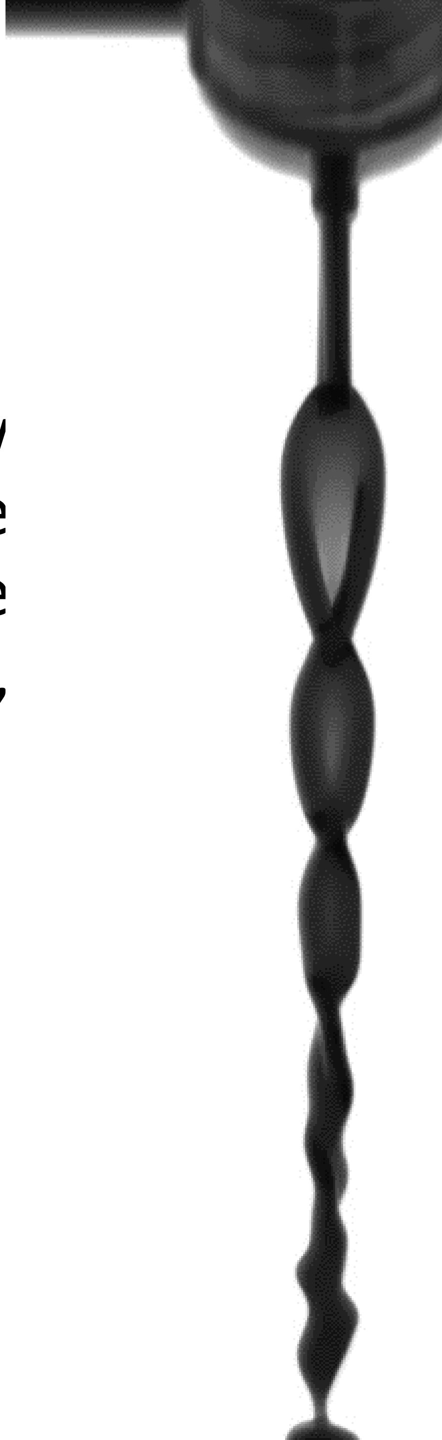
№10
Quaint jet

Reporter: Anastasiia Chervinskaia

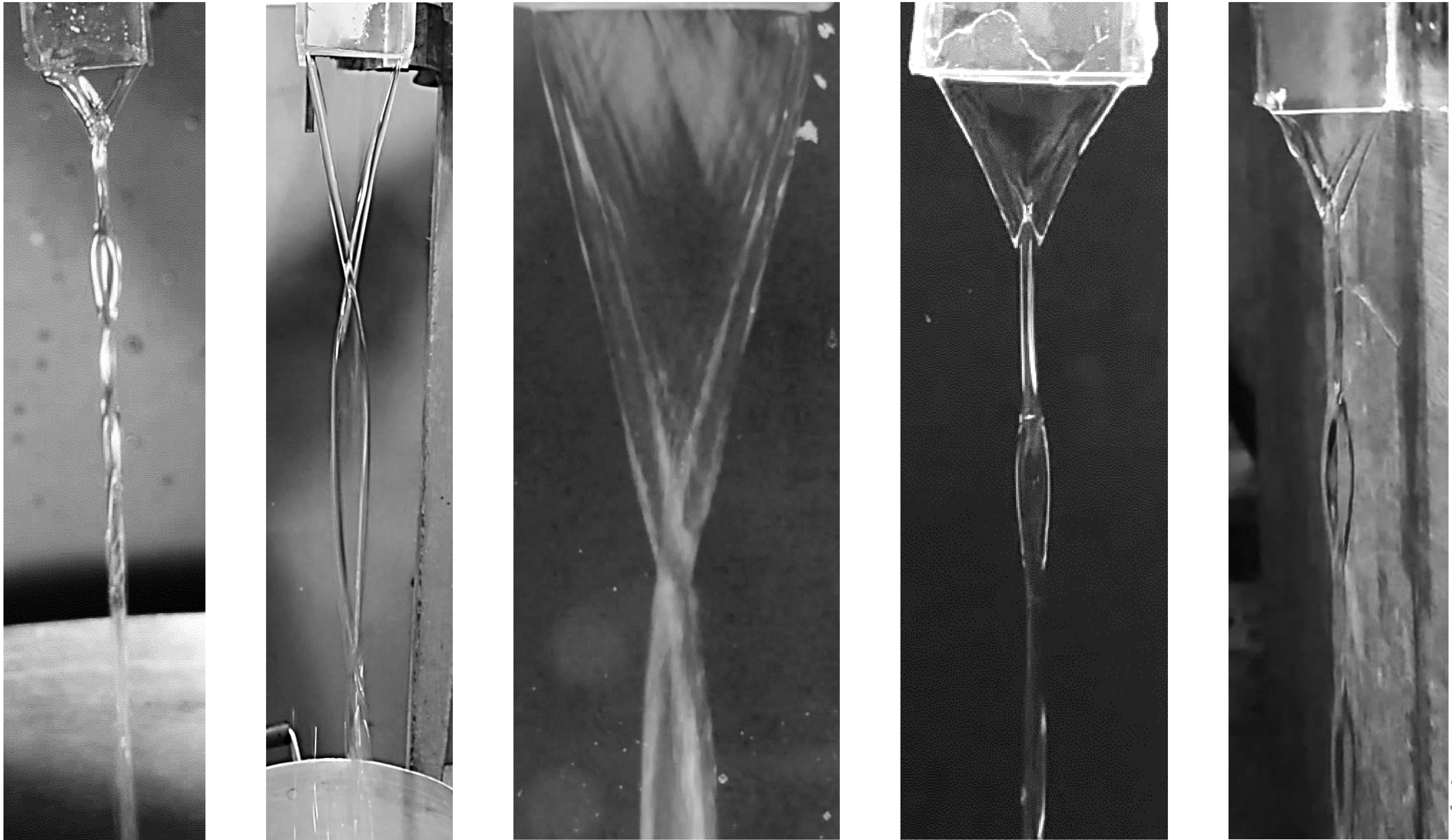
Team: Russia, Voronezh

The problem

When water is forced through a **thin slit**, the flow sometimes takes the shape of a **helix**. Describe the phenomenon and explain the dependence of the aspect **ratio(s)** of the helix on the fluid parameters, parameters of the flow and the shape of the nozzle.



The phenomena



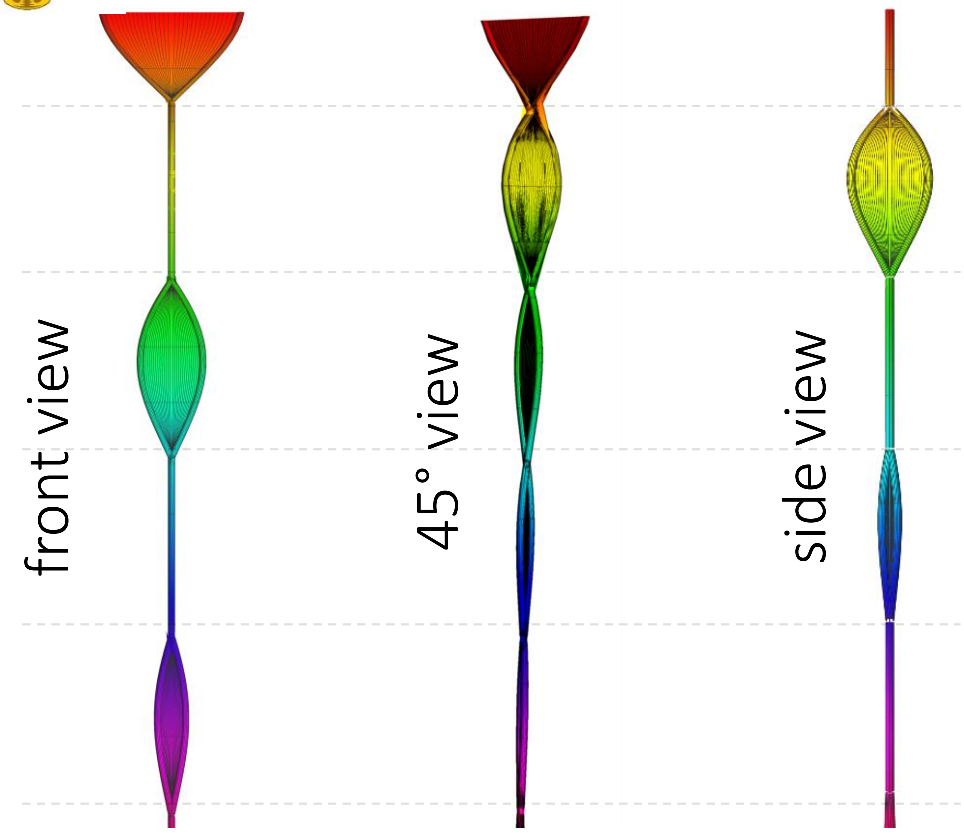
Plan of the report

Qualitative description of the phenomena

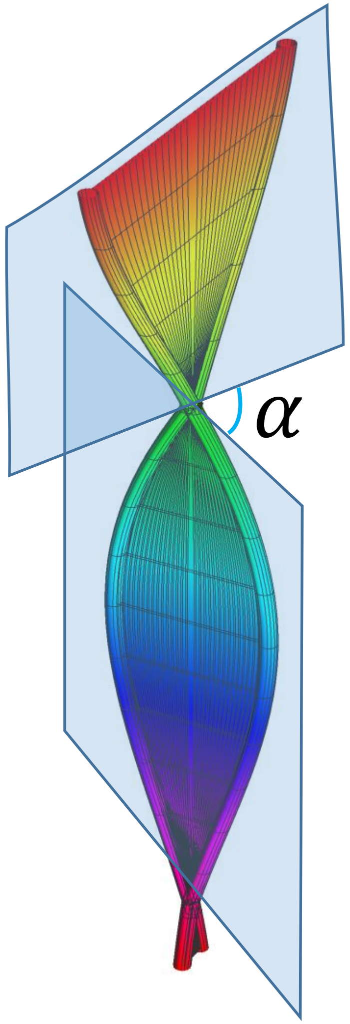
Linear model of the process

Nonlinear effects

Description of helix



chain-like structure



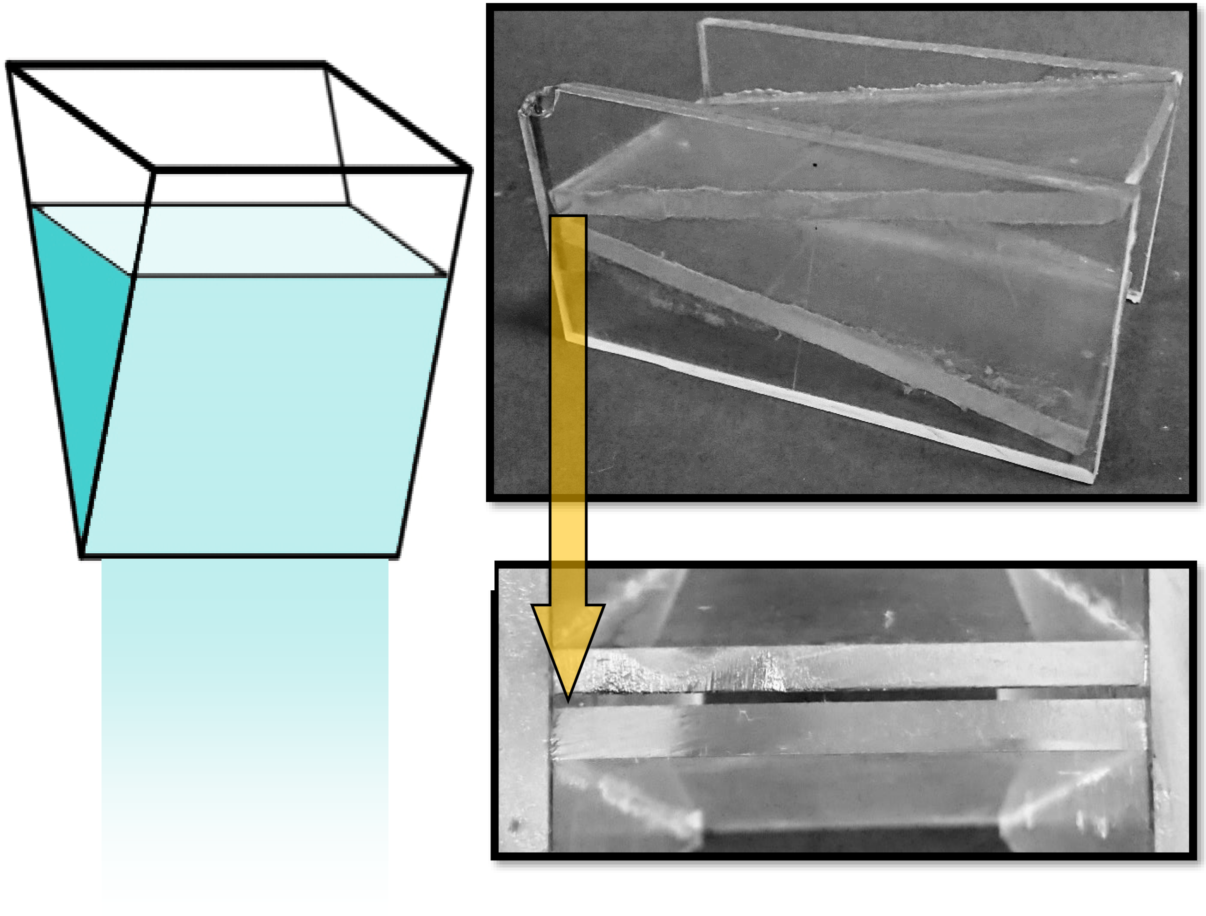
α – the angle between the planes

$$\alpha = 90^o + \alpha_0$$

α_0 –“twisting angle"

