Sound thermometer

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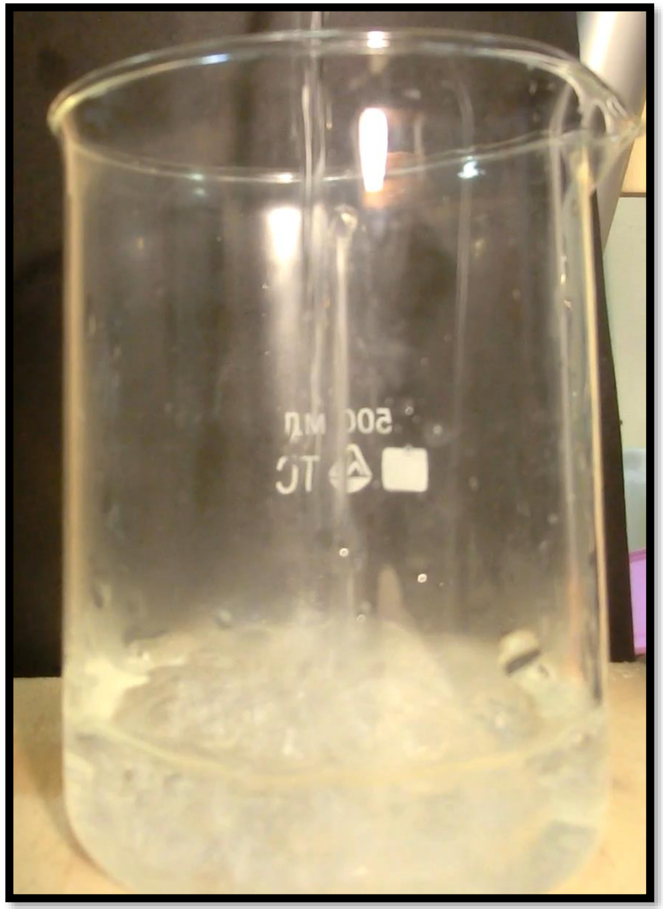
The problem

Devise a method to obtain the temperature of a fluid by listening to the sound emitted when it is poured into a cup. State the precision, accuracy and the limits of your method as well as the important parameters of the fluid.

Sound of pouring

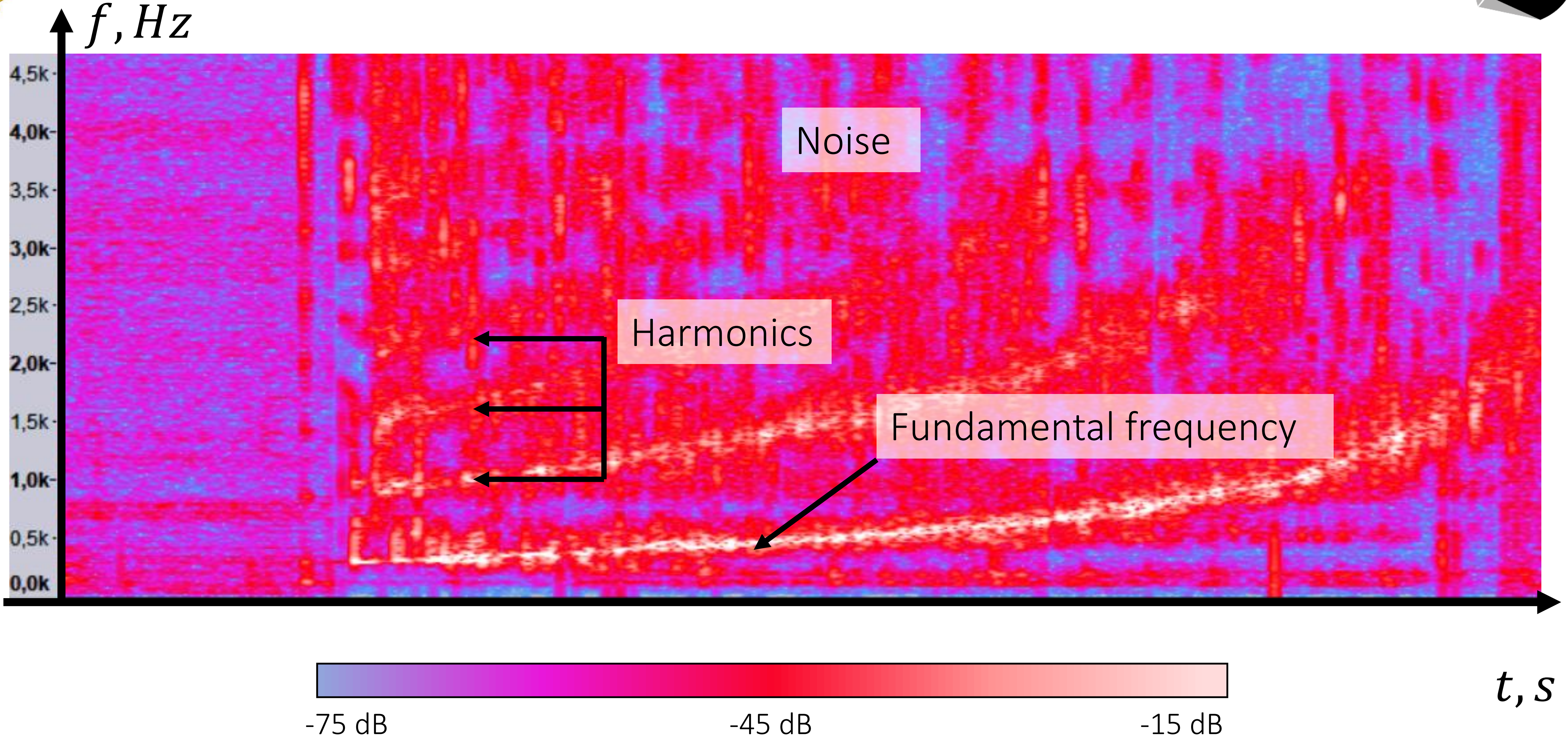


Hot

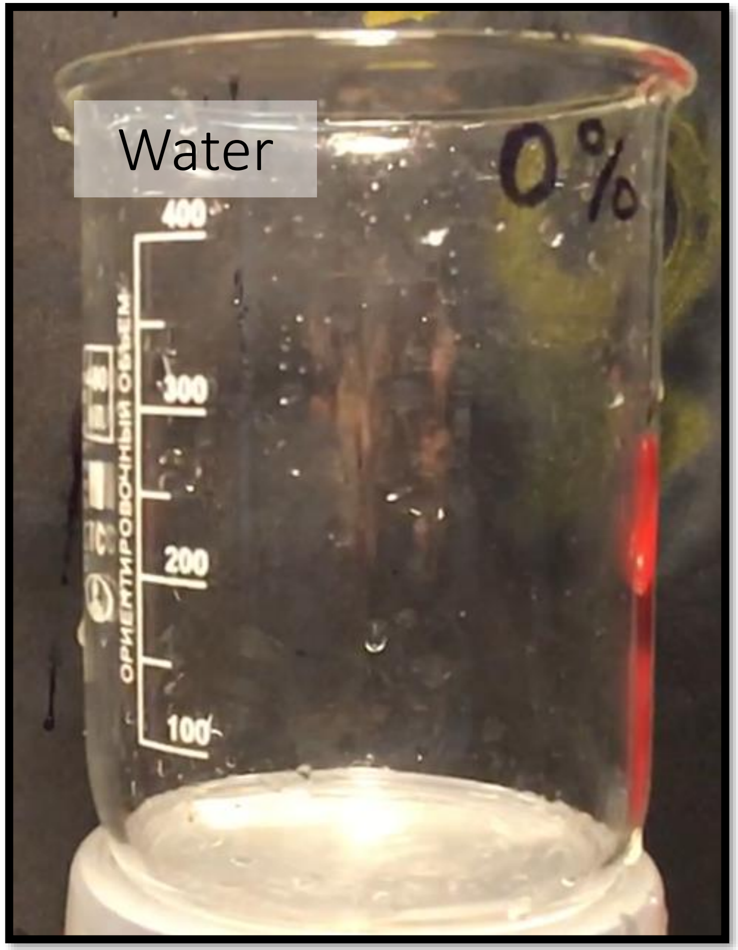
Cold

Theoretical explanation

Sound of pouring



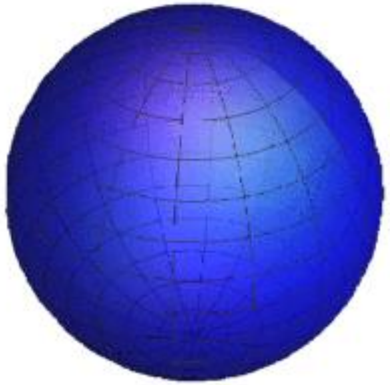
Sound emitters



The main sound emitters are bubbles

Oscillations of bubbles in the water

Minnaert resonance: oscillations of air



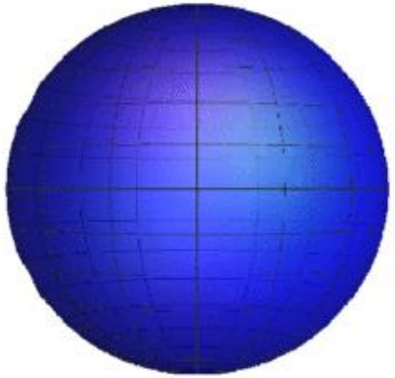
$$f = \frac{1}{2\pi r}\sqrt{\frac{3\gamma p_0}{\rho}}$$

f – fundamental frequency
γ – adiabatic coefficient of air

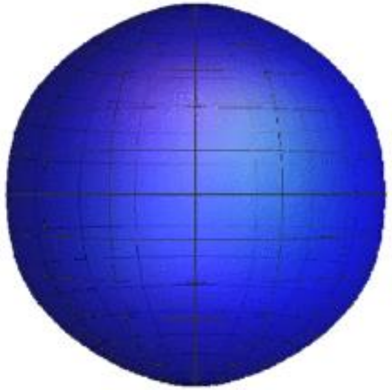
p_0 – pressure of fluid
ρ – density of fluid
r – radius of bubble

Oscillations of surface

$n = 1$



$n = 3$



$$f_n^2 \approx \frac{1}{4\pi^2}(n-1)(n+1)(n+2)\frac{\sigma}{\rho r^3}$$

n – number of harmonic
σ – surface tension

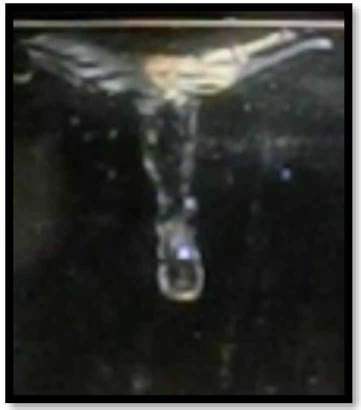
Applicable for small bubbles (r < 1 mm)

