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Neutrosophic Random Variables

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Abstract: In this paper, general definition of neutrosophic random variables is introduced and its properties are presented. Concepts of probability distribution function, cumulative distribution function, expected value, variance, standard deviation, mean deviation, rth quartiles, moments generating function and characteristic function in crisp logic are generalized to neutrosophic logic. Many solved problems and applications are presented which show the power of the study and show the ability of applying the results in various domains including quality control, stochastic modeling, reliability theory, queueing theory, decision making, electrical engineering, … etc.

Keywords: Expected Value; Variance; Standard Deviation; Probability Density Function; Cumulative Distribution Function; Moments Generating Function; Characteristic Function; Neutrosophic Logic.

1. Introduction

Neutrosophic logic is an extension of intuitionistic fuzzy logic by adding indeterminacy component (I) where $I^2 = I, \dots, I^n = I, 0 \cdot I = 0$; $n \in N$ and I^{-1} is undefined [1], [2]. Neutrosophic logic has wide applications in many fields including decision making [3], [4], [5], machine learning [6], [7], intelligent disease diagnosis [8], [9], communication services [10], pattern recognition [11], social network analysis and e-learning systems [12], physics [13], [14], … etc.

In probability theory, F. Smarandache defined the neutrosophic probability measure as a mapping $NP: X \to [0,1]^3$ where X is a neutrosophic sample space, and defined the probability function to take the form $NP(A) = \big(ch(A), ch(neutA), ch(antiA)\big) = (\alpha, \beta, \gamma)$ where $0 \le \alpha, \beta, \gamma \le 1$ and $0 \le \alpha + \beta + \gamma \le 3$ [15], also researchers introduced many neutrosophic probability distributions like Poisson, exponential, binomial, normal, uniform, Weibull, …etc. [2], [16], [17], [18]. Researchers also presented the concept of neutrosophic queueing theory in [19], [20] that is one branch of neutrosophic stochastic modelling. Researchers also studied neutrosophic time series prediction and modelling in many cases like neutrosophic moving averages, neutrosophic logarithmic models, neutrosophic linear models, … etc. [21], [22], [23].

Neutrosophic logic has solved many decision-making problems efficiently like evaluating green credit rating, personnel selection, … etc. [24], [25], [26], [27].

In this paper we will suggest a generalization to classical random variable to deal with imprecise, uncertainty, ambiguity, vagueness, enigmatic adding the indeterminacy part to its form, then we will find several characteristics of this neutrosophic random variable including expected value, variance, standard deviation, moments generating function and characteristic function and study its properties.

This extension lets us build and study many stochastic models in the future that help us in modelling, simulation, decision making, prediction and classification specially in the cases of incomplete data and indeterminacy.

2. Terminologies

We present here some basic definitions and axioms of neutrosophic logic and neutrosophic probability.

2.1 Some definitions

Definition 1 [28]**:** Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $\{x, (\mu A(x), \delta A(x), \gamma A(x)) : x \in X\}$, where $\mu A(x)$, $\delta A(x)$ and $\gamma A(x)$ represent the degree of membership, the degree of indeterminacy , and the degree of non-membership respectively of each element $x \in X$ to the set A .

Definition 2 [29]**:** Let K be a field, the neutrosophic file generated by $\langle K \cup I \rangle$ which is denoted by $K(I) = \langle K \cup I \rangle$.

Definition 3 [2]**:** Classical neutrosophic number has the form $a + bI$ where a,b are real or complex numbers and I is the indeterminacy such that $0 \cdot I = 0$ and $I^2 = I$ which results that $I^n = I$ for all positive integers n.

Definition 4 [15]**:** The neutrosophic probability of event A occurrence is $NP(A) = \big(ch(A), ch(neutA), ch(antiA)\big) = (T, I, F)$ where T, I, F are standard or nonstandard subsets of the nonstandard unitary interval $]\,^-0, 1^+[$.

