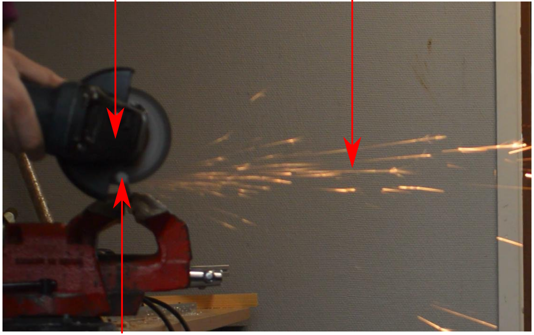
PROBLEM N°9
HALF-LIFE SPARKLES

Team Ecole polytechnique

The problem

Sparks caused by an angle grinder tend to fly over a certain distance and then to split into several smaller sparks. ***What causes them to split?*** *What is the condition for a split to occur? What influences the* ***distance before the split?*** *What will be the* ***distance distribution of the sparks to fly?***

What are we studying ?



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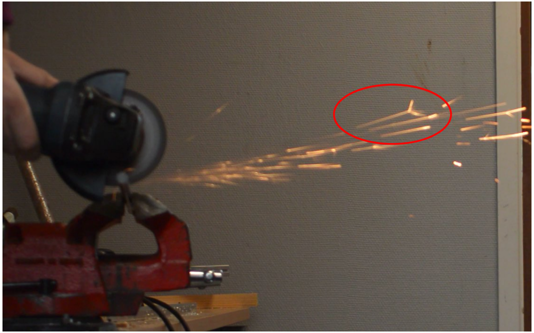

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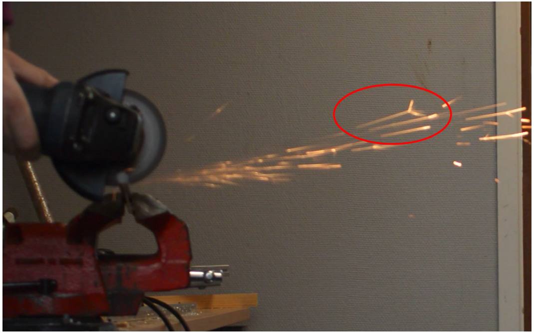


Experimental set up

High speed camera

Grinder and steel

Black background

Using different steels

Given in mass fraction

RS reference :	Low Carbon 1	Low Carbon 2	Medium Carbon	High Carbon
C	< 0.07	0.1	0.16 – 0.18	1.10 – 1.20
Cr	17 – 19	17 – 19		0.35 – 0.50
Ni	11	10		
	stainless			

Other metals like copper or aluminium don't produce sparks.

Observations with a high speed camera

Low Carbon 2 (C 0.1%) with 100N

The physics of sparks

With : $\mathbf{V_0}$: initial velocity, R_0 : initial radius, T_0 : initial temperature

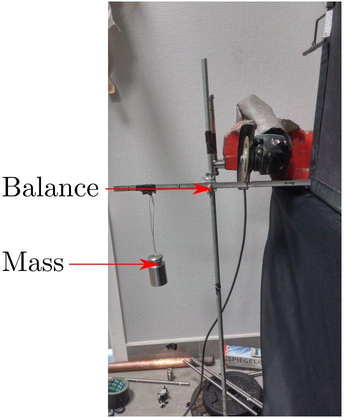
Newton's law

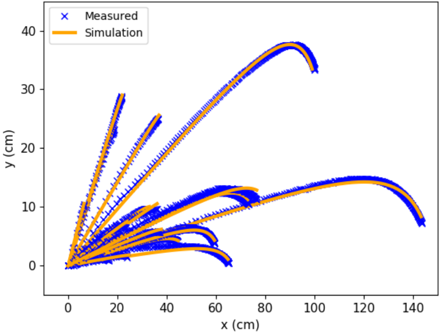
Newton's law :

$$\frac{4}{3}\pi\rho_{steel}R^3\frac{d\mathbf{V}}{dt} = -g\frac{4}{3}\pi\rho_{steel}R^3\mathbf{e_y} - \frac{1}{2}\rho_{air}\pi R^2V^2C_x(V,R)\mathbf{e_V}$$

The path of a spark only depends on R and $\mathbf{V}(t=0)$: by analysing the trajectories we can study these two variables.