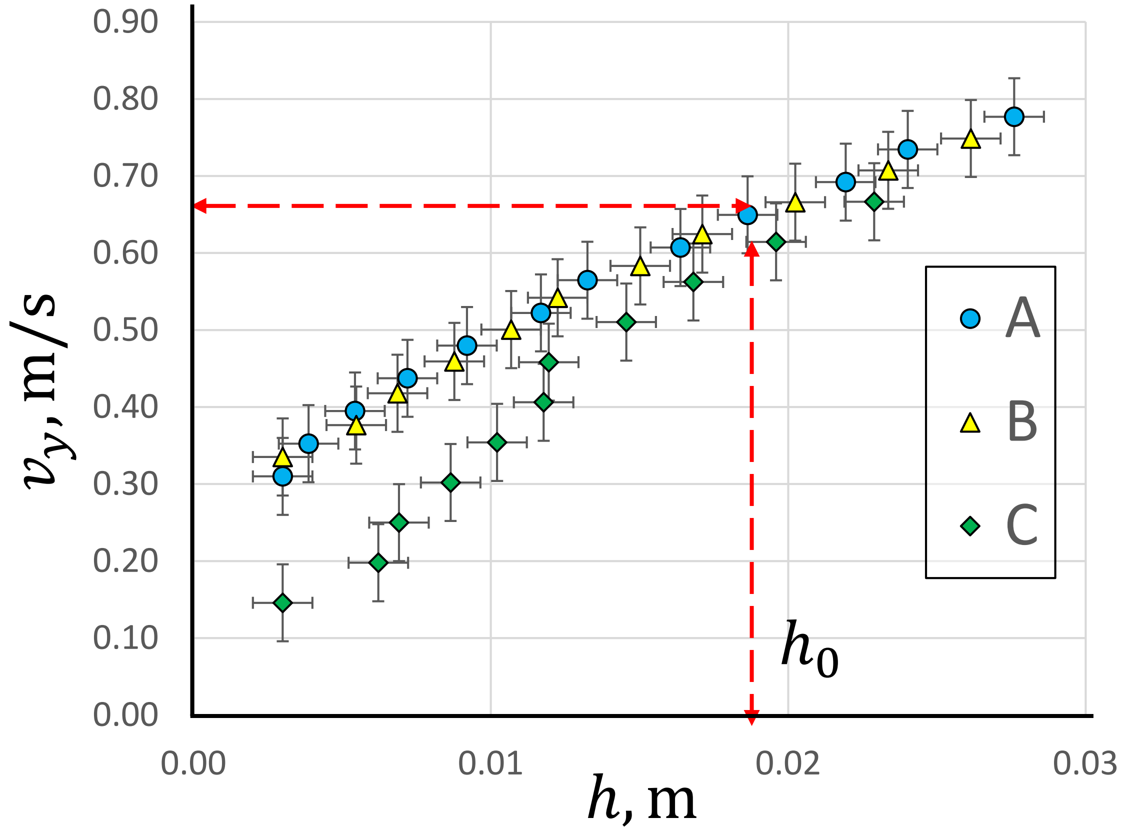
Experimental setup

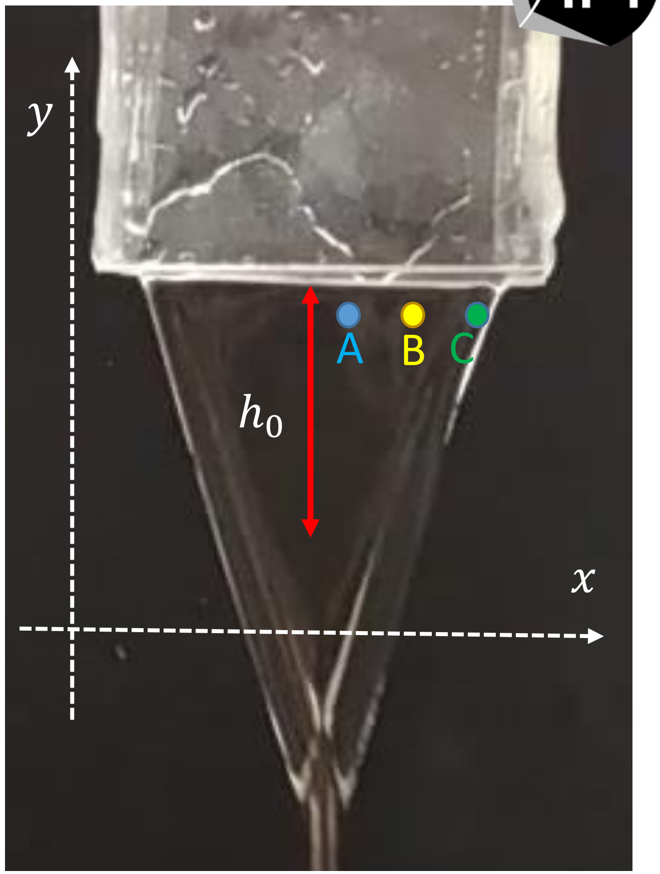
Scheme and photo



Work of the setup

Velocity profile of the jet

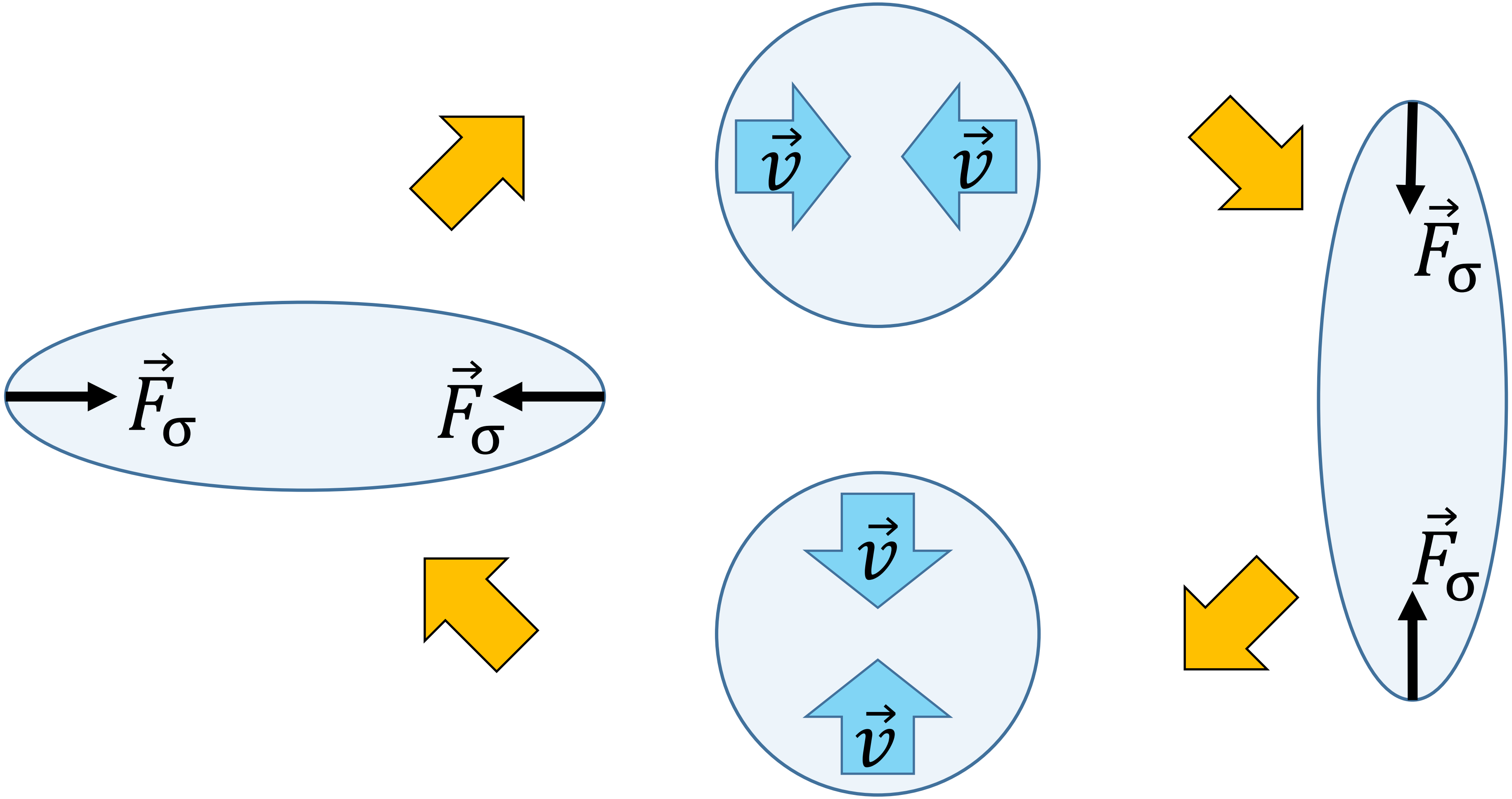




v_y almost doesn't depend on x-coordinate on the height h_0.

The motion of water layer can be considered as a whole.

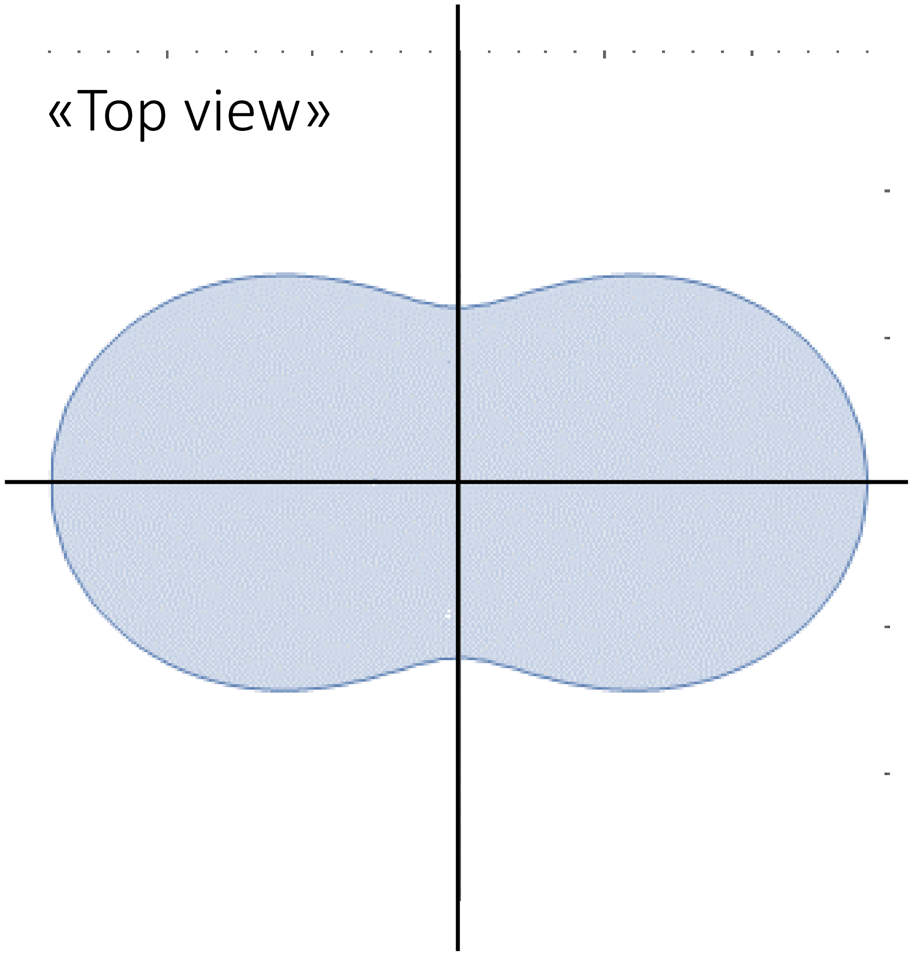
Forces acting on the stretched water layer