Among this paper, we will denote probability density function by PDF, probability mass function by PMF, cumulative distribution function by CDF, moments generating function by MGF, characteristic function by CF.

3. Neutrosophic Random Variables

In *[15]* Smarandache defined the neutrosophic random variable that it is a variable that may have and indeterminate outcome, in the following definition we are going to represent this indeterminacy by mathematical formula, then we are going to find the properties of neutrosophic random variable.

Definition 3.1: Neutrosophic Random Variable

Consider the real valued crisp random variable X which is defined as follows:

$$X : \Omega \to R$$

Where Ω is the events space. We define neutrosophic random variable X_N as the following:

$$X_N : \Omega \to R(I)$$

And:

$$X_N = X + I$$

Where I is indeterminacy.

3.1: PDF and CDF of neutrosophic random variables

Consider the neutrosophic random $X_N = X + I$, Where CDF of X is $F_X(x) = P(X \le x)$ then:

$$F_{X_N}(x) = F_X(x - I) \tag{1}$$

$$f_{X_N}(x) = f_X(x - I) \tag{2}$$

Proof:

$$F_{X_N}(x) = P(X_N \le x) = P(X + I \le x) = P(X \le x - I) = F_X(x - I)$$

By taking the derivative according to x we get:

$$f_{X_N}(x) = \frac{dF_{X_N}(x)}{dx} = \frac{dF_X(x - I)}{dx} \cdot \frac{d(x - I)}{dx} = f_X(x - I)$$

3.2: Expected value of neutrosophic random variable

Consider the neutrosophic random variable $X_N = X + I$, we can find its expected value as follows:

$$E(X_N) = E(X) + I \tag{3}$$

Proof:

If X is continuous then:

$$E(X_N)=E(X+I)=\int_x (x+I)f(x)dx=\int_x xf(x)dx+I\int_x f(x)dx=E(X)+I$$

Where $\int_x f(x)dx=1$ because it is a pdf

If X is discrete then:

$$E(X_N)=E(X+I)=\sum_x (x+I)f(x)=\sum_x xf(x)+I\sum_x f(x)=E(X)+I$$

Properties of expected value of a neutrosophic random variable

1. $E(aX_N+b+cI)=aE(X_N)+b+cI\,;a,b,c\in R$

Proof: straight forward.

2. If X_N,Y_N are two neutrosophic random variables, then $E(X_N\pm Y_N)=E(X_N)\pm E(Y_N)$

Proof: straight forward.

3. $E[(a+bI)X_N]=E(aX_N+bIX_N)=E(aX_N)+E(bIX_N)=aE(X_N)+bIE(X_N)\,;a,b\in R$

Proof: straight forward.

4. $|E(X_N)|\le E|X_N|$

Proof:

If X is continuous:

$$|E(X_N)|=\left|\int_x (x+I)f(x)dx\right|\le\int_x |(x+I)|f(x)dx=E|X_N|$$

Where $|f(x)|=f(x)$ because it is a PDF

If X is discrete:

$$|E(X_N)|=\left|\sum_x (x+I)f(x)\right|\le\sum_x |(x+I)|f(x)=E|X_N|$$

3.3: Variance of neutrosophic random variable

Consider the neutrosophic random variable $X_N=X+I$, we can prove that its variance is equal to X's variance, i.e.:

$$V(X_N)=V(X) \tag{4}$$

Proof:

Whatever is X_N, discrete or continuous we can write:

$$V(X_N)=E[X_N-E(X_N)]^2=E[X+I-E(X)-I]^2=E[X-E(X)]^2=V(X)$$

Example 3.1

Let X be a random variable with probability density function given as follows:

$$f_X(x)=2x\,;0\le x\le 1$$

(a) We will find PDF of $X_N=X+I$ then proof that it's a density function (it's integral equals to one)

(b) We will calculate the expected value of X_N.

(c) We will calculate the variance of X_N.

