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Problem 3
Ink Tree

❑ The problem

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- When a drop of ink is injected inside especially still water, or dropped very close to its surface, it firstly forms a ring of ink which then divides into smaller rings (see video). The process repeats again and again and forms a tree-like structure of ink. What is the maximal number of ring divisions that one can see and how does it depend on the important parameters?

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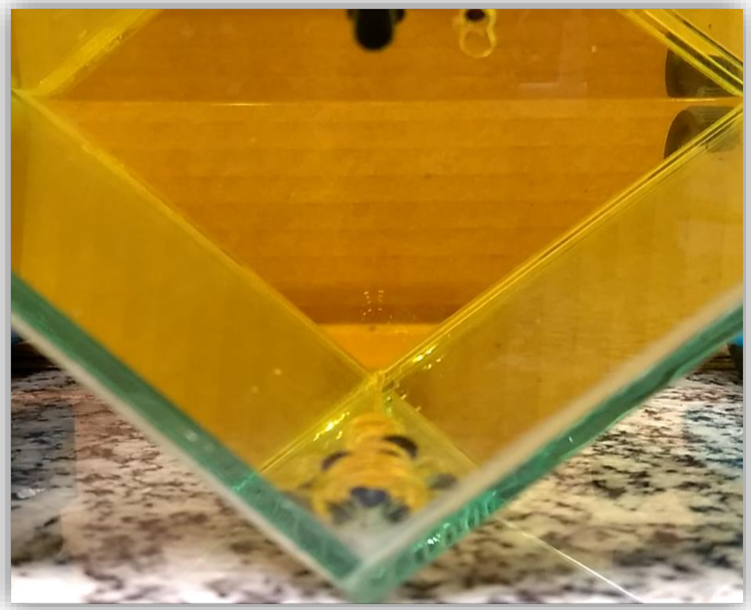
❑The phenomenon

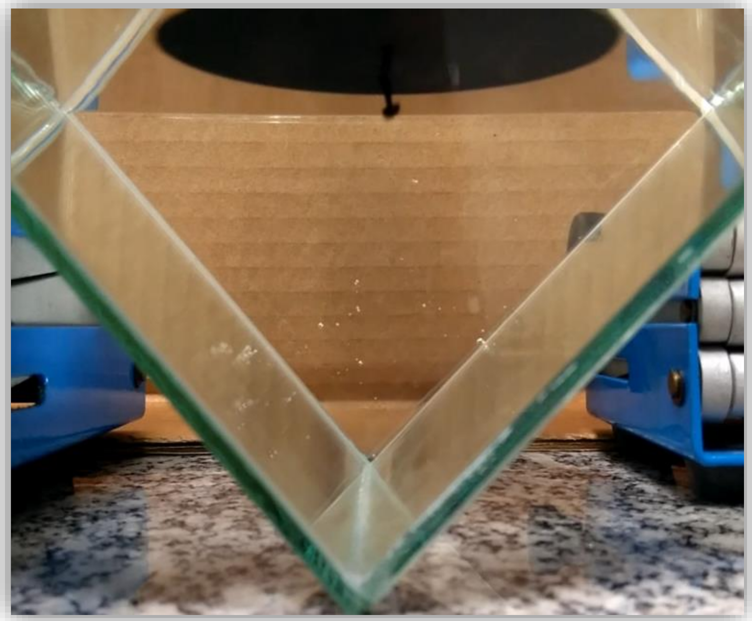
- Ink droplets injected in water
- Formation of vortex rings
- Rings divide into new rings
- Tree-like structure

❑Rings always formed?

- For immiscible liquids (oil) [1] :
  - Expressive superficial tension;
  - Minimization of contact area;
  - Droplets remain spherical;



- For miscible liquids (water):
  - No superficial tension;
  - Droplets do not remain spherical;
  - Formation of rings!



[1] THOMSON, Joseph John et al. On the formation of vortex rings by drops falling into liquids, and some allied phenomena,1886.

