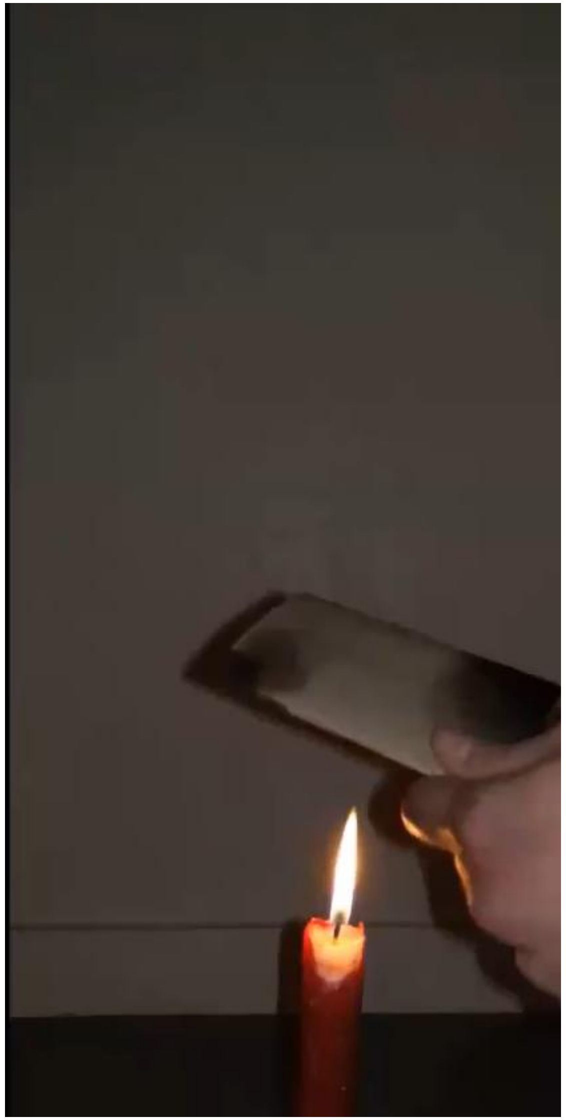
PROBLEM N°8

CANDLE LIGHTING TRICK

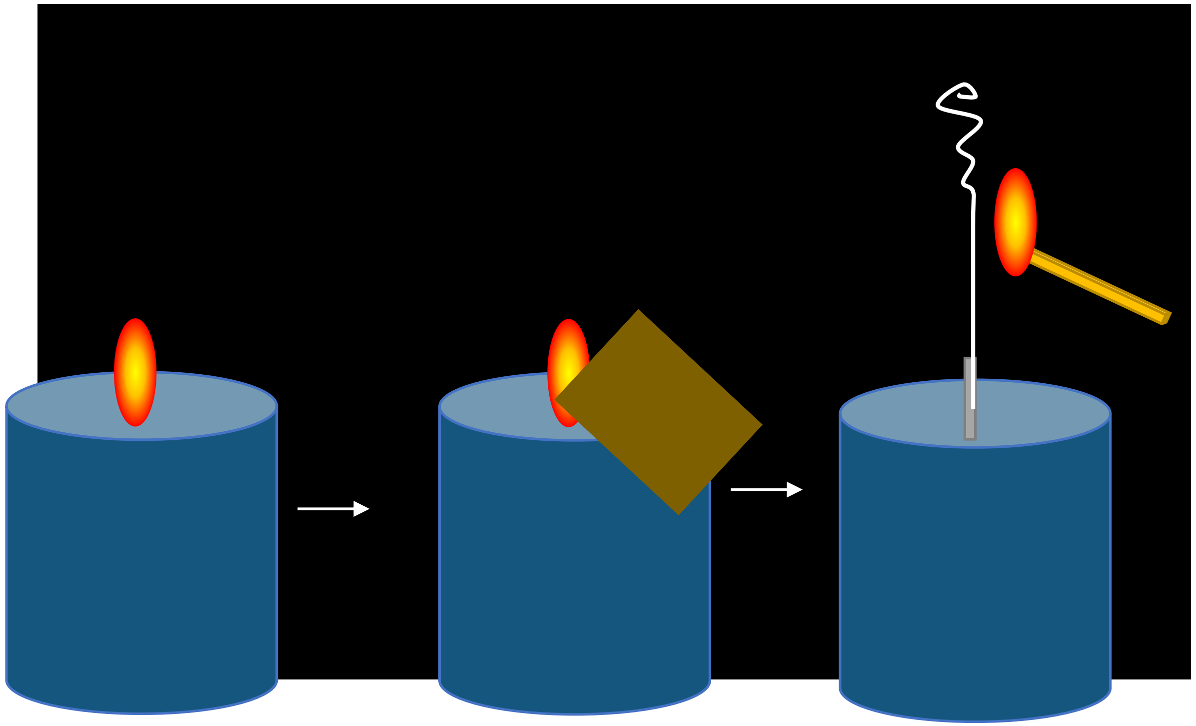
Team Ecole polytechnique

Problem

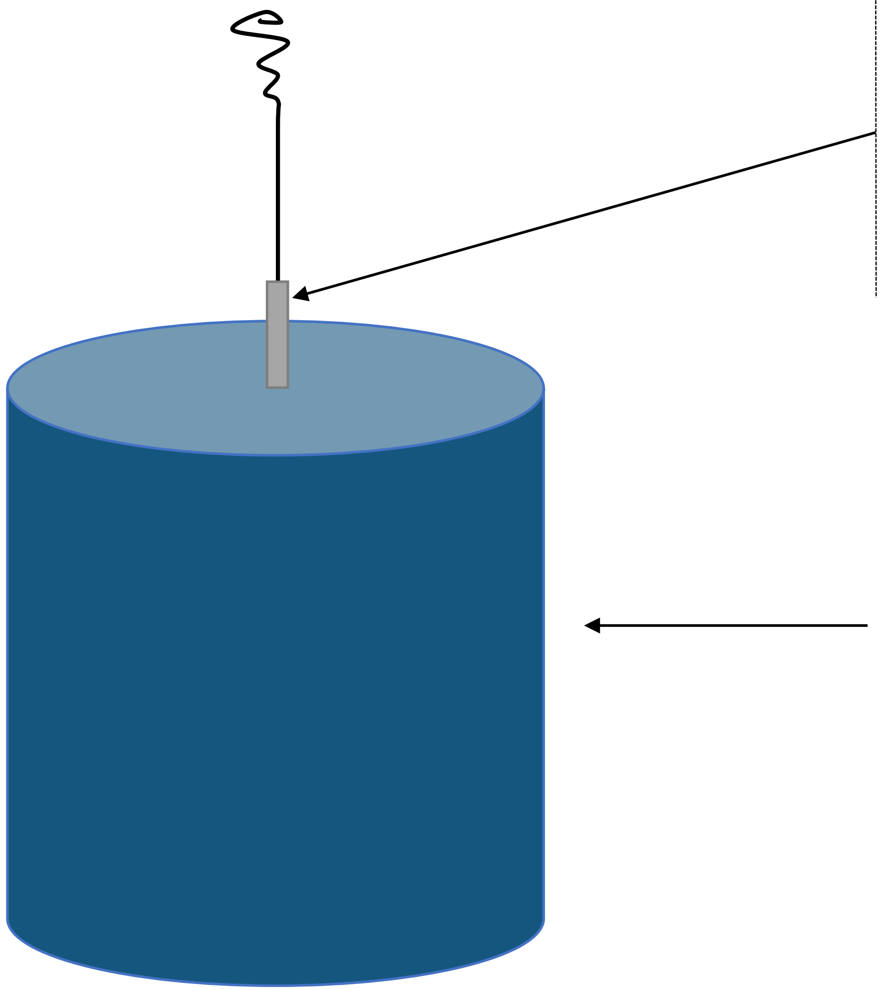
It is possible to relight a candle that has just been blown out by lighting the smoke that is created in the process (see video). Indeed, the smoke contains vaporized wax which is the substance that burns in the flame in the first place. What is the **maximum distance** (between the match and the candle) from which one can relight the candle? **Identify the important parameters** and find **how they influence** this maximum distance.



