Theodor Kaluza's Theory of Everything: revisited

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Abstract
Using a metric based on solutions for the scalar of a 5-dimensional Kaluza model in the Einstein field equations allows to derive a convergent series of particle energies, to be quantized as a function of the fine-structure constant, α, with limits given by the energy values of the electron and the Higgs vacuum expectation value. The value of α can be given numerically by the gamma functions of the integrals involved, extending the formalism to N-dimensions yields a single expression for the electroweak coupling constants. The series expansion of the energy equation provides quantitative terms for Coulomb, strong and gravitational interaction. A scalar field term in the field equations gives a value for the cosmological constant in the correct order of magnitude.
The model can be expressed ab initio without use of free parameters.

1 Introduction
General theory of relativity (GTR) is a fundamental concept connecting energy and energy related phenomena with the geometry of space-time, established by Einstein for gravitational effects in 4 dimensions [1]. In 1921 Kaluza demonstrated that GTR may be unified with Maxwell's equations of electromagnetism in a 5 dimensional model [2], mainly known as Kaluza-Klein theory today, including the contributions of Klein [3] who introduced the idea of compactification and attempted to join the model with the emerging principles of quantum mechanics. This version became a progenitor of string theory. The classical Kaluza model was developed further as well [4], Wesson and coworkers elaborated a general non-compactified version to describe phenomena extending from particles to cosmological problems. The equations of 5D space-time may be separated in a 4D Einstein tensor and metric terms representing mass and the cosmological constant, Λ. Particles are photon-like in 5D, traveling on time-like paths in 4D. This version is known as space-time-matter theory [5]. Both successor theories focus more on general relationships than providing quantitative results for specific phenomena such as particle energies.

The model described in the following evolved from a heuristic approach and does not attempt to give a complete solution for a 5D theory but to demonstrate that Kaluza's ansatz provides very simple, parameter-free and in particular quantitative solutions for a wide range of phenomena. The basic equations will be picked from the existing literature. The main innovation will be to interpret the equations in their entirety as related to electromagnetism which essentially means using an electromagnetic constant in place of the gravitational term, G/c_0^4, in the field equations. The framework of Kaluza's equations suggests to use either or and while Kaluza noticed the incompatibility of the constant G with the energy scale of particles he seemingly did not inquire thoroughly into the alternate possibility. This shift in order of magnitude will create a space-time curved strong enough to fit the effects of electromagnetism and to localize a photon in a self trapping kind of mechanism. Gravitational phenomena will be recovered via a series expansion of the energy equation.

GTR provides a flexible system of 2nd order differential equations which may reproduce quantum mechanical expressions such as the Klein-Gordon-equation for spinless particles [5]. To include rotation /spin, the basic theory has to be extended, examples are Einstein-Cartan or Twistor models. Spin is implicitly introduced in Kaluza's ansatz as well, since electrodynamics allows solutions for circular polarized light.

For the model presented here it might be helpful to use the following visualization: a photon with its intrinsic angular momentum interpreted as having its E-vector rotating around a central axis of propagation [1] will be transformed into an object that has the - still rotating - E-vector constantly oriented to a fixed point, the origin of the local coordinate system used, resulting in an SO(3) object with point charge properties [2]. The vectors E, B and V of the propagation velocity are supposed to be locally orthogonal and subject to the standard Maxwell equations, however, on the background of an appropriately curved space-time.

The use of an electromagnetic constant in natural units in the field equations of the GTR / Kaluza framework

1 Angular momentum J = 1, symmetry SO(2) as projected in propagation direction;

2 Neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of reversed E-vector orientation and opposite polarity.

and the assumption that the objects considered retain photon-like properties will be the only fundamental assumptions needed in this model.

The basic proceeding will be as follows:

Kaluzas 5D equations may be arranged to give

1.) Einstein-like equations for space-time curved by an electromagnetic stress-energy tensor plus a scalar field term, Φ, (equ. (5)),

2.) Maxwell equations where the source depends on the scalar field,

3.) a wave-like equation connecting the scalar with the electromagnetic tensor (equ. (6)).

Using solutions for the scalar Φ of 3.) as ansatz in a general 4D metric will yield electroweak coupling constants as geometric coefficients in 2, 3 and 4 spatial dimensions and a convergent series of particle energies quantized as a function of the fine-structure constant, α, with its limits given by electron and the Higgs VEV energy. The series expansion of the incomplete Γ-function in the energy expression for a point charge will include a term which at short range yields effects associated with strong interaction, at long range gives a quantitative term for gravitational interaction. The scalar field term of 1.) may be considered to be a natural candidate for the cosmological constant, Λ, which will give a result in a correct order of magnitude if the basic coefficients of this model will be used.

The relation of the masses e, μ, π with α was noted first in 1952 by Y.Nambu [6]. M.MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [7].

To focus on the more fundamental relationships some minor aspects of the model, including topics such as mean life time and magnetic moment, are exiled to an appendix, related topics to be marked as [A]. Typical accuracy of the calculations presented is in the order of 0.001-0.0001 [3]. QED corrections are not considered in this model.

