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A STUDY ON THE FORCES THAT CAUSE THE UNIVERSE'S ACCELERATING EXPANSION

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Abstract:

When people calculate the forces inside the universe at a large scale, gravitational force is often focused, while the radiation pressure forces of stars are often neglected. This article reveals after calculations that, at a large distance, the radiation pressure force is larger than the gravitational force inside the universe. This will cause the universe to expand at increasing speeds. In addition, calculation also shows that cosmic rays pressure force is a stronger force than radiation pressure force in accelerating the expansion of the universe. Other factors including gamma ray bursts can also be important causes of growing universe expansion. The causes of universe expansion provided by this article is original and innovative. This scientific discovery can be used to replace the space expansion theory to explain the accelerating expansion of the universe. Moreover, The relationship between the total radiation pressure force (F) of a star and the power (P) of the star can be described by equation: F=P/C. Thus we can roughly calculate the overall power the universe needs to sustain an accelerating expansion.

Key Words: universe expansion; space expansion; radiation pressure force; light pressure; gravity; gravitational force; cosmic ray; star; particle; equation; supernova; black hole; neutron star; gravitational wave event; gamma ray burst; sunlight pressure; universe; cosmology; milky way; galaxy; main sequence; red giant; planet; interstellar dust; momentum; impulse; kinetic energy;

Main Text:

Inside the universe, for a star like the sun, it gravitational force will help pull the matters together, causing the universe to concentrate. While it is often neglected that the sunlight pressure force is also an important force and can cause the matter inside universe to expand. The sunlight pressure force, or star light pressure force, means the radiation pressure force which sometimes just called the force of light.

In near distance, the gravitational force is much larger than the light pressure force. But at a hugely remote distance, the gravitational force becomes smaller because it is inversely proportional to distance, while the total force of sunlight pressure remains the same no matter how far it is. For example, if we build a huge spherical shell structure which completely covers all the sunlight, no matter its diameter is twice the diameter of the sun or its diameter is 2 billion times the diameter of the sun, the collective force of the sunlight pressure imposed upon the inner surface of the spherical structure is exactly the same as long as there is no other matters blocking the sunlight inside the spherical structure.

Therefore, this article aims to calculate the gravitational forces of the sun towards matters of the universe at different distance of the universe and compares it with the total force of sunlight pressure.

Firstly, let's examine the total gravity force of the sun towards the universe. This means how much gravity force the sun imposes upon matter inside the universe to pulls the universe together/towards the sun, so the directions of the force is towards the sun.

Whereas the total mass of the sun M_{sun} is 2×10^{30}kg, the total mass of the universe $M_{universe}$ is 1×10^{53}kg, and the radius of the universe R is approximately 4.4×10^{26}m.

Because the entire observable universe is a BIG BALL of space filled with matter and energy with the radius of $R=4.4\times10^{26}$m. And we can see the part of the universe around the sun with a radius of $R/2=2.2\times10^{26}$m as a SMALL BALL. Now let's focus on the part of universe outside of the SMALL BALL. So the shape of this part of the universe is a HOLLOW BALL. And we can call this part of universe as the HOLLOW BALL universe.

Thus let's examine the total gravity force of the sun towards the HOLLOW BALL universe. Apparently the total volume of the HOLLOW BALL universe is 7/8 of total volume of the universe. On a very large scale, all matter inside the universe can be seen as approximately evenly distributed, so the total mass of the HOLLOW BALL universe $M_{hollow\ ball}$ is 7/8 of the mass of the entire universe ($M_{universe}$) $= 7/8 \times 1 \times 10^{53} = 8.75 \times 10^{52}$kg.