Experimental setup



Important parameters



1 Length of the wick

2 Thickness of the wick

3 Type of wick

4 Type of wax

Characteristic of wax

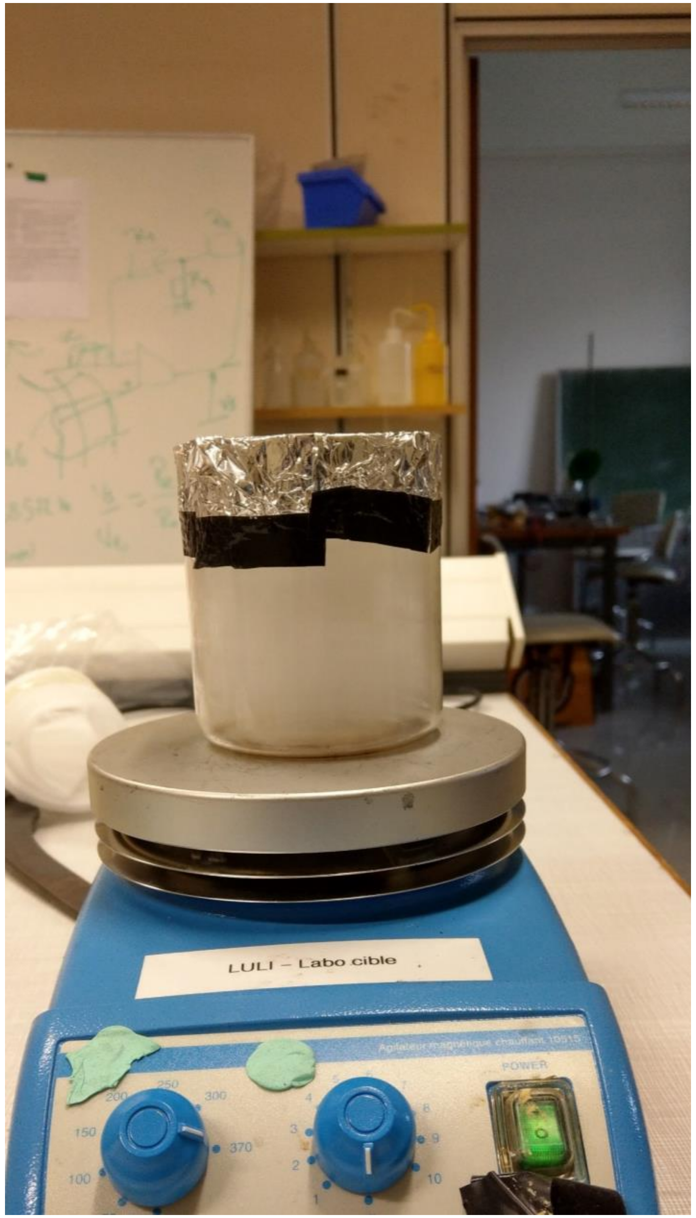
Characteristics	Paraffin	Beeswax
Composition	Consists of mainly alkanes(C_{18} to C_{40})	Consists of esters(60%) and alkanes, alkanoic acids, alkenes
Density/gcm^{-3}	0.88 – 0.92	0.958 – 0.970
Boilling point/°C	380.0-390.0	369.0-371.0
Latent heat of vaporisation/Jg^{-1}	$\sim$172	~ 242

Why can we light up the smoke?

There must be sufficient vapour concentration to light up the smoke

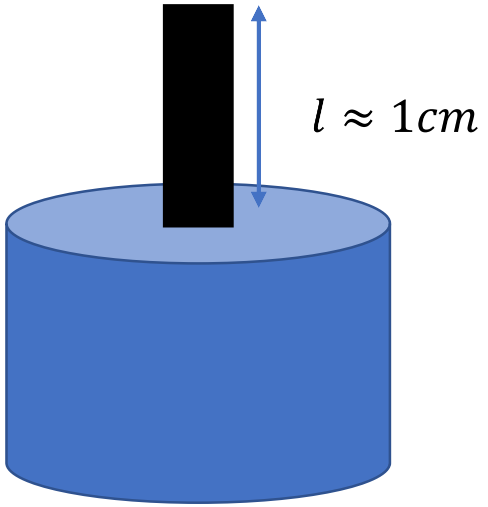
Minimum concentration

Type of wax	Minimum concentration to relight/g/m^3
Paraffin	120
Beeswax	150



Thermal plume

$$Initial\ concentration = \frac{\sigma T^4 \times \pi r^2}{latent\ heat\ of\ vaporisation\ \times \pi r^2 l} \approx 1kg/m^3$$



Behavior of the smoke

Paraffin wax, thin wick

Similar behaviour for all different kinds of candles used

Instability

Instability on the boundary between air and smoke

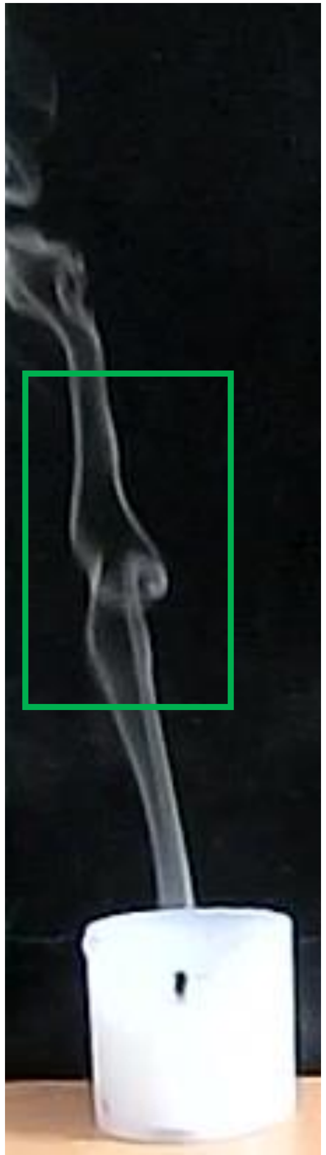
Squire instability

$$\omega_{i,max} \approx 0{,}2\frac{\Delta U}{\delta}$$

Maximum distance $\approx \frac{2\pi\delta}{0{,}2}$

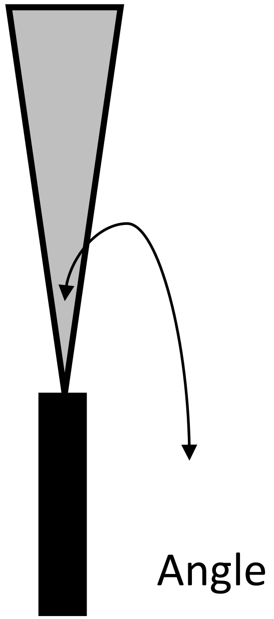
$$\delta = 10^{-3} \sim 10^{-2}\ m$$

$$\boxed{3cm < z_{max} < 30\ cm}$$



Source: François Charru. *Instabilités hydrodynamiques*

Cone shape and angle



$Angle \sim 12 - 20°$

Theoretical model:Boussinesq plume

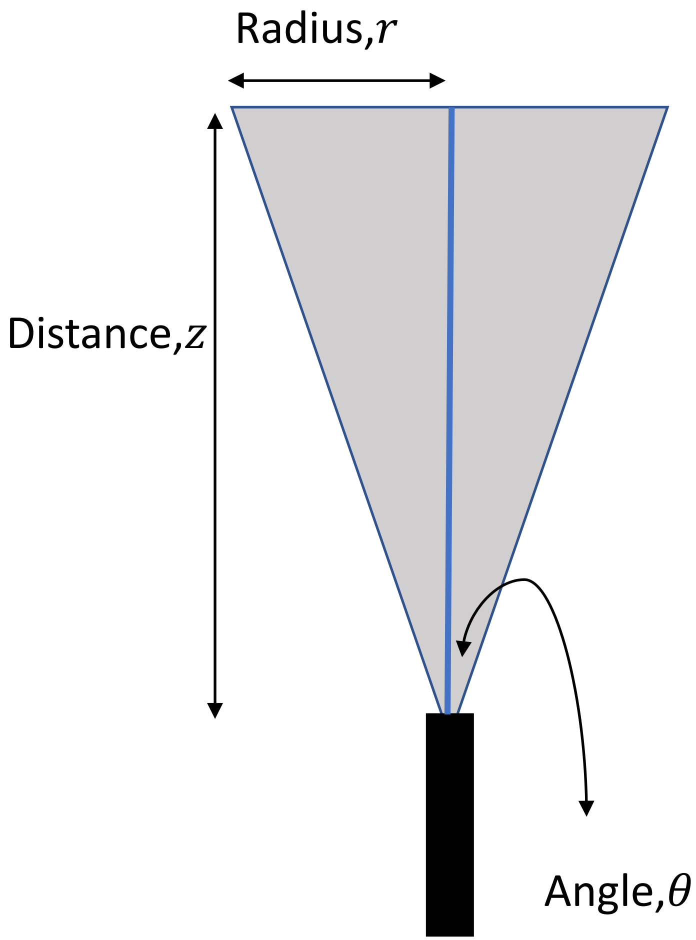
Assumption: No diffusion of wax vapour

Conservation of mass of wax vapour:

$$dz\pi r_i^2 \rho_i = dz\pi r_f^2 \rho_f$$

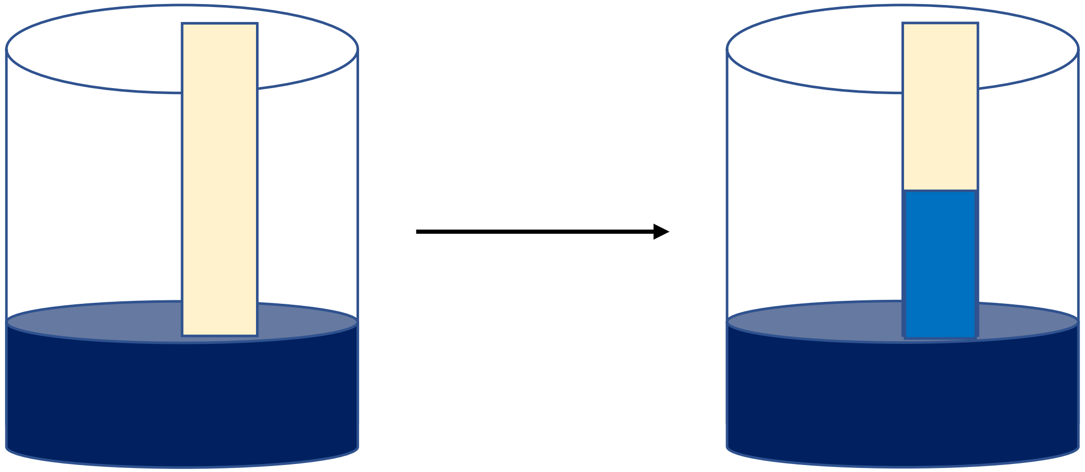
$$\rho_f = \rho_i \frac{r_i^2}{r_f^2} \approx 100 g/m^3$$

What is the initial concentration?