Surface energy converts into the kinetic energy and vice versa

Qualitative explanation. “Links of the chain”

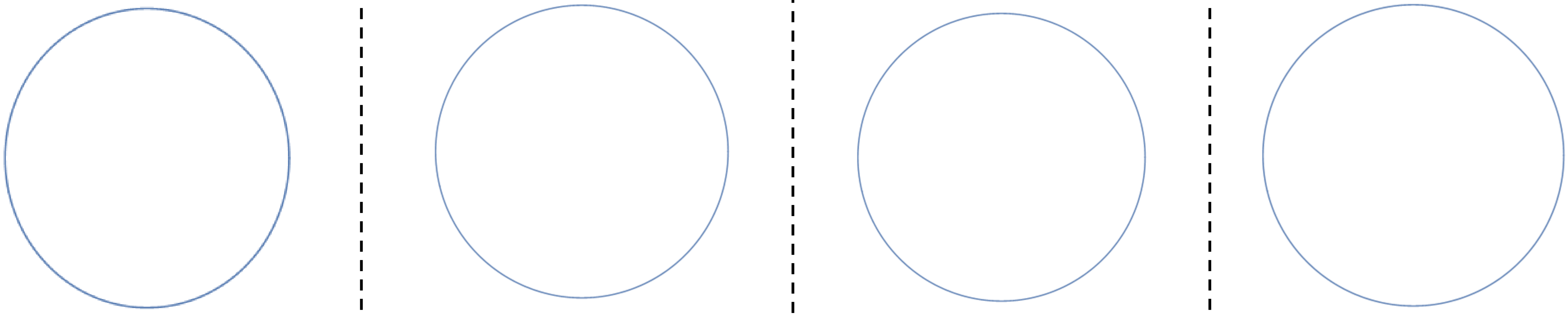
«Top view»



Inside the jet

The oscillations of water layer create helix-like shape of jet

Number of mode



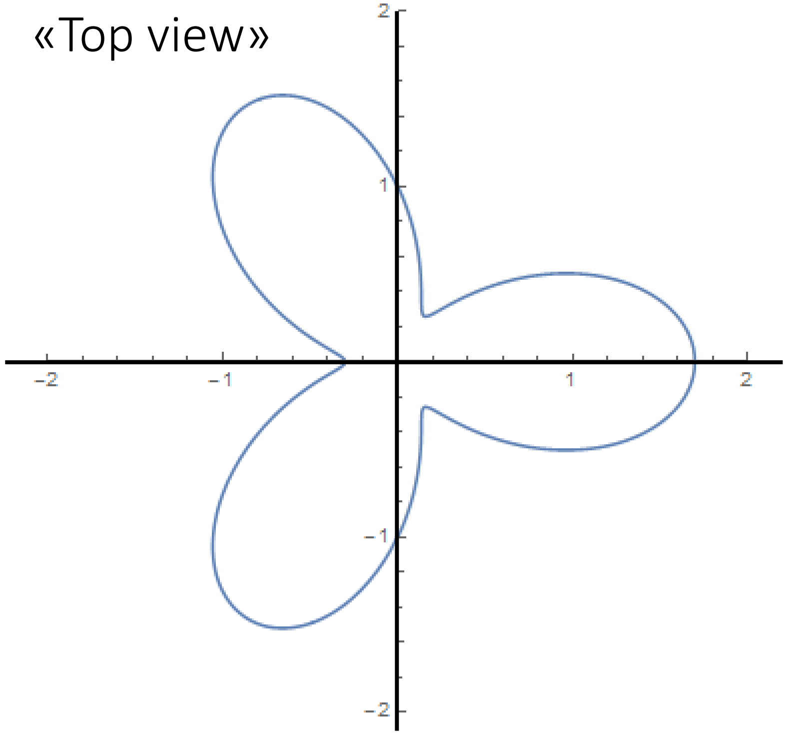
$n = 2$ | $n = 3$ | $n = 4$ | $n = 5$

The higher modes can appear for other shapes of the orifice

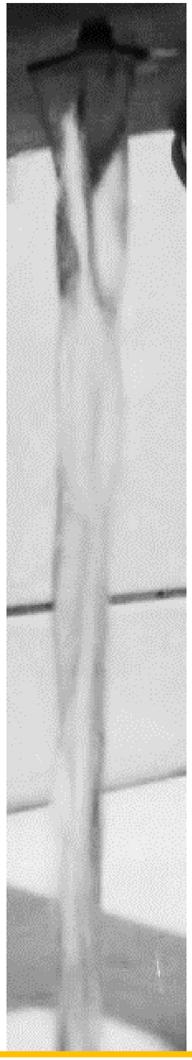
Experiment with 3-d mode

Nozzle

«Top view»



Experiment



Numeric modeling

Similar oscillation can take place on the third mode.

Weber number

Weber number — dimensionless number, describing a measure of the relative importance of the fluid's inertia compared to its surface tension.

$$We = \frac{\rho L v^2}{\sigma}$$

L is the characteristic length

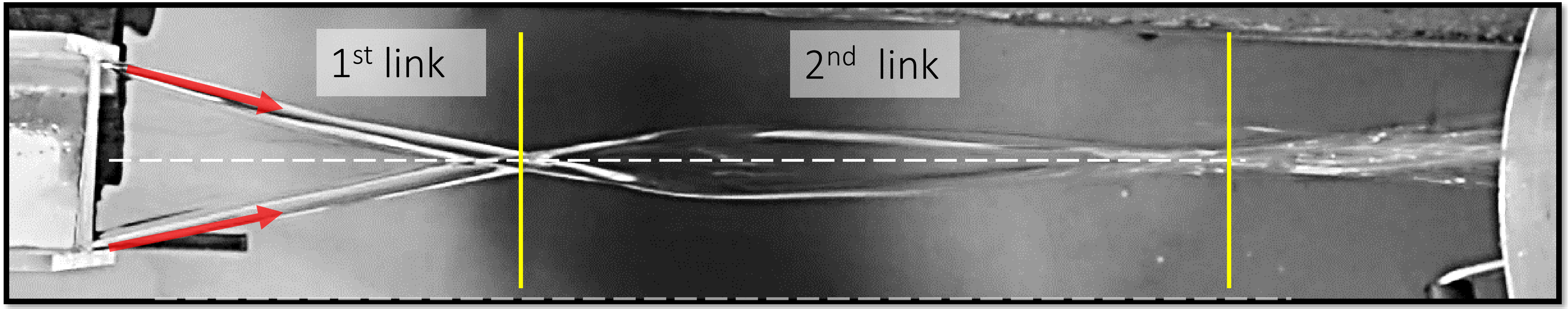
v is the velocity of the fluid

ρ is the density of the fluid

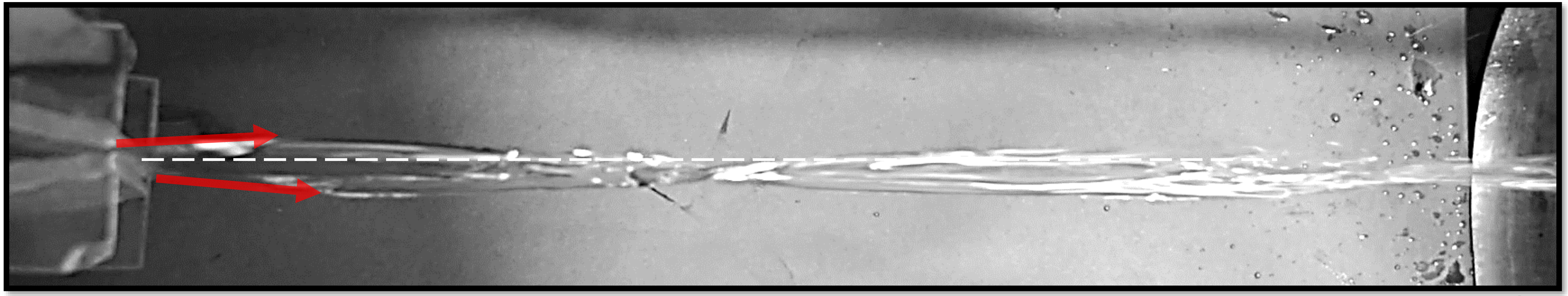
σ is the coefficient of the surface tension

Twisting

Front view



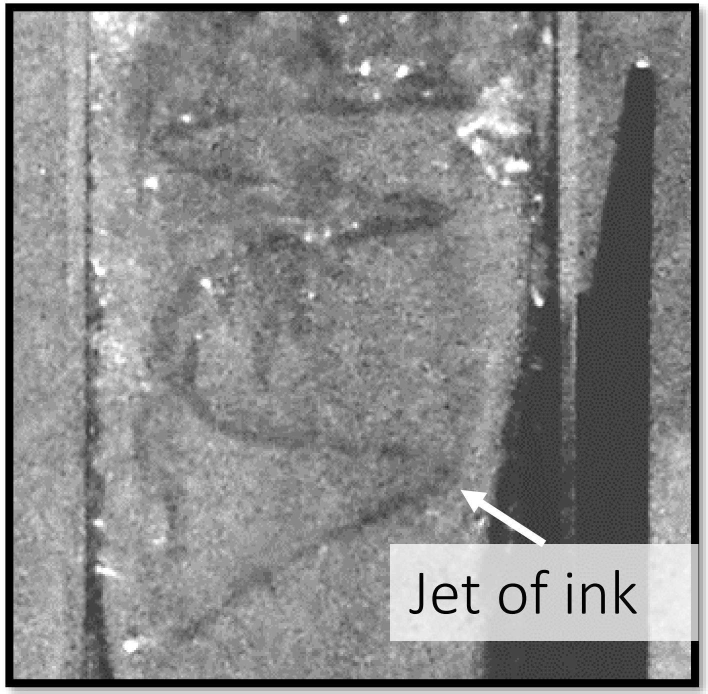
Side view



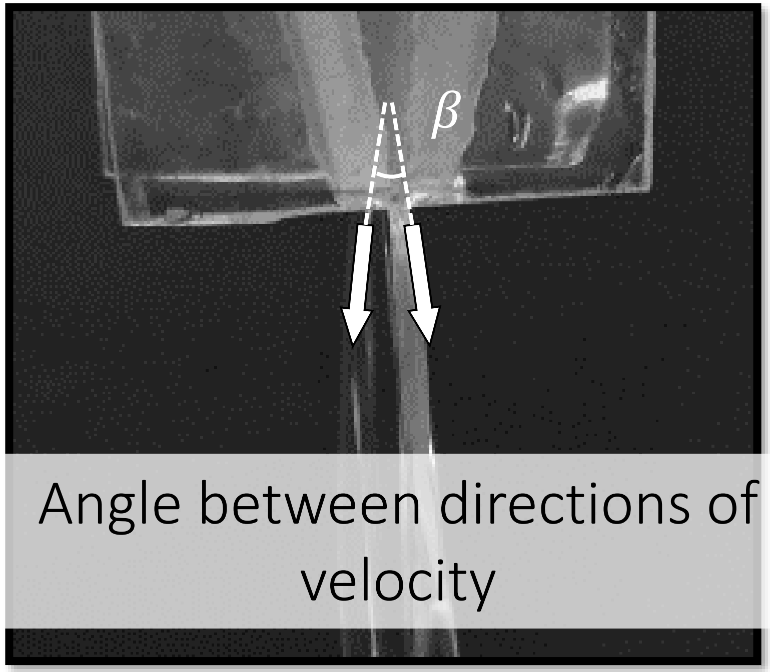
Qualitative explanation. Twisting

1. Initial vorticity.

a. in the funnel



b. because of slit's asymmetry



$$\Gamma = \Gamma_0 \quad \Gamma = \int \vec{v}\,\vec{dl} \approx \omega_0 S$$

Γ is the circulation ω_0 is the initial angular velocity S is the initial area of jet cross-section

