Problem 13

Egg white pearls

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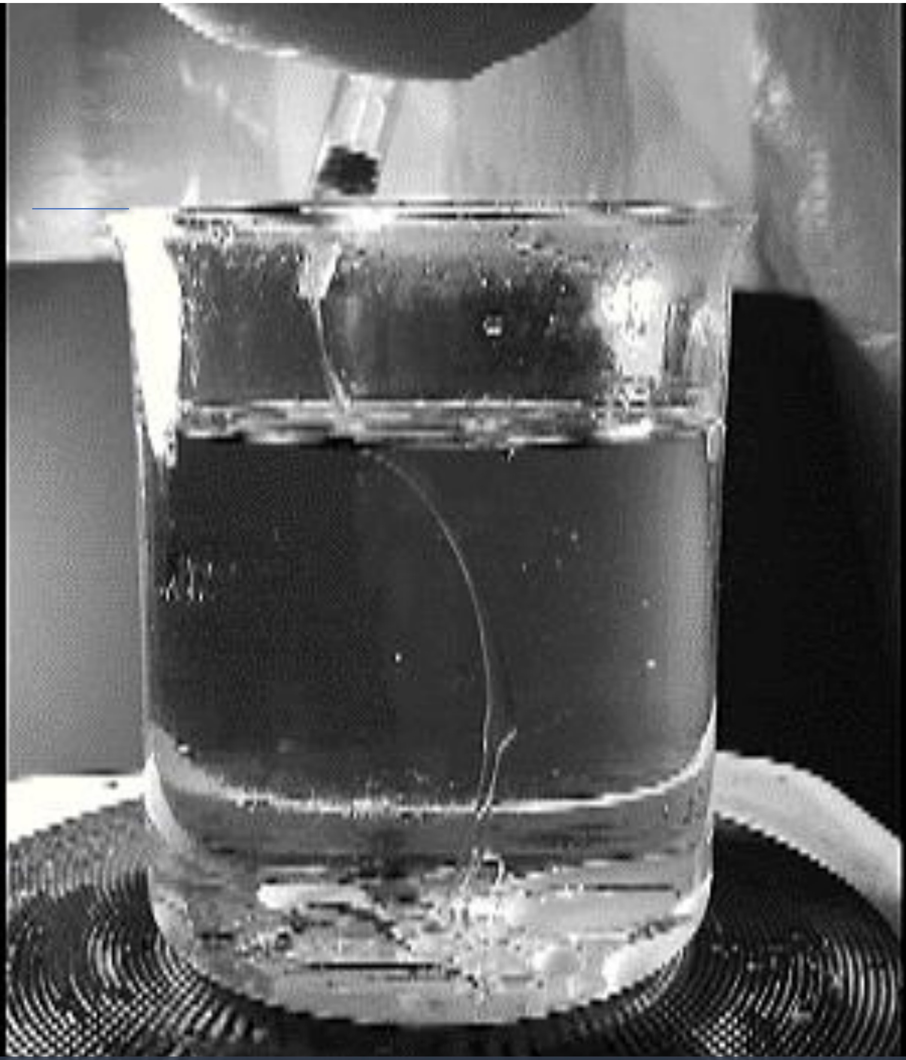
Problem statement

Egg whites are separated from the yolk and put into a syringe. From the syringe, the egg white is ejected into heated oil while the tip is in motion (see video). How does the size of the egg white pearls produced depend on the various parameters such as the temperature of the oil, ejection and motion speed, nozzle diameter or the non-Newtonian properties of egg whites?

Observed phenomena

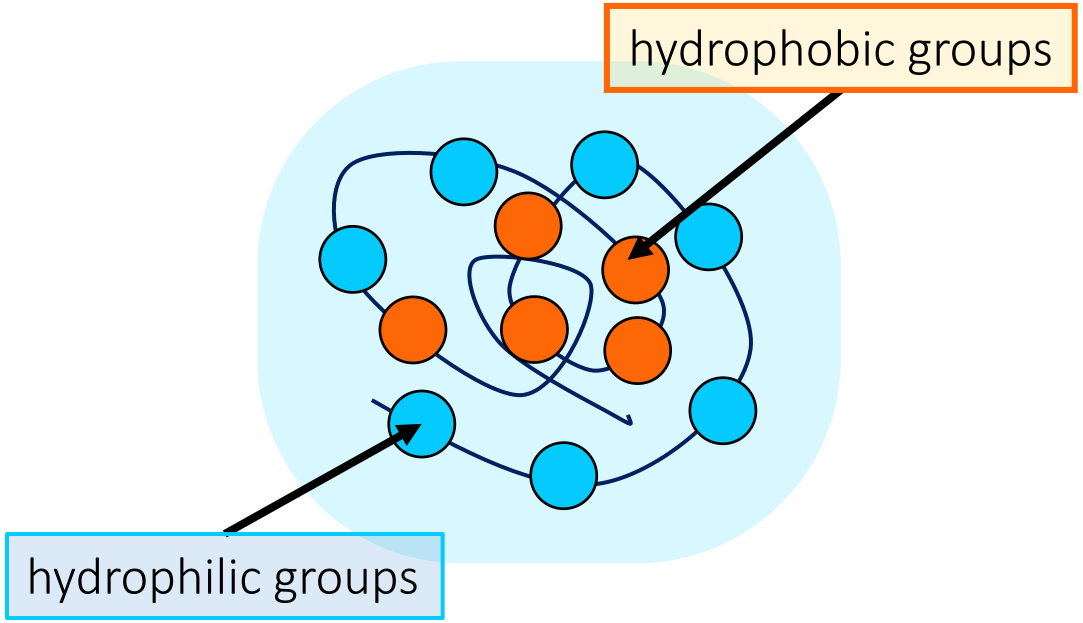


The bulges appear on the jet of egg white, and then they transform into separate drops

Properties of egg white

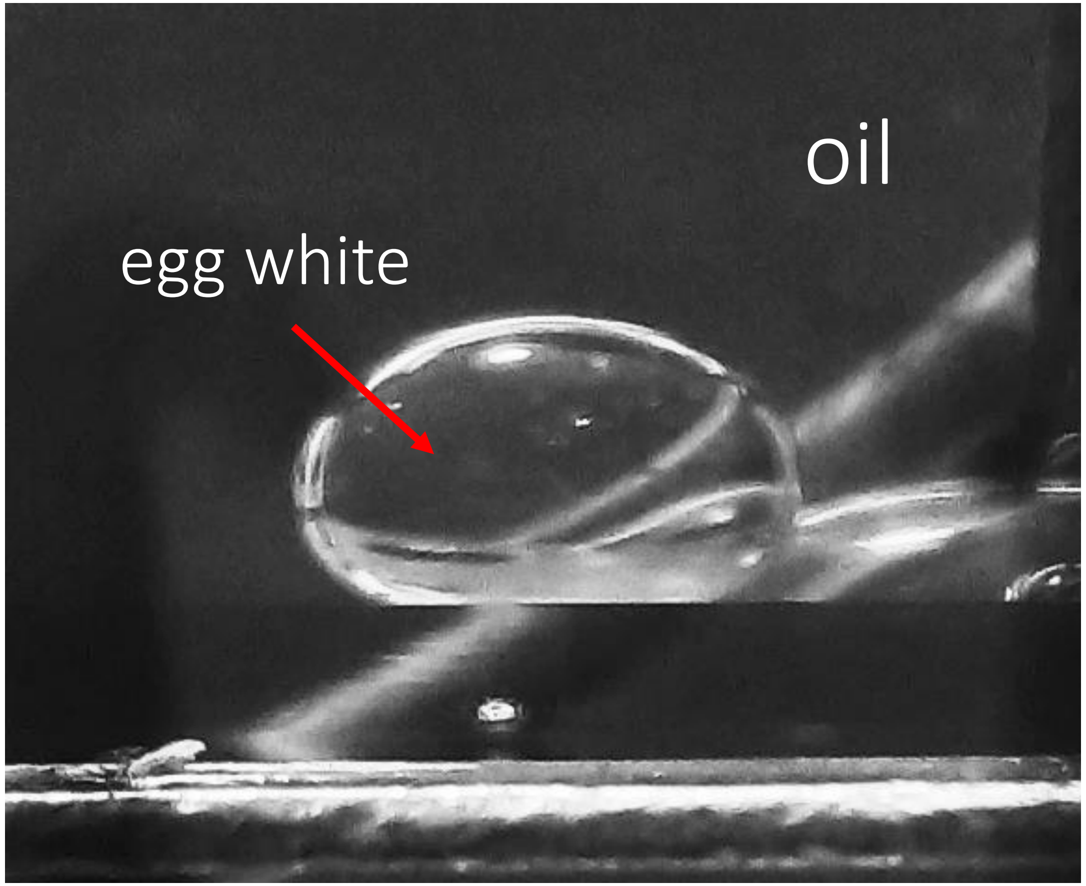
Basic physics

Egg white compound

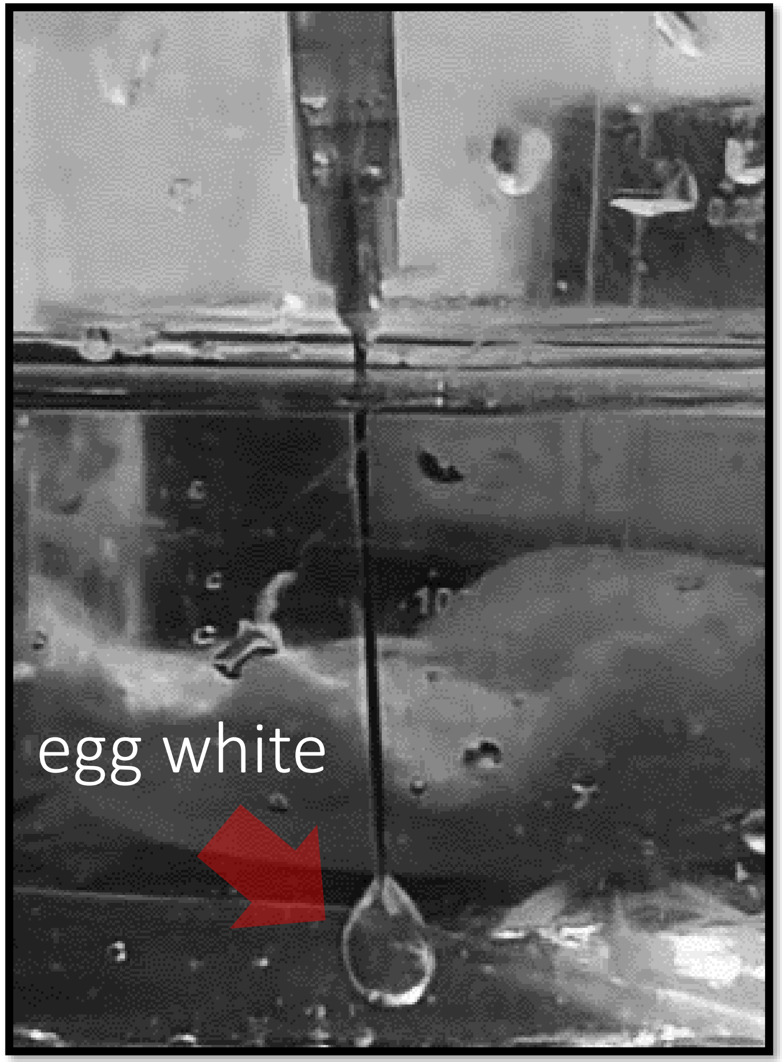


~ 90% of water
~ 10% of proteins
(mainly ovalbumin)

Interfacial tension between oil and egg white



Surface tension measurement (egg white)



1 ml of egg white (V) was injected into the syringe

The egg white was dripped in oil

$$\sigma = \frac{V(\rho_{egg} - \rho_{oil})g}{\pi N d}$$

d – diameter of needle bevel

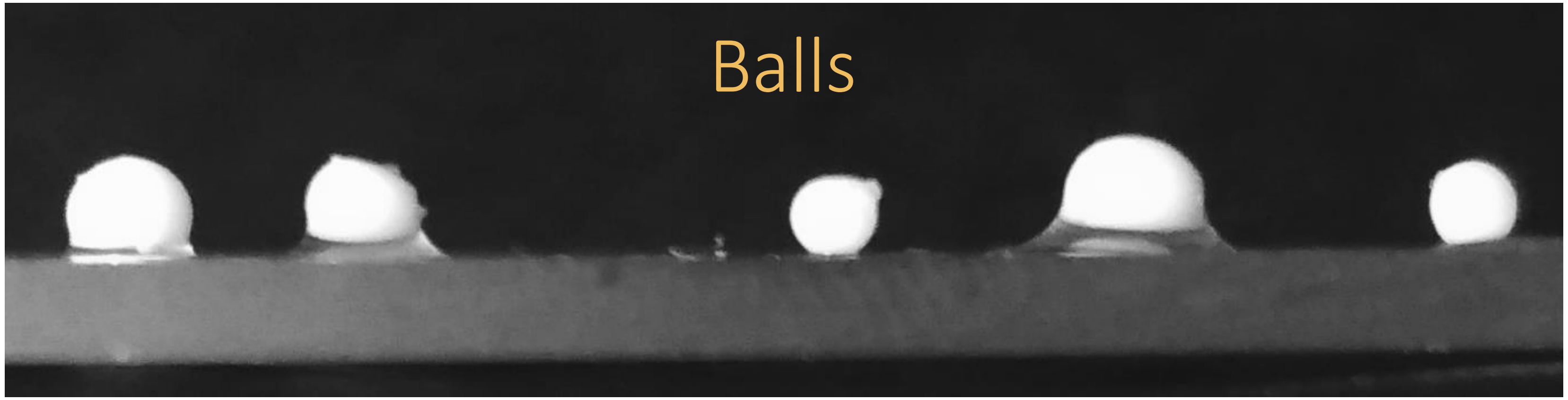
N – number of drops

$$\sigma_{egg\ white} = 32 \cdot 10^{-3}\ {}^{N}/_{m}$$

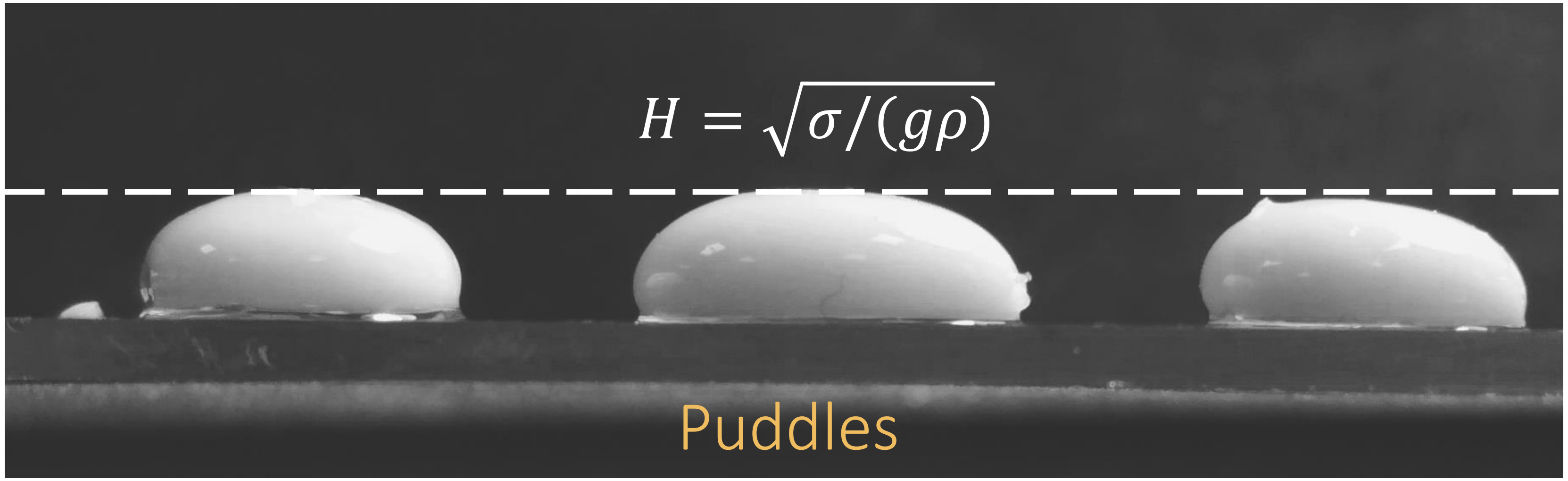
$\rho_{egg} = 1054\ kg/m^3$ $\rho_{oil} = 927\ kg/m^3$

