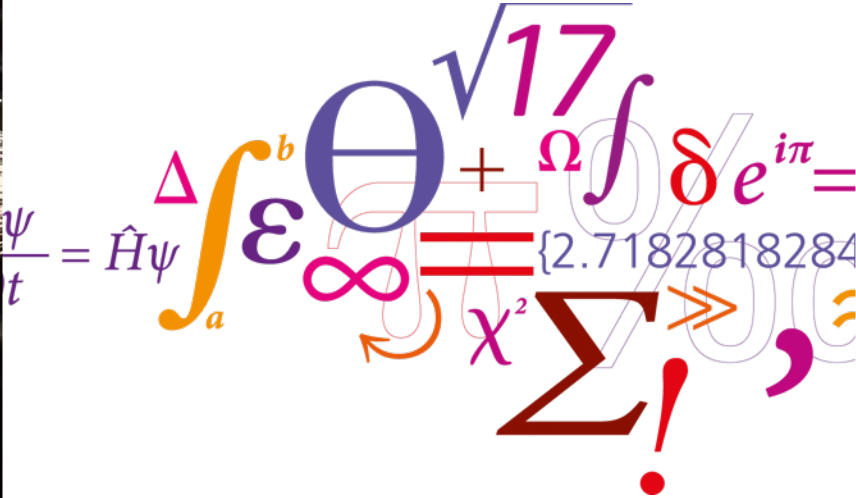
Half-life sparkles

”Sparks caused by an angle grinder tend to fly over a certain distance and then to split into several smaller sparks. What causes them to split? What is the condition for a split to occur? What influences the distance before the split? What will be the distance distribution of the sparks to fly?”



Frederik Laust Durhuus, Danish Team

DTU Physics
Department of Physics

Spark Emission

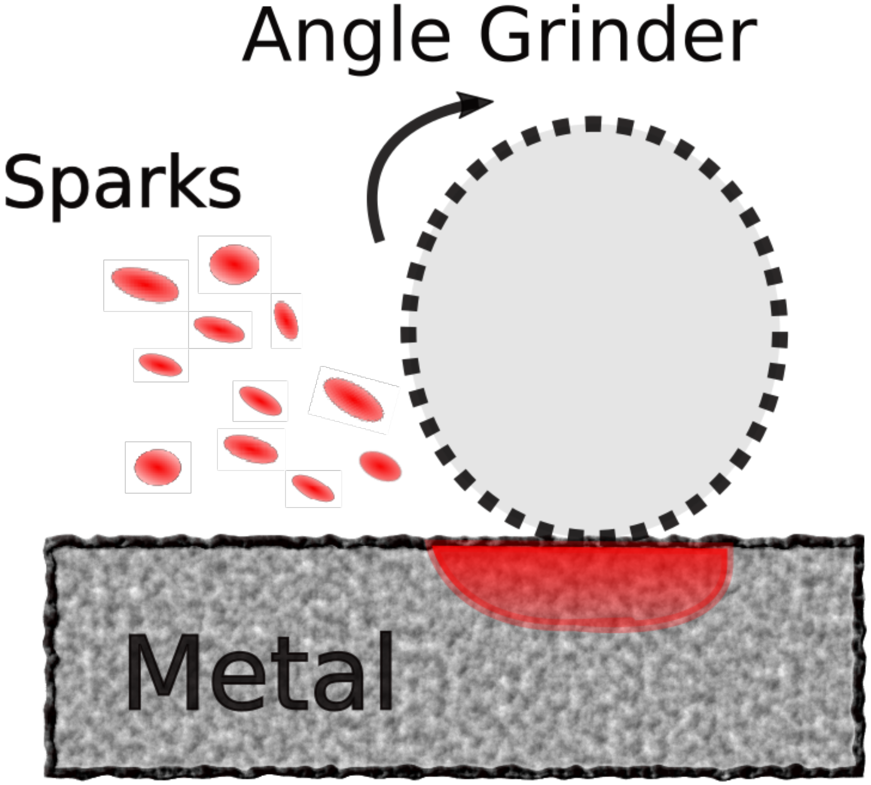
$$v(0) : P_v(v)$$

$$r(0) : P_r(r)$$

- Local friction heating
- Velocity and size distributions
- Power transferred

$$P = F\omega R$$

- Microstructure of wheel and metal
- Grain size



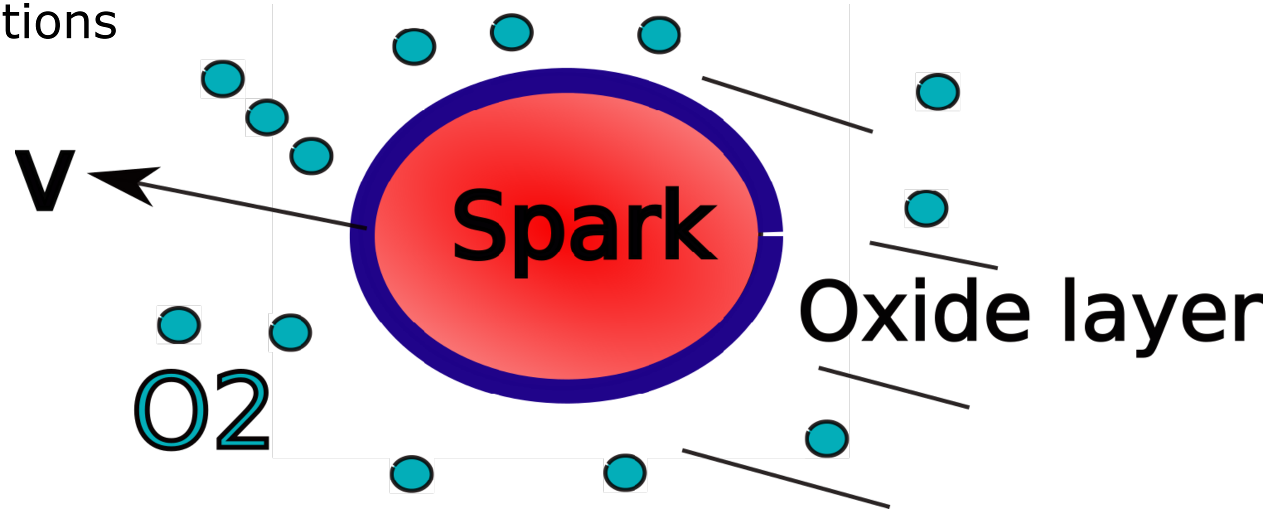
Flying spark

Speed dominated by air resistance: $m\dot{v} = -bv$

Oxide thickness[1]: $h_{ox}(t) = \sqrt{k_0 t e^{-Q/2RT}} = \sqrt{k_{ox} t}$

- Oxidative heating and surface cooling
- Exothermic reactions
- Auto-ignition

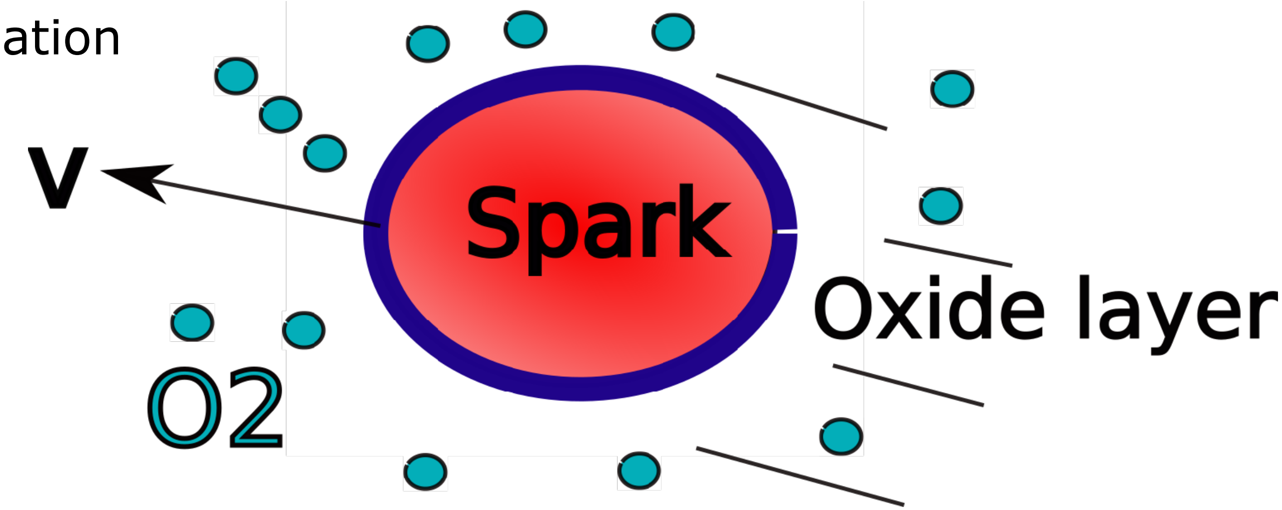
h_{ox} : Oxide thickness[m]
k : Rate constant[m^2/t]
b : Damping parameter[kg/s]
Q : Activation energy[J/mol]



[1]: Chen, et. Al. "Review of the high-temperature oxidation of iron and carbon steels in air or oxygen."