So according to gravity equation, $F=(GM_aM_b)/r^2$, the gravity force between the sun and the HOLLOW BALL universe $F_{hollow\ ball}$ is:

$(GM_{sun}M_{hollow\ ball})/(R/2)^2 > F_{hollow\ ball} > (GM_{sun}M_{hollow\ ball})/R^2$, therefore:

$(6.67\times10^{-11}\ N\bullet m^2/kg^2\times2\times10^{30}kg\times8.75\times10^{52}kg)/(2.2\times10^{26}m\times2.2\times10^{26}m) > F_{hollow\ ball} > (6.67\times10^{-11}\ N\bullet m^2/kg^2\times2\times10^{30}kg\times8.75\times10^{52}kg)/(4.4\times10^{26}m\times4.4\times10^{26}m)$

So, $F_{hollow\ ballMAX}=2.4\times10^{20}N > F_{hollow\ ball} > 6.0\times10^{19}N=F_{hollow\ ballMIN}$

So, we can roughly consider

$F_{hollow\ ball} \approx F_{hollow\ ballAVERAGE} = (F_{hollow\ ballMAX}+F_{hollow\ ballMIN})/2=1.5\times10^{20}N$

Then,

Let's examine the total gravity force of the sun towards the entire milky way. The total mass of the milky way $M_{milky\ way}$ is estimated to be roughly 4×10^{41}kg. The distance between the sun and the core of the milky way R' is approximately 2.7 light years ($2.7\times9.46\times10^{15}$m $=2.55\times10^{16}$m). If we suppose that all mass of the milky way is located in the core of the milky way, then the gravity between the milky way and the sun is roughly:

$F_{milky\ way}$=（$GM_{sun}M_{milky\ way}$）/ R'^2=（6.67×10^{-11} N•m²/kg²×2×10^{30}kg×4×10^{41}kg）/（2.55×10^{16}m × 2.55×10^{16}m）= 8.2×10^{28}N

Now, let's examine the total radiation pressure force of the sun.

We already know that the pressure exerted by sunlight on objects at 1 AU from the sun is:

p = 4.53N/km^2.

When the light is fully reflected, the p can be 4.53N/km^2×2=9.06N/km^2, and the total area of the surface of the virtual ball with radius of 1AU is roughly:

$S=4\pi r^2=\pi D^2=4\times3.14\times1.5\times10^8\times1.5\times10^8=2.8\times10^{17}$km^2.

So the total force of pressure of sunlight $F_{pressure}=p\times S=2.54\times10^{18}$N when the sunlight is fully reflected. This is the force of the sun to push matters of the universe away from the sun. This force seems not so big. However, the current sun is in the stage of main-sequence star of its life time with a current age of 4.5 billion years. And it is becoming brighter all the time. After 5 billion years when it's main-sequence star stage come to an end, it will become 2.2 times brighter than it is now. Then, it will become a red giant star which will last for 2 billion years with the brightness 3000 times larger than it is now.

So the average force of the pressure of sunlight throughout its life time can be roughly:

$F_{total\ pressure}$= ($F_{pressure}$ × (4.5 billion years + 5 billion years) + $F_{pressure}$ × 3000 × 2 billion years) / (4.5 billion years + 5 billion years + 2 billion years) = (2.54×10^{18}N × (4.5+5) + 2.54×10^{18}N × 3000×2) / (4.5+5+2) = (12.35×10^{18}N + 7800×10^{18}N)/11.5=1.39×10^{21}N

Now we compare the forces:

Firstly,

$F_{total\ pressure}>F_{hollow\ ball}$. and, $F_{total\ pressure}/F_{hollow\ ball}\approx F_{total\ pressure}/F_{hollow\ ballAVERAGE}$ = 1.39×10^{21}N / 1.5×10^{20}N = 9.27.

As mentioned earlier, $F_{hollow\ ball}$ is the gravity force of the sun imposed upon the HOLLOW BALL universe. And $F_{total\ pressure}$ is the radiation pressure force when lights are fully reflected. If there is no reflection, ratio of the pressure force versus gravity force will be half, that is, 4.64.

So, even if only 1/4 of the total sunlight is absorbed or reflected by the matters, including stars, planets, black holes, and interstellar dusts, inside the HOLLOW BALL universe, the light pressure force of the sun will be bigger than the gravity force of the sun. So it means that the sun generally will push the HOLLOW BALL universe away.