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❑ Fluids

- First of all, the ink must be miscible in water;
- We must also have: $\rho_{ink} > \rho_{water}$
- Then the actual values for density and viscosity changes the evolution of the phenomenon

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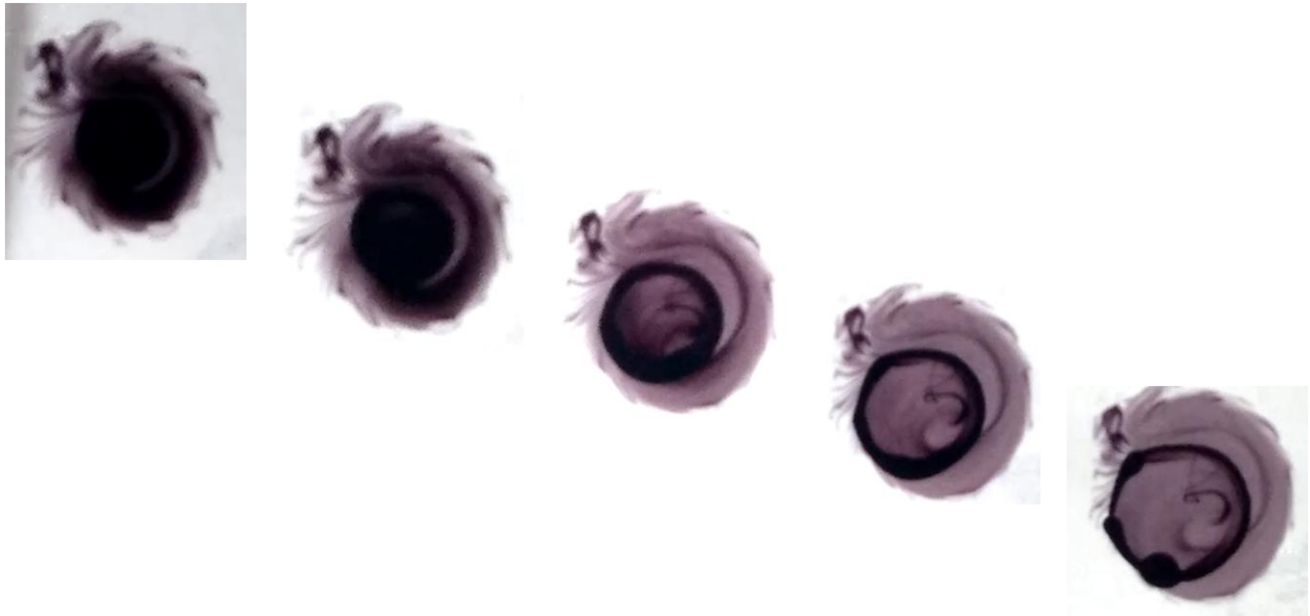
❑ The reasons why

- Viscous flow: horizontal vortex motion;
- Droplet gets flat;
- Water penetrates the flattened ink drop;
- The plane droplet opens in a ring – energetically more favorable due to vorticity!
- Kelvin-Helmholtz instability!