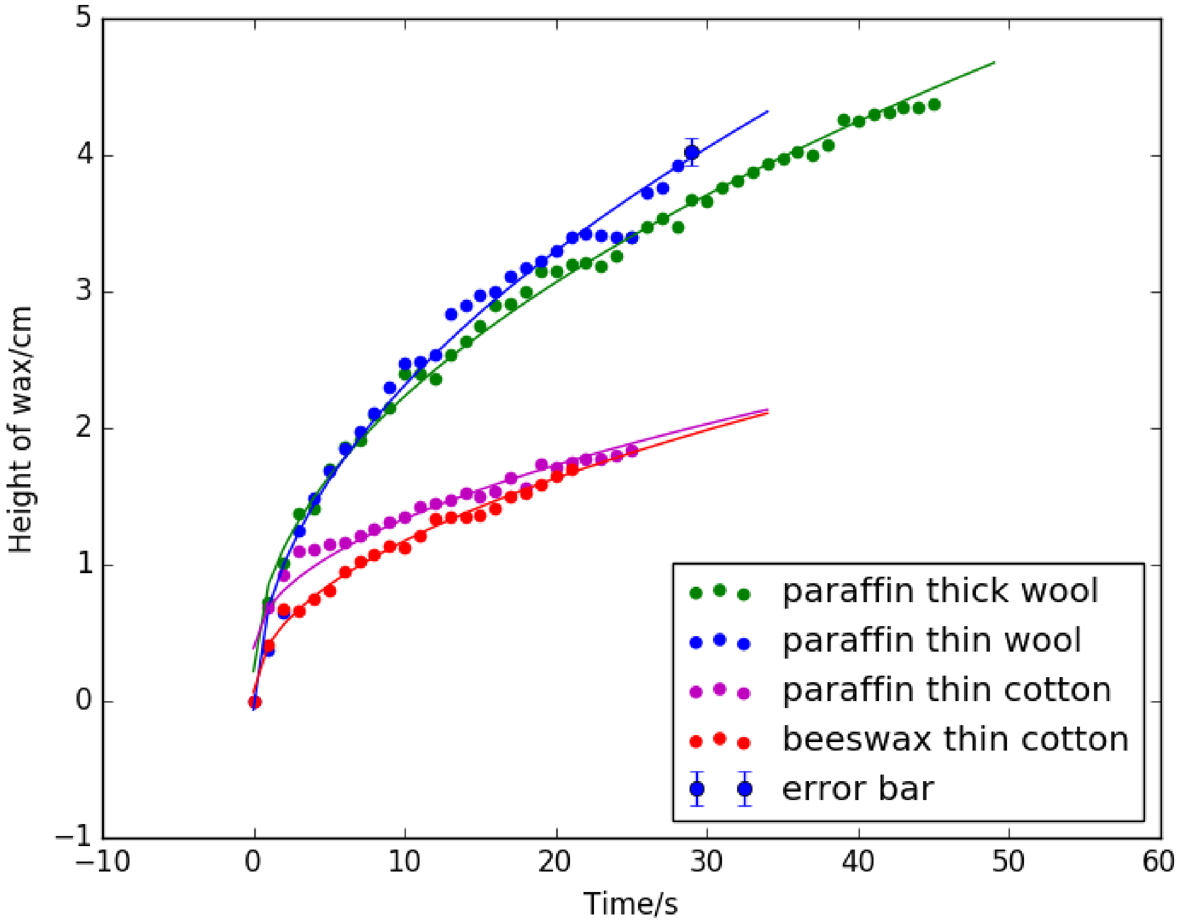


Maximum capillary length

Type of wax ,type of wick ,thickness of wick	Maximum capillary length/cm
Paraffin, cotton,0.3cm	2.1±.0.1
Paraffin,wool,0,3cm	4.0±.0.1
Paraffin, wool, 0.5cm	4.3±.0.1
Beeswax,cotton, 0,3cm	1.9±.0.1



Change of capillary height



Lucas-Washburn Law for porus medium :

$$Z = A\sqrt{t}$$

$$dV = \pi r^2 dZ$$

$$flow\ rate\ at\ a\ particular\ height = \frac{dV}{dt} \propto \frac{1}{z}$$

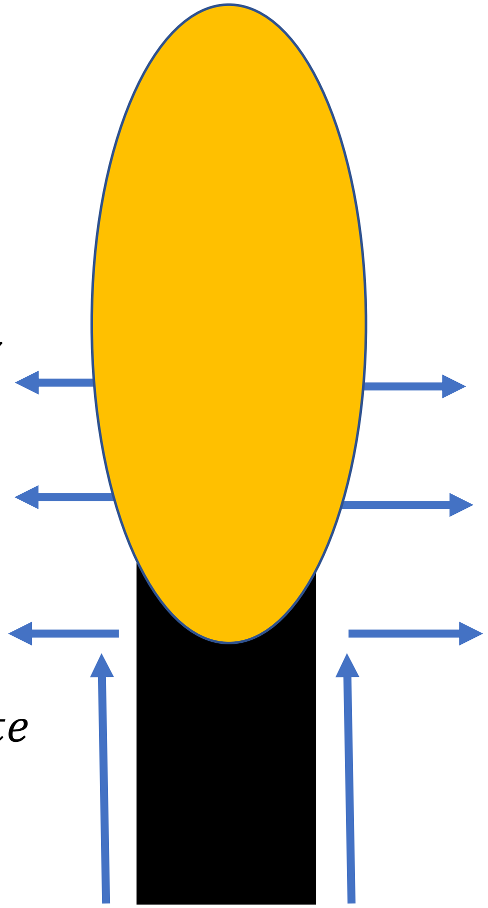
Burning rate

$$flow\ rate\ at\ a\ particuar\ height = \frac{dV}{dt} = \frac{\pi r^2 A^2}{2z}$$

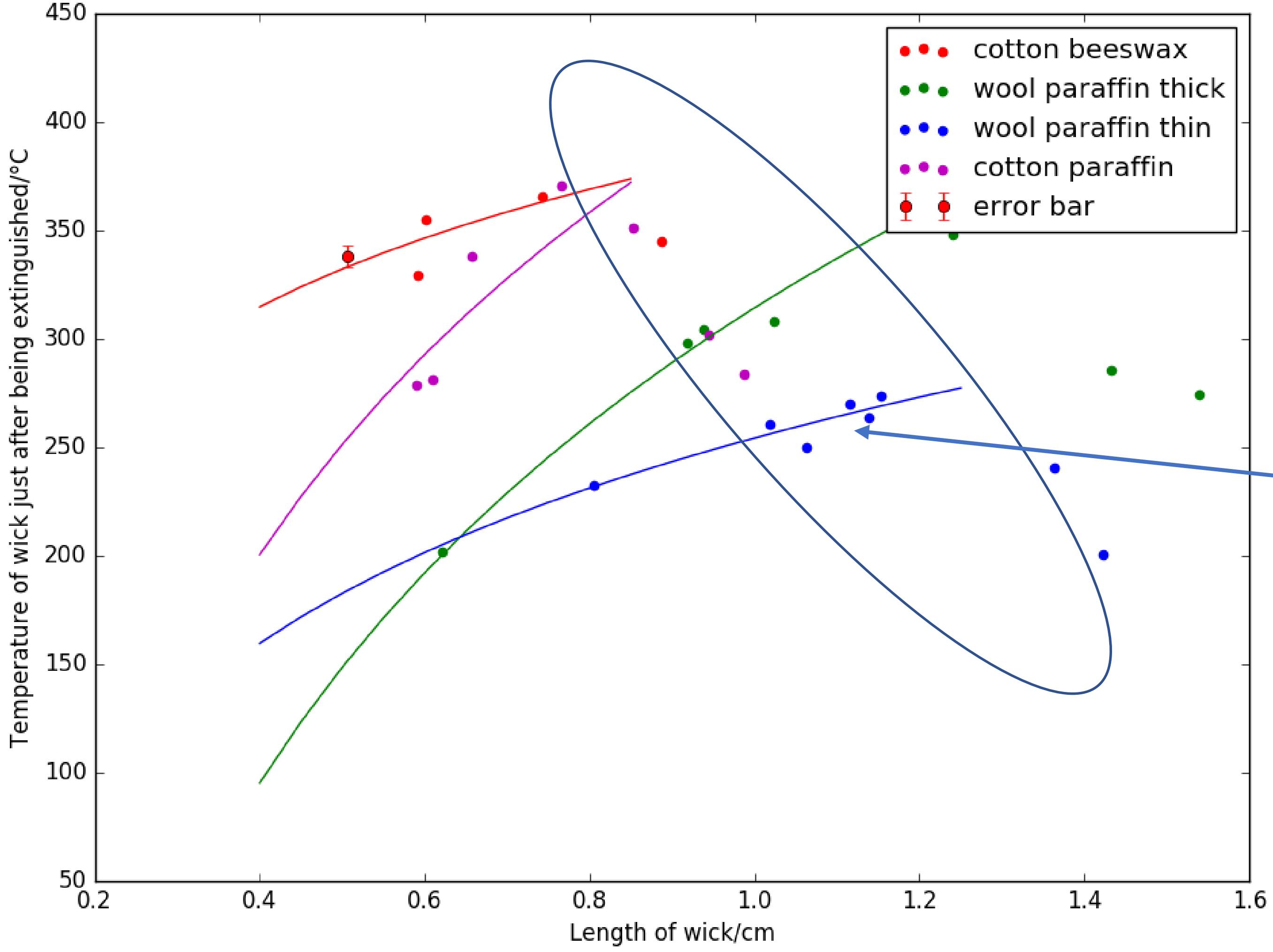
$$burning\ rate\ at\ height\ z = flow\ rate\ \times\ area = \frac{\pi r^2 A^2}{2z} \times 2\pi r dz$$