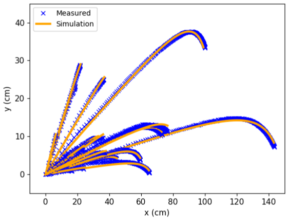
Fitting radius and initial velocities for different forces





$50N$

Fitting radius and initial velocities for different forces



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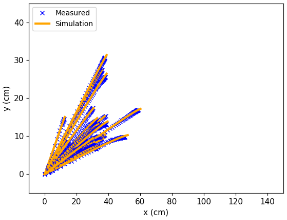
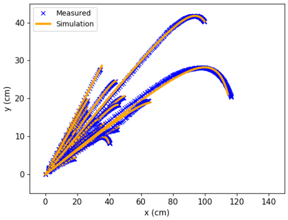


75N



100N

Radius and initial velocity of the sparks

Experimental values for R and $\mathbf{V}(t=0)$ (Medium Carbon steel) :

	radius (μm)	initial velocity ($m.s^{-1}$)
$50N$	50 ± 20	30 ± 10
$75N$	50 ± 10	40 ± 9
$100N$	50 ± 20	33 ± 5

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- $\mathbf{V}(t=0) = 34 \pm 8 m.s^{-1}$

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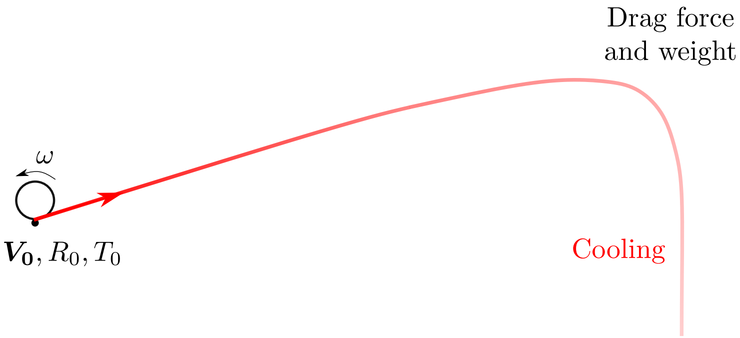
Tangential speed on the edge of the angle grinder : $40 m.s^{-1}$

The physics of sparks



With : $\mathbf{V_0}$: initial velocity, R_0 : initial radius, T_0 : initial temperature

Theory on convective cooling

Hypothesis :

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Newton's law of cooling :

$$\frac{dT}{dt} = -\frac{3}{Rc_{steel}\rho_{steel}} h(V)(T - T_{air})$$

For a spark at the beginning of the trajectory $h \simeq 2000 \ W.m^{-2}.K^{-1}$

With :

- T : temperature of the spark
- T_{air} : temperature, 300 K
- R : radius of the spark, 50 μm
- c_{steel} : heat capacity, $4.4.10^2 \ J.kg^{-1}.K^{-1}$
- ρ_{steel} : density, $8.10^3 \ kg.m^{-3}$

Initial temperature of the spark

The law of cooling gives :

$$T = (T_0 - T_{air})\exp\left(-\frac{t}{\tau}\right) + T_{air} \text{ with } \tau = 30\ ms$$

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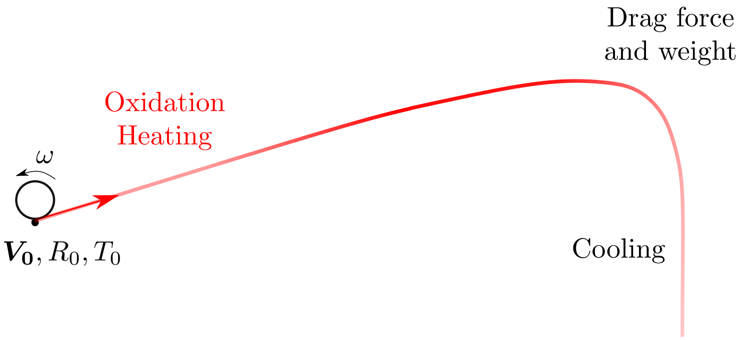
This is unless we consider oxidation of sparks.