The shape of egg-pearls

Volume



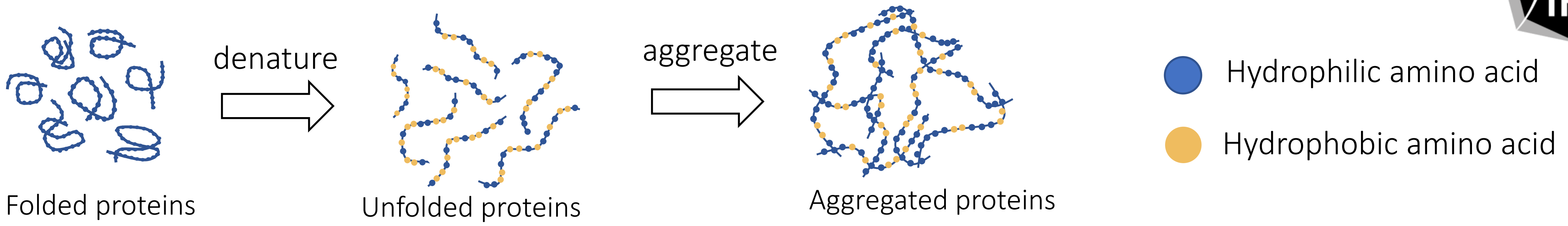
$E_{surf} \gg E_{grav}$

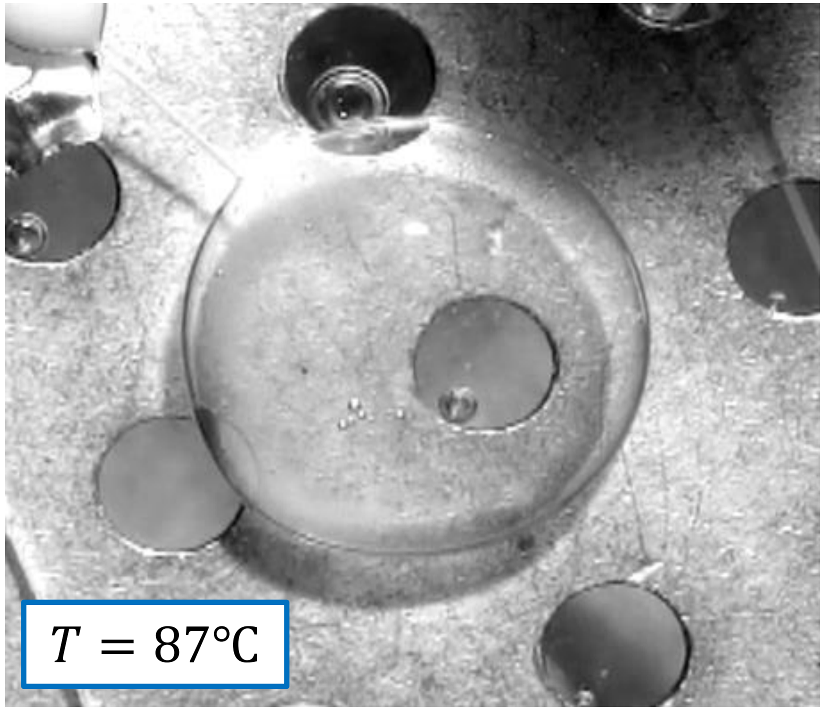


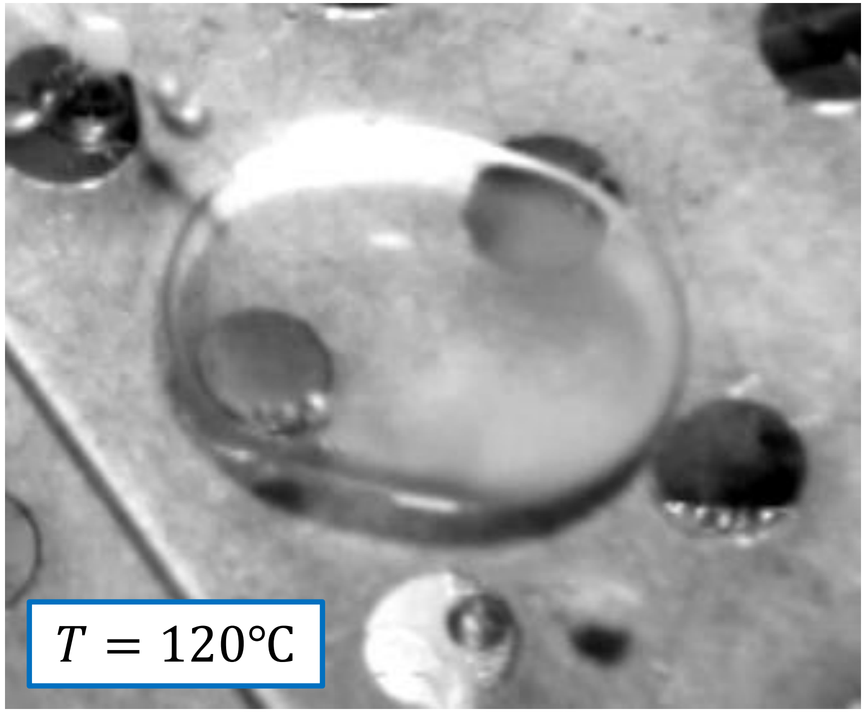
$E_{surf} \approx E_{grav}$

The height of the pearls does not depend on the volume

Heating and denaturation



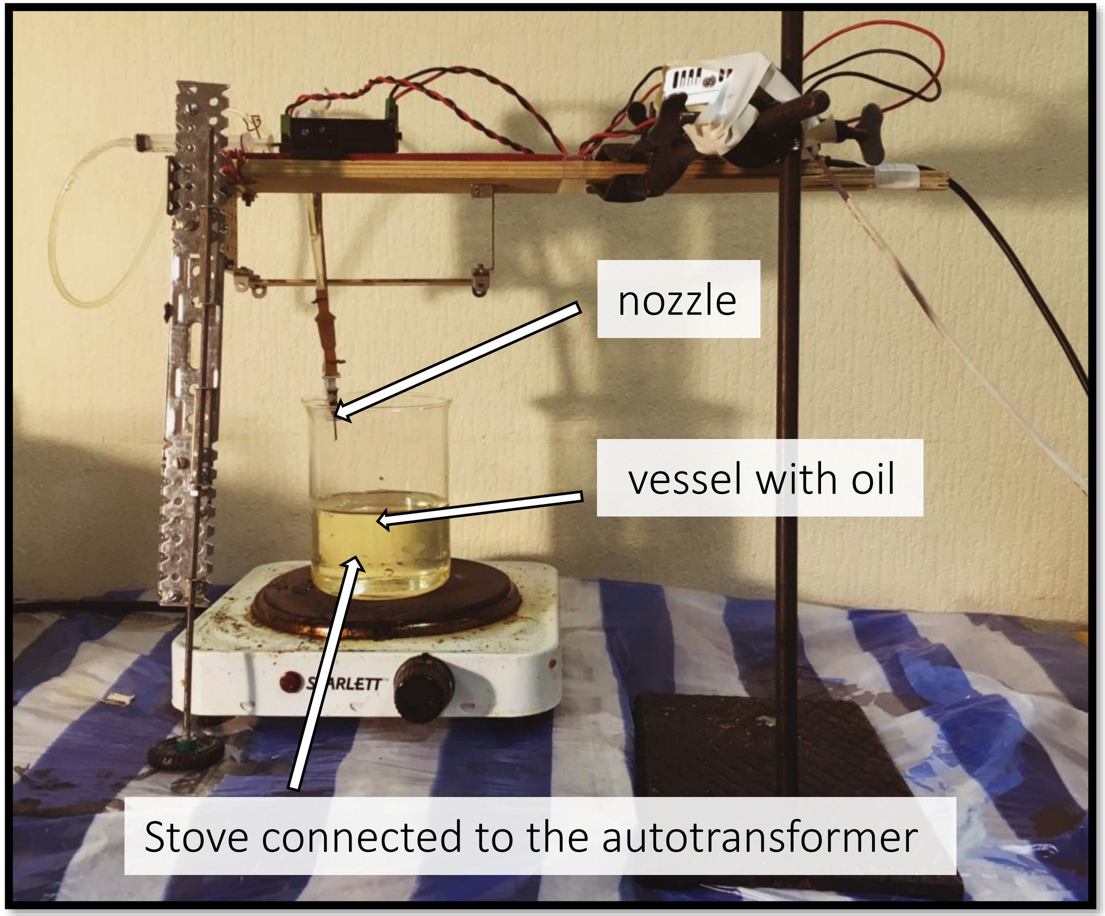


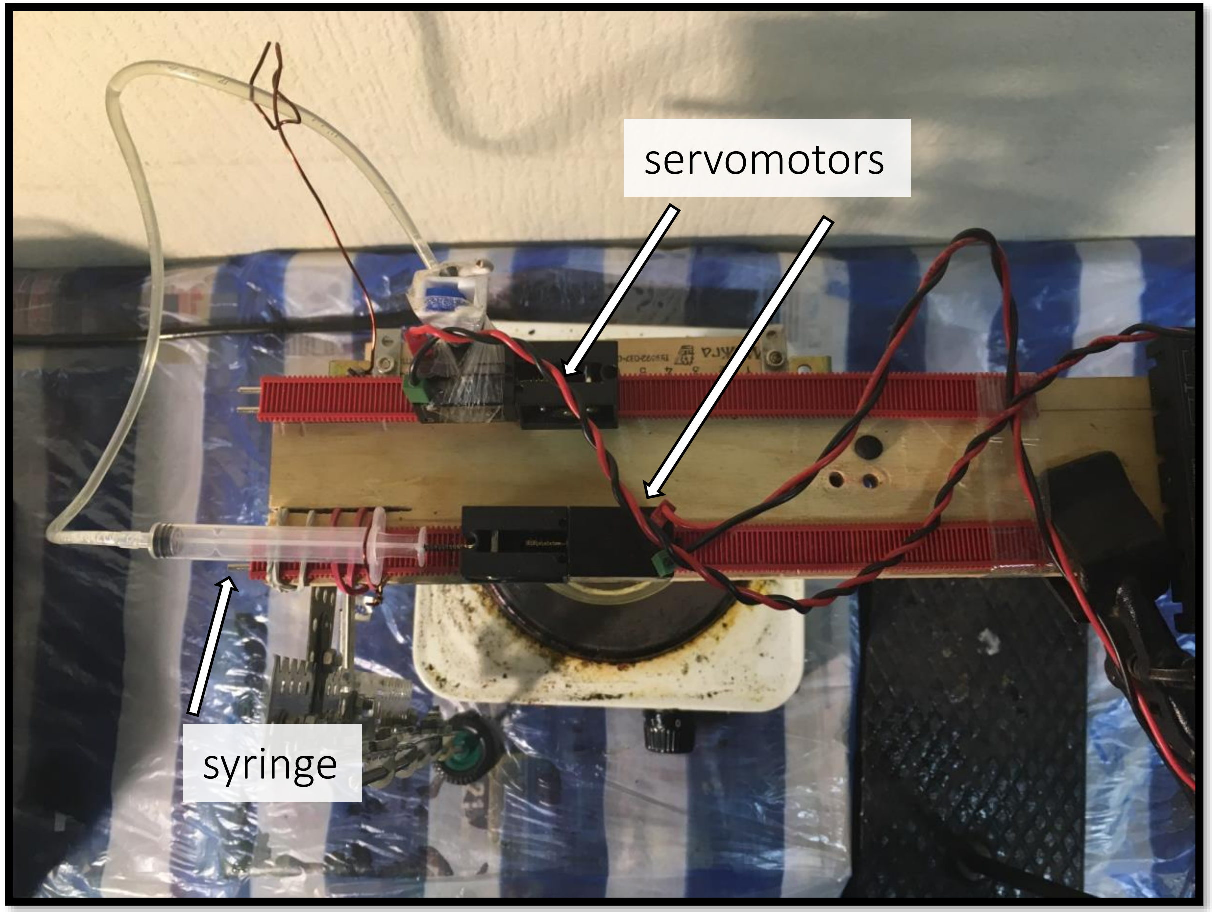


Working range of temperatures: $70 - 95$ °C

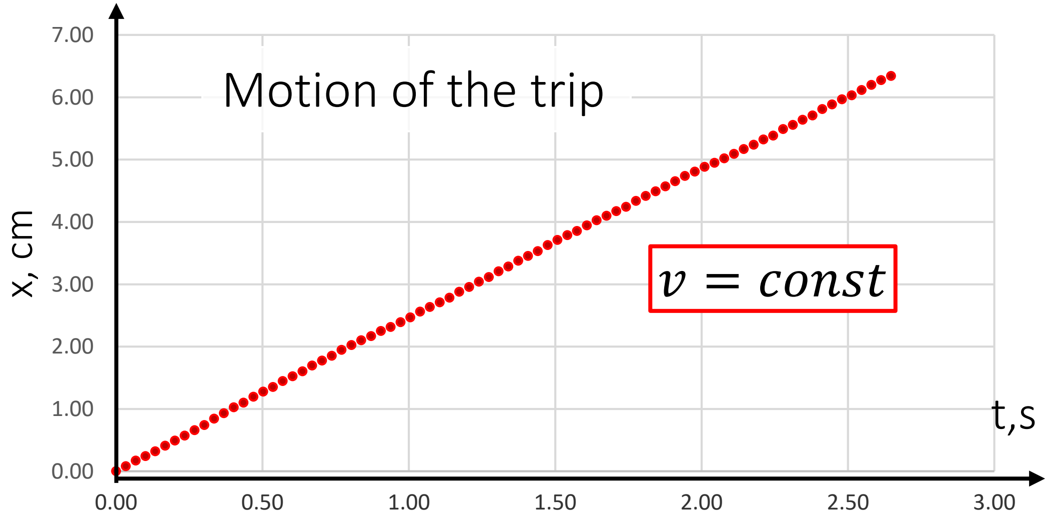
Experimental technique

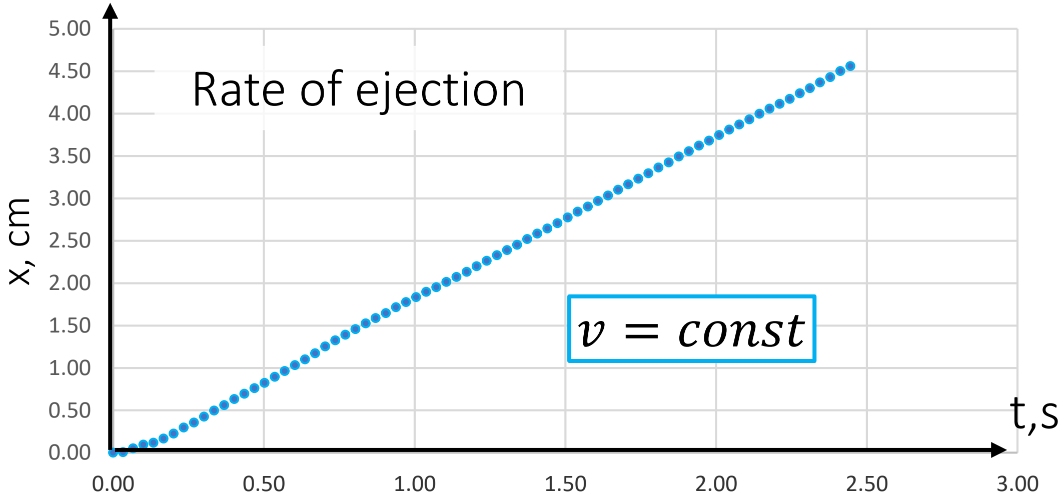
Experimental setup

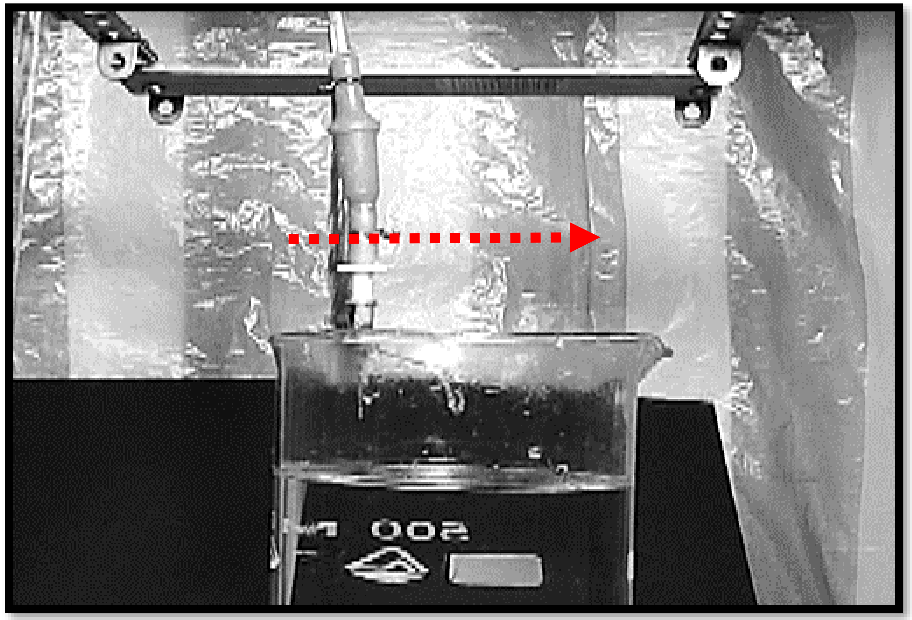


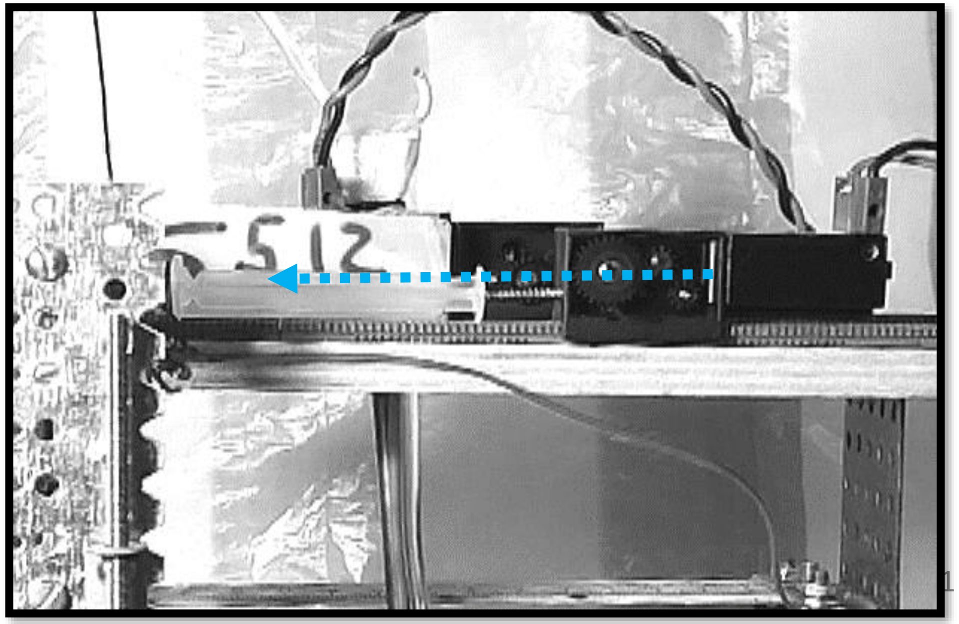


Tracking of motion of the tip and rate of ejection



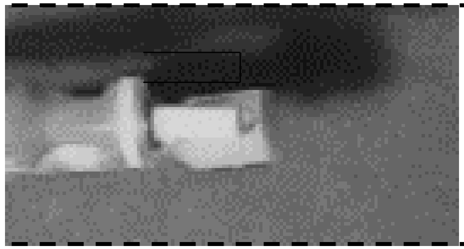






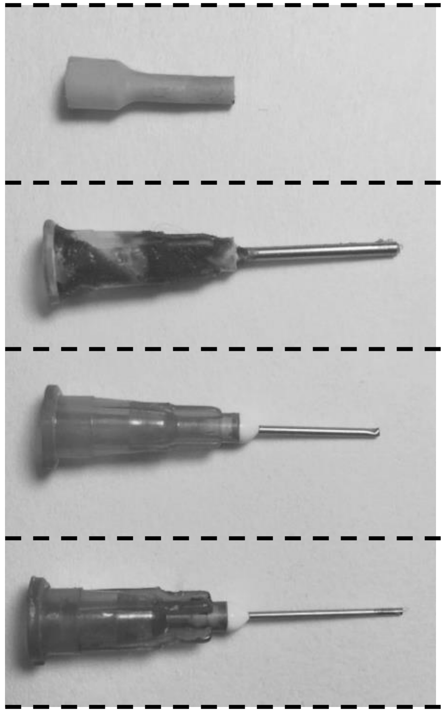
Experimental setup

Diameter of the orifice



2.3 mm

Nozzles



1.5 mm

0.975 mm

0.535 mm

0.365 mm

Parameters of the setup

Velocity of the piston in the syringe: $0.1\ cm/s - 2.1\ cm/s$

The velocity of the jet: $0.2 - 3\ m/s$

Velocity of the tip's motion: $1\ cm/s - 2.5\ cm/s$

Parameters of oil

Density: $927\ kg/m^3$

Viscosity $(T = 23℃)$: $0.073\ Pa \cdot s$

Temperature-viscosity dependence:

$$\mu(T) = \mu_0 \exp(E_a/RT)$$

Formation of the pearls

Videos

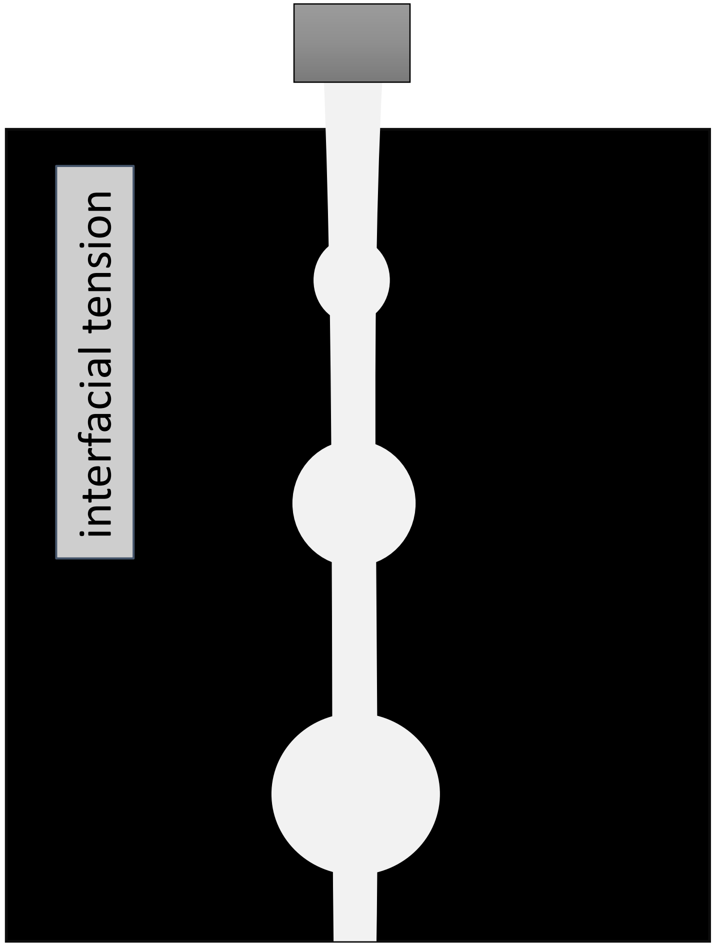
Rayleigh-Plateau instability

Bending instability

Turbulent breakup

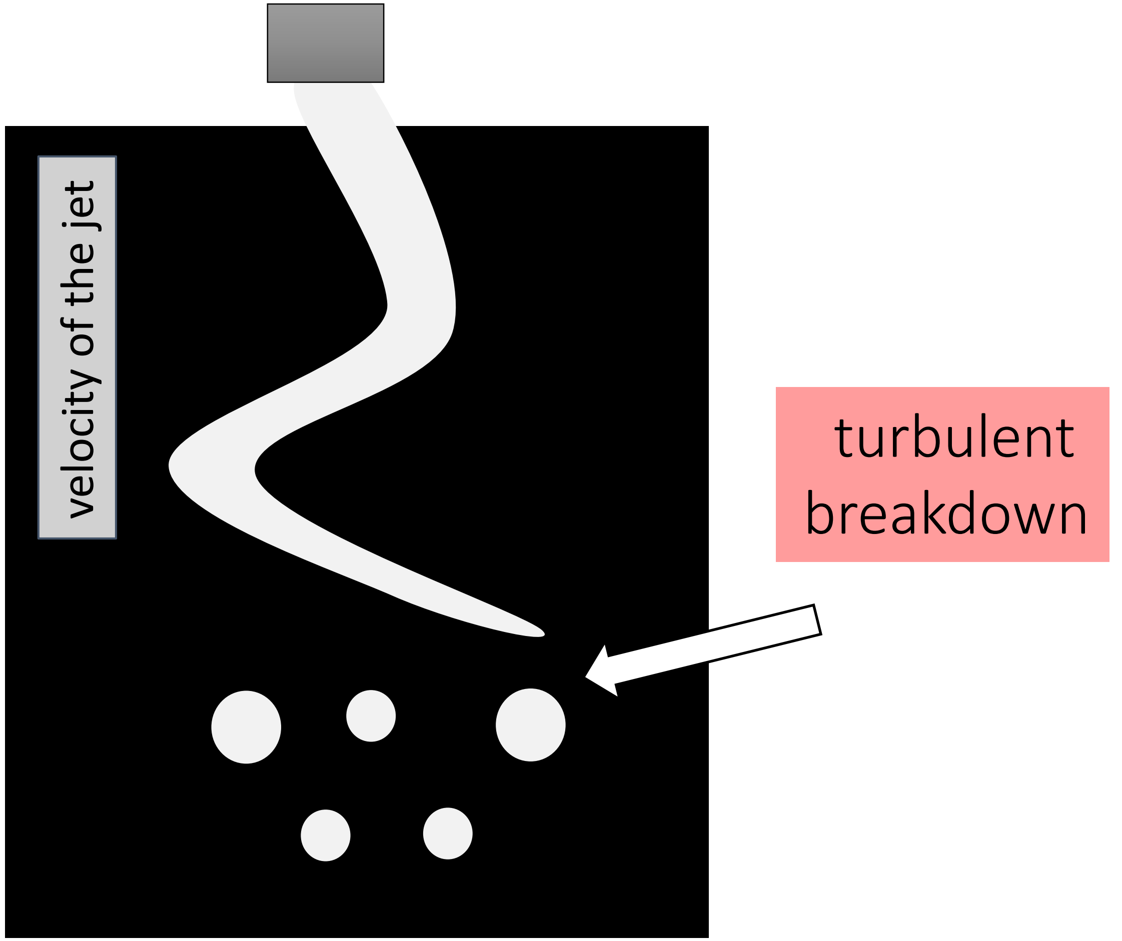
Rate of ejection

Two mechanisms of pearls formation



Rayleigh-Plateau instability

VS



Bending instability

Weber number

Weber number — dimensionless number, describing a measure of the relative importance of the fluid's inertia compared to its surface tension.