Solution:

(a)

Using equation (1):

$$f_{X_N}(x)=f_X(x-I)=2(x-I);0\le x-I\le 1$$

$$f_{X_N}(x)=2x-2I\ ;\ I\le x\le 1+I$$

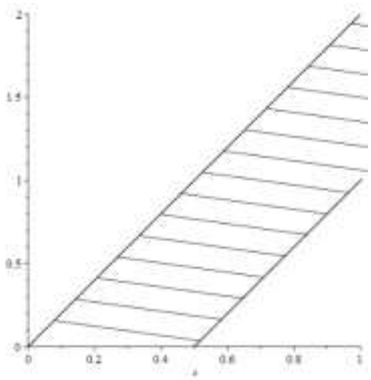

Fig (1)

Let's prove that $f_{X_N}(x)$ is a density function

$$\int_{I}^{1+I} (2x-2I)dx = [x^2-2Ix]_I^{1+I} = (1+I)^2 - 2I(1+I) - I^2 + 2I^2 = 1 + 2I + I^2 - 2I - 2I^2 - I^2 + 2I^2$$

$$= 1 + 2I + I - 2I - 2I - I + 2I = 1$$

(b) using equations (3), (4):

$$E(X_N) = E(X) + I = \int_0^1 2x^2 dx + I = \frac{2}{3} + I$$

(c)

$$V(X_N) = V(X) = \int_0^1 \left(x - \frac{2}{3}\right)^2 2x dx = \frac{1}{18}$$

3.4: Mean deviation of neutrosophic random variable:

The mean deviation of neutrosophic random variable denoted by $M.D(X_N)$ is:

$$\boldsymbol{M.D(X_N) = M.D(X) = E|X - E(X)|} \quad (5)$$

Proof:

$$M.D(X_N) = E|X_N - E(X_N)| = E|X + I - E(X+I)| = E|X + I - E(X) - I| = M.D(X)$$

3.5: The rth quartile of neutrosophic continuous random variable:

The rth quartile of neutrosophic random variable denoted by Q_N^r is:

$$\boldsymbol{\int_{-\infty}^{Q_N^r} f_{X_N}(x)dx = \frac{r}{4}; r = 1, 2, 3} \quad (6)$$

We call Q_N^1, Q_N^2 and Q_N^3 the neutrosophic first, second and third quartiles respectively.

Example 3.2

Let X_N be the neutrosophic random variable defined in example 3.1, let's calculate it's 3 quartiles.

Solution:

We have

$$f_{X_N}(x) = 2x - 2I \;;\; I \le x \le 1 + I$$

So, using equation (6):

$$\int_{I}^{Q_N^r} (2x - 2I)dx = \frac{r}{4}; r = 1,2,3$$

For $r = 1$ we get:

$$\int_{I}^{Q_N^1} (2x - 2I)dx = \frac{1}{4}$$

$$[x^2 - 2Ix]_I^{Q_N^1} = \frac{1}{4}$$

$${Q_N^1}^2 - 2IQ_N^1 - I^2 + 2I^2 = \frac{1}{4}$$

$${Q_N^1}^2 - 2IQ_N^1 + I = \frac{1}{4}$$

Solving the neutrosophic equation respect to Q_N^1 we get:

$$\Delta = b^2 - 4ac = 4I - 4\left(I - \frac{1}{4}\right) = 4I - 4I + 1 = 1$$

$$(Q_N^1)_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2I - 1}{2} = -\frac{1}{2} + I$$

Rejected because $I \le x \le 1 + I$.

$$(Q_N^1)_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2I + 1}{2} = \frac{1}{2} + I$$

Accepted.

For $r = 2$ we get:

$${Q_N^2}^2 - 2IQ_N^2 + I = \frac{2}{4} = \frac{1}{2}$$

Solving the neutrosophic equation respect to Q_N^2 we get:

$$\Delta = b^2 - 4ac = 4I - 4\left(I - \frac{1}{2}\right) = 4I - 4I + 2 = 2$$

$$(Q_N^2)_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2I - \sqrt{2}}{2} = -\frac{\sqrt{2}}{2} + I$$

Rejected.

$$(Q_N^2)_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2I + \sqrt{2}}{2} = \frac{\sqrt{2}}{2} + I$$

Accepted.