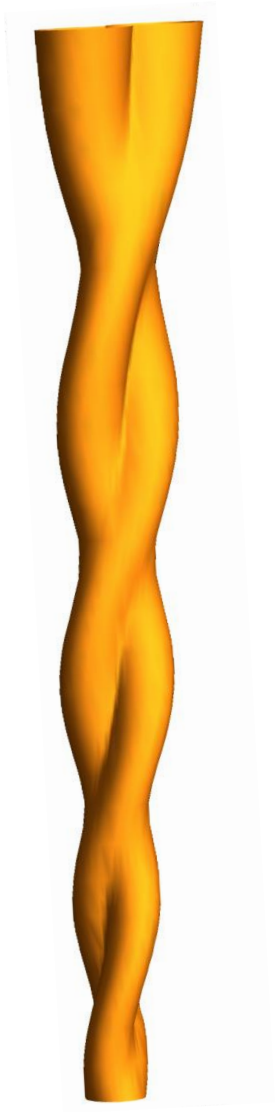
The appearance of twisting

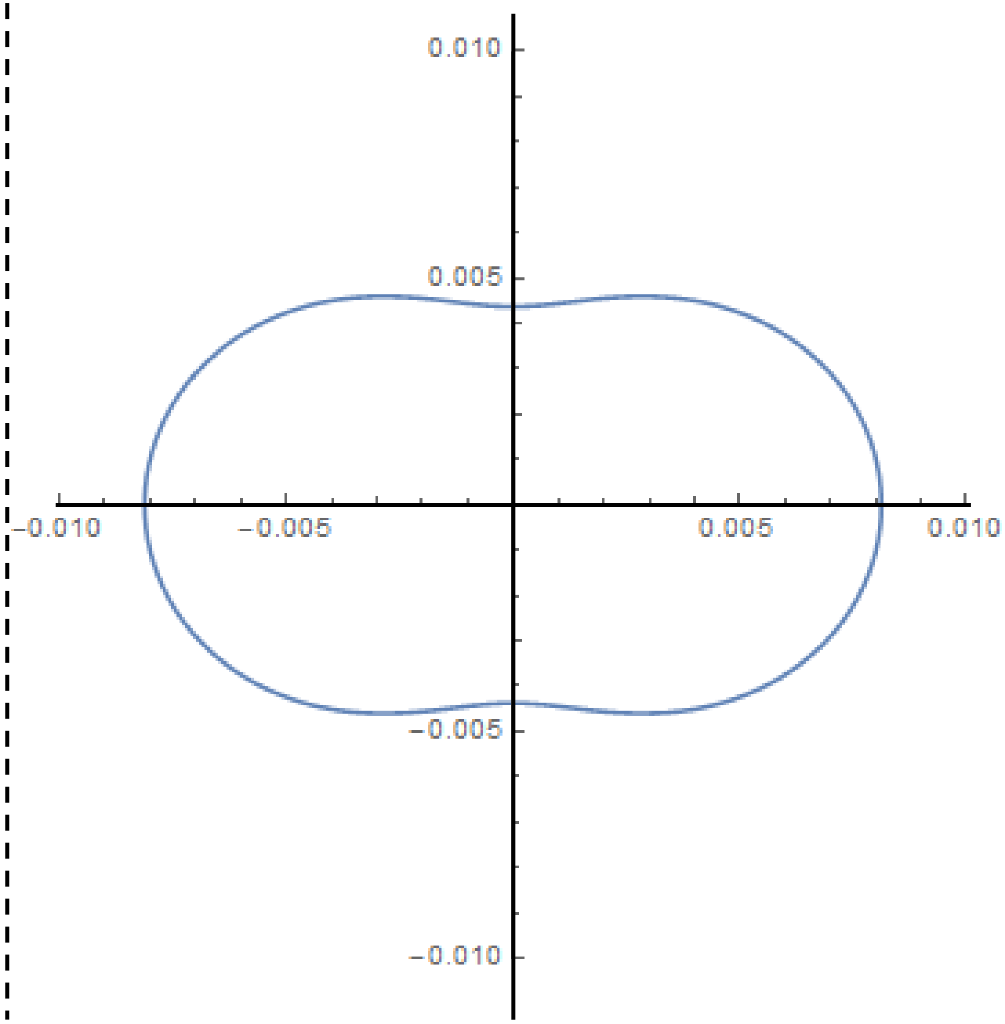
Circulation is constant $\Gamma = \mathrm{const}$

The less is the area, the bigger is angular velocity $\omega_0 = \frac{\Gamma}{S}$

The vertical velocity increases, and the cross area decreases $v_y \uparrow, \quad S \downarrow$

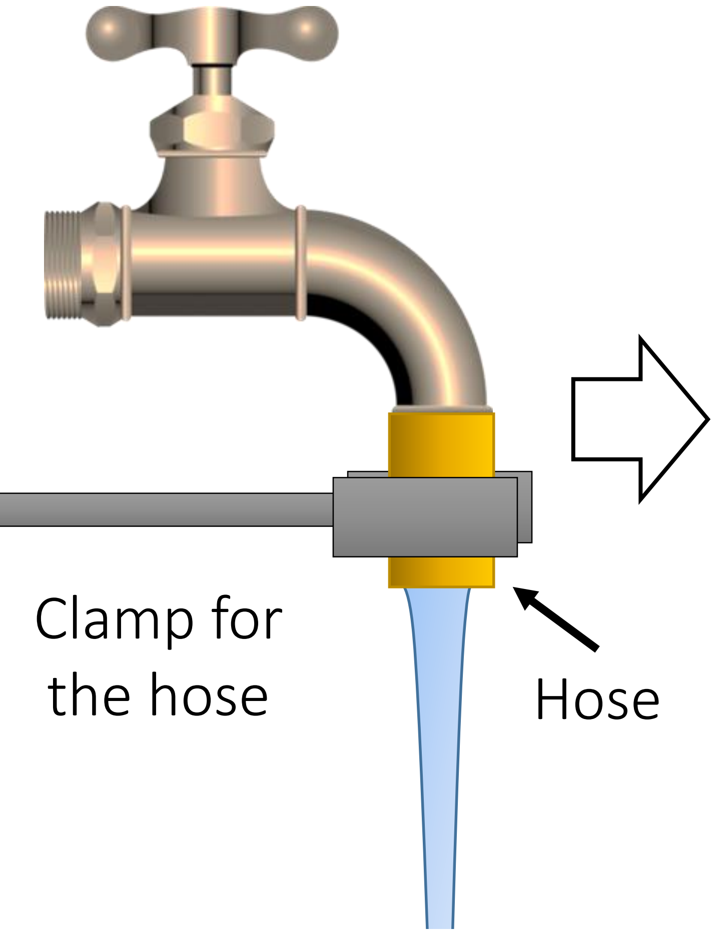
Angular velocity increases $\omega_0 \uparrow$