$$We = \frac{\rho L v^2}{\sigma}$$

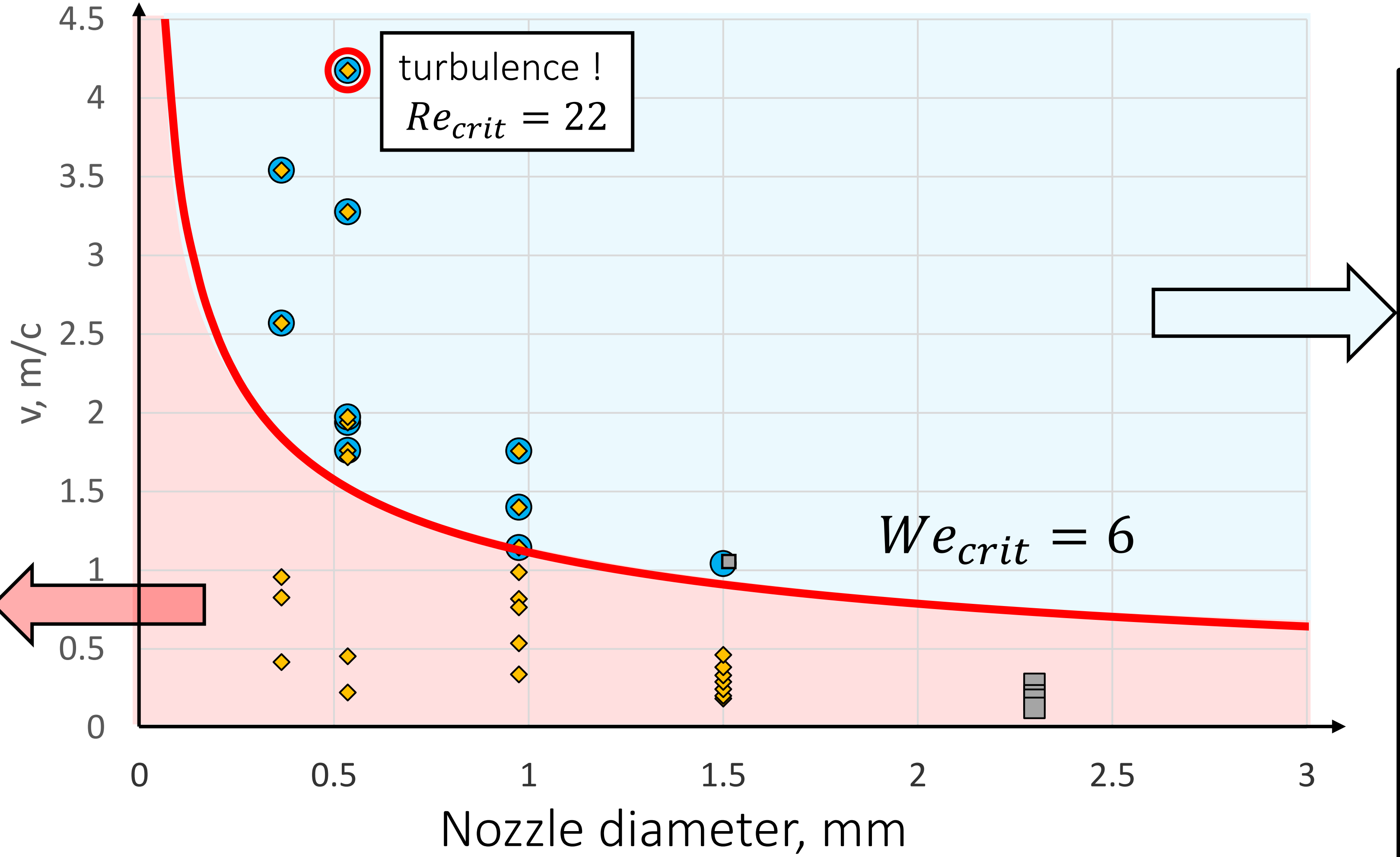
L characteristic length

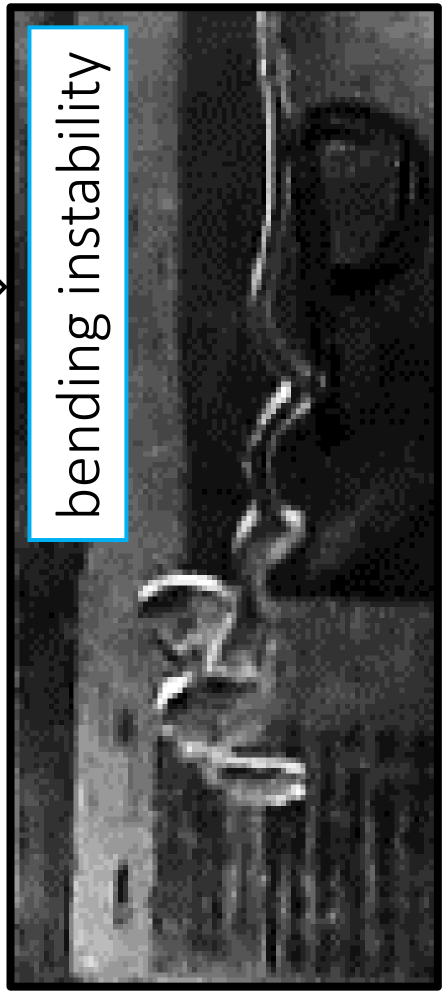
v velocity of the fluid

σ coefficient of the surface tension

ρ density of the fluid

Modes of the flow

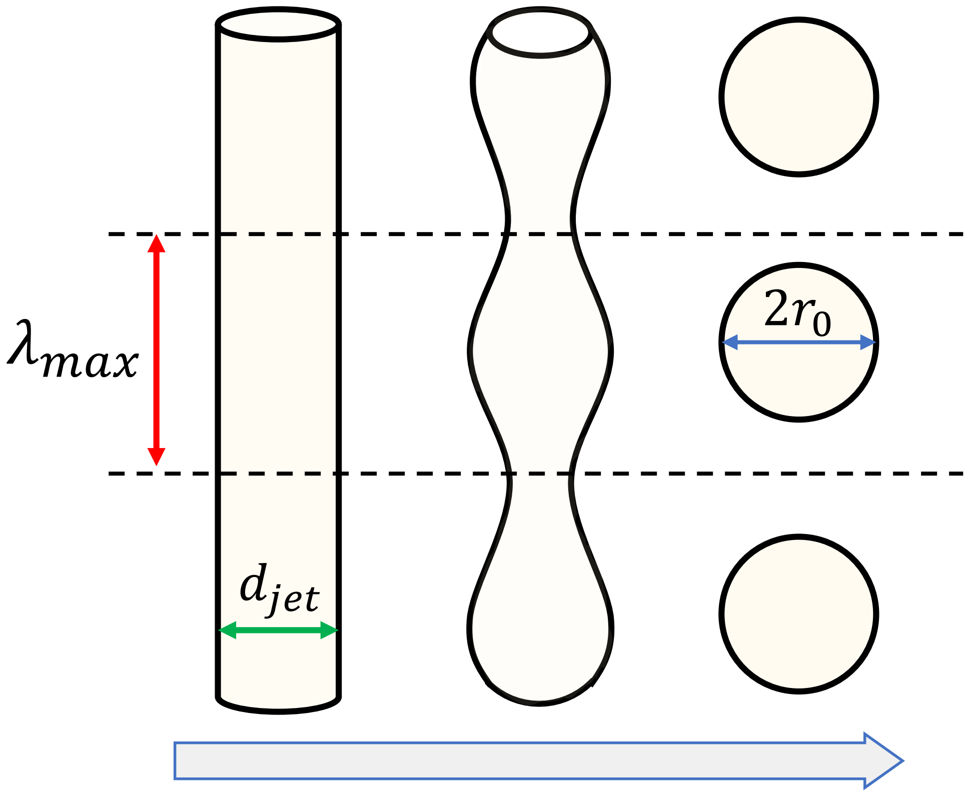




Rayleigh-Plateau instability   bending instability   no pearls

Rayleigh-Plateau instability

The main mechanism of pearls formation



From volume conservation

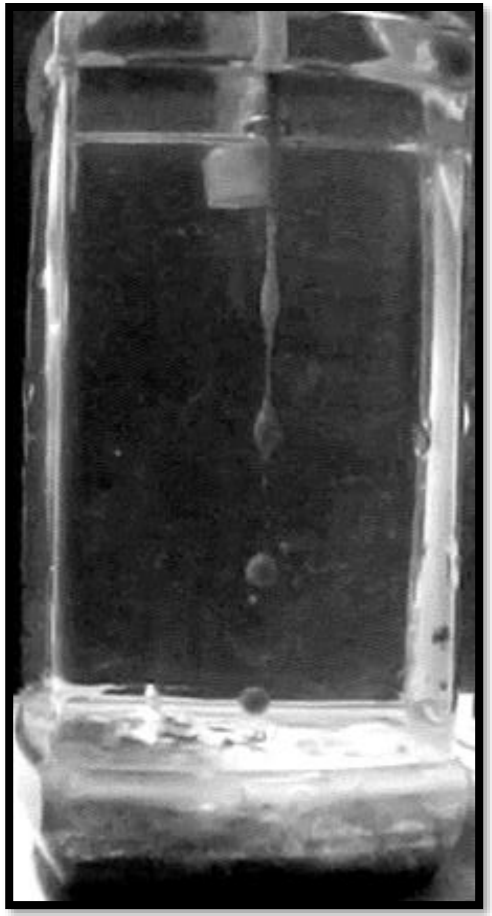
$$\frac{1}{4}\lambda_{max}\pi d_{jet}^2 = \frac{4}{3}\pi r_0^3$$

The size of pearls is determined by the instability wavelength (λ_{max})

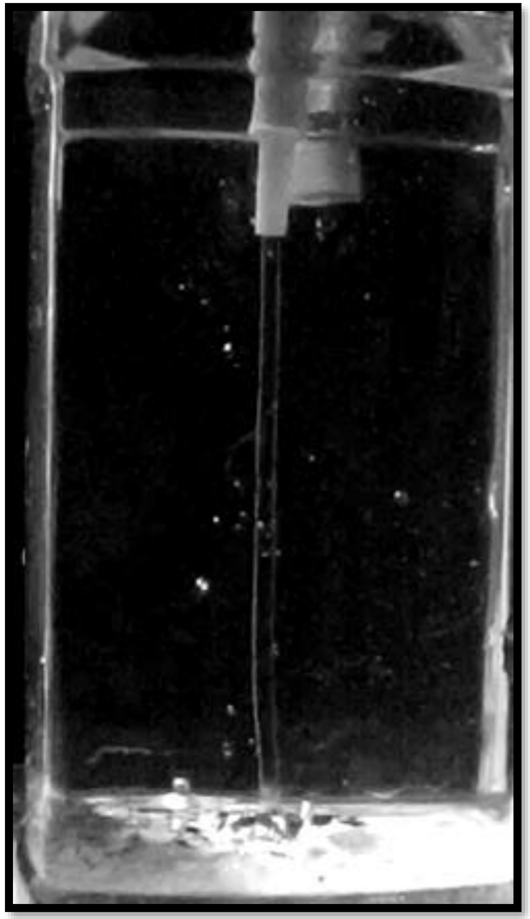
Observations

Egg white

Glycerol



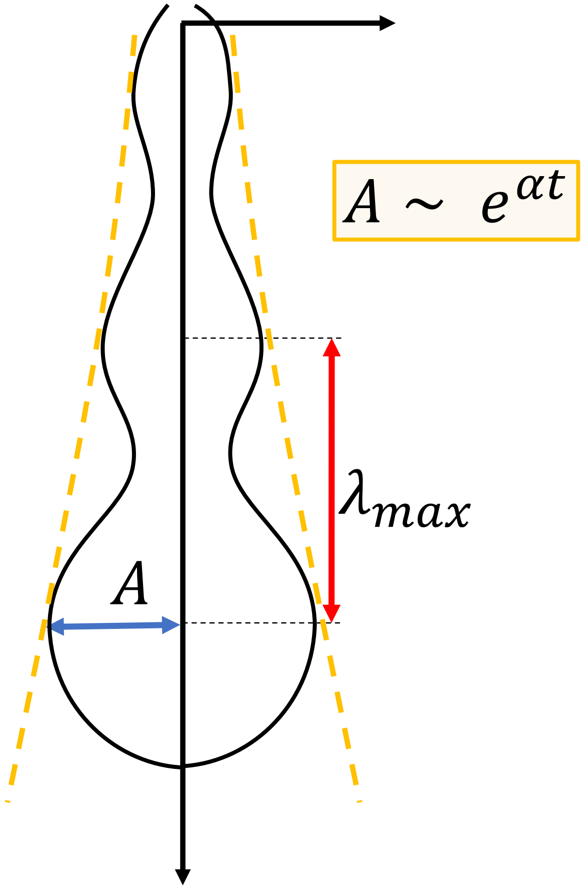
Water



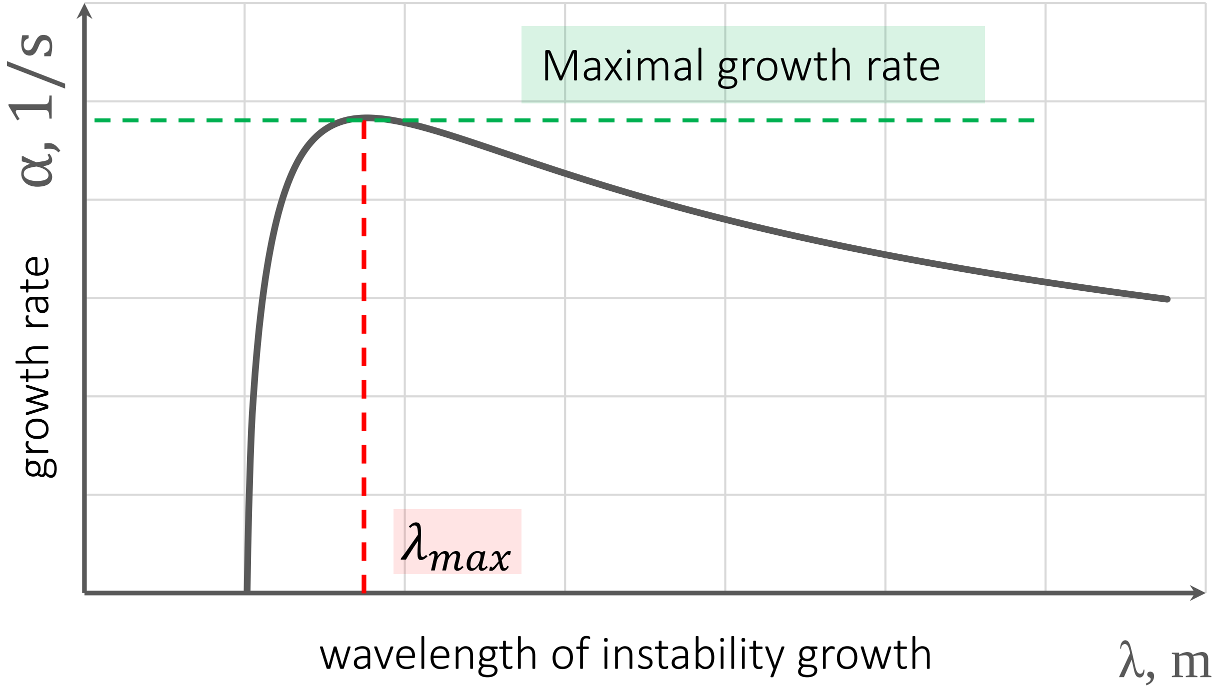
The phenomena is observed for any pair of immiscible fluids

Dispertion relation

Amplitude of perturbation grows exponentially

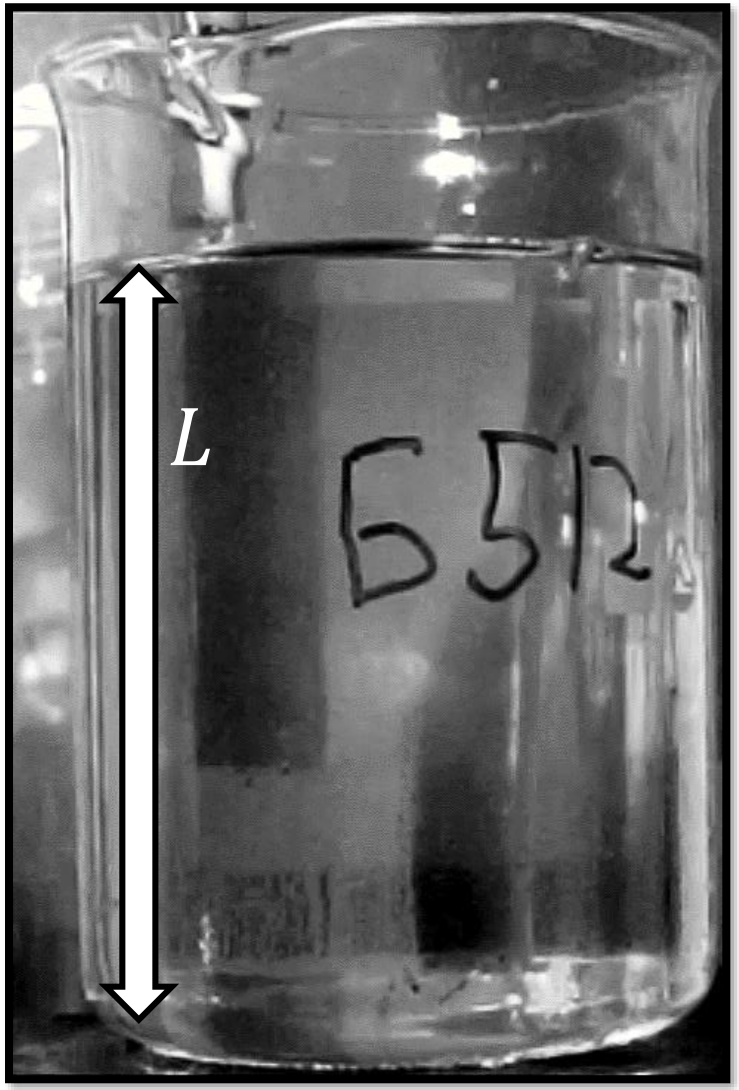


Dispersion relation



The instability grows on the wavelength, corresponding to the maximal growth rate