❑ Rayleigh-Taylor instability

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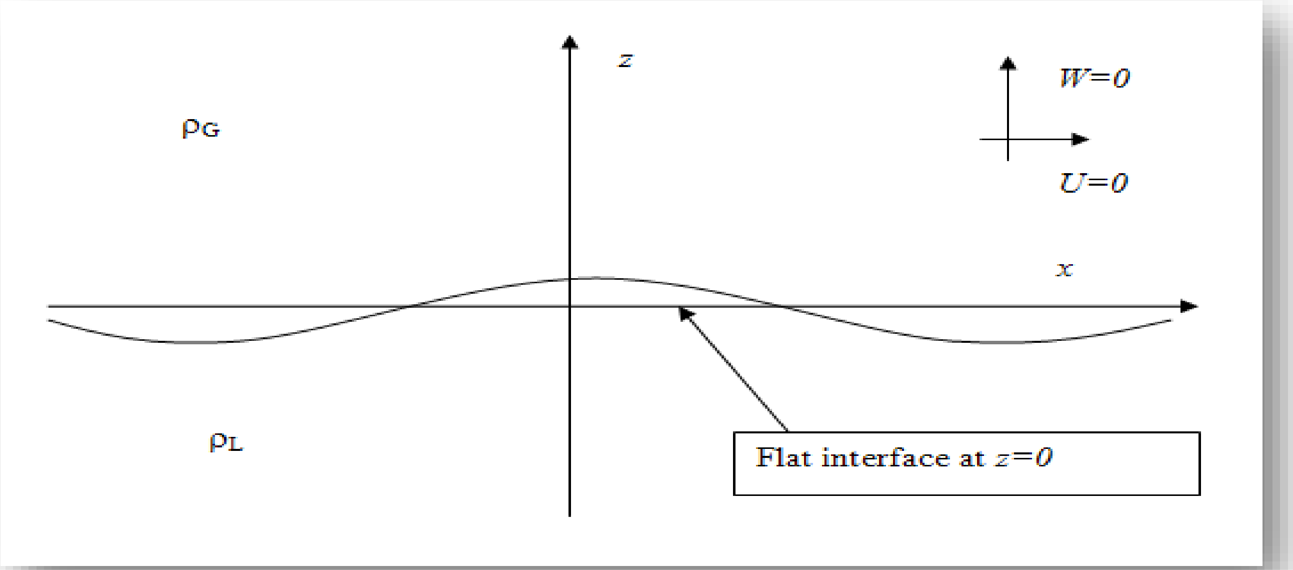
- Two fluids of different density;
- Droplets of the denser one are injected in the less denser;
- Perturbed interface:
- Denser droplets go down as the lighter fluid goes up



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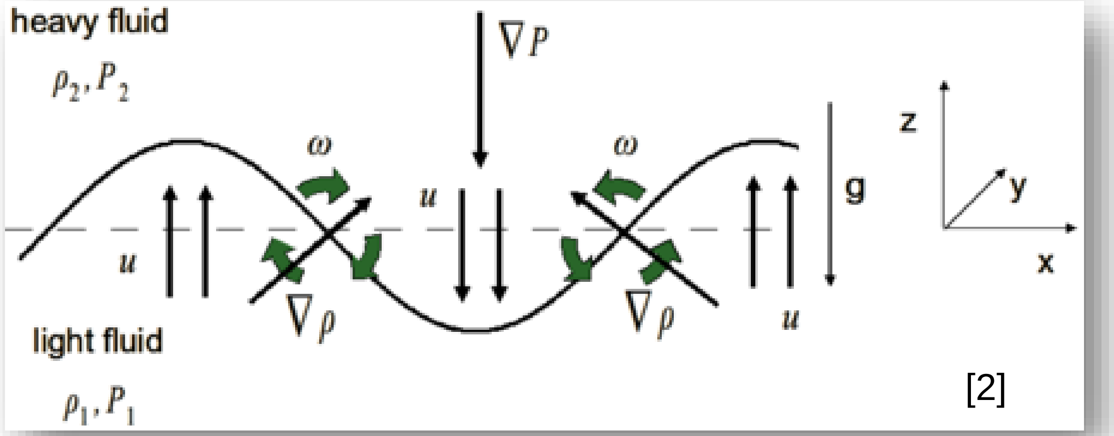
❑ Baroclinic torque

- Vorticity as the ink falls through the water.
  - Baroclinic torque!
  - Misalignment of the pressure and density gradients at the perturbed interface:

$$\frac{\partial \boldsymbol{w}}{\partial t} + (\boldsymbol{v} \cdot \nabla)(\boldsymbol{w}) = \frac{1}{\rho^2} \nabla \rho \times \nabla \mathrm{p}$$



Equations

w: *vorticity*

ρ: *fluid density*

p: *pressure*

[2] Roberts, M.S. , Experiments and simulations on the incompressible, Rayleigh-Taylor instability with small wavelength initial perturbations, 2012