For $r = 3$ we get:

$${Q_N^3}^2 - 2IQ_N^3 + I = \frac{3}{4}$$

Solving the neutrosophic equation respect to Q_N^3 we get:

$$\Delta = b^2 - 4ac = 4I - 4\left(I - \frac{3}{2}\right) = 4I - 4I + 6 = 6$$

$$(Q_N^3)_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2I - \sqrt{6}}{2} = -\frac{\sqrt{6}}{2} + I$$

Rejected.

$$(Q_N^3)_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2I + \sqrt{6}}{2} = \frac{\sqrt{6}}{2} + I$$

Accepted.

3.6: MGF of neutrosophic random variable

Consider the neutrosophic random $X_N = X + I$ then its MGF will be:

$$M_{X_N}(t) = e^{tI} M_X(t) \tag{7}$$

Proof:

$$M_{X_N}(t) = E(e^{tX_N}) = E\left(e^{t(X+I)}\right) = E(e^{tX}e^{tI}) = e^{tI}E(e^{tX}) = e^{tI}M_X(t)$$

Properties:

1. $M_{X_N}(0) = 1$

Proof: Straight forward.

2. $\frac{dM_{X_N}(0)}{dt} = E(X_N)$

Proof:

$$\frac{dM_{X_N}(t)}{dt}|_{t=0} = \frac{de^{tI}M_X(t)}{dt}|_{t=0} = \frac{de^{tI}}{dt}M_X(t)|_{t=0} + \frac{dM_X(t)}{dt}e^{tI}|_{t=0} = Ie^{tI}M_X(t)|_{t=0} + M'_X(t)e^{tI}|_{t=0}$$
$$= IM_X(0) + M'_X(0) = I + E(X) = E(X_N)$$

3. $\frac{d^n M_{X_N}(t)}{dt^n}|_{t=0} = E(X_N^n)$

Proof: Straight forward.

4. If $Y_N = (a+bI)X_N + c + dI$ then $M_{Y_N}(t) = e^{(c+dI)t}e^{t(a+b)I}M_X\big((a+bI)t\big)$

Proof:

$$M_{Y_N}(t) = E(e^{tY_N}) = E\left(e^{t[(a+bI)X_N+c+dI]}\right) = E\left(e^{t(a+bI)(X+I)}\,e^{(c+dI)t}\right) = e^{(c+dI)t}E\left(e^{t(a+bI)(X+I)}\right)$$
$$= e^{(c+dI)t}E\left(e^{t(aX+aI+bIX+bI^2)}\right) = e^{(c+dI)t}e^{t(a+b)I}E\left(e^{t(a+bI)X}\right)$$
$$= e^{(c+dI)t}e^{t(a+b)I}M_X\big((a+bI)t\big)$$

Theorem 3.5 CF of Neutrosophic Random Variable

Consider the neutrosophic random $X_N = X + I$ then its CF will be:

$$\varphi_{X_N}(t) = e^{itI}\varphi_X(t)\ ;\ i = \sqrt{-1} \tag{8}$$

Proof:

$$\varphi_{X_N}(t) = E(e^{itX_N}) = E\left(e^{it(X+I)}\right) = E(e^{itX}e^{itI}) = e^{itI}E(e^{itX}) = e^{itI}\varphi_X(t)$$

Properties:

1. $\varphi_{X_N}(0) = 1$

Proof: Straight forward.

2. $|\varphi_{X_N}(t)| \le 1$, which means that CF always exists.