Possibility of pearls observation



$$\tau = \frac{1}{\alpha}$$ - characteristic time of Rayleigh-Plateau instability growth

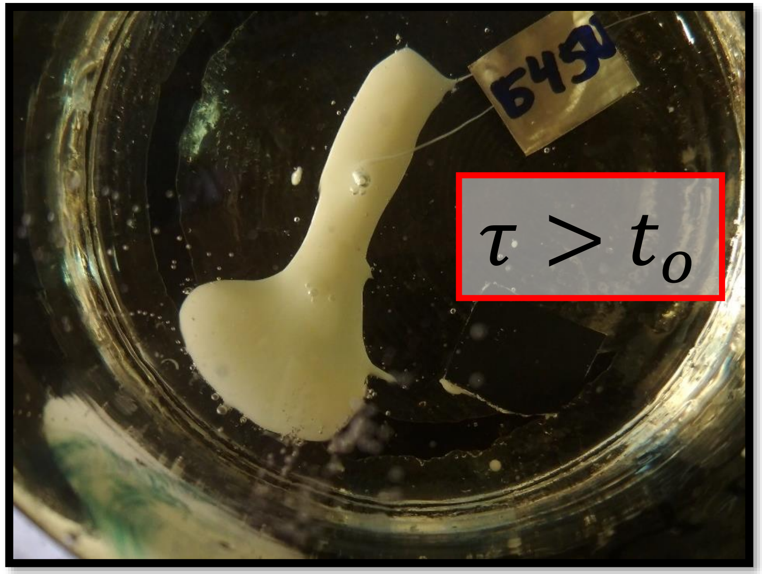
$$t_o = \frac{L}{v}$$ - characteristic time of observation (the time for the instability to grow up)

the condition for pearls to appear

$$\tau < t_o$$

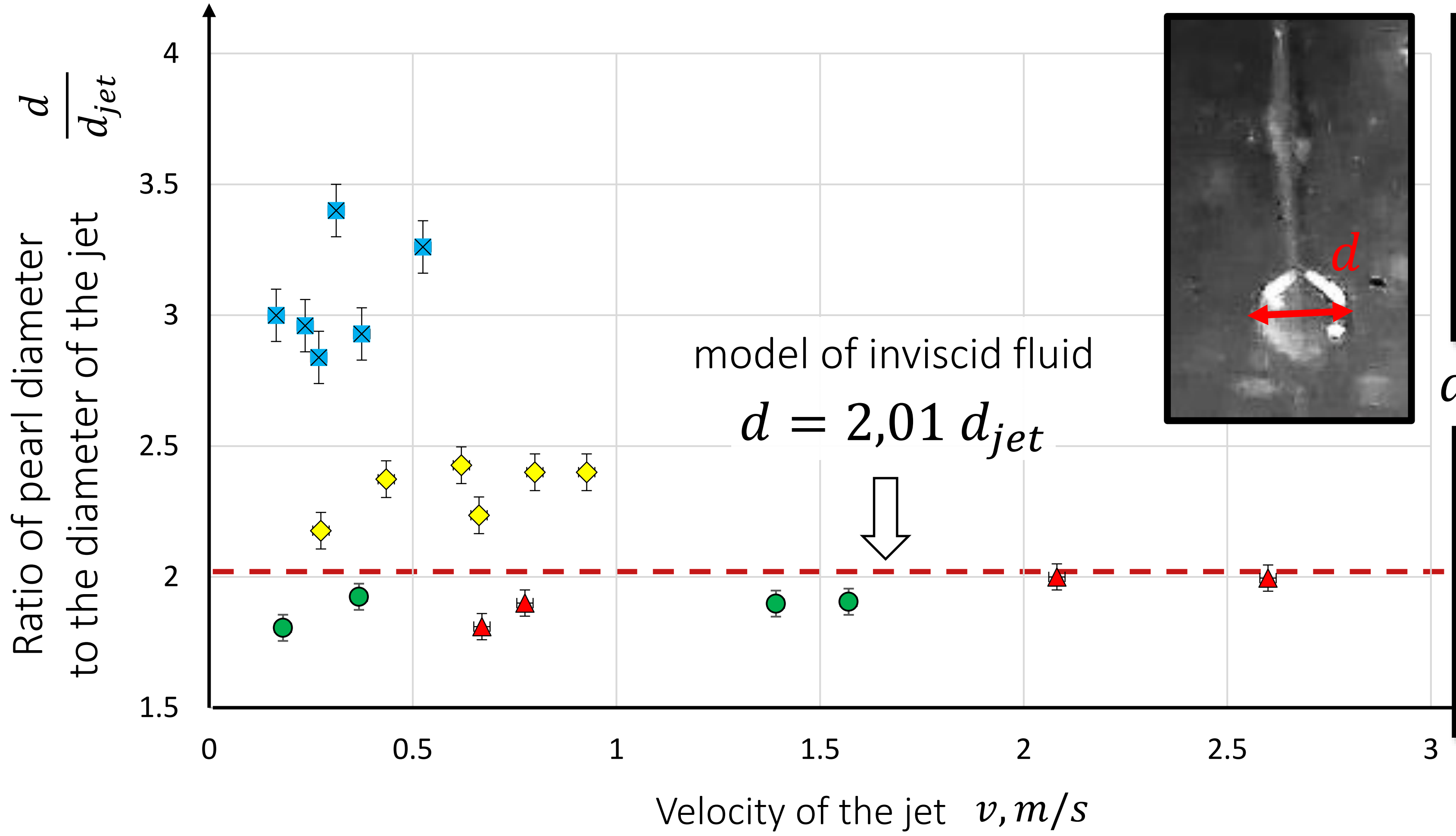
α – growth rate of instability

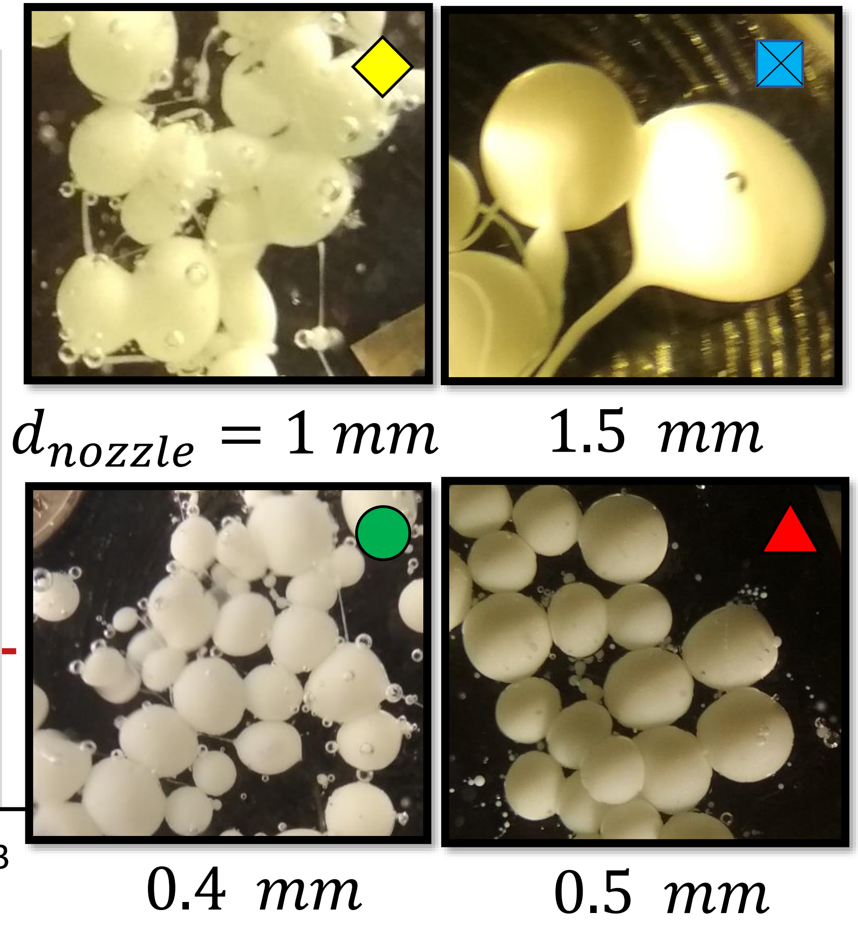
v – velocity of the jet



Size of the pearls

Simplest model VS experiment





The model of RP-instability for inviscid fluids doesn’t work for big diameters of jets!

The size of pearls (Rayleigh-Plateau instability)

For two fluids with arbitrary viscosity ratio ($\mu_A \sim \mu_B$):

Growth rate of instability

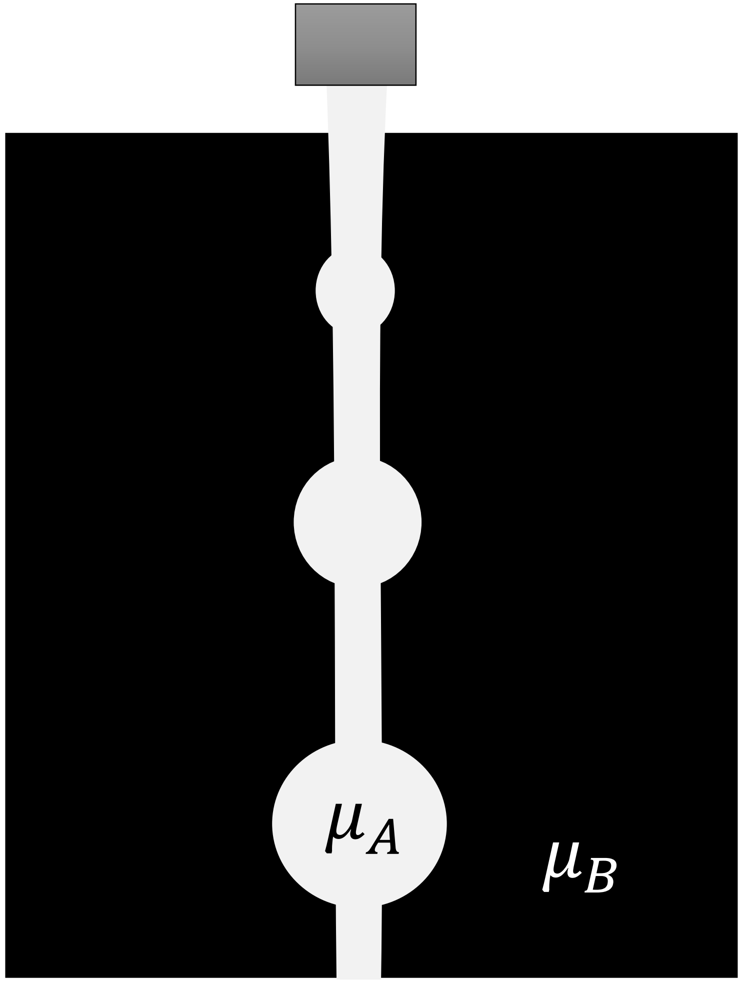
$$\alpha^2 = \frac{\sigma(1 - k^2 r_{jet}^2)}{2a\mu_B} \Phi\left(kr_{jet}, \frac{\mu_A}{\mu_B}\right)$$

σ coefficient of the surface tension

$k = \frac{2\pi}{\lambda}$ wave number

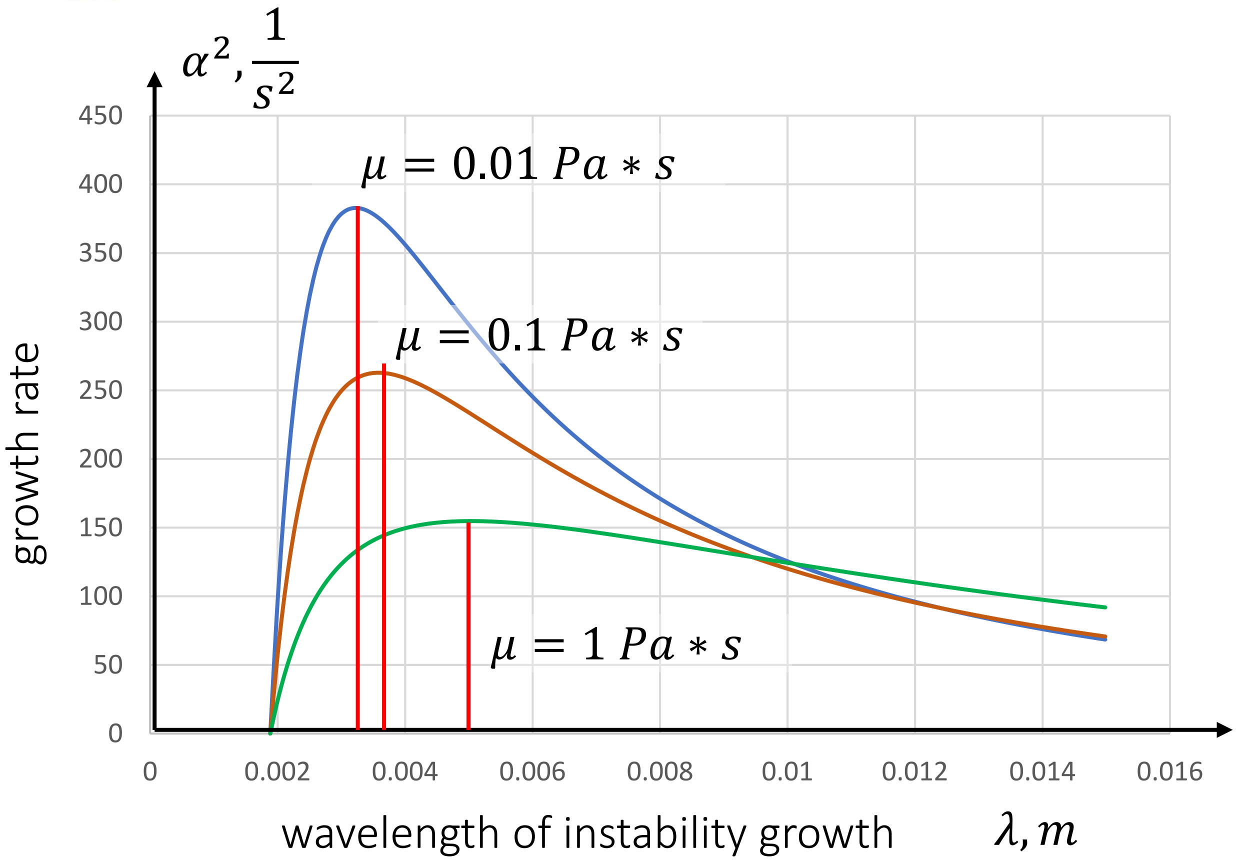
$r_{jet} = \frac{d_{jet}}{2}$ radius of the jet

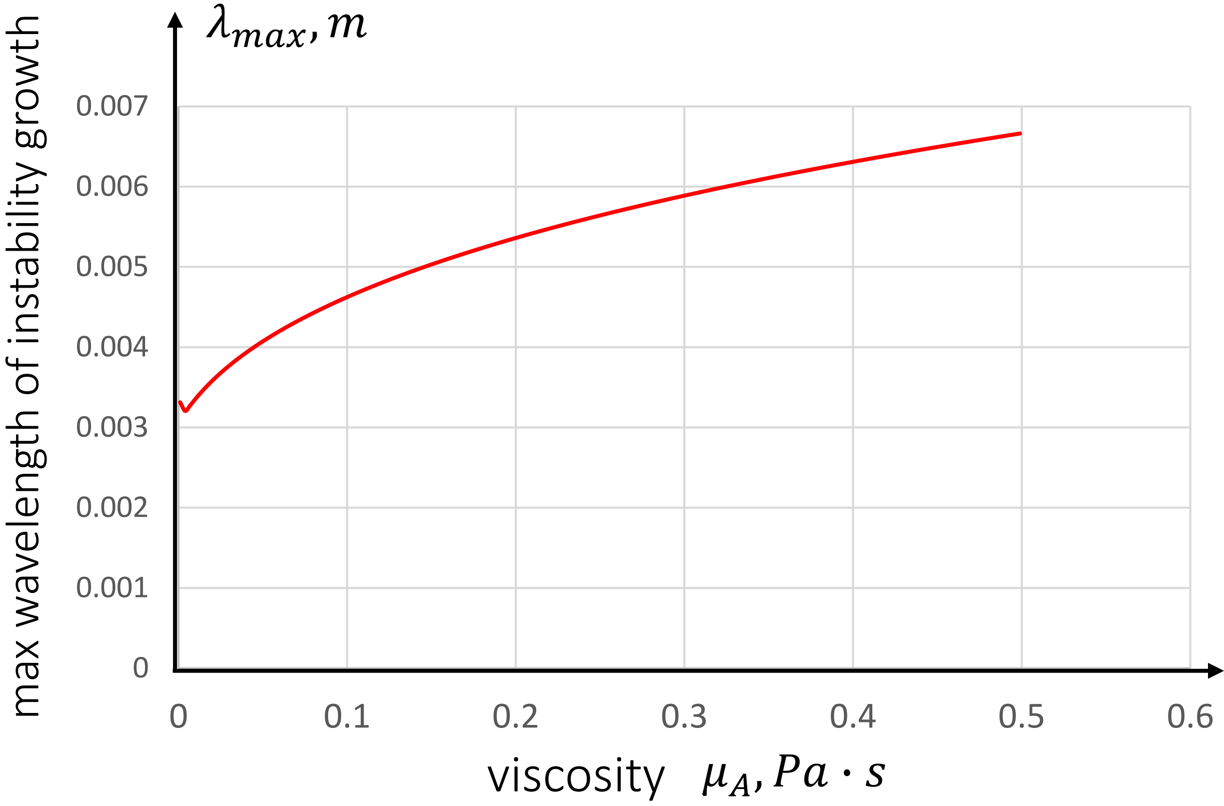
See the definition of Φ in appendix slides



Tomotika, S (1935). "On the Instability of a Cylindrical Thread of a Viscous Liquid Surrounded by Another Viscous Fluid". *Proceedings of the Royal Society of London A*. **150**: 322–337.

