Candle lighting trick

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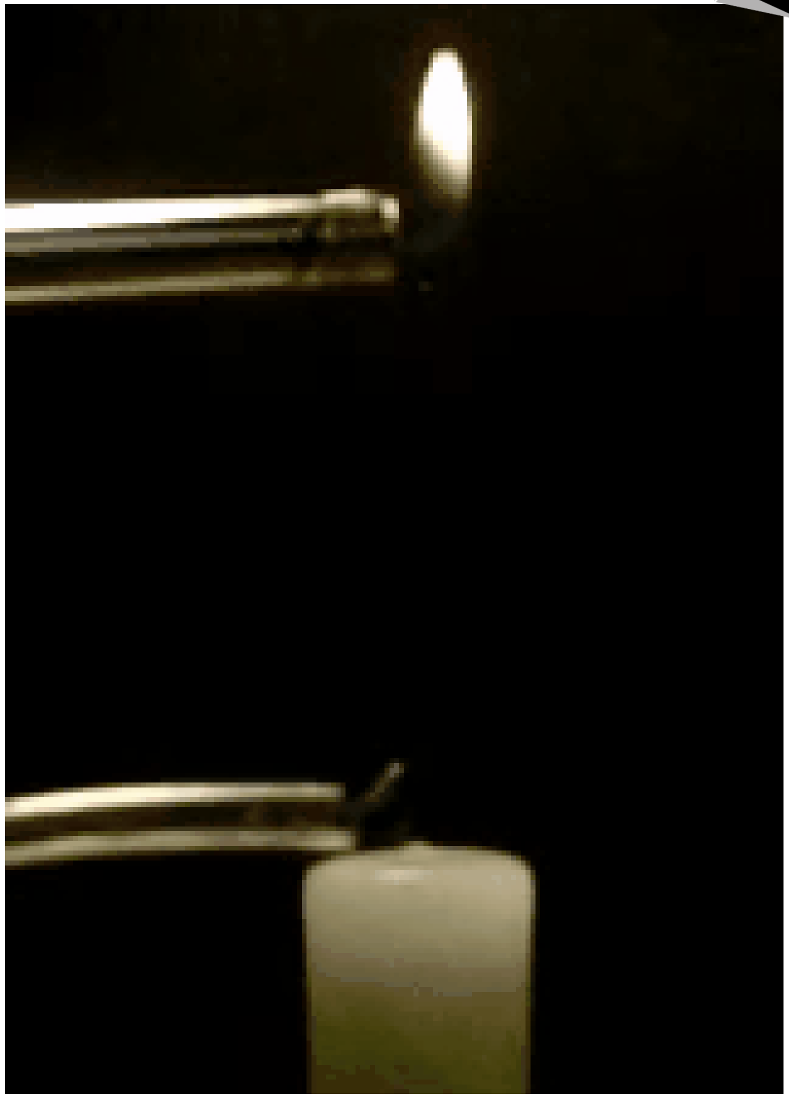
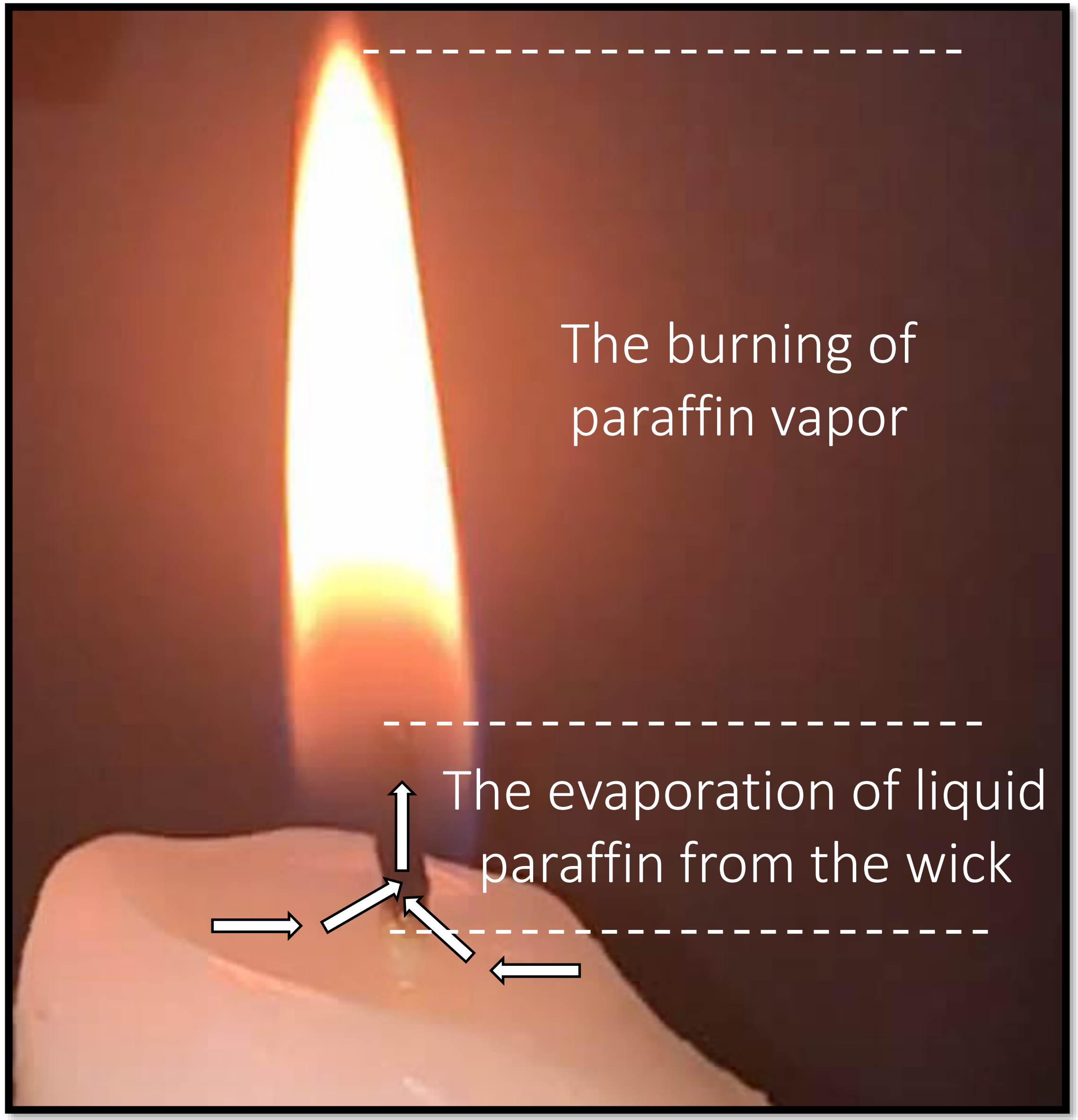
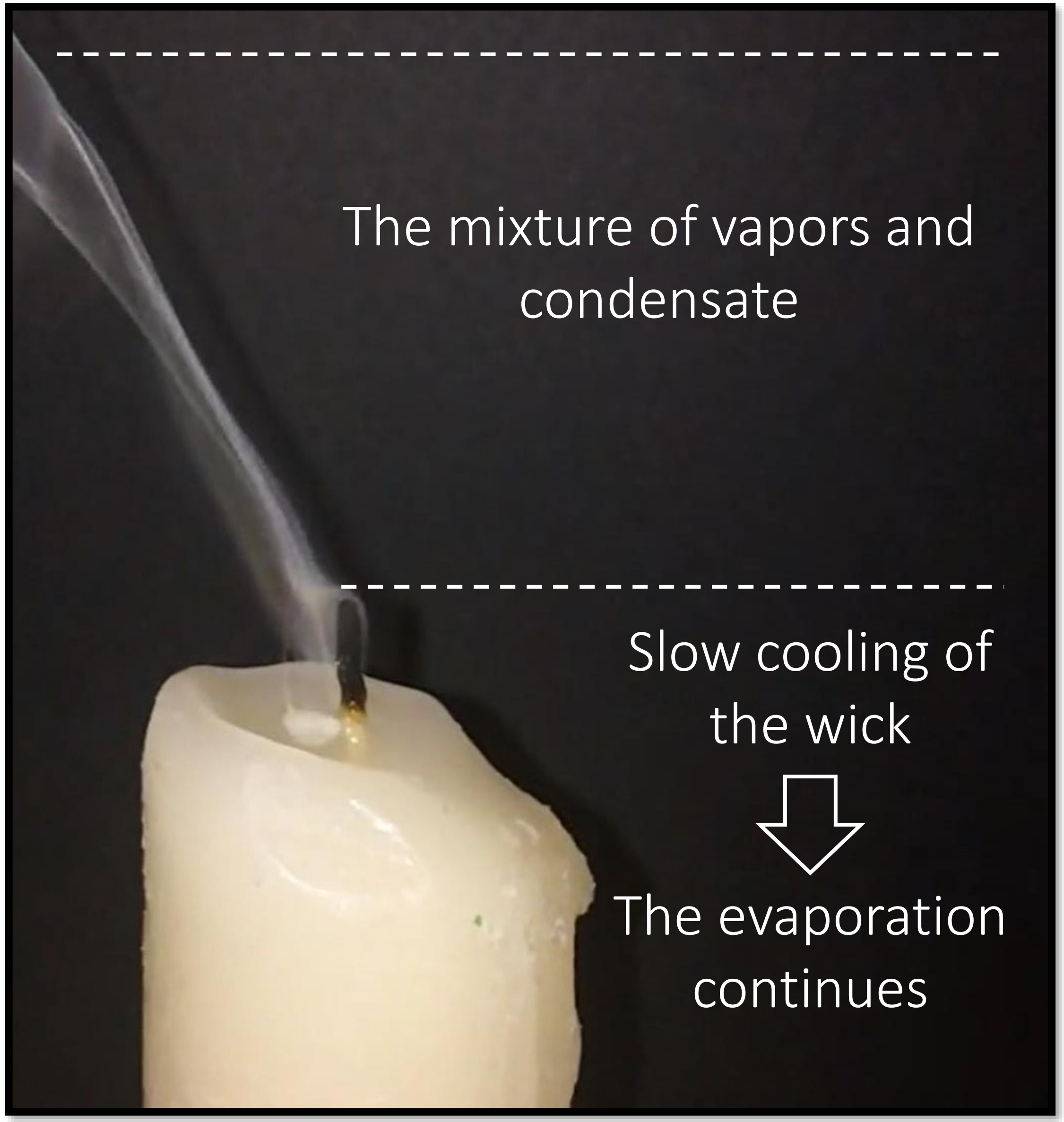
Statement of the task

It is possible to relight a candle that has just been blown out by lighting the smoke that is created in the process (see video). Indeed, the smoke contains vaporized wax which is the substance that burns in the flame in the first place. What is the maximum distance (between the match and the candle) from which one can relight the candle? Identify the important parameters and find how they influence this maximal distance.