Split explanations

- Solid state fracture:
  - Thermal expansion mismatch
  - Growth stress from oxidation
- Phase transitions:
  - Melting
  - Local sublimation
- Outwards pressure:
  - Explosive reactions



Spark temperature[2]

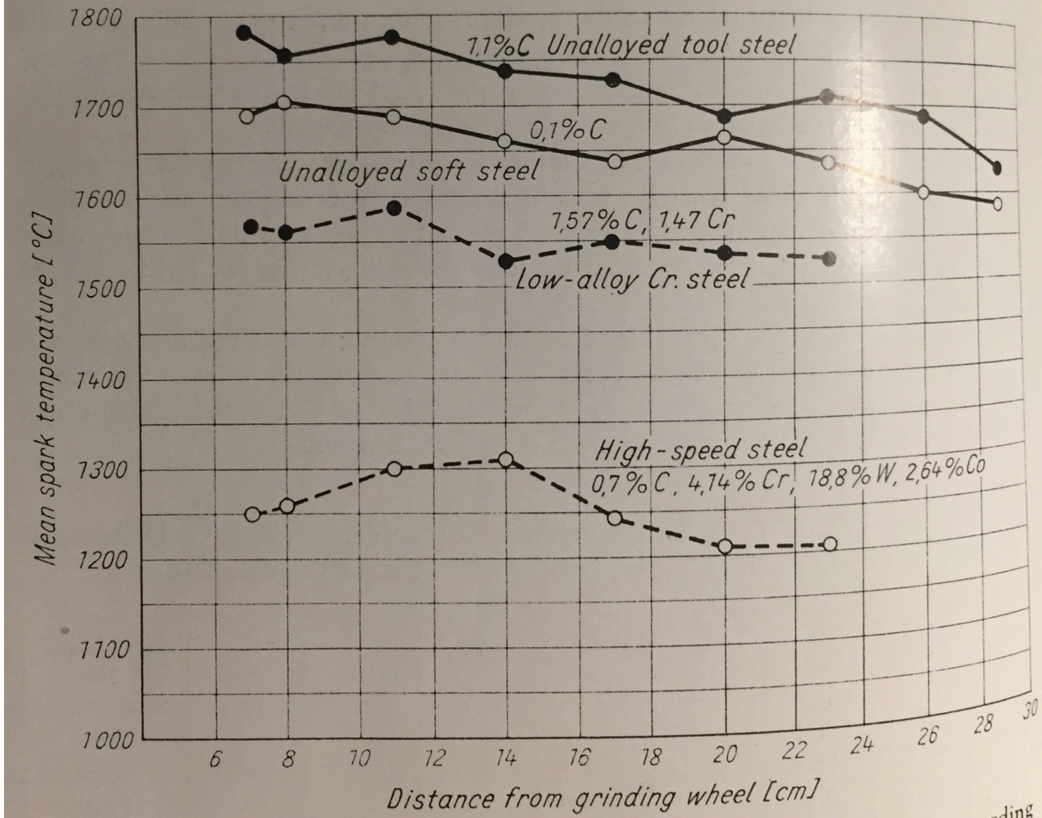


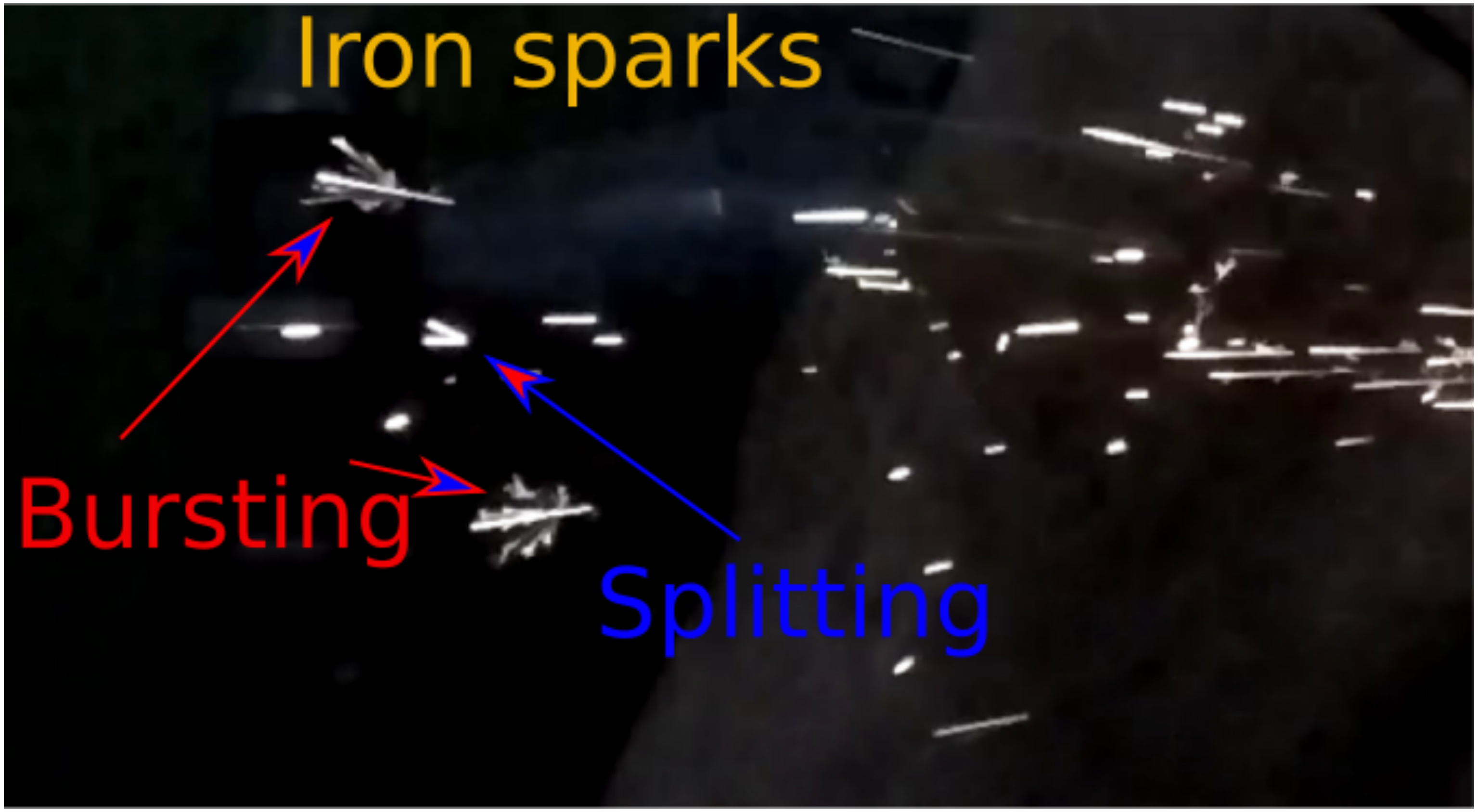
FIG. 1.1 Spark Temperatures Versus Distance from the Grinding Wheel (according to RIEZLER and HARDT)

- Mean temperatures
- Distance variation due to oxidation

[2]: Tschorn G., "Spark Atlas of Steels"