And what is worth noticing is that because interstellar dusts covers very large space with relatively small mass, they may play a critical role in the process. Because for a big star or planet, it is apparent that the pull of the gravitational force of our sun will be much bigger than the radiation pressure force push of the sun. So it is very much possible that much of the light pressure of the sun is imposed on the dusts of galaxies in the vast space inside the universe and pushes the dusts away from the sun. And then the dusts will help pull the celestial bodies near to the dusts away from the sun. This kind of pushing upon the galaxy dusts may push the dusts out of the galaxy, but because the galaxy also rotates, this rotation can cause the dusts to rotate around the center of the galaxy so this keep the dusts remain inside the galaxy. As a result, the dusts will have a constant pulling effect upon the galaxy when they are pushed by the radiation pressure force of the sun.

Secondly,

And apparently, $F_{milky\ way}$ is much larger than $F_{total\ pressure}$. As mentioned earlier, $F_{milky\ way}$ is the gravity force of the sun imposed upon the milky way. Because the milky way is very near to the sun, the gravity force is much bigger than the light pressure force. So the sun generally pulls the galaxy instead of pushing it away.

As a result, the sun will generally push the far universe farther and pull near universe nearer.

But how far is far? To what extent will the sun push the "far" universe away?

Now let's imagine, there is a smaller ball with the sun as the center inside the SMALL BALL. The radius of the smaller ball is $R/2/2=1.1\times10^{26}$m, half of the radius of SMALL BALL. We can call this smaller ball as SMALL BALL'. Now let's focus on the part of the universe inside the SMALL BALL and outside of the SMALL BALL'. This part of the universe is also a HOLLOW BALL, thus we can call it HOLLOW BALL'universe. Apparently, the mass of HOLLOW BALL'universe $M_{hollow\ ball'}$ is $1/8\times7/8\times M_{universe}=1/8M_{hollow\ ball}$, where $M_{hollow\ ball}$ is the total mass of the HOLLOW BALL universe, while the distance of the HOLLOW BALL'universe to the sun is half of the distance of the HOLLOW BALL universe to the sun.

So according to gravity equation, F=（GM_aM_b）/r² the gravity force between the sun and the HOLLOW BALL' universe $F_{hollow\ ball'}$ is:

$(GM_{sun}M_{hollow\ ball'})/(R/4)^2 > F_{hollow\ ball'} > (GM_{sun}M_{hollow\ ball'})/(R/2)^2$, therefore:

$(6.67\times10^{-11}\ N\bullet m^2/kg^2\times2\times10^{30}kg\times8.75\times10^{52}kg\times1/8)/(2.2/2\times10^{26}m\times2.2/2\times10^{26}m) > F_{hollow\ ball'} > (6.67\times10^{-11}\ N\bullet m^2/kg^2\times2\times10^{30}kg\times8.75\times10^{52}kg\times1/8)/(4.4/2\times10^{26}m\times4.4/2\times10^{26}m)$

So, $1/2\times F_{hollow\ ballMAX}=F_{hollow\ ball'MAX}=1.2\times10^{20}N > F_{hollow\ ball'} > 3.0\times10^{19}N = F_{hollow\ ball'MIN}=1/2\times F_{hollow\ ballMIN}$

So, we can roughly consider $F_{hollow\ ball'}\approx F_{hollow\ ball'AVERAGE} = (F_{hollow\ ball'MAX} + F_{hollow\ ball'MIN})/2 = 0.75\times10^{20}N = 1/2\times F_{hollow\ ballAVERAGE}$

And we already know that the average radiation pressure force of the sun is $F_{total\ pressure}=1.39\times10^{21}N$,

So, $F_{total\ pressure}>F_{hollow\ ball'}$. And, $F_{total\ pressure}/F_{hollow\ ball'}\approx F_{total\ pressure}/F_{hollow\ ball'AVERAGE} = 1.39\times10^{21}N / 0.75\times10^{20}N = 18.53$.