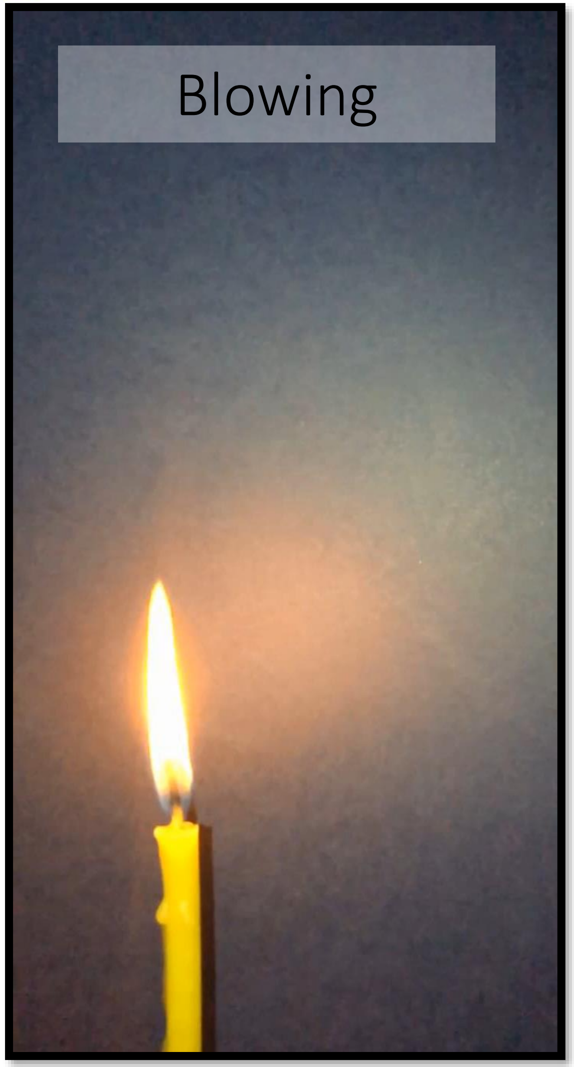


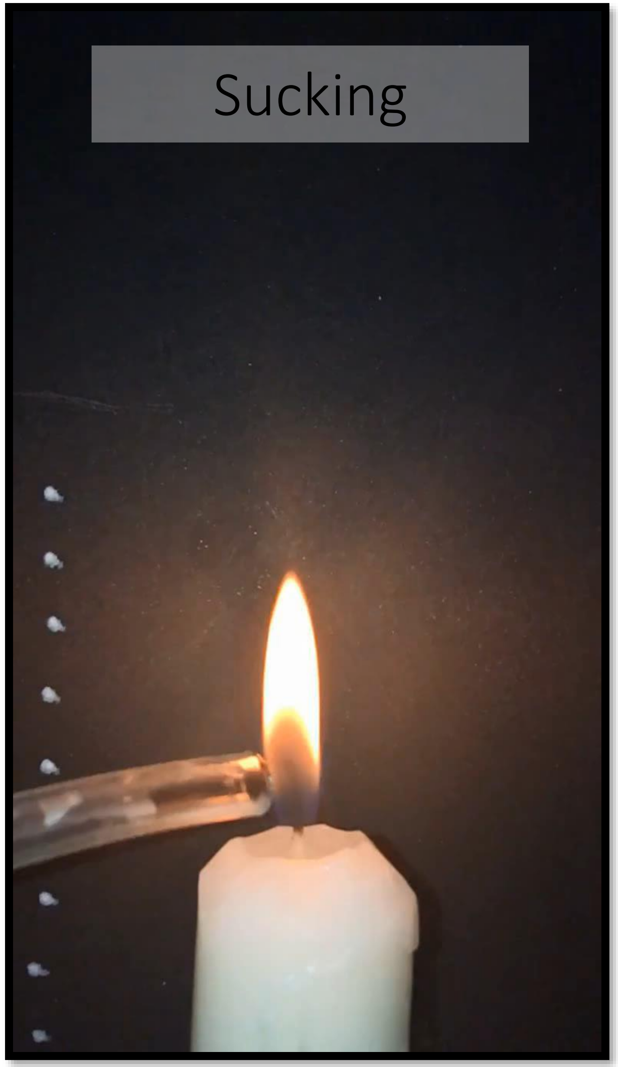
Qualitative explanation





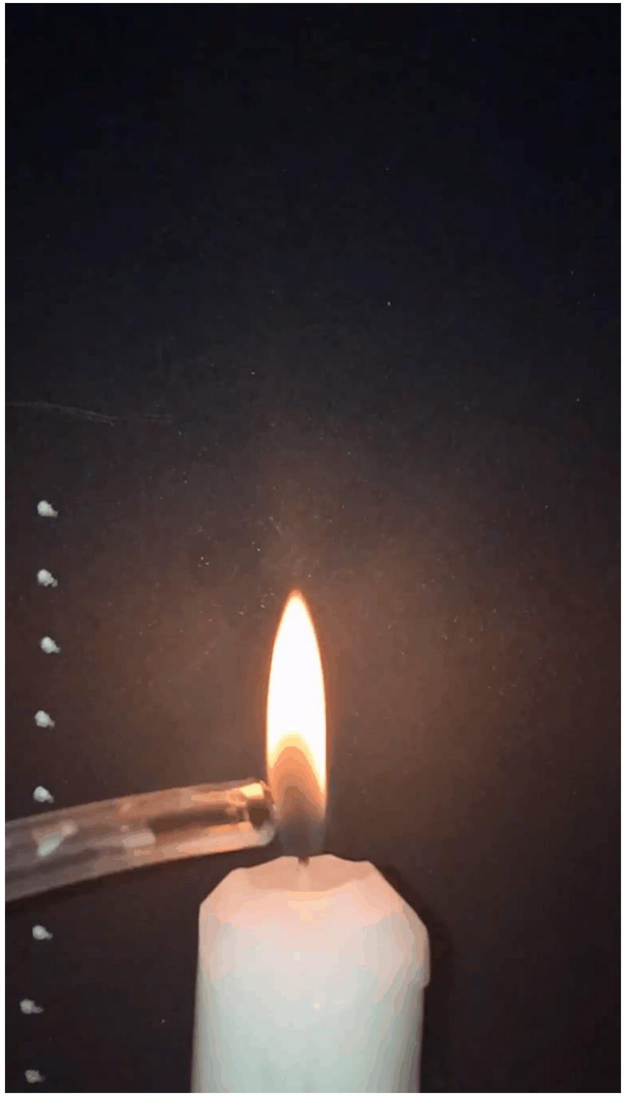
The way of extinguishing the flame



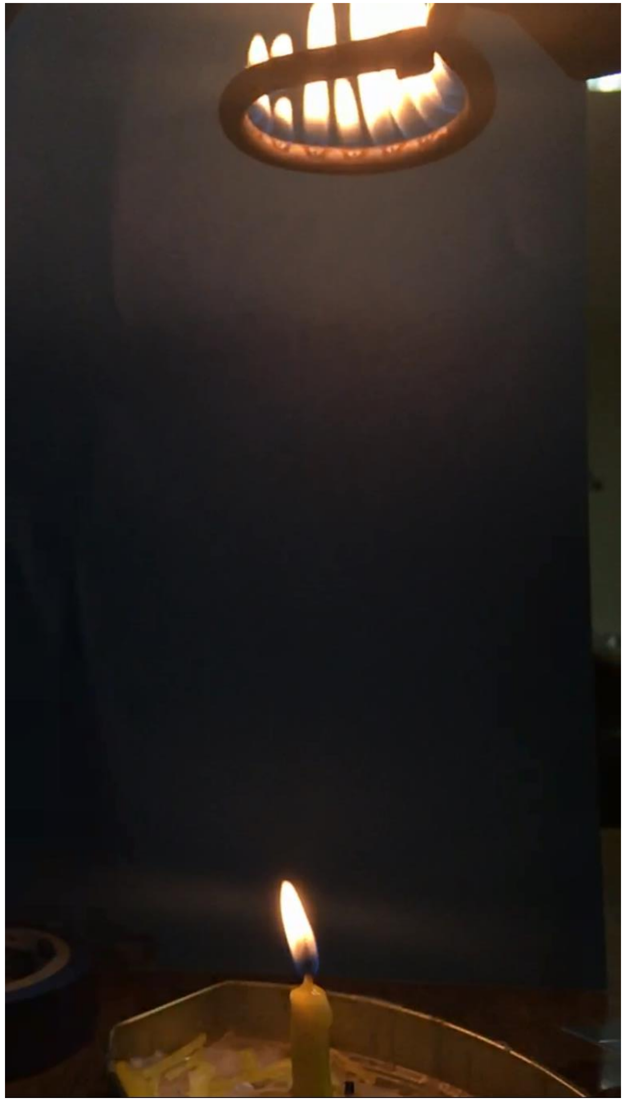


Sucking gives stable reproduceable results

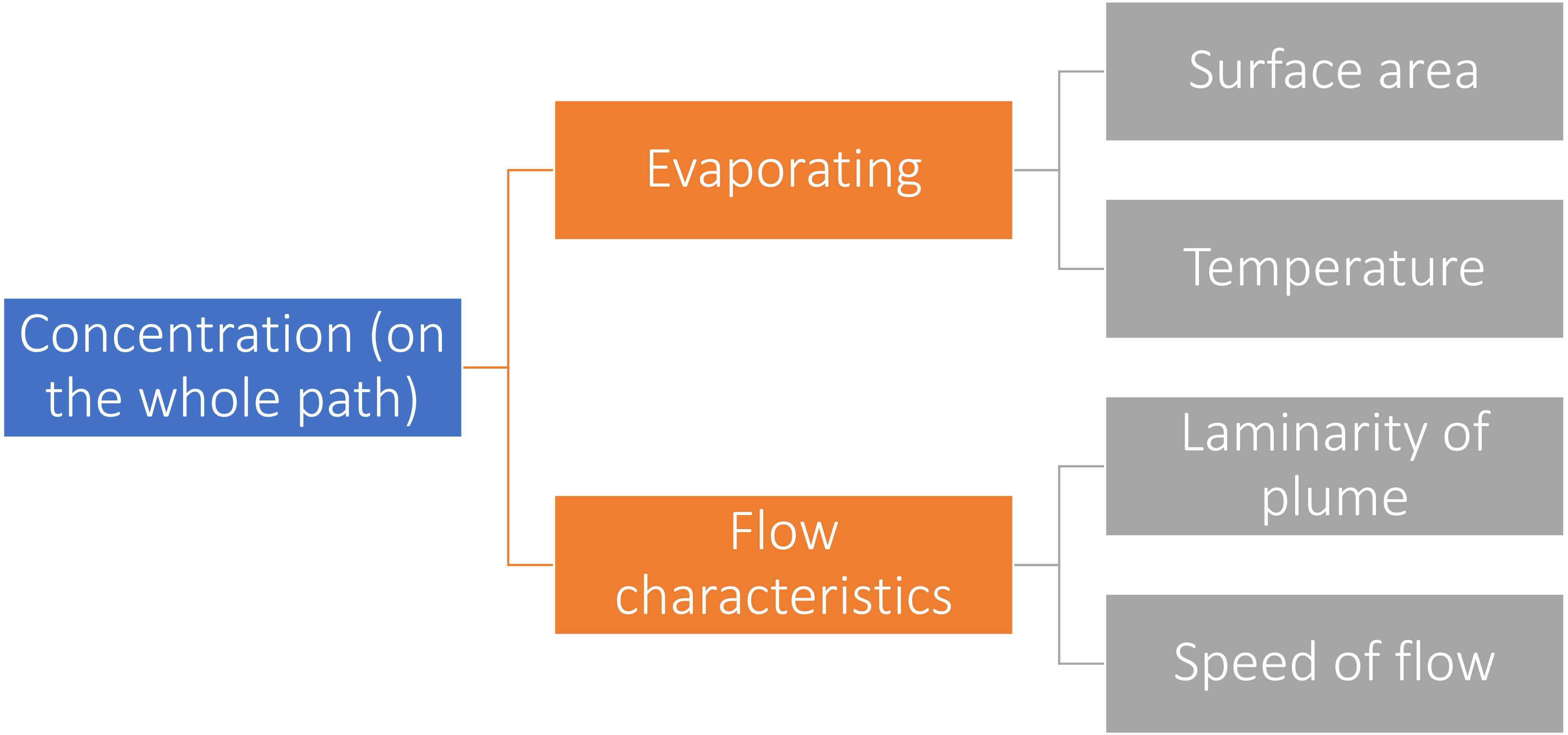
Experimental setup



Such burner catches smoke better than lighter



The limiting factors for the distance of relighting

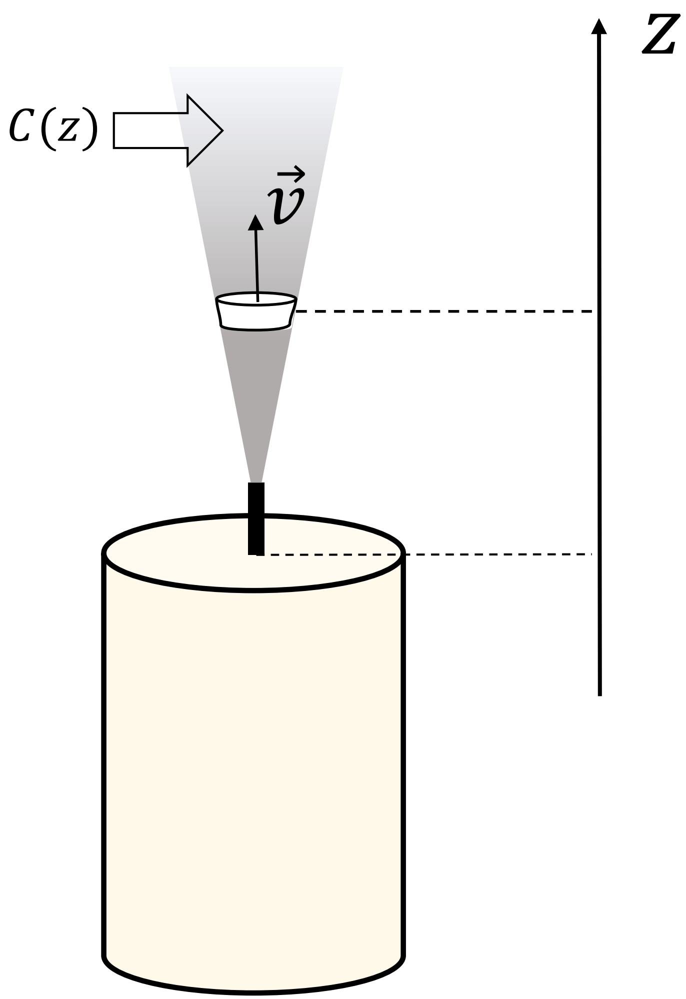


Wicks

- Diameters:
  - 1.2 mm (default)
  - 0.6 mm
  - 1.3 mm
  - 1.5 mm
  - 1.9 mm
  - 2.6 mm
  - 3.1 mm
  - 3.7 mm
  - 4.1 mm
- Length of all wicks except default was 1 cm
- We varied length of default wick

Concentration of paraffin vapor

The concentration of paraffin vapor



Concentration of paraffin vapor

$$C(z) = \frac{4KS}{\pi v(z) D^2(z)}$$

K – rate of evaporation from the unit of area of the wick
v – the velocity of the convection flow
r – radius of the convective flow
S – area of the wick

Parameters of the convective flow

The velocity of convective flow

$$v = \sqrt{\frac{2gd\Delta T}{T}}$$

ΔT - temperature difference between the wick and outer space
d – diameter of the wick
T – ambient temperature