W. Moss, H. Yeh "Sounding Liquids: Automatic Sound Synthesis from Fluid Simulation"

Bubbles forming

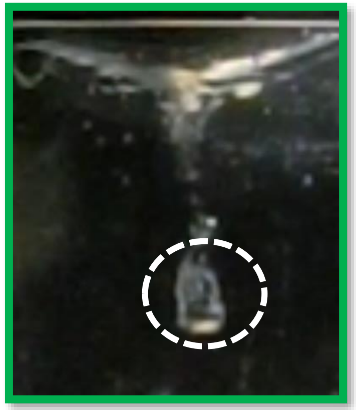
480 fps

Column



Collapse

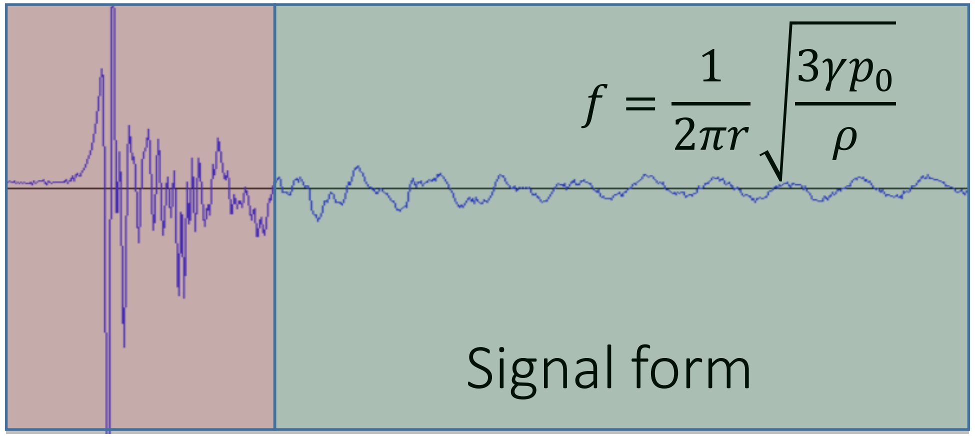
Bubble



Frequency of resonator of column

$$f' = \frac{c}{4L}$$

c – speed of sound



$$f'_{exp} = 4782 \text{ Hz}$$

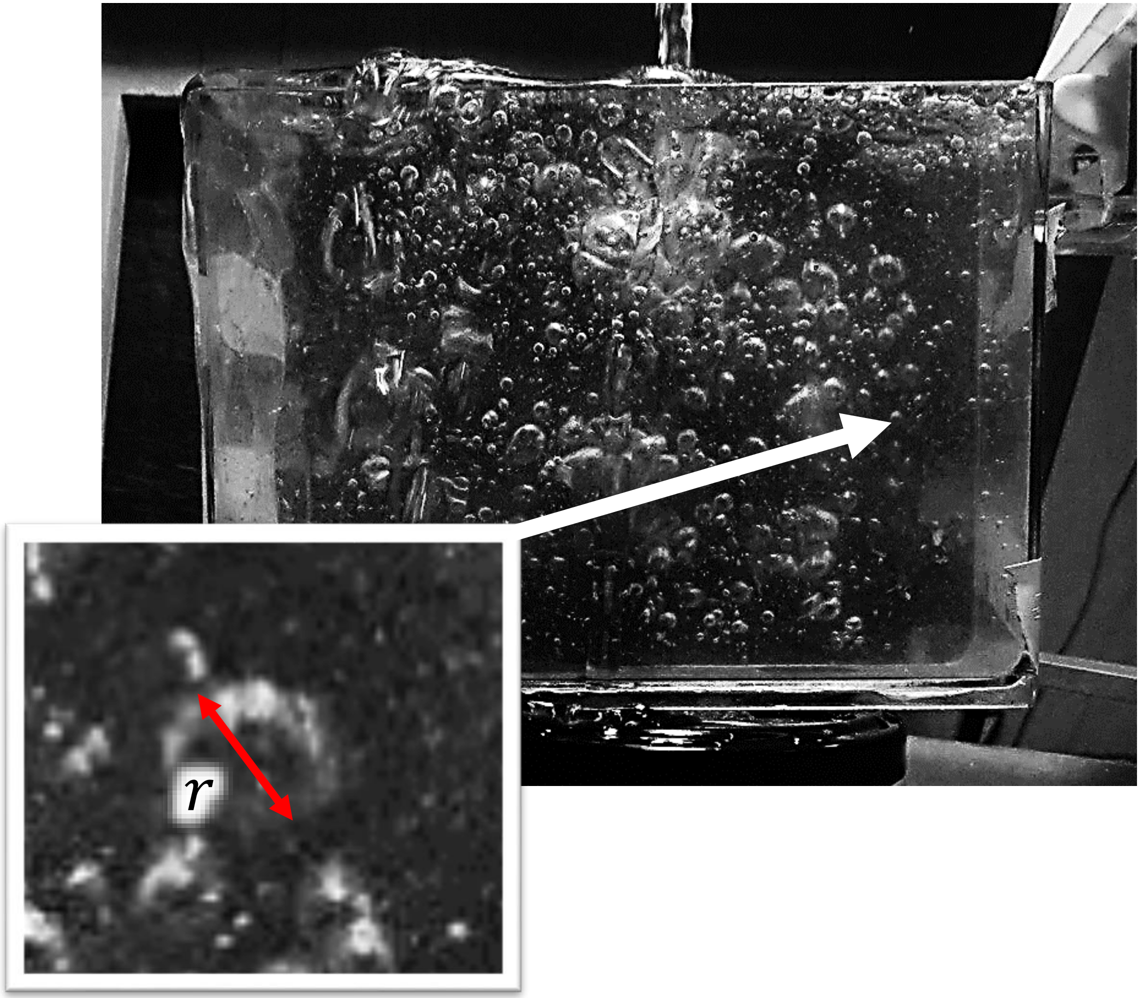
$$f'_{theor} = 4000 - 5000 \text{ Hz}$$

$$f_{exp} = 1016 \text{ Hz}$$

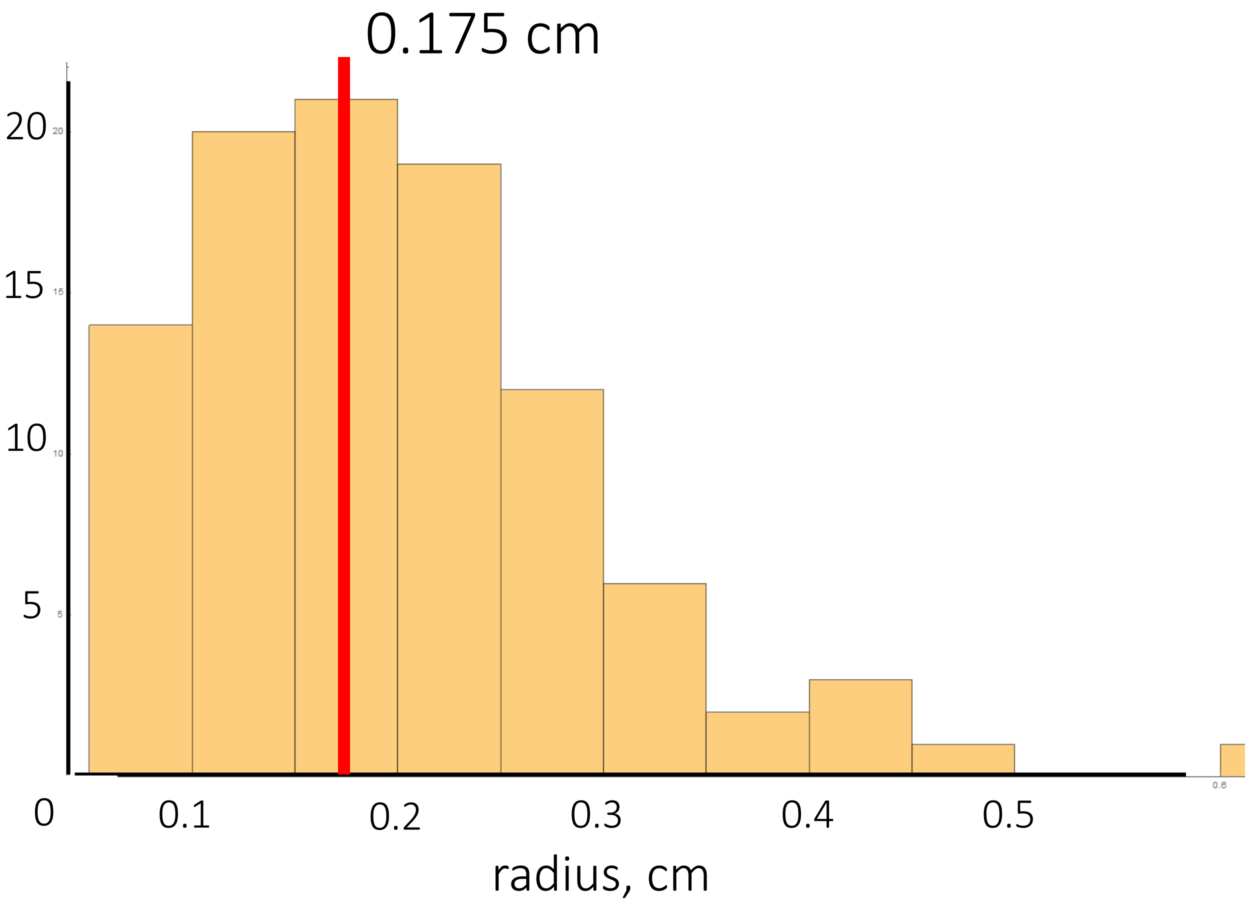
$$f_{theor} = 800 - 1600 \text{ Hz}$$

Radius distribution

Bubbles of different radii

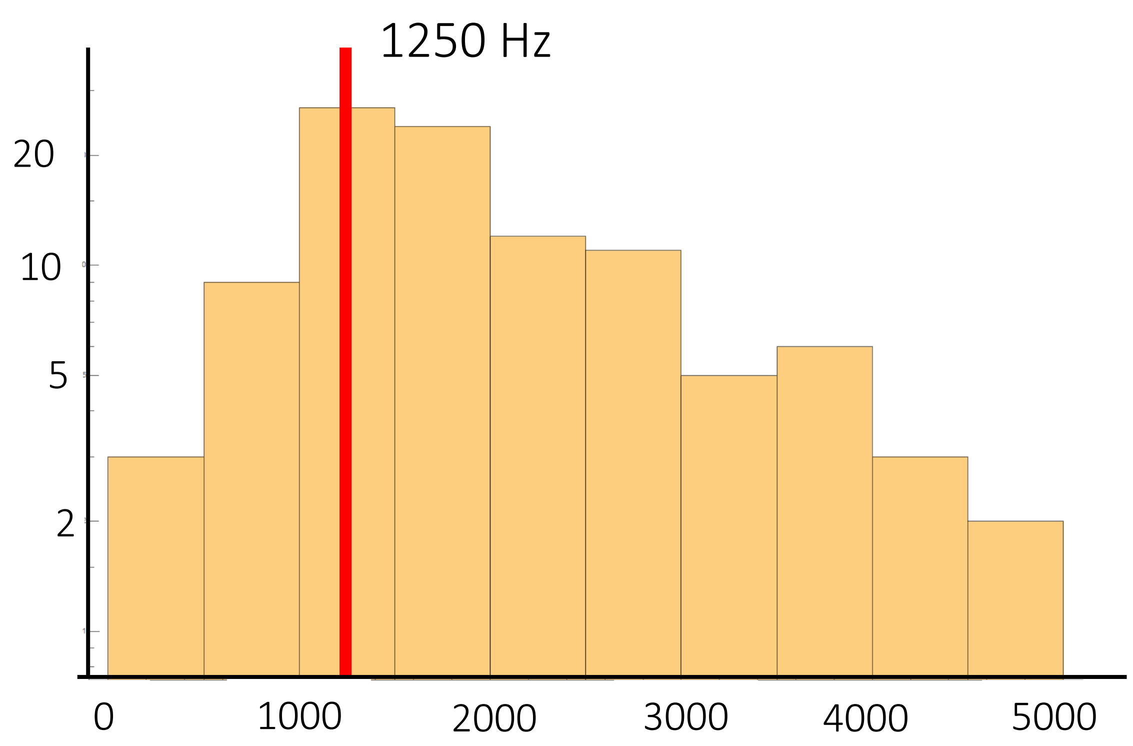