So, even if only 1/8 of the total sunlight is absorbed or reflected by the matters, including stars, planets, black holes, and interstellar dusts, inside the HOLLOW BALL'universe, the light pressure force of the sun will be bigger than the gravity force of the sun. So it means that the sun generally will push the HOLLOW BALL'universe away.

Then we can repeat the same method. Let's further imagine there is a even smaller ball with the sun as the center inside SMALL BALL', and the radius of the even smaller ball is half of the radius of the SMALL BALL'.

We can call this even smaller ball as SMALL BALL''. Now let's focus on the part of the universe inside the SMALL BALL' and outside of the SMALL BALL''. This part of the universe is also a hollow ball, thus we can call it HOLLOW BALL''universe. Apparently, the mass of HOLLOW BALL''universe $M_{hollow\ ball''}$ is $=1/8M_{hollow\ ball'}$, where $M_{hollow\ ball'}$ isthe total mass of the HOLLOW BALL'universe, while the distance of the HOLLOW BALL''universe to the sun is half of the distance of the HOLLOW BALL'universe to the sun.

So according to gravity equation, $F=(GM_aM_b)/r^2$

The gravity force between the sun and the HOLLOW BALL'' universe $F_{hollow\ ball''}$ is:

$(GM_{sun}M_{hollow\ ball''})/(R/8)^2 > F_{hollow\ ball''} > (GM_{sun}M_{hollow\ ball''})/(R/4)^2$,

So, $1/2\times F_{hollow\ ball'MAX}=F_{hollow\ ball''MAX}=0.6\times10^{20}N>F_{hollow\ ball''}>1.5\times10^{19}N=F_{hollow\ ball''MIN}=1/2\times F_{hollow\ ball'MIN}$

So, we can roughly consider $F_{hollow\ ball''}\approx F_{hollow\ ball''AVERAGE}=(F_{hollow\ ball''MAX}+F_{hollow\ ball''MIN})/2 = 0.375\times10^{20}N = 1/2\times F_{hollow\ ball'AVERAGE}$

And the average radiation pressure force of the sun is $F_{total\ pressure}=1.39\times10^{21}N$,

So, $F_{total\ pressure}>F_{hollow\ ball''}$.

And, $F_{total\ pressure}/F_{hollow\ ball''}\approx F_{total\ pressure}/F_{hollow\ ball''AVERAGE}=1.39\times10^{21}N/0.375\times10^{20}N=37.07$

So, even if only 1/16 of the total sunlight is absorbed or reflected by the matters, including stars, planets, black holes, and interstellar dusts, inside the HOLLOW BALL''universe, the light pressure force of the sun will be bigger than the gravity force of the sun. So it means that the sun generally will push the HOLLOW BALL''universe away.

Therefore, by repeating the same method, we can continuously divide the universe into smaller hollow ball universes and examine them. The conclusions of the further examinations will be like:

So, even if only 1/32 of the total sunlight is absorbed or reflected by the matters inside the HOLLOW BALL'''universe, the sun generally will push the HOLLOW BALL'''universe away.

So, even if only 1/64 of the total sunlight is absorbed or reflected by the matters inside the HOLLOW BALL''''universe, the sun generally will push the HOLLOW BALL''''universe away.

So, even if only 1/128 of the total sunlight is absorbed or reflected by the matters inside the HOLLOW BALL'''''universe, the sun generally will push the HOLLOW BALL'''''universe away.

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In the end, we can divide and examine probably 90% or 95% or 98% of the total universe, and find that, for the hollow universes we examine, the push of radiation pressure force of the sun is always bigger than the pull of the gravitational force of the sun. And, according to this repeated method, the sum of the amount of the sunlight absorbed or reflected by the matters inside all of these hollow universes can be less than HALF of the total sunlight(because 1/4+1/8+1/16+1/32+1/64+ 1/128......<1/2). So the sun actually always pushes the entire universe away until the galaxies are so near to the sun that this same method can no longer be used. Or we can put it like this: for most part of the universe where its matters are not near from the sun and can be considered as evenly distributed inside it, if sufficient proportion of the sun's radiation pressure force imposes on it, the sun pushes it away; for the rest part of the universe that is quite near to the sun and cannot be considered evenly distributed with matters, the condition will be different.