Diameter of the flow

$$D(z) = \frac{z^{1/2}}{Q^{1/4}}\left(\frac{\nu^3 k}{\alpha g \beta}\right)^{1/4}$$

Q – the power of candle
ν – kinematic viscosity of air
α – thermal diffusivity
β – thermal expansion coefficient
k - thermal conductivity

v doesn't depend on distance, $D \sim \sqrt{z}$ ⇒ the concentration decreases with height as $C \sim 1/\sqrt{z}$

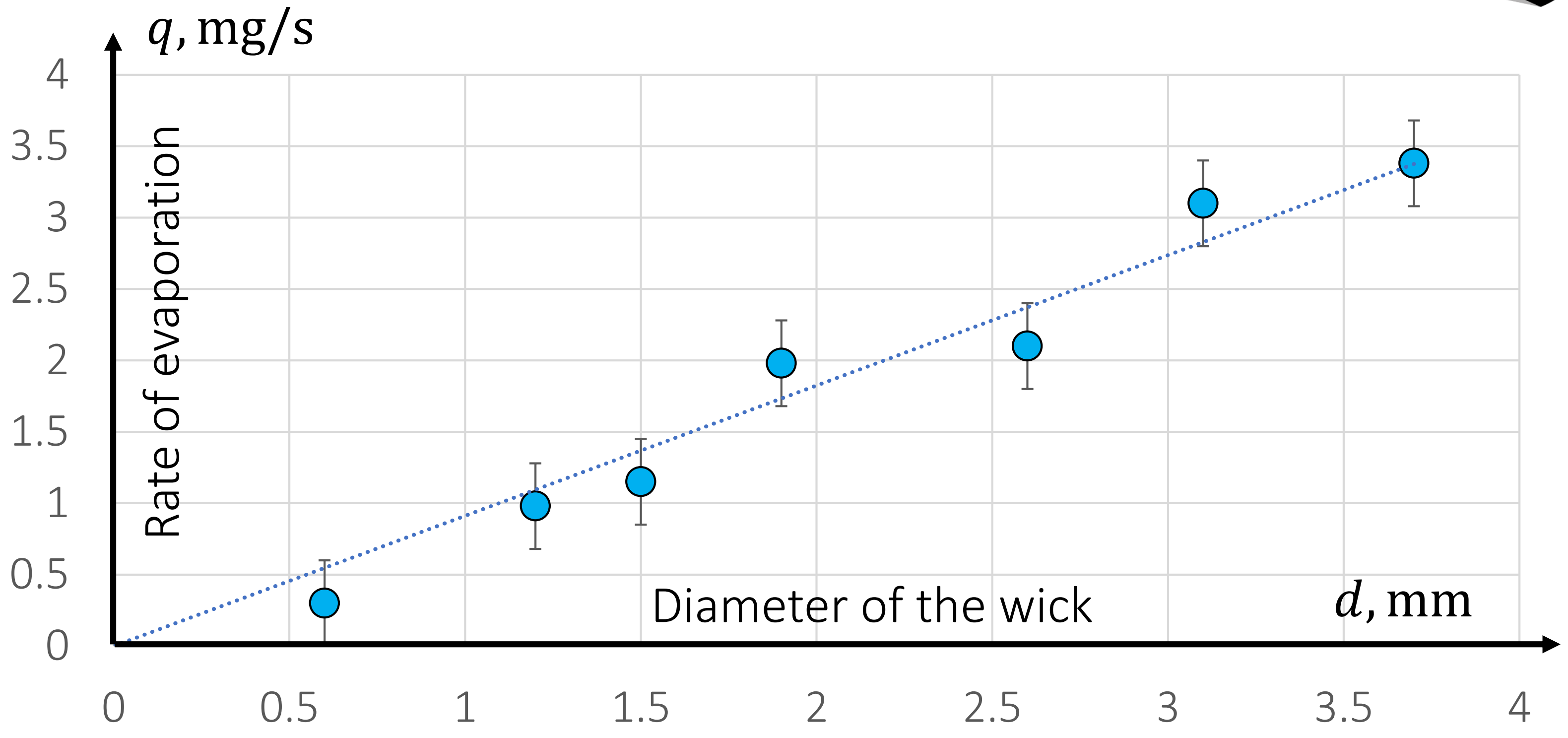
Rate of evaporation over time

We weighed candle

The candled burned for 10 minutes

We weighed candle again

We calculated rate from the difference in weight

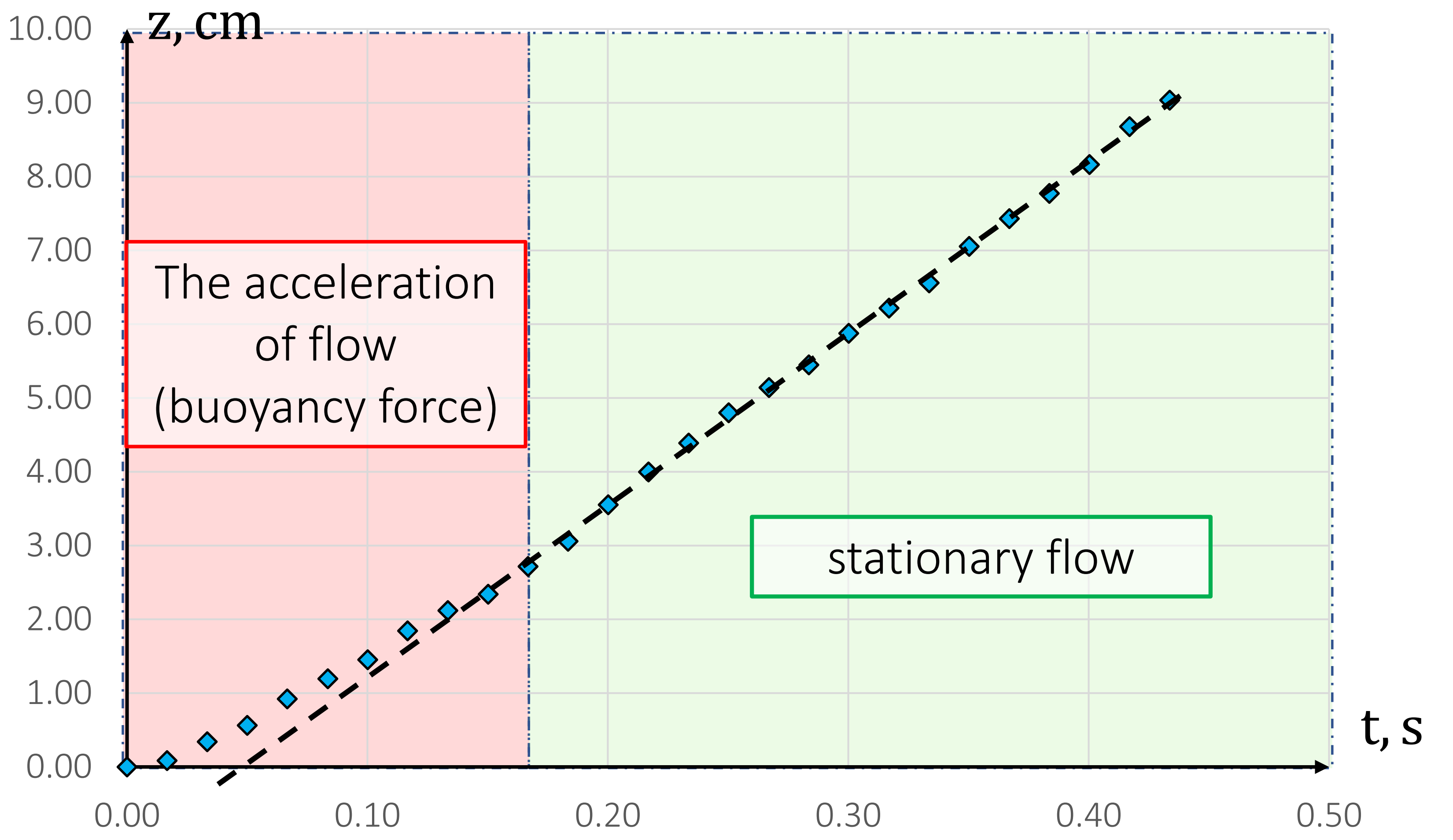


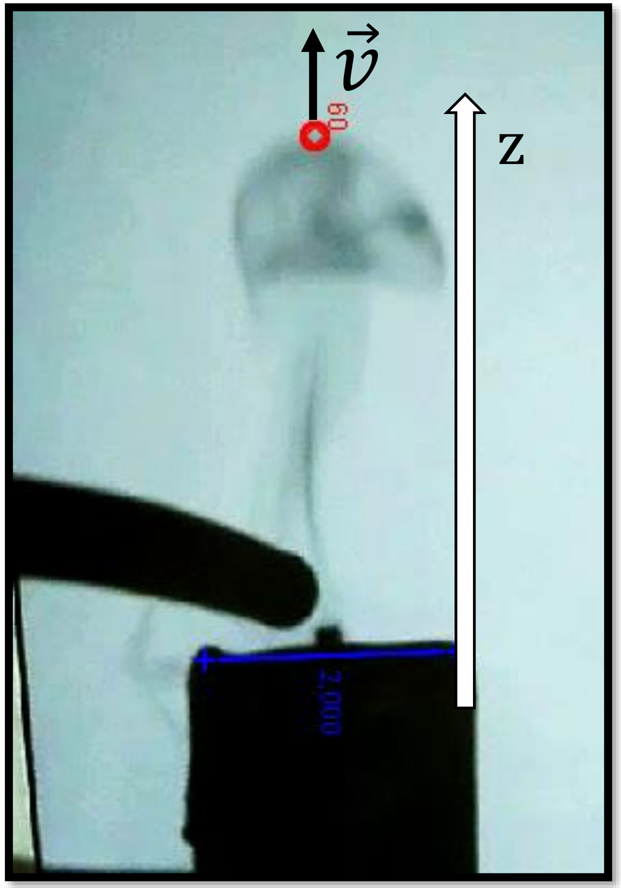
Rate of evaporation from the unit of area of the wick