Distribution of bubbles radii

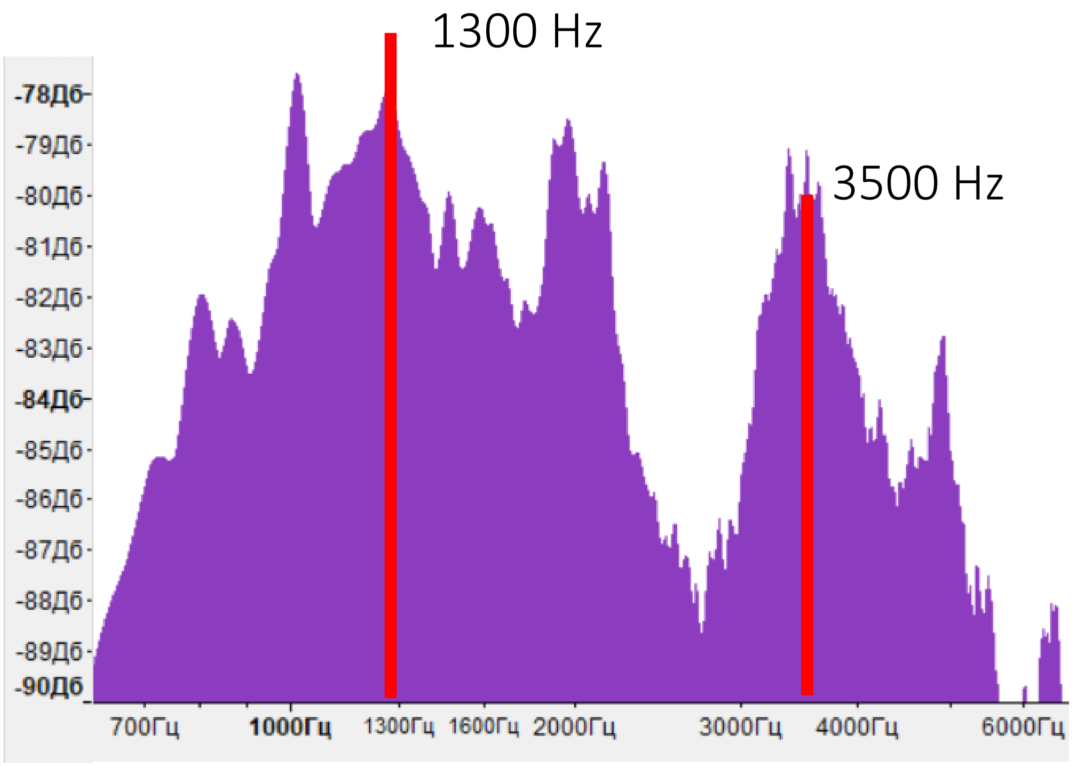


Spectrum of sound

Generated spectrum



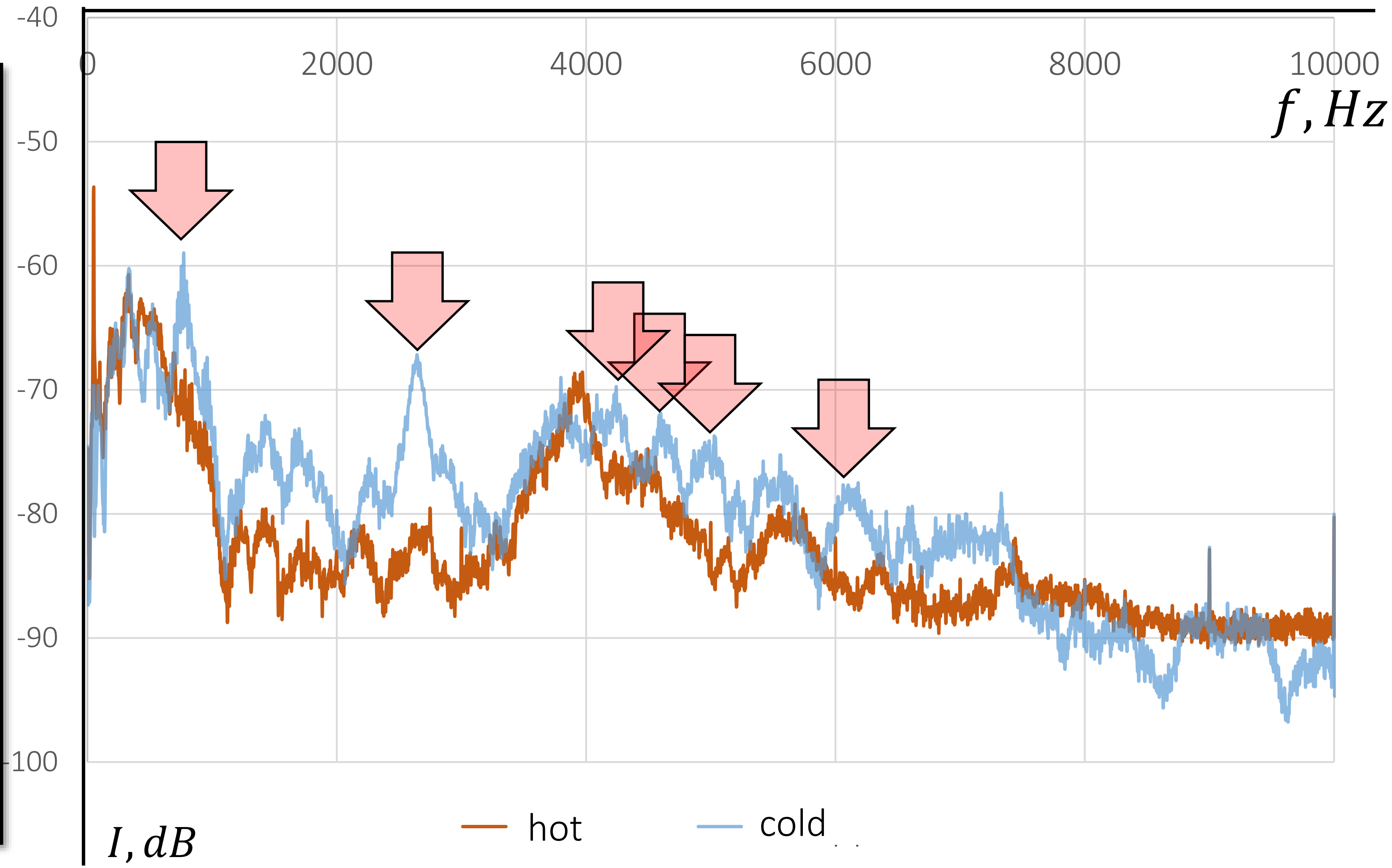
Real spectrum



Spectrum is partially continuous → sound is produced by bubbles

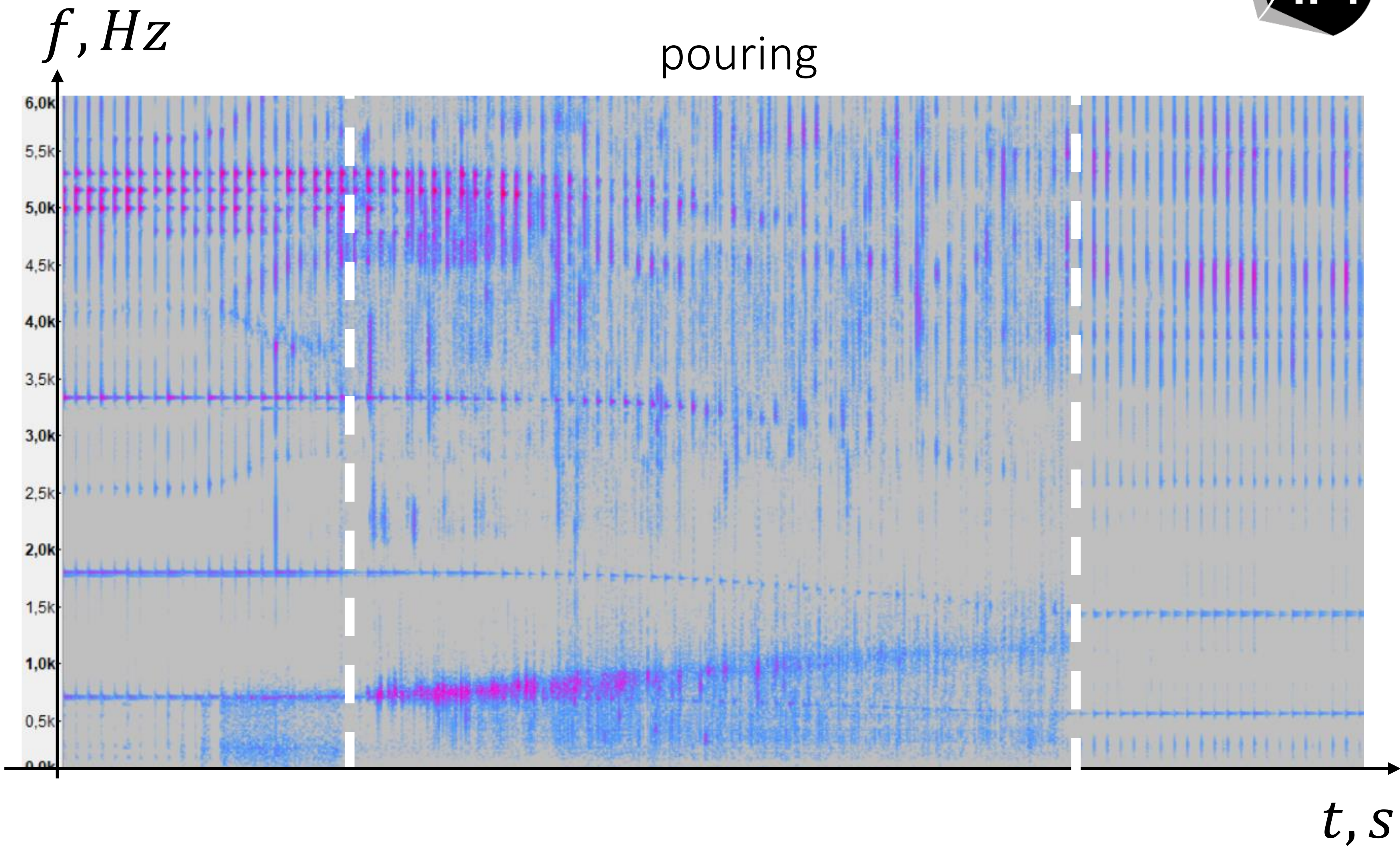
Software: Wolfram Mathematica, Audacity

Difference between hot and cold water



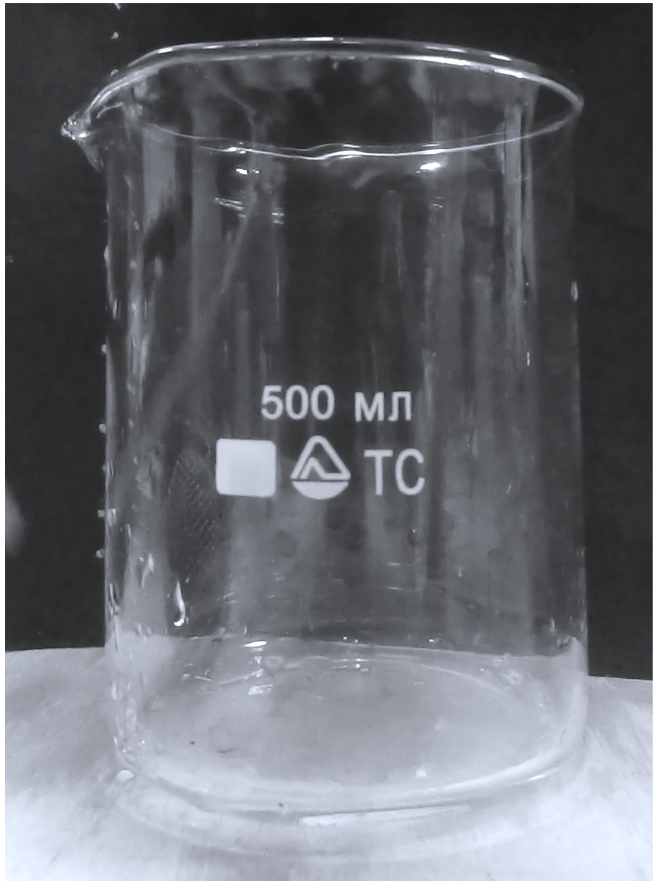
Average spectrum of the whole pouring

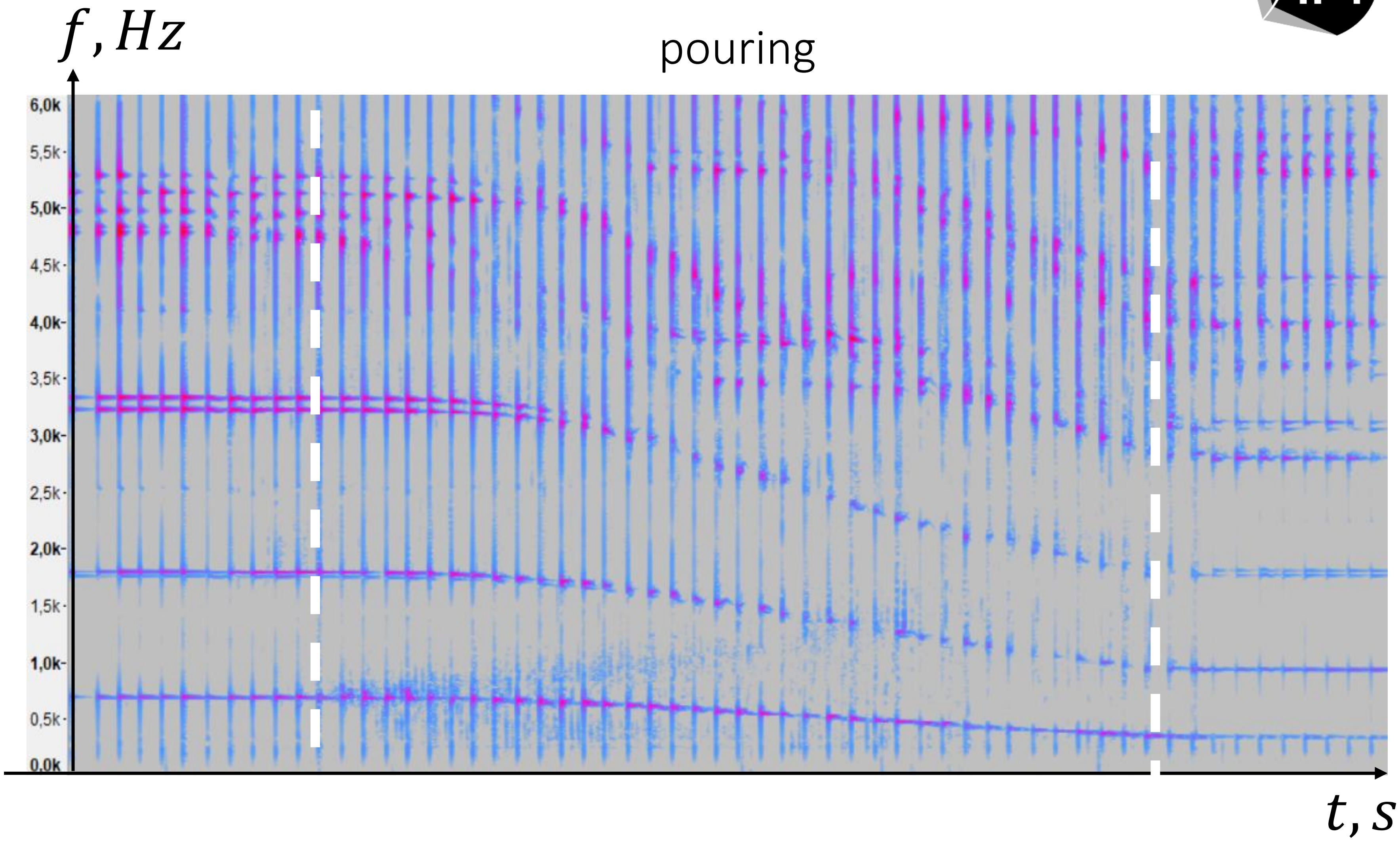
Sound of walls



Rayleigh scattering on microbubbles

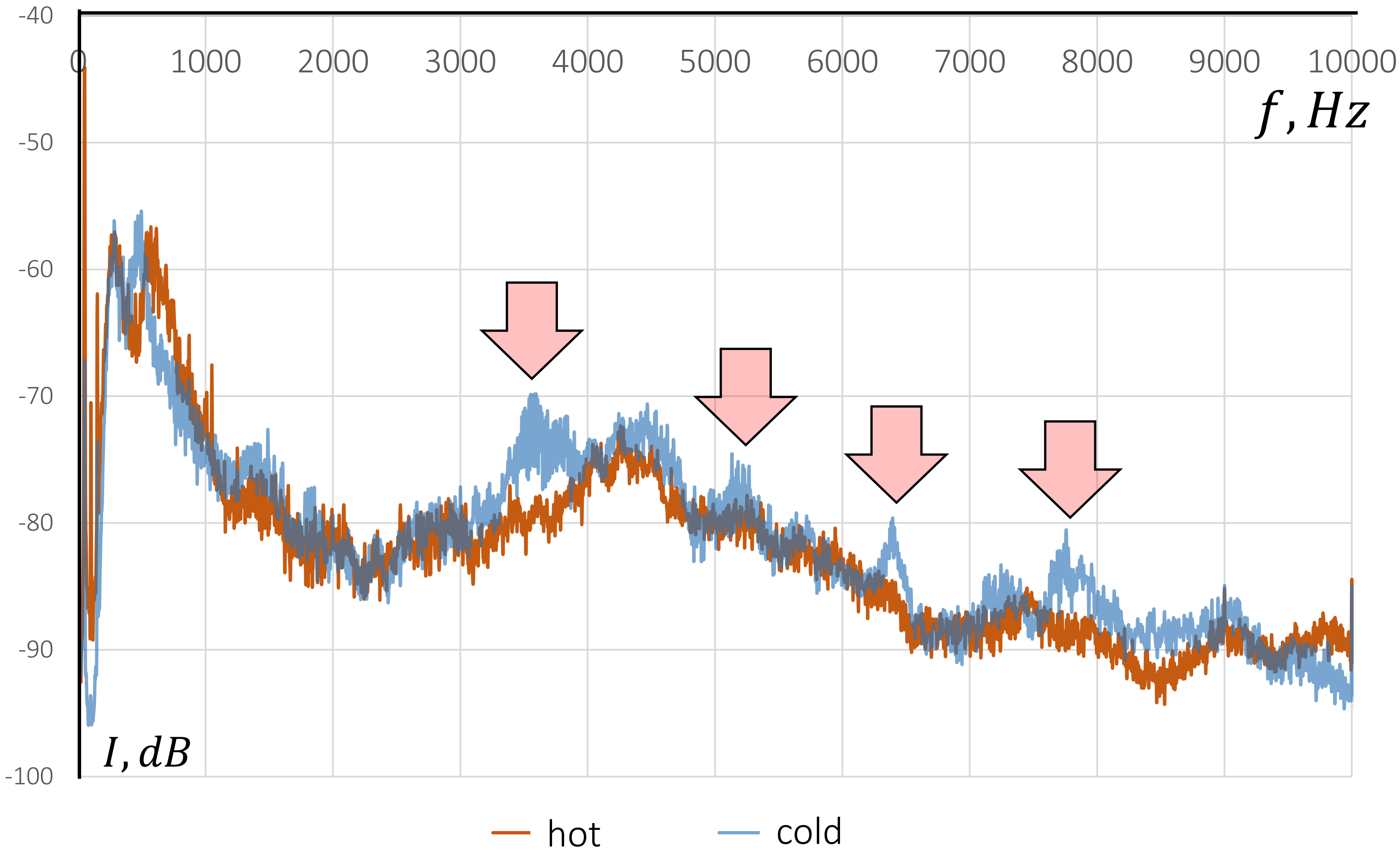
Sound of walls





Less bubbles → less scattering