$$total\ burning\ rate = \int_a^l \frac{\pi r^2 A^2}{2z} \times 2\pi r\ dz = \pi^2 r^3 A^2 \ln\frac{l}{a}$$

$$Temperature\ and\ initial\ concentration \propto total\ burning\ rate$$



Temperature and length of wick



Wick is too long and heat loss from conduction plays a role

Maximum concentration

Conditions	Approximate length of wick for maximum concentration/cm
Cotton Beeswax	0.8±.0.1
Cotton Paraffin	0.85±.0.1
Wool Paraffin Thick	1.4±.0.1
Wool Paraffin Thin	1.3±.0.1

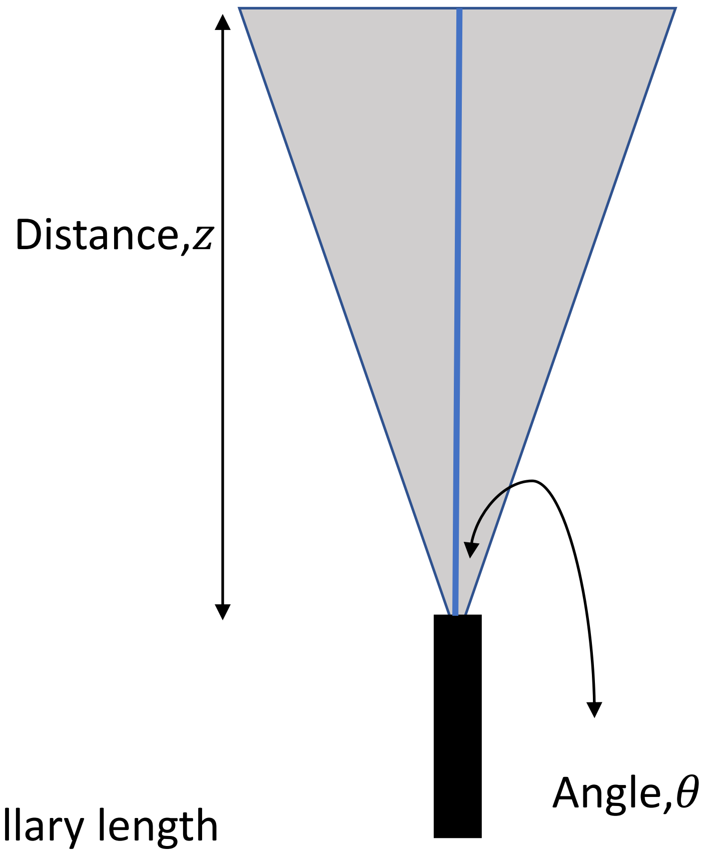
Approximate length $\approx 0.3 - 0.4$ of maximun capillary length

Theoretical model: Maximum height

$$dz\pi r_i^2 \rho_i = dz\pi r_f^2 \rho_f$$

$$\rho_i \propto \pi^2 r^3 A^2 \ln\frac{l}{a}$$

$$z_f^2 \propto \frac{r^5}{tan\theta^2 \rho_f}\pi^2 A^2 \ln\frac{l}{a}$$

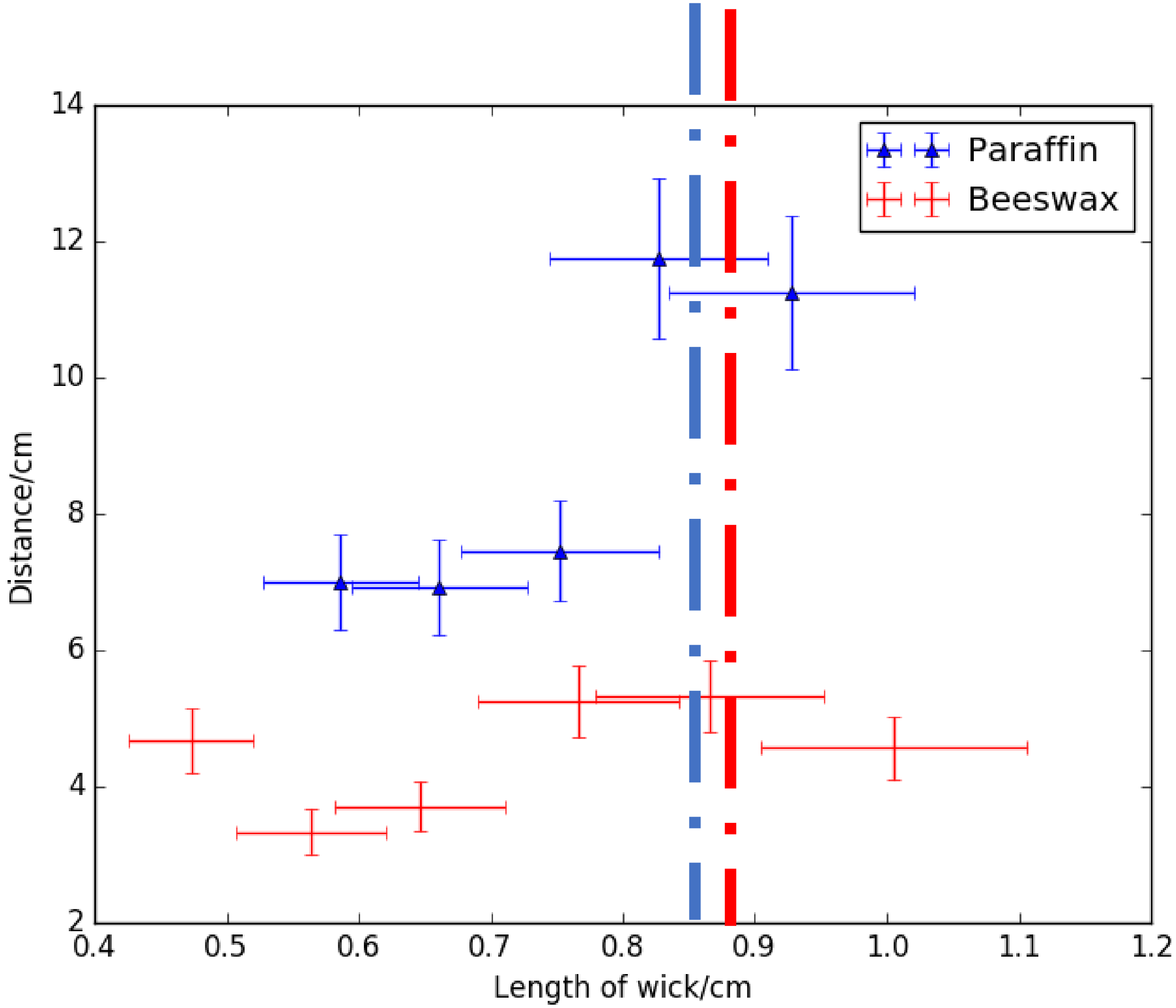


Maximum height when $l = 0.3 - 0.4$ of maximun capillary length

Type of wax

Prediction:
Paraffin will have a larger height because it has a steeper gradient(A)