The physics of sparks



With : $\mathbf{V_0}$: initial velocity, R_0 : initial radius, T_0 : initial temperature

Oxidation of iron

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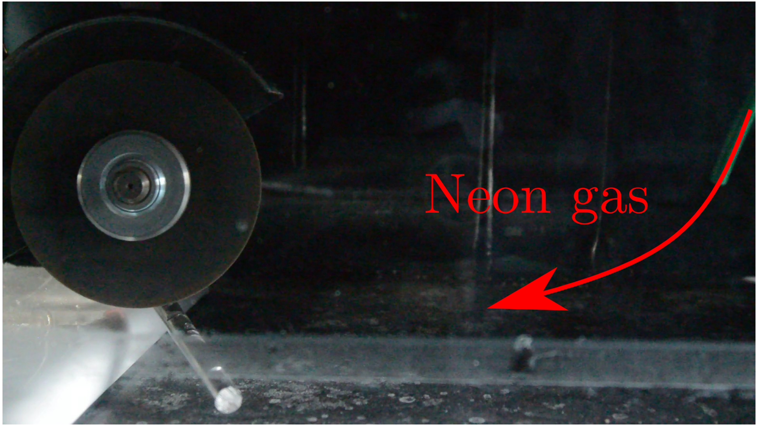
So there is enough energy stored in each spark, to heat it adiabatically by $\Delta T = 9.6.10^3\ K$ no matter it's size

The characteristic time of the diffusion of oxygen inside the spark is $T_{diffusion} = 8\ ms$, meaning the energy is slowly released allowing the spark to keep on glowing.

There are no sparks if we replace oxygen with neon



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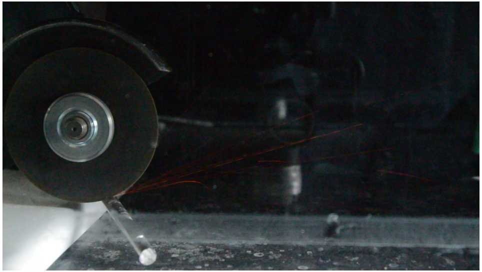
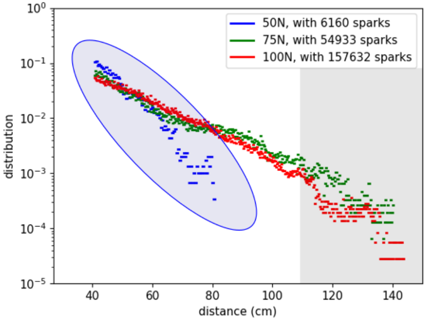


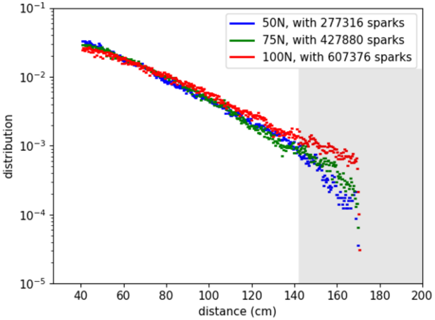
With neon

With oxygen

Distance distribution of the sparks

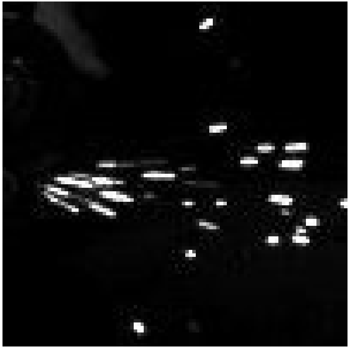


Low Carbon 2 (C 0.1%)



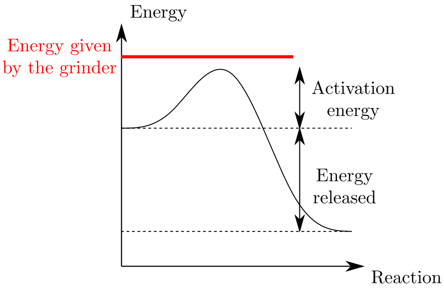
Medium Carbon (C 0.16 – 0.18%)