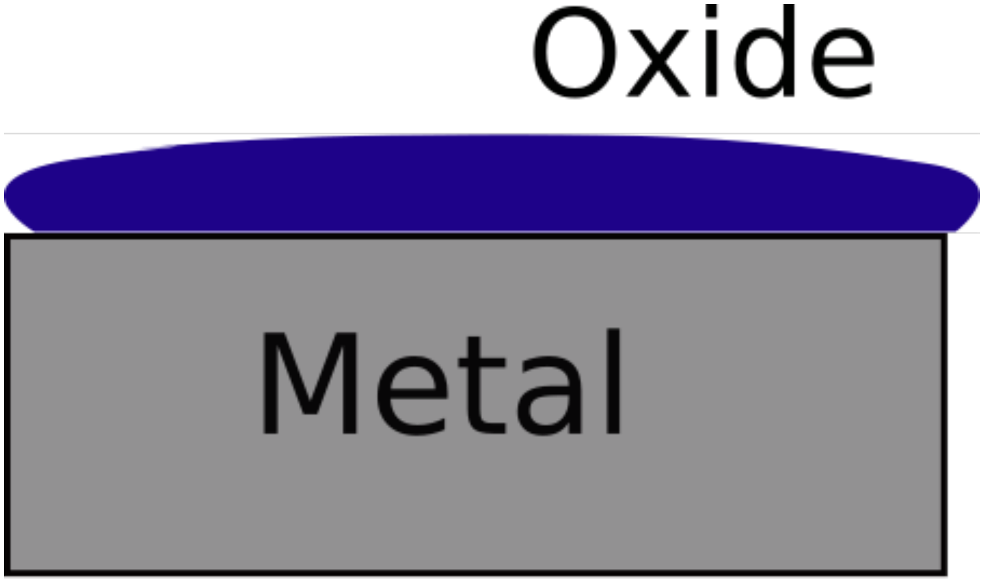
2 separate effects

- Bursting: Explosive reactions.
- Splitting: Solid fracture



Growth stress split 1

- Rapid surface oxidation



Growth stress split 2

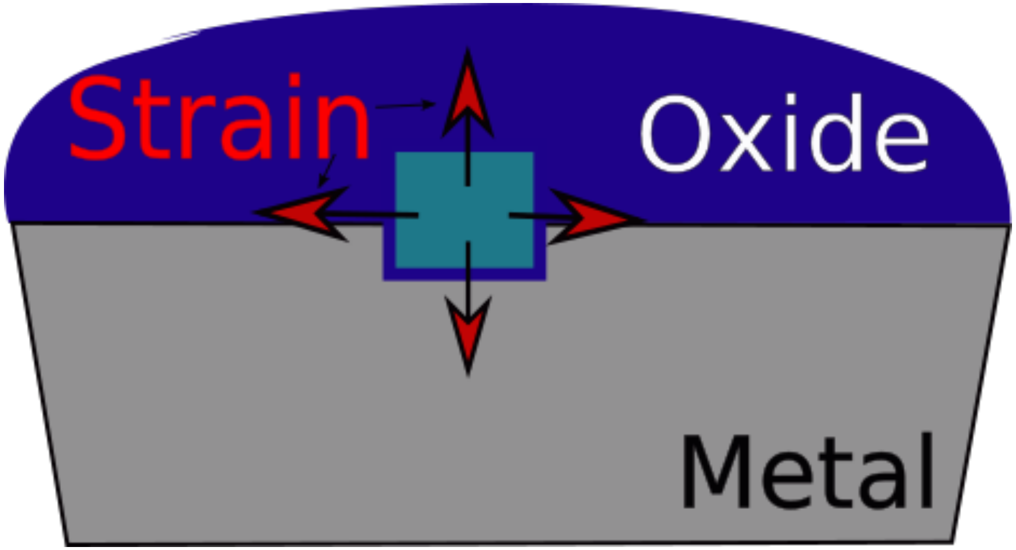
- Rapid surface oxidation

$$\dot{\epsilon}_{growth} \propto e^T \dot{h}_{ox}$$

- Growth strain

$$e^T = \sqrt[3]{\frac{V_{M,ox}}{V_{M,m}}} - 1$$

- Deformation of metal



Growth stress split 3

E : Young modulus[Pa]

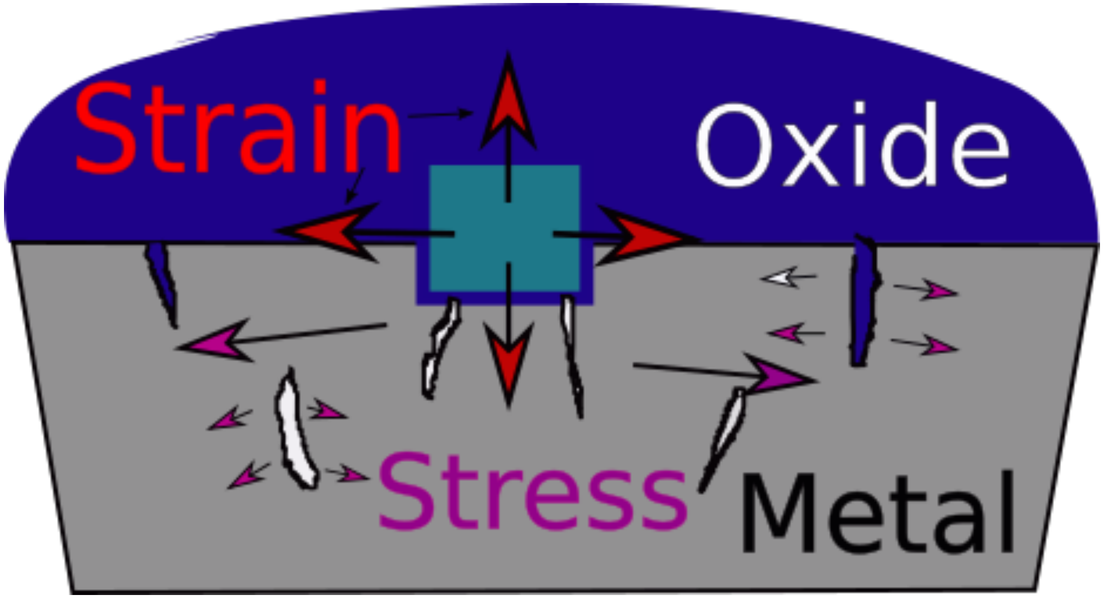
ν : Poisson ratio

r_s : Spark radius[m]

- Rapid surface oxidation
- Growth strain
- Deformation of metal
- Stress build up[2]
- Spreading cracks
- Brittle fracture

$$\dot{\epsilon}_{growth} \propto e^{T} \dot{h}_{ox}$$

$$\sigma_m \propto \frac{k_{ox}}{r_s} \frac{E_{ox}}{1-\nu_{ox}} t$$ [3]

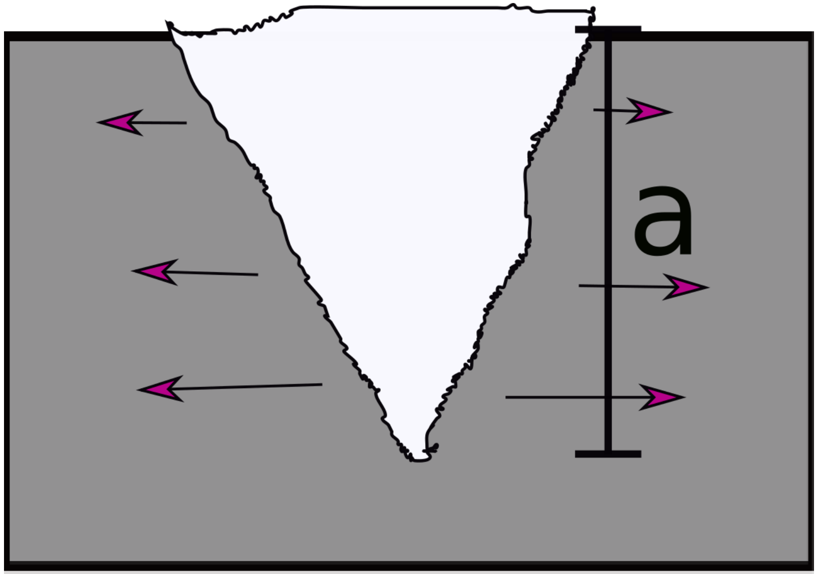


[3]: Panicaud et. Al. "On the growth strain origin and stress evolution prediction during oxidation of metals"

Fracture condition

Stress intensity factor[4]: $K_I = \alpha\sqrt{\pi a}\sigma$

Subcritical crack growth[5]: $\dot{a} = A\mathrm{e}^{-Q_0/k_b T} K^n$



Brittle Fracture[3]: $$a_{frac} = \frac{K_{Ic}^2}{\alpha^2\sigma^2\pi}$$

α : Geometric parameter

A : Constant

Q_0 : Activation energy[J]

[4]: D Roylance "Introduction to Fracture Mechanics"

[5]: Bazant et. Al. " Subcritical crack growth law and its consequences for lifetime statistics and size effect of quasibrittle structures"

Statistics of fracture

f : Single crack fracture probability
N_d : Defect number
P_F : Spark fracture probability
ρ : Defect density$[\mathrm{m}^{-3}]$
V : Spark volume$[\mathrm{m}^{3}]$

- Independently growing cracks

$$P_F(t) = 1 - \prod_{i=1}^{N_d} [1 - f(t)] \overset{[6]}{\Rightarrow} P_F(t) \approx 1 - \exp[-\rho V f(t)]$$

- Self-similarity: $f(t) \propto t^m$ $\quad m \in [1,3]$
- Stress effect[5]: $f(t,\sigma_0) \propto t^m \sigma_0^{nm}$ $\quad n \in [10,30]$

