Origami-Launcher

Lucia Härer
Team FAUltiere
Germany

Task

Folded paper structures such as the **Miura-ori origami** can be programmed to exhibit a wide range of elastic properties depending on their crease and defect patterns. **Design and build an origami cannon** to **vertically** launch a standard **Ping-Pong ball** using only a **single uncut sheet of A4 paper (80g/m²)**. How is the height of the ball elevation related to the folding pattern? Optimize your design to achieve the **maximum height** possible.

Overview

THEORY

model **energy** stored folding pattern

estimate **take-off velocity** of the ball

model **trajectory**

→ jumping height

compare

compare

EXPERIMENT

measure **spring constant** of crease

measure force pattern exhibits when compressed

→ **energy**

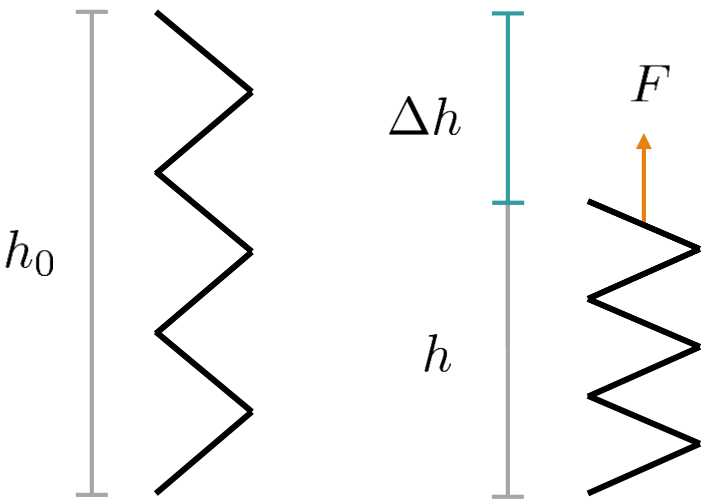
measure **jumping height**

1. Models for folding patterns

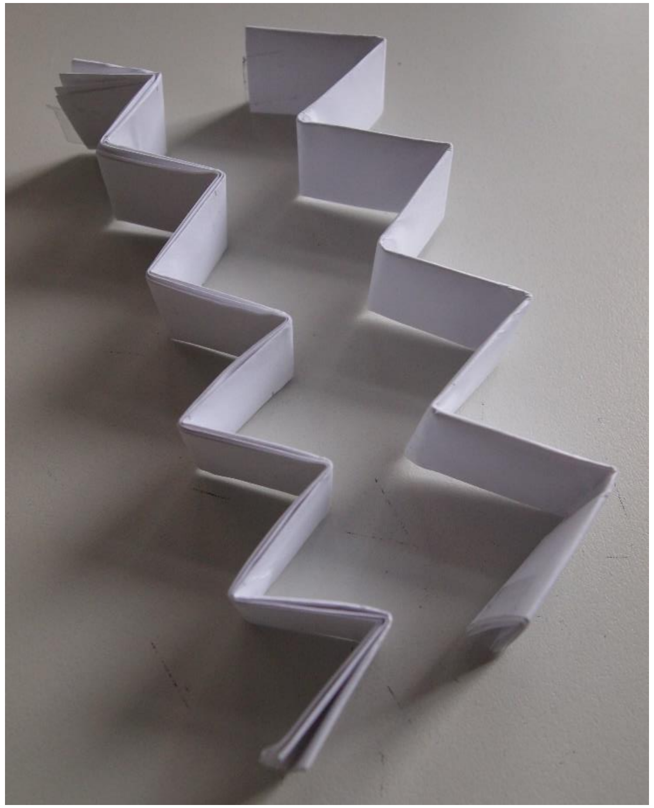
Goal: Find **energy** $E(\Delta h)$ **stored folding pattern** when compressed by Δh

ZIG-ZAG FOLDING

model as **spring**: $F = -K\Delta h \Rightarrow E = \frac{K}{2}\Delta h^2$



A4 paper is folded lenghtwise (3 times), then creases are added



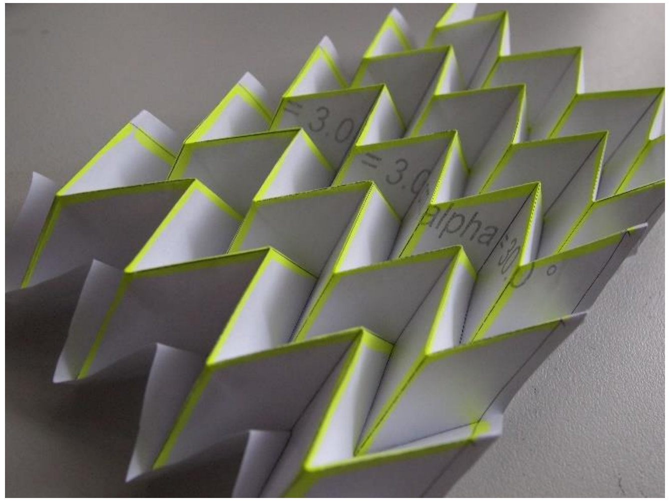
1. Models for folding patterns

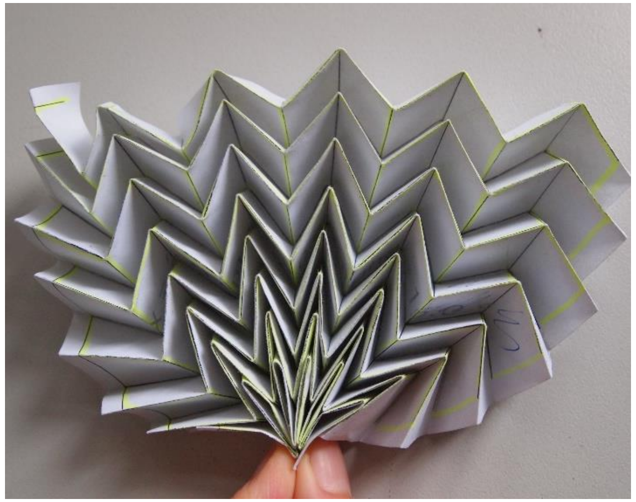
https://de.wikipedia.org/wiki/Miura-Faltung#/media/File:Miura-ori.gif

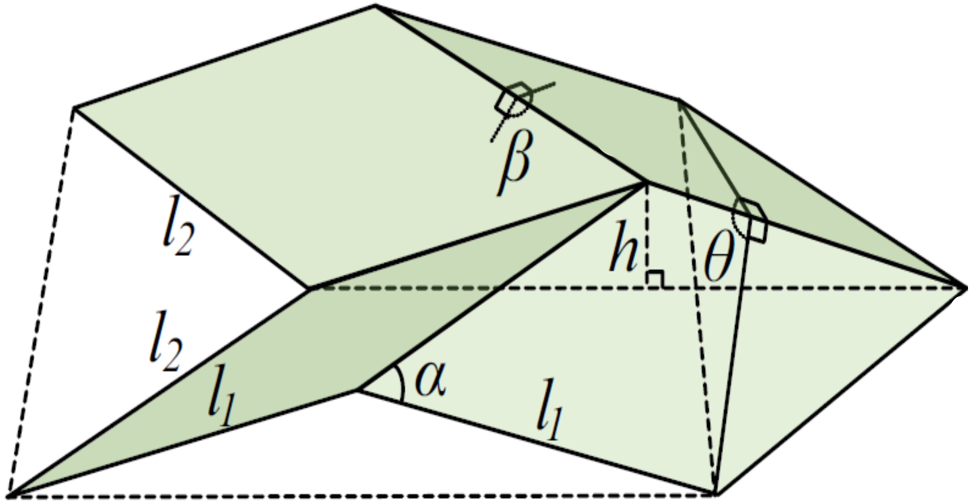
MIURA-ORI

is a mechanical meta-material

- Specific pattern is **parameterized** by $\boldsymbol{l_1, l_2, \alpha}$ and number on unit cells **N**
- State is **fully described** by $\boldsymbol{\theta}$







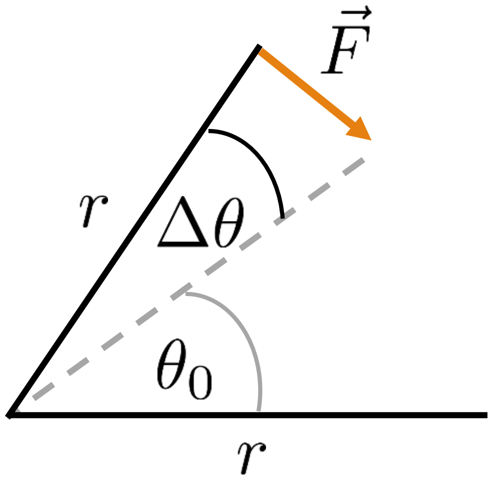
Z. Y. Wie et al.; Geometric Mechanics of Periodic Pleated Origami; PRL 110 215501, (2013)