Some sparks don't ignite



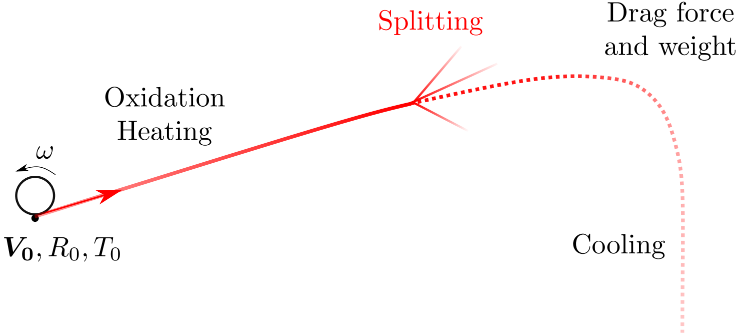
Low Carbon 2 (C 0.1%) with $50N$

Activation energy



Sparks don't light up

The physics of sparks



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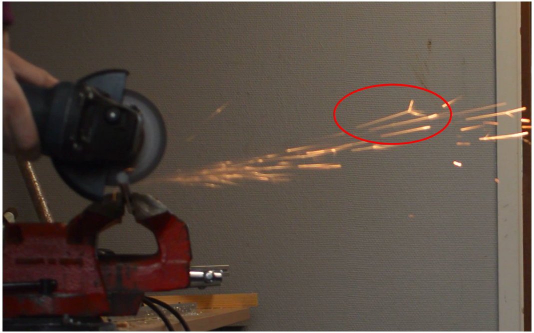
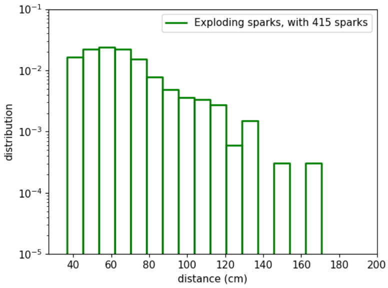


Observations with a high speed camera

Medium Carbon (C $0.16 - 0.18\%$) with $100N$

Distance distribution of the sparks to fly

Steel under study : Medium Carbon (C 0.16 – 0.18%)



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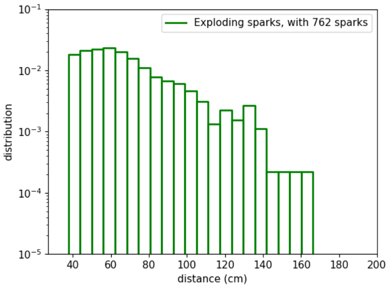

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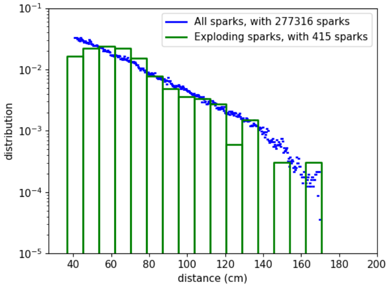

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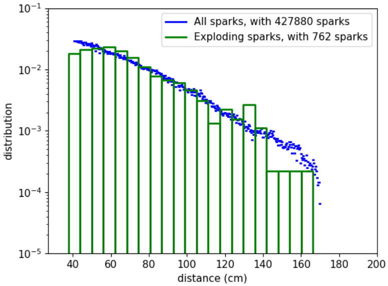

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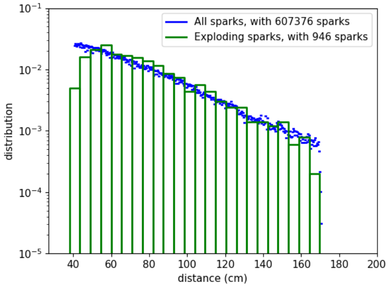