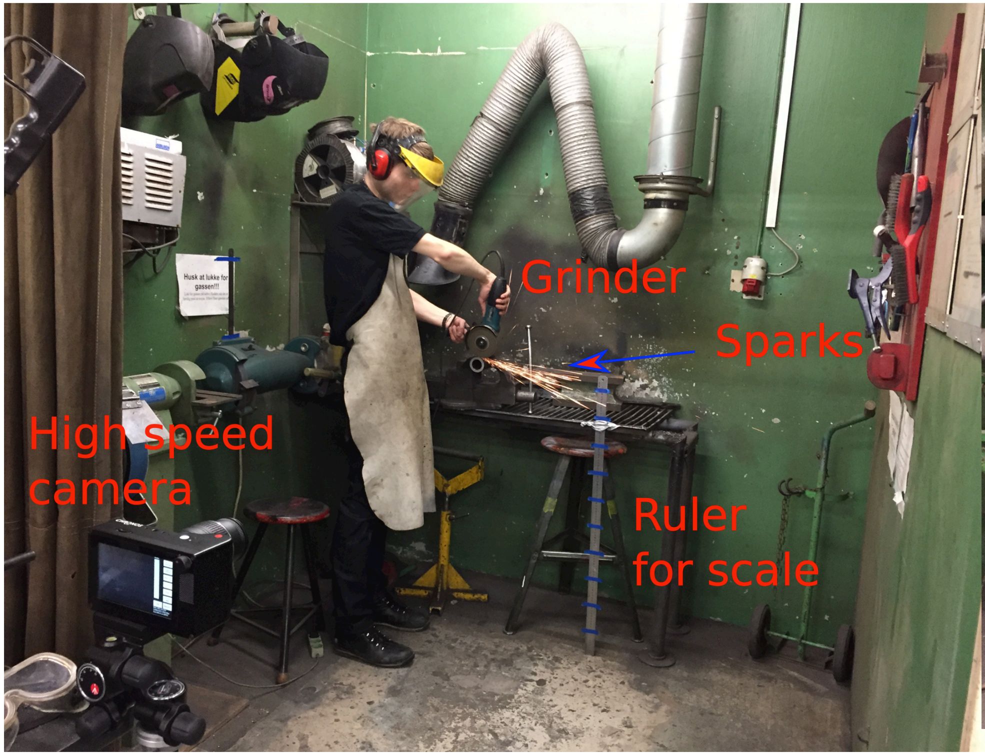
- Weibull distribution: $P_F(t,\sigma) = 1 - \exp\left(-A t^m \bar{\sigma}^{nm}\right)$

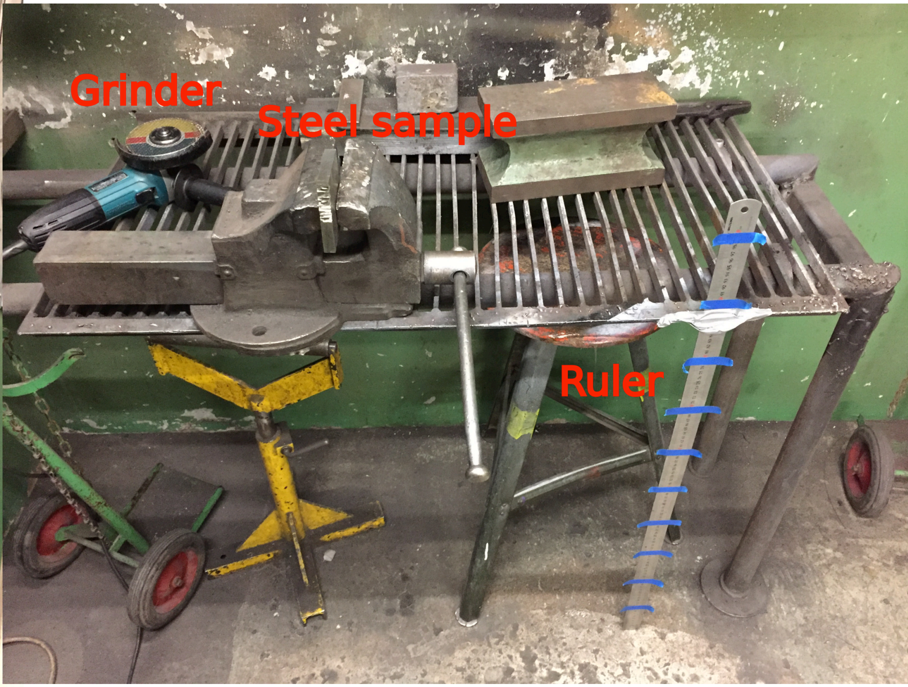
[6]: Alava et. Al "Statistical Models of Fracture"

[5]: Bazant et. Al. " Subcritical crack growth law and its consequences for lifetime statistics and size effect of quasibrittle structures"

Experimental setup

- Grinder: 11000RPM, R=6.7cm
- Steel and stainless steel rectangels. Iron cylinder
- 1503 FPS camera.