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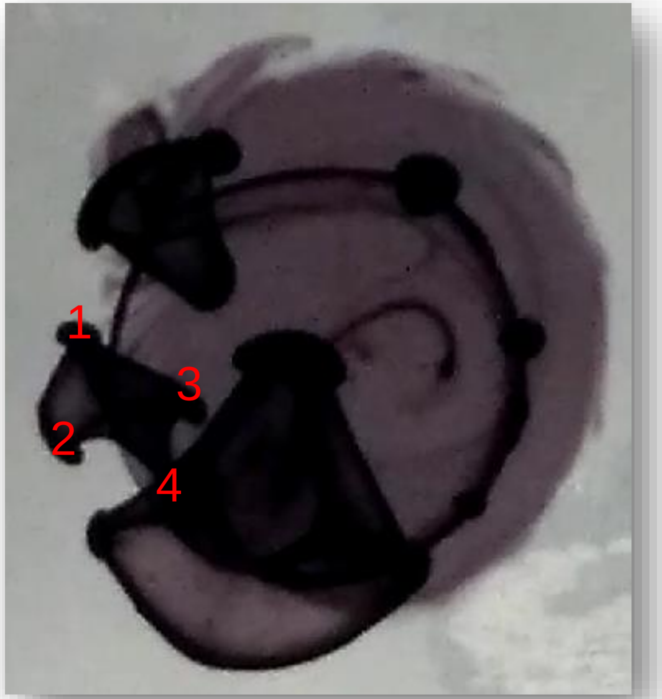
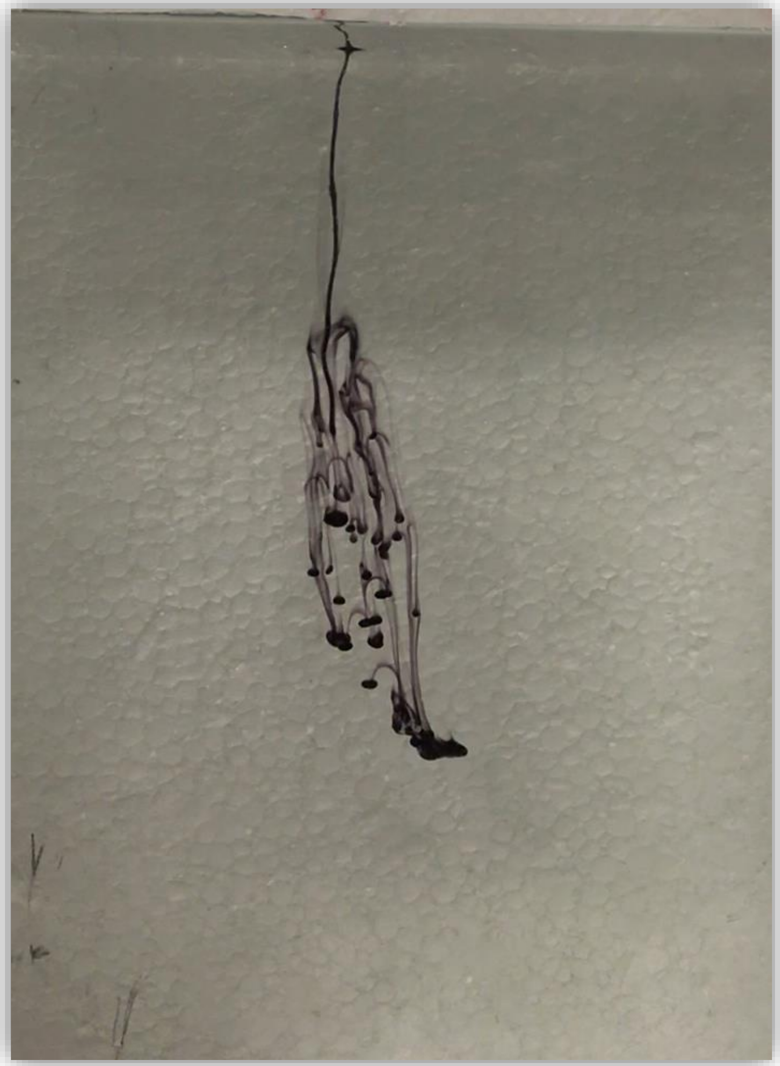
❑ Ring fissioning

- Ring is itself unstable, therefore it fissionates again;
- Multiple openings
  - Not always three!