2 Calculation

2.1 System of natural units

It is common to define natural electromagnetic units by referring them to the value of the speed of light. The same will be done here, thus subscript c will be used. The freedom in defining the units will be used to obtain a constant appropriate to replace G/c_0^4 in the Einstein field equations (EFE). Retaining SI units for length, time and energy the electromagnetic constants may be defined as:

$$c_0^2 = (\varepsilon_c\, \mu_c)^{-1} \tag{1}$$

with $\varepsilon_c = (2.998\text{E+8 [m}^2\text{/Jm]})^{-1} = (2.998\text{E+8})^{-1}$ [J/m]

$\mu_c = (2.998\text{E+8 [Jm/s}^2\text{]})^{-1} = (2.998\text{E+8})^{-1}$ [s²/Jm]

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307\text{E-28}$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671\text{E-36}$ [J²]. In the following $e_c = 3.110\text{E-18}$ [J] and $e_c/\varepsilon_c = 9.323\text{E-10}$ [m] or related terms (including 4π) may be used as natural unit of energy and length.

The constant G/c_0^4 [m/J] in the EFE will be replaced by:

$$(8\pi)G/c_0^4 \quad => \quad \approx \; -\frac{1}{\varepsilon_c} \tag{2}$$

in an accordingly modified field equation:

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{1}{\varepsilon_c} T_{\alpha\beta} \tag{3}$$

2.2 Kaluza theory

Kaluza theory is an extension of general relativity to 5D space-time with a metric given as [5, equ. 2.2]:

$$g_{AB} = \begin{bmatrix} (g_{\alpha\beta} - \kappa^2 \Phi^2 A_\alpha A_\beta) & -\kappa \Phi^2 A_\alpha \\ -\kappa \Phi^2 A_\beta & -\Phi^2 \end{bmatrix} \tag{4}$$

In (4) roman letters correspond to 5D, greek letters to 4D, (ct, r, ϑ, φ, 5th coord.) = (x0, x1, x2, x3, x4), κ^2 corresponds to the constant in the field equation (3) [4], A is the electromagnetic potential. In the context of the

3 Including e.g. errors due to the numerical approximation of Γ-functions.

4 $\kappa^2 = 16\,\pi G/c_0^4 \;=>\; \kappa_c^2 \approx -2/\varepsilon_c$; the unit system of 2.1 gives e.g. terms $1/\varepsilon_c T^{EM} \sim 1/\varepsilon_c(\varepsilon_c E^2 + B^2/\mu) = (E^2 + c_0^2 B^2)$[m^{-2}];

static approach of this model A will be assumed to be represented by the electric potential, $\varphi(r) \sim \rho/r \sim e_c/(4\pi\varepsilon_c r)$. Assuming 5D space-time to be flat, i.e. $R_{AB} = 0$, gives for the 4D-part of the field equations [5, equ. 2.3]:

$$G_{\alpha\beta} = \frac{\kappa^2 \Phi^2}{2} T_{\alpha\beta}^{EM} - \frac{1}{\Phi}\left(\nabla_\alpha(\partial_\alpha \Phi) - g_{\alpha\beta}\Box\Phi\right) \tag{5}$$

From $R_{44} = 0$ follows:

$$\Box\Phi = -\frac{\kappa^2 \Phi^3}{4} F_{\alpha\beta} F^{\alpha\beta} \tag{6}$$

In the following only the diagonal part of (4) and only derivations with respect to r of a spherical symmetric coordinate system will be considered. Equation (6) will be used to obtain an ansatz for a metric to get a solution of the 00-component in (3).

A function Φ_N

$$\Phi_N \approx \left(\frac{\rho}{r}\right)^{N-1} e^{\nu} = \left(\frac{\rho}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^{N}\right) \qquad \text{with} \quad \nu = -\left(\frac{\rho}{r}\right)^{N} \tag{7}$$

yields solutions for an equation of general type of (6), where the term of highest order of exponential N, given by Φ'', $\sim \rho^{3N-1}/r^{3N+1}$ may be interpreted to provide the terms for $A' \sim \varphi' \sim \rho/r^2$ (using [5, equ. 6.76] for the 2nd term in (8), $g_{00} \sim e^{2\nu}$ (see [A4], [A5])):

$$\Phi_N'' \sim \left(\frac{\rho^{3N-1}}{r^{3N+1}}\right) e^{\nu} \sim \mathbf{\Phi_N^3} e^{-2\nu} (\boldsymbol{A_0}')^2 \approx \left[\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r}}\right)^{\boldsymbol{N-1}} \boldsymbol{e^{\nu}}\right]^{\mathbf{3}} e^{-2\nu} \left(\frac{\boldsymbol{\rho}}{\boldsymbol{r^2}}\right)^{\mathbf{2}} = \left(\frac{\rho}{r}\right)^{3N-3} e^{\nu} \left(\frac{\rho}{r^2}\right)^2 \tag{8}$$

$R_{44} = 0$ does not have to be obeyed strictly and is secondary to condition $R_{AB} = 0$. The significance of (8) lies in providing the relation of exponential and pre-exponential factor and first of all in the requirement to contain $A \sim \rho/r$ in the terms for Φ_N. [5]

2.3 Example for metric, point charge energy

In the following equ. (7) with N = 3, which is supposed to represent 3 spatial dimensions, see chpt. 2.6, will be used as ansatz in a general metric.