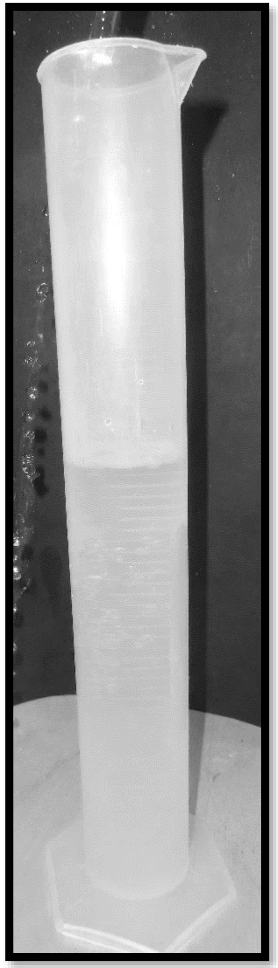
Difference between hot and cold water

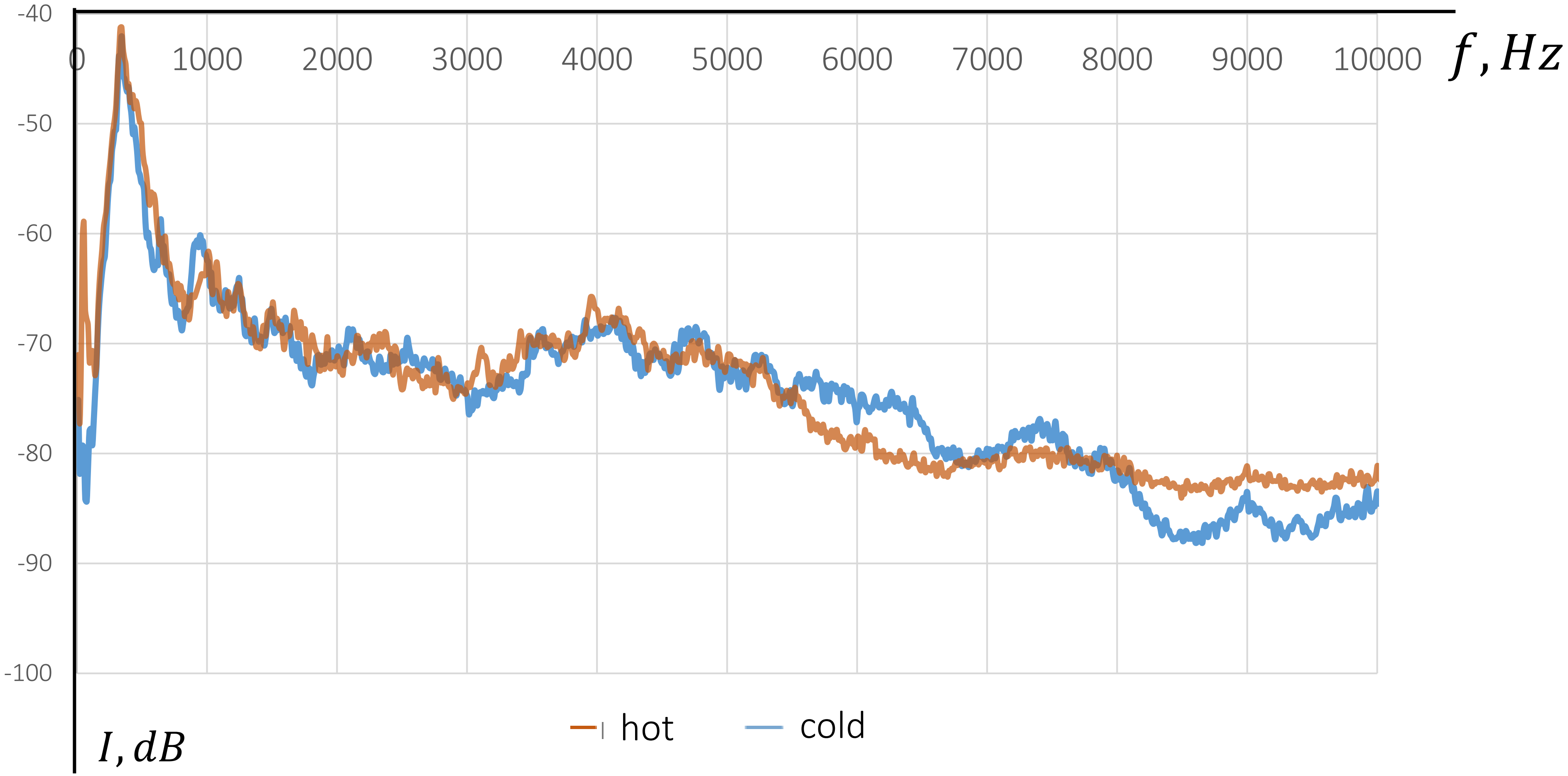


Average spectrum of the whole pouring

Pouring into plastic cylinder

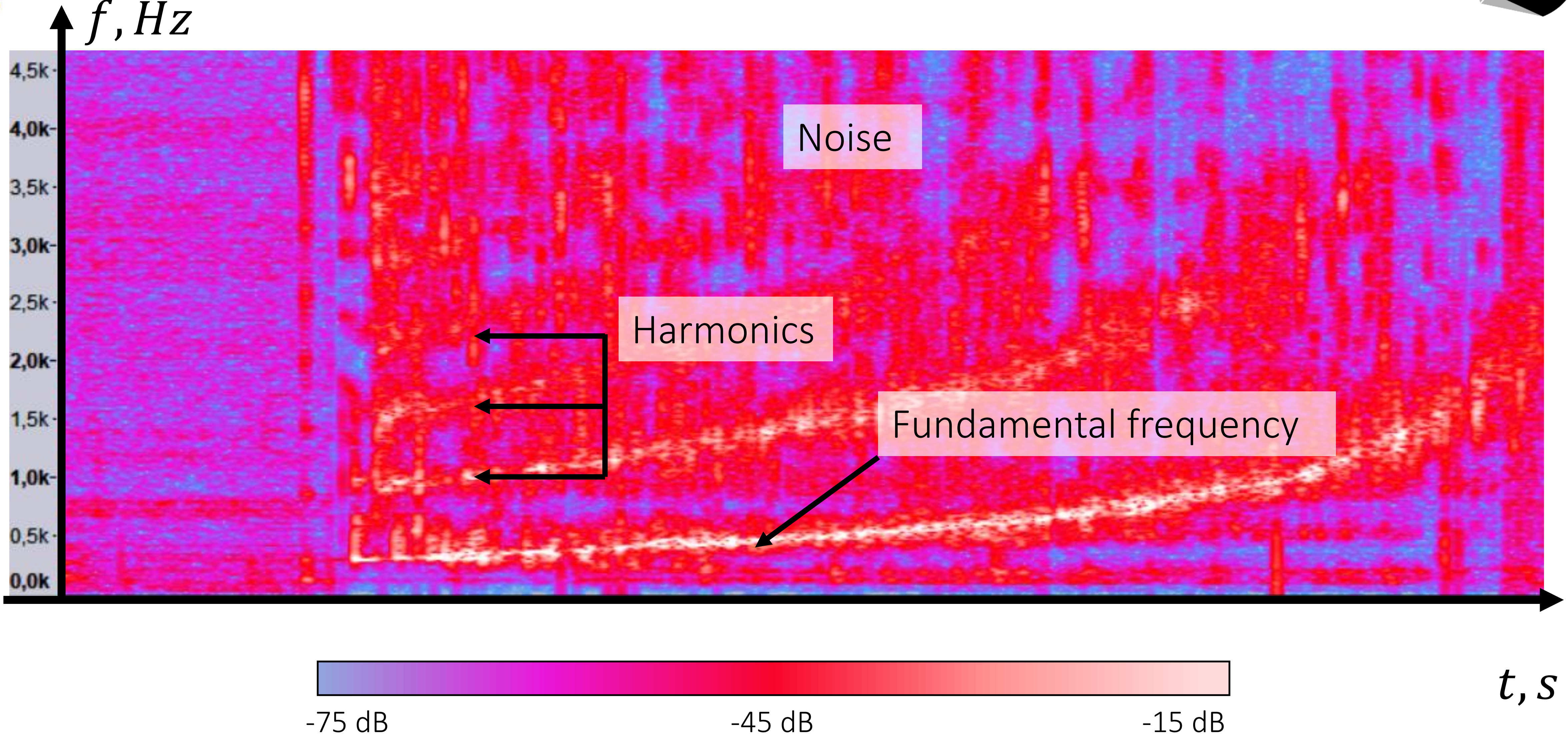
Plastic cylinder



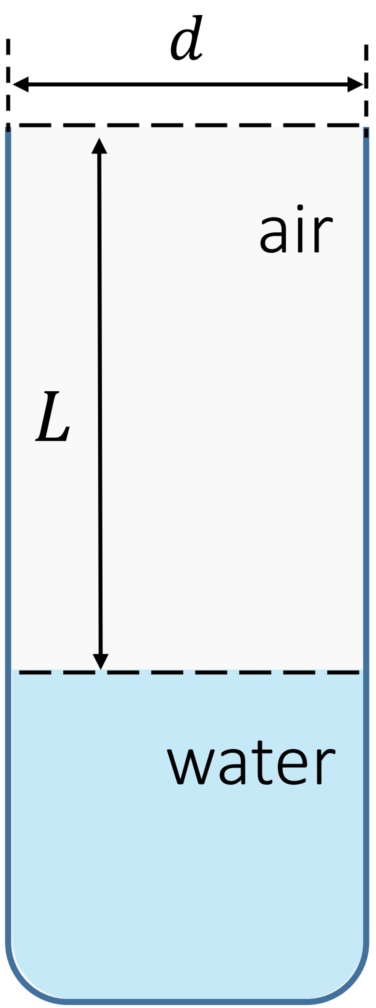


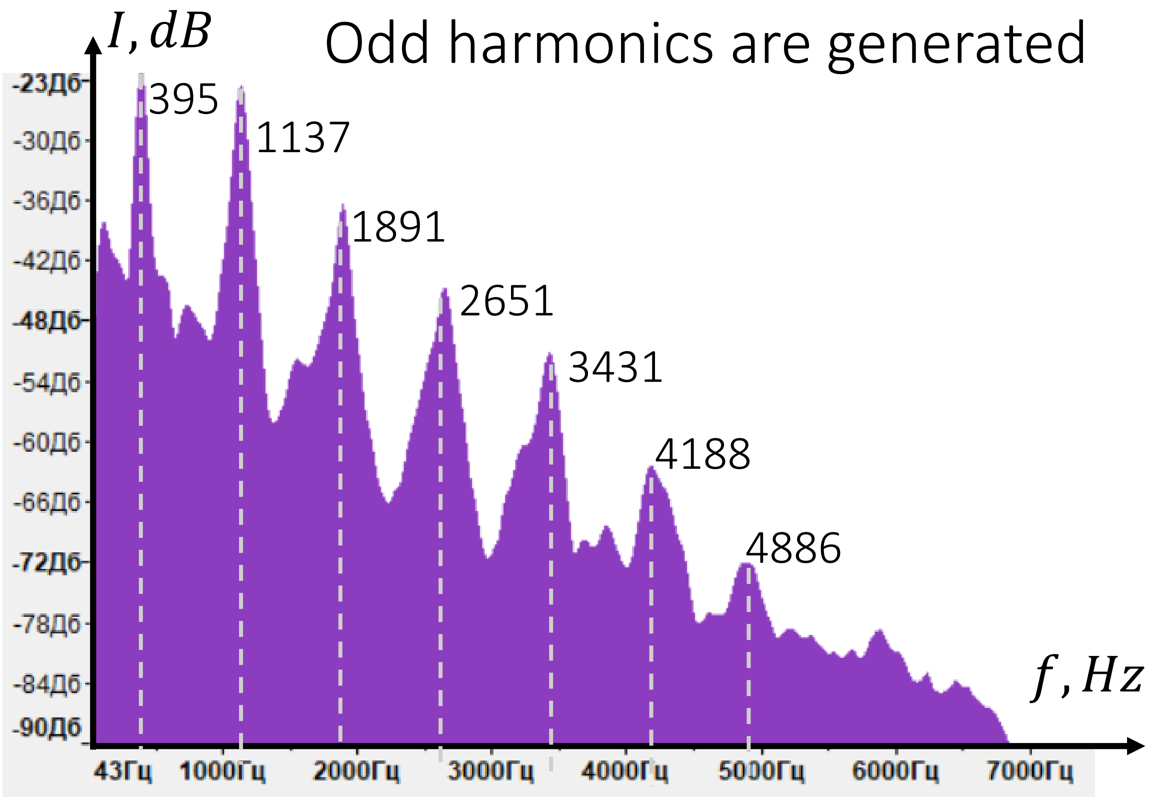
This vessel has low Q value→ no peaks

Sound of pouring



Resonance chamber





fundamental frequency

$$f = \frac{c}{4L + 0.2d}$$

c – speed of sound in the air
$0.2d$ – end correction

Air between surface of water and top of cup resonates as a pipe with one sealed end.