Dispersion relation for different μ_A

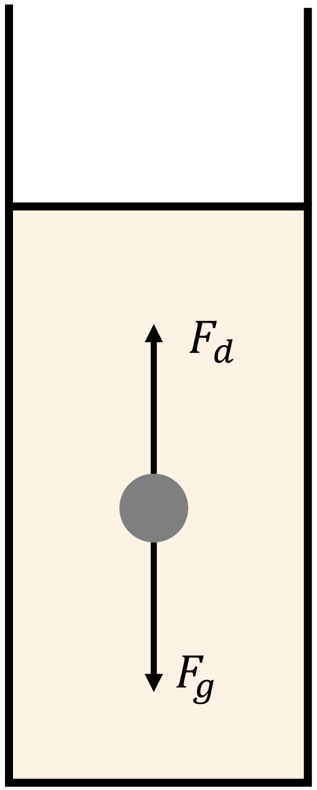




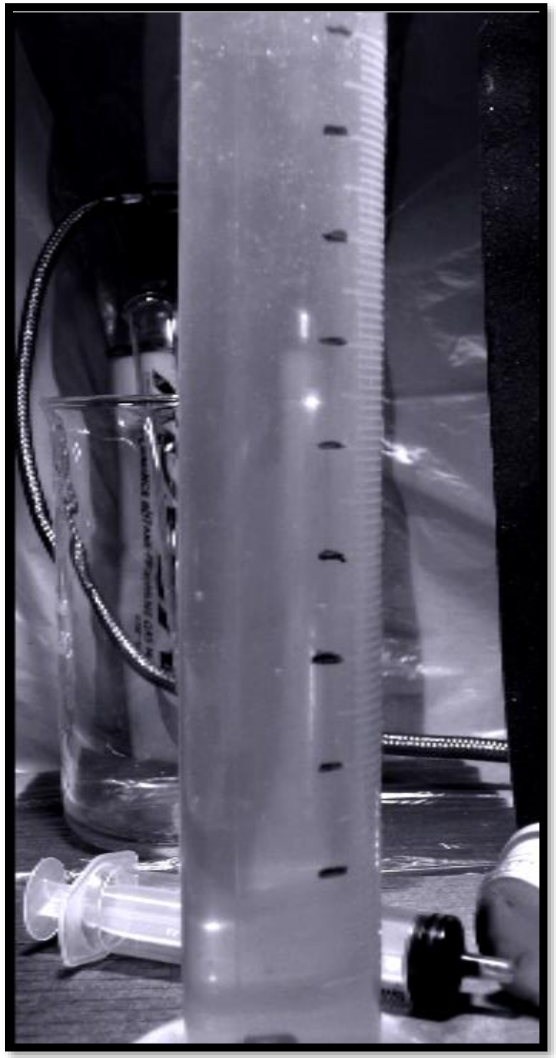
If $\mu_A \gg \mu_B$, the wavelength of maximal rate of growth increases

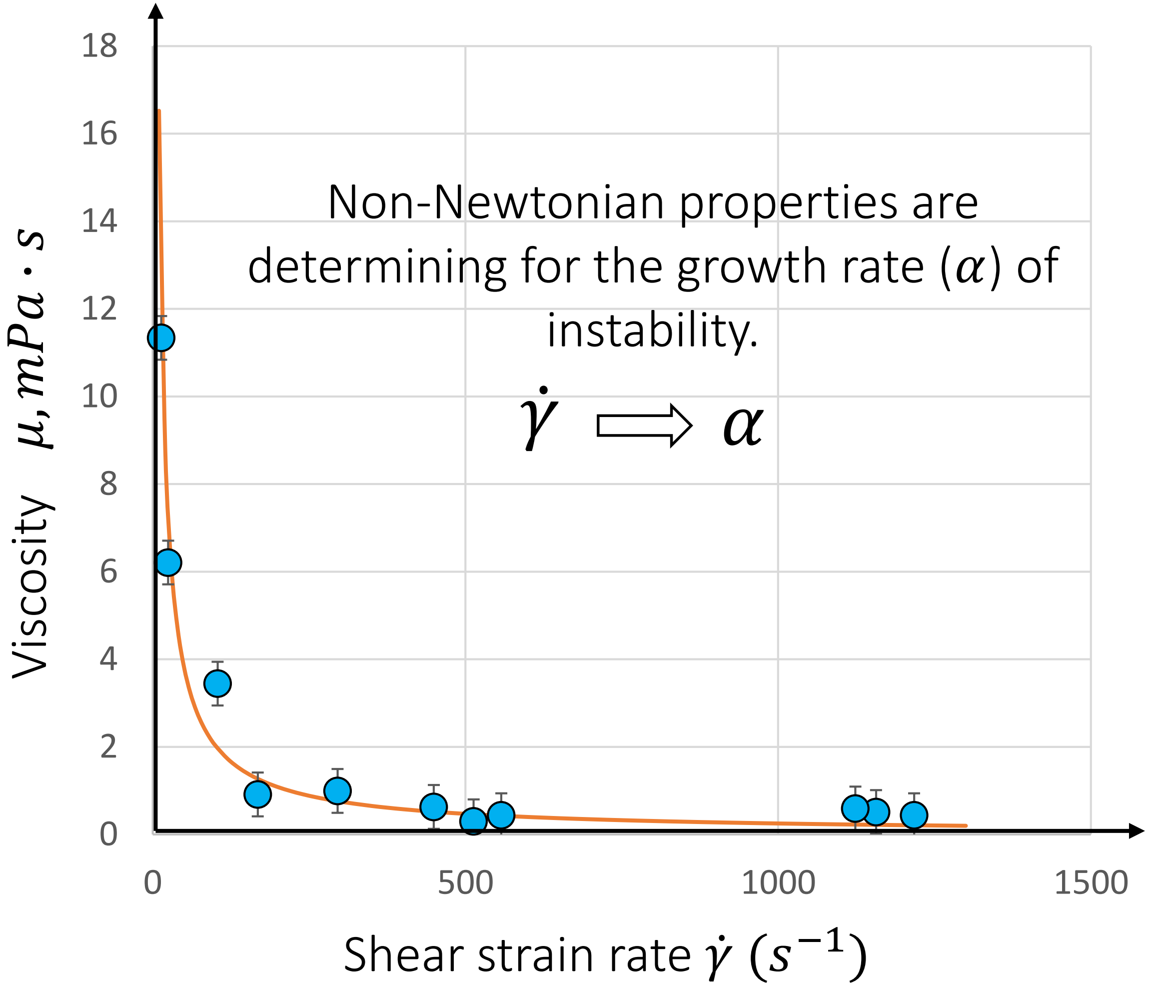
Rheology of eggs

Falling ball viscometer



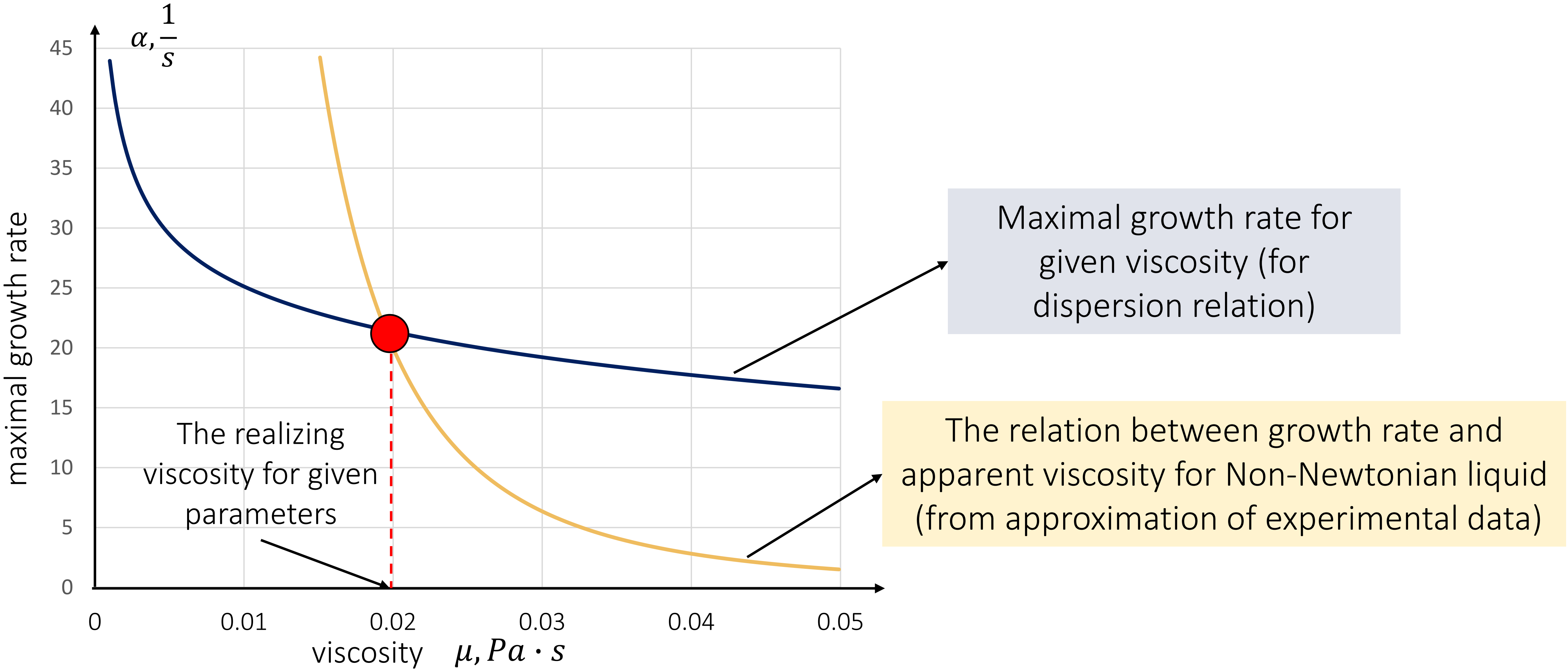
The method is based on measuring the rate of incidence of small balls in the test fluid



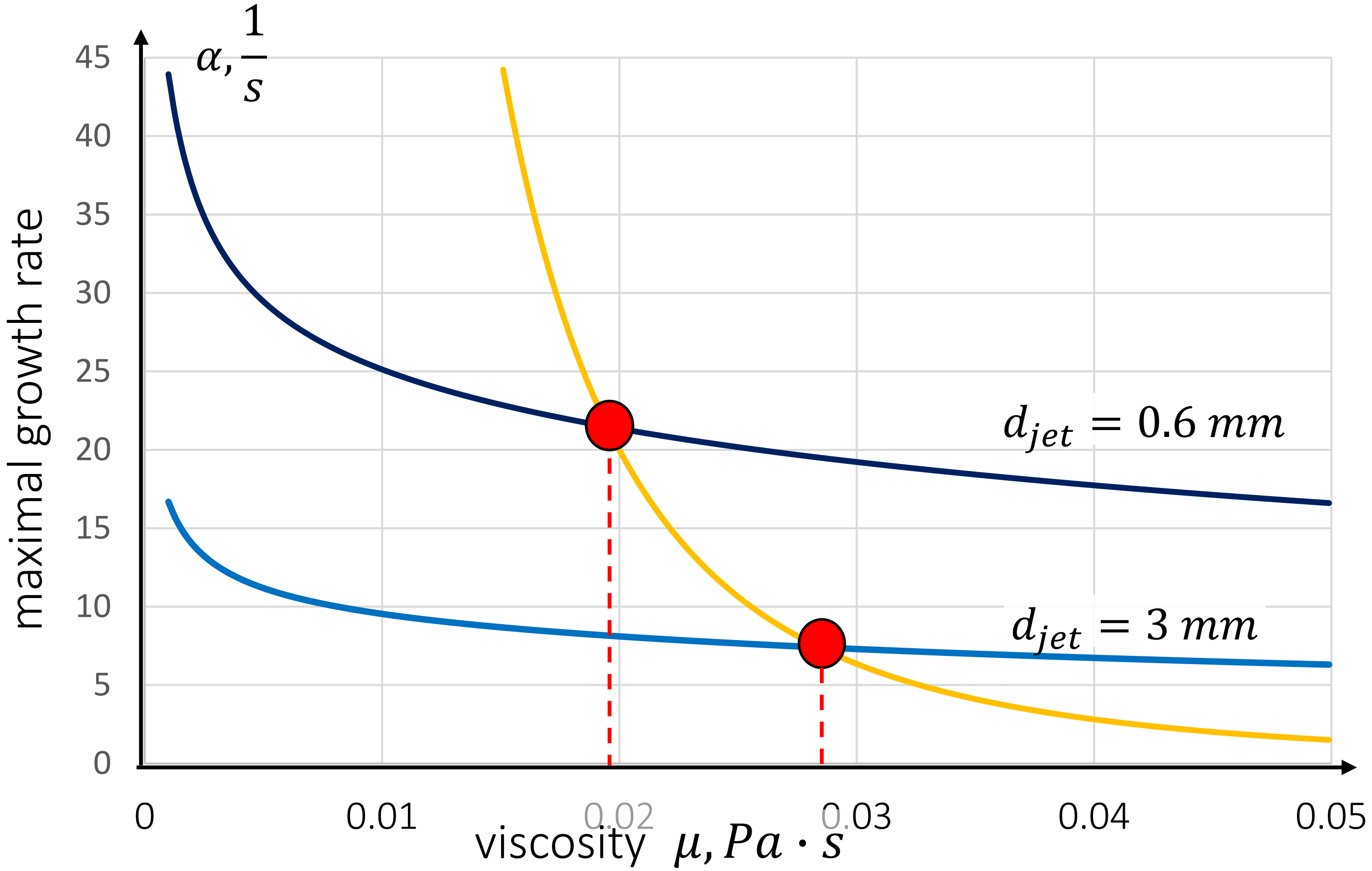


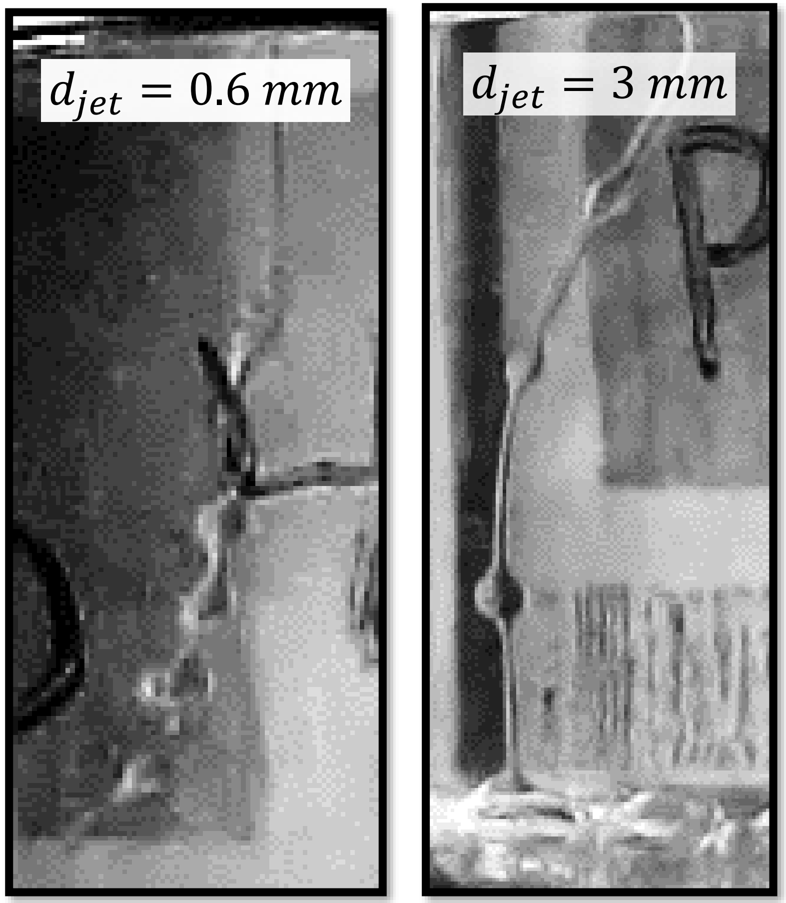
shear-thinning liquid

The determination of apparent viscosity for given parameters



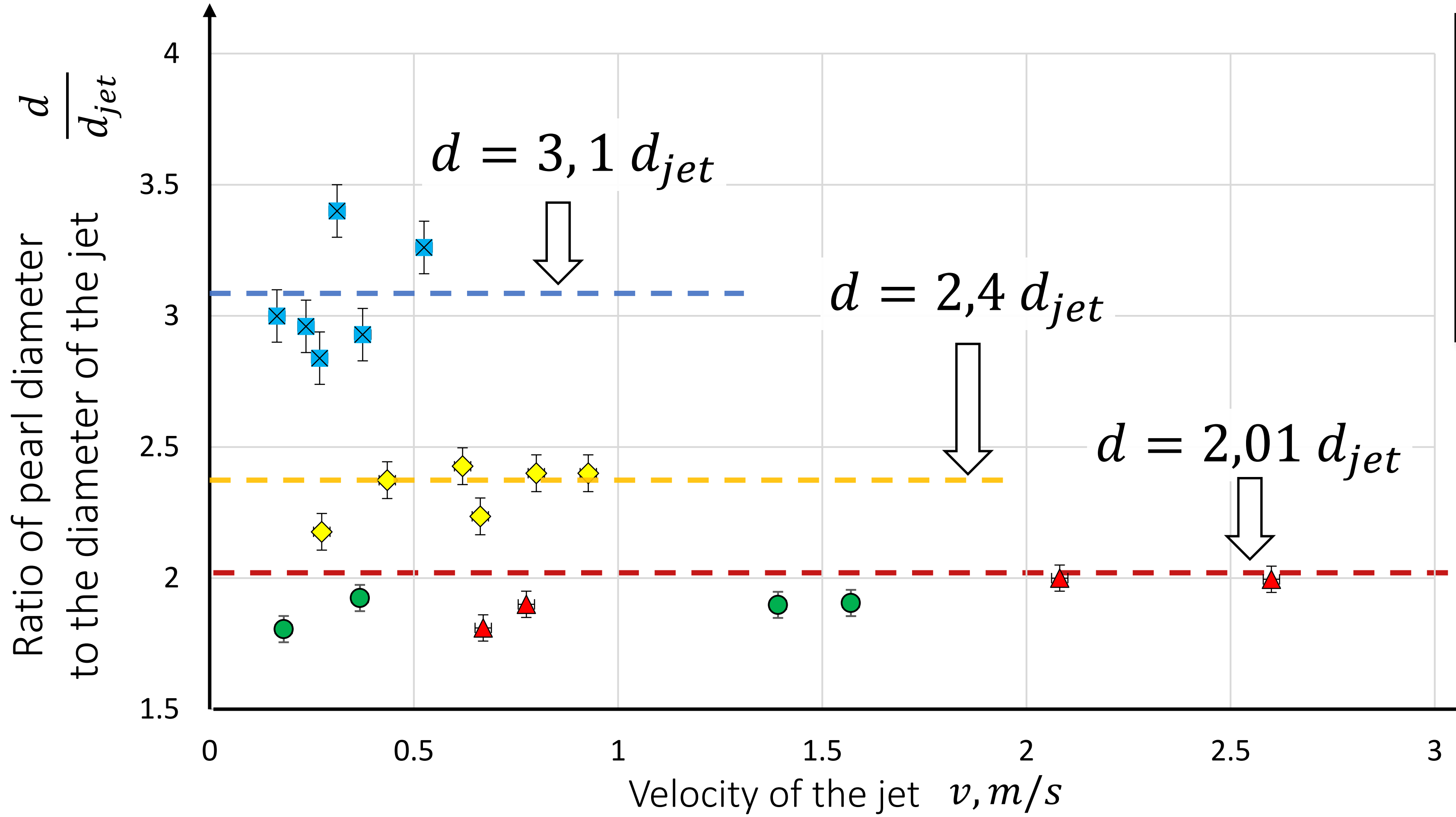
The influence of Non-Newtonian properties

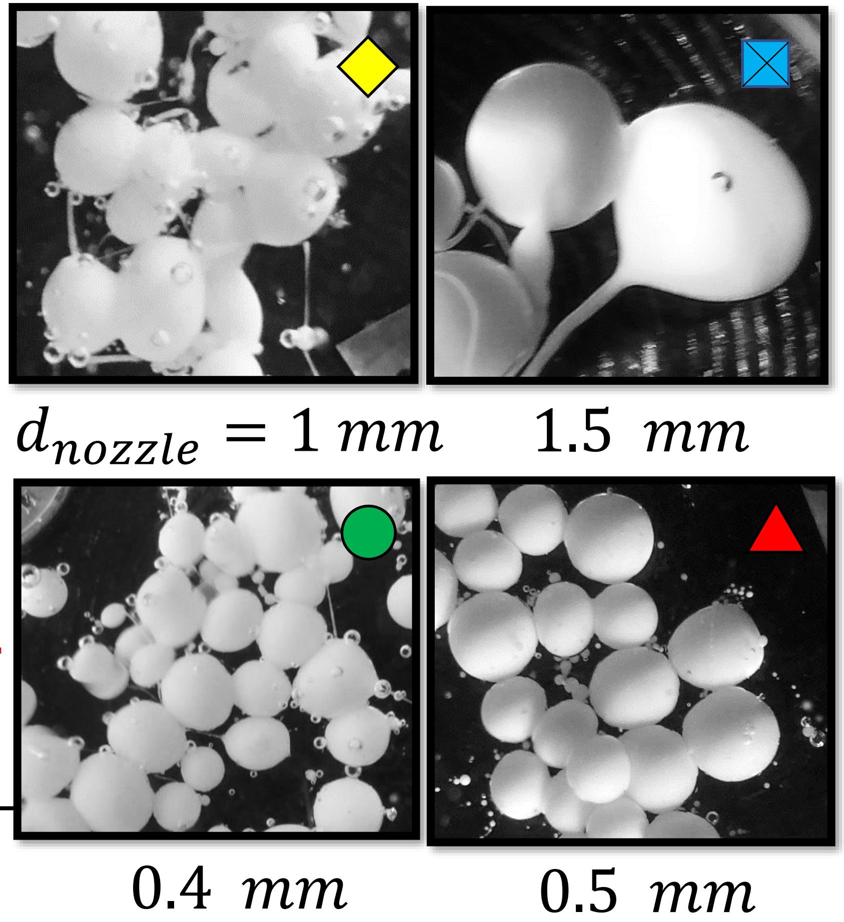




The bigger is the diameter of the jet, the bigger is apparent viscosity.
With the growth of viscosity, the wavelength increases.