There are a lot of possible solutions using such an ansatz and one may choose the one that fits experimental observations best. The following specific examples are intended to serve as proof of concept, some aspects considered for choosing them include:

a) a spherical symmetric coordinate system and metric will be used, with opposite sign of time and radial component, yet some additional freedom in angular components,

b) in particular, since rotation (SO(3)) of an E-vector with extension in angular direction will result in some kind of self interaction increasing with r ->0 *unless* space(-time) is curved in such a way as to prevent this, the r^2-term in the angular coordinates has to be canceled, implying positive curvature and an expansion of curved space-time with r^2 at any given r, i.e. $R(r) \sim -1/r^2$, has to hold for the Ricci scalar,

c) it might be necessary to differentiate between ρ in the exponent and the prefactor, resulting in terms $(\rho^*/r)^{N-1}\exp(-(\rho/r)^N)$, $\rho^* \neq \rho$,

d) simplicity [6].

In [A4] the solution for G_{00} of two examples of a metric of type

$$\begin{aligned} g_{\alpha\beta} = &\left[\left(\frac{\rho^*}{r}\right)^2 \exp\left(-a\left(\frac{\rho}{r}\right)^3\right)\right]^{\boldsymbol{p}}, \ -\left[\left(\frac{\rho^*}{r}\right)^2 \exp\left(-b\left(\frac{\rho}{r}\right)^3\right)\right]^{\boldsymbol{p}}, \ -/+\left[\left(\frac{\rho^*}{r}\right)^2 \left(-c\left(\frac{\rho}{r}\right)^3\right)\right]^{\boldsymbol{q}} r^2, \\ &-/+\left[\left(\frac{\rho^*}{r}\right)^2 \left(-c\left(\frac{\rho}{r}\right)^3\right)\right]^{\boldsymbol{q}} r^2 \sin^2\vartheta \end{aligned} \tag{9}$$

will be given in detail for p = 1, q = 0 and p = 2, q = 1 [7].

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5 Using terms of Φ_N for canceling of similar terms of other $R_{\alpha\beta}$ components may in fact increase the resources to obtain a specific solution.

6 including dimensionality: a 5D solution should refer to a flat 5D space-time [5], thus a 4D metric might be sufficient as solution;

7 Terms with p = 2 correspond to Φ being squared in g_{44} of (4).

Coefficients ρ will be defined as $\rho^* = e_c/(4\pi\varepsilon_c)$, $\rho \sim \alpha(n)\,\rho^*$ (cf. equ. (22)).
For both cases the Einstein tensor component G_{00} will be (with $av = 2v = (\rho/r)^3$):

$$G_{00} = -/+\ \rho^{*2}/r^4\, e^{2v} \tag{10}$$

and using equ. (2)f will give (w = energy density):

$$-/+\frac{\rho^{*2}}{r^4}e^{2v} \approx -\frac{w}{\varepsilon_c} \quad => \quad \frac{\varepsilon_c \rho^{*2}}{r^4}e^{2v} \approx +/-\, w \tag{11}$$

The volume integral over (11) gives the particle energy according to:

$$W_n = \varepsilon_c \rho^{*2} \int_0^{r_n} \frac{e^{2v}}{r^4}\, d^3r = 4\pi\varepsilon_c\rho^{*2}\int_0^{r_n}\frac{e^{2v}}{r^2}\, dr \tag{12}$$

Solutions for integrals over e^v times some function of r can be given by:

$$\int_0^{r_n} \exp\left(-(\rho_n/r)^3\right) r^{-(m+1)} dr = \Gamma\left(m/3, (\rho_n/r_n)^3\right)\frac{\rho_n^{-m}}{3} = \int_{(\rho_n/r_n)^3}^{\infty} t^{\frac{m}{3}-1} e^{-t} dt\ \frac{\rho_n^{-m}}{3} \tag{13}$$

with m = {..-1;0;1;..}. The term $\Gamma(m/3, (\rho_n/r_n)^3)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind [8]. In the range of values relevant in this work, for m ≥ 1 the complete gamma function $\Gamma_{m/3}$ is a sufficient approximation, for m ≤ 0 the integrals have to be integrated numerically, requiring an integration limit, see 2.4. Equation (12) will give:

$$W_{n,\,elstat} = 4\pi\varepsilon_c\rho^{*2}\int_0^{r_n}\frac{e^{2v}}{r^2}\, dr = b_0\,\Gamma(1/3, (\rho_n/r_n)^3)\,\rho_n^{-1}/3 \approx b_0\,\Gamma_{1/3}\,\rho_n^{-1}/3 \tag{14}$$

resulting in the integral for the energy of a point charge term modified by e^{2v}. Particles are supposed to be electromagnetic objects possessing photon-like properties, thus it will be assumed that particle energy has equal contributions of electric and magnetic energy, i.e.

$$W_n = 2W_{n,,elstat} = 2W_{n,mag} = W_{n,elstat} + W_{n,mag} \approx 2\, b_0\, \Gamma_{1/3}\, \rho_n^{-1}/3\ . \tag{15}$$