1. Models for folding patterns

Model **single crease** as **torsional spring**

$\vec{r} \times \vec{F} = -D\vec{\Delta\theta}$

$F = -D\frac{l}{r}\Delta\theta \Rightarrow E = \frac{1}{2}Dl\Delta\theta^2$



Sum up energy over full unit cell

$$E = ND(l_1\Delta\theta^2 + l_2\Delta\beta^2)$$

$$\Rightarrow E(h) = 2D\frac{LH}{\sin\alpha}\left[\frac{1}{l_2}\left(\sin^{-1}\frac{h}{H} - \sin^{-1}\frac{h_0}{H}\right)^2 + \frac{1}{l_1}\left(\sin^{-1}\eta - \sin^{-1}\eta_0\right)^2\right]$$

where $\eta = \frac{h\cos\alpha}{\sqrt{H^2 - h^2\sin^2\alpha}}$

Changed variables:

$\beta \mapsto \theta,\ \theta \mapsto h$

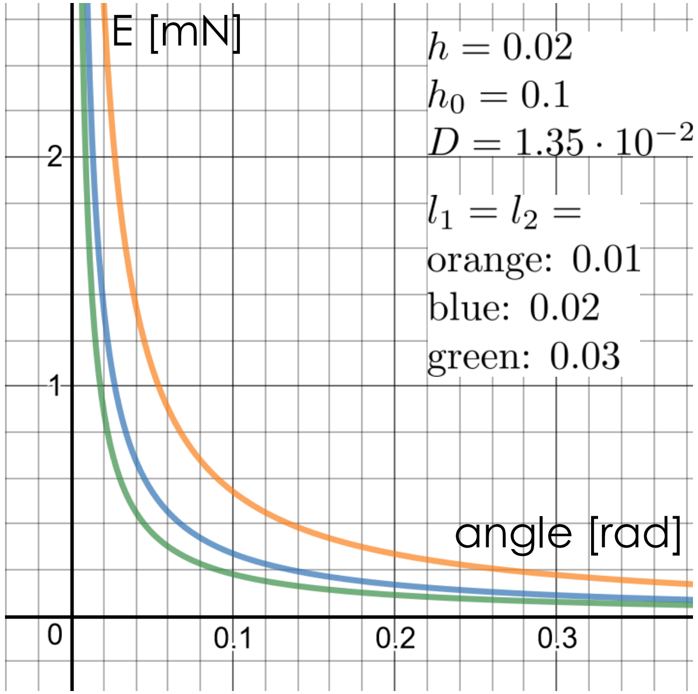
$N \mapsto L, H$ (dimensions of A4 paper, $L = 29.7$ cm, $H = 21$ cm)

1. Models for folding patterns

What Miura-Ori pattern can store **the most energy**?

$$E(h) = 2D\frac{LH}{\sin\alpha}\left[\frac{1}{l_2}\left(\sin^{-1}\frac{h}{H} - \sin^{-1}\frac{h_0}{H}\right)^2 + \frac{1}{l_1}\left(\sin^{-1}\eta - \sin^{-1}\eta_0\right)^2\right]$$

- $E(h) \sim \frac{1}{l_1}, \frac{1}{l_2}$ → Choose **small l_1 and l_2**
- $E(h) \sim \frac{1}{\sin\alpha}$ → Choose **small α**



Selected patterns for experiment:

2x2 cm 45°	2x2 cm 60°	2x2 cm 75°	3x3 cm 60°

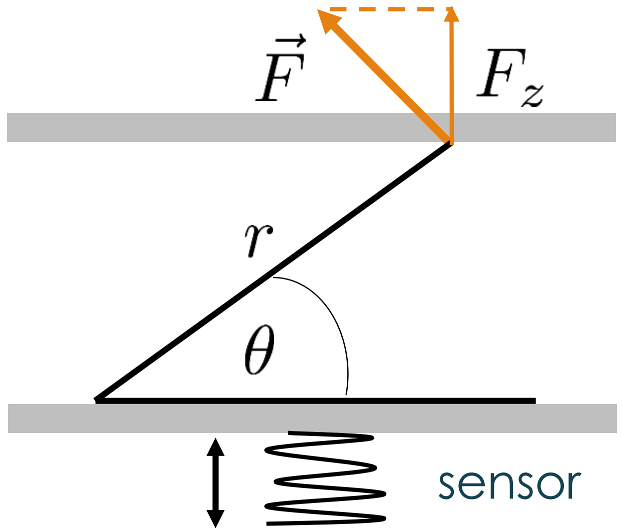
material: paper A4, 80 $[\frac{\text{g}}{\text{m}^2}]$

2. Spring constant (single crease)

Determine D to complete model for $E(h)$
→ measure spring constant of a single crease

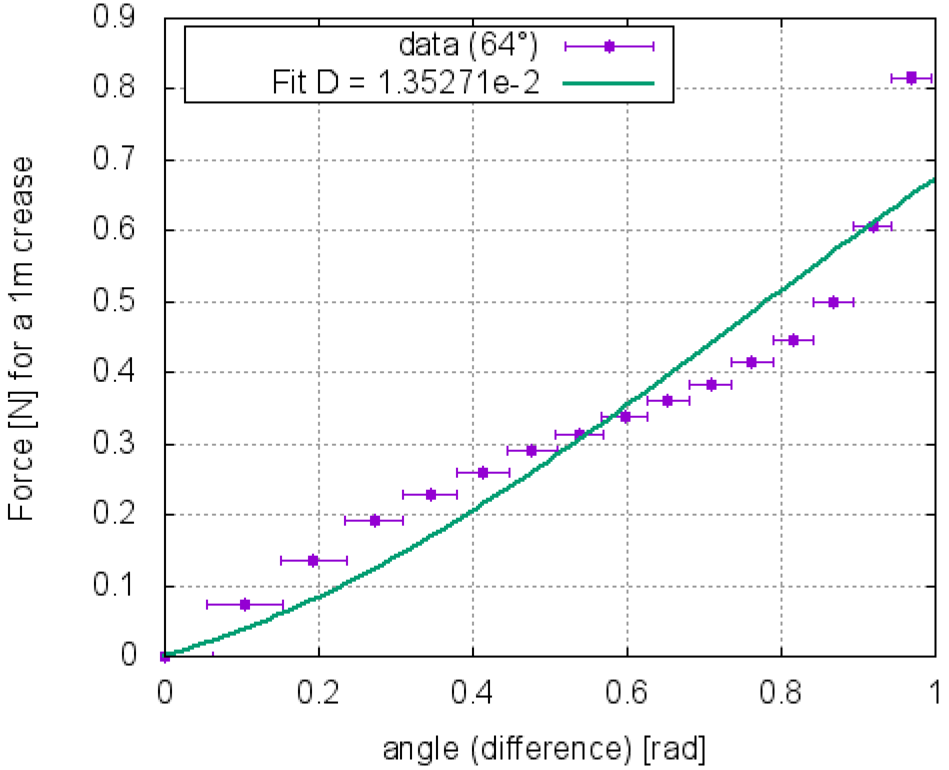
Setup measures only force in z-Direction

$$F_z = D\frac{l}{r}(\theta - \theta_0)cos\theta$$



Determine D
from **Fit**

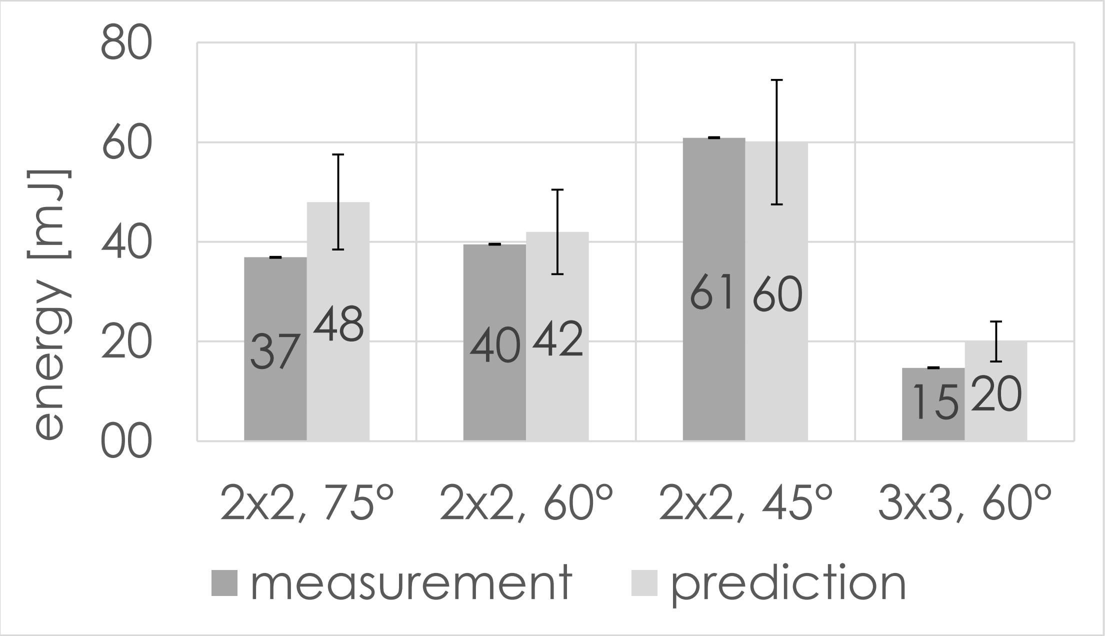
$$D = (1.4 \pm 0.3) \cdot 10^{-2}\,\frac{\mathrm{N}}{\mathrm{rad}}$$