$$q = SK \Rightarrow K = 0.0295 \frac{kg}{m^2 s}$$

S – surface area of the wick

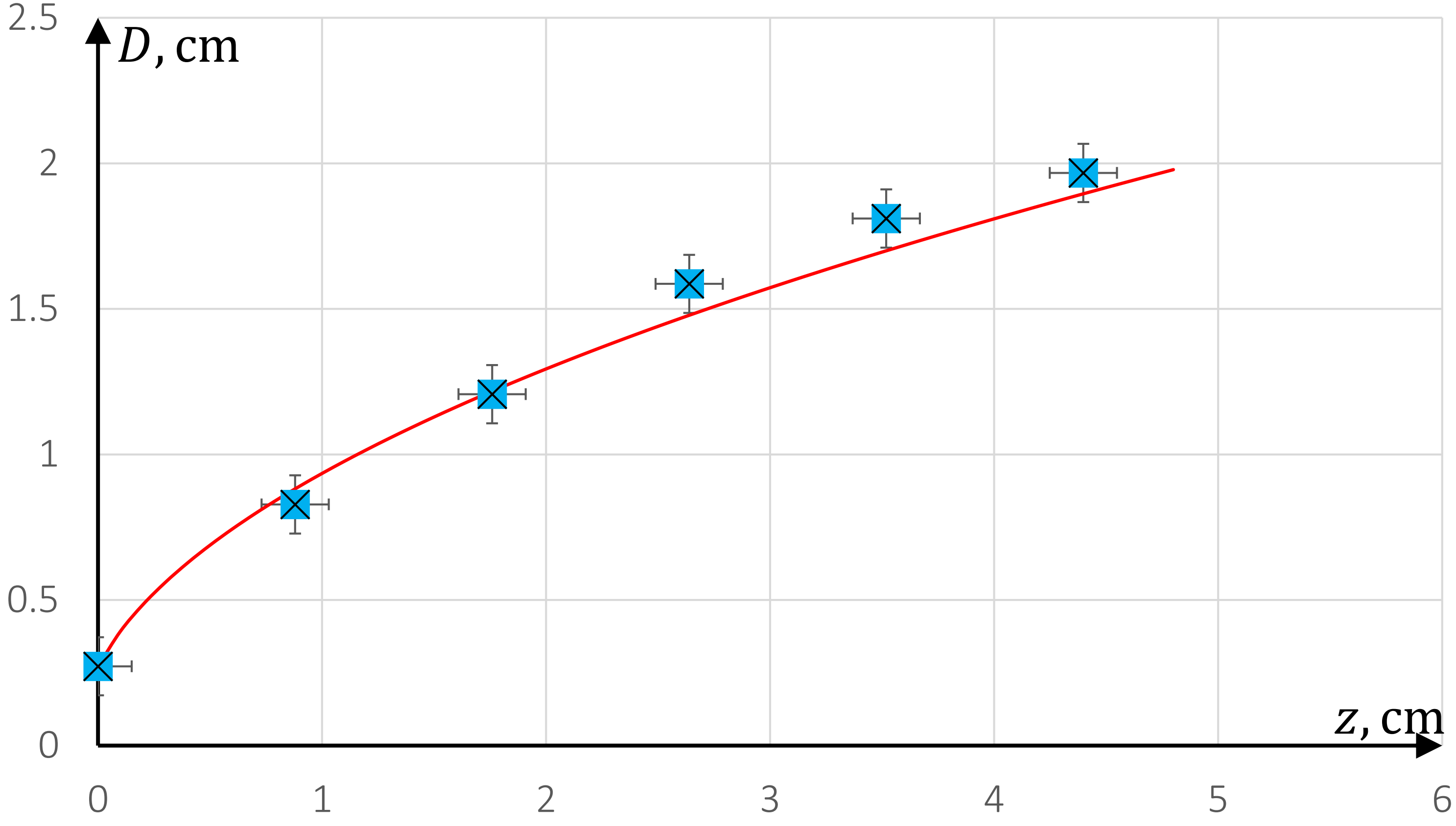
Velocity of convective flow

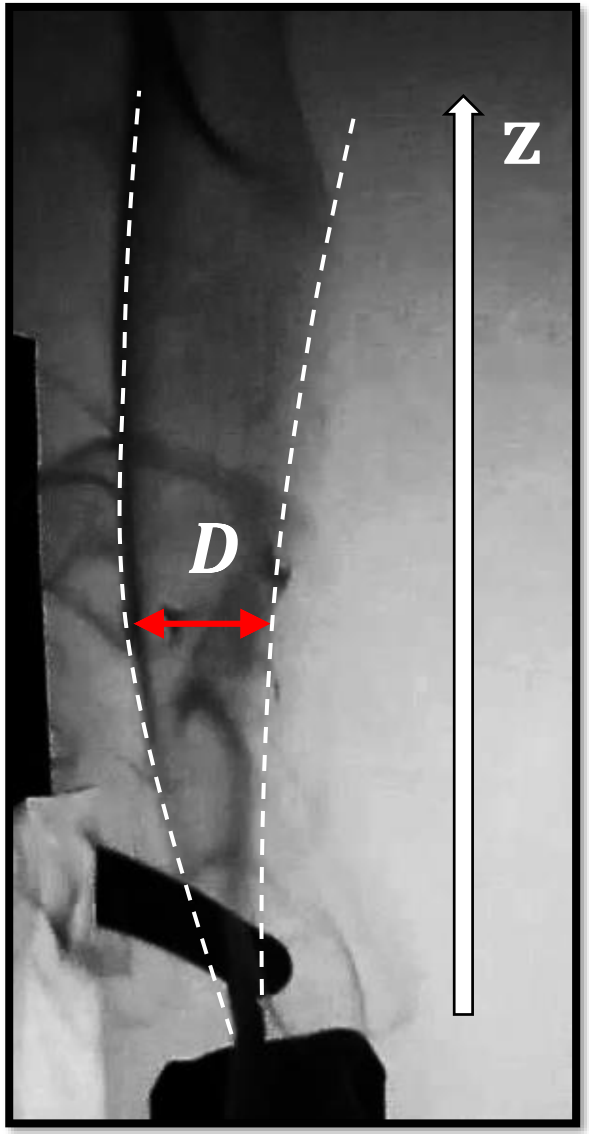




$v_{teor} = 23\ \text{cm/s}$	$v_{exp} = 27\ \text{cm/s}$

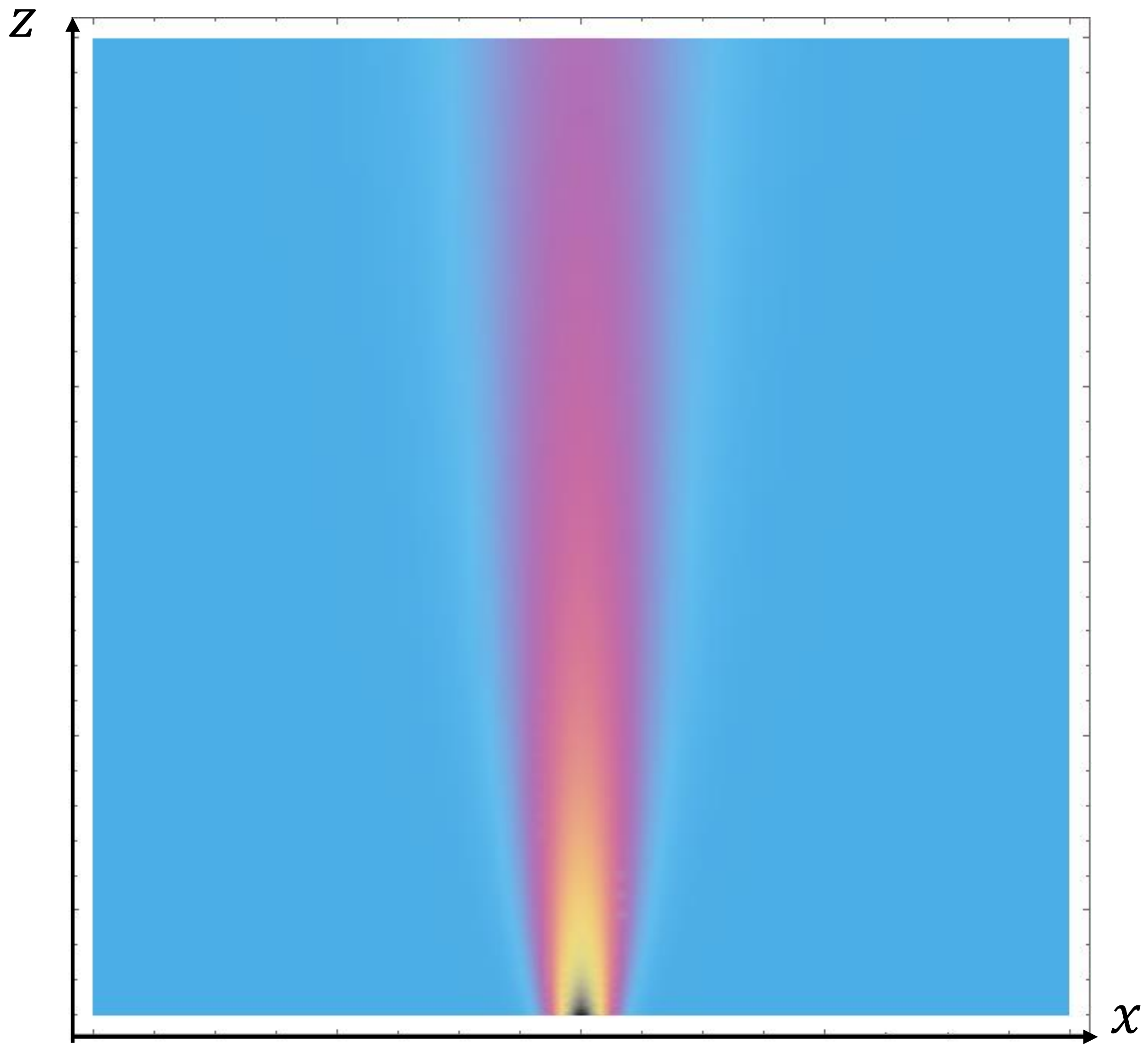
The expansion of convective flow





The diameter of convective flow expands as $\sqrt{z}$

The dependence of concentration on the coordinate



Model of **Gaussian distribution** for the concentration in the laminar plume

$$C(x,z) = \frac{M}{\sqrt{4\pi D z/v}} \exp\left(-\frac{(x-x_0)^2}{4\pi D(z/v - t_0)}\right)$$

D – diffusion coefficient (laminar diffusion)
M – normalization constant

M is defined from

$$C(z) = \int C(x,z)\,dx$$

$\mathrm{C}(z)$ is defined from previous considerations

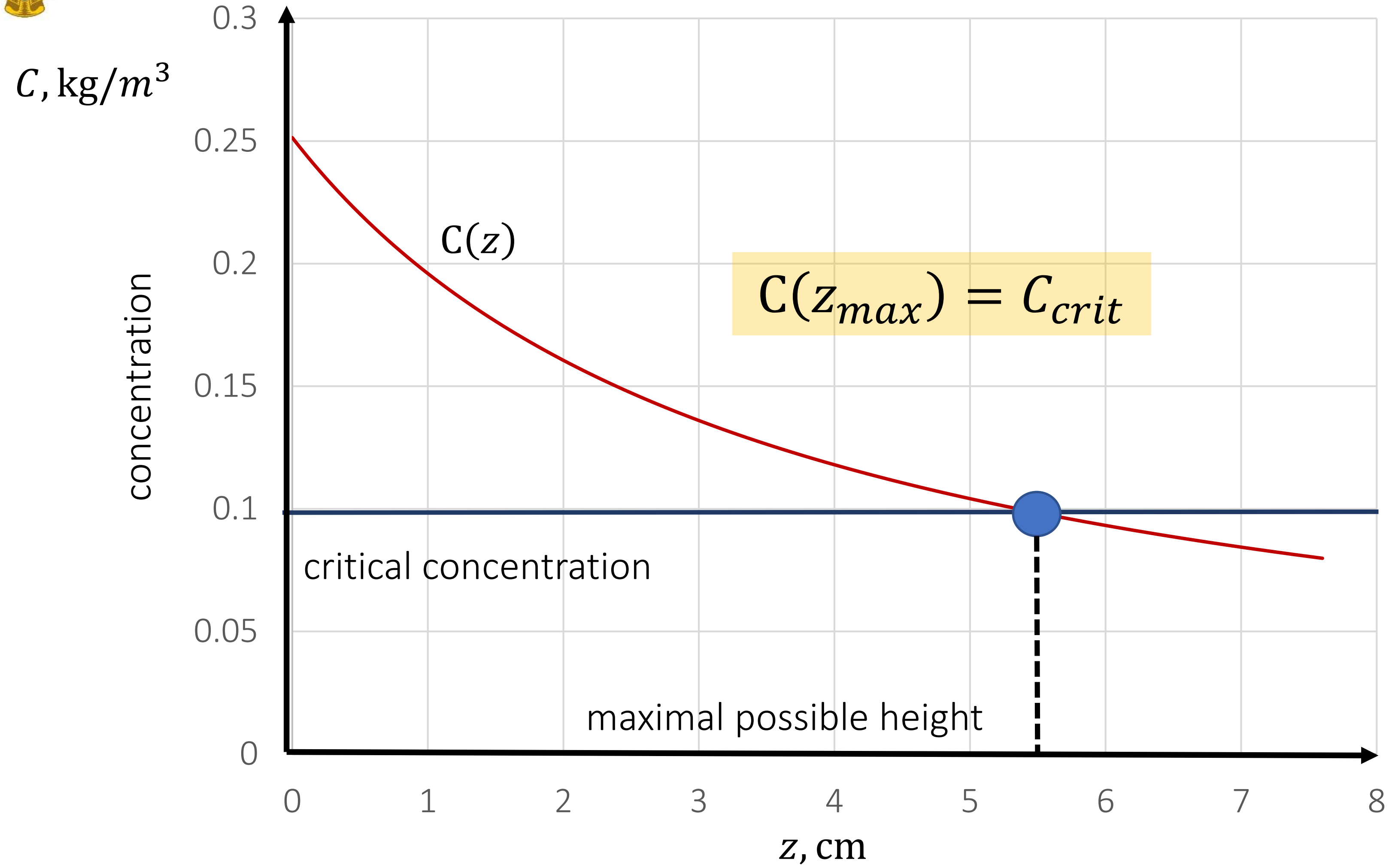
The critical concentration of paraffin vapor for burning

Me measured the maximal time after which the candle already can’t be burned

$$\tau_{cr} = 2.5\ s$$

$$C_{min} \approx 0.1\ kg/m^3$$

The condition for relighting (through concentration)

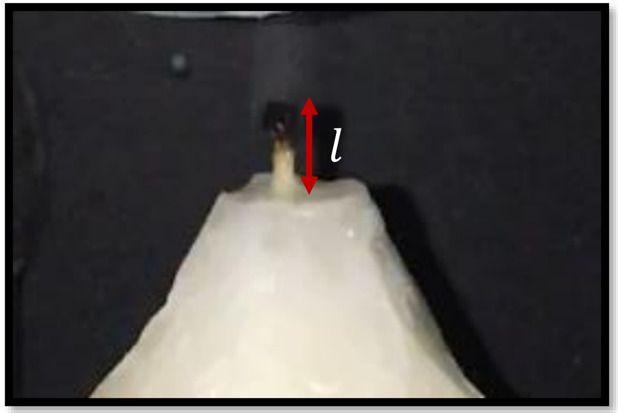


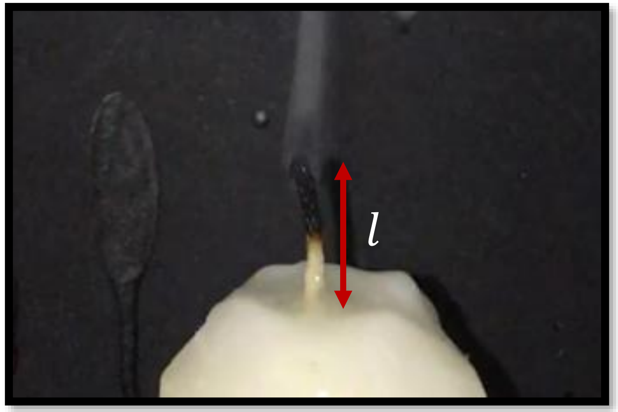
$$C(z) = \frac{2l^{3/4}K^{3/4}}{z^{\frac{1}{2}}(\pi cd(T-T_0))^{\frac{1}{4}}\left(\frac{\alpha g\beta}{\nu^3 k}\right)^{\frac{1}{4}}}$$