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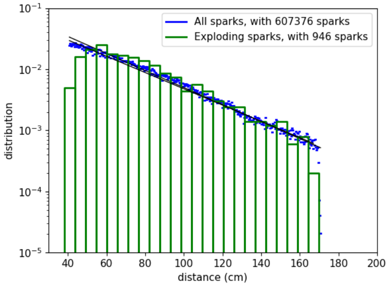


$100N$

Exponential law : $f(d) = \frac{1}{\lambda} e^{-\frac{d}{\lambda}}$

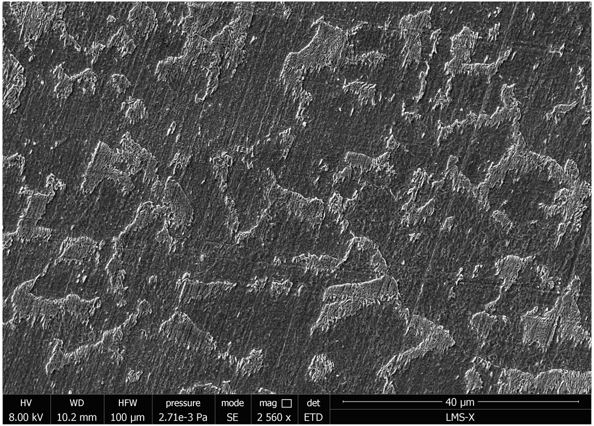
Experimental values for λ (given in cm) :

	all sparks	exploding sparks
$50N$	23.8 ± 0.4	24 ± 2
$75N$	25.5 ± 0.3	24 ± 2
$100N$	30.9 ± 0.3	33 ± 2

The coefficient λ increase with the force, and is the same for the two distributions

What causes the sparks to split ?

Microstructure of steel :



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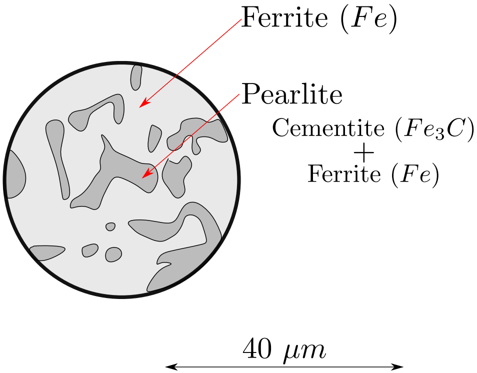

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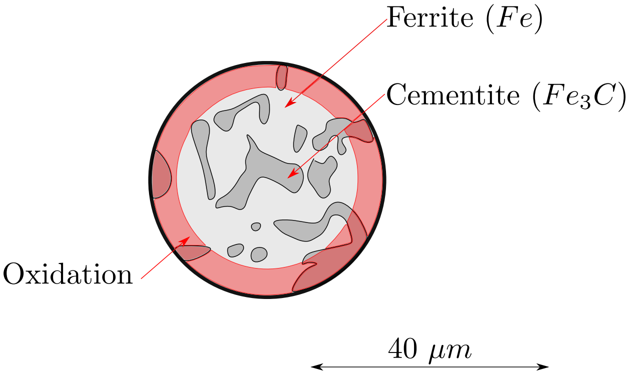
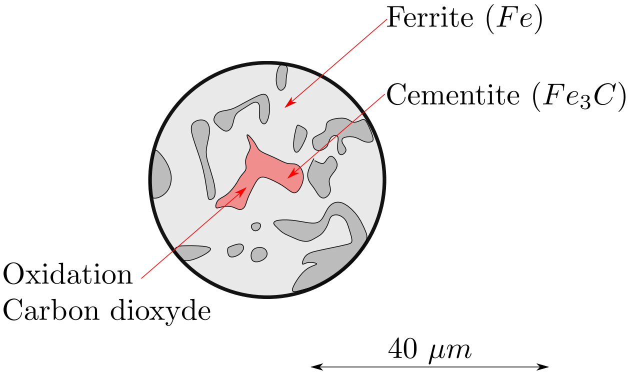


What causes the sparks to split ?