Speed of sound

$$c = \sqrt{\frac{\gamma R T}{M}}$$

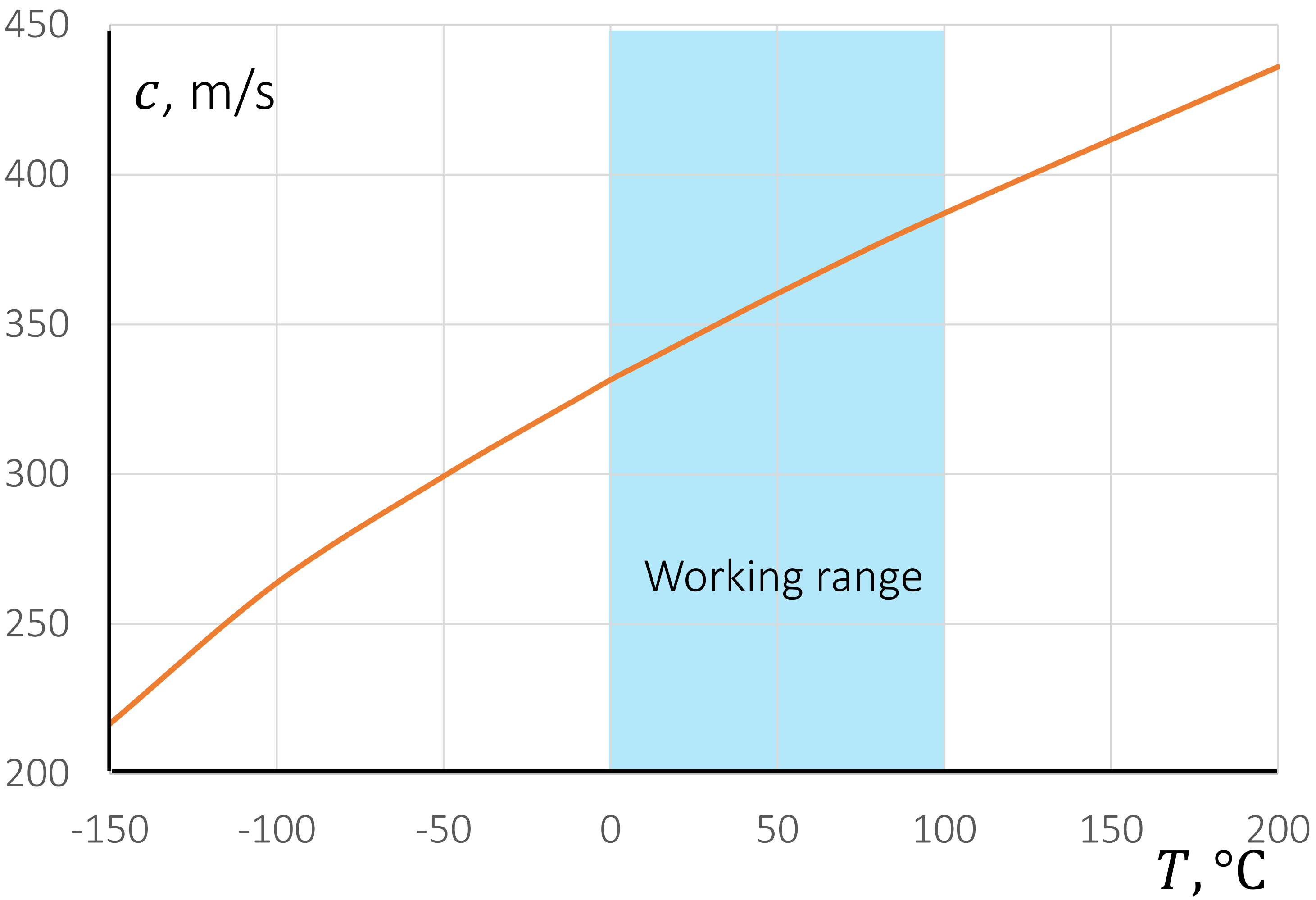
c –speed of sound

$\gamma = \frac{7}{5}$ –adiabatic coefficient

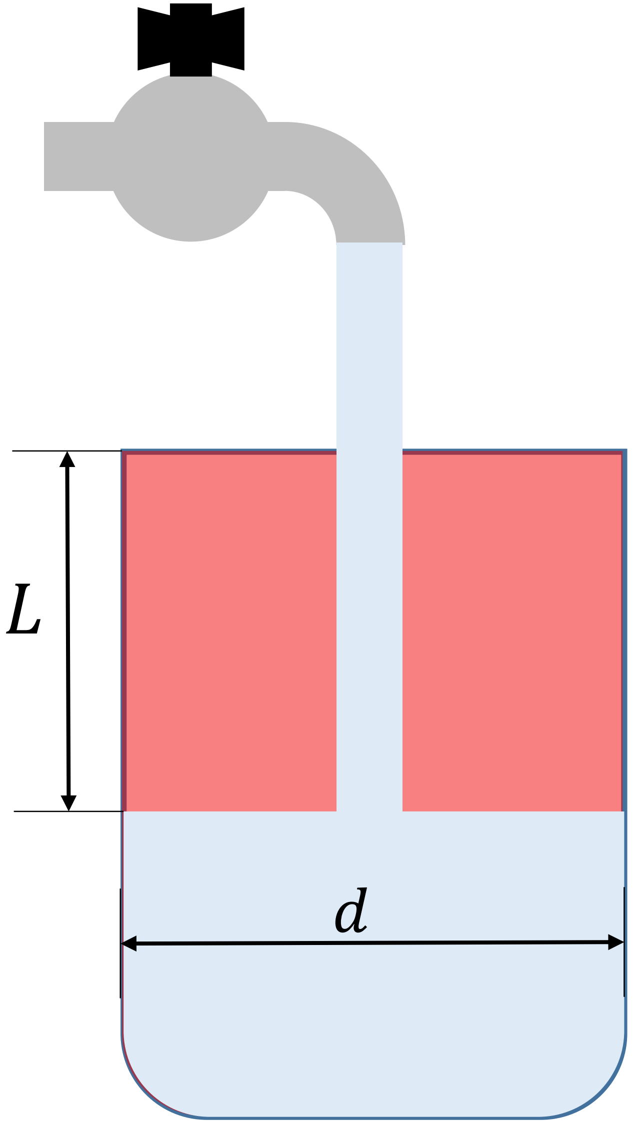
R –molar gas constant

M – molar mass

T –thermodynamic temperature



Heating of air in the cup



Hot water heats air due to evaporation and convection

⇩

Speed of sound increases

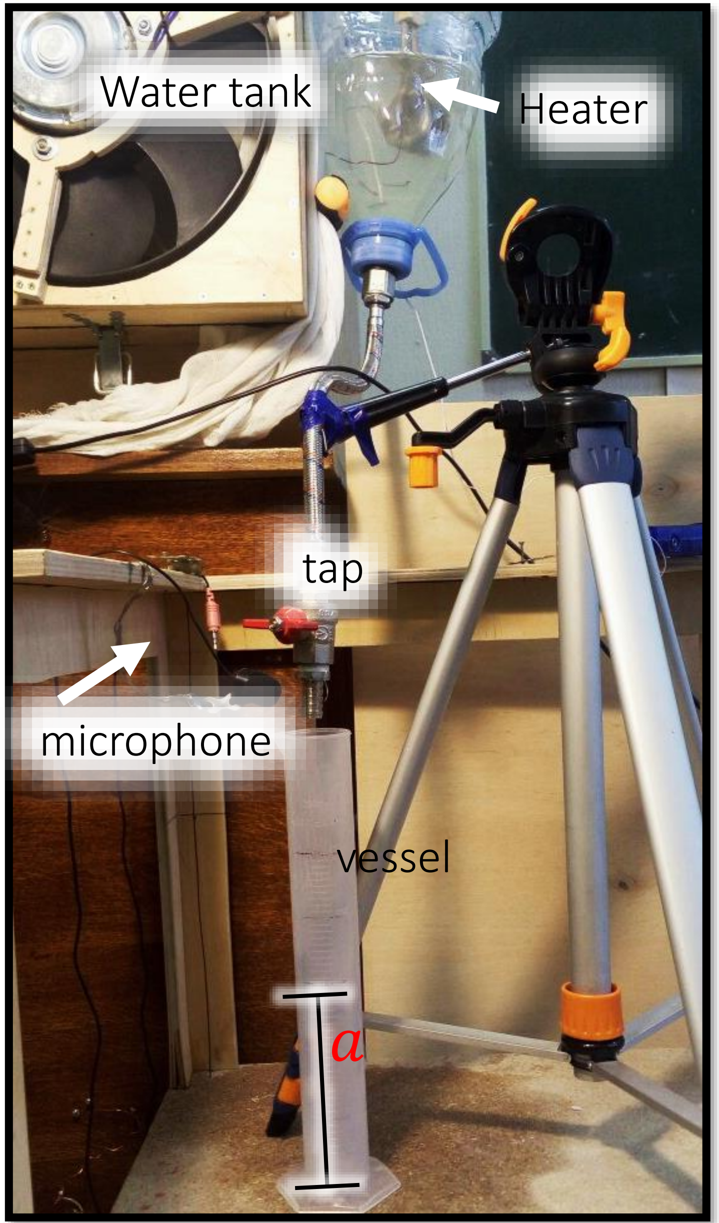
⇩

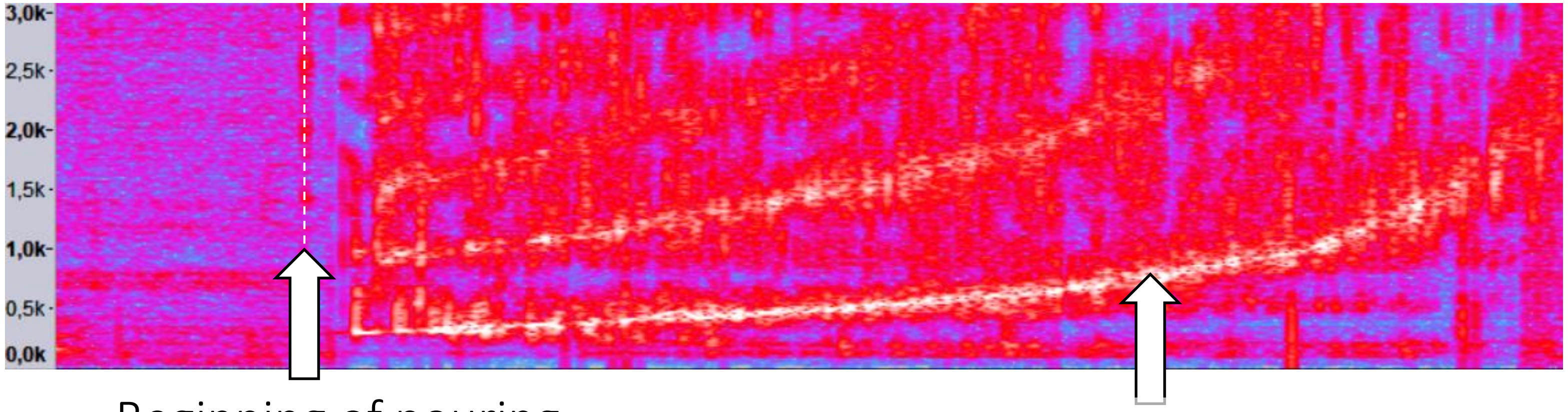
Fundamental frequency of resonator is $f \sim \sqrt{T}$

