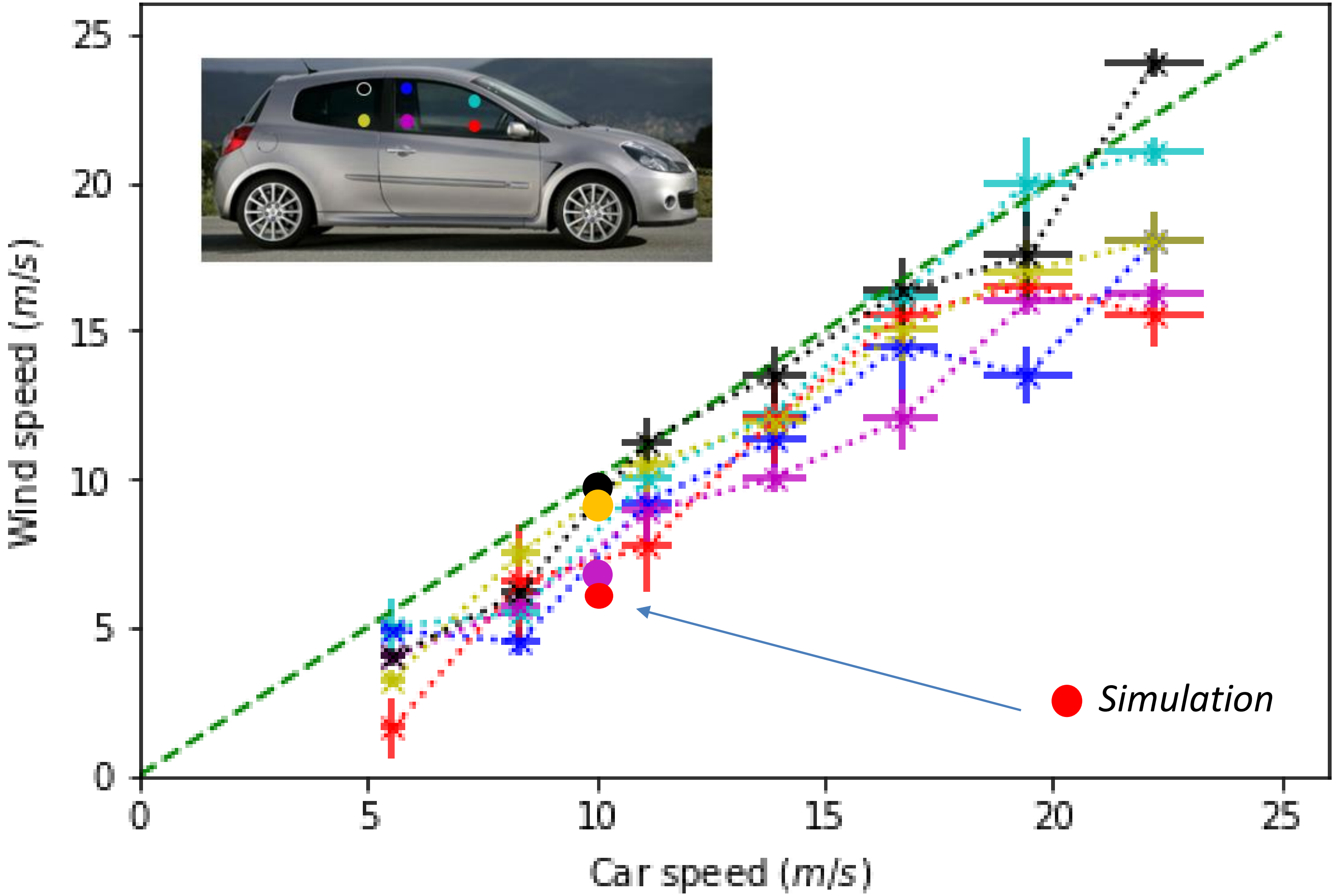


Marginal winds occur on the car !

And the wind on the car window ? Value ?

anemometer

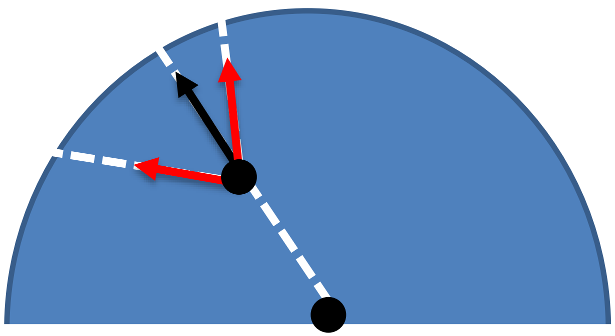
And the wind on the car window ?
Value ?



Why is there wind going up the car window ?

Qualitative argument

Model



Shorter path !

Direction of the drop ? Any wind