2.4 Angular momentum, coefficient σ

Euler integrals of (13) with m ≤ 0 require a lower integration limit, $(\rho_n/r_n)^3$ that may be derived from the condition for angular momentum |J| = 1/2 [ħ].
A simple relation with angular momentum J for spherical symmetric states will be given by applying a semi-classical approach using

$$J = r_2\, x\, p(r_1) = r_2 W_n(r_1)/c_0 \tag{16}$$

with $W_{kin,n} = 1/2\ W_n$, using term $2b_0$ of equ. (15) as constant factor, integrating over a circular path of radius $|r_2| = |r_1|$ and particle radius, r_n, or for the Euler integral, $(r_n/\rho_n)^3$, as integration limit. Equation (13) will give for m = 0:

$$|J| = \int_0^{r_n}\int_0^{2\pi} J_n(r)\, d\varphi\, dr = 4\pi\frac{b_0}{c_0}\int_0^{r_n} e^{2v} r^{-1} dr = 4\pi\alpha\hbar\int_0^{r_n} e^{2v} r^{-1} dr = \frac{4\pi}{3}\frac{b_0}{c_0}\int_{(\rho_n/r_n)^3}^{\infty} t^{-1} e^{-t} dt \equiv 1/2\ [\hbar] \tag{17}$$

to obtain J=1/2 the integral over $e^{2v}r^{-1}$ of (17), *has to* yield $\alpha^{-1}/8\pi$.

$$\int_0^{r_n} e^{2v} r^{-1} dr = 1/3\int_{(\rho_n/r_n)^3}^{\infty} t^{-1} e^{-t} dt \equiv \frac{\alpha^{-1}}{8\pi} \approx 5.45 \tag{18}$$

Relation (18) may be used for a numerical calculation of the integration limit, $(\rho_n/r_n)^3$, representing spherical symmetry and J = 1/2 [9].
The existence of an integration limit implies a differential equation of a general type:

$$-\frac{r}{}\frac{d^2 e^v}{dr^2} + \frac{\rho^3}{r^3}\frac{de^v}{dr} - \frac{\rho^3}{\sigma\ r^4}e^v = 0 \tag{19}$$

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8 Euler integrals yield positive values, the absolute sign used for e.g. $|\Gamma_{-1/3}|$ is due to the sign convention of Γ-functions.
9 The geometric expression of σ_0 as given in (23), [A6] may conversely used as starting point to define J, see 2.6 as well.

with a solution

$$e^{v} = \exp\left(-\left(\frac{\rho_n^3}{r^3}+\left[\left(\frac{\rho_n^3}{r^3}\right)^2-4\,\frac{\rho_n^3}{\sigma r^3}\right]^{0.5}\right)/2\right) \tag{20}$$

in place of the simple exponential of (7). From (20) follows:

$$r_n = (\sigma\rho_n^3/8)^{1/3} \tag{21}$$

For higher angular terms, l -> ∞, σ will approach ≈1, (20) will approximate e^v of equ. (7)ff. Equation (7) may serve as an excellent approximation of (20) for *any* σ if σ will be included in the parameter ρ_n:

$$2v = -(\rho_n/r)^3 \approx -2\sigma\alpha(n)(e_c/(4\pi\varepsilon_C r))^3 \tag{22}$$

Coefficient α(n) will be a particle specific term, coefficient σ an integration limit term related to symmetry that may be given in various useful expressions for spherical symmetry as σ_0 (see [A1]):

$$\sigma_0 = 8\,(r_n/\rho_n)^3 = (1.5133\ \alpha^{-1}\ 2/3\ |\Gamma_{-1/3}|)^3 = 1.5133^3\ \sigma^* = 8\left(\frac{4\pi|\Gamma_{-1/3}|^3}{3}\right)^3 = 1.772E+8\ [-] \tag{23}$$

2.5 Photon energy

In the following a term for length expressed via the Euler integral of (13) will be introduced for $\lambda_{C,n}$:

$$\lambda_{C,n} = \int_0^{\lambda_{C,n}} e^{2v}dr = \rho_n/3\int_{(\rho_n/\lambda_{C,n})^3}^{\infty} t^{-4/3}e^{-t}dt \approx \Gamma(-_{1/3,}(\rho_n/\lambda_{C,n})^3)\ \rho_n/3 \tag{24}$$

In the limit $(\rho_x/r_x)^N$ ->0

$$\Gamma(-1/N_{,}(\rho_x/r_x)^N) = \int_{(\rho_x/r_x)^N}^{\infty} t^{-(1/N+1)}e^{-t}dt \approx N\,(\rho_x/r_x)^{-1} \tag{25}$$

holds. Equation (25) inserted in the right side of (24) gives back $\lambda_{C,n}$, however, (24) may be seen as expressing $\lambda_{C,n}$ in terms useful for this model, i.e. ρ_n and Γ-functions, if equ. (25) is used for calculation of the incomplete Γ-function, using the integration limit $(\rho_n/r_n)^3$ = 8/σ according to chpt. 2.4 in the Euler integral. This gives in good approximation:

$$\lambda_{C,n} \approx 36\pi^2\ \rho_n/3\ |\Gamma_{-1/3}| \tag{26}$$

With (26) energy of a photon can be expressed by:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = \frac{hc_0}{\int^{\lambda_{C,n}} e^{2v}dr} = \frac{3hc_0}{36\pi^2|\Gamma_{-1/3}|\rho_n} \tag{27}$$

2.6 Fine-structure constant, α

The energy of a particle is assumed to be the same in both photon and point charge description. Equating (15) with (27) and rearranging to emphasize the relationship of α with the gamma functions ($\Gamma_{1/3}$ = 2.679; $|\Gamma_{-1/3}|$ = 4.062) gives as first approximation (note: h => ħ):

$$\frac{4\pi\Gamma_{1/3}|\Gamma_{-1/3}|}{0.998} = \frac{9hc_0}{18\pi b_0} = \frac{\hbar c_0}{b_0} = \alpha^{-1} \tag{28}$$

The agreement may be improved by using better approximations of the incomplete Γ-functions involved.