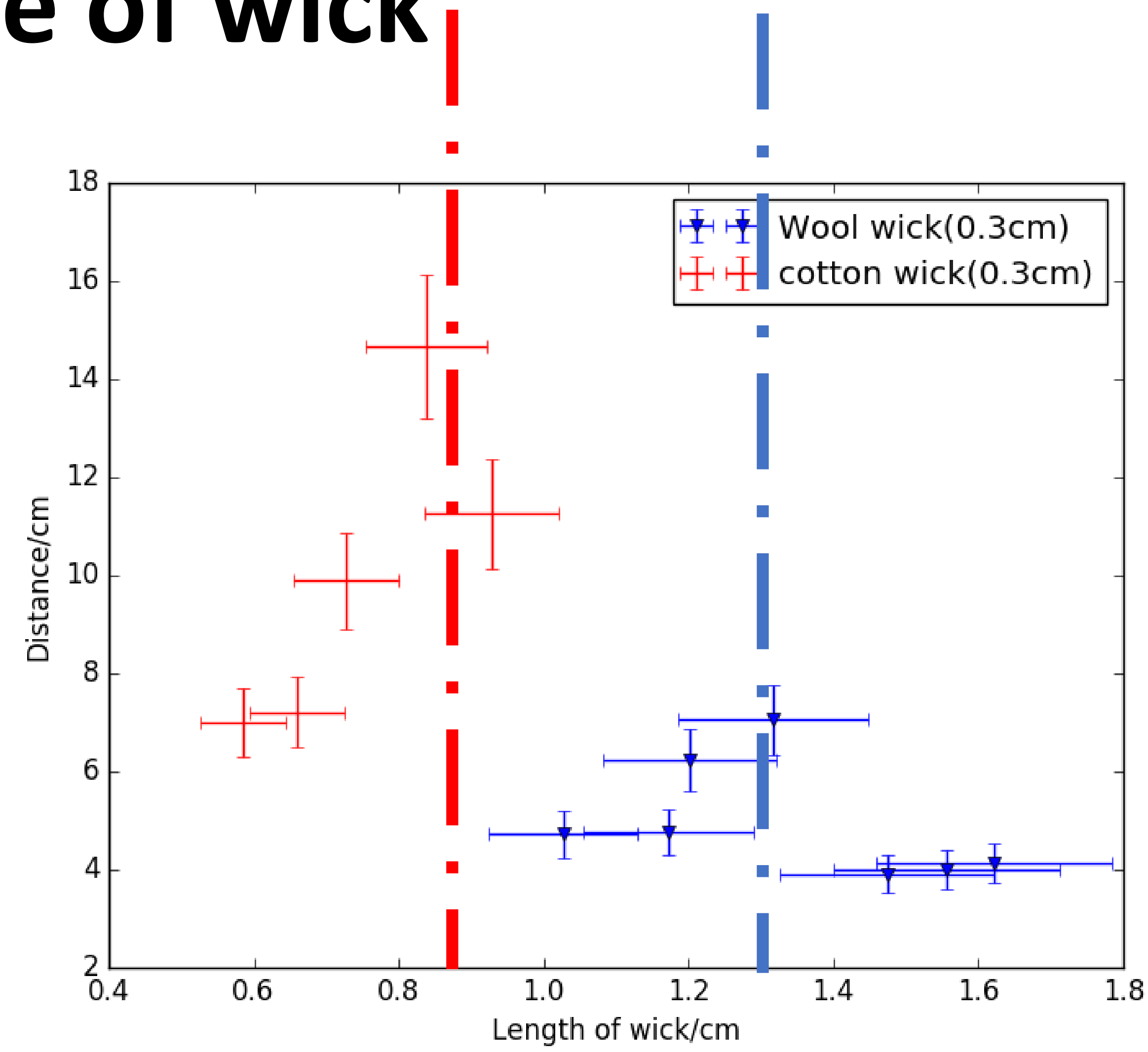
$$z_f^2 \propto \frac{r^5}{tan\theta^2 \rho_f} \pi^2 A^2 \ln \frac{l}{a}$$



Type of wick

Prediction:
Cotton will have a larger height because it has a steeper gradient(A)

$$z_f^2 \propto \frac{r^5}{tan\theta^2 \rho_f} \pi^2 A^2 \ln \frac{l}{a}$$

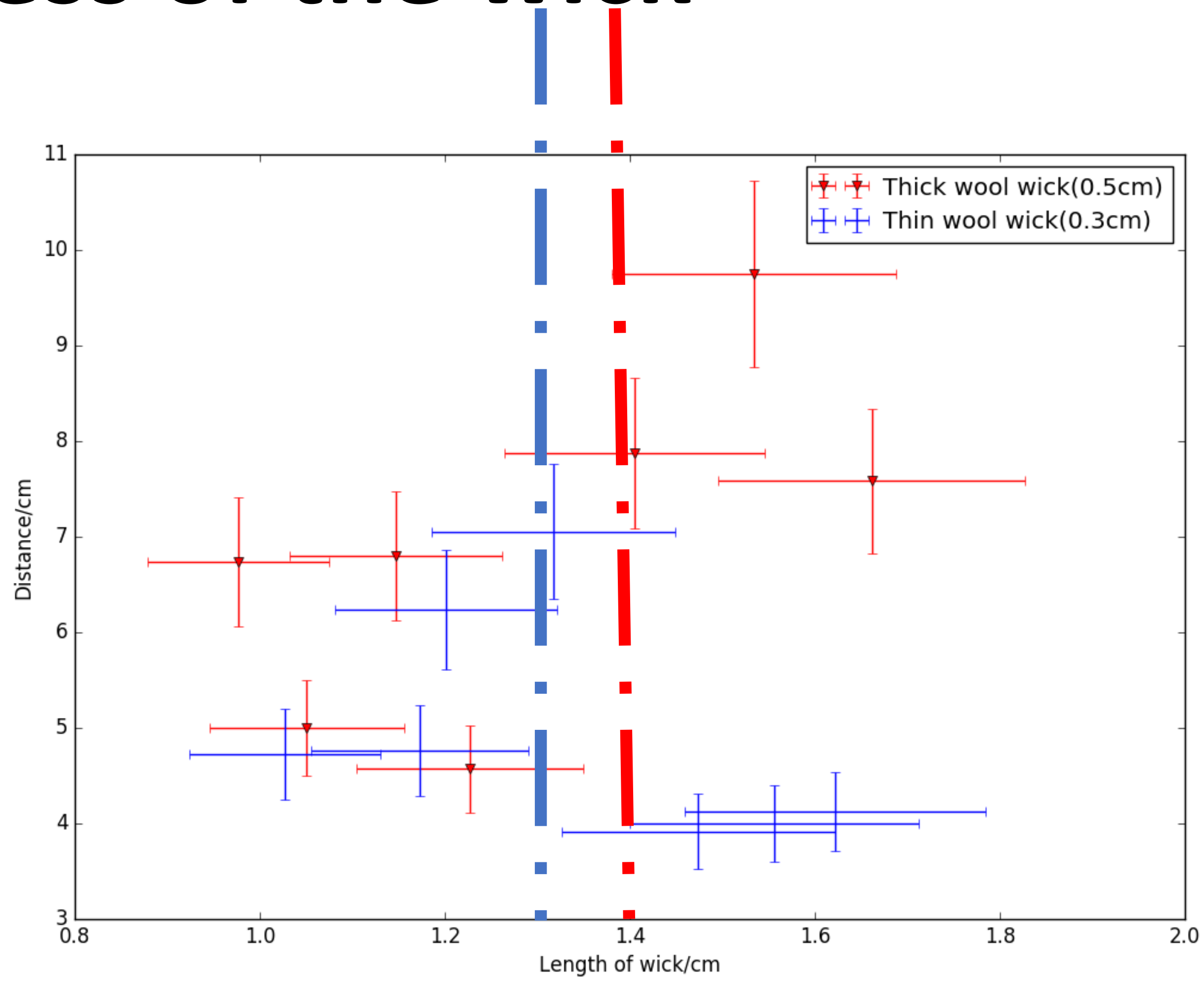


Data from paraffin wax

Thickness of the wick

Prediction:
Wick with a larger radius will have a larger height

$$z_f^2 \propto \frac{r^5}{tan\theta^2 \rho_f} \pi^2 A^2 \ln\frac{l}{a}$$

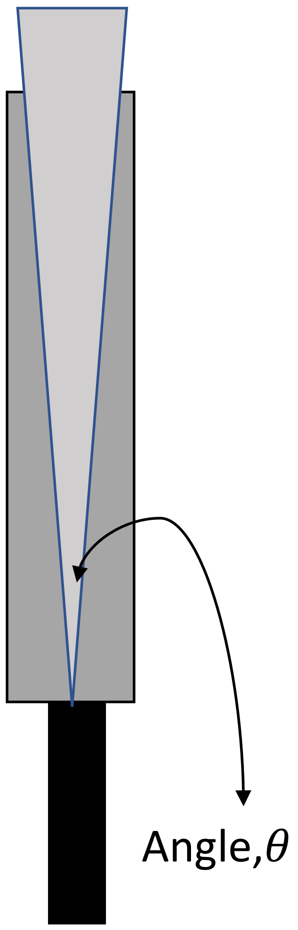


Data from paraffin wax

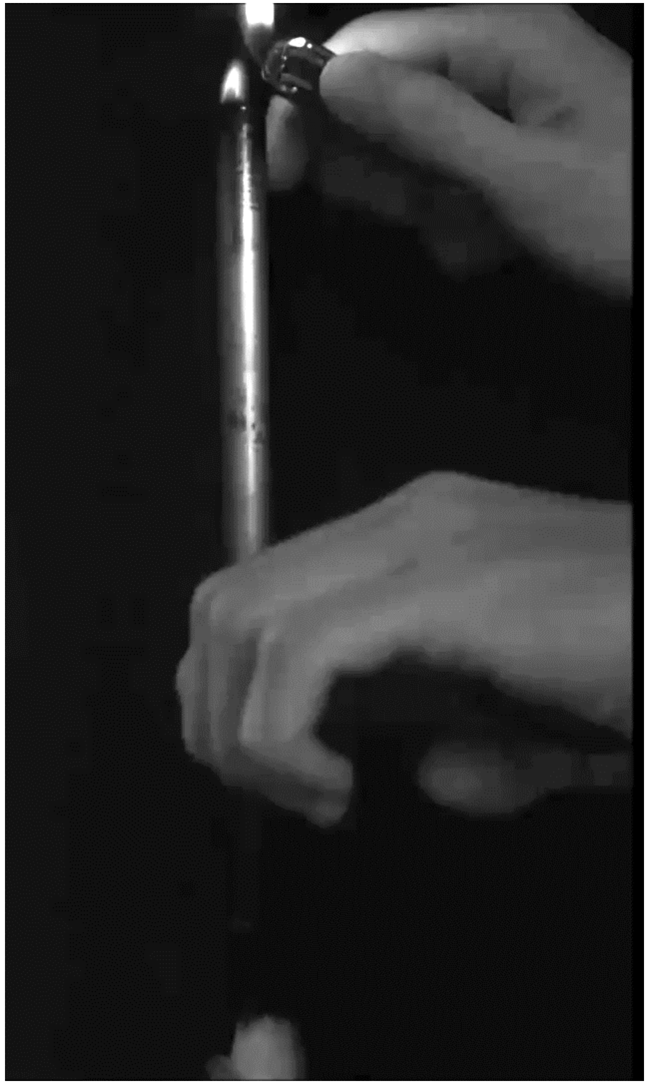
Can we do better?