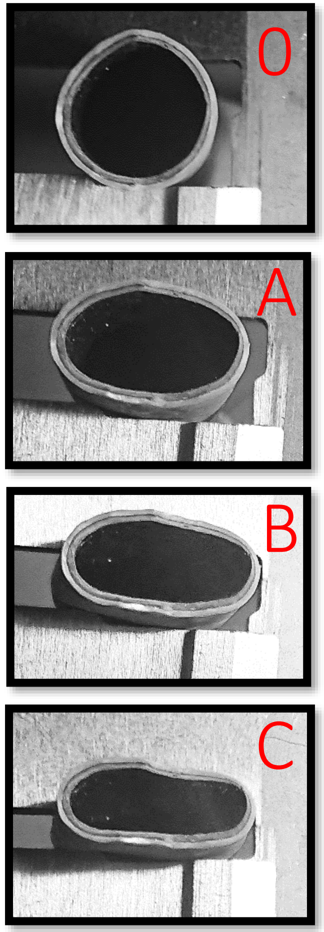


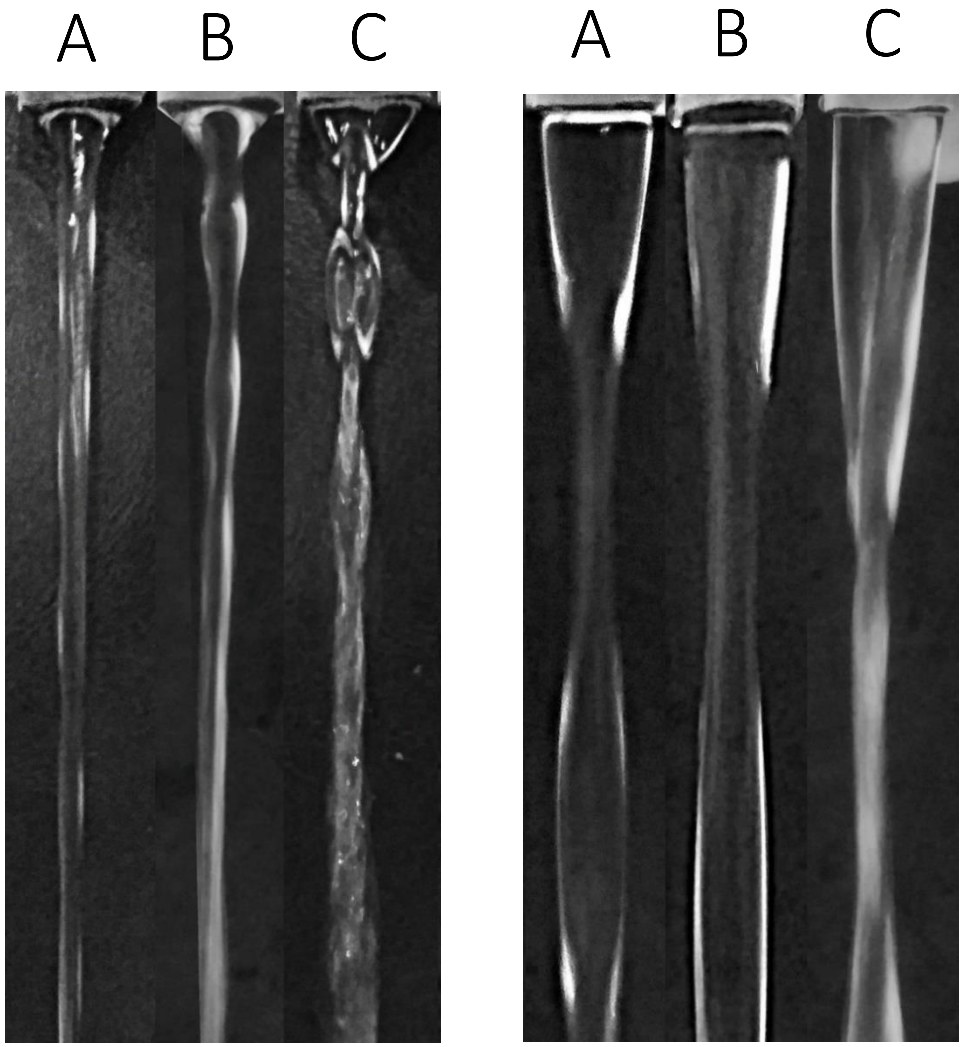
Experimental setup – the hose

Tap without aerator

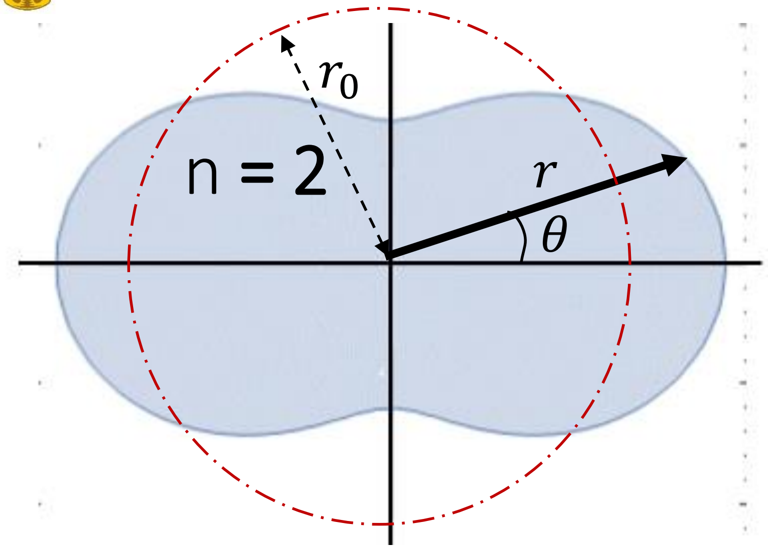


Varying the shape of hose





The frequency of oscillations of liquid cylinder



$r = r_0(1 + \alpha_n \cos n\theta \cos \omega t)$

Kinetic energy

$$L = \frac{\rho}{2}\int v^2 dV - \sigma S$$

Potential energy

Eqution of oscilltion

$$\ddot{\alpha}_n + \omega^2 \alpha_n = 0$$

linearization

n = 2

Frequency for the second mode

$$\omega^2 = \frac{6\sigma}{\rho r_0^3}$$

σ is the surface tension

$r_0(z)$ – hydraulic radius of the jet

ρ – density of the liquid

The hydraulic radius *vs* height

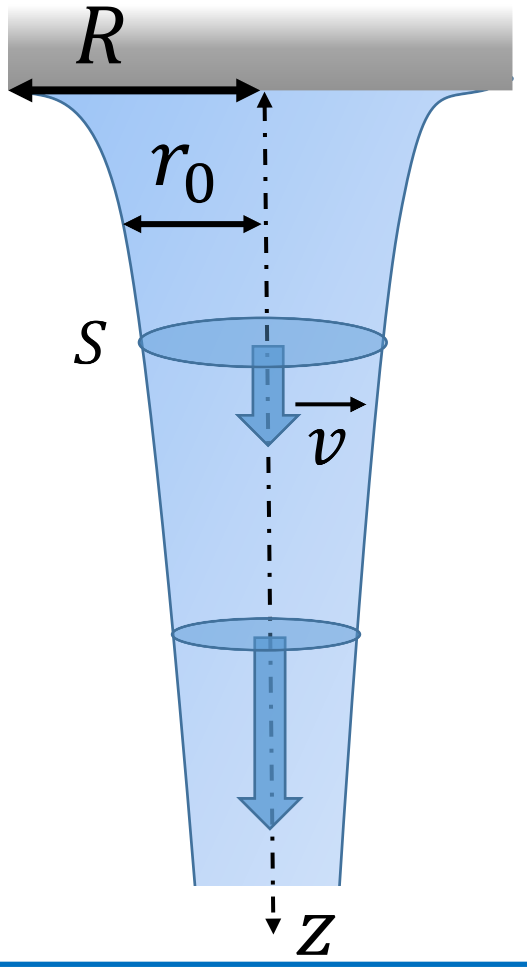
Velocity of the layer

$$v = \sqrt{v_0^2 + 2gz}$$

S – is the cross-section area of the jet

Continuity equation

$$vS = const$$



$$r_0 = R\left(1 + \frac{2gz}{v_0^2}\right)^{-1/4}$$

R – hydraulic radius of the orifice

v_0 - initial velocity of the jet

Hydraulic radius decreases with height
The frequency of oscillations increases

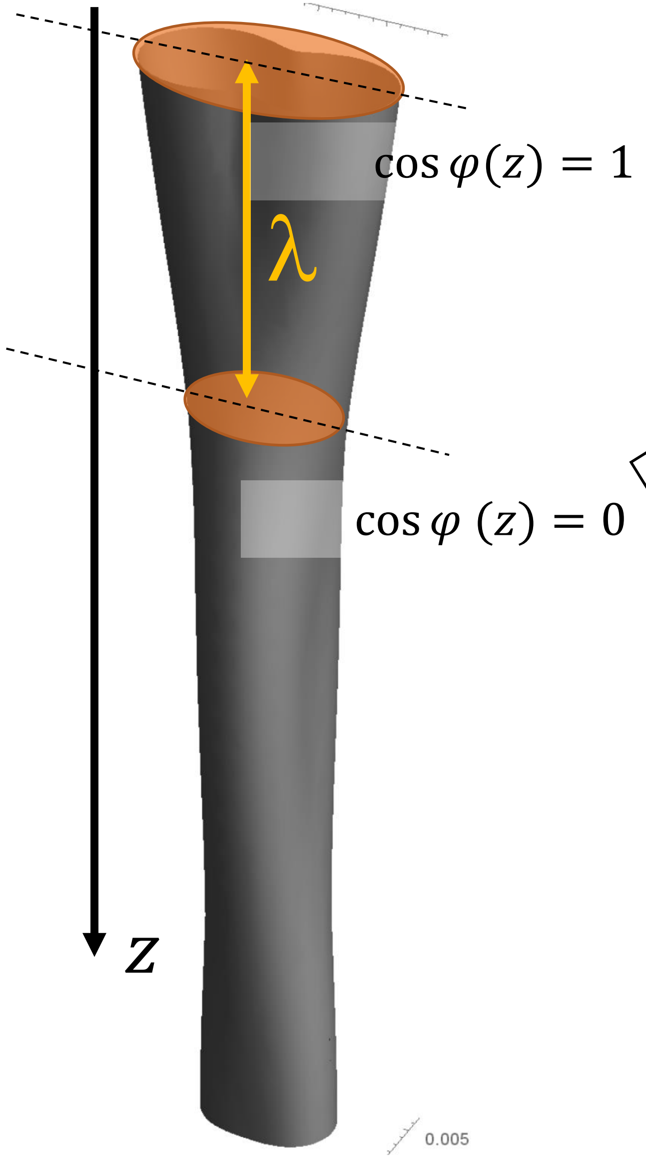
The aspect ratio of the helix

Frequency

$$\omega = \sqrt{\frac{6\sigma}{\rho R^3}}\left(1 + \frac{2gz}{v_0^2}\right)^{3/8}$$

Time

$$t(z) = \frac{v_0}{g}\left(\sqrt{1 + \frac{2gz}{v_0^2}} - 1\right)$$



Phase

$$\varphi(\lambda) = \int_0^\lambda \omega(z)\frac{\partial t}{\partial z}dz = \frac{\pi}{2}$$

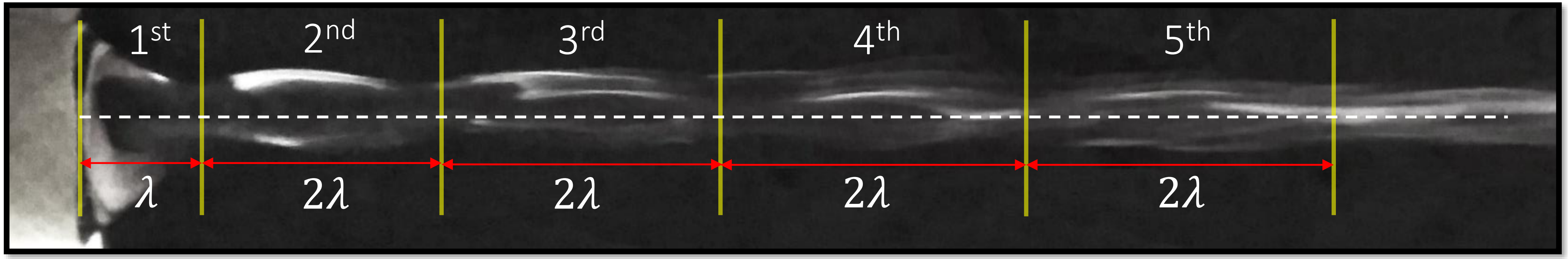
$$\frac{\lambda}{R} \approx 0{,}95\sqrt{We}$$