The concept for calculating the fine-structure constant α may be extended directly to 4 and - with some additional assumptions - to 2 dimensions, based on the integral over the N-dimensional point charge term modified by a generalized exponential term, Ψ_N, with N ={2; 3; 4}:

$$\Psi_N(r) = \exp\left(-\left(\frac{x}{r}\right)^N\right) \tag{29}$$

This will give a point charge term (S_N = geometric factor for n-dimensional surface, in case of 3D: 4π):

$$\int_0^r \Psi_N(r)r^{-2(N-1)}\ d^N r = S_N\int_0^r \Psi_N(r)r^{-(N-1)}dr \sim S_N\int_0^r \Phi_N dr \tag{30}$$

that will be multiplied by a complementary integral

$$\int_0^r \Psi_N(r) r^{(N-3)} dr \tag{31}$$

to yield a dimensionless constant. This results in (see [A6]):

$$\alpha_N^{-1} = \frac{(2\pi)^{\delta(N-2)}}{(2\pi)^{(N-2)}} \int_0^r \Psi_N(r) r^{-(N-1)} dr \int_0^r \Psi_N(r) r^{(N-3)} dr \tag{32}$$

with N = {2; 3; 4} or in terms of the Γ-functions:

$$\alpha_N^{-1} = S_n \frac{\Gamma_+(\Psi_N)\Gamma_-(\Psi_N)}{N^2 \ arg(\Gamma(\Psi_N))^2} \tag{33}$$

with $\Gamma_{+/-}$ (Ψ_N) being the positive and negative Γ-functions attributed to the integrals over Ψ_N and arg($\Gamma(\Psi_N)$) being the argument of the Γ-functions attributed to Ψ_N [10], i.e. the three coupling constants of the electroweak charges g', e and g can be combined in a single function of spatial dimension only [11].

Dimension - space	coupling constant	Value of *inverse* of coupling constant, α_N^{-1}	
4D	α(g)	$2\pi^2\Gamma_{+1/2}\,\lvert\Gamma_{-1/2}\rvert\ 4/16 = \pi^3 =$	31.006
2D	α(g')	$2\pi\ \Gamma(0, 8/\sigma_{2D})^2/4 = \pi^4 =$	97.409
3D	α(e)	$4\pi\ \Gamma_{+1/3}\lvert\Gamma_{-1/3}\rvert\ 9/9 = 4\pi\ \Gamma_{+1/3}\lvert\Gamma_{-1/3}\rvert =$	137.036

Table 1: Values of electroweak coupling constants

The ratio of α_e and α_g represents the Weinberg angle, θ_W, and may be expressed as:

$$\sin^2\theta_W = \frac{\alpha_e}{\alpha_g} = \frac{\pi^2}{4\,\Gamma_{1/3}|\Gamma_{-1/3}|} = 0.2263 \tag{34}$$

(Experimental values: PDG [8]: $\sin^2\theta_W = 0.2312$, CODATA [9]: $\sin^2\theta_W = 0.2223$). The mass ratio of the W- and Z-bosons will be given by cos $\theta_{W,calc} = (m_W/m_Z)_{calc} = 0.8796 = 0.998\ (m_W/m_Z)_{exp}$ [10].

2.7 Quantization with powers of $1/3^n$ over α

In a general case ρ_n may be given as product of $\rho^* = e_c/(4\pi\varepsilon_c)$ [m], factor 2σ and a partial product of particle specific dimensionless coefficients, α_n, of succeeding particles representing the ratio ρ_{n+1} / ρ_n (cf. (22)):

$$\rho_n \sim \rho' \Pi_{k=1}^n \alpha_k = \rho' \Pi_n \qquad n = \{1;2;..\} \tag{35}$$

Index n will indicate spherical symmetric solutions and serve in the following as equivalent of a radial quantum number. For the angular terms of Φ(r, ϑ, φ), to be indicated by index l, only rudimentary results exist, their contribution has to be incorporated in parameter σ. Inserting (35) in the product of the point charge and the photon expression of energy, (15) and (27), gives for the square of energy $W_n^2 = W_{pc,n} W_{phot,n}$:

$$W_n^2 = 2 b_0 h c_0 \frac{\int^{r_n} e^{2\nu} r^{-2} dr}{\int^{\lambda_{C,n}} e^{2\nu} dr} \sim \frac{1}{\rho_n^2} \sim \frac{\alpha_1^1\alpha_2^1.....\alpha_n^1}{\alpha_1^3\alpha_2^3....\alpha_n^3} \tag{36}$$