$$z_f^2 \propto \frac{r^5}{\tan\theta^2 \rho_f} \pi^2 A^2 \ln\frac{l}{a}$$

Use the tube to minimise the angle



With a tube



Maximum length is 75cm

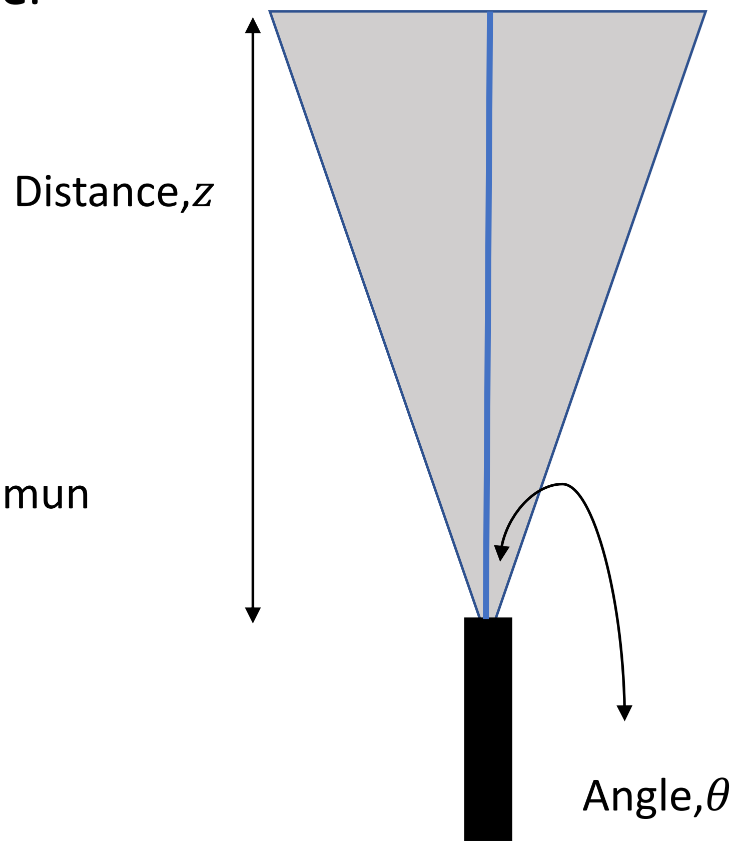
Conclusion

What is the **maximum distance** (between the match and the candle) from which one can relight the candle? Identify the **important parameters** and find how they **influence this maximum distance**.

Maximum height:75cm

$$z_f^2 \propto \frac{r^5}{tan\theta^2 \rho_f} \pi^2 A^2 \ln \frac{l}{a}$$

Maximum height for a particular candle is when $l \approx 0.3 - 0.4$ of maximun capillary length of the wick



Bibliographie

Sinaringati S, Putra N, Amin M, et al“THE UTILIZATION OF PARAFFIN AND BEESWAX AS HEAT ENERGY STORAGE IN INFANT INCUBATOR.” *ARPN Journal of Engineering and Applied Sciences*, Jan. 2016, pp. 800–804.

Squire instability

$$\omega_{i,max} \approx 0,2\frac{\Delta U}{\delta}$$

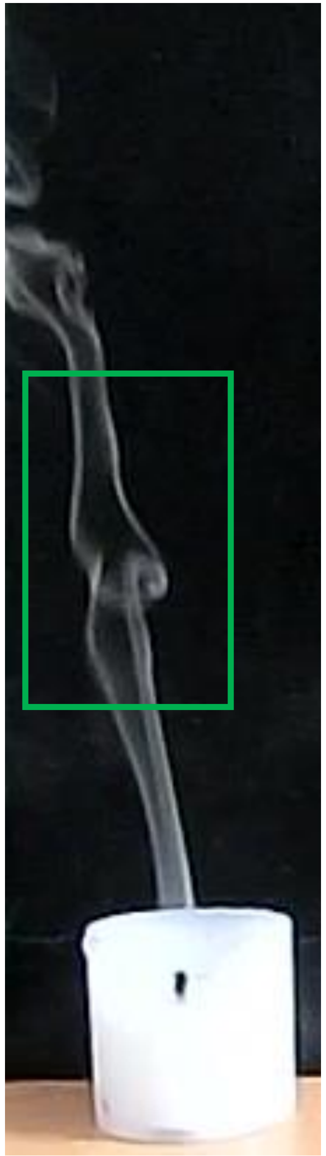
$$T_{min} \approx \frac{2\pi}{0,2}\frac{\delta}{\Delta U}$$

$\delta \sim 10^{-3}\ m$

$\Delta U \sim 0,1$ à $0,3\ m.s^{-1}$

$$T_{min} \sim 0,1 \text{ à } 0,3\ s$$

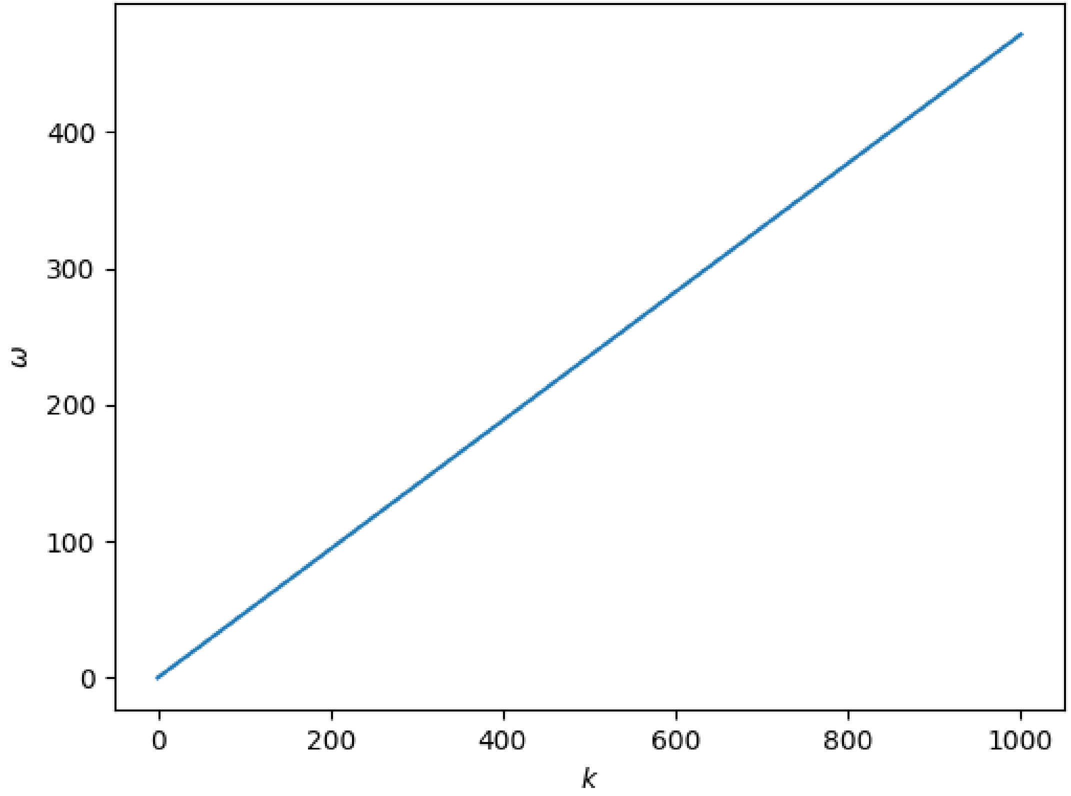
$$\boxed{z_{max} < 11,2\ cm}$$



Source: François Charru. *Instabilités hydrodynamiques*

Kelvin-Helmoltz instability

$$\omega = k\sqrt{\frac{\rho_1\rho_2(U_1 - U_2)^2}{(\rho_1+\rho_2)^2} + \frac{\rho_1-\rho_2}{\rho_1+\rho_2}.\frac{g}{k} + \frac{\sigma}{\rho_1+\rho_2}k}$$