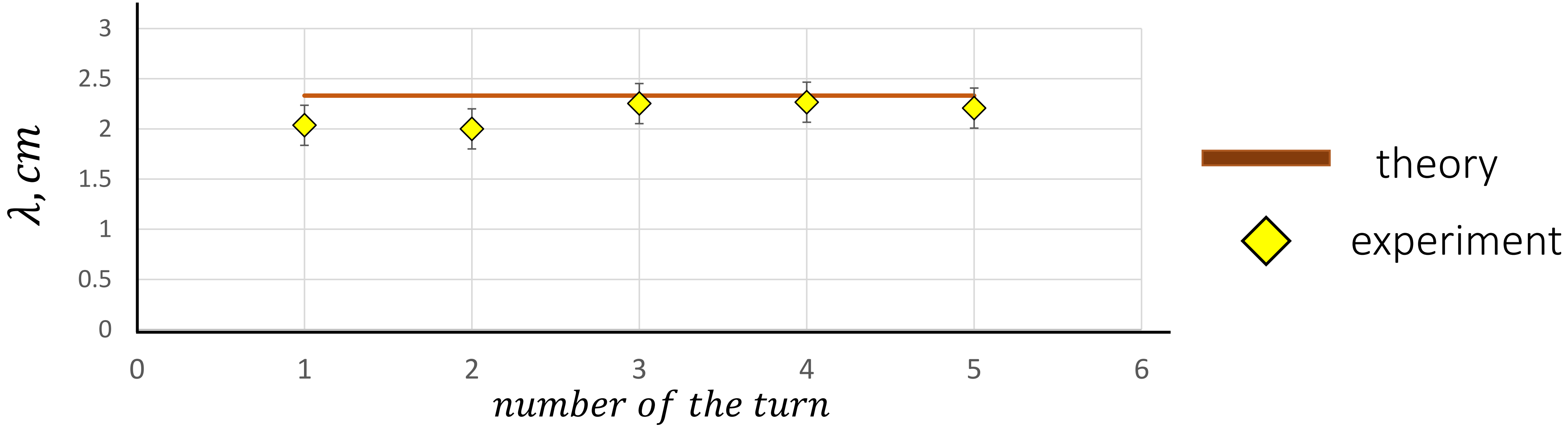
We – Weber number

R – hydraulic radius of the orifice

λ - pitch of a helix

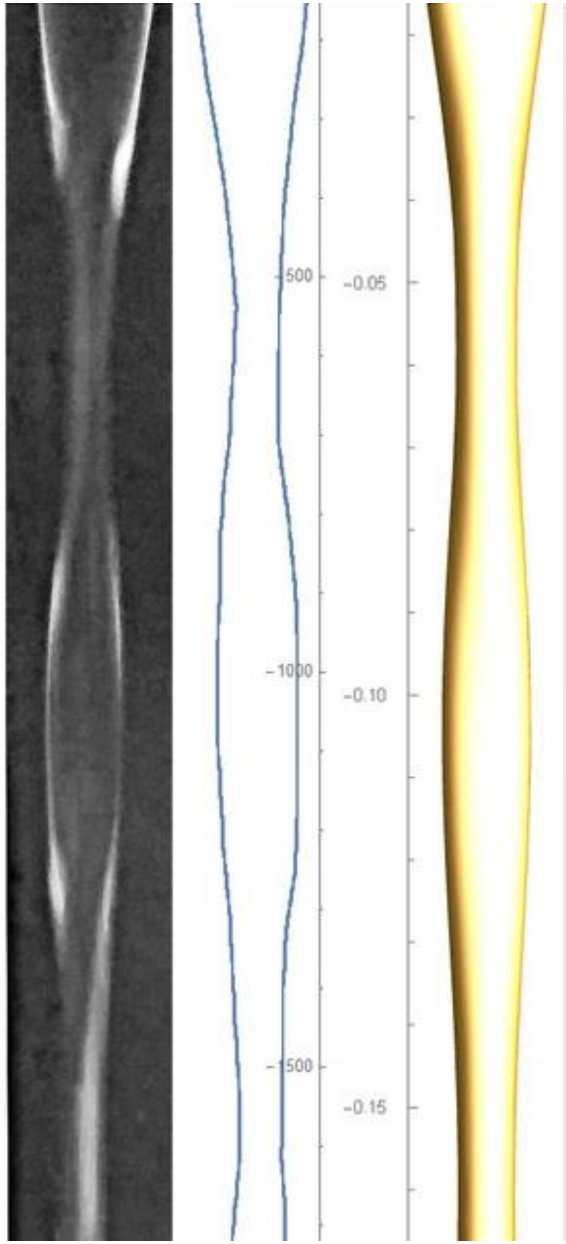
Helix pitch vs number of the turn

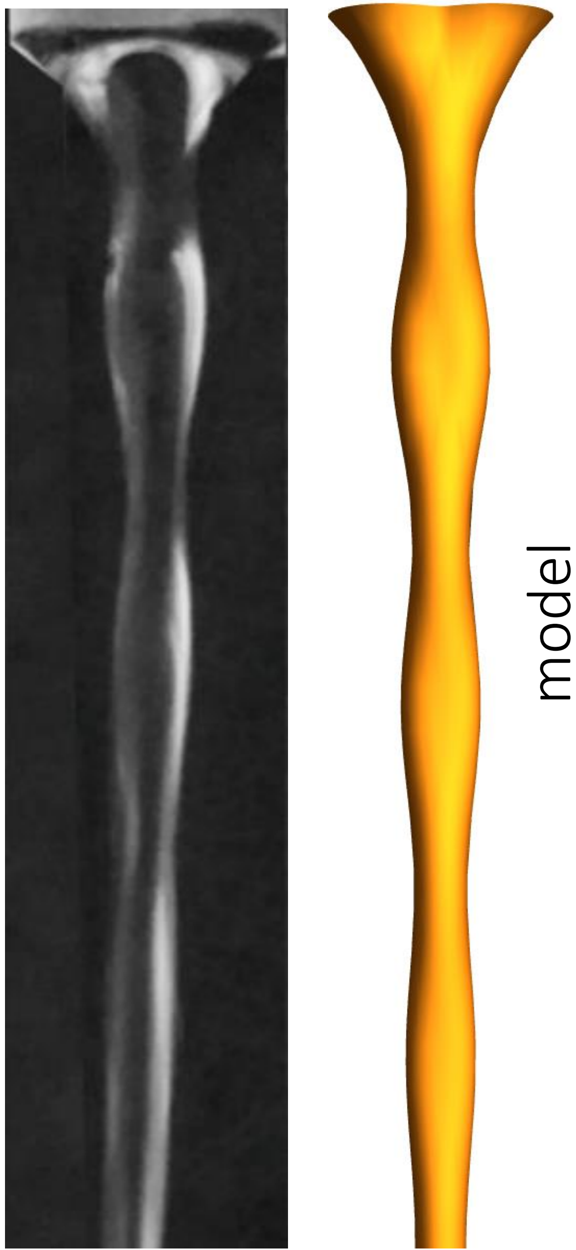




The length of the link doesn't depend on the number of turn

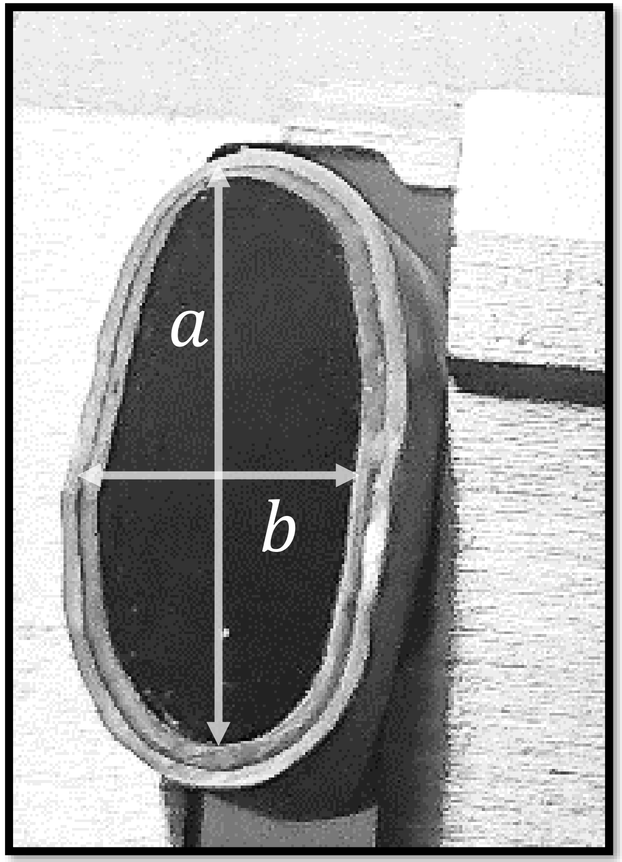
The experiment vs numeric model





Role of nonlinearity

linear case

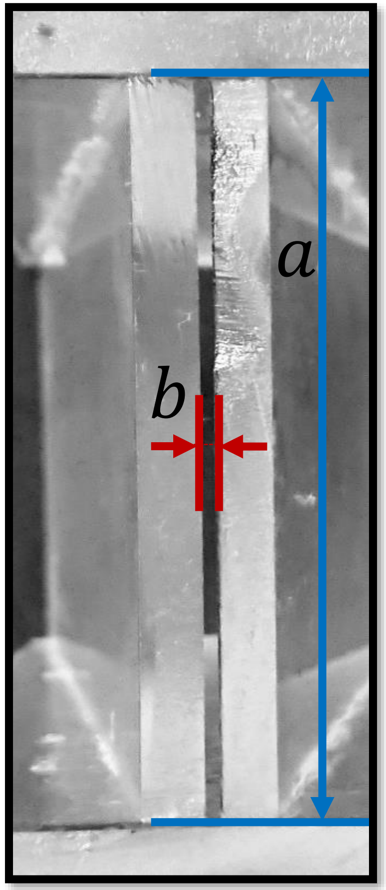


$\varepsilon \simeq 0.5$

Relative elongation

$$\varepsilon = \frac{a-b}{a+b}$$

nonlinear case



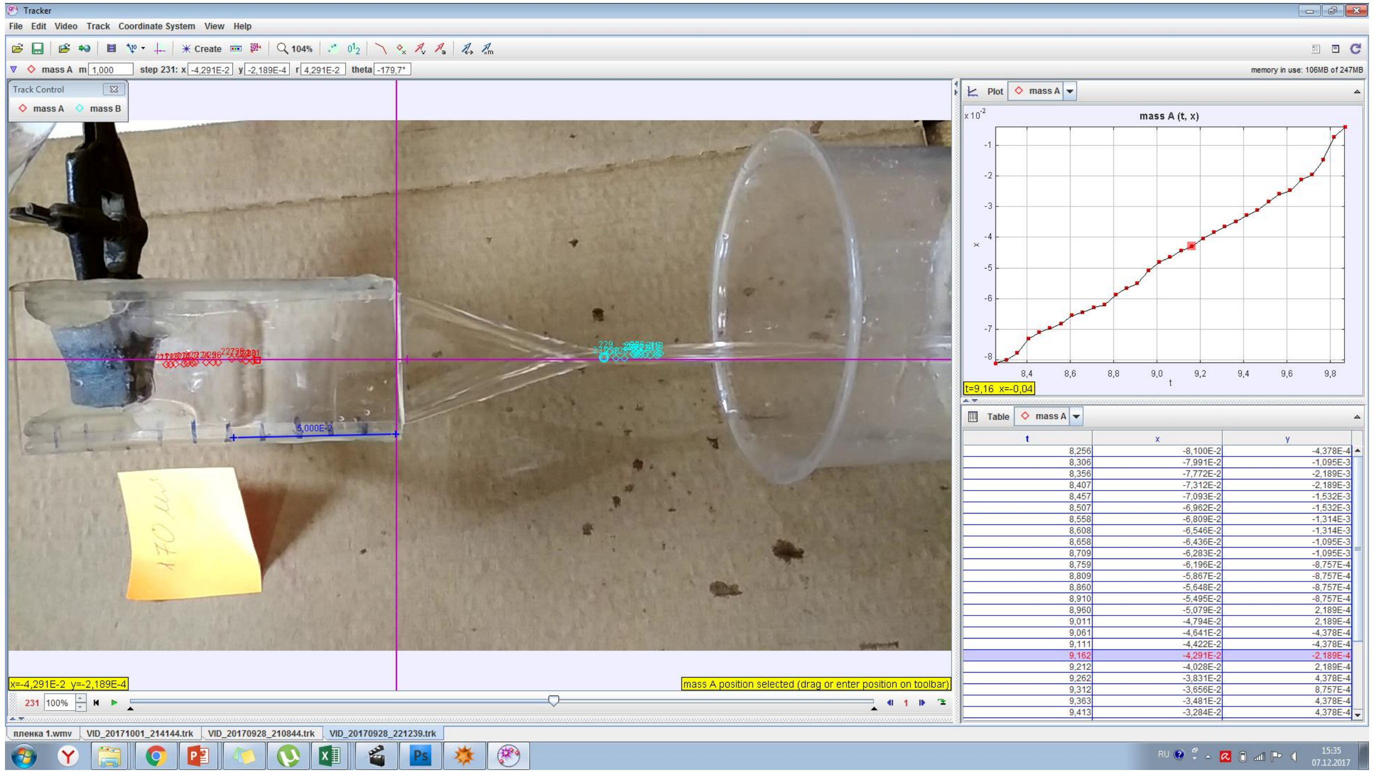
$\varepsilon \simeq 0.95$

The technique of the experiment

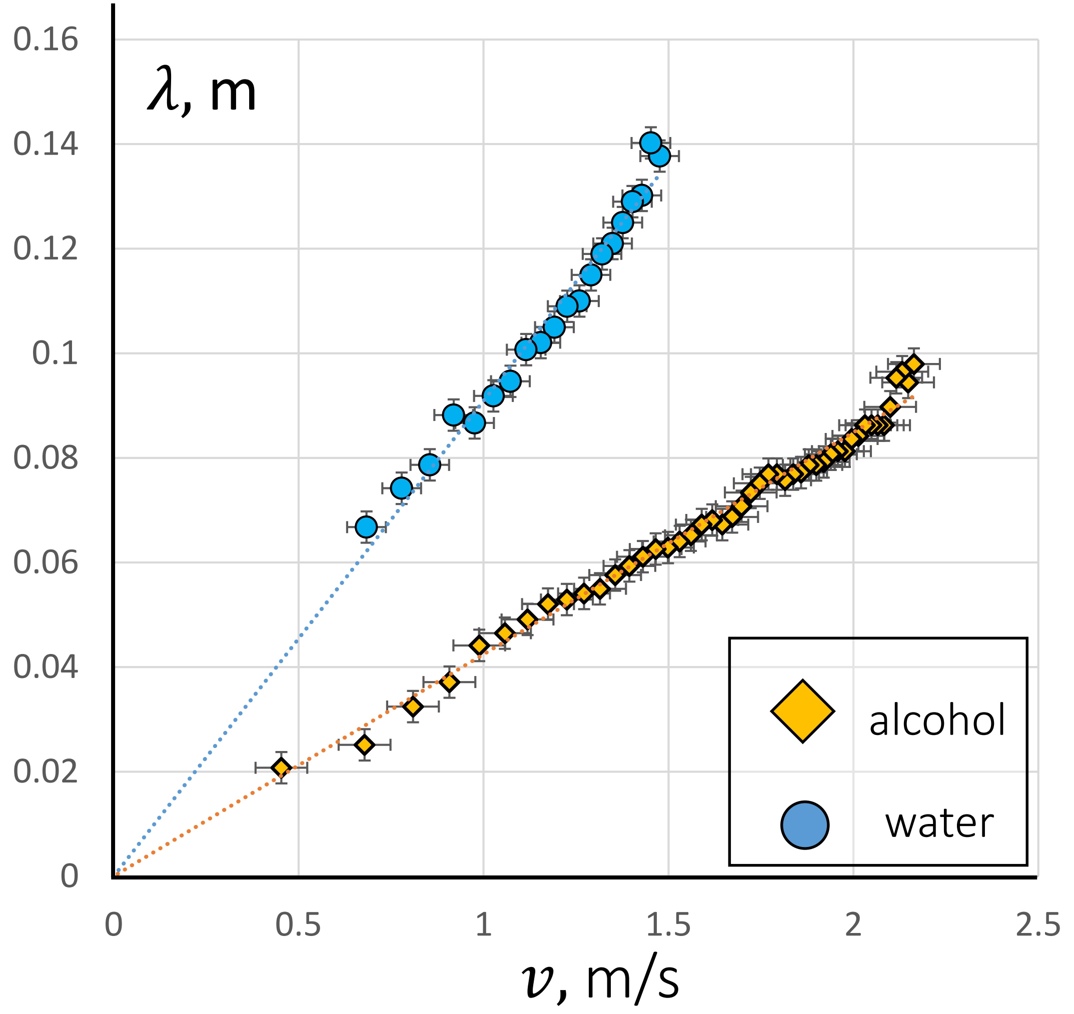
Using Tracker (software for video analyses) we determine the height of water in the funnel

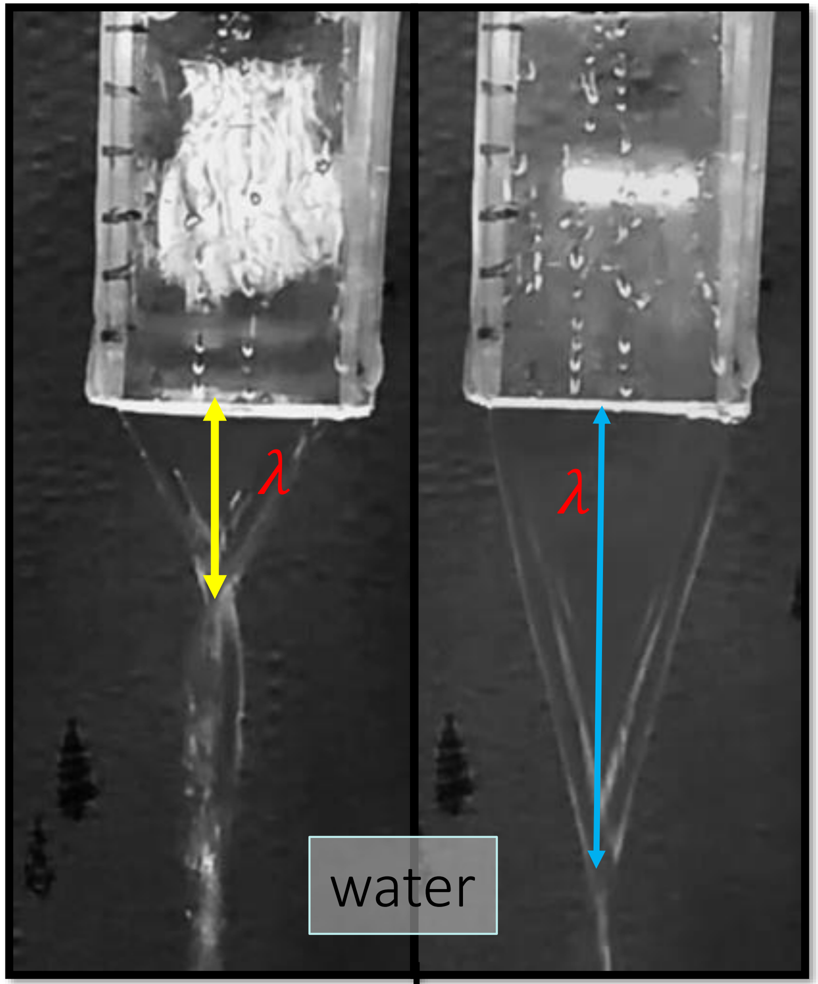
The flow rate was calculated for the geometry of funnel