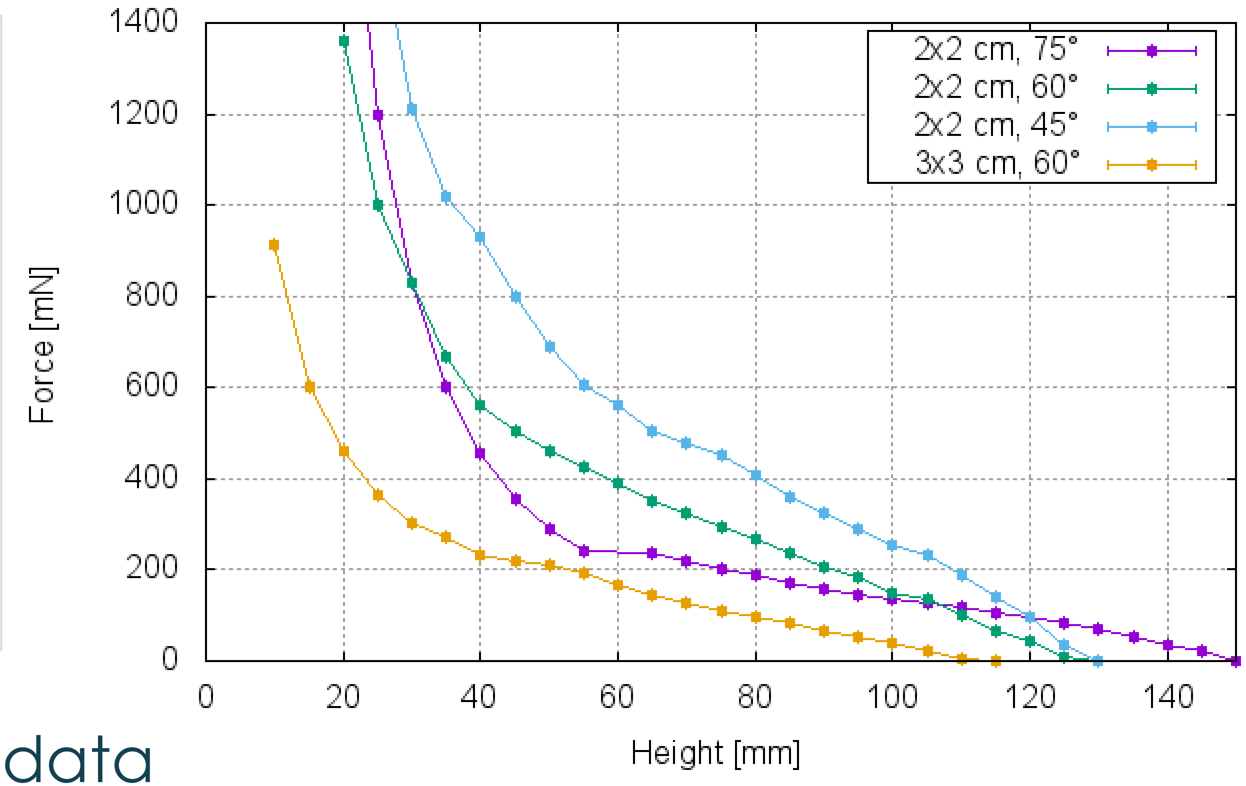


3. Energy stored in folding pattern

MIURA-ORI

Compare modelled $E(h)$ to measurement (measure $\vec{F}(h)$ and integrate)





→ model **agrees** with experimental data

3. Energy stored in folding pattern

ZIG-ZAG FOLDING

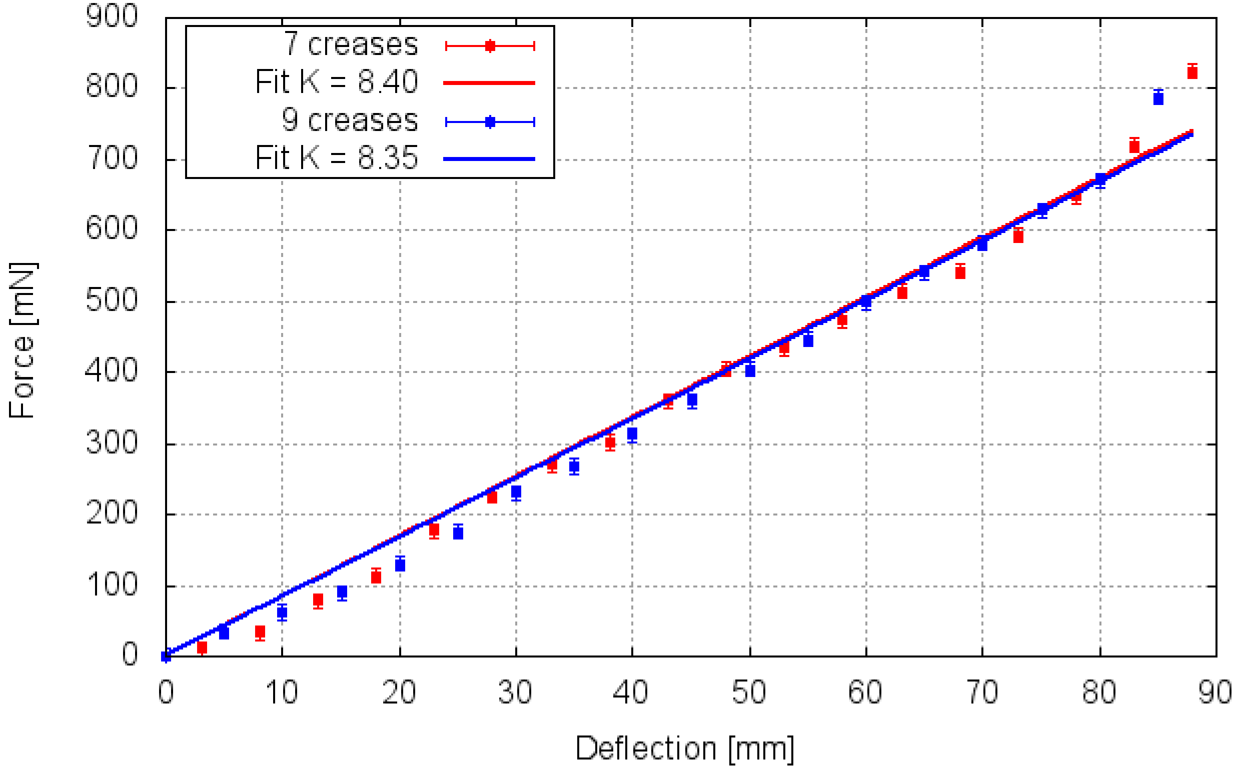
Energy [mJ] for $h = 2\ \mathrm{cm}$

7 creases	9 creases
47.2±0.1	44.4±0.1

Determine K $[\frac{\mathrm{N}}{\mathrm{m}}]$ by fitting

$$F = -K\Delta h$$

7 creases	9 creases
8.4±0.2	8.3±0.2



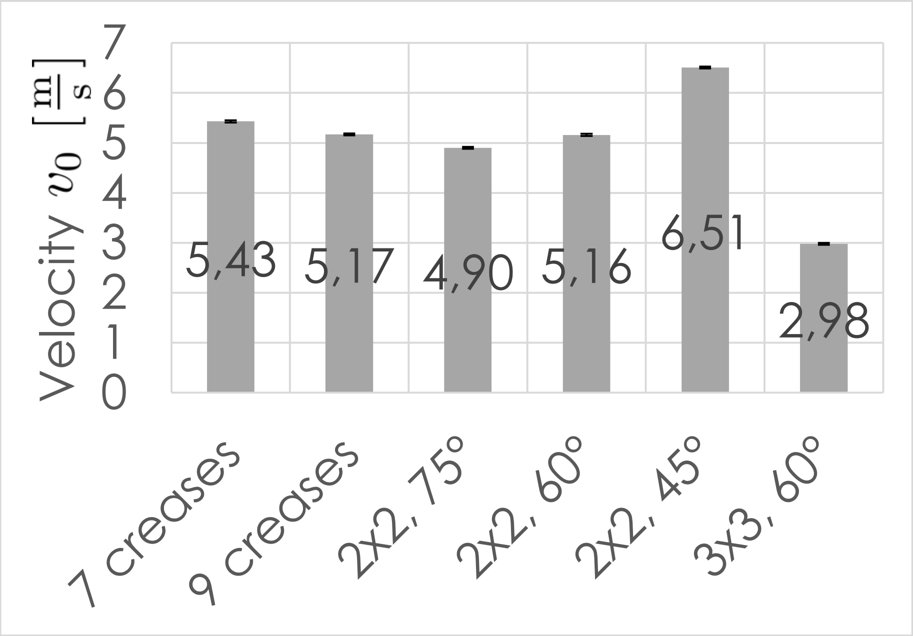
4. Take-off velocity

Estimate take-Off velocity from $E(h)$ obtained by experiment

Consider losses due to **expansion** and **jumping of pattern**

$$E_{\text{start}} = E - E_{\text{expand}} - E_{\text{jump}} = E - Mgh_{\text{jump}} - \frac{1}{2}Mg\Delta l \overset{!}{=} \frac{1}{2}mv_0^2$$