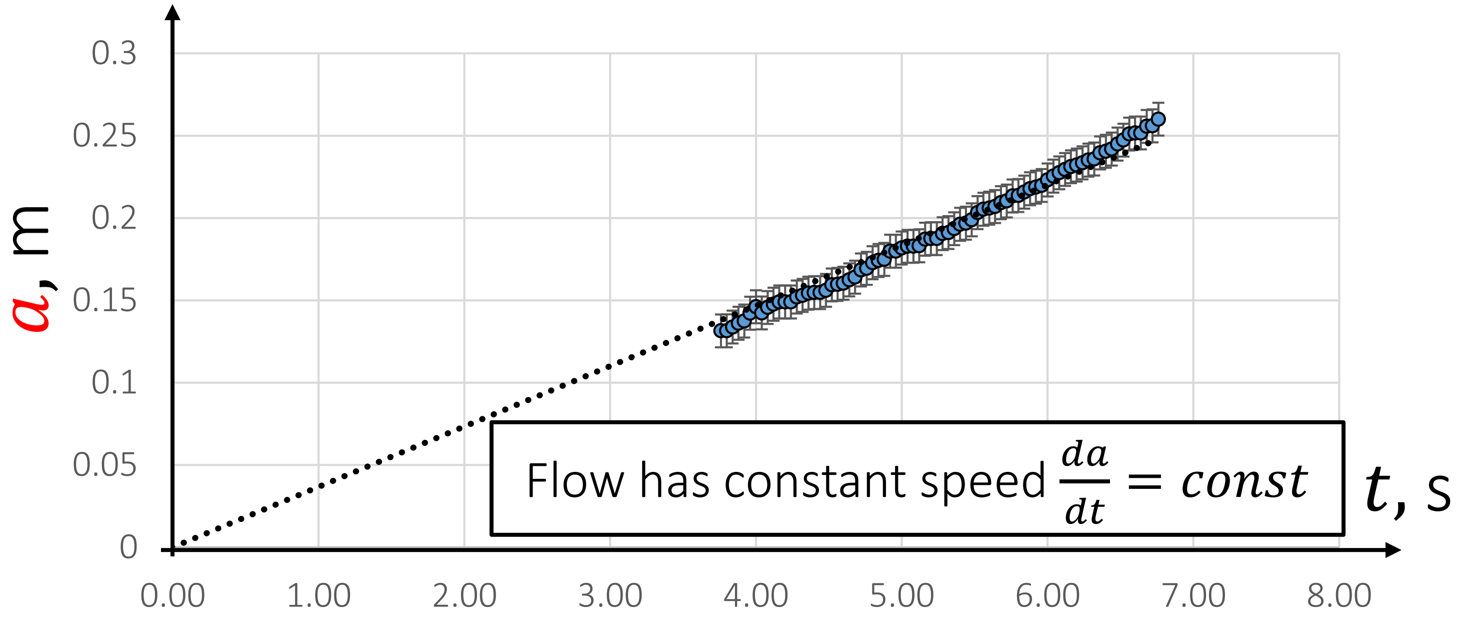
$$f = \frac{c}{4L + 0.2d}$$

Method of measurement

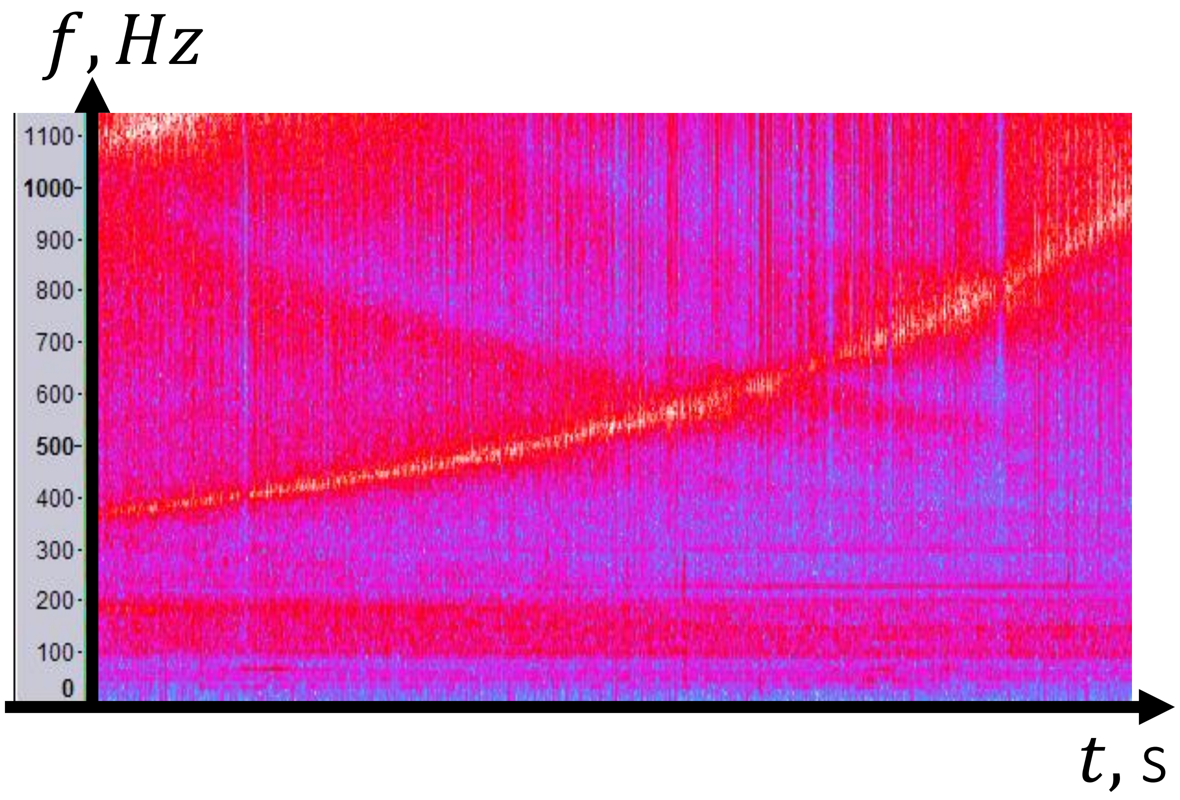
Experimental setup

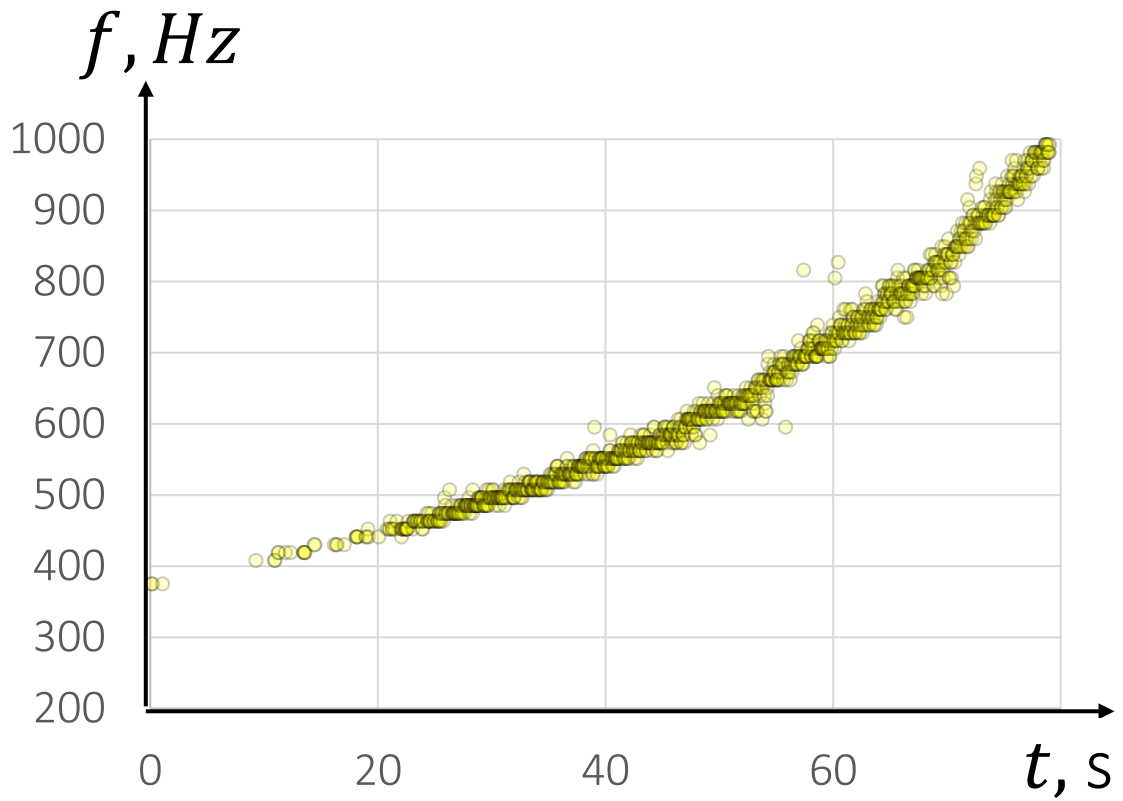






Data processing

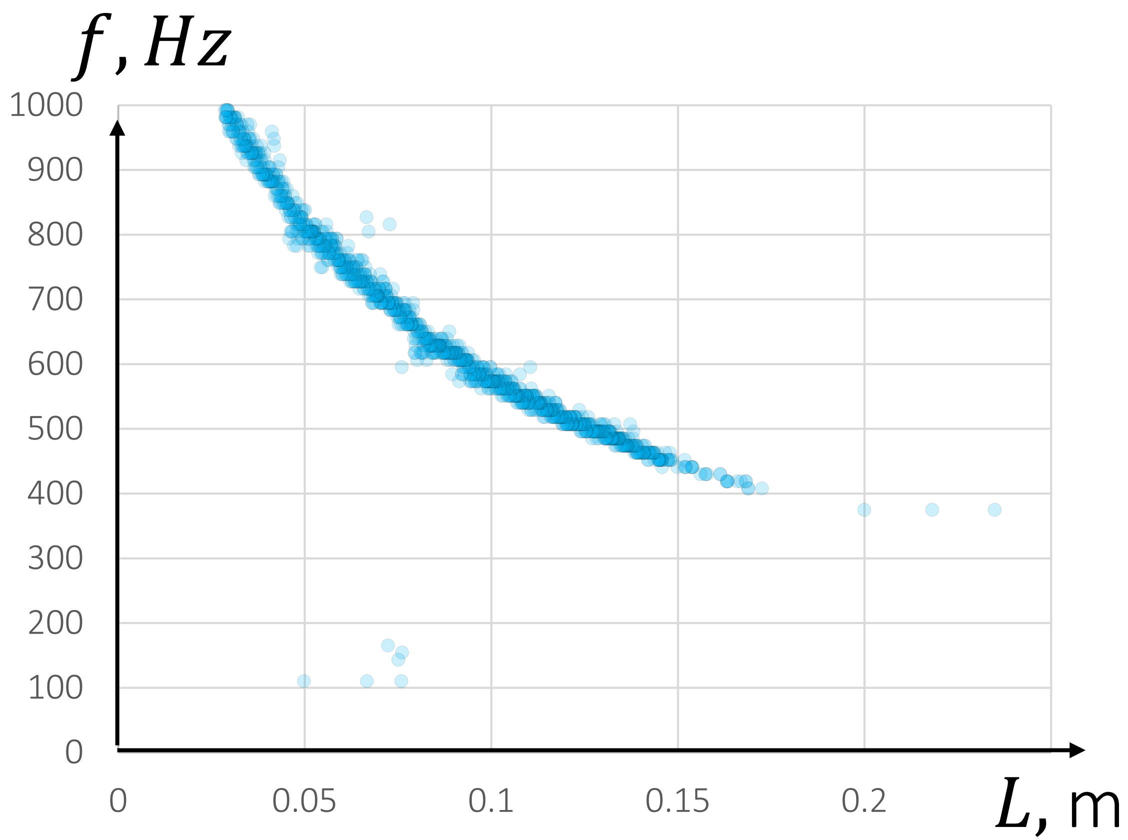


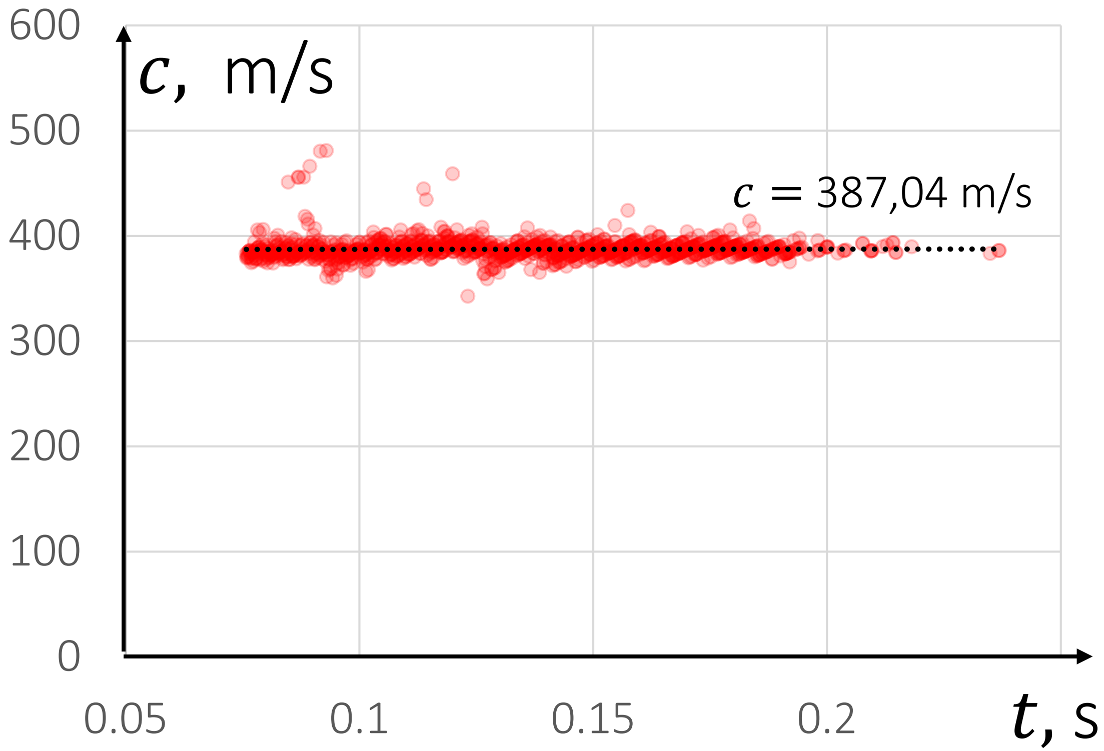


We extract fundamental frequency from the sound

Software: Wolfram Mathematica

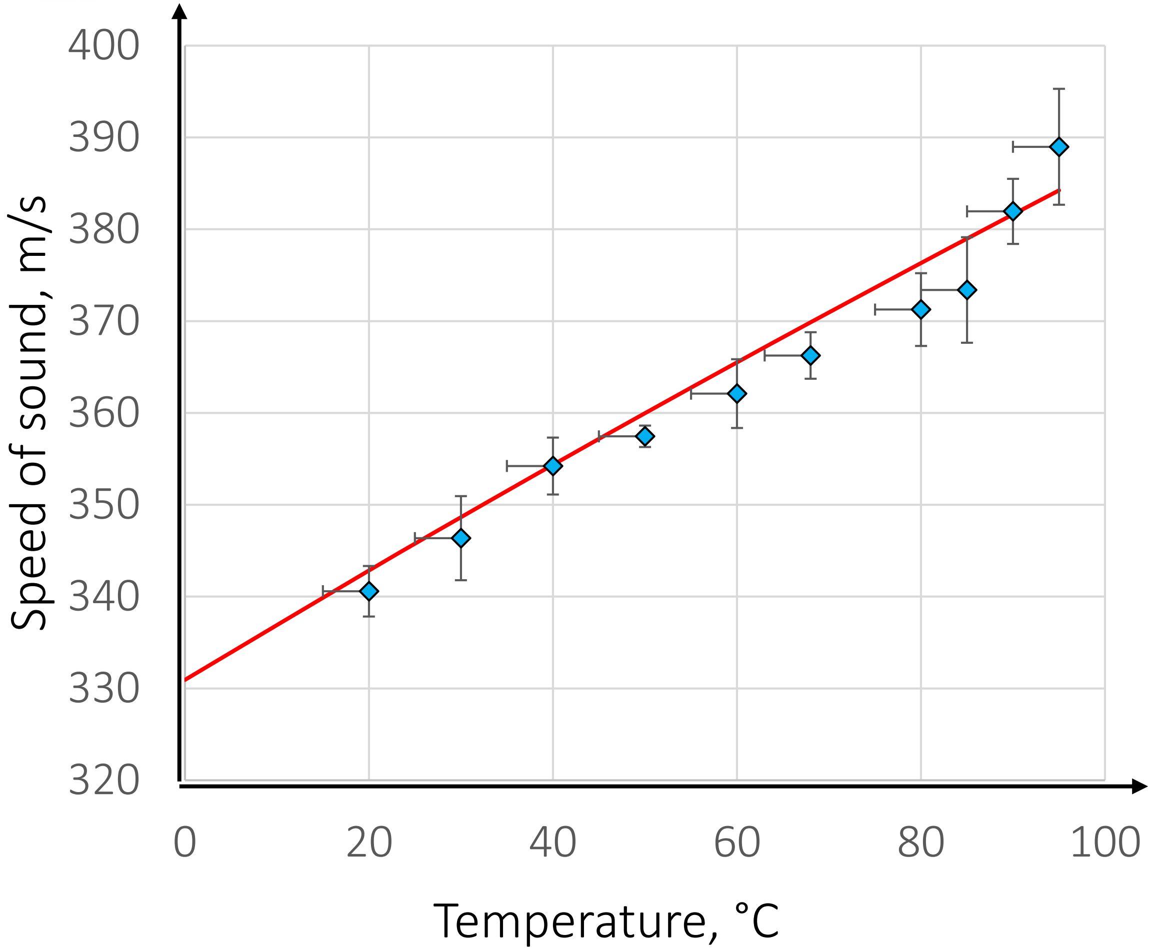
Data processing

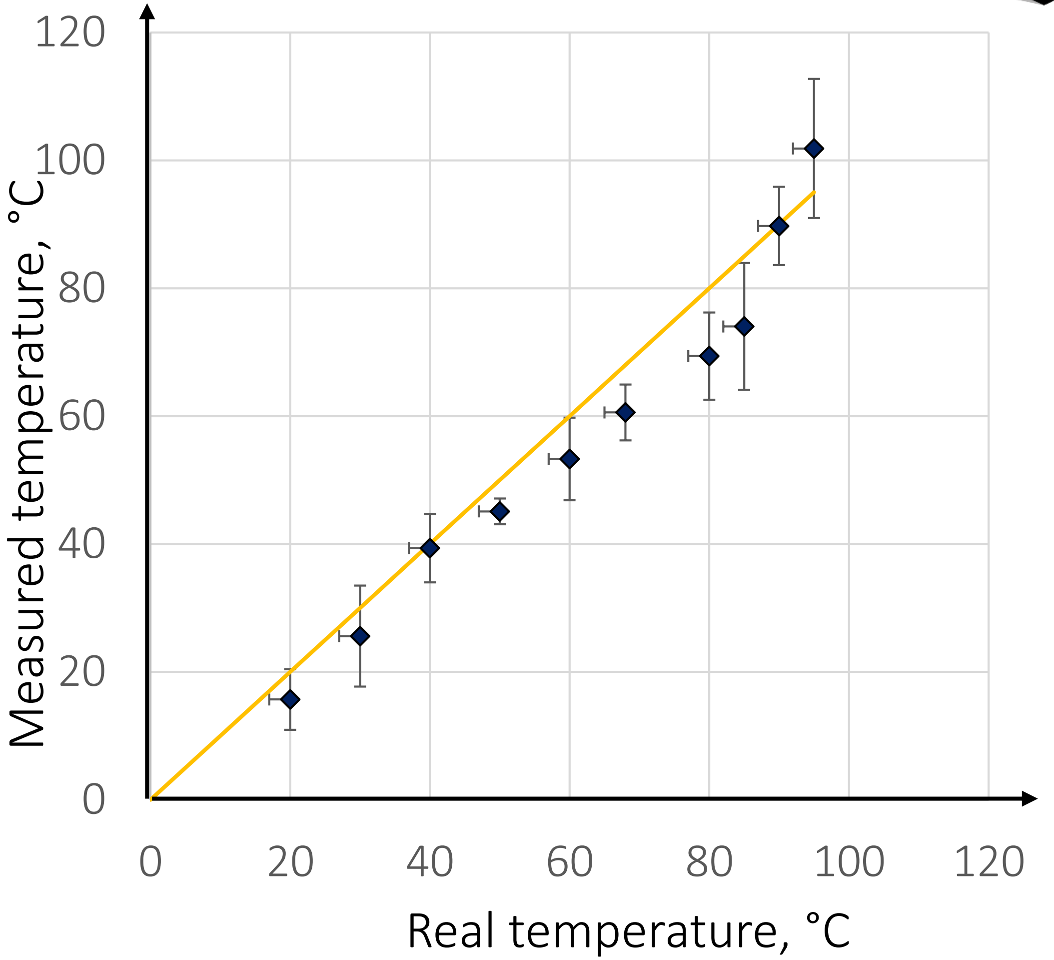




We find speed of sound from $f = \frac{c}{4L+0.2d}$

Results analysis (water)





We calculate temperature from $c = \sqrt{\frac{\gamma RT}{M}}$

Investigation of accuracy and working range