Using Tracker measured the length of the first link



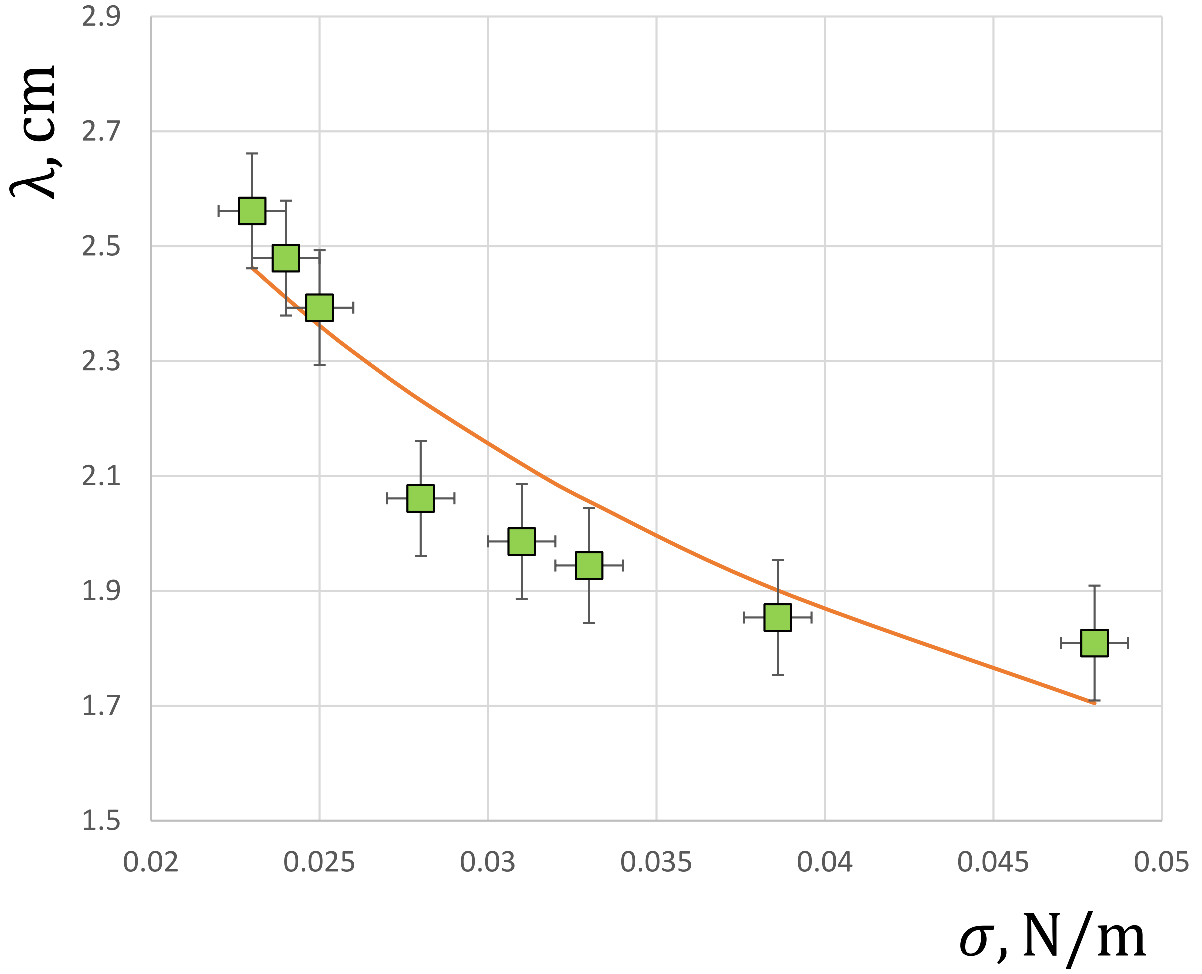
First link length vs velocity

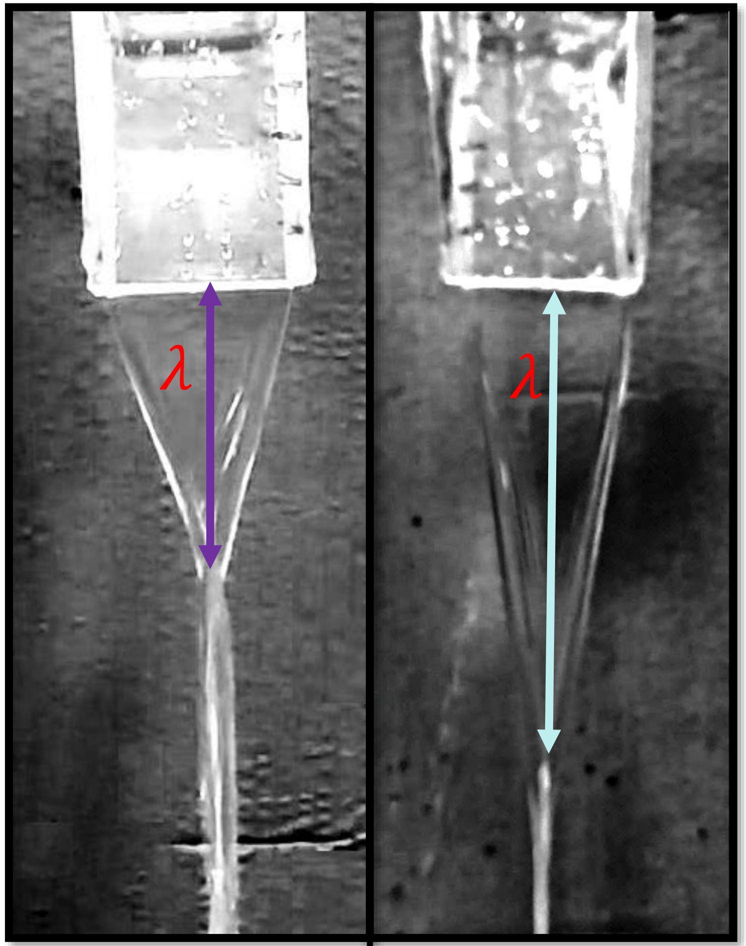




$v = 0{,}7\ m/s$ | $v = 1{,}5\ m/s$

First link length vs surface tension

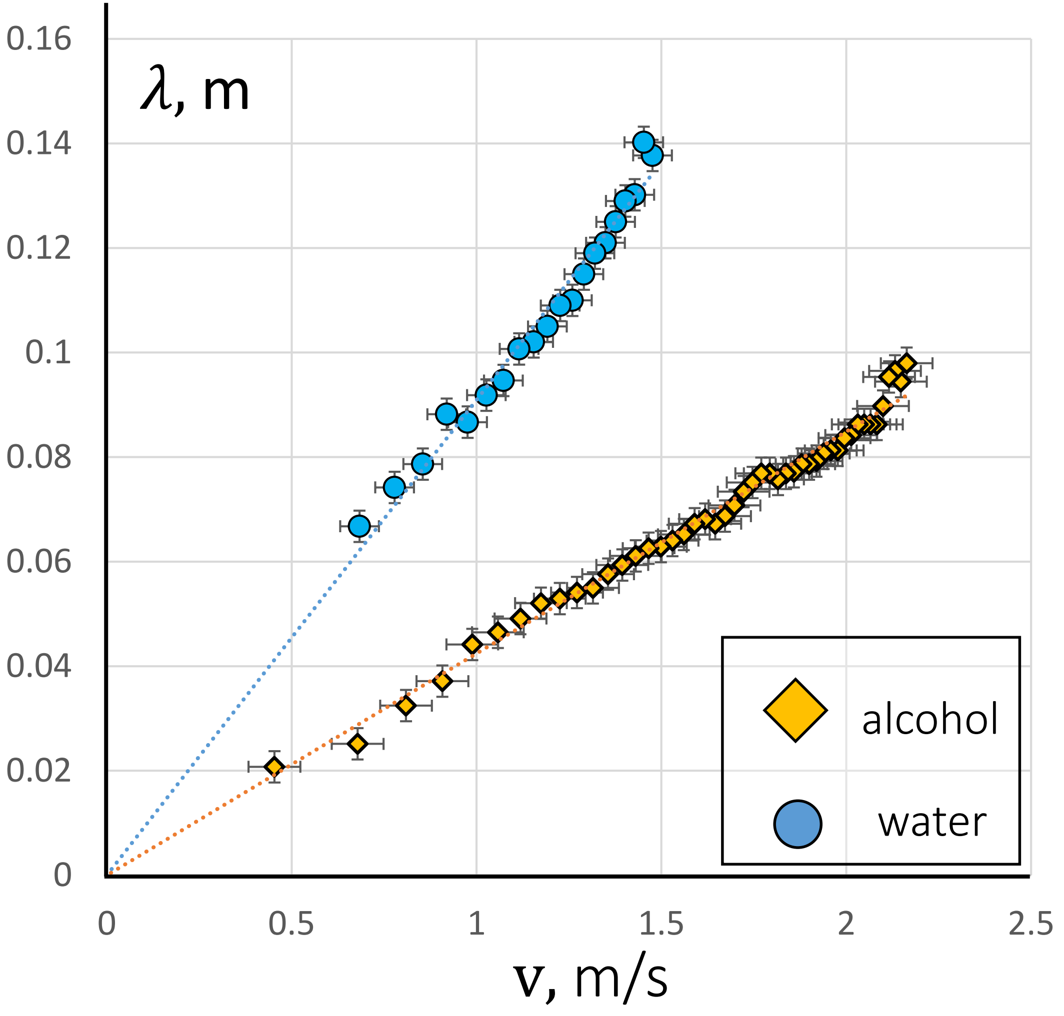


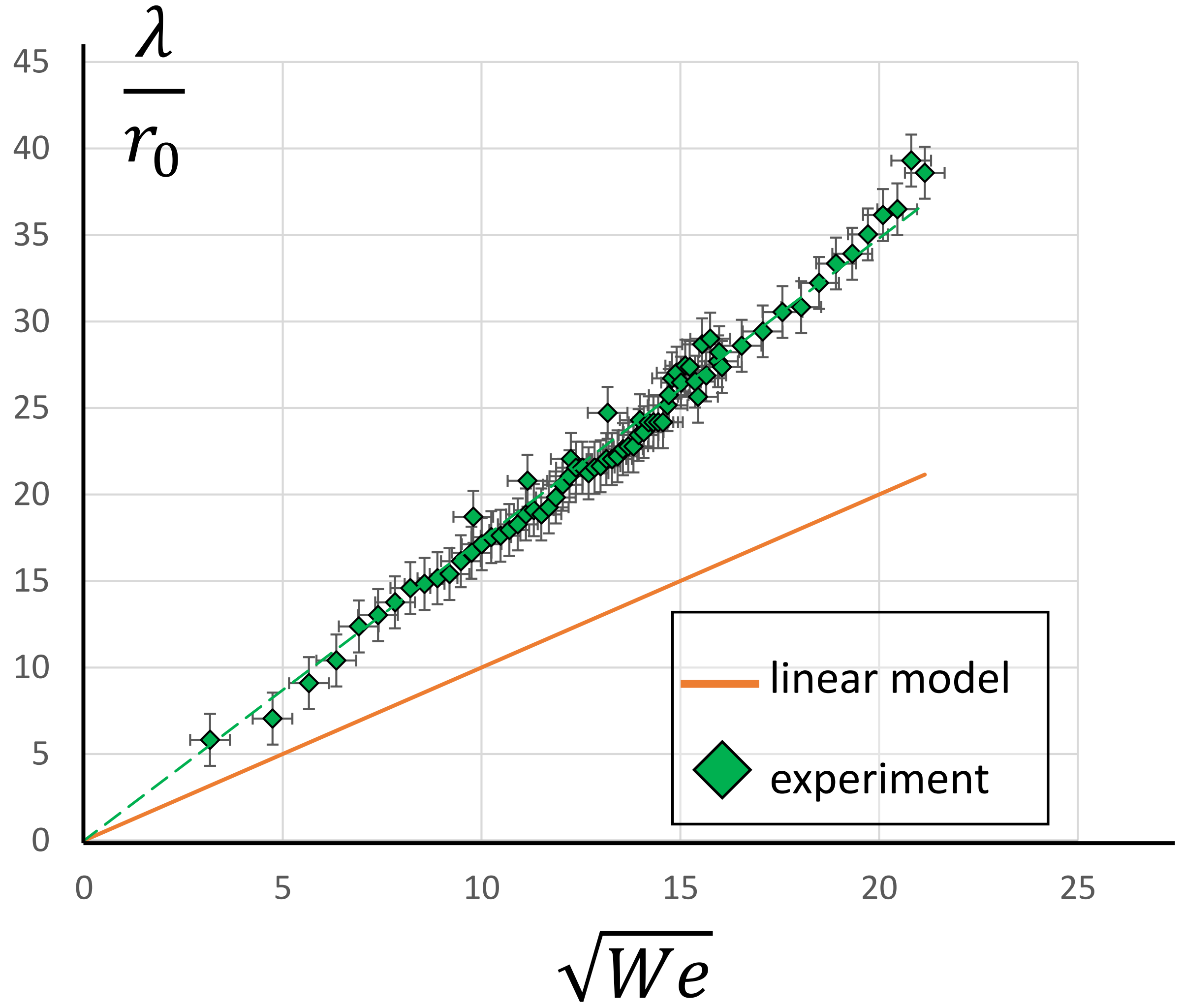


water
$\sigma = 0{,}073\ N/m$

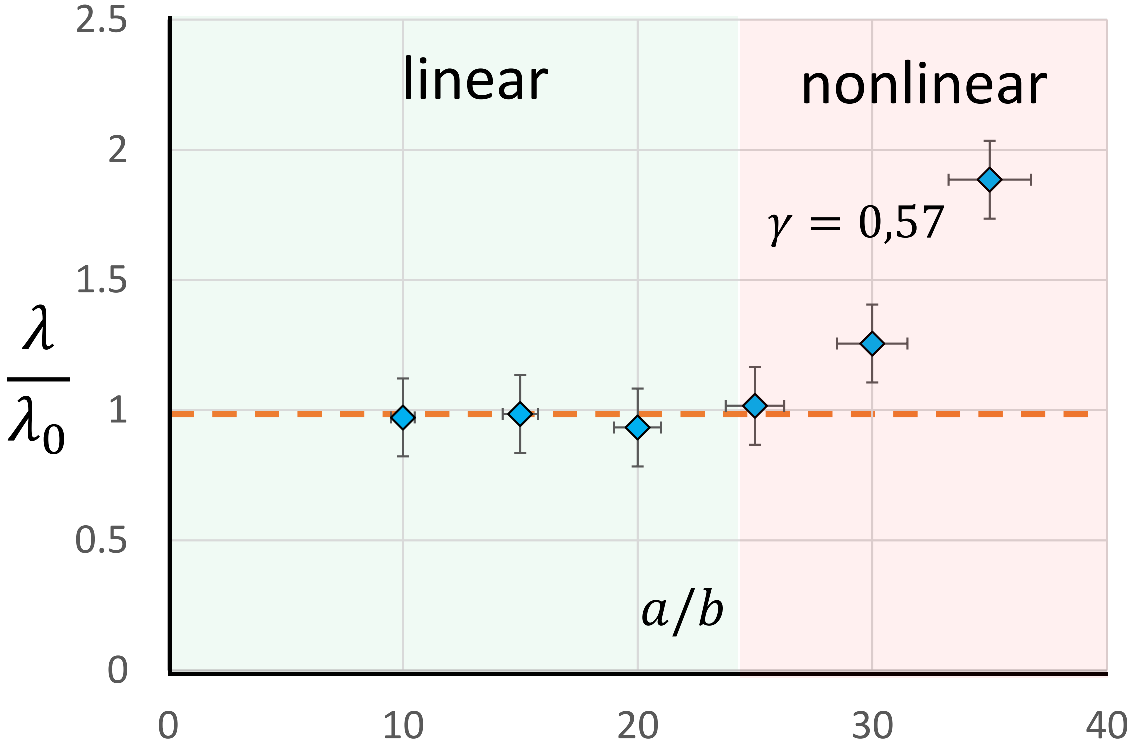
alcohol
$\sigma = 0{,}022\ N/m$

The dependence on We





The first link elongation for the contraction of the slit



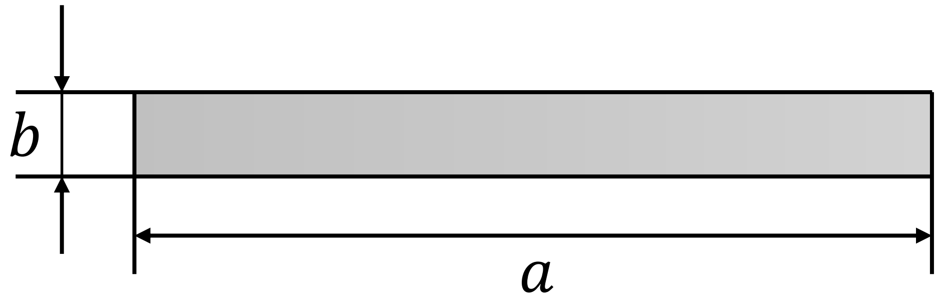
modification of the frequency

$$\omega = \omega_0[1 - \gamma\varepsilon^2 + o(\varepsilon^4)]$$

$$\frac{\lambda}{\lambda_0} \simeq \frac{1}{1 - \gamma\varepsilon^2}$$

λ – length of the first link (experiment)

λ_0 – length of the first link (linear model).



The increase of amplitude leads to the increase of the period, and, correspondingly, the length of the links.

Damping oscillations

Damped oscillator

$$\ddot{\alpha} + \gamma\dot{\alpha} + \omega^2\alpha = 0$$