❑ Maximum number of rings

- Gravity directs the ink flow downwards: prefered diffusion direction;
- Drag and ink formation process slow down the flow;
- At some point, no more directed flow;
- Whole tree moves together, no more rings;



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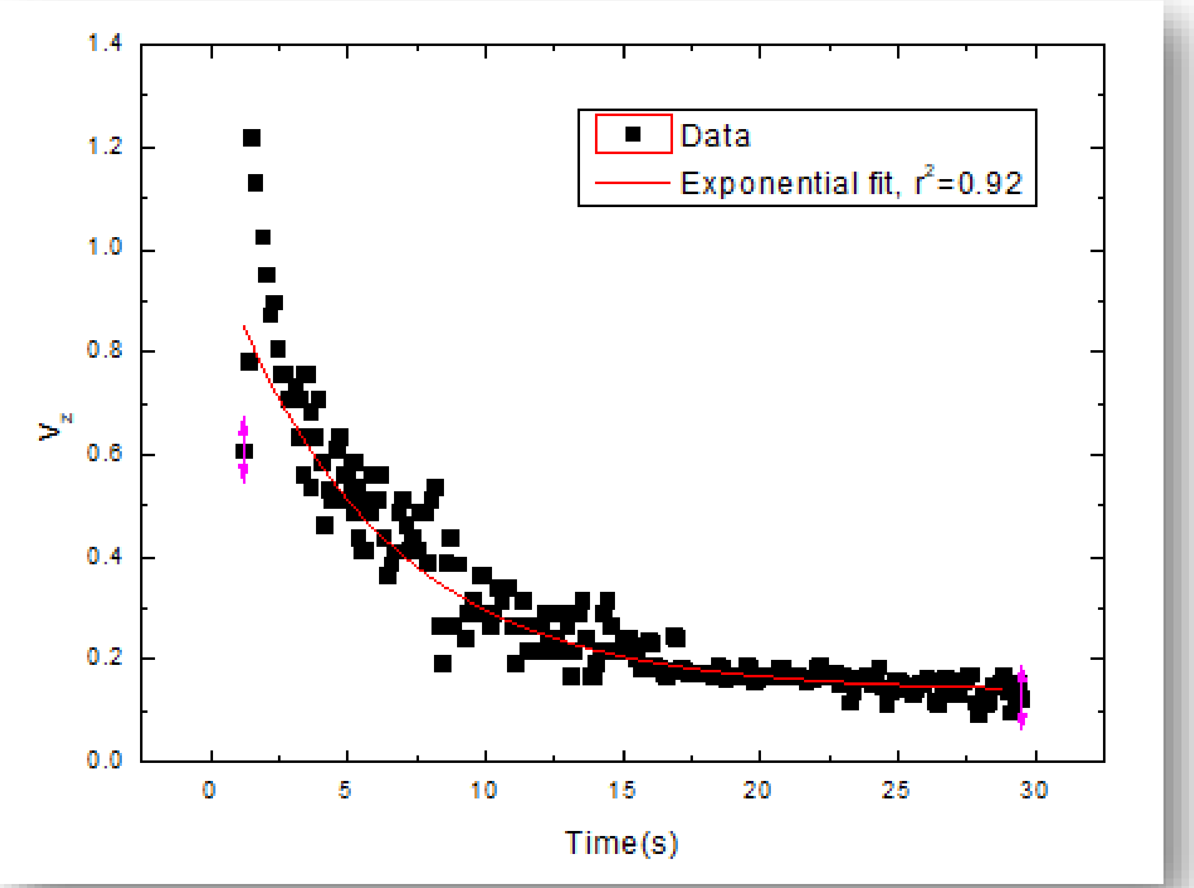
❑Velocity profile