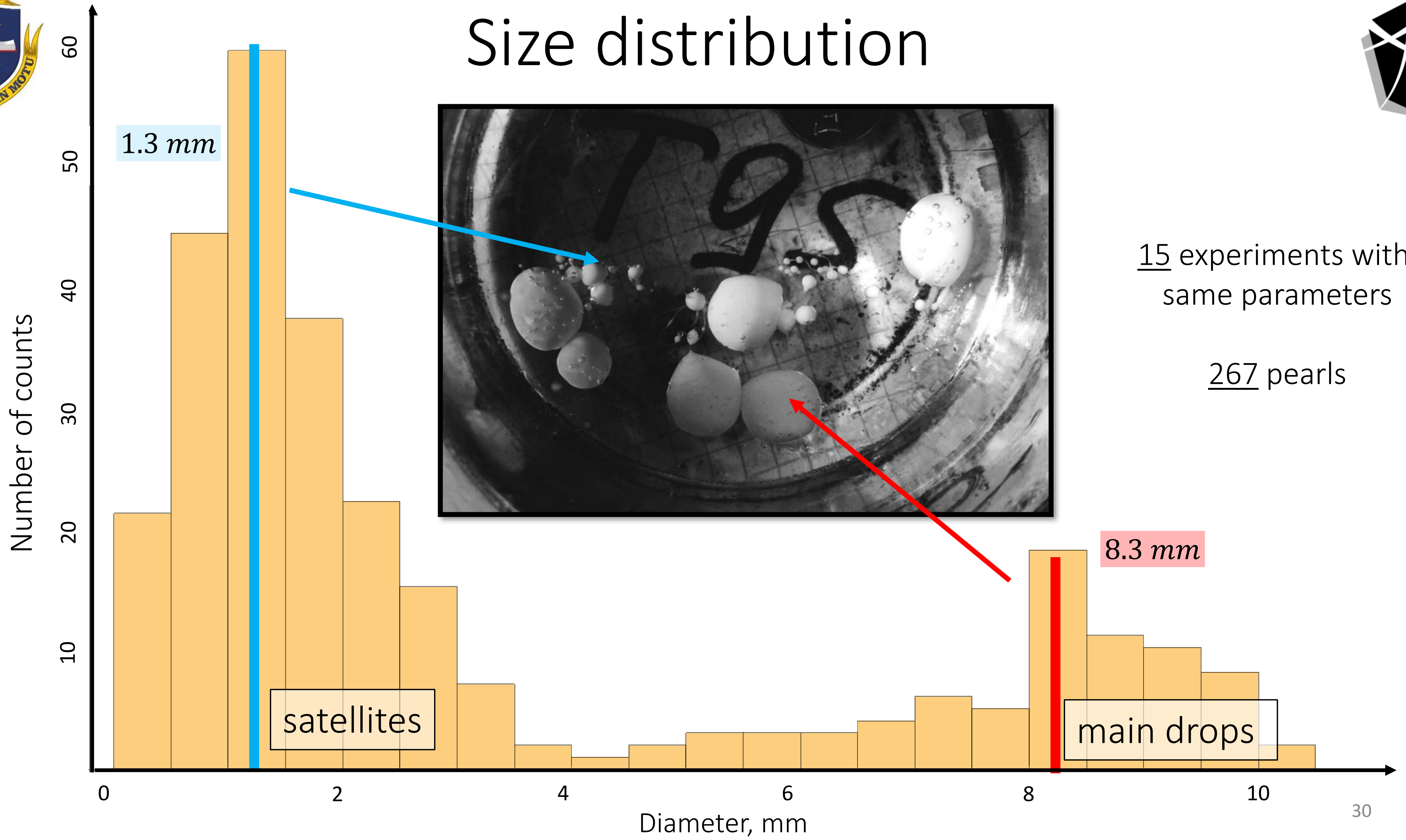
Simplest theoretical estimation of size





The Non-Newtonian properties of egg white determine the size of pearls for big diameters of jets.

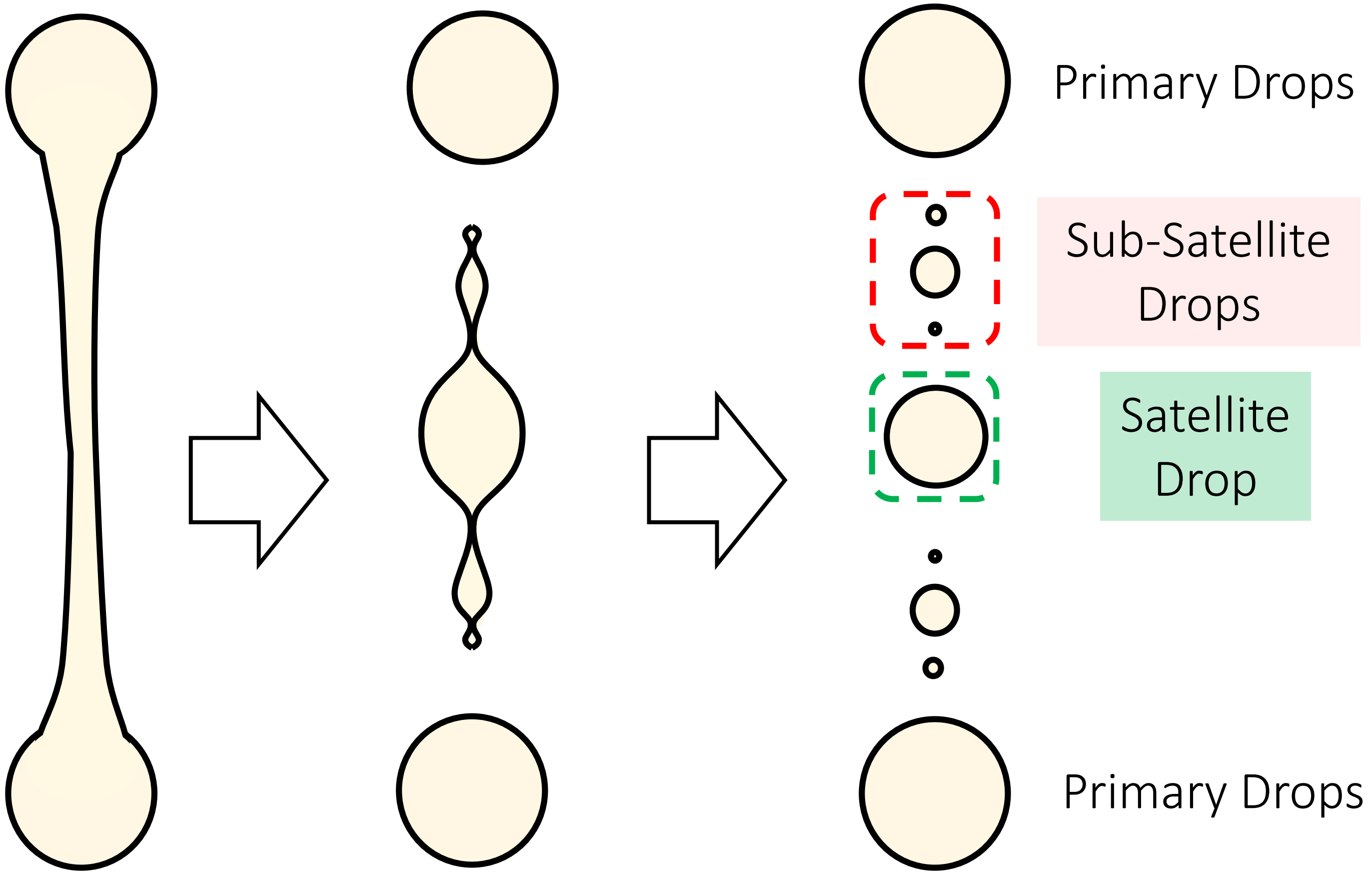
Size distribution

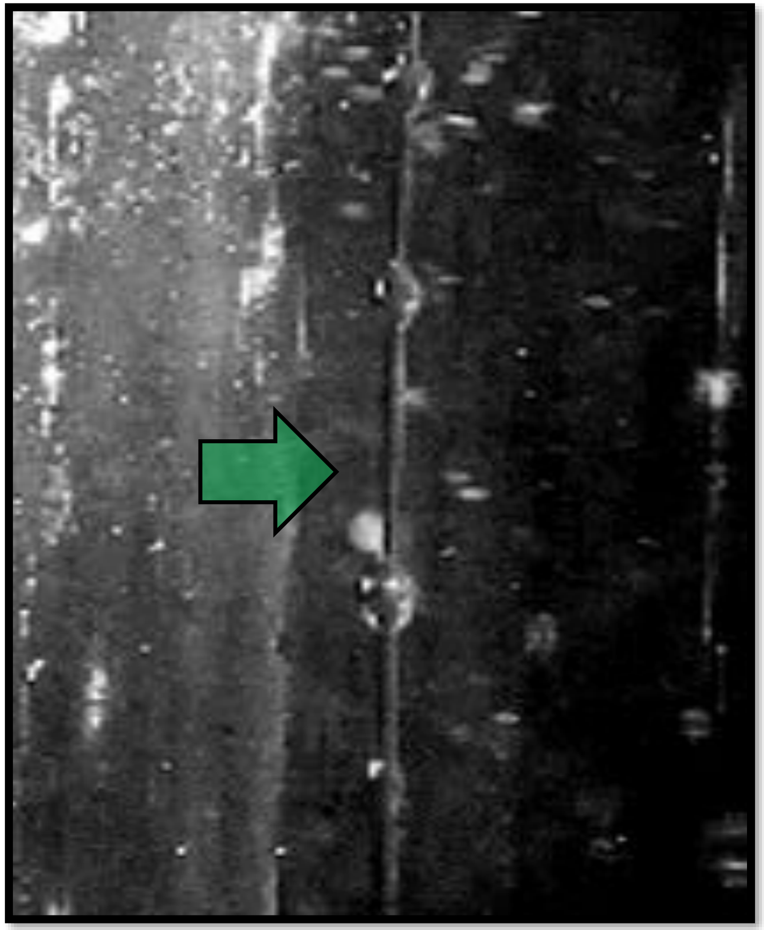


15 experiments with same parameters

267 pearls

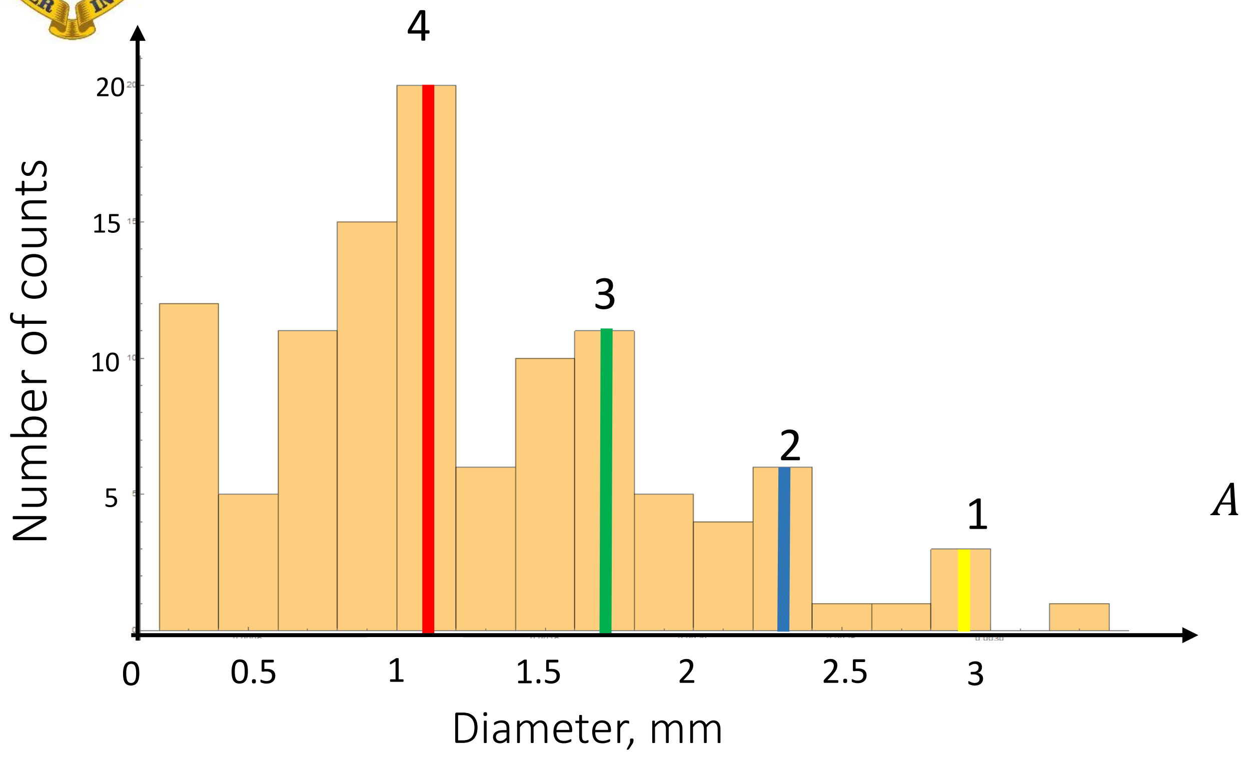
Satellite and subsatellite formation

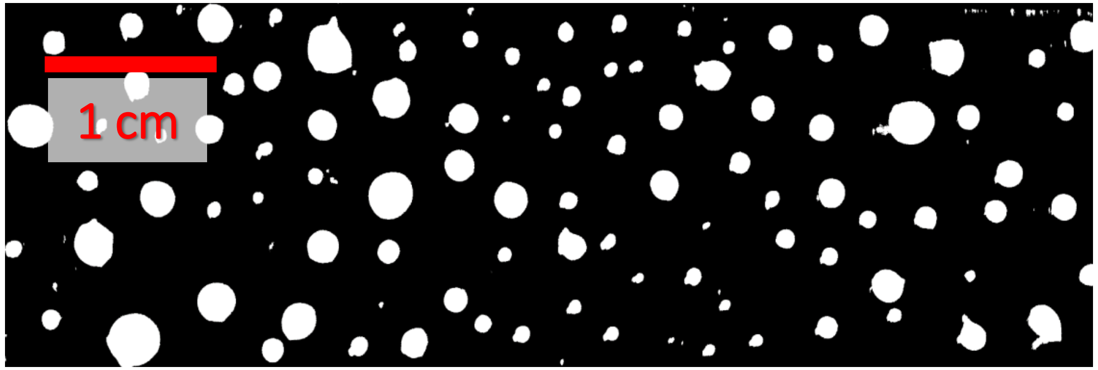




The satellite drop is formed from the filament because of nonlinear effects.