Therefore, although all above calculations are simplified and rough calculations, we can still conclude:

- The sun's interaction with the universe matter very near to the sun is mainly governed by gravity force, so matters tend to concentrate together to the sun.
- The sun's interaction with the universe matter not very near to the sun is more governed by light pressure force, so matters tend to go away from the sun.

- Stars similar to the sun are all like the sun in above mentioned 2 conclusions: they pull nearby matters and push distant matters away.

These conclusions are exactly in line with the current condition of the universe: nearby matters come together and form galaxies or celestial bodies, and faraway matters are going away from each other at increasing speeds. Hence, the current expansion of the universe.

So, the radiations of the sun and other stars are at least one of the causes that push the universe to expand. It is worth mentioning that, in early life of the universe, when matters were closer to each other than now, there were probably more huger stars which radiate at much stronger rate, e.g., at a rate more than dozens or hundreds or thousands times stronger. And it is possible that these radiation are now still travelling or being reflected more than once in the universe and be at work to expand the universe.

In addition to the push of the radiation pressure force, there are other forces inside universe causing the universe to expand. Another known force is the pressure force of cosmic rays.

Cosmic rays are different from rays of light. They are not electromagnetic radiations but particles. Actually they have nearly the same composition as ordinary interstellar gas but travels at a extremely high speed, nearly the speed of light. In short, radiations should be considered as energy but cosmic rays can be considered as matter. In considering cosmic rays, we can reasonably suppose that cosmic rays are isotropic inside the universe so we can use the data on earth to draw conclusions and use the conclusions on the entire universe. Because the cosmic rays are considered matters and radiations are considered energy, part of the energy of cosmic rays are kinetic energy while the energy of radiation is mainly electromagnetic energy. As a result, the cosmic rays with the same energy as electromagnetic radiation has a larger effect than electromagnetic radiation in pushing the earth, or any other matter in the universe. Here, in studying the energy, momentum, and impulse of cosmic rays, the author found an equation describing the relationship between the collective radiation pressure force (F) of a star and the power (P) at which the star generates energy. This equation holds for any star in the universe.

Firstly, for particles travelling nearly at the speed of light

$E=Pt=MC^2 \approx MV^2$

$Ft=MV-M\times 0=MV=MV^2/V$

When V equals the speed of light C:

$Ft=MV^2/V=Pt/V=Pt/C$

Therefore:

$F=P/C$

This equation can be used on any star. When a star generates nuclear fusion energy at the rate of power P, then its collective radiation pressure force F=P/C. The author checked this equation to be usable. Welcome other physicists to verify its validity. If we use this equation, we can roughly calculate the total power that the universe needs to accelerate expansion when we only consider radiation pressure force as the cause for universe expansion. According to the previous calculation, $F_{total\ pressure}/F_{hollow\ ball} \approx 4.64$ when radiation is not reflected but fully absorbed. So when the sun's radiation are absorbed by the matter inside the universe, the pushing of radiation pressure force is roughly 4.64/2 =2.32 times bigger than the pulling of gravitational force. Because the nuclear fusion power of the sun P_{sun} is 3.72×10^{26}W. IfA% of the total radiation is absorbed by matter inside universe, B%of the total radiation is reflected, and we suppose that reflections happen only once, then the nuclear fusion power the universe need for sustainable expansion will be:

$P_{universe}=M_{universe}/M_{sun}\times P_{sun}/2.32/(A\%+2\times B\%)$

So, $P_{universe}=(1\times 10^{53}kg)/(2\times 10^{30}kg)\times 3.72\times 10^{26}W/2.32/(A\%+2\times B\%)$

So, $P_{universe}=8.02\times 10^{48}W/(A\%+2\times B\%)$.