Oxidation of cementite : $Fe_3C + 3O_2 \longrightarrow Fe_3O_4 + CO_2$



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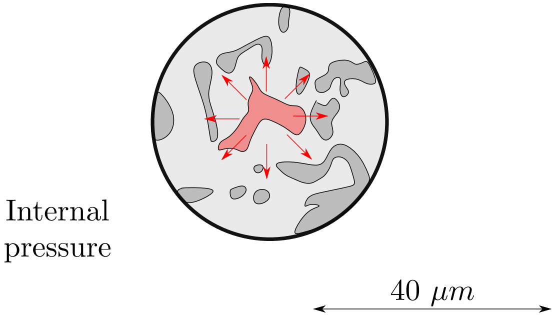

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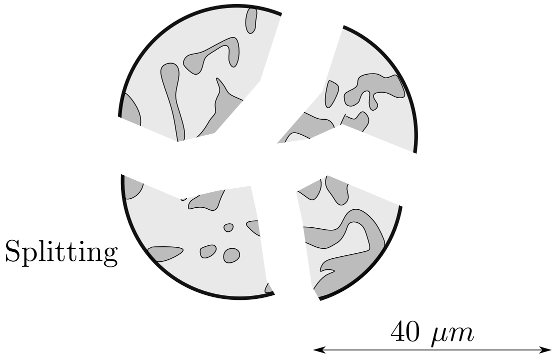
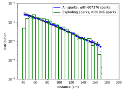


Observation of an explosion

Conclusion

- **What causes them to split ?**

Splitting



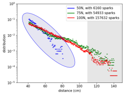
$$f(d) = \frac{1}{\lambda} e^{-\frac{d}{\lambda}}$$

$$\lambda = 24 - 33\ cm$$

Conclusion

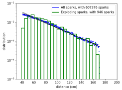
- **What causes them to split ?**
- **What influences the distance before the split ?**

Oxidation Heating



$\Delta T_{max} = 9.6.10^3 K$
$\tau_{diffusion} = 8\ ms$

Splitting



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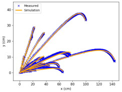
Drag force and weight



$R_0 = 50 \pm 20\ \mu m$
$V_0 = 34 \pm 8\ m.s^{-1}$

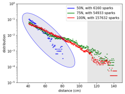
Cooling

$\tau_{cooling} = 30\ ms$
$T_{min} \simeq 1000\ K$

Conclusion

- What causes them to split ?
- What influences the distance before the split ?
- What will be the distance distribution of the sparks to fly ?

Oxidation Heating



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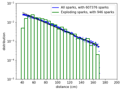

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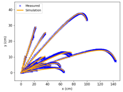

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About sparks

Observations :

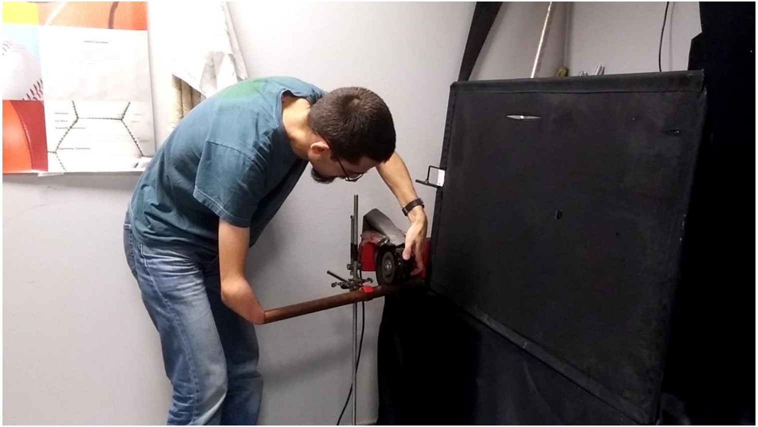
- Steel produces sparks, but not aluminium nor copper
- Sparks need oxygen

Element	Melting point (K) pure	Melting point (K) oxide	Young's modulus (GPa)	Conductivity ($W.m^{-1}.K^{-1}$)
Fe	1811	1642 – 1870	211	80.4
Al	933	2327	70	237
Cu	1358	1505 – 1599	110 – 128	401