- Agrees with theory!
- Velocity in function of time due to the effect of an opposing linear force

$$v_z = v_0 + A_1 e^{-bt}$$

$v_0 = (0.14 \pm 0.0078)$ [pixels]

$A_1 = (0.8 \pm 0.028)$

$b = (5.8 \pm 0.26)$[s]

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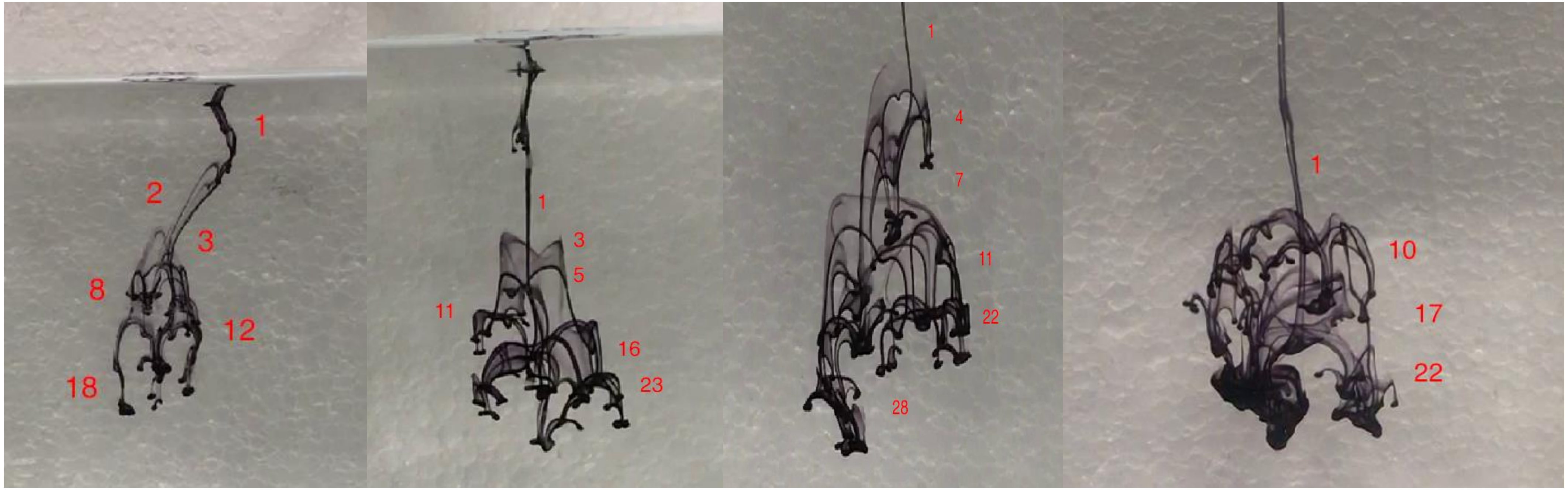
❑Distribution of ring divisions

- We counted the number of divisions in each ring formation for a fixed temperature:



- Parameter varied: quantity (volume) of ink ionjected into water and entry velocity.

❑ Important parameters

- Radial separation (opening of the tree) as a function of temperature.