where m mass of the mass
M mass of the pattern



5. Trajectory

Determine **maximal jumping height** $h_{\max}$ by modelling the trajectory

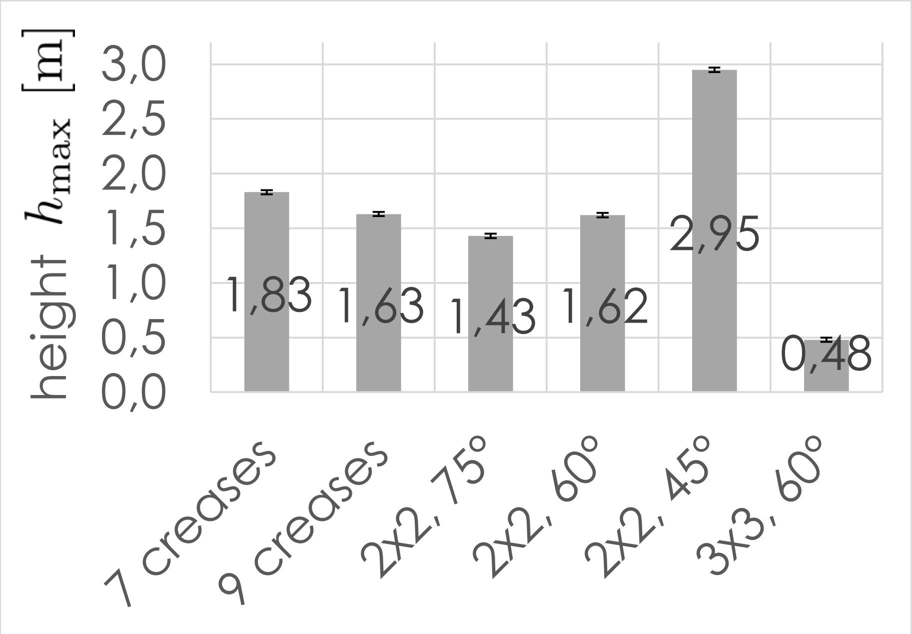
Assume **Newtonian air resistance**

$$m\dot{v} = -mg + Rv^2 \Rightarrow v(t) = -v_\infty \tanh\left(\frac{gt}{v_\infty} - \operatorname{artanh}\left(\frac{v_0}{v_\infty}\right)\right)$$

With the critical velocity $v_\infty = -\sqrt{\frac{mg}{R}}$

and $R = \frac{1}{2} c_w A \rho$

$$v(t) \overset{!}{=} 0 \Rightarrow h_{\max} = -\frac{v_\infty^2}{g} \ln \sqrt{1 - \frac{v_0^2}{v_\infty^2}}$$



6. Measurement of jumping height

SETUP

- metal bars **restrict** movement of Ping-Pong ball **to z-direction**
  - bar diameter 1 cm, bars separated by 3 cm
  - standard Ping-Pong ball: $m = 2.7\ \mathrm{g},\ d = 40\ \mathrm{mm}$
- pattern is compressed to $h = 2\ \mathrm{cm}$, ball is released

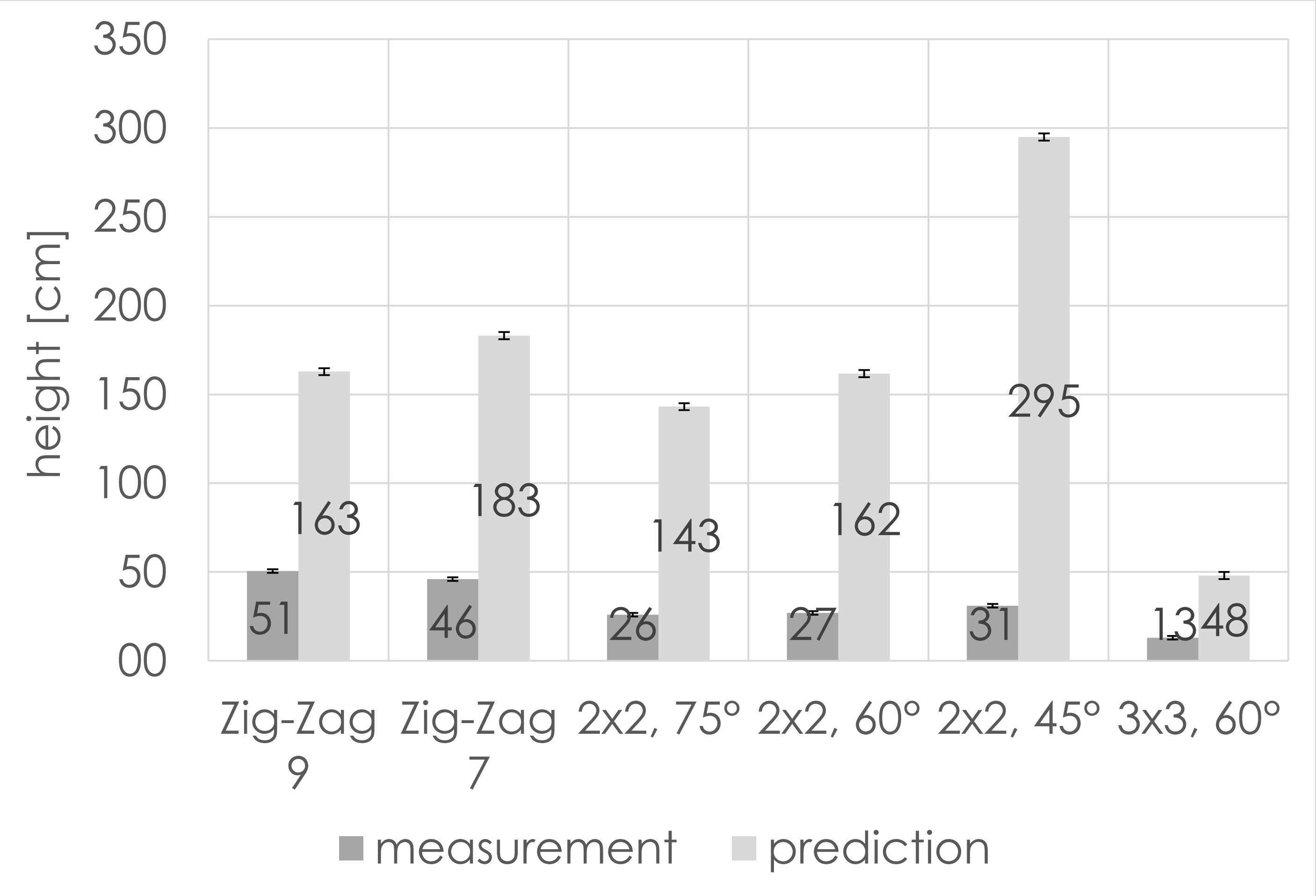
MEASUREMENT

Trajectory analysis with Viana (colour tracking)
→ determine $h_{\max},\ v_0$

6. Measurement of jumping height

RESULTS

- $h_{\max}$ is **not** reached
- **Zig-Zag** springs produce **biggest heights** (absolute and percental)
- **qualitative** behaviour is **reproduced** for Miura-Ori patterns

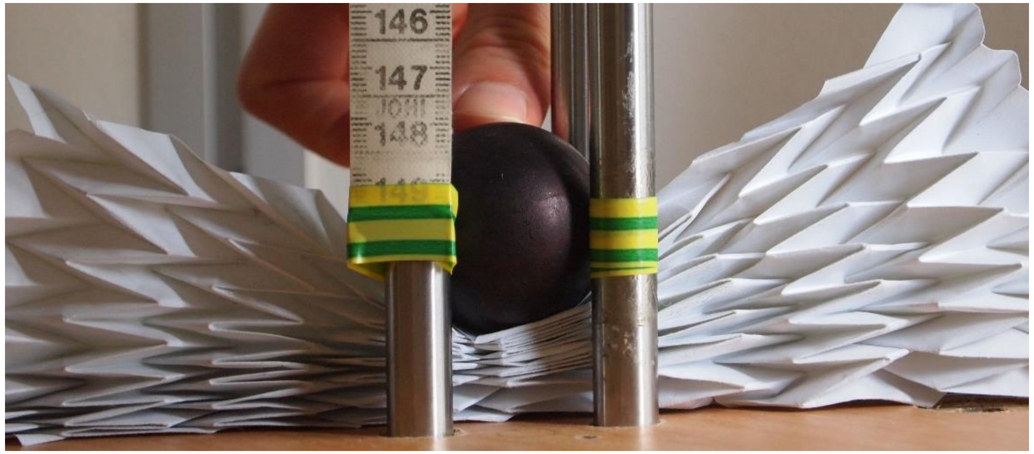


6. Measurement of jumping height

DISCUSSION

- additional energy losses: **rotation** of the ball, **friction** (metal bars) if ball isn't launched completely vertically
- **mechanical instability:** patterns with **narrow** and/or long **base area**, stabilizing can lead to additional friction

  Miura-Ori: small α
  → narrow, long base area
  → paper is **not compressed uniformly**
- paper creases **wear out**



Summary

WHAT IS THE BEST FOLDING PATTERN?