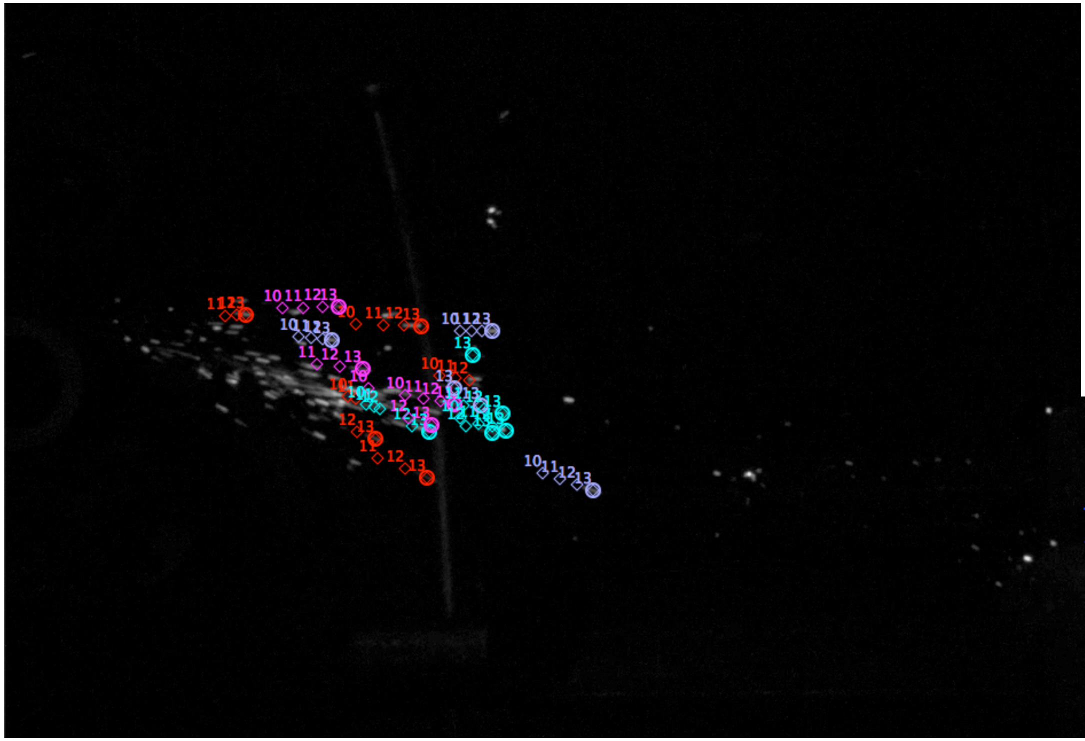


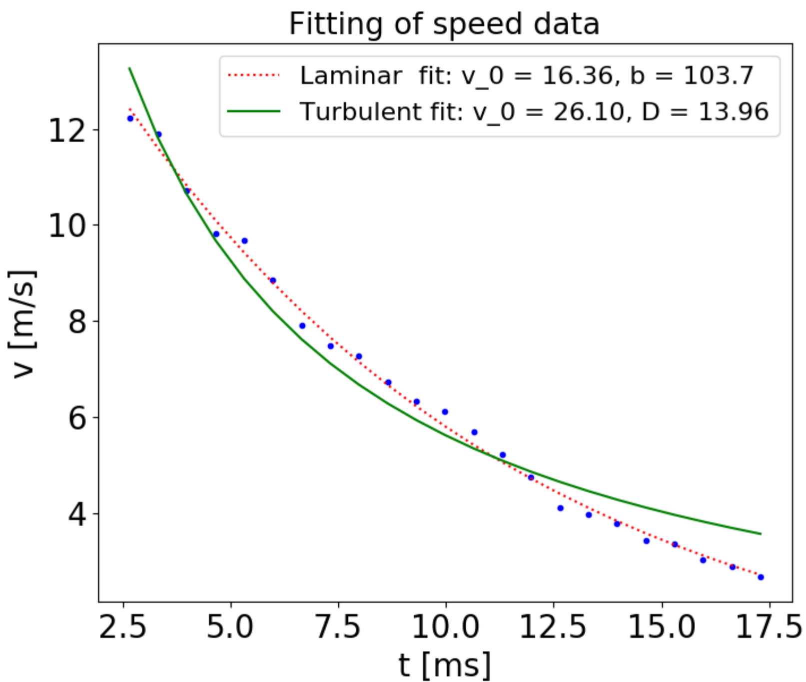
Experiments

Camera handled by Sif Fink
Arnbjerg Nielsen

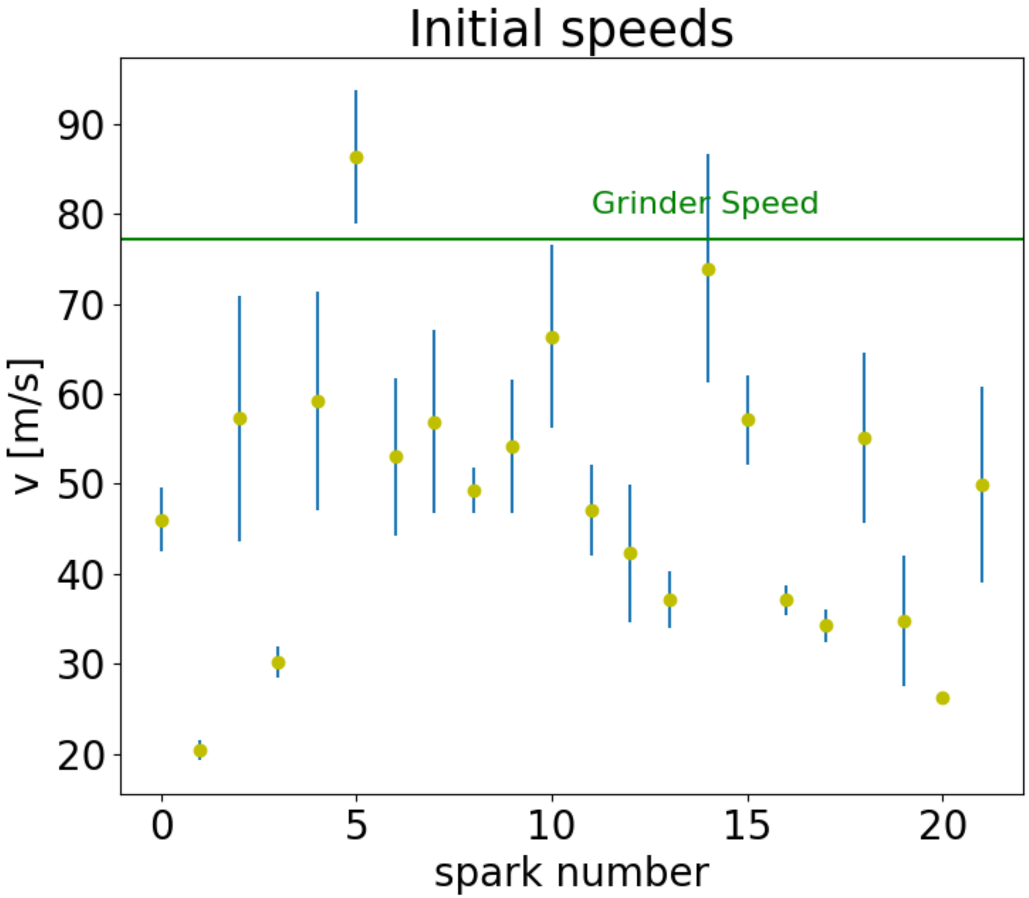
Video Analysis

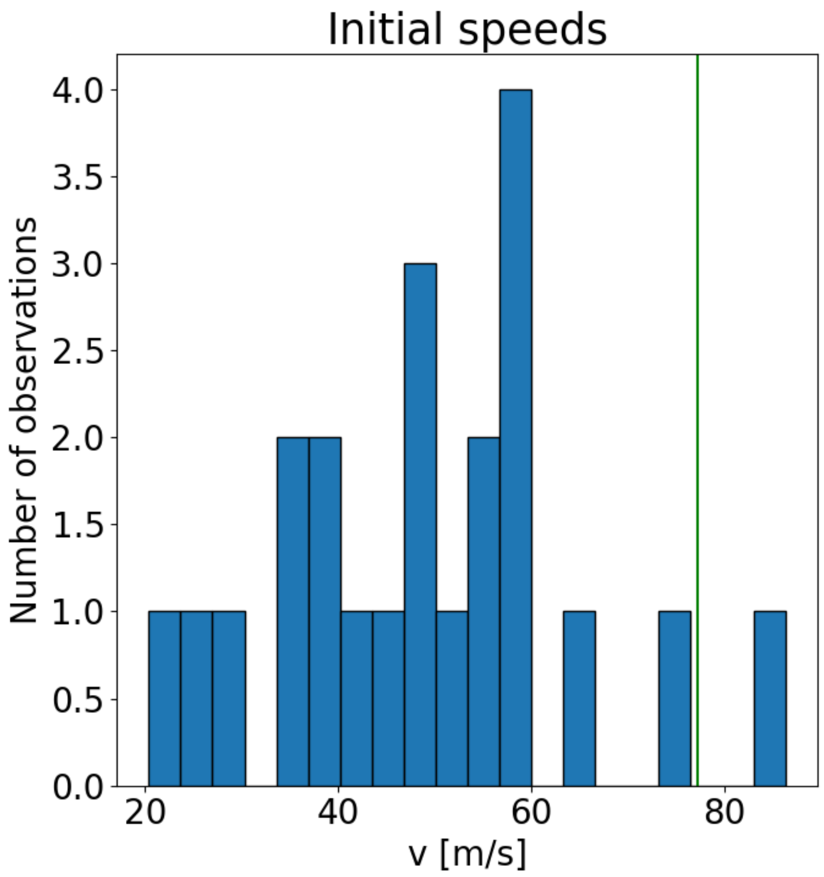
- Tracker. Marking sparks frame by frame.
- Best fit for stokes drag