Proof:

$$|\varphi_{X_N}(t)| = |E(e^{itX_N})| \le E|e^{itX_N}| = E|\cos tX_N + \sin tX_N| = E|1| = 1$$

3. $\frac{d\varphi_{X_N}(t)}{dt}|_{t=0} = iE(X_N)$

Proof:

$$\frac{d\varphi_{X_N}(t)}{dt}|_{t=0} = \frac{de^{itI}\varphi_X(t)}{dt}|_{t=0} = \frac{de^{itI}}{dt}\varphi_X(t)|_{t=0} + \frac{d\varphi_X(t)}{dt}e^{itI}|_{t=0}$$
$$= iIe^{itI}\varphi_X(t)|_{t=0} + \varphi'_X(t)e^{itI}|_{t=0} = iI\varphi_X(0) + \varphi'_X(0) = iI + iE(X) = i\big(I + E(X)\big)$$
$$= iE(X_N)$$

4. $\frac{d^n\varphi_{X_N}(t)}{dt^n}|_{t=0} = i^nE(X_N^n)$

Proof: Straight forward.

5. $\varphi_{X_N}(t) = M_{X_N}(it)$

Proof: Straight forward.

Example 3.3

Let X_N be the neutrosophic random variable defined in example 3.1 and let's find:

(a) $M_{X_N}(t)$.

(b) $E(X_N)$ Depending on properties of $M_{X_N}(t)$

(c) Conclude $\varphi_{X_N}(t)$ formula.

Solution

(a) Using equation (7):

$$M_{X_N}(t) = e^{tI}M_X(t)$$

But:

$$M_X(t) = \int_0^1 e^{tx}2x\,dx = \frac{2(te^t - e^t + 1)}{t^2}$$

So:

$$M_{X_N}(t) = e^{tI}\frac{2(te^t - e^t + 1)}{t^2} = 2\frac{te^{t(1+I)} - e^{t(1+I)} + e^{tI}}{t^2}$$

(b) Using proved properties of $M_{X_N}(t)$ we get:

$$M'_{X_N}(t) = 2\frac{t^2\left(e^{t(1+I)} + (1+I)e^{t(1+I)}t - (1+I)e^{t(1+I)} + Ie^{tI}\right) - 2t\left(te^{t(1+I)} - e^{t(1+I)} + e^{tI}\right)}{t^4}$$
$$= 2\frac{t\left(e^{t(1+I)} + (1+I)e^{t(1+I)}t - (1+I)e^{t(1+I)} + Ie^{tI}\right) - 2\left(te^{t(1+I)} - e^{t(1+I)} + e^{tI}\right)}{t^3}$$
$$= 2\frac{te^{t(1+I)} + (1+I)e^{t(1+I)}t^2 - (1+I)te^{t(1+I)} + Ite^{tI} - 2te^{t(1+I)} + 2e^{t(1+I)} - 2e^{tI}}{t^3}$$

$$M'_{X_N}(0) = \frac{2}{3} + I = E(X_N)$$

(c) Using the proved property that $\varphi_{X_N}(t) = M_{X_N}(it)$ we get:

$$\varphi_{X_N}(t) = M_{X_N}(it) = 2\frac{ite^{it(1+I)} - e^{it(1+I)} + e^{itI}}{-t^2}$$

4. Applications and Future Research Directions

The results that are presented in this paper can be applied to define several concepts in neutrosophic probability theory that are not defined and not studied yet including stochastic processes, reliability theory models, quality control techniques, …etc. where all depend on the concept of neutrosophic random variables and it's properties. Also, these results can be applied in stochastic modelling and random numbers generating which is very important in simulation of probabilistic models.

We are looking forward to study the properties of neutrosophic probability distributions like Pareto, Gaussian, Gamma, Beta, … etc. when the distribution of random variables changes to $X_N = X + I$ i.e., when the random variable contains an indeterminant part so we can model and simulate many stochastic problems including arrivals and departures to services stations, lifetimes of units in manufacturing systems, loss models, …etc.

5. Conclusions

In this research, we firstly obtained a general definition of neutrosophic random variables, concepts of probability distribution function and cumulative distribution function. We focused on the neutrosophic representation and proved some properties. In addition, we showed the ability of applying the results in various domains including quality control, stochastic modeling, reliability theory, queueing theory, electrical engineering, …etc.

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