Lateral view 10°C

Top view 10°C

$$D \alpha T$$

Lateral view 25°C

Top view 25°C

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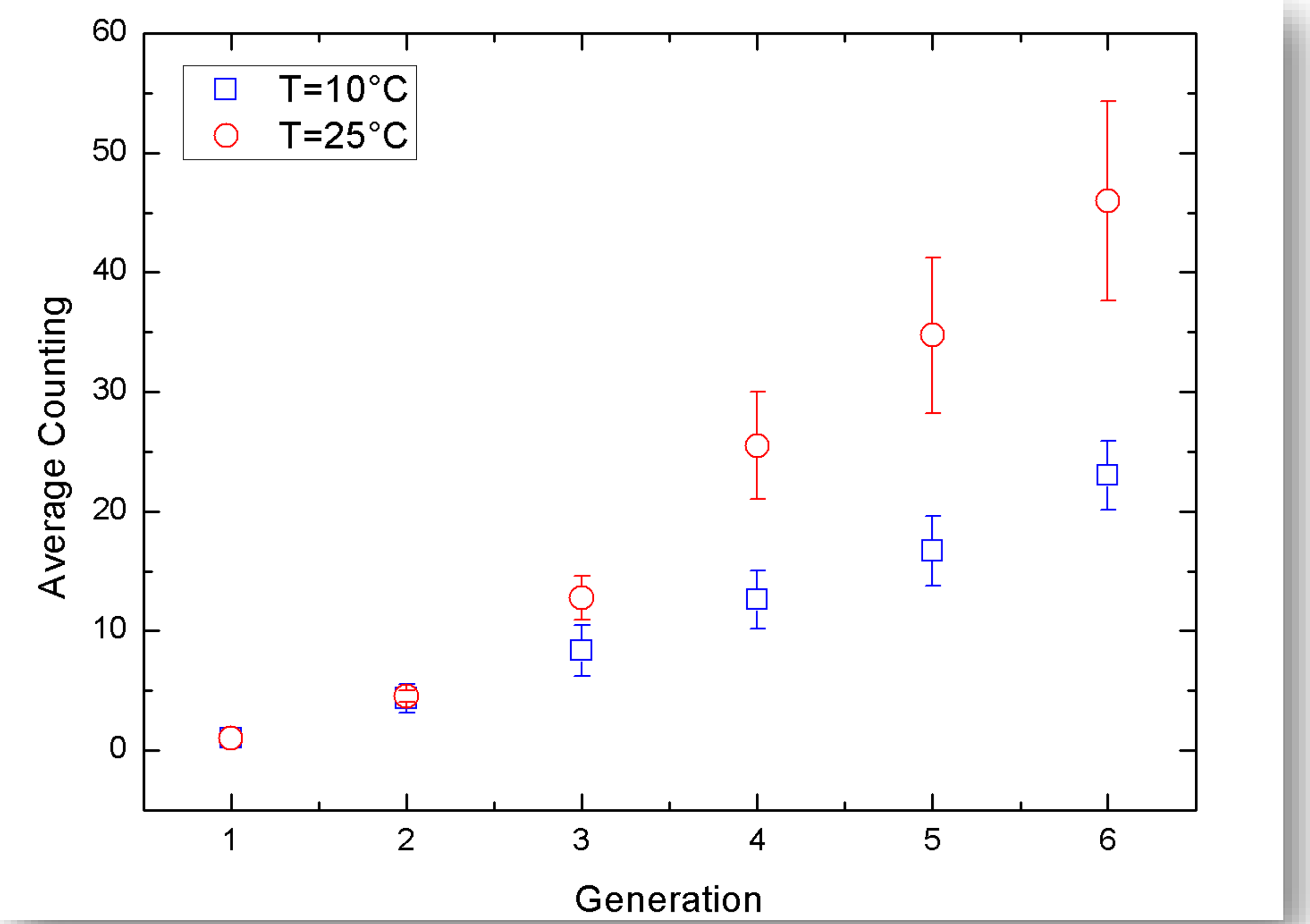
❑Distribution for ring divisions for different temperatures



Conclusion/Summary

- Rings created only for miscible fluids;
- Once in water, ink moves in a thread;
  - Viscous drag makes it flat;
  - Further drag makes it a ring;
- Rayleigh-Taylor instability;
  - Excessive surface energy brakes the ring;
- The process repeat;
- Flow in z direction dissipates mechanical energy in z;
- Dependence on temperature and boundary conditions;

❑References

- [1] THOMSON, Joseph John et al. On the formation of vortex rings by drops falling into liquids, and some allied phenomena,1886.
- [2] Roberts, M.S. , Experiments and simulations on the incompressible, Rayleigh-Taylor instability with small wavelength initial perturbations, 2012

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❑ Boundary conditions

- Influence from walls
  - Ink interacting with walls;
  - Thread is redirected;
  - For greater control: experiments performed far from walls