The last expression of (36) is obtained by expanding the product $\Pi_n{}^2$ included in $\rho_n{}^2$ of (35) with $\Pi_n{}^1$.
The only non-trivial solution for W_n^2 where all intermediate particle coefficients cancel out and W_n becomes a function of coefficient α_1 only is given by a relation $\alpha_{n+1} = \alpha_n^{1/3}$:

$$W_n^2 \sim \frac{\alpha_1 \wedge (3/3^n)}{\alpha_1^3} \qquad n = \{1;2;..\} \quad ^{12} \tag{37}$$

10 I.e. in 4, 3 and 2D $\Gamma_{+/-}$ (Ψ_N) will be $\Gamma_{+/-1/2}$, $\Gamma_{+/-1/3}$ and $\Gamma(0, 8/\sigma_{2D}) = 7.872 \approx (2\pi^3)^{0.5}$ (numerical calculation); arg($\Gamma(\Psi_N)$) will be 1/2, 1/3, and for 2D *ad hoc* arg(Γ(0)) = 1;

11 As with all calculations in this work the calculation for coupling constants refers to a rest frame and thus corresponds to an IR limit. The geometric character of the “constants” implies that their values are subject to relativistic effects in other reference frames.

Including the other factors contained in (36) gives the square of (15) (term in square brackets cancels via (28):

$$W_n^2 = 2b_0 hc_0 \frac{\int\limits_{\lambda_{C,n}}^{r_n} e^{2\nu} r^{-2} dr}{\int e^{2\nu} dr} = \frac{4\pi b_0^2}{\alpha} \frac{\int\limits_{\lambda_{C,n}}^{r_n} e^{2\nu} r^{-2} dr}{\int e^{2\nu} dr} = \frac{4 b_0^2 \Gamma_{1/3}^2}{9[\alpha\, \boldsymbol{4\pi \Gamma_{1/3}|\Gamma_{-1/3}|}]\rho_n^2} = 2\, b_0\, \Gamma_{1/3}\, \rho_n^{-1}/3 \qquad (38)$$

According to chpt 2.4 ρ_n has to include additional Γ-, α-terms, suggesting to test such a term as candidate for α_1. Identifying α_1 as $\alpha_1 = \alpha$ and comparing with experimental particle data shows that an expression for particle energies can be given using the muon as reference state, with (37) given as:

$$\left(\frac{\alpha\wedge(3/3^n)}{\alpha^3}\right)^{0.5} = \frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} = \Pi_{k=1}^{n} \alpha\wedge(-3/3^k) \qquad n = \{1;2;..\} \qquad (39)$$

and the corresponding term for particle energies *relative* to the muon state will be:

$$W_n/W_\mu = \Pi_{k=1}^{n} \alpha\wedge(-1/3^k) \qquad n = \{1;2;..\} \qquad (40)$$

The partial product of (40) may be extended to include the electron by inserting *ad hoc* an additional factor ≈ 3/2 to represent an irregularity due to the energy ratio of e, μ, $W_\mu/W_e = 1.5088\ \alpha^{-1}$ (see 2.4, [A1]). In chpt. 2.11 it will be demonstrated that a fundamental relationship exists between the electron and the Planck energy, implying the electron to correspond to a ground state term. With W_e as ground state W_n would be given by (36)ff relative to the electron state as:

$$W_n/W_e \approx \frac{3}{2}\Pi_{k=1}^{n}\alpha\wedge(-3/3^k) = \frac{3}{2}\ \Pi_n^{-1} \qquad n = \{1;2;..\} \qquad (41)$$

for spherical symmetric states, see table 2. The electron coefficient in the exponential ν, equ. (22), and the energy term, equ. (40), would be given as:

$$\nu \sim \alpha_e^3 \approx (3/2)^3 \alpha^9 \quad \text{and} \quad W_e \sim \alpha_e^{-1} \approx 2/3\ \ \alpha^{-3} \qquad (42)$$

2.8 Upper limit of energy

Non-spherical particle states should exhibit lower values of σ (and r_n). The variable part in σ is given by the term $(1.5133\ \alpha^{-1})^3$ in equ. (23), leaving the minimum for σ, defined by the Γ-term in the integral expression for length, (24)f, and the integers in (23) to be:

$$\sigma_{min} = (2/3\ |\Gamma_{-1/3}|)^3 \qquad (43)$$

The maximum angular contribution to W_{max} would be:

$$\Delta W_{max,\ angular} = 1.5133\ \alpha^{-1} \qquad (44)$$

The limit of the partial product in (41) for a given l is $\alpha^{-1.5}$, the limit term of ≈ 3/2 by 1.5066 [A1], thus according to (41) and (44), the maximum energy will be $W_{max} = W_e\ 1.5066{*}1.5133\ \alpha^{-2.5}$ = 4.103E-8 [J] (=1.041 Higgs vacuum expectation value, VEV = 246GeV = 3.941E-8 [J] [11]).

In the simple visualization sketched in the introduction the “rotating E-vector” might be interpreted to cover the whole angular range in the case of spherical symmetric states while an object with one angular node, as represented by the spherical harmonic Y_1^0 or an atomic p-orbital, might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the angular limit case, l -> ∞, a state of minimal angular extension representing the original vector, however, extending in both directions from the origin and featuring parity p= -1. Considering only „half“ such a state, extending in one direction only and having p = +1, would feature an energy of 1.024 W_{Higgs}, the energy value of the Higgs boson.