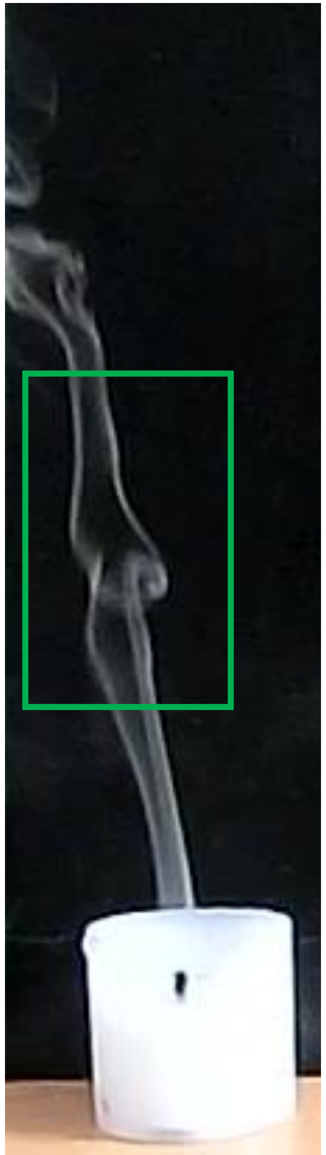
$$\lambda < l$$

$$k > \frac{2\pi}{l}$$

$$\omega > C\frac{2\pi}{l}$$

$$T < \frac{l}{C}$$

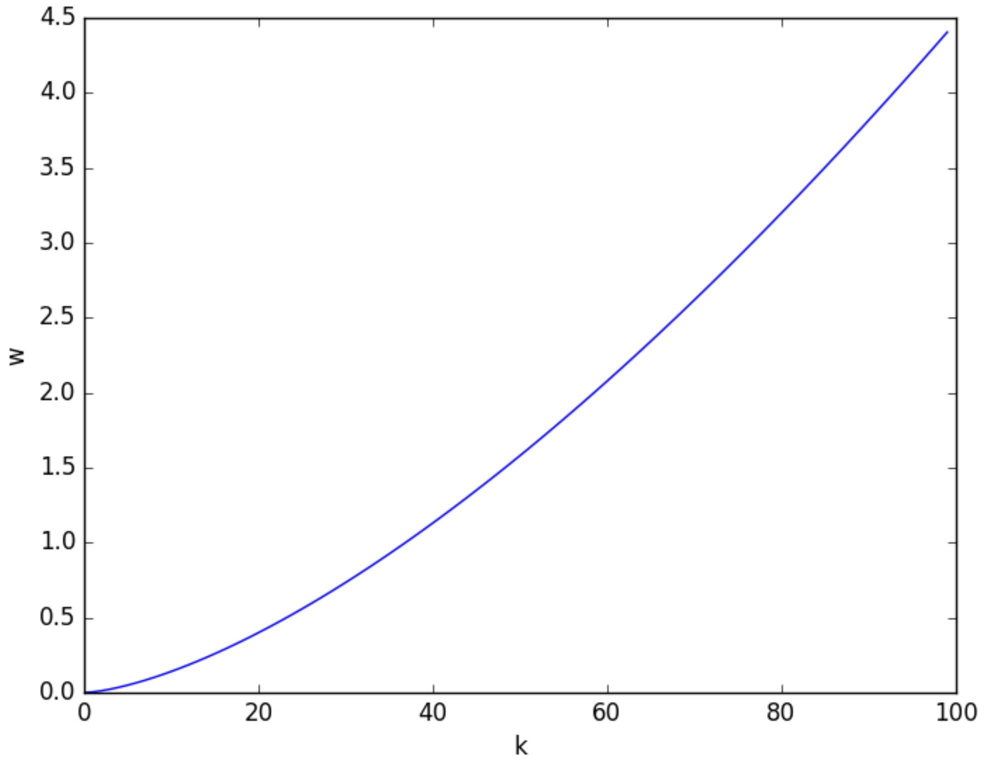
Instability happens almost instantly which is not what we observe



Classical squire instability

$$\omega = \frac{uk}{1+\frac{2\alpha}{kh}}\left(1 + \sqrt{\frac{2\sigma}{\rho u^2 h}\left(1+\frac{2\alpha}{kh}\right) - \frac{2\alpha}{kh}}\right)$$

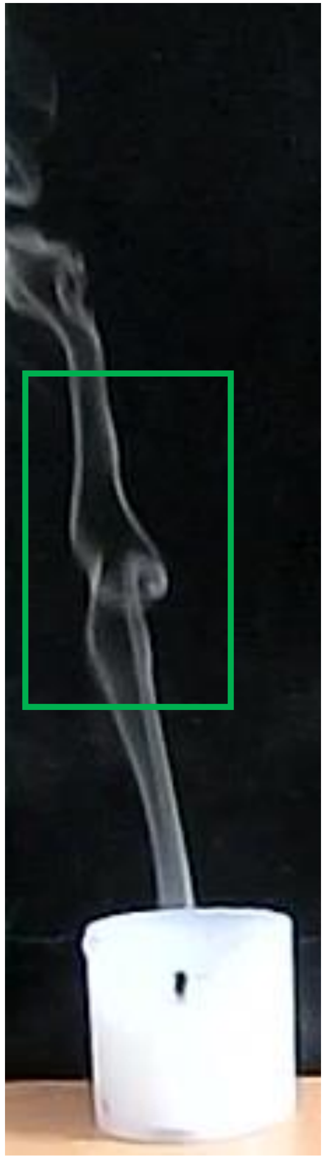
$$\omega = \frac{uk}{1+\frac{2\alpha}{kh}}\sqrt{\frac{2\alpha}{kh}}$$



$$\frac{2\alpha}{kh} > 10$$

$$\omega \approx \frac{uk\sqrt{kh}}{\sqrt{2\alpha}}$$

Instability happens almost instantly which is not what we observe

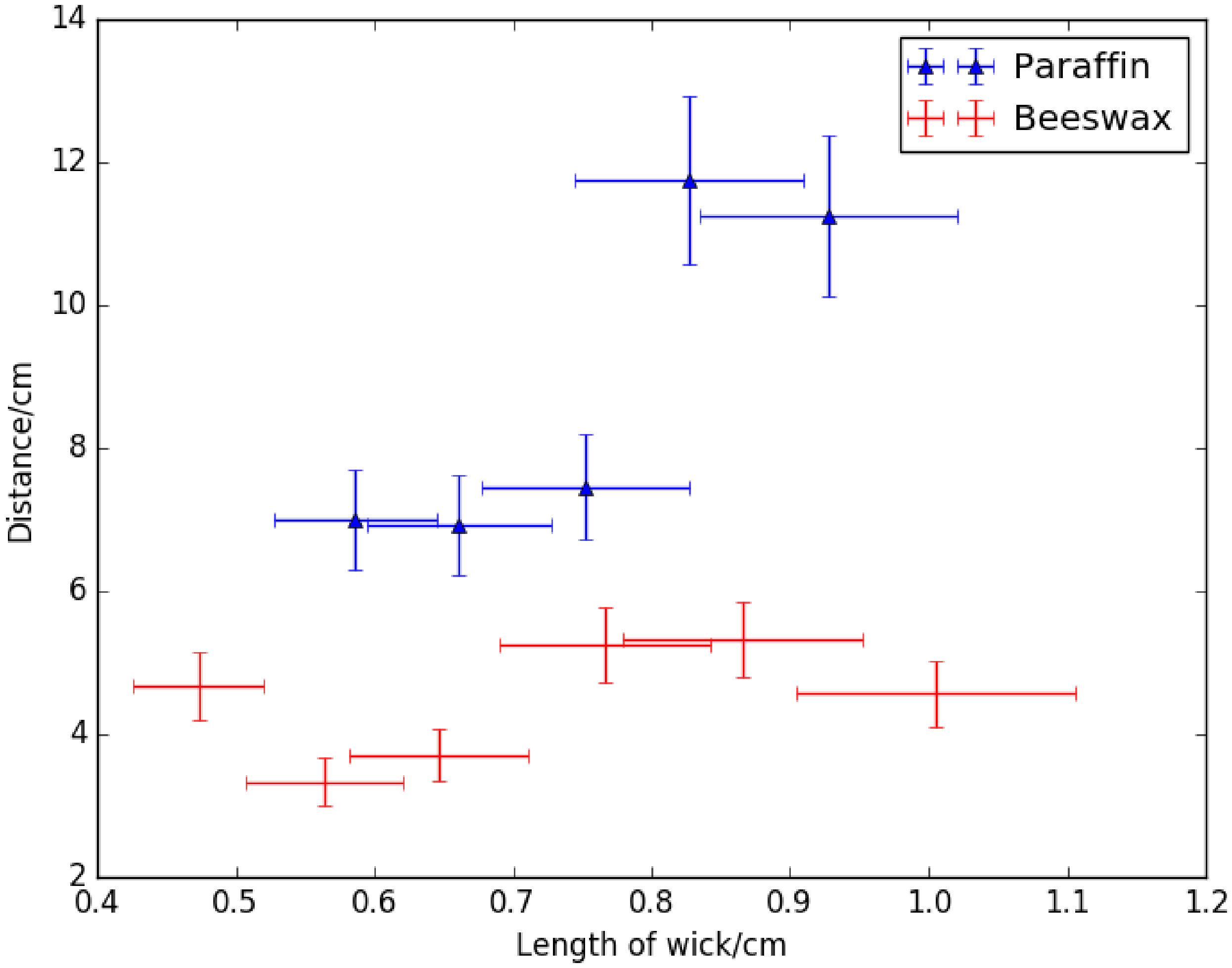


Type of wax

Prediction:
Paraffin will have a larger height because it has a steeper gradient(A)