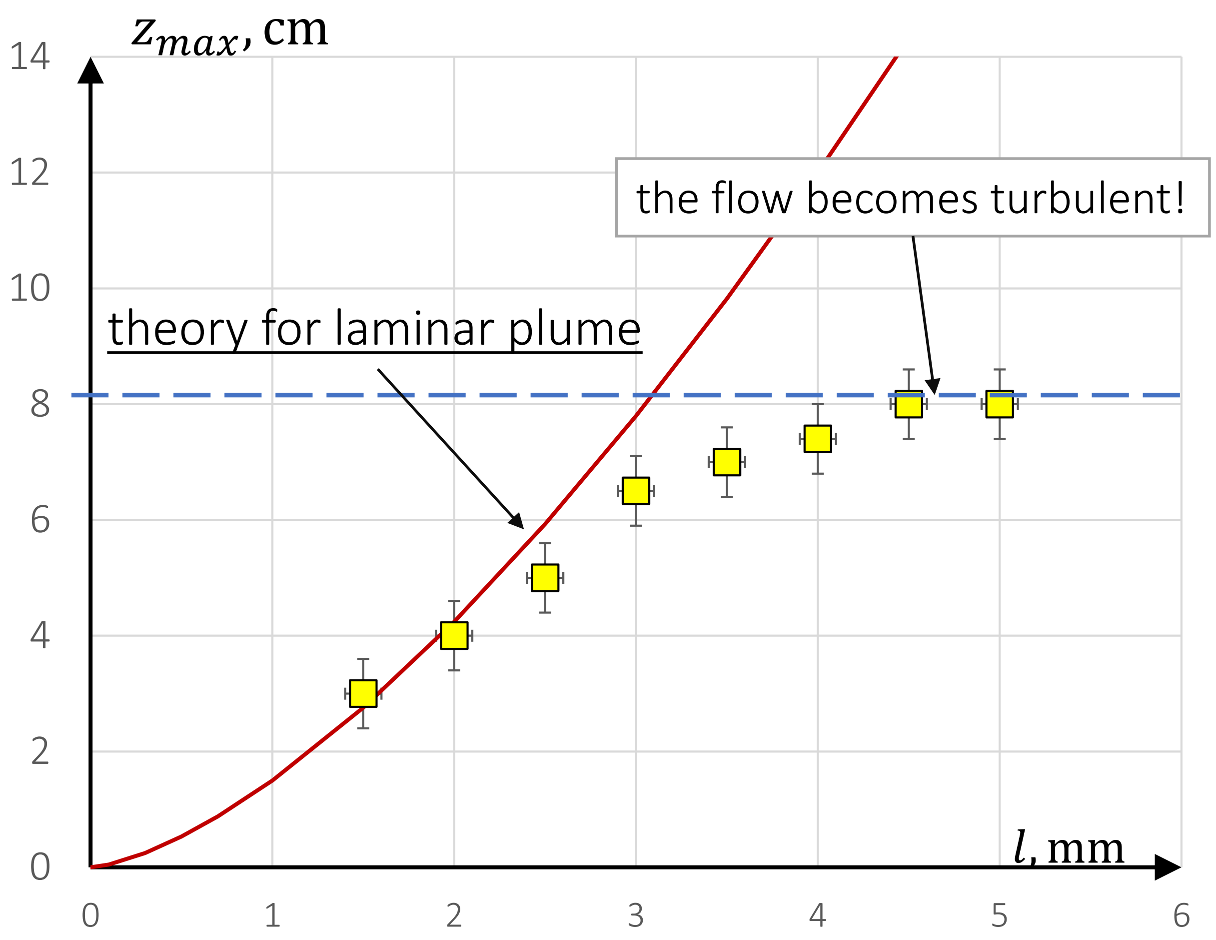
T – temperature of the wick

The condition for relighting of the candle – the concentration on the certain height should be higher than critical!

Maximal height of lighting vs length of the wick







Theory diverges with experiment for high concentrations, because of plume's turbulence

Laminarity of the plume

Laminarity of the plume

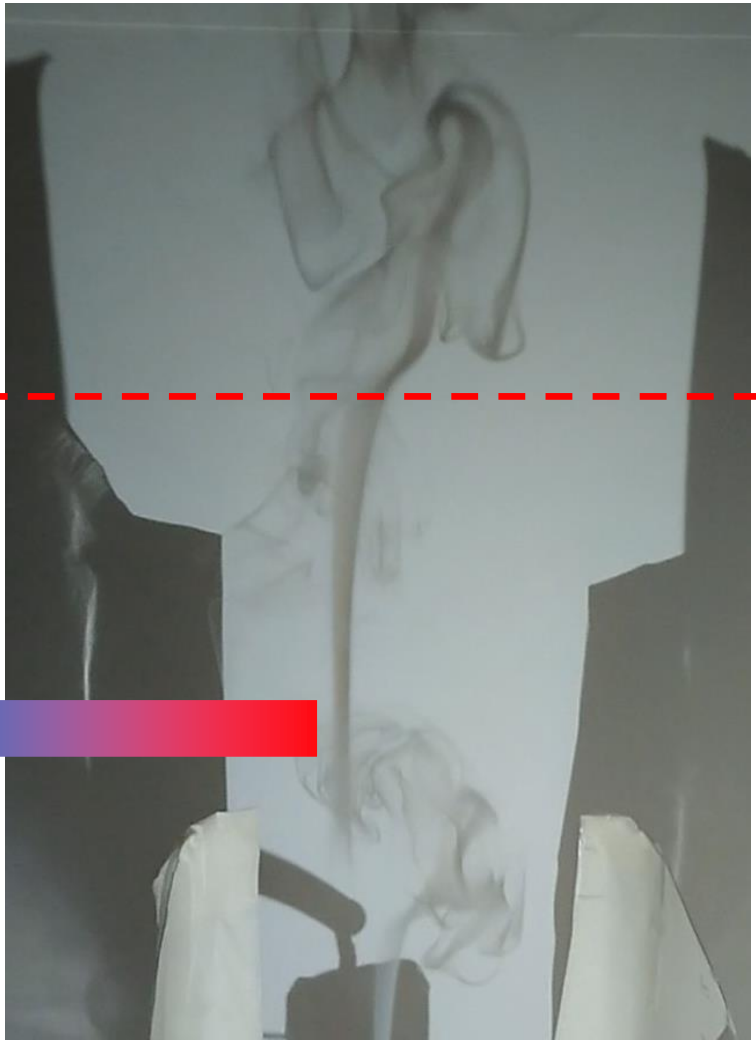
Re_{crit}

Velocity increases, diameter increases

$v \uparrow \quad D \uparrow$

Reynolds number increases

$Re \uparrow$

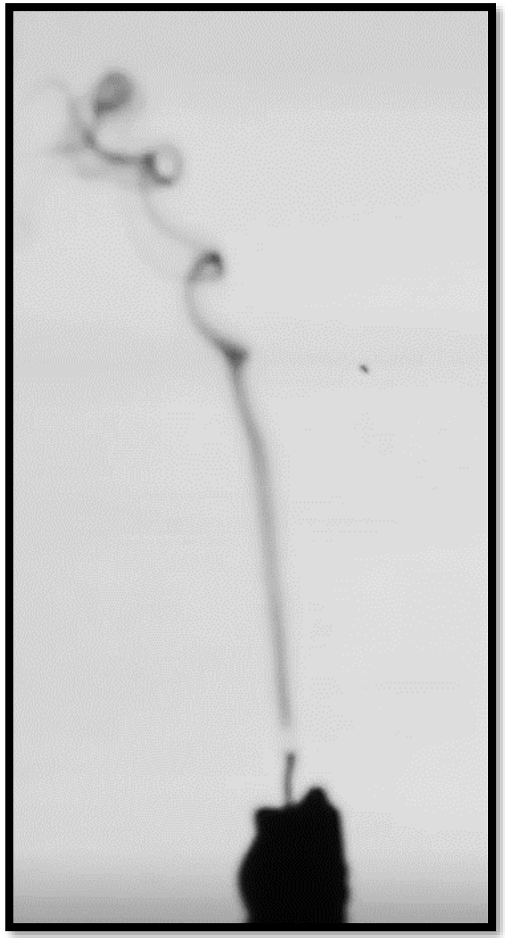


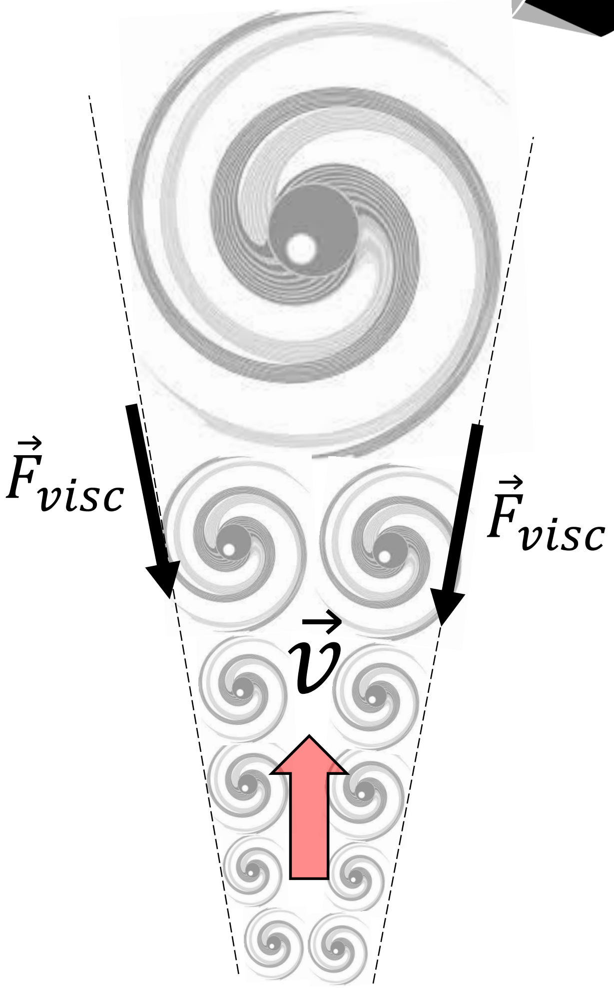
Turbulent mode

Laminar mode

Rate of diffusion increases dramatically due to turbulence

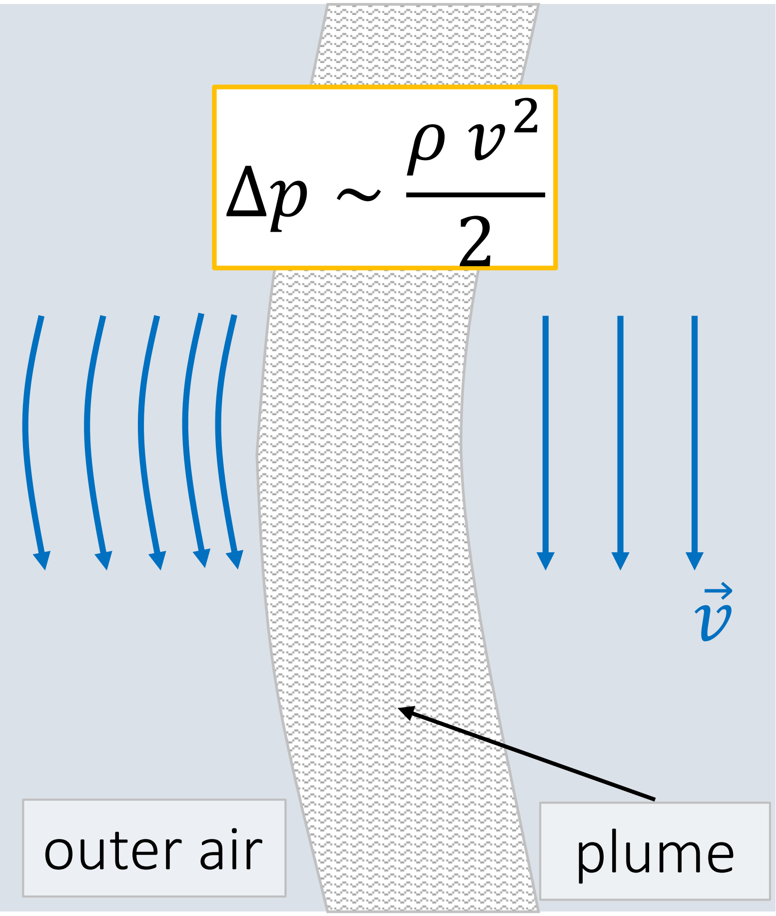
Shadow method





The instability begins to grow because of **Kelvin-Helmholtz instability**.