2.9 Other non-spherical symmetric states

Except for the limit case of 2.8 angular solutions for particle states are not known yet and to extend the model to such states assumptions have to be made.

Assuming the angular part to be related to spherical harmonics and exhibiting the corresponding nodes would give the analog of an atomic p-state for the 1st angular state, Y_1^0. With the additional assumption that $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3}$ (V = volume) is applicable for non-spherically symmetric states as well, this would give

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12 For illustration purposes with $\alpha_1 = \alpha$, n = 4: $\dfrac{\alpha^1\alpha^{1/3}\alpha^{1/9}\boldsymbol{\alpha}^{\mathbf{1/27}}}{\boldsymbol{\alpha}^{\mathbf{3}}\alpha^1\alpha^{1/3}\alpha^{1/9}} = \dfrac{\boldsymbol{\alpha}^{\mathbf{1/27}}}{\boldsymbol{\alpha}^{\mathbf{3}}}$

$W_1^0/W_0^0 = 3^{1/3} = 1.44$. A second partial product series of energies corresponding to these values (denoted y_1^0) approximately fits the data, see tab. 2.
A change in angular momentum has to be expected for a transition from spherical symmetric states, y_0^0, to y_1^0 which is actually observed with $\Delta J = \pm 1$ except for the pair μ/π with $\Delta J = 1/2$.

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient (energy-term) Π_n^{-1} equ (41)	W_{calc}/W_{Lit}	J	r_n [fm] equ (21)
Planck	(-1,∞)	1.0 E+21*	$(2/3\ \alpha^{-3})^3\ 3/2\ \alpha^{-1}\ 2$ source term, relative to e !	0.9994 rel. to e !	-	-
e^{+-}	**0, 0**	**0.51**	**$2/3\ \alpha^{-3}$**	**1.0001**	**1/2**	**1412**
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**1.0001**	**1/2**	**6.83**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}\ 3^{1/3}$	1.0919	0	4.74
K		495	see [A3]		0	
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**0.9934**	**0**	**1.32**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})\ 3^{1/3}$	1.0124	1	0.92
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})\ 3^{1/3}$	1.0029	1	0.92
K*		894			1	
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.0017**	**1/2**	**0.76**
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.0004**	**1/2**	**0.76**
η'		958	see [A3]		0	
Φ^0		1019	see [A3]		1	
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**1.0107**	**1/2**	**0.63**
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**1.0047**	**1/2**	**0.61**
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**1.0026**	**3/2**	**0.59**
Ξ		1318			1/2	
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})\ 3^{1/3}$	0.9797	3/2	0.53
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})\ 3^{1/3}$	0.9725	3/2	0.45
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})\ 3^{1/3}$	1.0047	3/2	0.43
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})\ 3^{1/3}$	1.0025	1/2	0.40
Higgs	∞,∞ **	1.25 E+5	$(\alpha^{-9/2})\ 3/2\ \alpha^{-1}/2$	1.0230	0	0.006
VEV	∞,∞ **	2.46 E+5	$(\alpha^{-9/2})\ 3/2\ \alpha^{-1}$	1.04	0	0.003

Table 2: Particle energies for $\mathbf{y_0^0}$ **(bold)**, y_1^0 [13]; col.2: radial, angular quantum number; col.3: energy values of [12] except* (see (48)); col. 4: α-coefficient, *including (2/3) α^{-3} of electron (42)*; col.5: W_{calc} calculated using the slightly more precise [A2 (62)f] in place of (41), (50); ** see 2.8; Blanks in the table are discussed in [A3].

2.10 Expansion of the incomplete gamma function $\Gamma(1/3, \rho_n/r^3)$, strong interaction term

The series expansion of $\Gamma(1/3,(\rho_n/r_n)^3)$ in the equation for calculating particle energy (14)f gives [13]:

$$\Gamma(1/3,\ (\rho_n/r)^3) \approx \Gamma_{1/3} - 3\left(\frac{\rho_n}{r}\right) + \frac{3}{4}\left(\frac{\rho_n}{r}\right)^4 \tag{45}$$

and for $W_n(r)$:

$$W_n(r) \approx W_n - 2b_0\frac{3\rho_n}{3\rho_n r} + 2b_0\frac{3}{4}\frac{\rho_n^4}{3\rho_n r^4} = W_n - \frac{2b_0}{r} + b_0\frac{\rho_n^3}{2r^4} \tag{46}$$

The 2^{nd} term in (46) drops the particle specific factor ρ_n and gives twice [14] the electrostatic energy of two elementary charges at distance r. The 3^{rd} term is an appropriate choice for the 0^{th} order term of the differential

13 up to Σ^{*0} all resonance states given in [12] as **** included; Exponents of -9/2 for Δ and tau are equal to the limit of the partial product in (41); r_n calculated with (21); 1.5133 approximated by 3/2;
14 Due to adding up the electromagnetic contributions in (15): $W_n = 2W_{n,el} = 2W_{n,mag} = W_{n,el} + W_{n,mag}$

equation (cf. (19), [A1]) as potential energy term. It is supposed to be responsible for the localized character of a particle state and may be identified with the “strong force” of the standard model as observable e.g. in particle scattering.
According to this model it is suggestive to interpret strong interaction as evidenced in scattering events to be due to overlap of wave function Ψ depending on: 1) comparable size and energy of wave functions, 2) sufficient *net* overlap. Condition 1) prevents neutrino or electron to exhibit effective interaction with hadrons, condition 2) prevents interaction of the tauon which is at the end of the partial product series for y_1^0 and should exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign [15].