- **Zig-Zag folding**: $h_{\max} = 0.5\ \mathrm{m}$ theoretically up to $2.95\ \mathrm{m}$
- The more creased paper, the more energy can be stored
  → Choose $\boldsymbol{\alpha}, \boldsymbol{l_1}, \boldsymbol{l_2}$ **small** (Miura-Ori), but: **buckling**, **uneven compression**

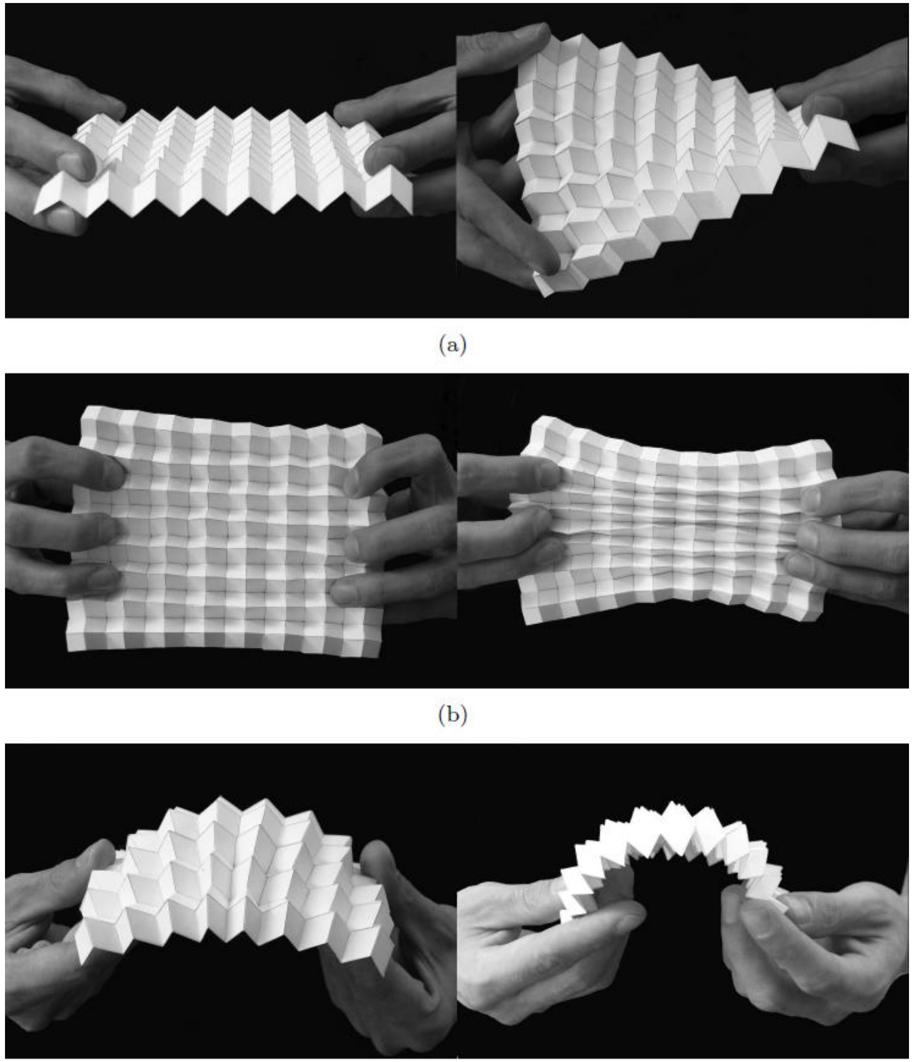
MATHEMATICAL MODELS

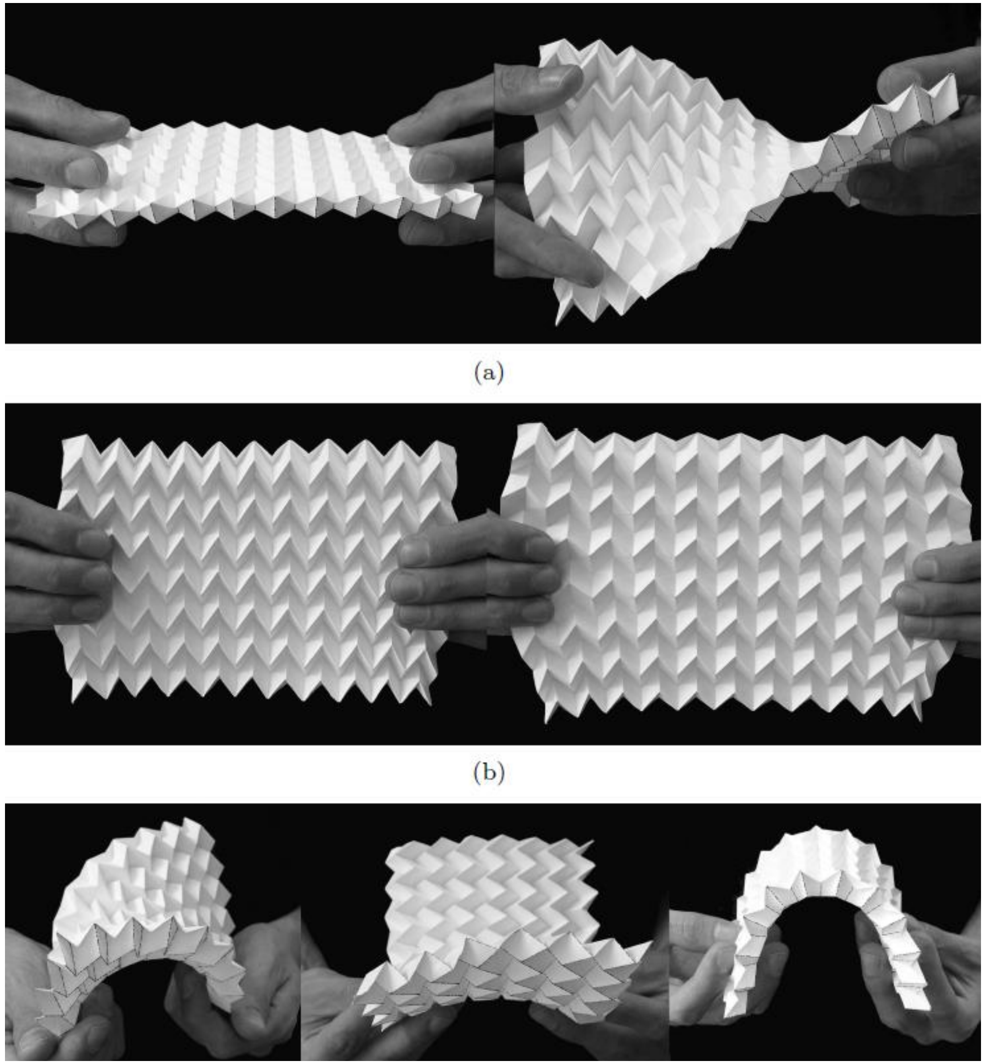
- $E(h)$: **agrees** with experiment
  crease = torsion spring
- $h_{\max}$: theoretical heights **not reached**, **qualitative** behaviour **confirmed**
  Newtonian air resistance, expansion and jumping

References

- M. Schenk, S. D. Guest: “Origami Folding: A Structural Engineering”, 5OSME, 2010
- *Z. Y. Wie et al.; Geometric Mechanics of Periodic Pleated Origami; PRL 110 215501, (2013)*
- *Quantamagazine.org/the-atomic-theory-of-origami-20171031/*

Out of plaine deformation





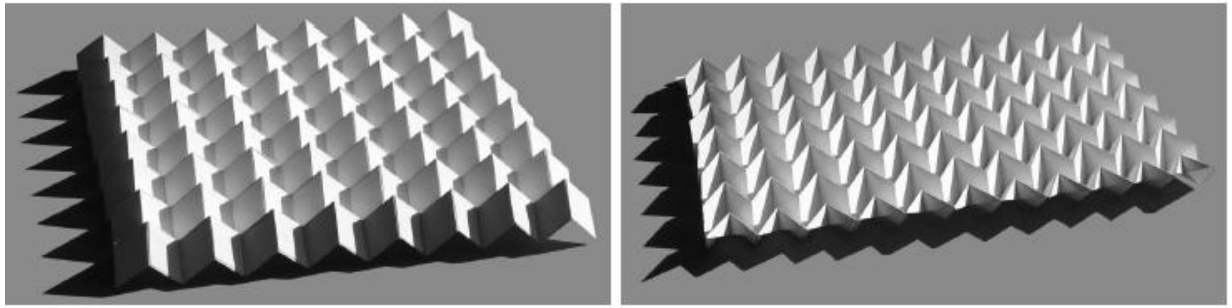
M. Schenk, S. D. Guest: "Origami Folding: A Structural Engineering", 5OSME, 2010