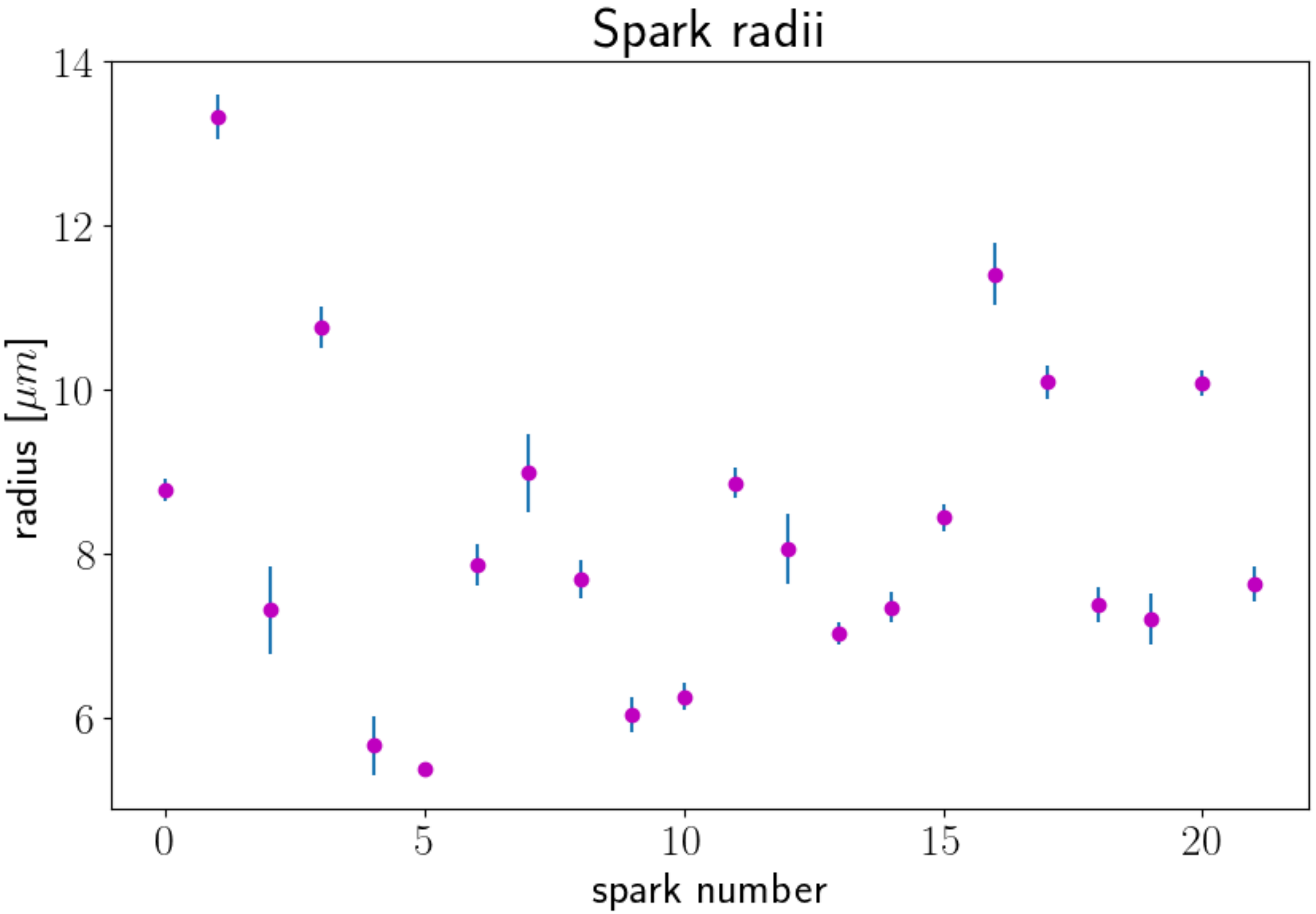
Speed distribution





Size distribution

$$Re \approx 50$$

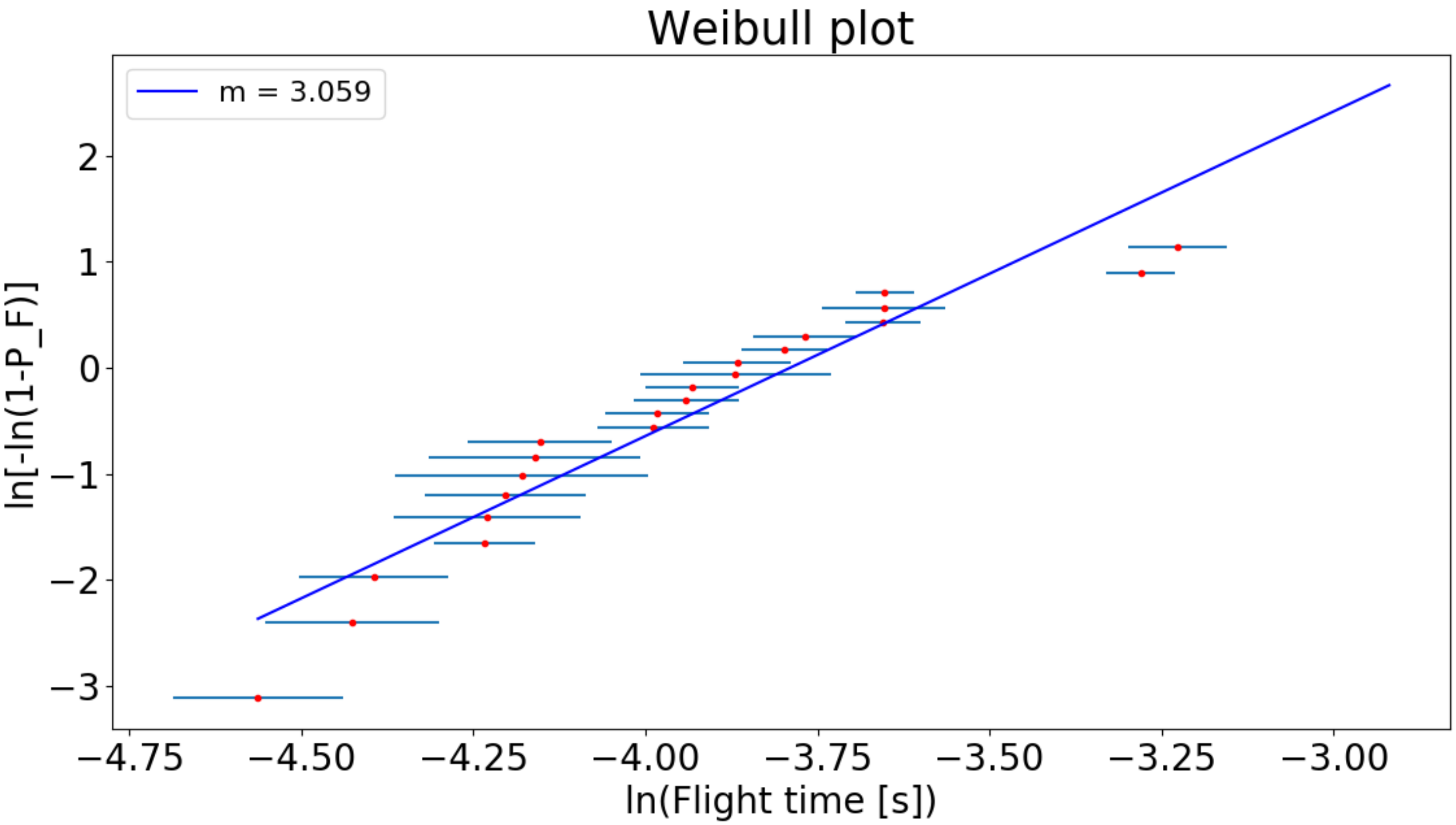


Fit to Weibull distribution

New measurement series

Expectation: Straight line and $m \in [1,3]$

$$P_F = 1 - \mathrm{e}^{-At^m} \Rightarrow \ln\left[-\ln\left(1-P_F\right)\right] = m \ln t + \ln A$$



Conclusion

- Split caused by oxidative growth stress
- Split at critical crack length
- Distance depends on: temperature, spark size, initial speed, oxidation kinetics, elastic properties and material microstructure
- Grinder pressure and speed is essential
- Weibull distribution of times

Bibliography 1

https://en.wikipedia.org/wiki/Fracture#Ductile_fracture

https://en.wikipedia.org/wiki/Toughness

[1]: Chen, R. Y., and W. Y. D. Yeun. "Review of the high-temperature oxidation of iron and carbon steels in air or oxygen." *Oxidation of metals* 59.5-6 (2003): 433-468.

[2]: Tschorn G., ”Spark Atlas of Steels”. (1963) VEB edition Leipzig

[3]: Panicaud, B., J. L. Grosseau-Poussard, and J. F. Dinhut. "On the growth strain origin and stress evolution prediction during oxidation of metals." *Applied surface science* 252.16 (2006): 5700-5713

[4]: Roylance, David. "Introduction to fracture mechanics." *Massachusetts Institute of Technology, Cambridge* 1 (2001).

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[5]: Le, Jia-Liang, Zdeněk P. Bažant, and Martin Z. Bazant. "Subcritical crack growth law and its consequences for lifetime statistics and size effect of quasibrittle structures." *Journal of Physics D: Applied Physics* 42.21 (2009): 214008.

[6]: Alava, Mikko J., Phani KVV Nukala, and Stefano Zapperi. "Statistical models of fracture." *Advances in Physics* 55.3-4 (2006): 349-476.

Appendix 1: Size from velocity

$$m\dot{v} = -bv \Rightarrow v(t) = v_0 \mathrm{e}^{-bt/m}$$

- Stokes drag and spherical sparks:

$$\frac{b}{m} = \frac{6\pi\mu_{air} r_{spark}}{\frac{4}{3}\pi r_{spark}^3 \rho_{spark}} \Rightarrow r_{spark} = 3\sqrt{\frac{\mu_{air}}{2\rho_{spark}}}$$

Appendix 2: Flight time and initial velocity

$$\frac{dr}{dt} = v_0 \mathrm{e}^{-bt/m} \Rightarrow r(t_a) - r(t_b) = \frac{v_0 m}{b}\left(\mathrm{e}^{-bt_b} - \mathrm{e}^{-bt_a}\right)$$

- For launch time $r(t_L) = 0$

$$\mathrm{e}^{-bt_L/m} = \frac{br(t_a)}{v_0 m} + \mathrm{e}^{-bt_a/m}$$

$$\Rightarrow t_L = \frac{m}{b} \ln\left[\frac{v_0 m}{br(t_a) + \mathrm{e}^{-bt_a/m}}\right]$$

$$\Delta t = t_{final} - t_L$$

$$v_L = v_0 \mathrm{e}^{-bt_L/m}$$

Appendix 3: Stress model assumptions[2]

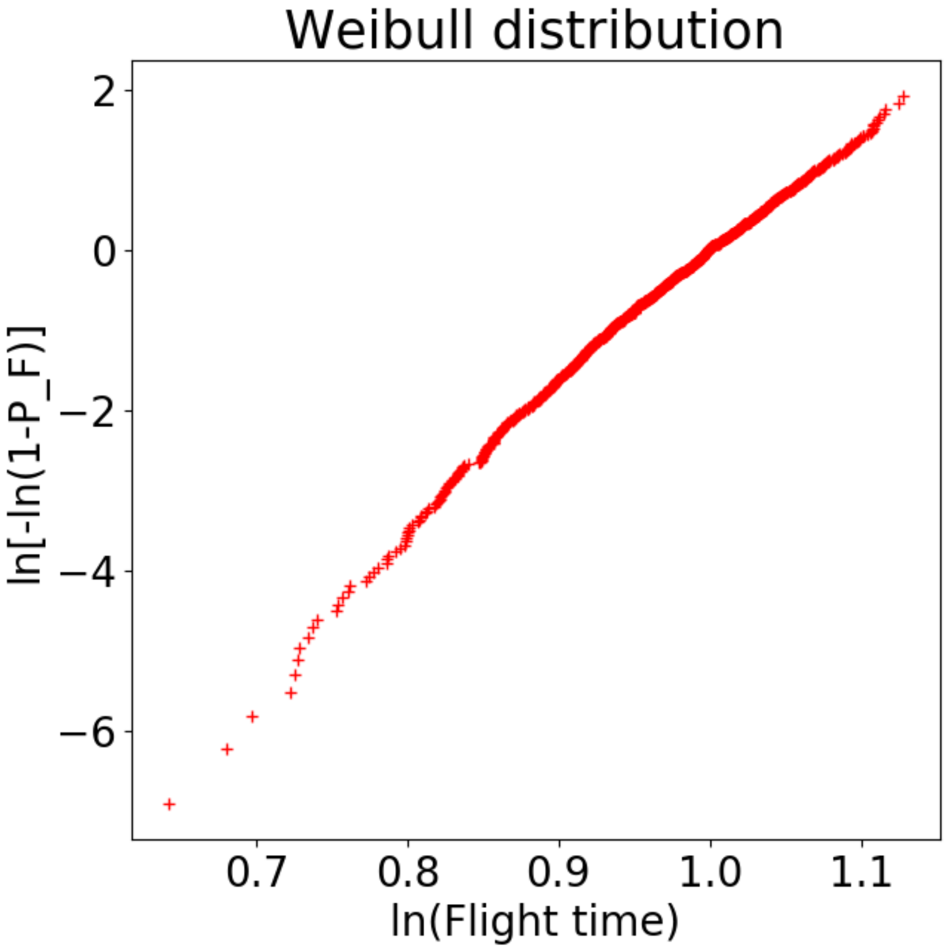
- Rectangular geometry
- Single oxide layer with continuity at interface
- No non-linear effects, e.g. Buckling
- Parabolic oxidation kinetics
- Constant temperature
- Isotropic system