Conditions for the jet to form bubbles

Characteristic time of Rayleigh-Plateau instability growth

$$t_c = \sqrt{\frac{\rho R^3}{\sigma}} \qquad t_v = \frac{\eta R}{\sigma}$$

The minimal time for dividing into droplets

$$t > \max[t_c, t_v]$$

ρ – density of water

R – hydraulic radius

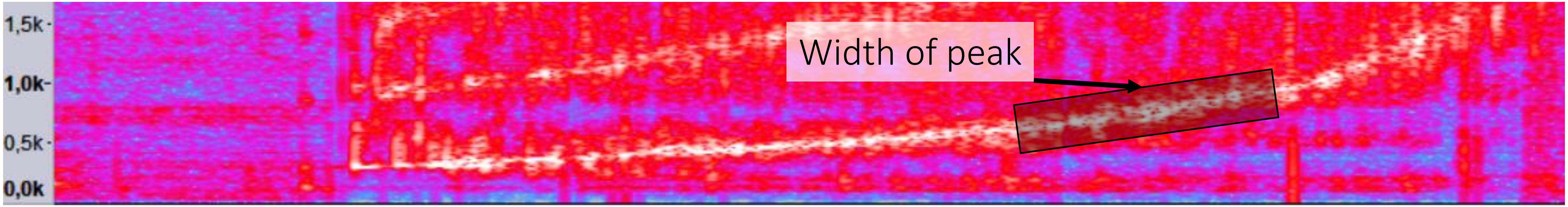
η – dynamic viscosity

Sources of errors

Random errors

Q value of resonance chamber

Irregularity of water surface

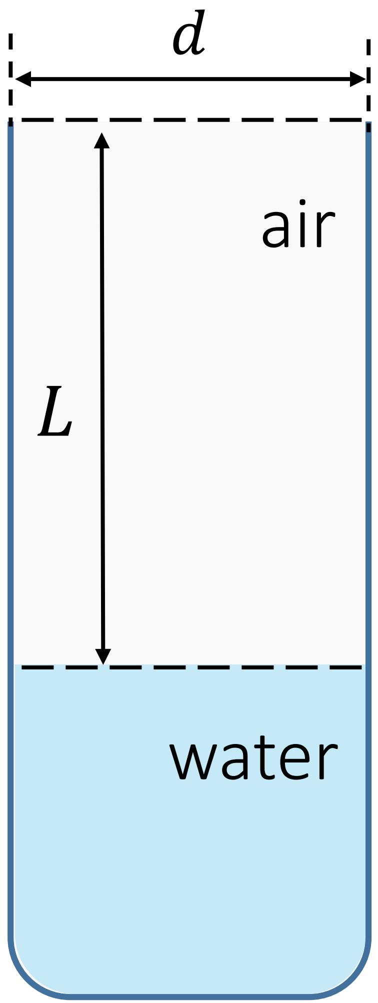


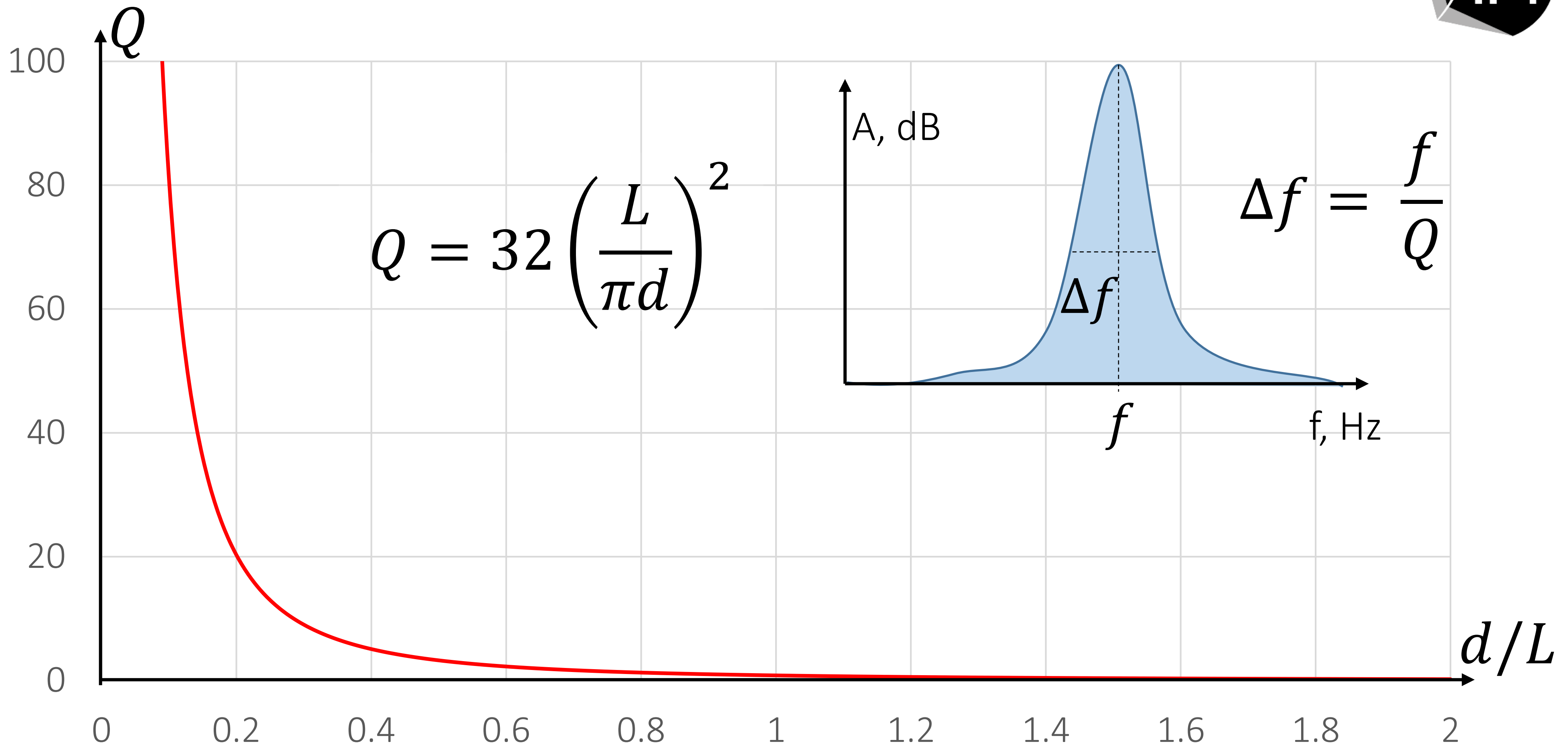
Systematic errors

Heat conduction

Heat loss

Quality factor of resonance chamber





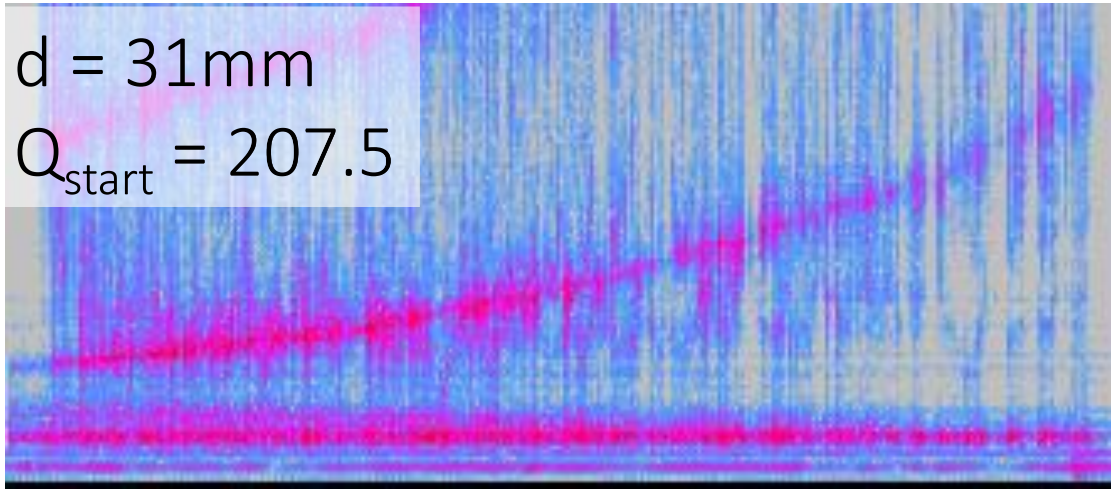
$$Q = 32\left(\frac{L}{\pi d}\right)^2$$