$$\alpha \sim \exp(-\gamma t) \qquad \gamma \sim \mu$$

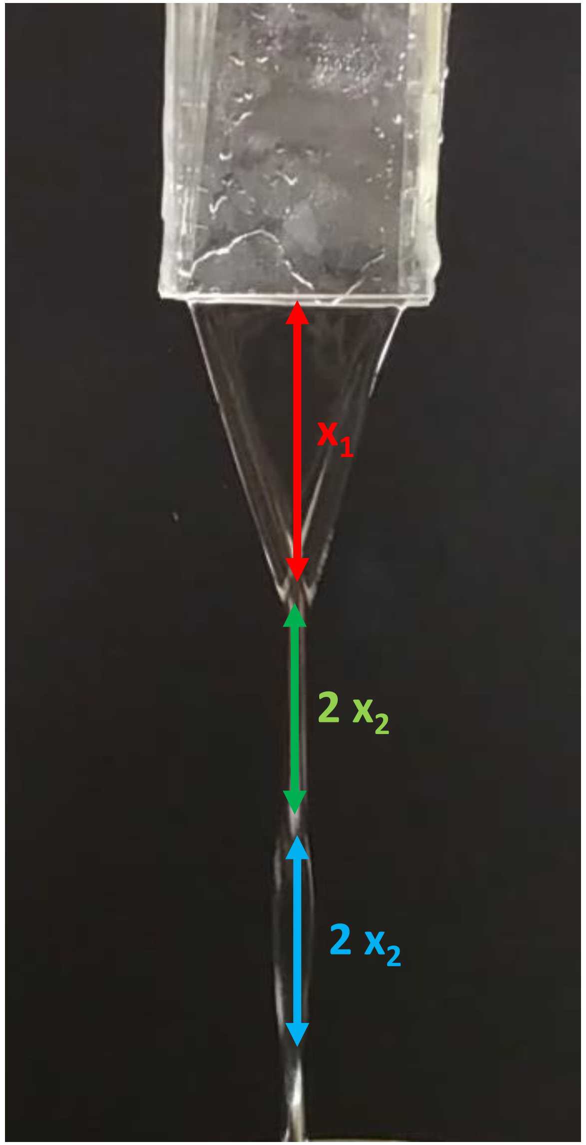
α – amplitude of oscillations $\qquad$ μ – dynamic viscosity

Only for nonlinear case

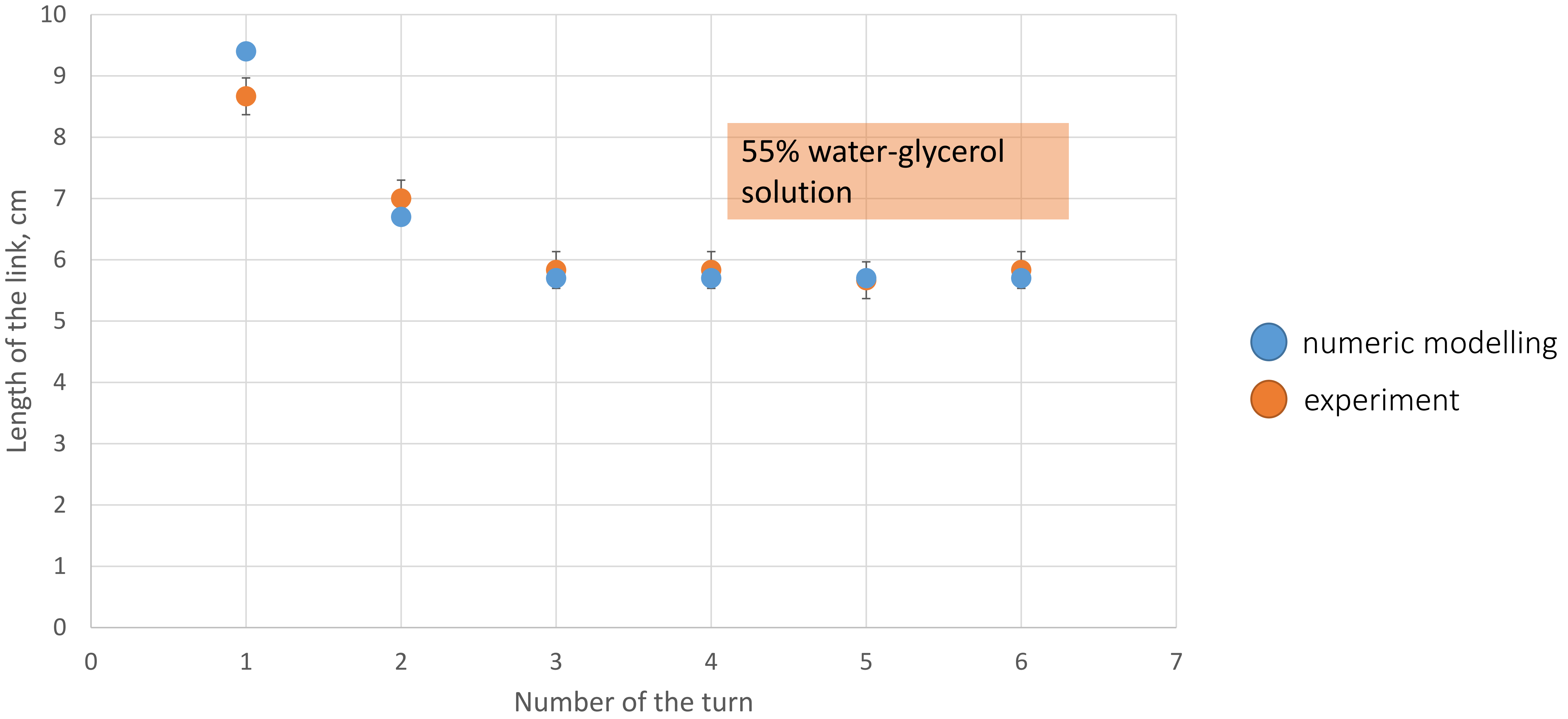
$$\omega = f(\alpha) \qquad \lambda = g(\alpha)$$

$$\alpha \downarrow \Rightarrow \lambda \downarrow$$

The length of links decreases with length
(in nonlinear case)



Length of the link vs number of the turn (nonlinear case)



Conclusions

- Qualitative explanation:
  - The cause of chain-links – oscillations of water layer
  - The cause of twisting – increase of initial vorticity.
- Aspect ratio of the helix is determined by Weber number and by the aspect ratio of the slit.
- The numeric model of the helix was created
- The dependence on viscosity, surface tension, geometry of jet and flow rate was investigated

References

1. N. Bohr, Determination of the surface-tension of water by the method of jet vibration
2. Lord Rayleigh, On the capillary phenomena of jets
3. J. Bush, On the collision of laminar jets: fluid chains and fishbones