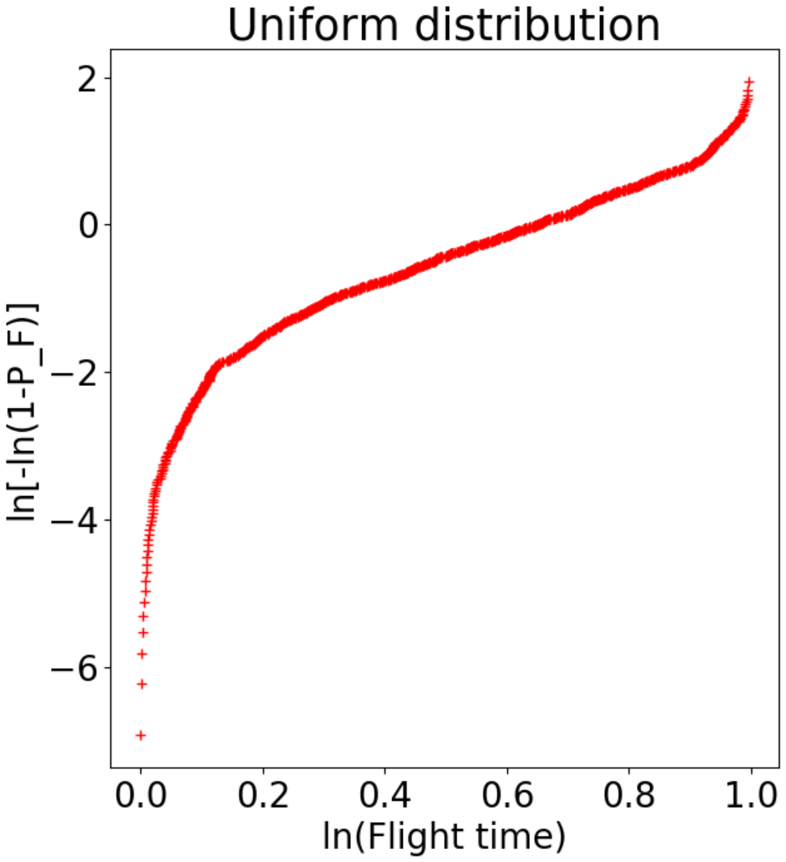
$$(\dot{\epsilon}_{elastic} + \dot{\epsilon}_{viscoelastic} + \dot{\epsilon}_{growth})_{ox} = (\dot{\epsilon}_{elastic} + \dot{\epsilon}_{viscoelastic})_{m}$$

[2]: Panicaud et. Al. "On the growth strain origin and stress evolution prediction during oxidation of metals"