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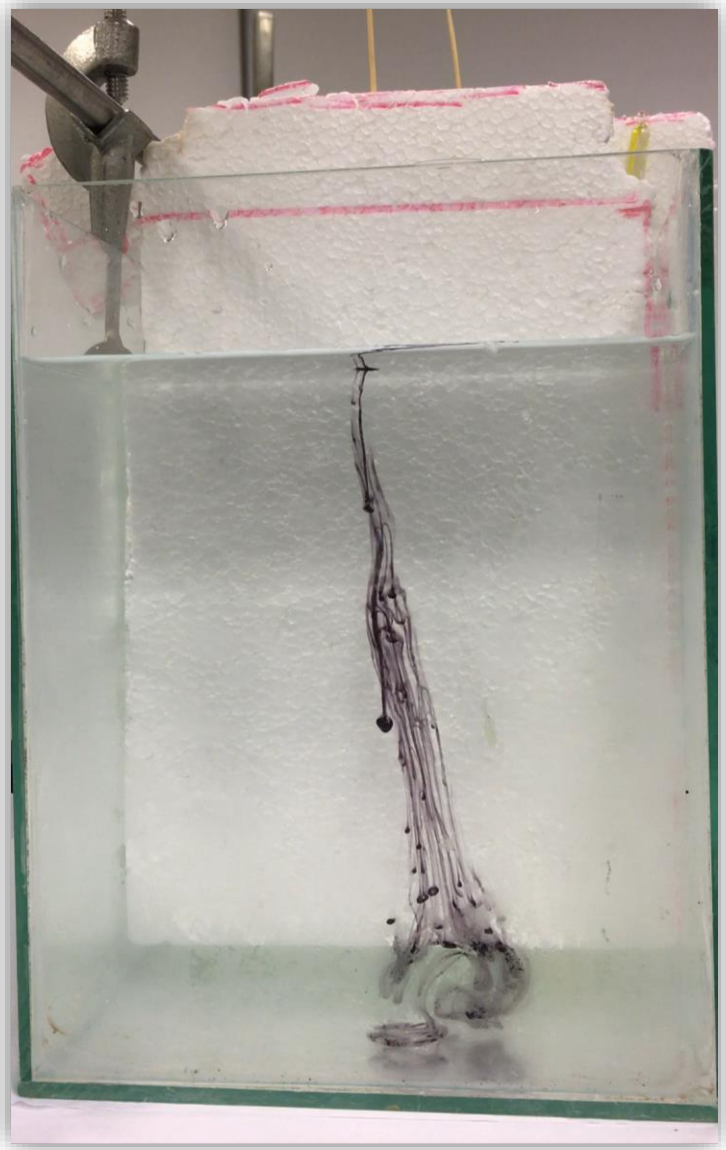
❑ Boundary conditions

- Influence from the bottom
  - The ink may reach it before the maximum number of rings is achieved;
  - Flow is reflected back upwards;
  - Further turbulence and convection stream;
  - For better control: experiments performed in a sufficiently tall recipient.



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❑ No more rings

- At some point, no further divisions are observed;
- The maximum number of rings is achieved.
- Explanation comes from Fick's Law

$$\boldsymbol{J} = -D\nabla\phi$$

Equations

$\boldsymbol{J}$: $diffusion\ flux$

D: $diffusivity$

ϕ: $ink\ concentration$

❑ No more rings

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- Diffusivity tensor: $D = D_{ij} = D^{ij}$, so that, in terms of components:

$$J^i = -D^{ij}\,\partial_j\phi$$

- Due to the action of gravity (at direction z), we can think of a effective diffusivity tensor, D^{eff}, such that:

$$D_{zz}^{eff} > D_{yy}^{eff} \text{ and } D_{zz}^{eff} > D_{xx}^{eff},$$

though it is reasonable to suppose $D_{xx}^{eff} = D_{yy}^{eff}$.

- Diffusion is more favorable at the direction of the flow.
- Drag makes $D_{xx}^{eff} = D_{yy}^{eff} = D_{zz}^{eff}$
- No longer a diffusion direction more favorable.

Ink diffuses equally in all directions: no more rings are formed.