Hypothesis :

- Iron heat more, enabling the ignition

Copper don't produce sparks



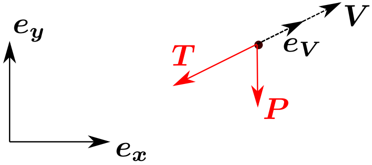
Forces acting on sparks

Hypothesis :

- Spherical sparks of constant radius

Forces :

- Weight : $\mathbf{P} = -g\frac{4}{3}\pi\rho_{steel}R^3\mathbf{e_y}$
- Drag : $\mathbf{T} = -\frac{1}{2}\rho_{air}\pi R^2V^2C_x(V,R)\mathbf{e_V}$



With :

- $\mathbf{V}$: velocity of the spark
- R : radius of the spark
- g : gravity, 9.81 $m.s^{-2}$
- ρ_{air} : density, 1.293 $kg.m^{-3}$
- ρ_{steel} : density, 8.10^3 $kg.m^{-3}$

The drag coefficient

We can express C_x as a fonction of the Reynolds number :

$$R_e = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho_{air} R V}{\eta}$$

With η the dynamic viscosity of air, $1.8.10^{-5}\ Pa.s^{-1}$

Typical values for sparks are $Re \simeq 200$, so we can approximate C_x with Van allen's law :

$$C_x = \frac{18.5}{Re^{0.6}}$$

The heat transfer coefficient

For forced convection around spheres, if $Re < 7.10^4$, we have an empirical formula [4] :

$$\frac{h(V)R}{\lambda_{air}} = 2 + 0.6 \left(\frac{\eta c_p}{4}\right)^{1/3} \left(\frac{\rho_{air} R V}{\eta}\right)^{1/2}$$

With :

- R : radius, $50^{-6}\ m$
- V : speed, $34\ m.s^{-1}$
- λ_{air} : heat conductivity, $2.6.10^{-2}\ W.m^{-1}.K^{-1}$
- c_p : isobaric heat capacity of air, $1.0.10^3\ J.kg^{-1}.K^{-1}$
- η : dynamic viscosity, $1.8.10^{-5}\ Pa.s^{-1}$
- ρ_{air} : density, $1.3\ kg.m^{-3}$

We get : $h \simeq 2000\ W.m^{-2}.K^{-1}$

[4.]Ph.Marty; Université Joseph Fourier, **Transferts thermiques convectifs**, 2012

The diffusion of oxygen inside the spark

Fick's law :

$$\frac{\partial n(t,\mathbf{r})}{\partial t} = \Delta\left(D(T,\mathbf{r})n(t,\mathbf{r})\right)$$

Where :

- n : concentration of oxygen
- D : diffusion coefficient

The diffusion coefficient of oxygen in pure iron

Dependance on the temperature [2, 3] :

$$D(T) = D^* \exp\left(\frac{-Q}{RT}\right)$$

Where : D^* is a preexponential factor and Q is an activation energy

Phase	$D^*(m^2.s^{-1})$	$Q(J.mol^{-1})$
α-iron ($T < 1183K$)	$(2.91^{+3.40}_{-1.57}) \times 10^{-7}$	$(89.5 \pm 7.2) \times 10^3$
γ-iron ($T > 1183K$)	$(1.30^{+0.80}_{-0.50}) \times 10^{-4}$	$(166 \pm 5) \times 10^3$

[2.]J.Takada, M.Adachi; Journal of Materials Science 21, **Determination of diffusion coefficient of oxygen in α-iron from internal oxidation measurements in Fe-Si alloys**, 1986
[3.]J.Takada, S.Yamamoto, S.Kikuchi, M.Adachi; Metallurgical Transactions 17A, **Determination of diffusion coefficient of oxygen in γ-iron from measurements of internal oxidation in Fe-Al alloys**

Temperature sensitivity of the camera

The spectral radiance of a blackbody is given by Planck's law :

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

To get the spectral energy density received by a pixel to form an image, we multiply it by the integration time of a pixel ($T = 10^{-3}\ s$), the half-surface of a spark ($S = 1.6.10^{-8}\ m^2$) and the solid angle of a pixel as seen by the spark at 1 m, taking into account the lens of 16 mm focal length ($\Omega = 3.1.10^{-7}\ Sr$).