Bending instability of plume



Length of laminarity

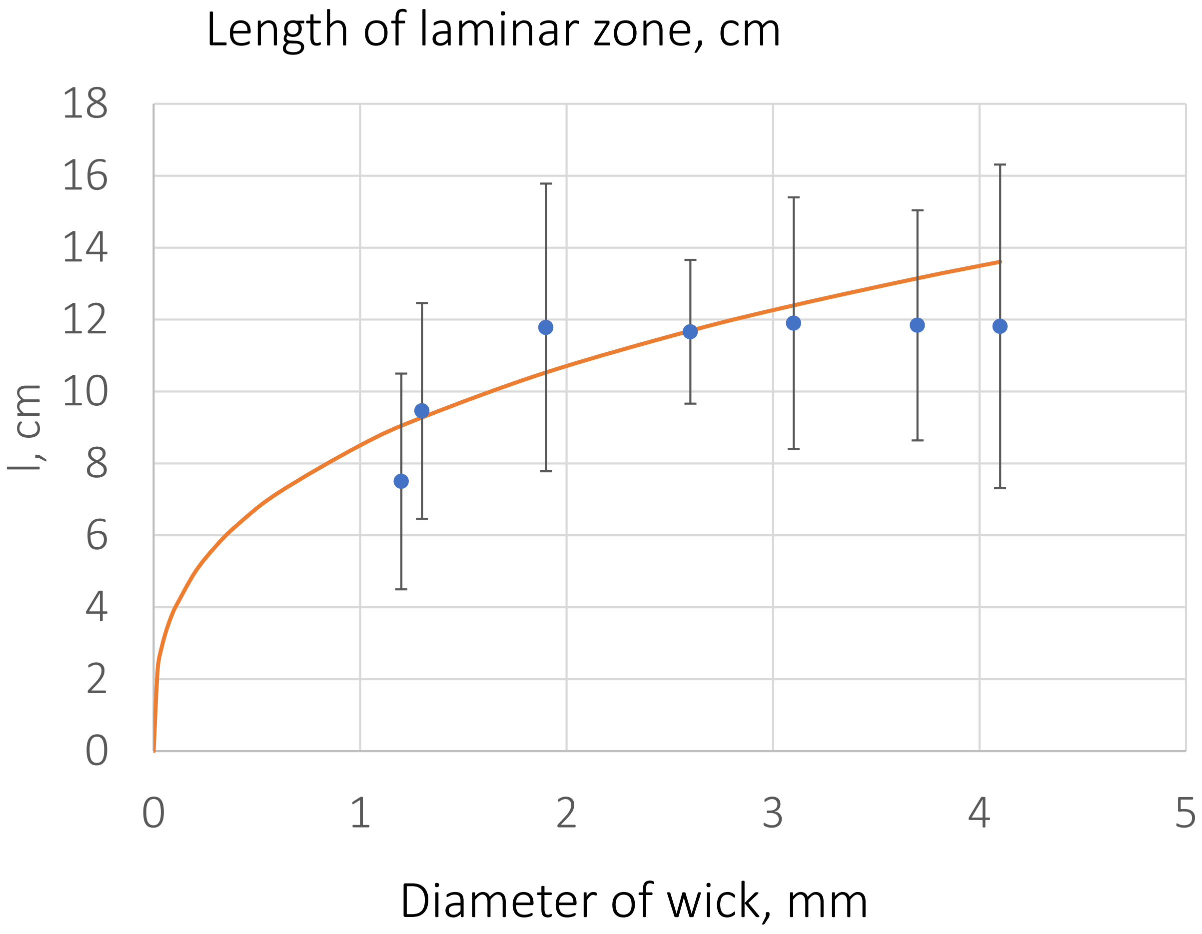
$$H = C(L)\frac{(3\mu D T_0)^{\frac{1}{3}}}{T^{\frac{1}{2}}\left(2g(T - T_0)\right)^{1/6}}$$

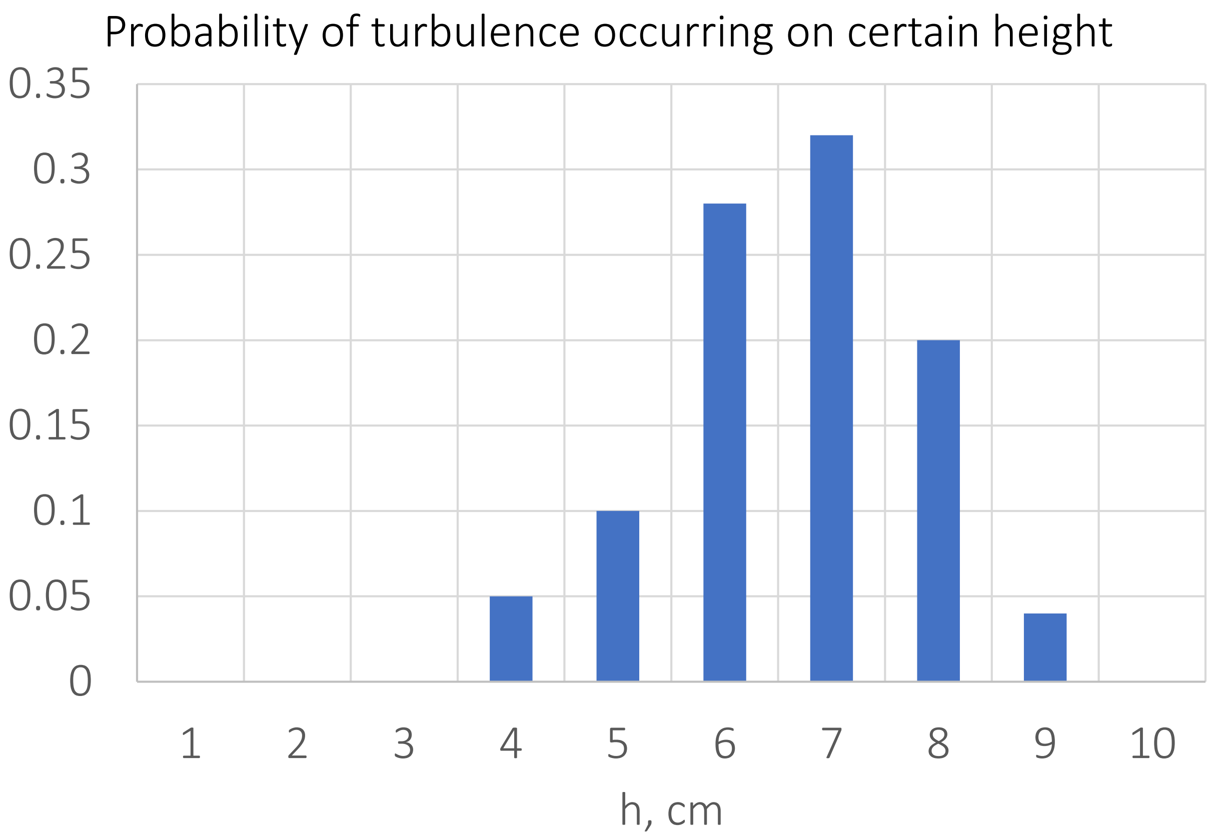
$C(L)$ - coefficient depending on the "size" of outer space
T_0 - ambient temperature

T – temperature of the wick
μ – dynamic viscosity of the air
D - characteristic size of the flow

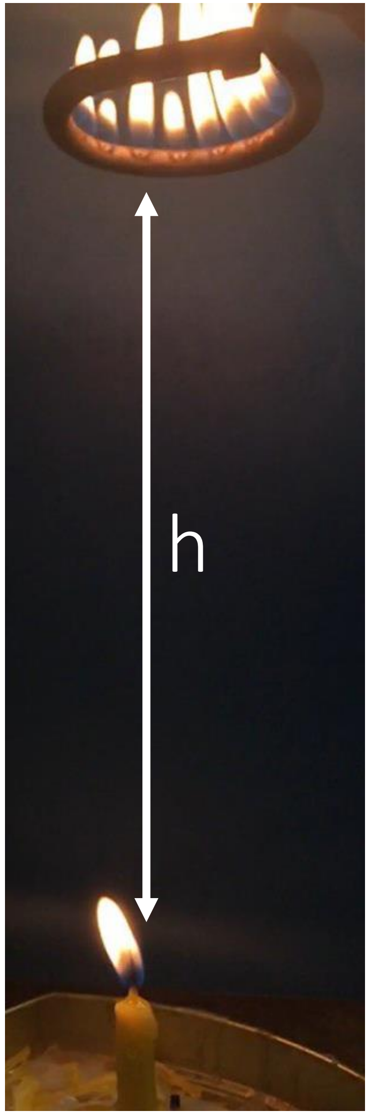
Small oscillations on the plume grow exponentially because of Bernoulli's law.

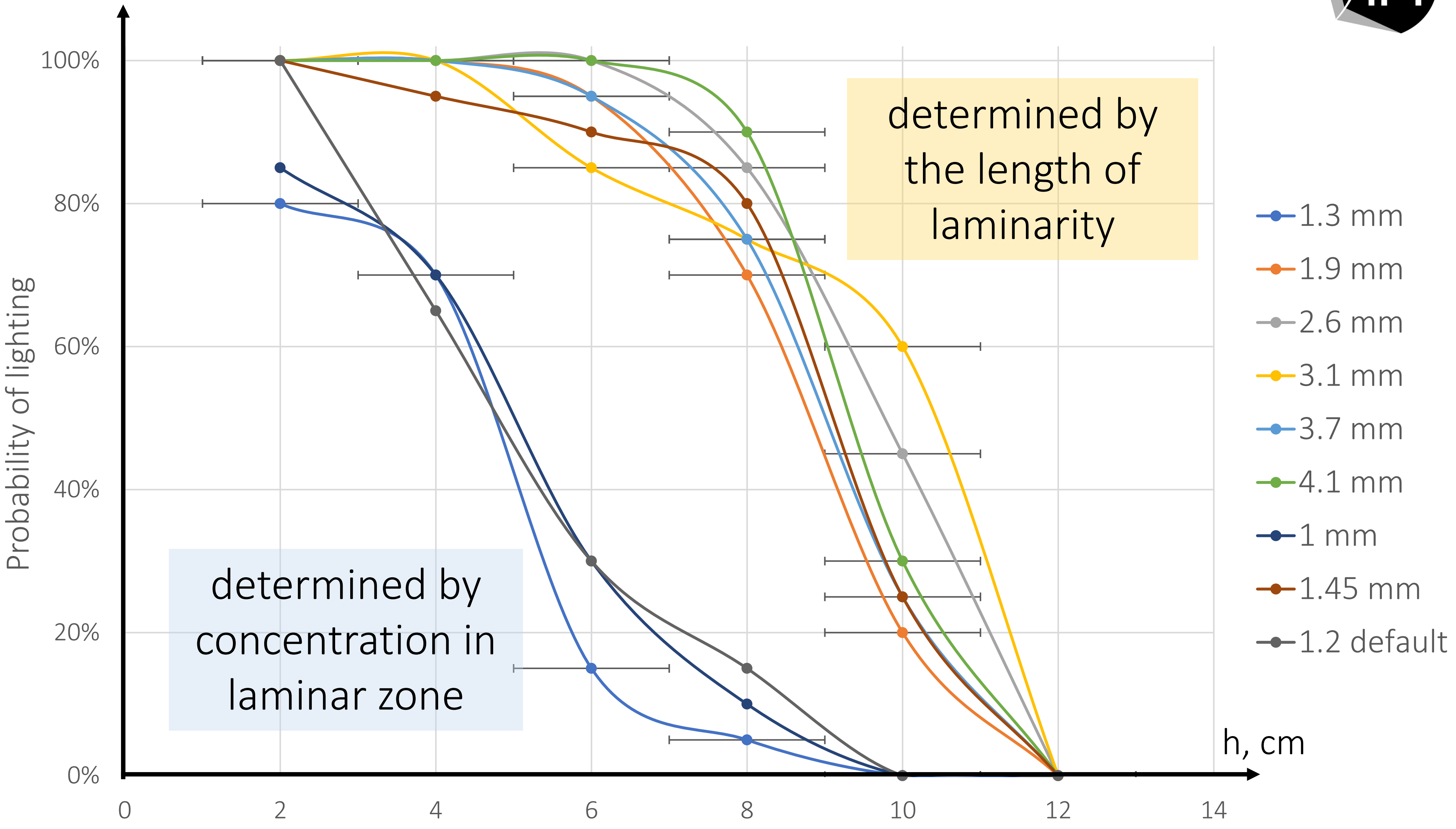
Dependence of length of laminar zone on diameter of wick