Other flat-folded patterns: eggbox

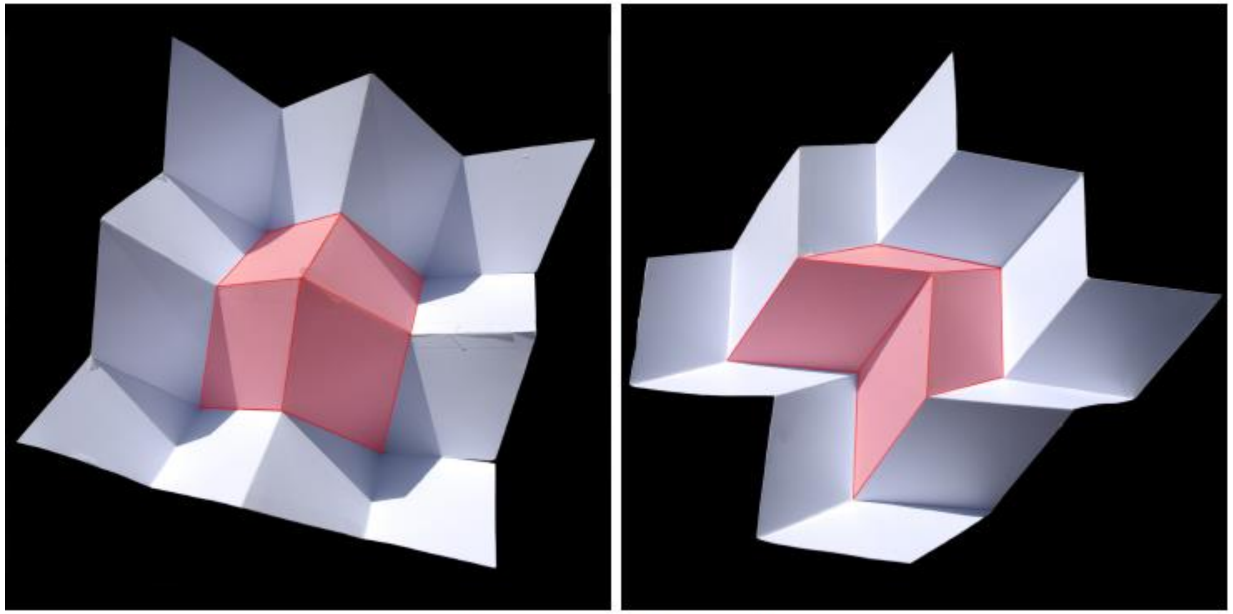
POISSONS RATIO

$$n = -\frac{e_{\text{trans}}}{e_{\text{long}}}$$

- Miura-Ori: negative for inplane deformation
- Eggbox: positive for inplane deformation



(a) overview of folded textured sheets

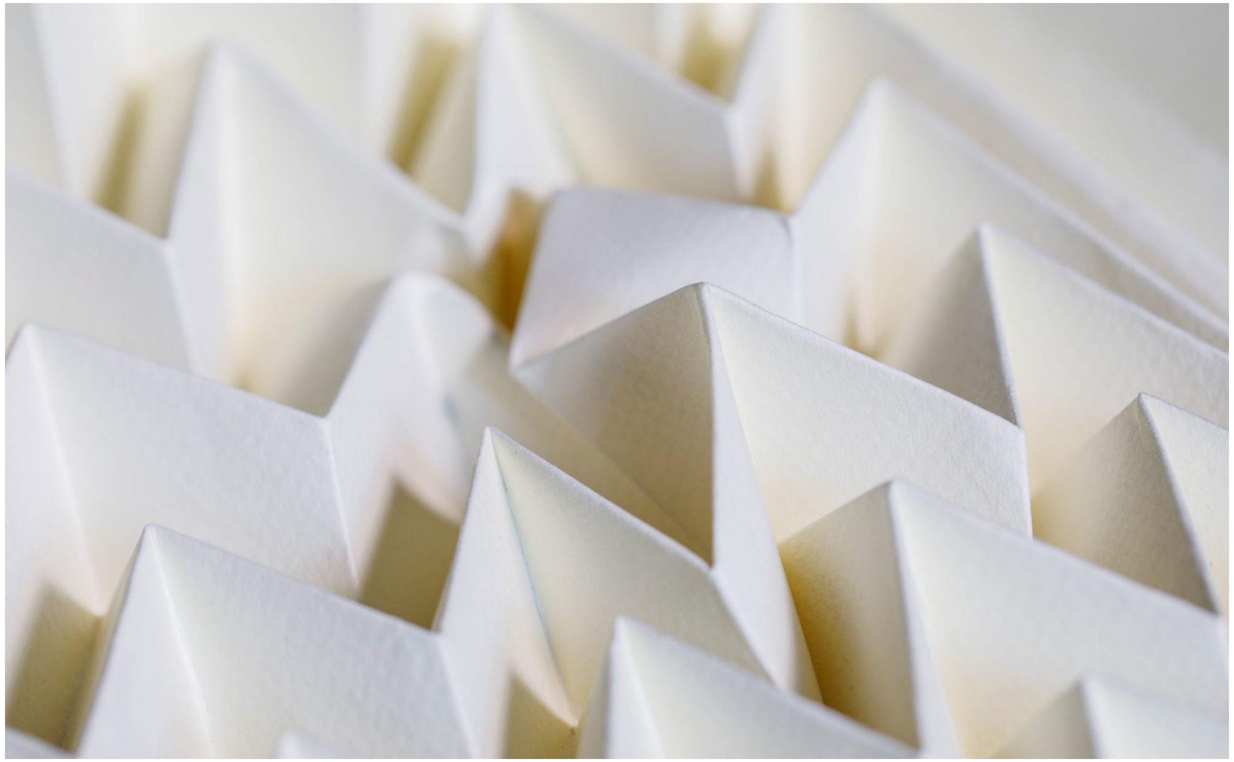


(b) close-up of unit cells

M. Schenk, S. D. Guest: "Origami Folding: A Structural Engineering", 5OSME, 2010

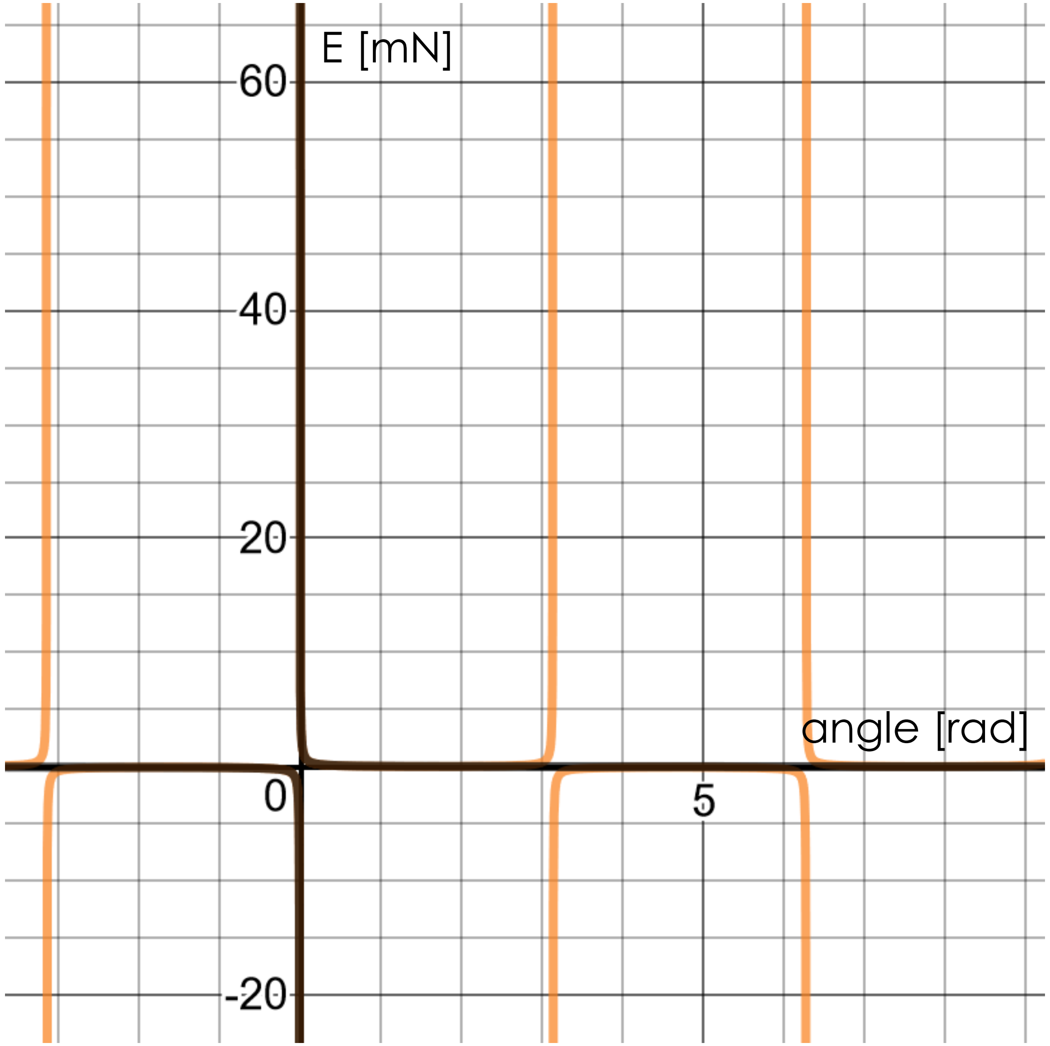
Defects

Defects **increase rigidity** and can be described as **excitations in as lattice** (quasiparticles)



Quantamagazine.org/the-atomic-theory-of-origami-20171031/

$$E(h) = 2D\frac{LH}{\sin\alpha}\left[\frac{1}{l_2}\left(\sin^{-1}\frac{h}{H} - \sin^{-1}\frac{h_0}{H}\right)^2 + \frac{1}{l_1}\left(\sin^{-1}\eta - \sin^{-1}\eta_0\right)^2\right]$$