Then dividing by $h\nu$ we get the spectral density of the number of photons received by the pixel.

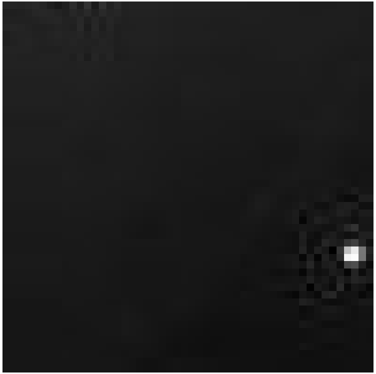
We normalise the resulting function with the quantum efficiency $QE(\nu)$ of the camera [6], measuring the proportion of photons converted in electrons as a function of their wavelength (49 % at 630 nm, then decreasing), so we get the spectral density of the number of electrons converted by the pixel.

Finally, we got to this integral for the number of electrons converted by the pixel as a function of the temperature of the spark :

$$N_{electrons}(T) = TS\Omega \int_{-\infty}^{+\infty} QE(\nu) \frac{B(\nu, T)}{h\nu}$$

The camera having a dark noise of $29e^-$ [6], the sparks are visible if $N(T) > 29$, wich gives around $T = 1100K$

The temperature of some sparks is not constant



Medium Carbon (C 0.16 – 0.18%) with $50N$

Possible explanation

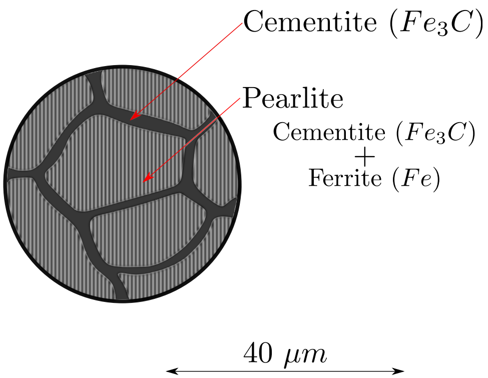
The characteristic appearance of the spark stream is apparently to be attributed chiefly to the oxidation of the carbon in the steel. [...] It appears probable that as the grinding wheel tears off small particles of steel the work done causes the temperature to rise. This rise in temperature may also be increased by a "pyrophoric" oxidation effect [...]. Oxygen and carbon react in the heated portions of the particles to produce C02 and perhaps CO. The oxide scale formed on a plain carbon steel is not very tenacious and easily flakes off. The gas which forms within the heated spherical particles escapes through this easily fractured skin and gives rise to the "series spark bursts." [...]As the carbon content of the steel is increased, the number and intensity of the "carbon bursts" increase.[1]

[1.]R.W.Buzzard; Bureau of Standards Journal of Research, ***The utility of the spark test as applied to commercial steels****, 1933*

Observations with a high speed camera

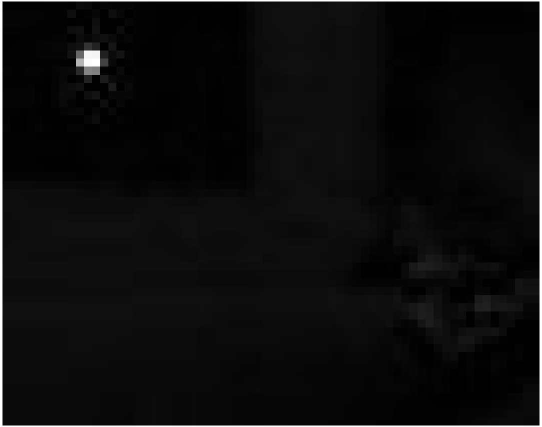
High Carbon (C $1.10 - 1.20\%$) with $100N$

The microstructure of steel change at 0.8 % carbon



[5.]M.F.Ashby, D.R.H.Jones; Dunod, **Matériaux : microstructures et procédés de mise en oeuvre**, 4ème édition

Splittings are much more complicated



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