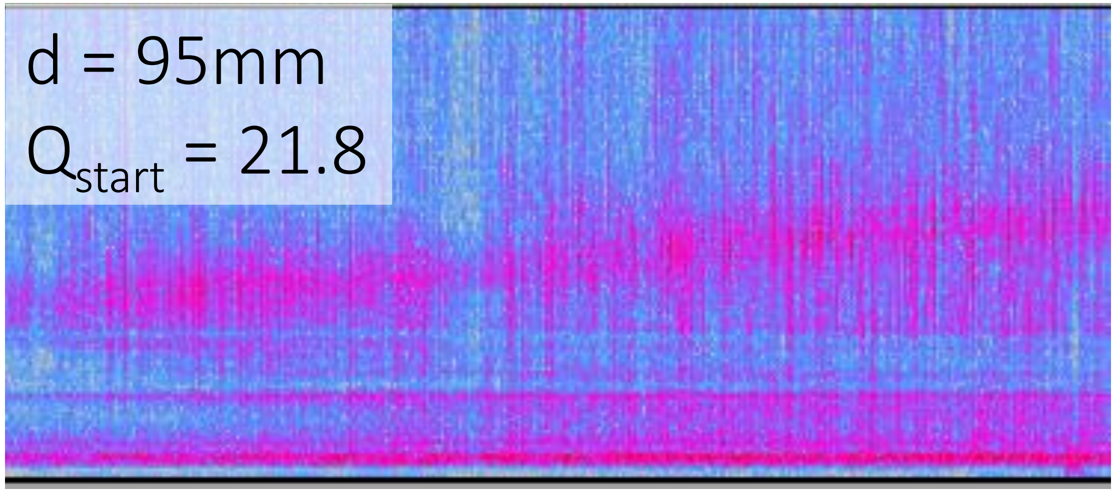
$$\Delta f = \frac{f}{Q}$$

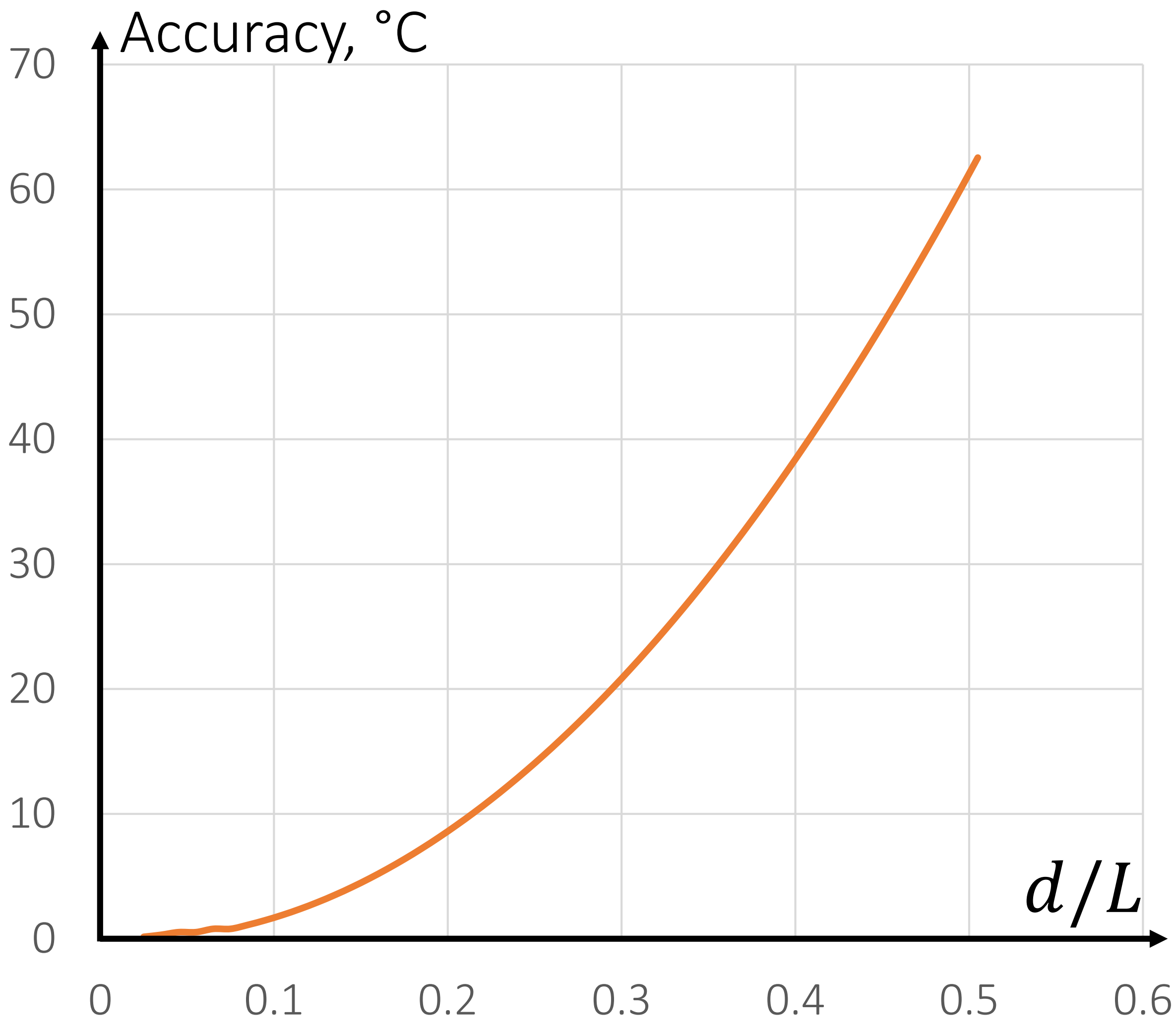
The more the Q factor, the more accurate is frequency determination

Johan Liljencrants, "Q value of a pipe resonator"

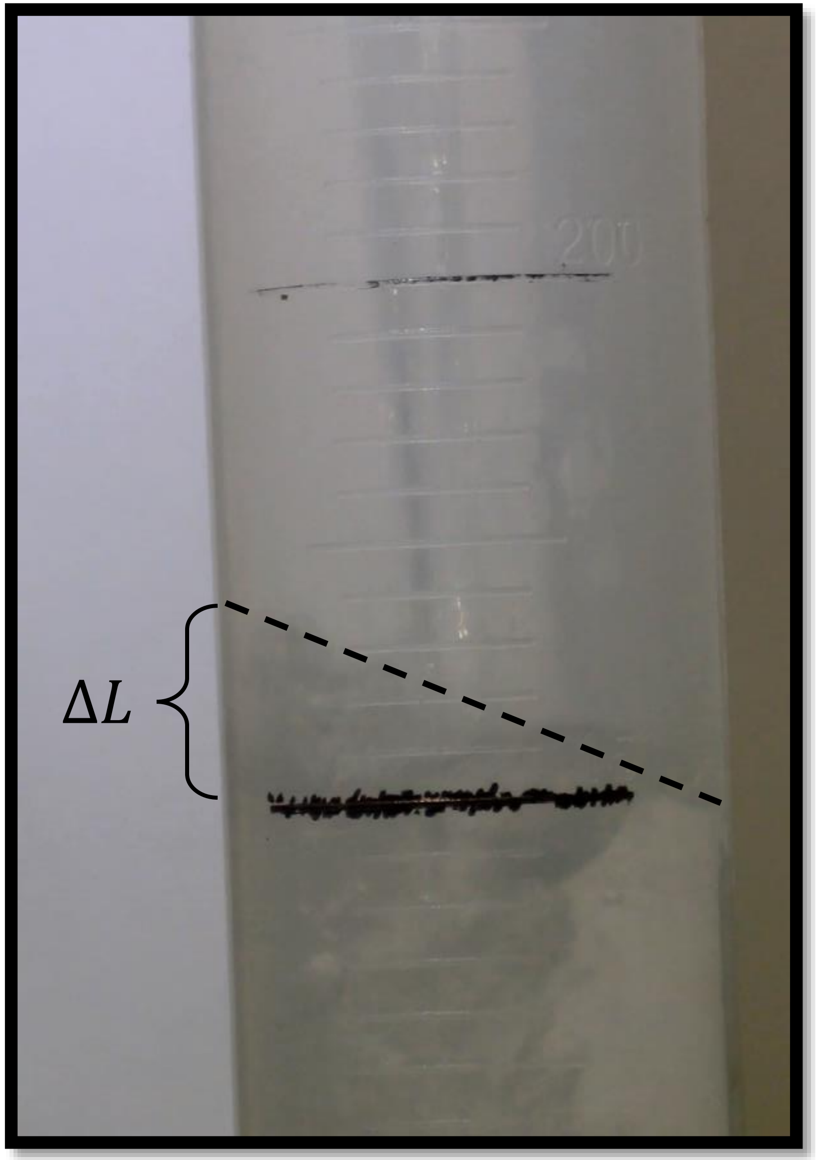
Random errors due to Q factor of vessel

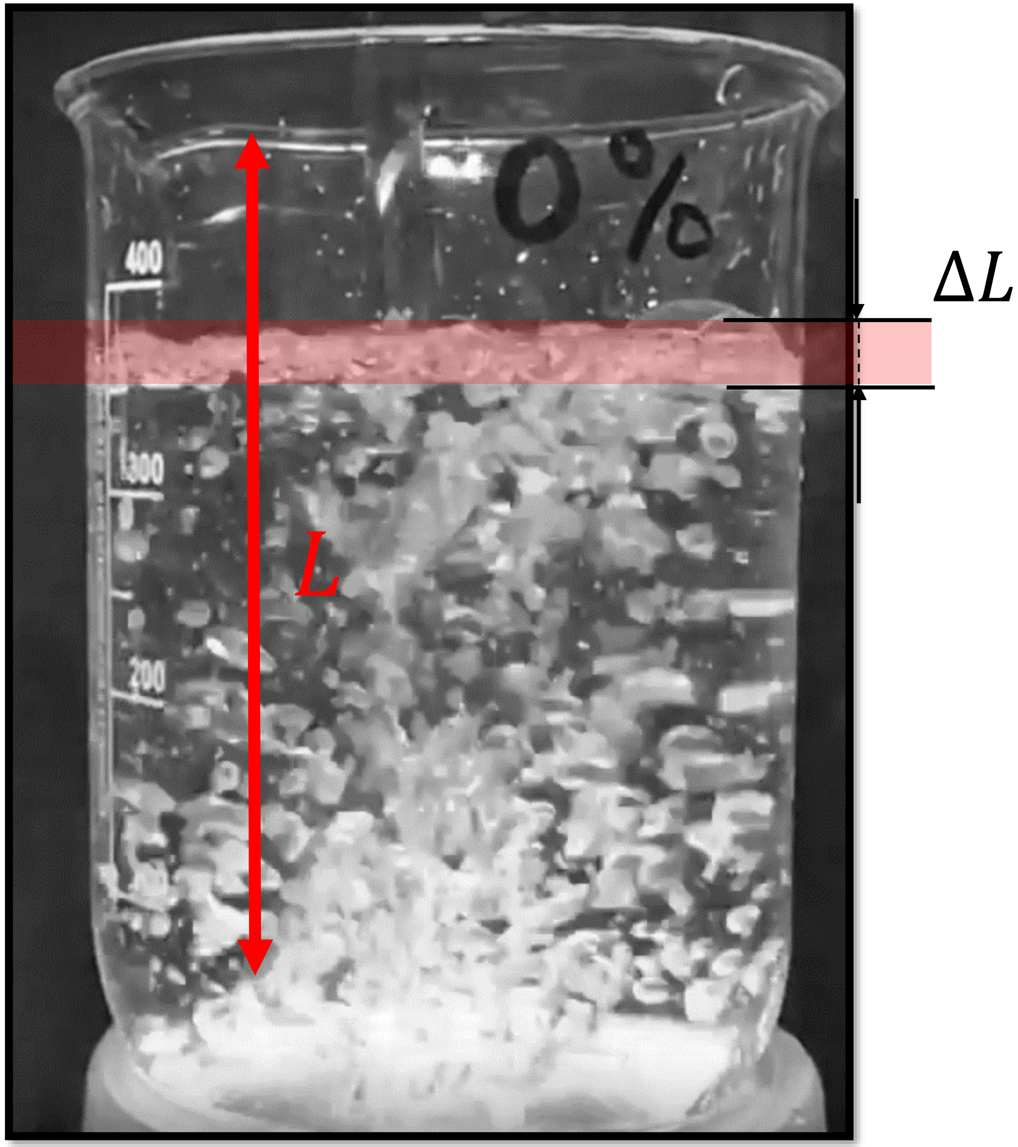






Irregularity of surface



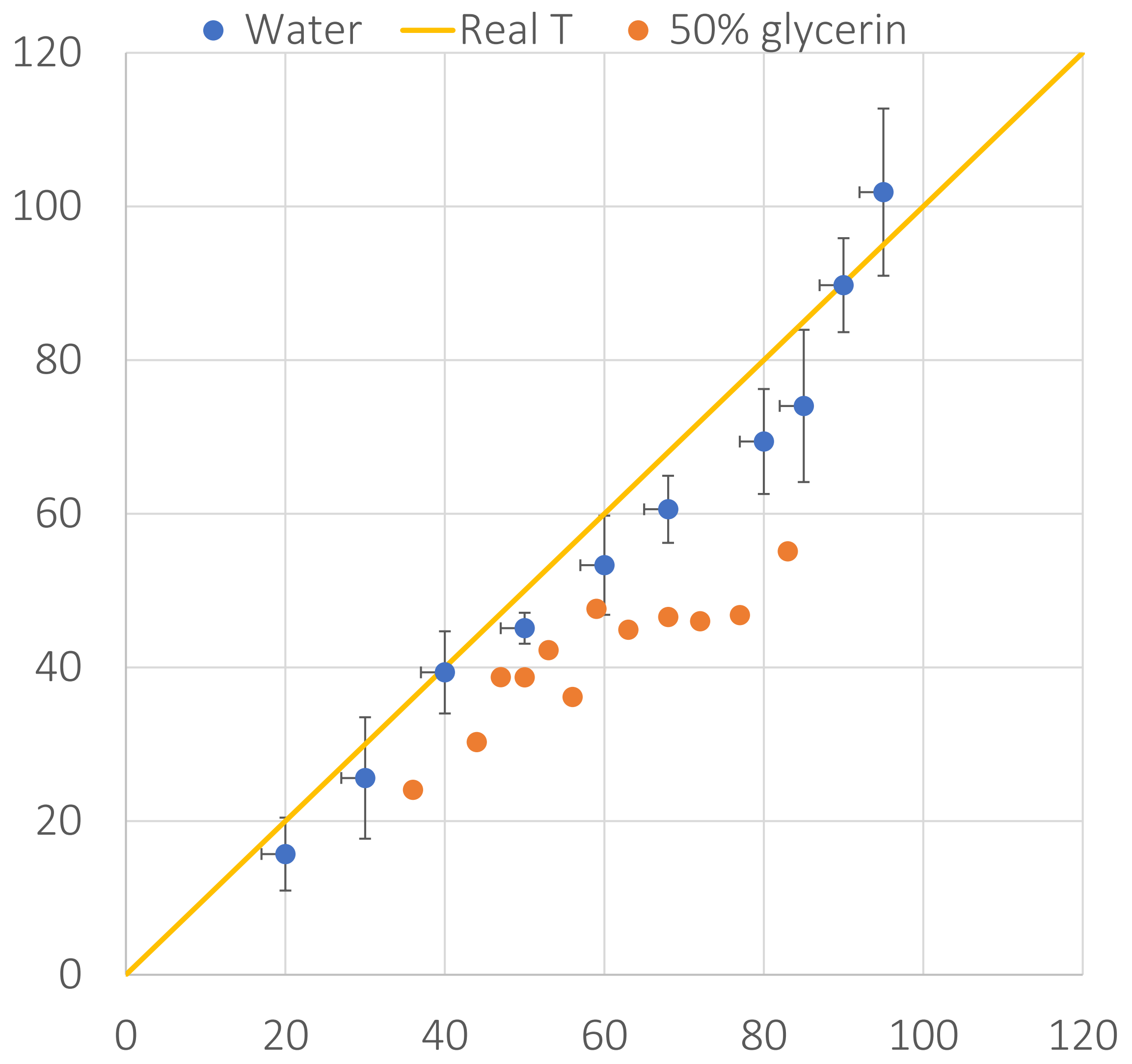


Systematic error due to heat conduction

Fluid heats walls and air slowly

Measured temperature is lower

Works for slowly evaporating fluids



Conclusions

- Sound is produced by bubbles and filtered by resonator in the cup
- We can hear difference between hot and cold water because of scattering of sound on bubbles in water
- We can find temperature by measuring peak frequency and calculating the speed of sound
- The main limitation of our method is presence of bubbles during pouring
- Accuracy is ±5,306°C for water
- Systematic error due to heat conduction is -12% for water and -23% for glycerin solution

Thank you for your attention