Only satellite and subsatellite

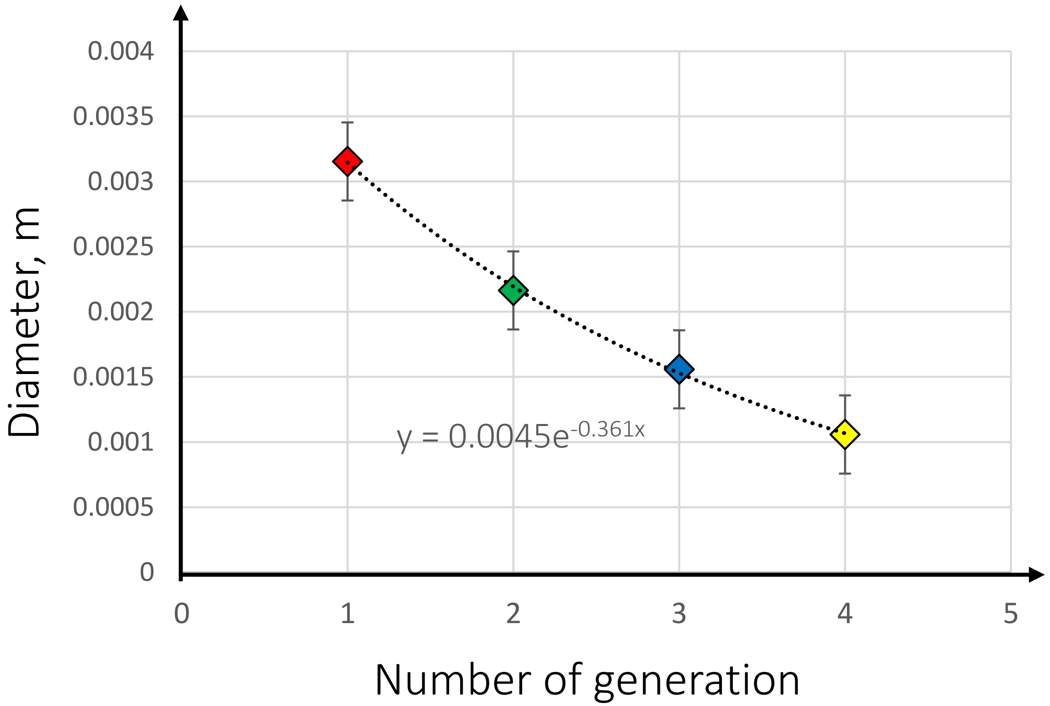




Recursive mechanism of subsatellite formation

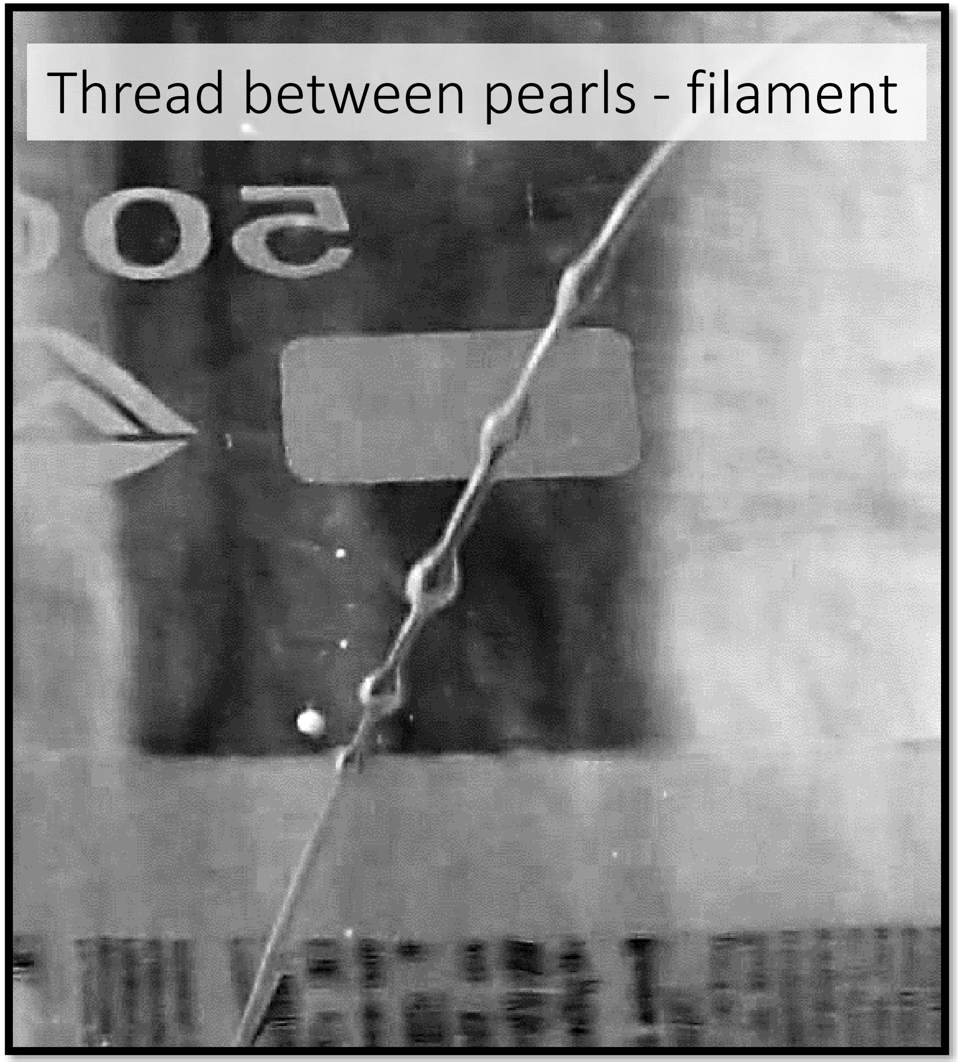
$$d_n = A * d_{n-1}$$

d_n – satellite diameter for the n-th generation



The features of pearls formation

«Bead-on-string» structure



"Bead-on-string" structure is characteristic for viscoelastic fluids

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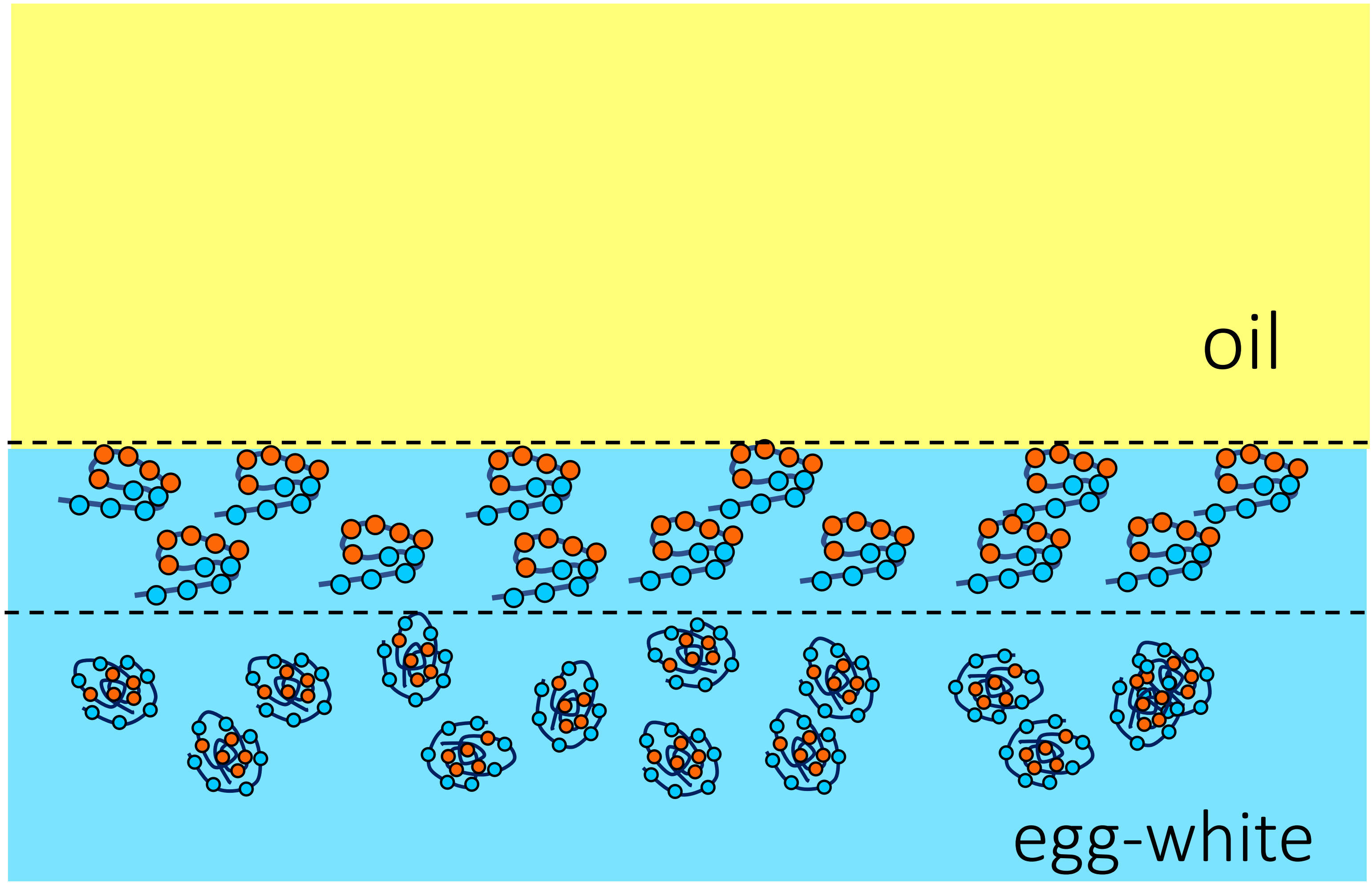
Viscoelasticity

Viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation

Adsorption of protein on the oil-water boundary

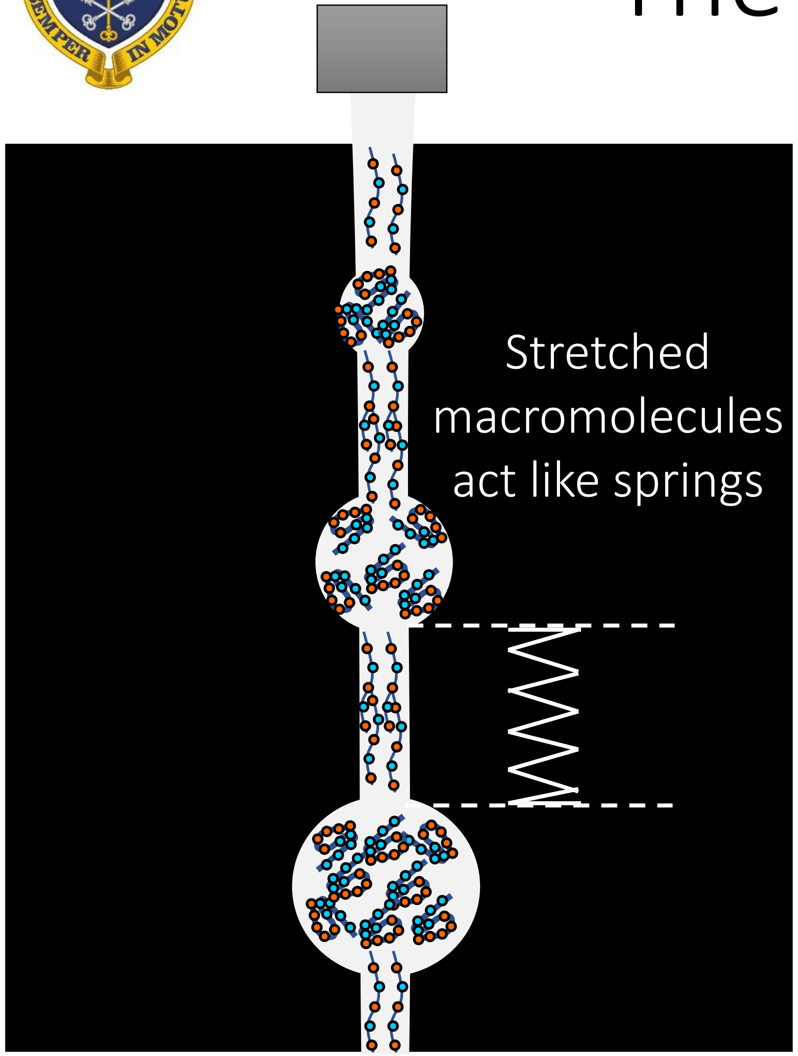


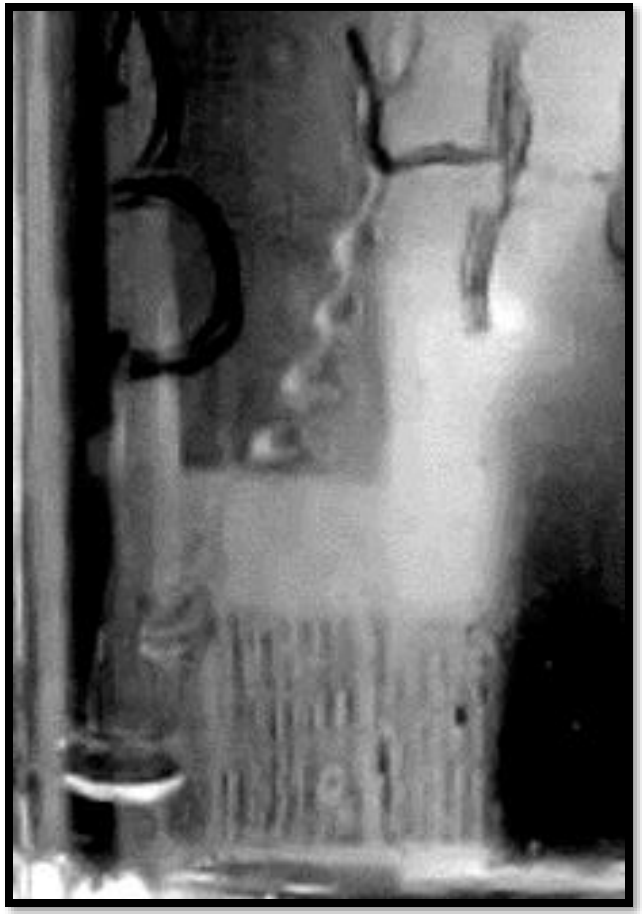
protein adsorbs on the oil-water boundary

viscoelastic multilayer appears

Baldursdottir, et. Al. (2010). Adsorption of proteins at the oil/water interface-Observation of protein adsorption by interfacial shear stress measurements. Colloids and surfaces. B, Biointerfaces.

The filaments and their role

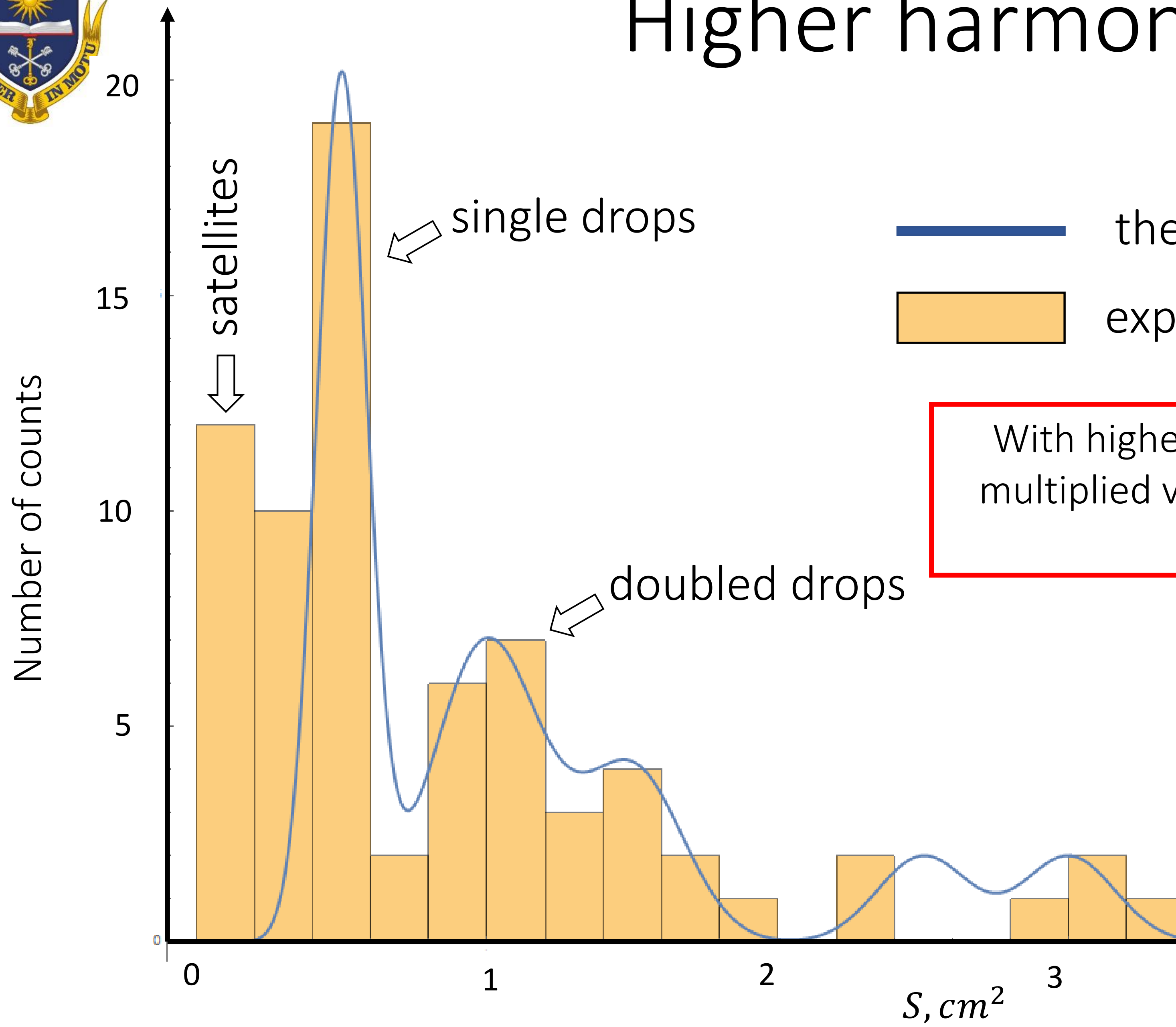



collision of beads


merging of beads on the bottom

Yarin, Alexander L. Free liquid jets and films: hydrodynamics and rheology. Longman Publishing Group, 1993.

Higher harmonics



With higher Deborah numbers the pearls with multiplied volumes appear (due to merging and collision)

Conclusions

- The main mechanism of pearls formation is Rayleigh-Plateau instability.
- The Non-Newtonian properties of egg-white significantly increases the size of pearls for high diameter of the jet.
- The small pearls, appearing among big ones, are satellites and form from the filament.
- The long-living filament connecting the pearls is explained by the viscoelastic multilayer of adsorbed protein on the oil-water interface.

Thanks for your attention!

Appendix

Deborah number

$$De = \frac{\theta}{t}$$

θ – relaxation time of viscoelastic multilayer

$De < De_{crit}$

the filaments destroy

$De > De_{crit}$

the filaments are important in dynamics of the process

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t – time scale of the process

observation time $t_o = \frac{L}{v}$

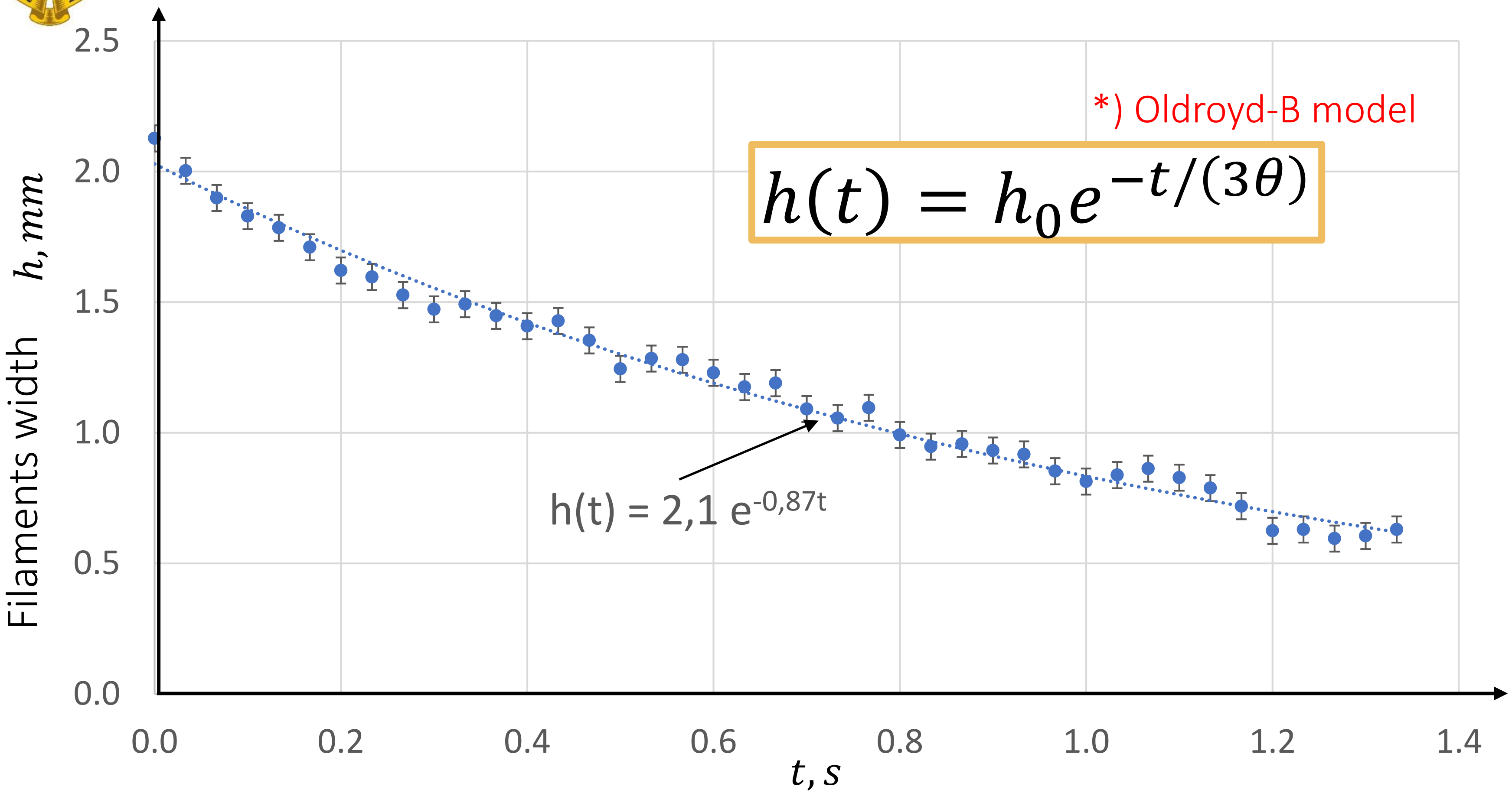
merging of beads on the bottom

$t_v = \frac{\mu d_{jet}}{\sigma}$ viscous RP instability time-scale

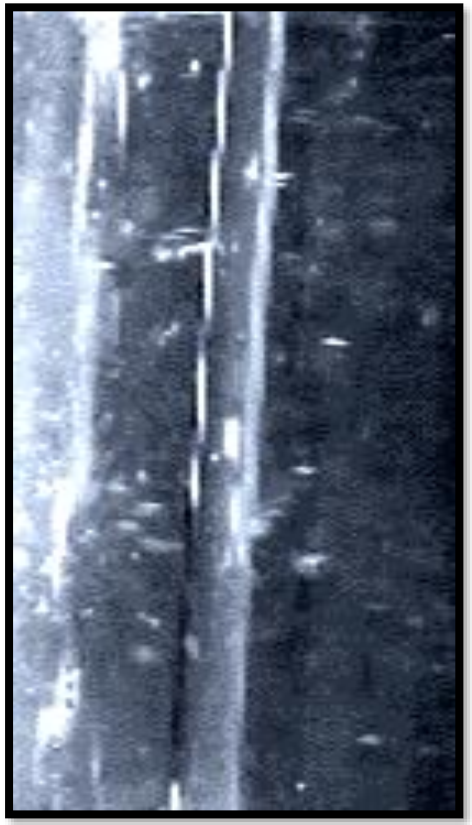
merging of beads on the jet

L – depth of oil layer v velocity of the jet d_{jet} – diameter of the jet μ – dynamic viscosity σ – surface tension