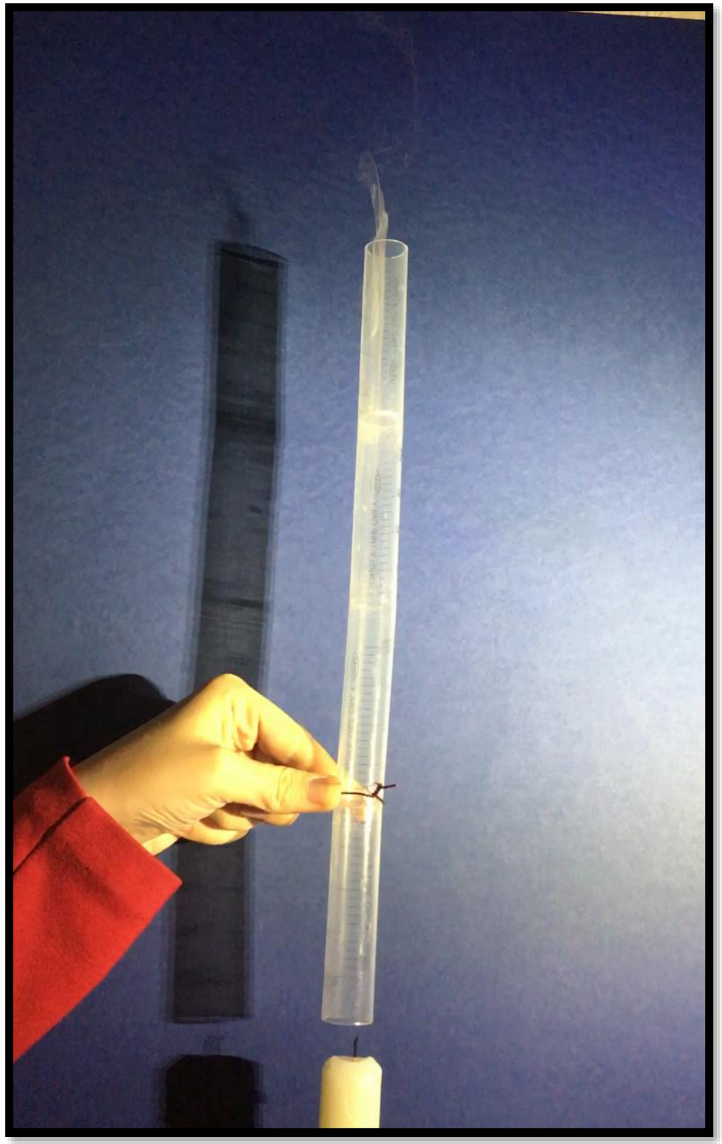


The maximum height

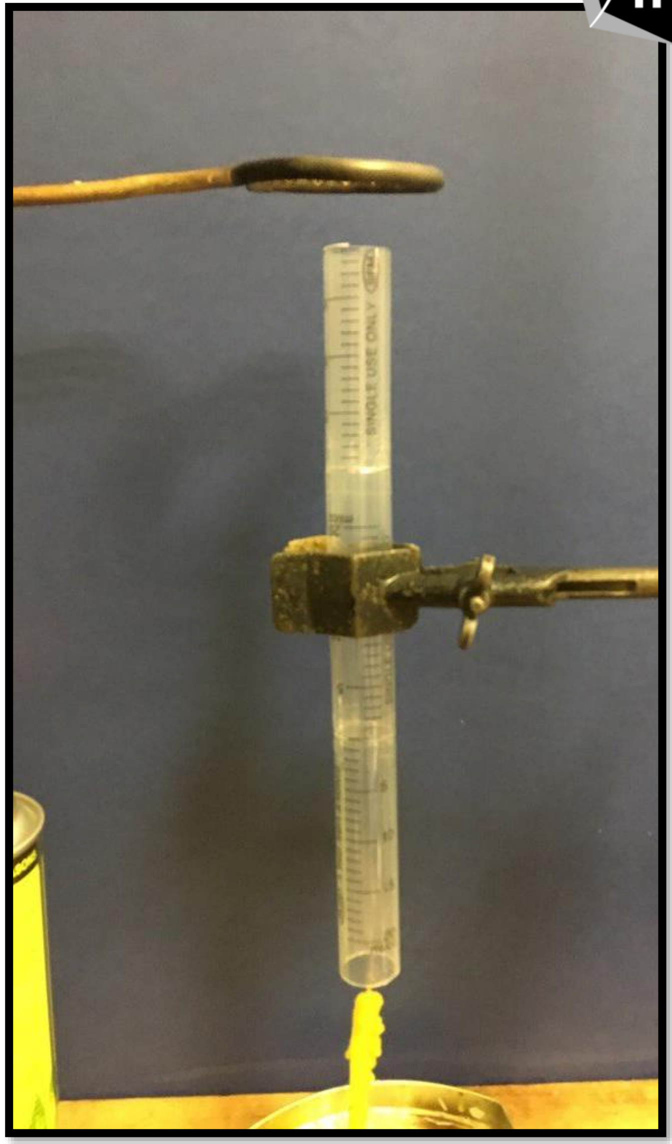




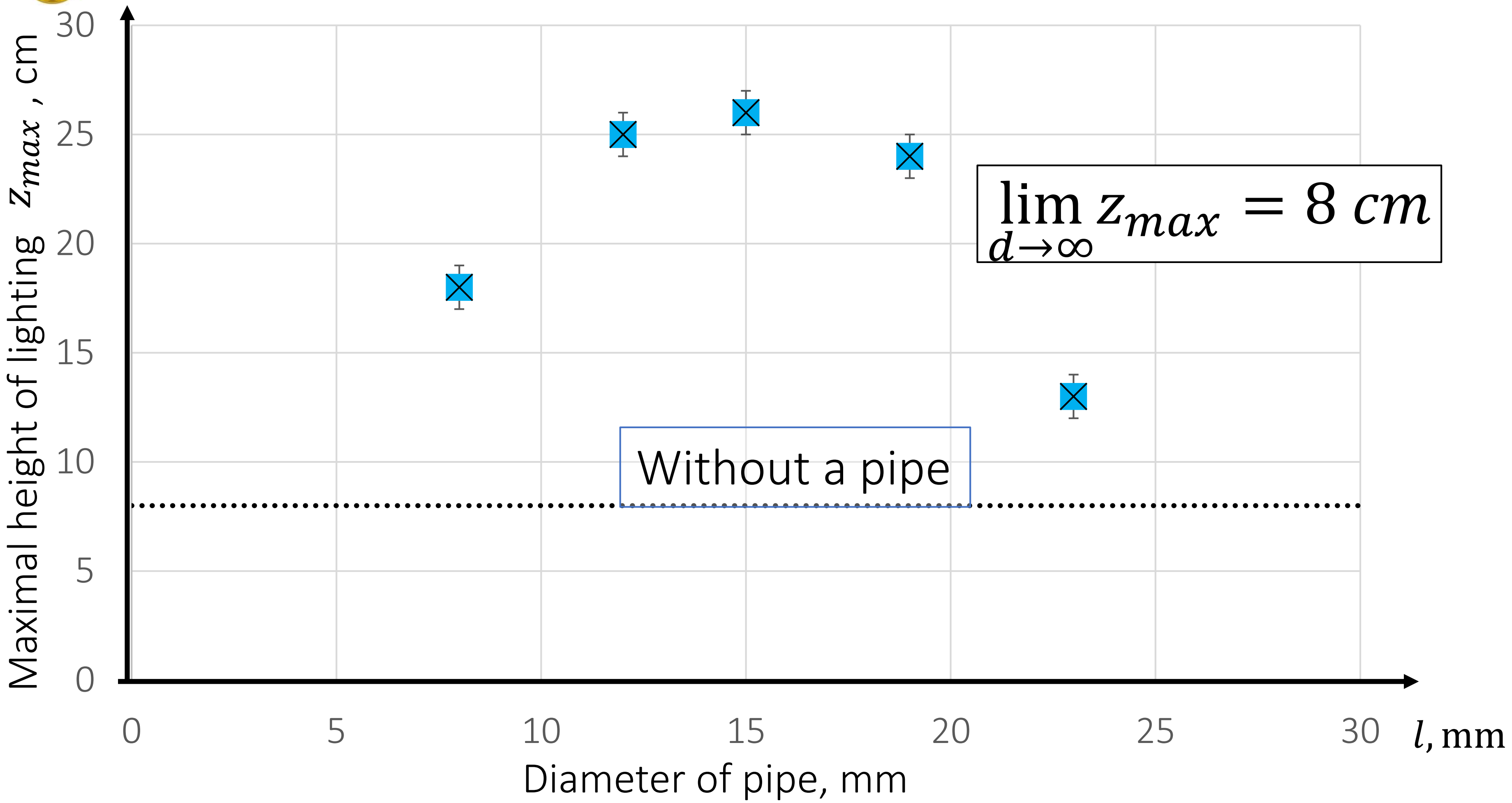
Laminar plume



Pipe laminarizes the plume



Maximal height of lighting (pipe case)



Even in laminar case we can't burn the candle on the certain height because of cooling of the wick.