$$z_f^2 \propto \frac{r^5}{tan\theta^2 \rho_f} \pi^2 A^2 \ln\frac{l}{a}$$

Type of wax	Number of measurements
Beeswax	12
Paraffin	28

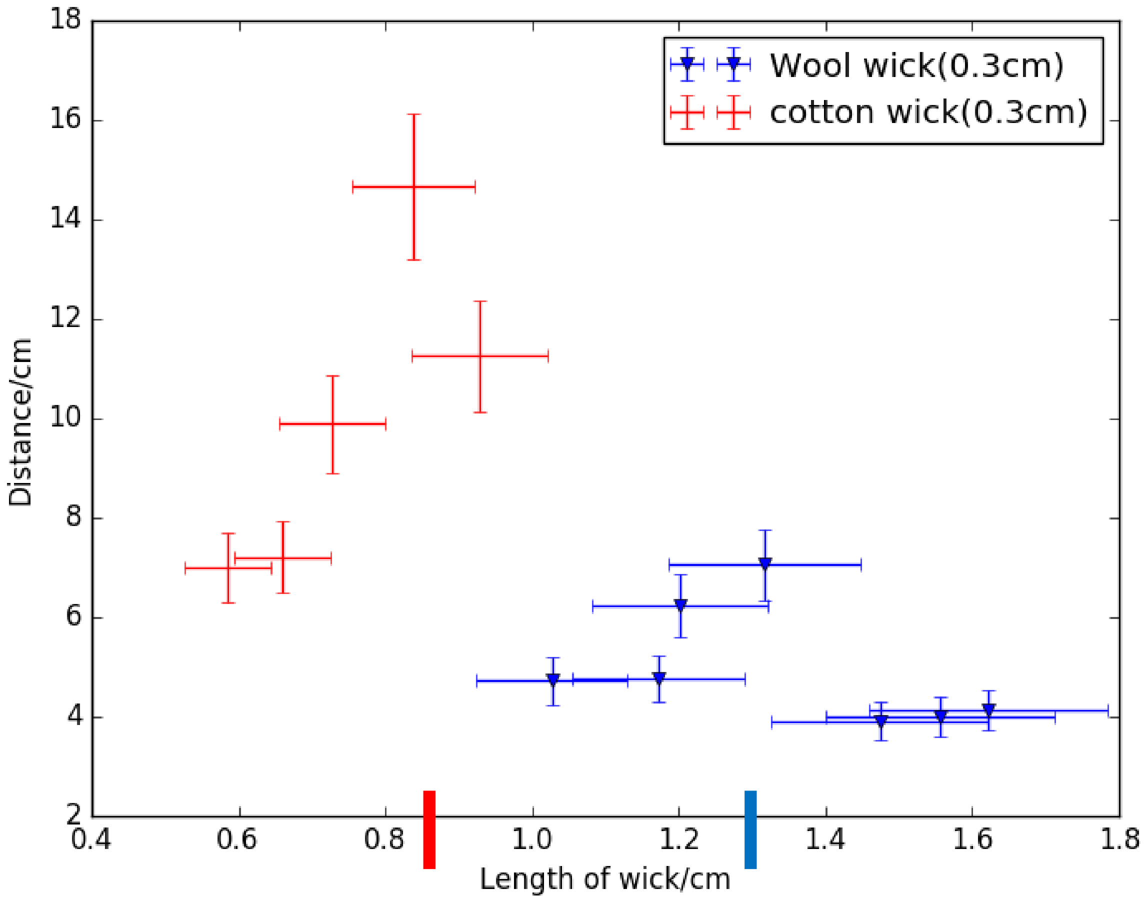


Type of wick

Prediction:
Cotton will have a larger height because it has a steeper gradient(A)

$$z_f^2 \propto \frac{r^5}{tan\theta^2 \rho_f} \pi^2 A^2 \ln \frac{l}{a}$$

Type of wick	Number of measurements
Wool	26
Cotton	28



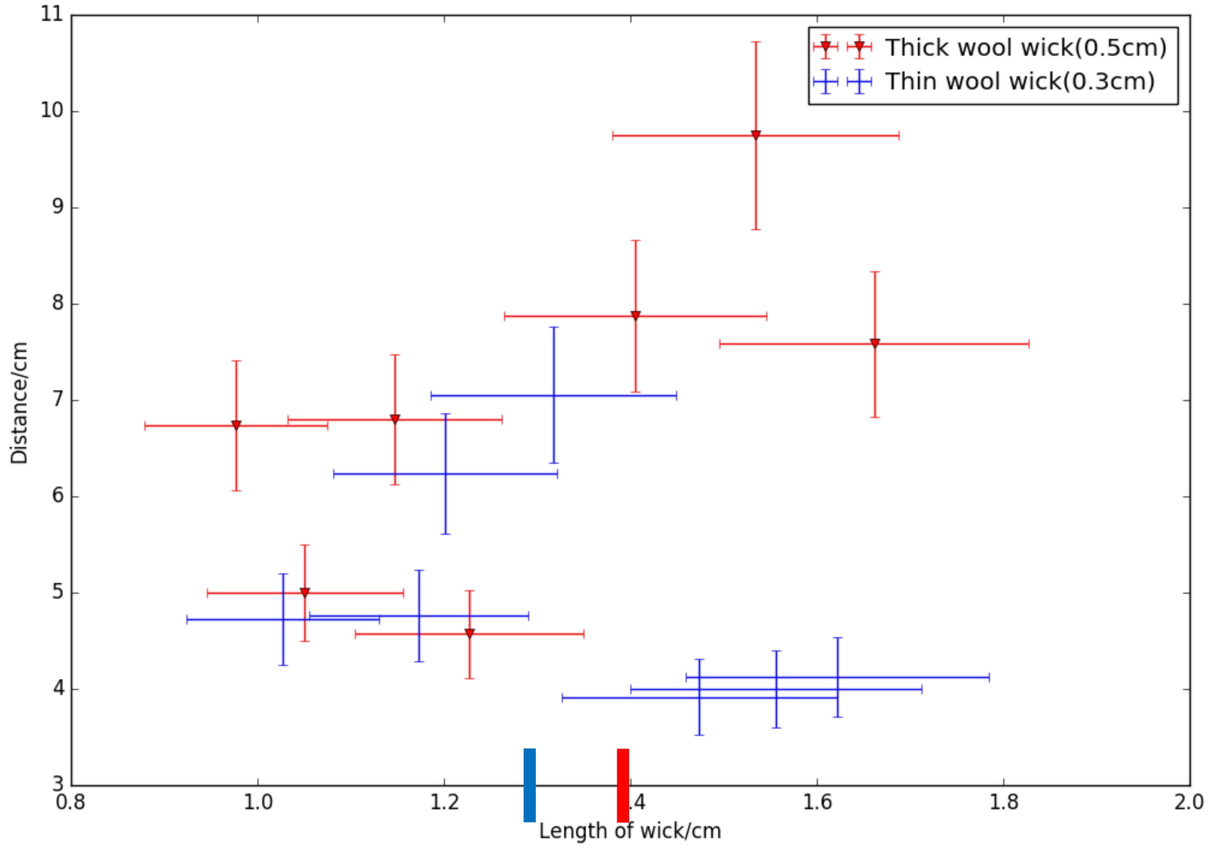
Data from paraffin wax

Thickness of the wick

Prediction:
Wick with a larger radius will have a larger height

$$z_f^2 \propto \frac{r^5}{tan\theta^2 \rho_f} \pi^2 A^2 \ln\frac{l}{a}$$

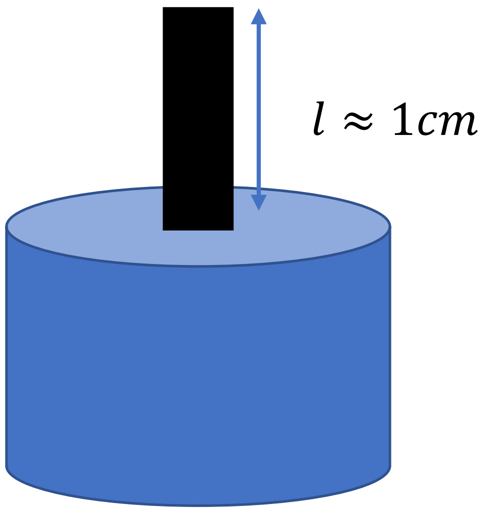
Thickness of wick	Number of measurements
Thick	20
Thin	26



Data from paraffin wax

Thermal plume

$$Initial\ concentration = \frac{\sigma T^4 \times \pi r^2}{latent\ heat\ of\ vaporisation\ \times \pi r^2 l} \approx 1kg/m^3$$



Constant velocity of 10-40cm/s

$Re = \frac{uz}{v} \approx 10^4 \sim 10^5$ ➔ not turbulent

Approximate length $\approx 0.3 - 0.4$ of maximun capillary length