The characteristic lifetime of filaments



$$h(t) = h_0 e^{-t/(3\theta)}$$



θ – relaxation time of viscoelastic layer

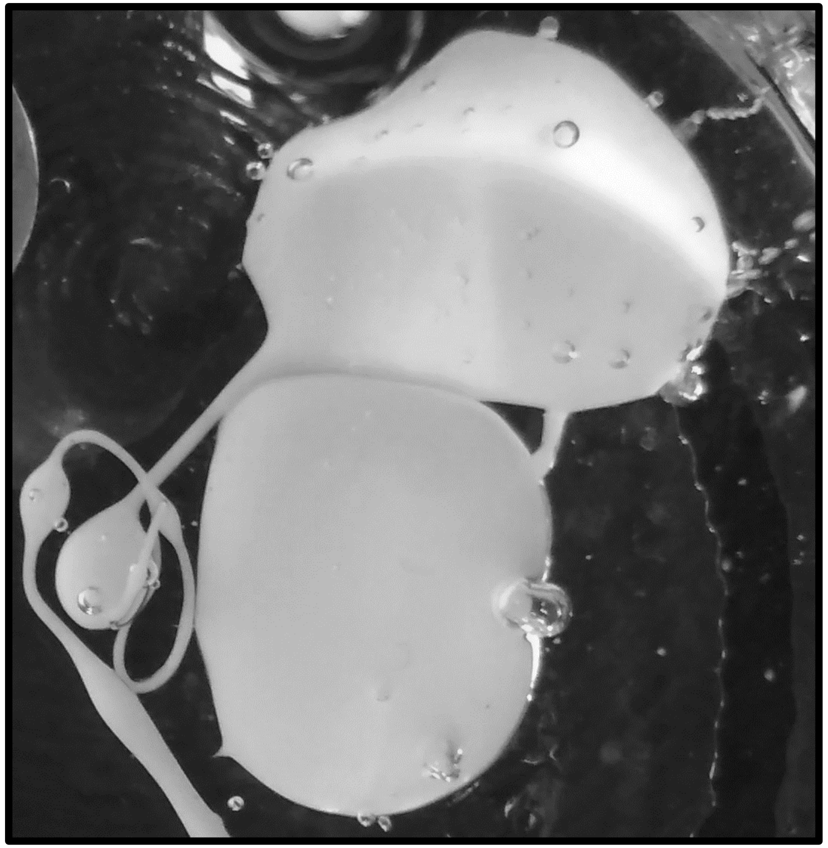
h_0 – initial filament width

$$\theta = 0{,}37\ s$$

Different Deborah numbers

$De = 0{,}75$

$De = 1{,}1$



$De = 3{,}2$

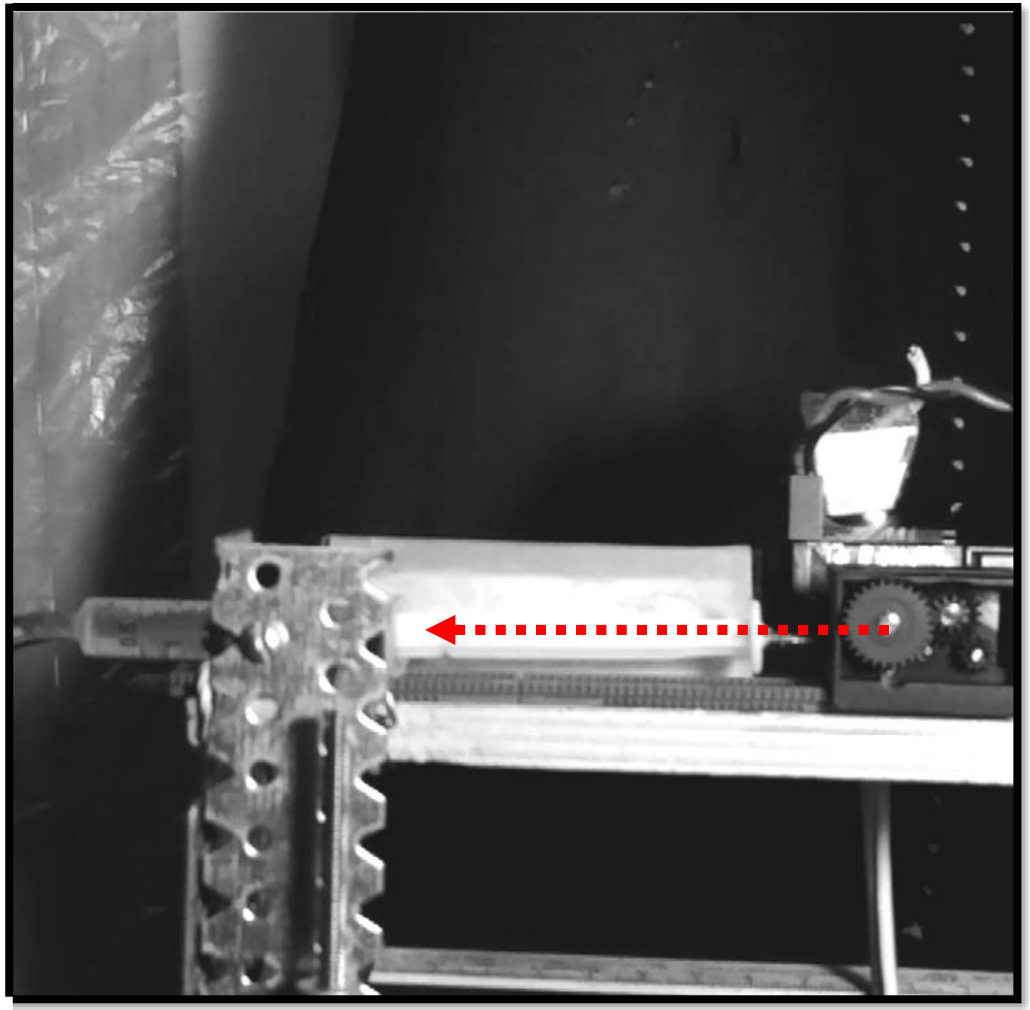
De

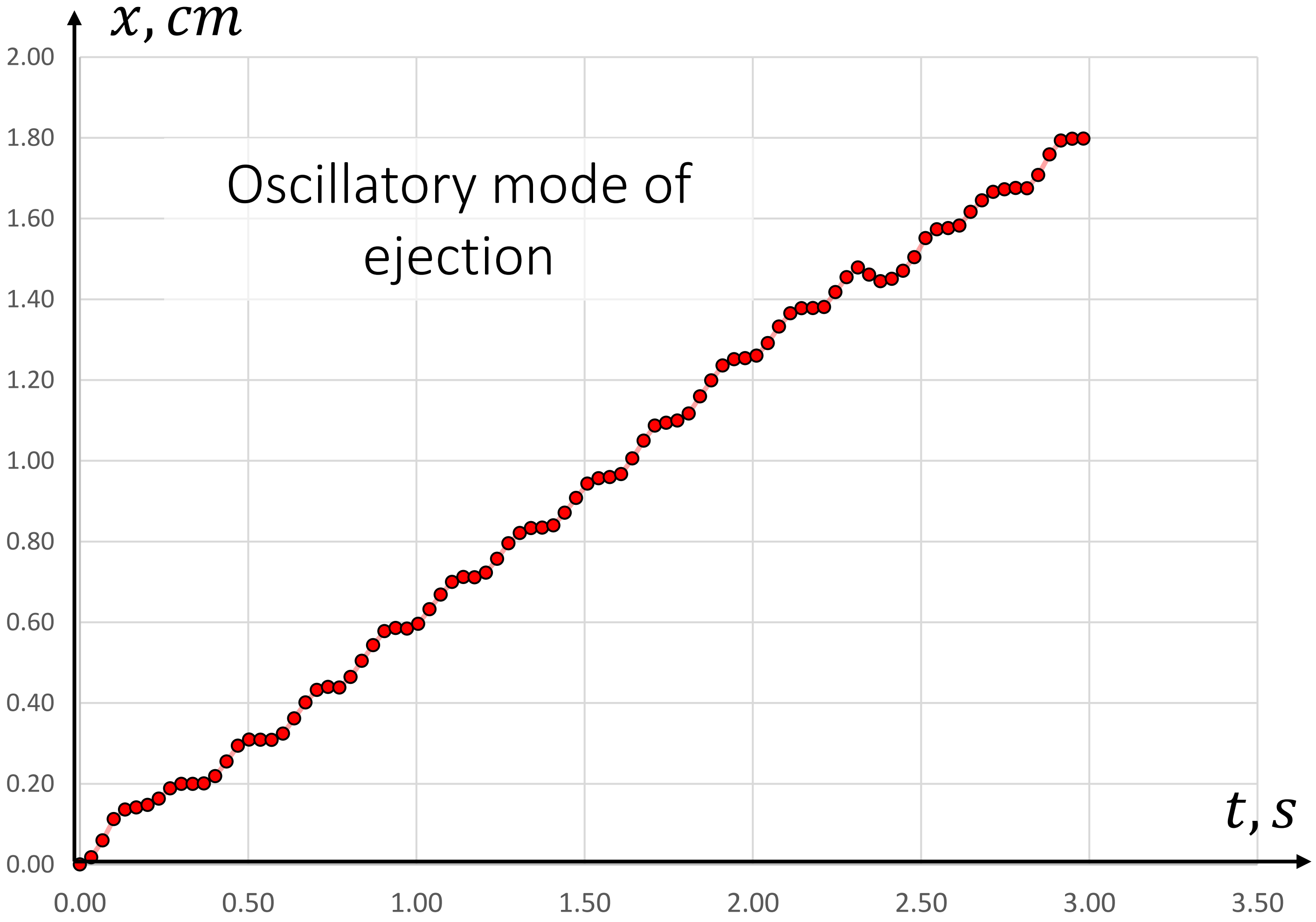
Specifics of ejection by hand

The pearls don’t appear for the uniform ejection for this set of parameters!

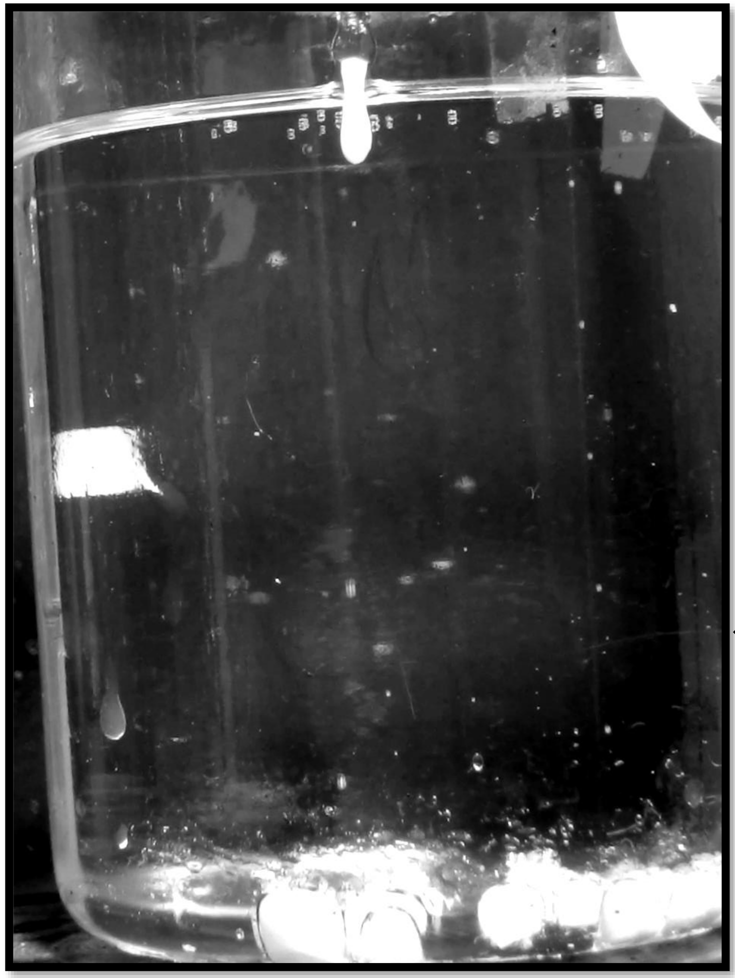
The uneven rate of ejection by hand produces initial perturbations which transform into pearls.

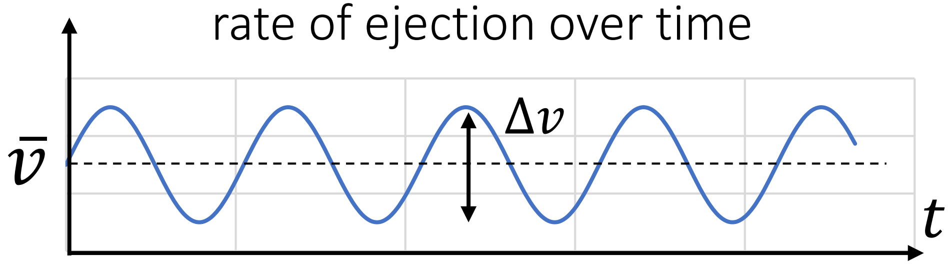
Model experiment with oscillating rate of ejection



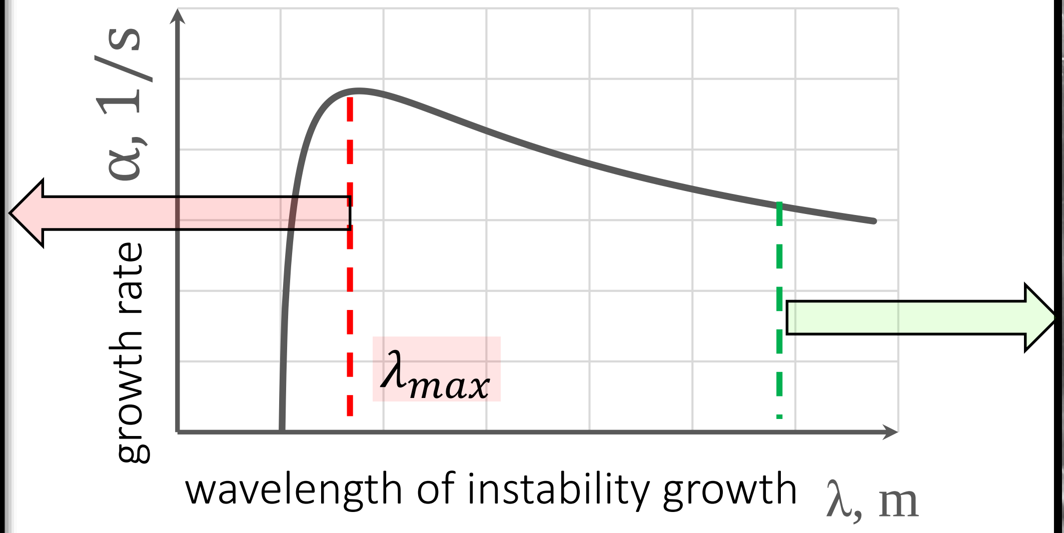


Resonance for perturbation excitement





$$\lambda = \bar{v} * T$$



Same amplitude of initial perturbations

resonance case

Far from resonance

Effect of horizontal motion of the tip

minimum possible horizontal velocity

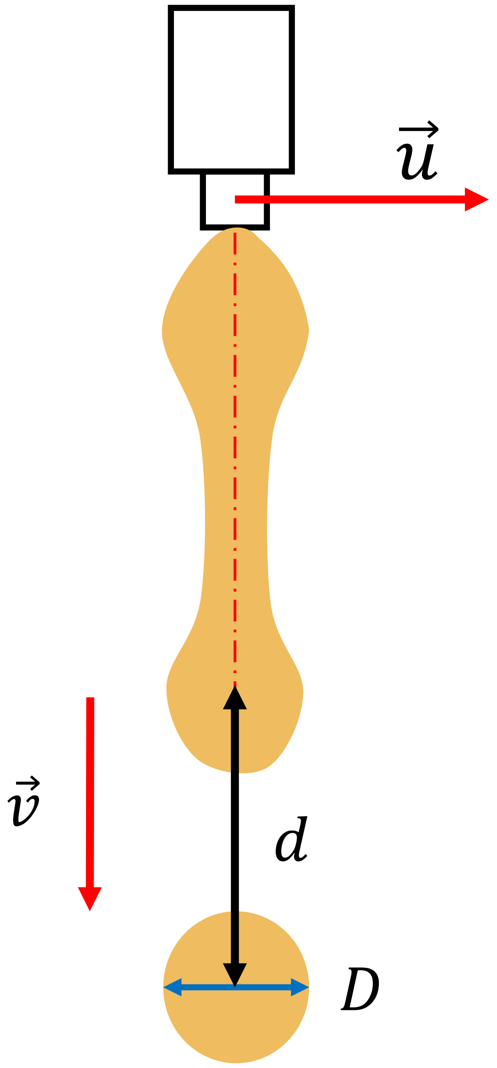
$$u_{min} = \frac{D\,v}{d}$$

u – horizontal velocity of the tip

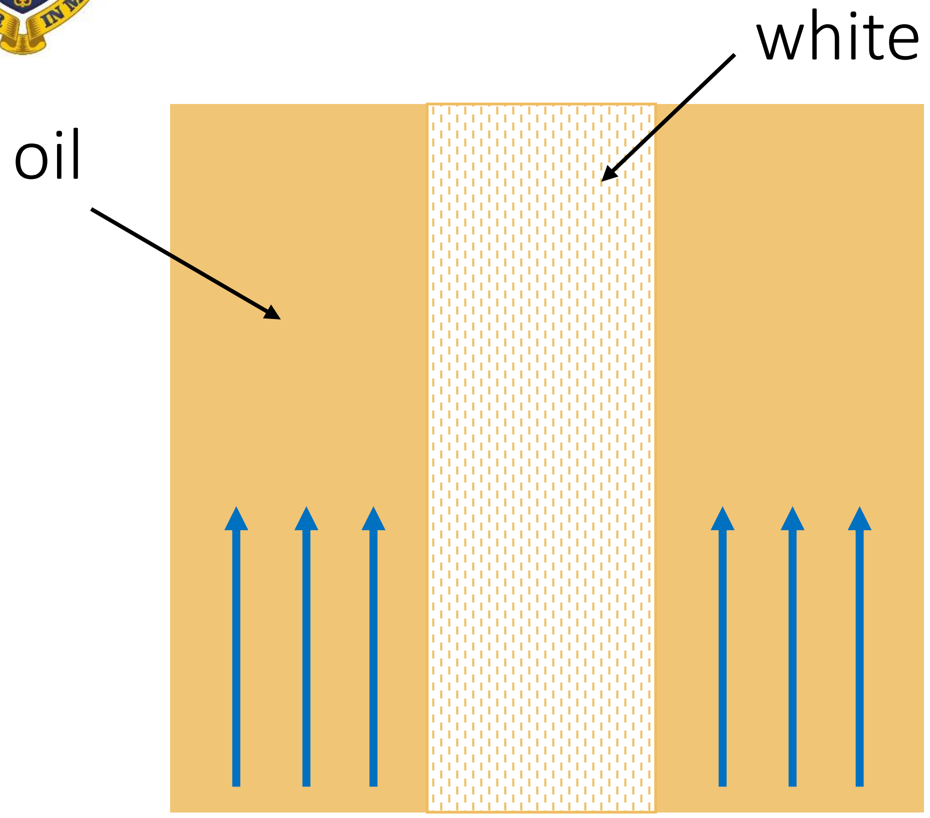
v – velocity of the jet

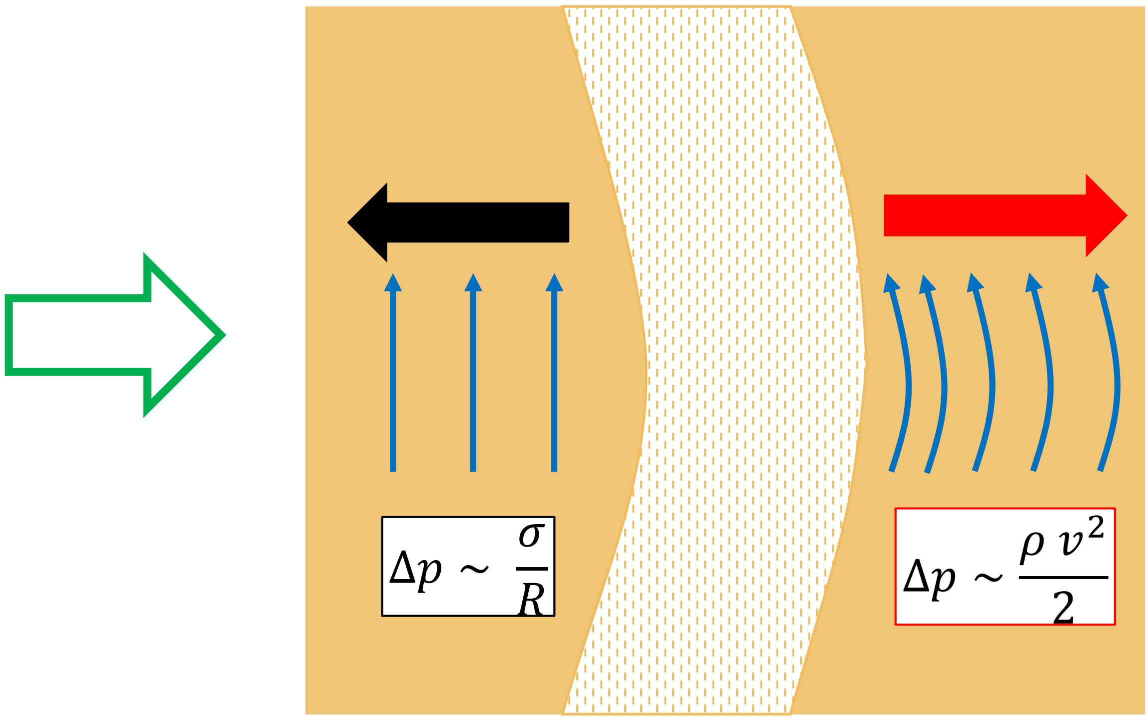
d – the distance between two neighboring drop

D – mean size of the pearl



Bending instability of jet





малое отклонение приводит уменьшению давления по закону Бернулли, оно порядка $\rho v^2/2$

возвращающей силой служит сила поверхностного натяжения, она порядка σ/R

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