Derivation of $E(h)$

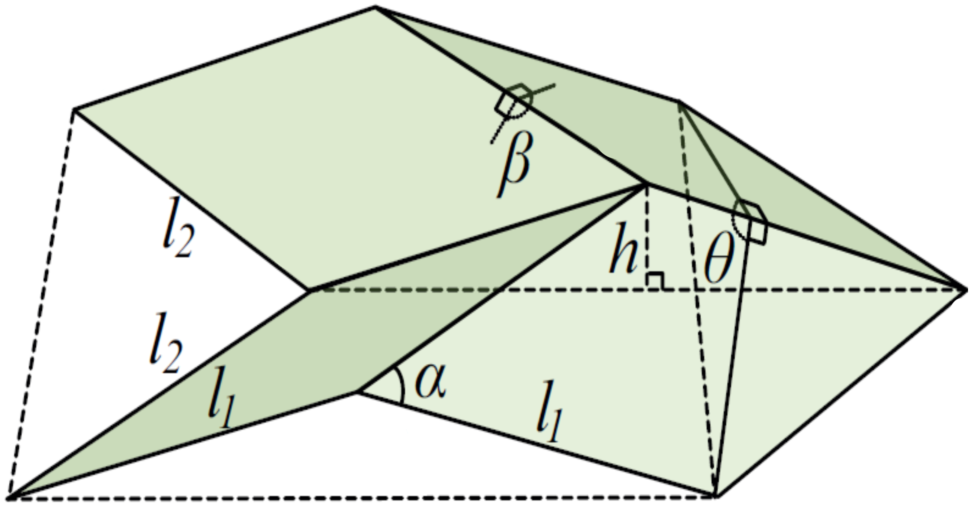
$$E = ND(l_1\Delta\theta^2 + l_2\Delta\beta^2)$$
$$\Rightarrow E(h) = 2D\frac{LH}{\sin\alpha}\left[\frac{1}{l_2}\left(\sin^{-1}\frac{h}{H} - \sin^{-1}\frac{h_0}{H}\right)^2 + \frac{1}{l_1}\left(\sin^{-1}\eta - \sin^{-1}\eta_0\right)^2\right]$$

where $\eta = \frac{h\cos\alpha}{\sqrt{H^2 - h^2\sin^2\alpha}}$

$$\beta = 2\sin^{-1}\left(\frac{l}{2l_1}\sin\frac{\theta}{2}\right) \qquad l = 2l_1\frac{\cos\alpha}{\sqrt{1-\sin^2\alpha\sin^2\frac{\theta}{2}}}$$

$$h = H\sin\frac{\theta}{2}$$

$$k_\theta = \frac{HL}{l_2\sin\alpha},\ k_\beta = \frac{HL}{l_1\sin\alpha}$$



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Flat folded origami

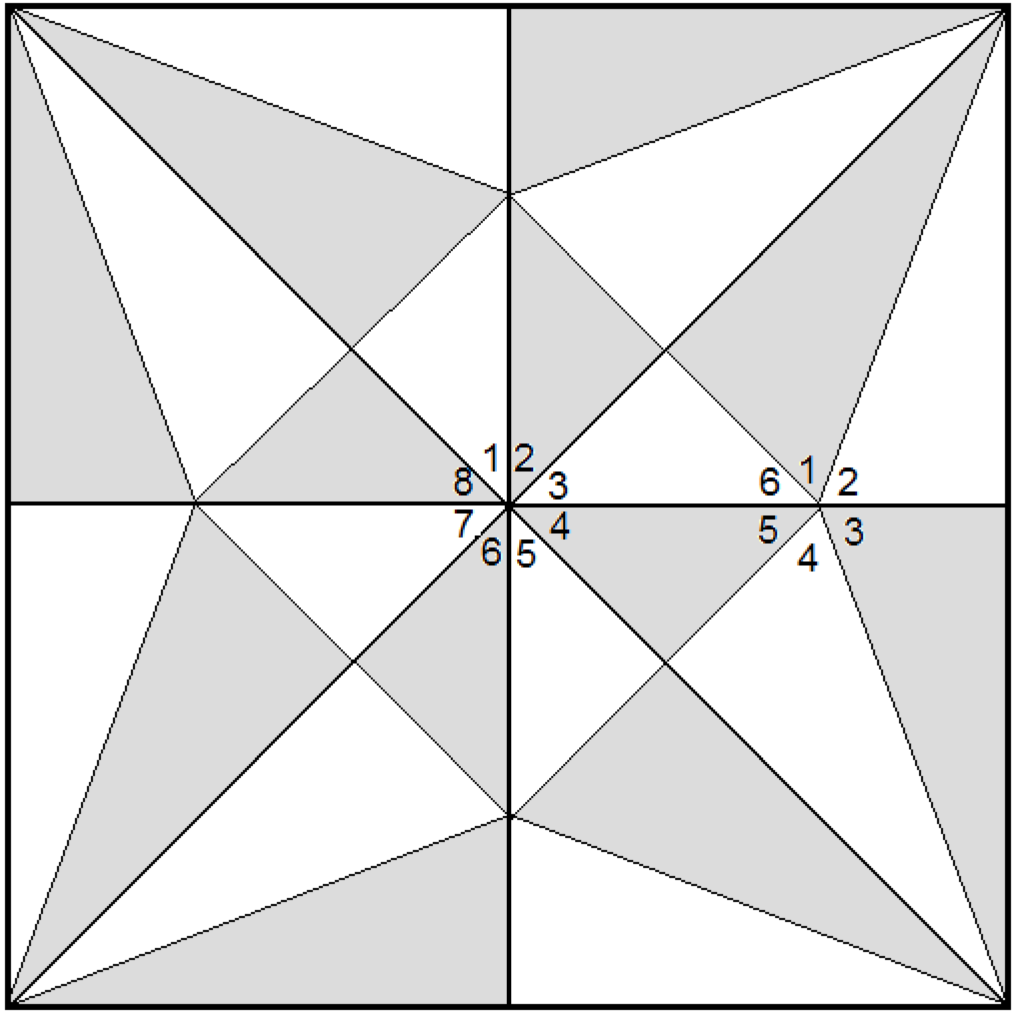
Maekawa Theorem

At every vertex in flat-folded origami, the difference between the number of mountain and valley creases is always two.

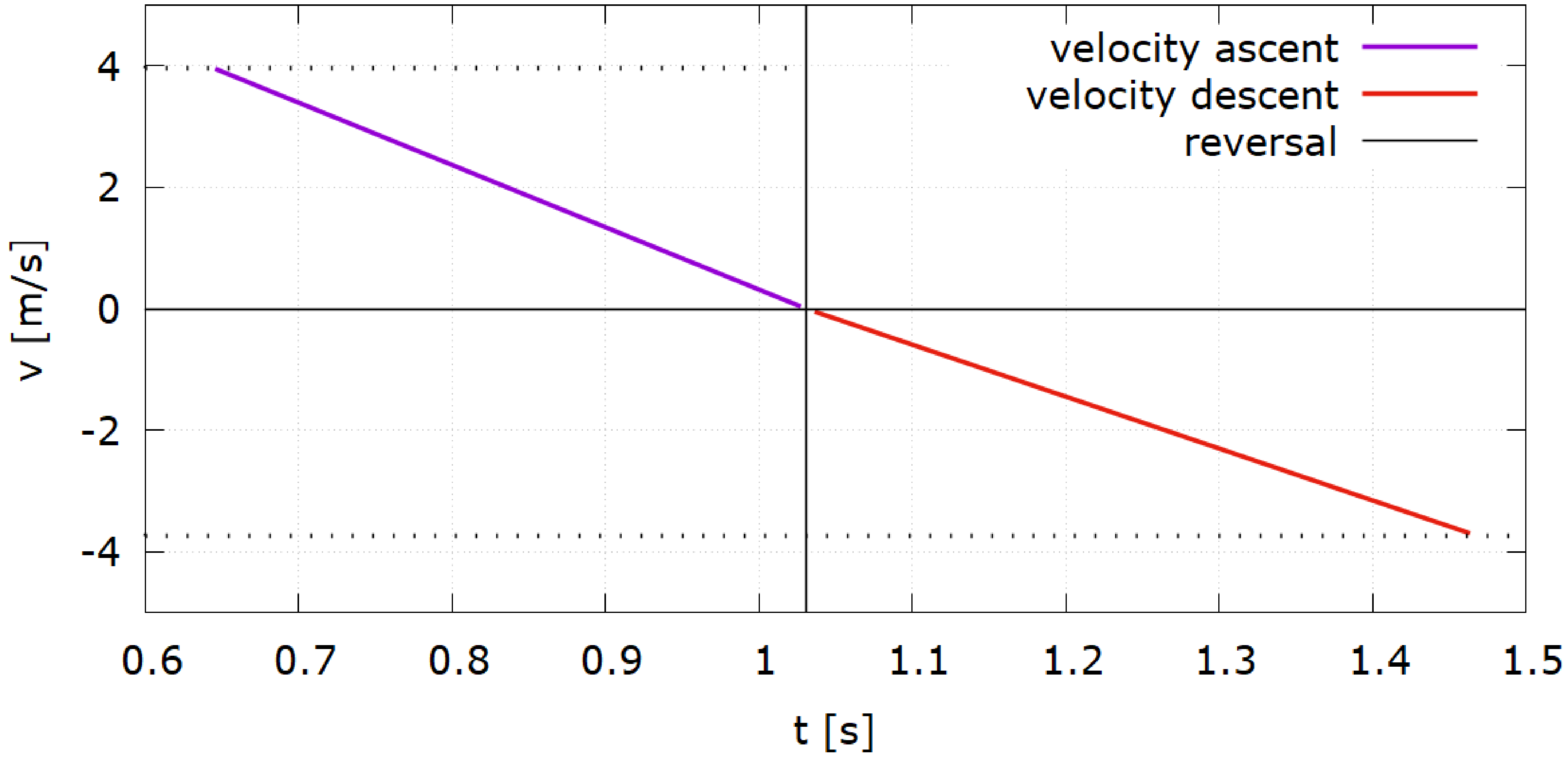
Kawasaki's theorem

Origami can only be flat-folded if the alternating sum of the angles at a single vertex adds to zero.