So if the actual nuclear fusion power of the universe now is larger than $P_{universe} = 8.02 \times 10^{48}W / (A\% + 2 \times B\%)$, the universe will increase its expansion. If the actual nuclear fusion power of the universe now is smaller than $P_{universe}=8.02\times 10^{48}W/(A\%+2\times B\%)$, we would need to look into the effect of other factors, such as that of the cosmic rays.

Now back to the discussion about cosmic rays. So when the speed of the particle is much lower than the speed of light, its energy is mainly kinetic energy, then:

$E=Pt=0.5\times MV^2$, so $2\times Pt=MV^2$

$Ft=MV-M\times 0=MV=MV^2/V$, so $FtV=MV^2$

So, $FtV=2\times Pt$

So, $F=2\times P/V>2\times P/C$

Therefore, for other particles whose speeds are less than the speed of light C, the pressure force they impose on their environment is larger than the force of radiation pressure. That is, at a given rate of power, the lower the particle's speed, the larger its force.

After calculation, the author finds that, when the speed of the particle of the cosmic rays is very close to the speed of light, i.e., when the energy of the cosmic rays are very high, its pushing force tend to be very close to the pushing force of the light with the same energy as the particle. For example, when a proton has an energy level of 10^{20}eV, its speed is very close to the speed of light, and its pushing force is almost the same as radiation pressure force.

And when the speed of the particle of the cosmic rays is much lower than the speed of light, i.e., when the energy of the cosmic rays are relatively lower, its pushing force is higher than the pushing force of the light with the same energy as the particle. For example, when a proton has an energy level of 10^{9}eV, its speed is 88% the speed of light, and its pushing force is roughly 1.6 times as the radiation pressure force.

According to current statistics, the total energy of cosmic rays the earth receives is almost the same as the total energy of the radiations the earth receives from the stars excluding the sun. Thus cosmic rays' total pushing force, or expanding force, upon the universe is greater than the pushing force, or expanding force, of the radiation pressure force. As a result, the total expanding force of the universe is much larger the contracting force, i.e., gravitational force. So the universe can only expand at an accelerating speed now.

Other possible causes that help the universe to expand can include supernova explosions, which are enormously huge release of energy within a very short period of time and can push the universe to expand. Usually supernova explosions are considered source of cosmic rays plus electromagnetic radiation. So we already discussed it. Also, all the gravitational wave events including mergers of black holes and neutron stars can lose enormous amount of mass and release extremely huge energies very quickly and help strongly push the universe to expand. The energy released during such events are probably gamma ray burst. This may add additional force to the total expanding force of the universe. In the future, more thorough and detailed computational models can be built to help calculate or simulate how much total force are there in the universe that cause the universe to expand. When all these causes are included and examined, it may be found that the forces that push the universe to expand could be larger than discussed here.

Additionally, so many cosmic activities are involved with mass losses, such as the nuclear fusion of the stars or the merger of the black holes, and the mass losses mean decreases of gravitational forces, no matter it is big and huge decrease or small and slow decrease. As a result, there are also less gravitational force that pulls the universe together. This also contributes to an increased expansion of the universe.

As a result, the above-mentioned causes may be used to replace the current space expansion theory in explaining the accelerating universe expansion.

Conclusions:

- At a near distance, the gravitational force of the stars towards the matters of the universe is much bigger than the star light pressure force. So this generally makes the nearby universe to concentrate.
- According to current data and calculation, at a large distance, the pushing of the stars' radiation pressure force is larger than the pulling of gravitational force inside the universe. Thus the radiation pressure force alone inside the universe will cause the entire universe to expand at an increasing rate.
- According to current data and calculation, the pushing of the cosmic rays' pressure force is bigger than the pushing of the stars' radiation pressure force in the universe. This will further accelerate the universe expansion.
- Other factors including gamma ray burst, probably resulted from the gravitational wave events, can also be important causes of universe expansion.
- The relationship between the total radiation pressure force (F) of a star and the power (P) of the star can be described by equation: F=P/C

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