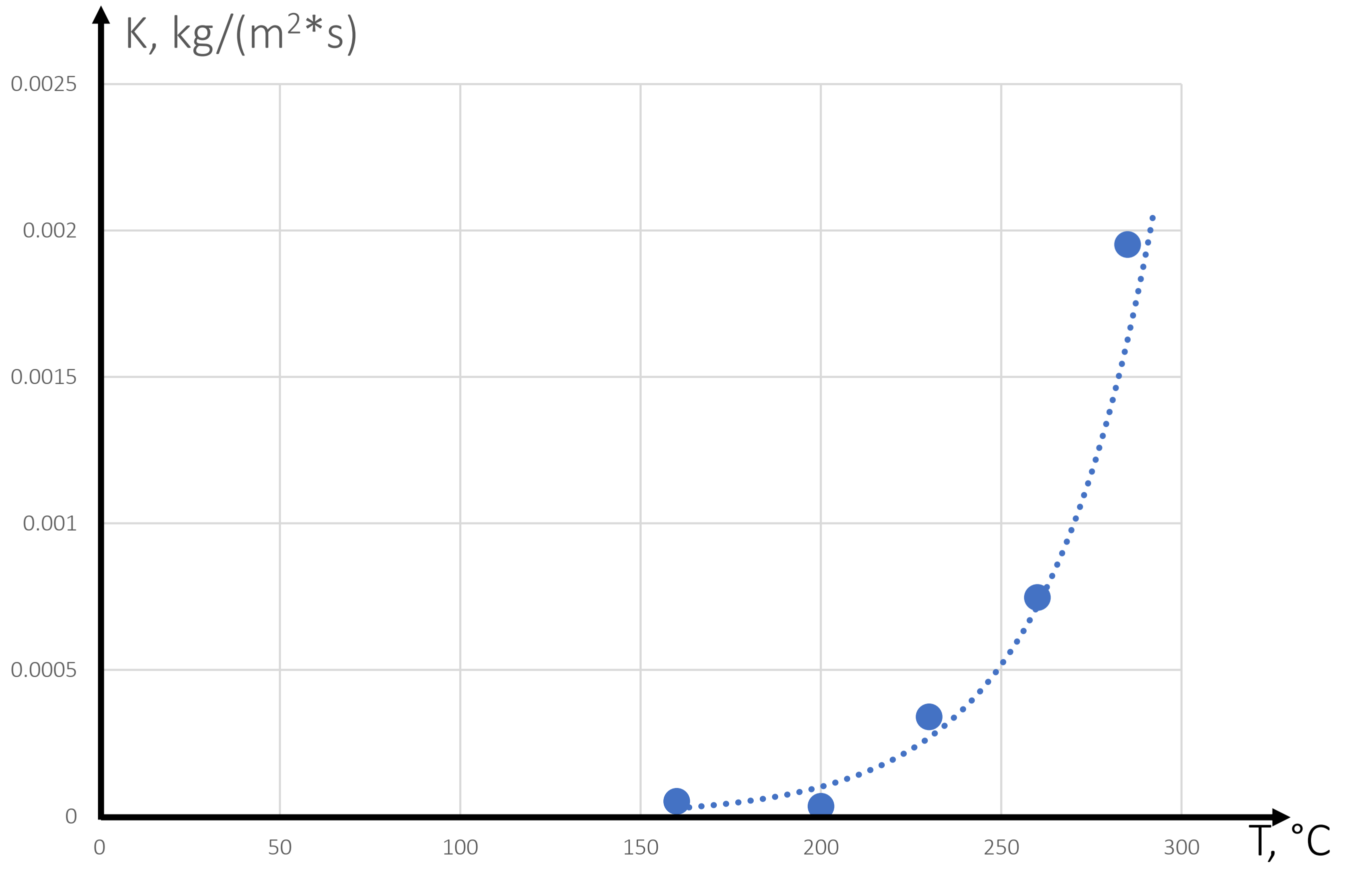
Evaporating rate on temperature

We heated paraffin to the required temperature

Temperature was controlled and constant

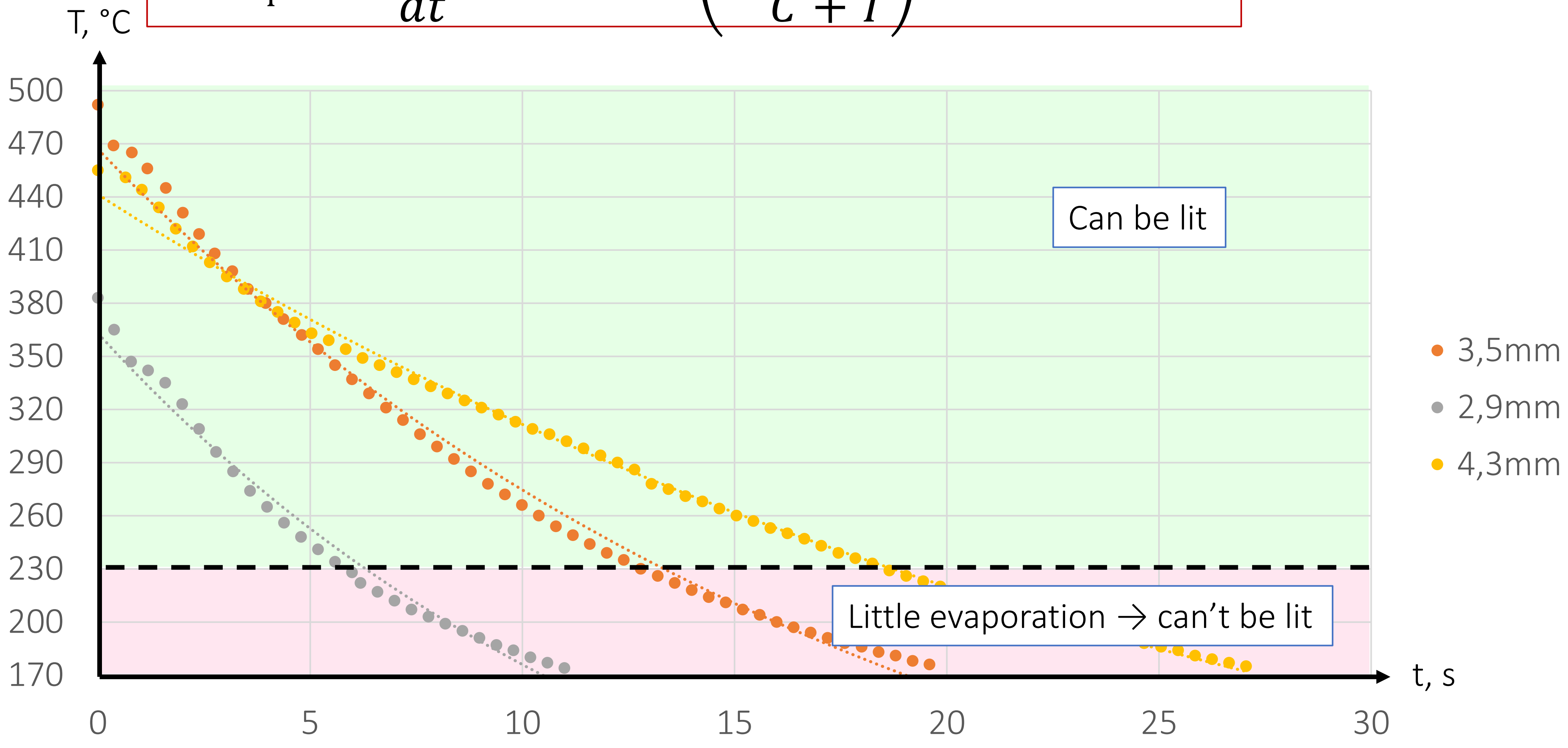
We weighed pot with paraffin 2 times with 10 minutes gap

We calculated rate from the difference in weight



Temperature of the wick after extinguishing

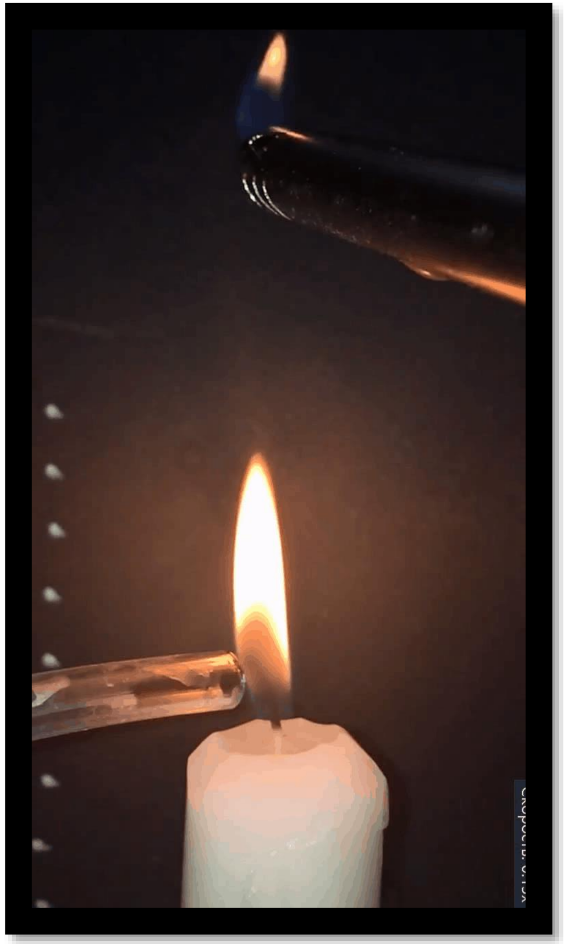
$$cV\rho_{\text{фитиля}}\frac{dT}{dt} = -\lambda S\exp\left(-\frac{B}{C+T}\right) - \beta S(T - T_0)$$

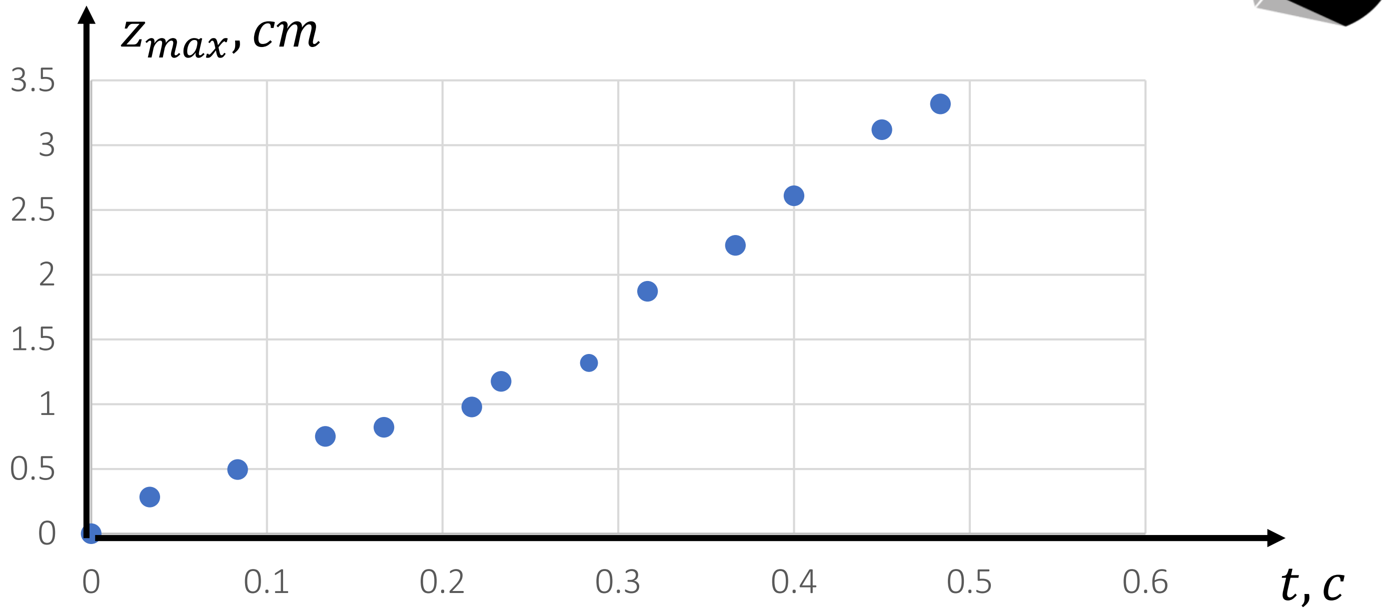


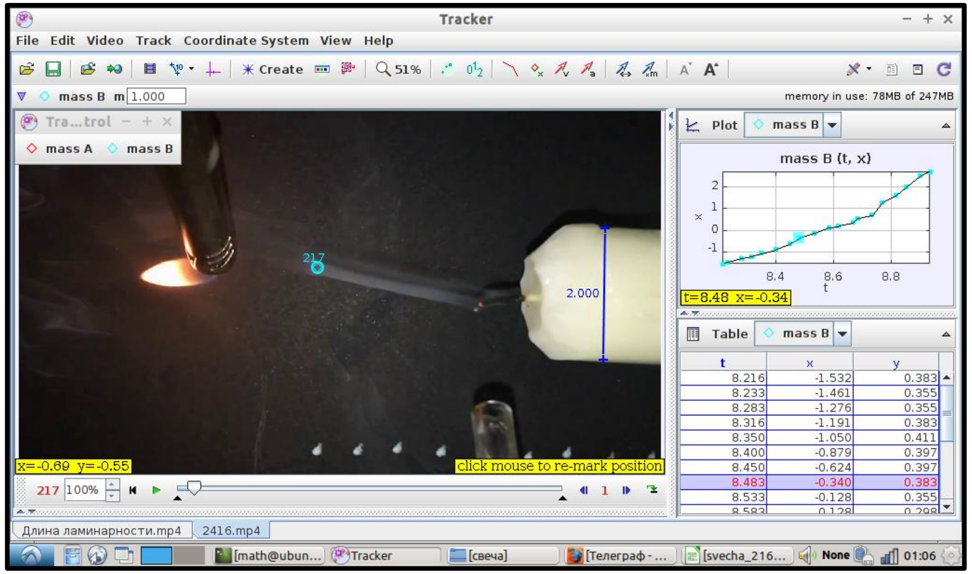
Measuring the speed of flame

$$v_{flame} = 9{,}1\ ^{\text{см}}/_{\text{с}}$$

Reference frame associated to observer





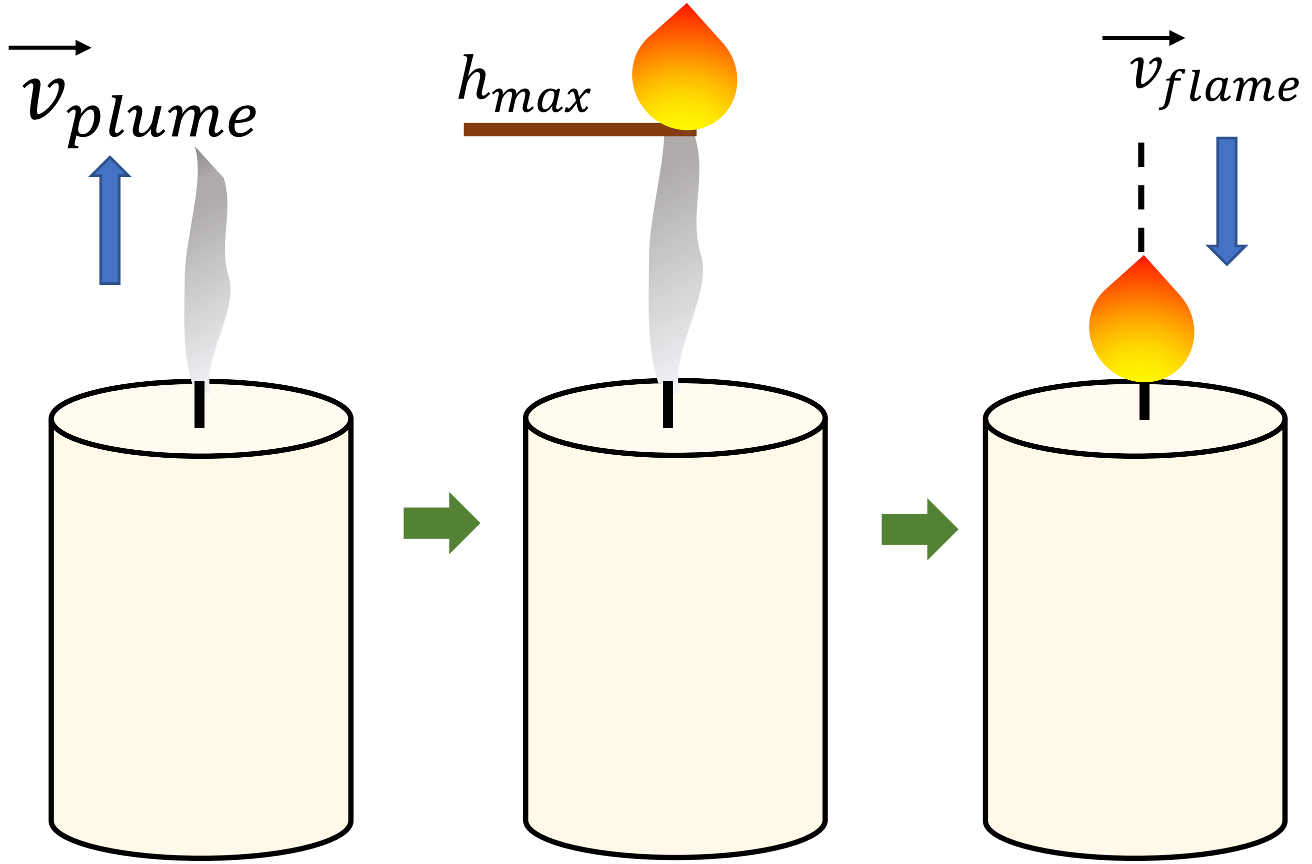


Estimation of maximal height of lighting for laminar mode

$$\frac{h_{max}}{v_{plume}} + \frac{h_{max}}{v_{plume}} \leq \tau_{crit}$$

$$h_{max} = \frac{\tau_{cr}\, v_{plume}\, v_{flame}}{v_{plume} + v_{flame}}$$

$$h_{max} = 26{,}5\ cm$$



Conclusions

Diameter of wick: 4.1 mm
Diameter of pipe: 18 mm
$z_{max} = 38\ cm$

1. The main parameters that limit the height of lighting are temperature of wick, length of laminar zone and
2. Maximal height of lighting default candle without pipe is 8 cm
3. Maximal height of lighting default candle with pipe is 26cm
4. Maximal reached height for the thickest wick is 38 cm

Thank you for your attention!