Naturalorigami.wordpress.com/2016/06/27/the-maekawa-theorem



Trajectory analysis with Viana



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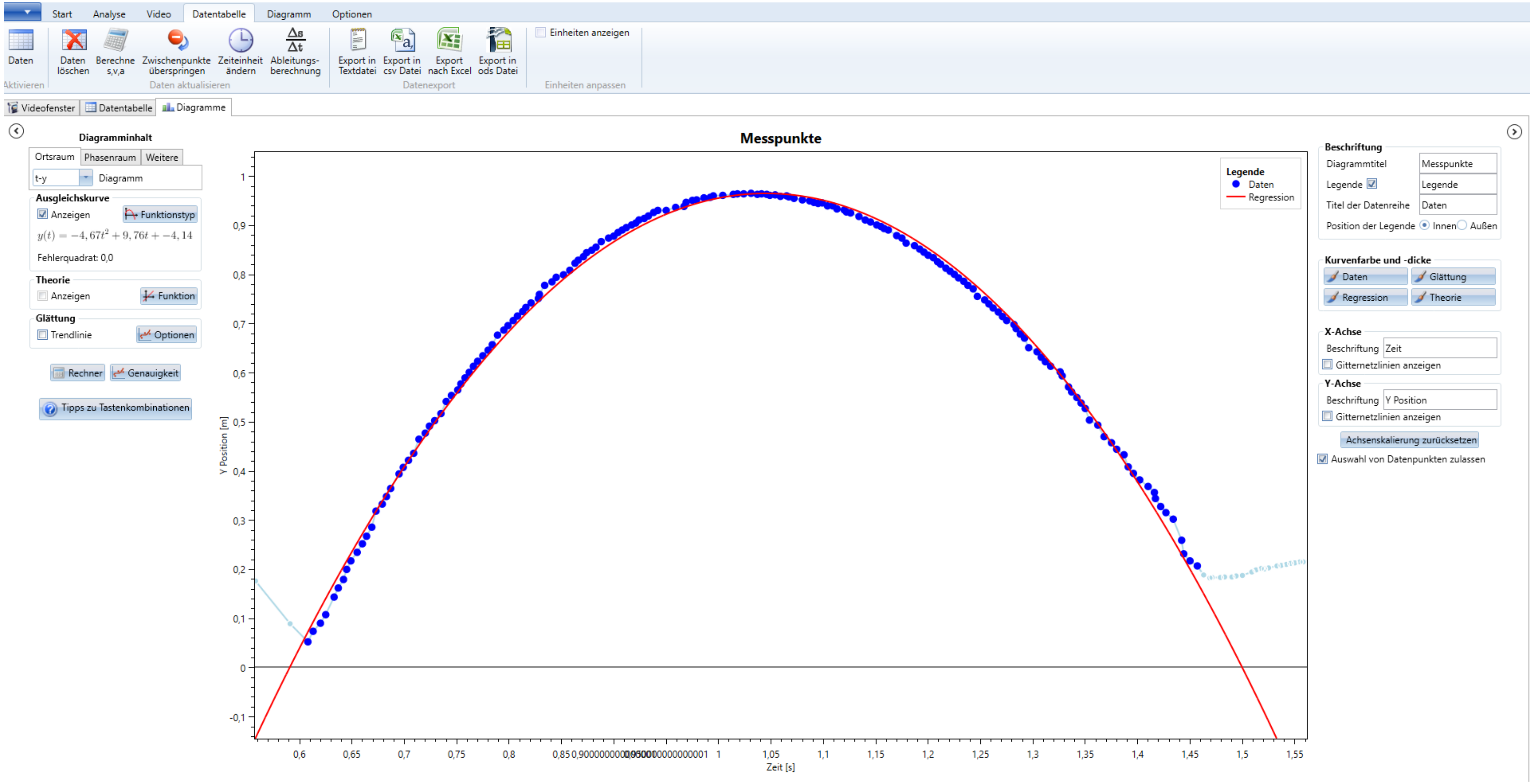
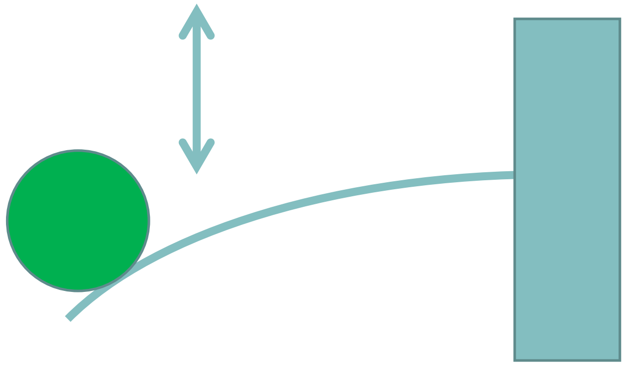


Additional ideas

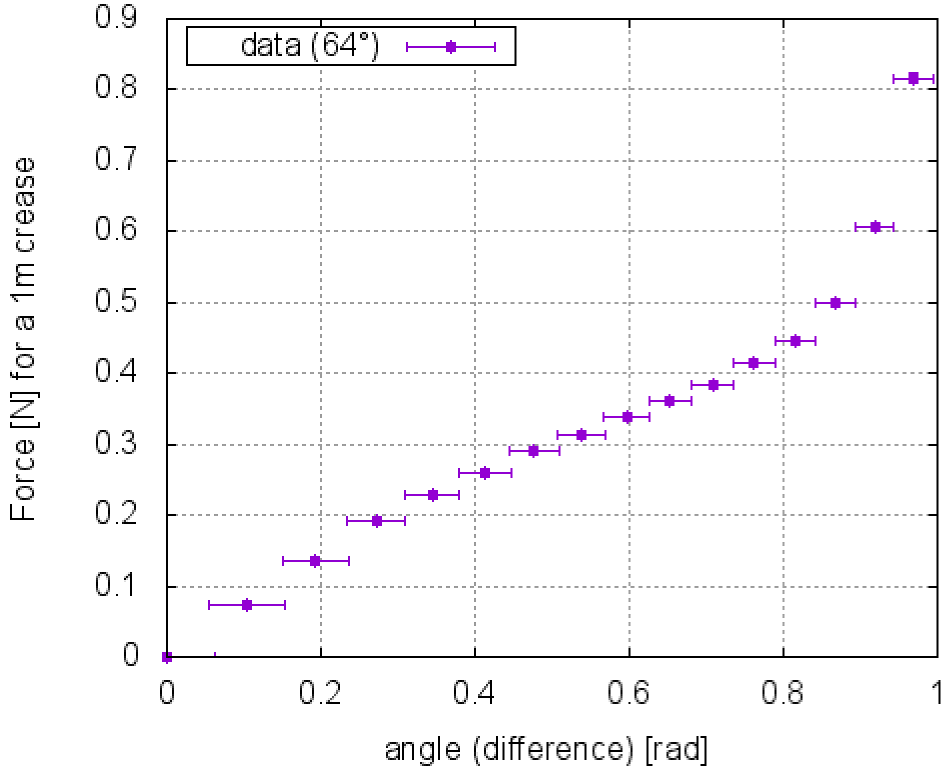


Alternative model for torsional spring

Obtain energy **stored in single crease** by integration

Sum up all creases of the pattern

Measurement: $E_{\mathrm{crease}} = 78.23 \ \frac{\mu\mathrm{J}}{\mathrm{cm}}$



similarities of energies in Zig-Zag patterns

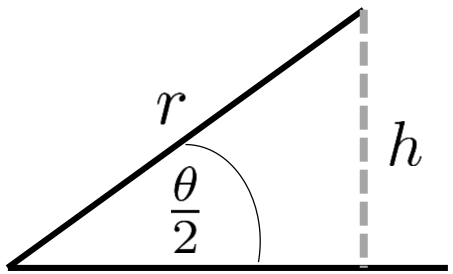
$$F = -K\Delta h = -K(h - h_0)$$

$$h = r\sin\frac{\theta}{2} \Rightarrow F = -Kr(\sin\frac{\theta}{2} - \frac{\theta_0}{2}) \sim r$$

7 creases: $F_{tot} = 7F,\ r = \frac{L}{7}$

9 creases: $F_{tot} = 9F,\ r = \frac{L}{9}$

→ factors cancel out



Energy loss due to expansion

$$E = \int F dh$$

$$F = -m(h)g = \frac{M}{l}hg \Rightarrow E = \int_0^l \frac{M}{l}hg\ dh = \frac{1}{2}\frac{M}{l}gl^2 = \frac{1}{2}Mgl$$