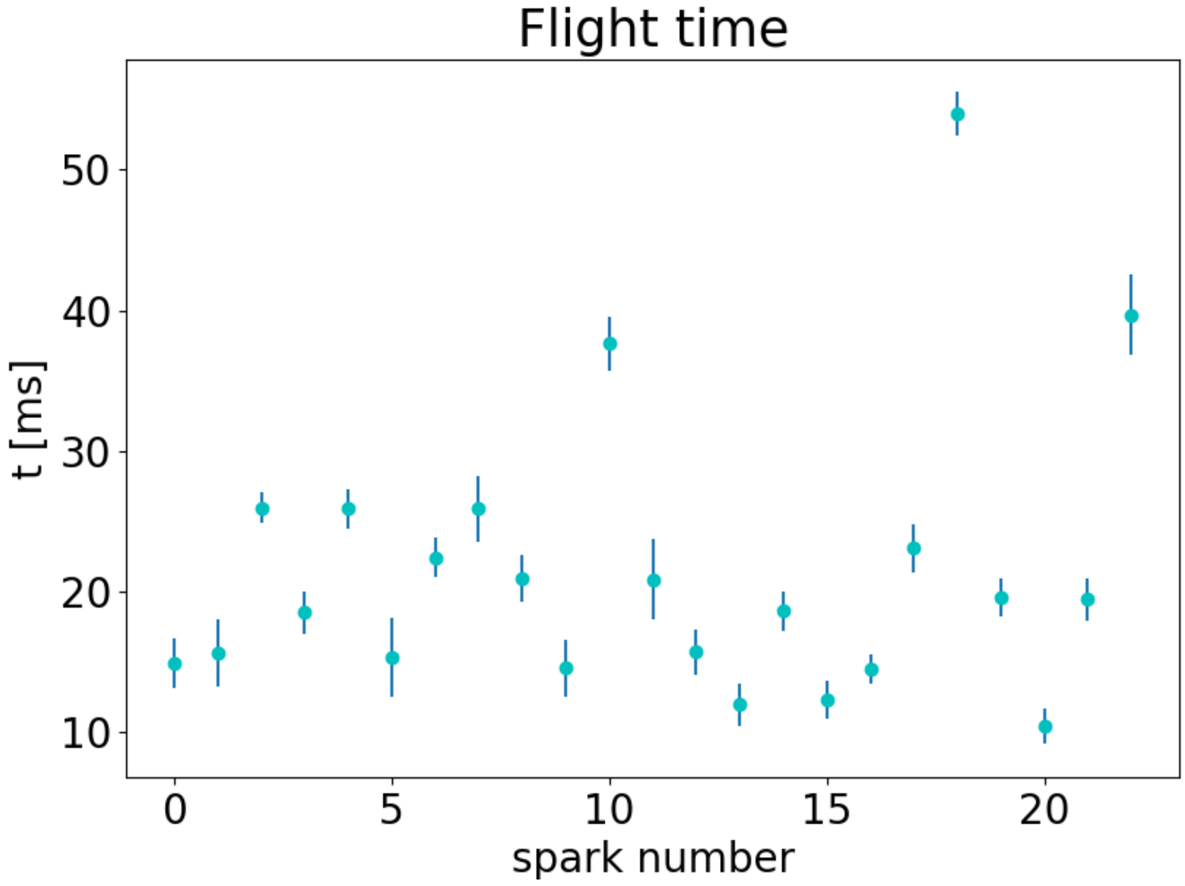
Appendix 4:
Weibull plot expectation

1000 points

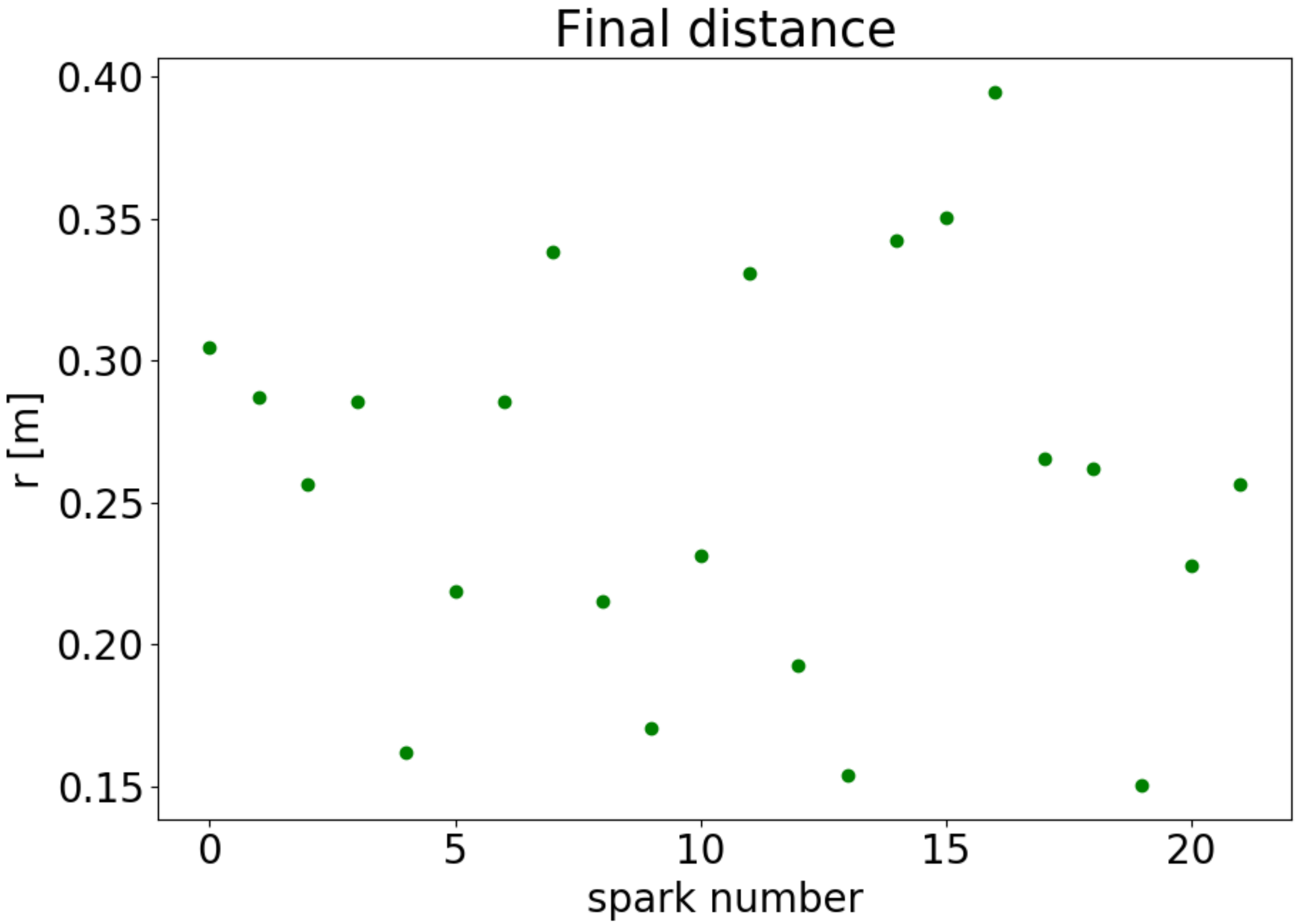

Weibull distribution


Uniform distribution

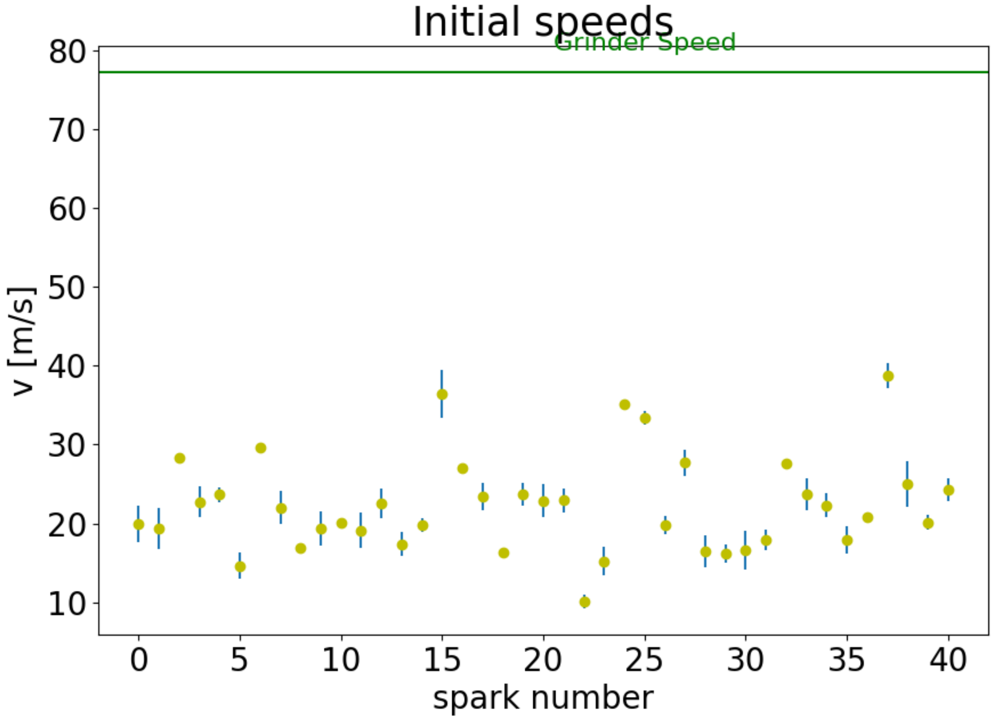
Appendix 5:
Flight time data (m2)

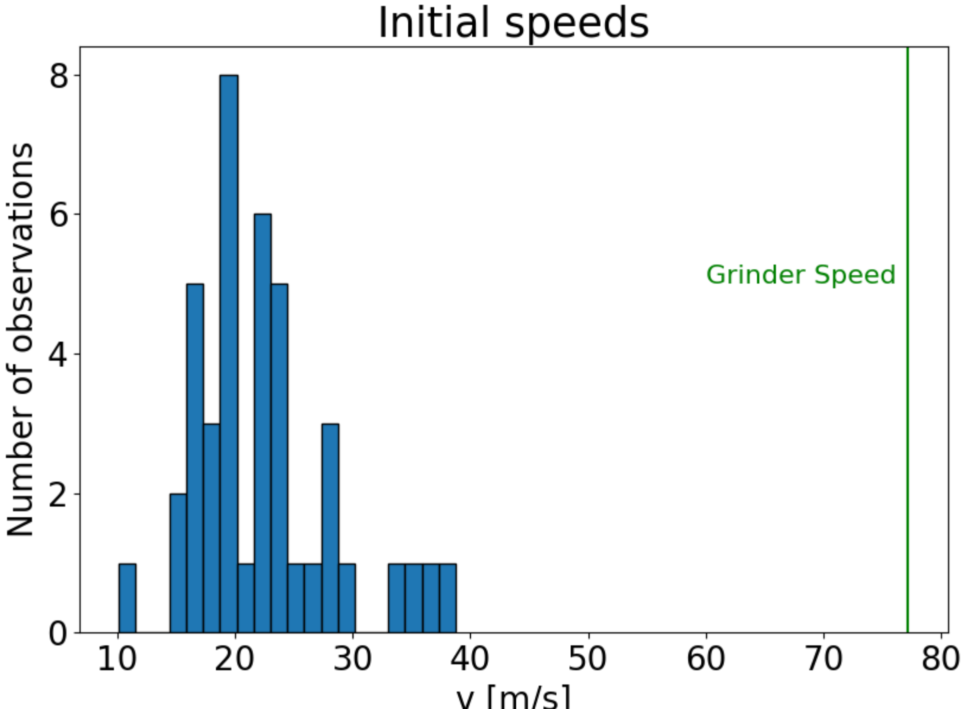


Appendix 6: Distance distribution (m1)



Appendix 7: m2 speed





Appendix 8: Spark characteristics of metals

Metal / Oxide	Spark behaviour	$\frac{V_{ox}}{V_m}$
Fe/Fe_3O_4	Frequent splits	2.16
Fe/Fe_2O_3	Frequent splits	2.15
Ti/TiO_2	Very frequent splits	1.78
Al/Al_2O_3	No sparks	1.30
W/WO_3	Sparks after long flight	3.40

https://en.wikipedia.org/wiki/Spark_testing#cite_note-davis-2

Appendix 9: Sparkler analogy

Appendix 10: Many sparks

Appendix 11: Reynolds calculation

$$\rho_{air} = 1.177 \frac{\mathrm{kg}}{\mathrm{m}^3} \qquad \mu_{air} = 1.846 \times 10^{-5} \frac{\mathrm{kg}}{\mathrm{m\ s}}$$

$$v = 40 \frac{\mathrm{m}}{\mathrm{s}} \qquad L_{char} = r_{spark} \approx 10 \mu\mathrm{m}$$

$$Re = \frac{v \rho L}{\mu} = 51$$

Appendix 12: Average stress

$$\bar{\sigma}(t) = \frac{1}{t}\int_0^t \sigma(t)\, dt$$

$$\propto \frac{1}{t}\int_0^t k_0 t \mathrm{e}^{-Q/RT} dt$$

$$\propto t\mathrm{e}^{-Q/RT}$$

Appendix 13: Expanded Model

$$\bar{\sigma}(t) \propto t\mathrm{e}^{-Q/RT}$$

$$P_F(t) = 1 - \exp(At^{m(n+1)}\mathrm{e}^{-Q/RT})$$

$$v_0(t,d) = \frac{bd}{1-\mathrm{e}^{-bt}}$$

$$P_d(d) = \int_0^\infty P_F(t) P_v(v_0[t,d])\, dt$$

$$P_v(v) = \delta(v - v_0) \Rightarrow P_d(d) = P_F(t[d, v_0])$$

$$= P_F\left(\frac{1}{b}\ln\left[\frac{v_0}{v_0 - bd}\right]\right)$$

Appendix 14: Experimental error sources

- Poorly known metal composition
- Sparks hitting ruler, rod or grinder screen
- Difficult to distinguish sparks close to grinder
- Varying and poorly known applied force