2.11 Gravitation

2.11.1 Planck scale

Gravitational effects may be recovered via the series expansion of chpt. 2.10, implying that the Coulomb term b_0 will be part of the expression for F_G, i.e. the ratio between gravitational and Coulomb force, e.g. for the electron, $F_{G,e}/F_{C,e}$ = 2.41E-43, should be a completely separate, self-contained term. This is equivalent to assume that gravitational interaction is a higher order effect with respect to electromagnetic interaction and as such should be of less or equal strength compared to the latter. This suggests to use the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \tag{47}$$

as definition for Planck terms, giving for the Planck energy, W_{Pl}:

$$W_{Pl} = c_0^2\,(b_0/G)^{0.5} = c_0^2\,(\alpha\hbar c_0/G)^{0.5} = 1.671\text{ E+8 [J]} \tag{48}$$

With definition (48) one may express a quantitative relationship for the ratio of W_e and W_{Pl} as:

$$1.0006\,\frac{W_e}{W_{Pl}} = \frac{\alpha_e^{+3}}{2\ \ \Delta W_{max,angular}} = 1.5133^2\,\alpha^{10}/2 = 4.903\text{ E-22} = \alpha_o \tag{49}$$

i.e. the relation between the electrostatic part of $W_{e,\,elst} = W_e/2$ and the electrostatically defined W_{Pl} is given by α_e^{+3}, i.e. the electron coefficient in e^v, (42), corresponding to an extension of relation (41) for spherical symmetric states beyond the electron, times the angular limit factor according to (44). In the next chapter a derivation will be given for this relation originating in the third term of the energy expansion (46).
With equ. (49) ρ_e^3 of the electron can be approximated by a particularly simple expression:

$$\rho_e^3 = 4\pi\,\sigma^*\alpha_0\,\rho^{*3} = \frac{\sigma_0{}^*\alpha_0}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 = 1.286\text{E-43 [m}^3\text{]}\quad{}^{16} \tag{50}$$

Using [A1 (59)] to express factor 1.5133 gives:

$$\left(\frac{W_e}{W_{Pl}}\right)^2 = \left(\frac{F_{G,e}}{F_{C,e}}\right)_{calc} \approx \left(\frac{1.5133^3\alpha^9}{1.5133\,\alpha^{-1}2}\right)^2 = \left(\frac{(4\pi)^2|\Gamma_{-1/3}|^4\alpha^{12}}{2}\right)^2 = 1.001^2\left(\frac{F_{G,e}}{F_{C,e}}\right)_{\exp} = \frac{G\,W_e^2}{c_0^4\,b_0} = \alpha_0^2 \tag{51}$$

Using (28) and [A2 (64)] for calculating W_e would turn G into a coefficient based on electromagnetic constants:

$$G_{calc} \approx \frac{c_0^4}{4\pi\varepsilon_c}\left(\frac{1}{3\pi^{2/3}}\left(\frac{|\Gamma_{-1/3}|}{\Gamma_{1/3}}\right)^4\alpha^{12}\right)^2 \approx \frac{c_0^4}{4\pi\varepsilon_c}\,\frac{2}{3}\,\alpha^{24} = 1.0008\,G_{\exp} \tag{52}$$

2.11.2 Virtual superposition states

Within this model particles might interact via direct contact in place of boson-mediated interaction. The particles are not expected to exhibit a rigid radius. Within the limits of charge and energy conservation a superposition of many states might be conceivable, extending the particle in space with radius ~ r_n, $\lambda_{C,n}$ etc. appropriate for energy of each virtual particle state (VS) [17], providing a source of energy at a distance r_{VS} from the primary particle and in turn contributing to the stress-energy tensor responsible for curvature of space-time that manifests itself in gravitational attraction.

15 As for energy density ~ W_m/W_n^4: e/p ~ E-13, μ/p ~ 6E-4; μ/π ~ 1/3, with r of (21) i.e. in case of μ/π some measurable effect should be expected; different symmetry may play an additional role.
16 Factor 2 of (22) included in α_0, see [A2];
17 Superposition states considered here are not virtual in a Heisenberg sense, energy is provided by the primary particle.

Virtual states are not supposed to consist of analogs of e.g. spherical symmetric states covering the complete angular range of 4π but to be an instantaneous, short term extension of the E-vector thus requiring the angular limit factor of (43).

A long range effect of the 3rd, the strong interaction term, of (46) may be exerted via virtual particle states. To estimate such an effect in first approximation the following will be used:

- the 3rd term of the energy expansion equ. (46) with ρ according to (35)ff, (50),
- the angular limit state of σ^*_{min} according to (43), $\sigma^*_{min} \approx 1$,
- $\rho^{*3} = (4\pi)^{-2} (e_c/\varepsilon_c)^3 \approx (\alpha^{-1} r_e)^3$, to represent the cube of a natural unit of length with value R.

For any VS at $r = \alpha^{-1}\, r_{VS} = \Pi_{VS} (\alpha^{-1} r_e)$, i.e. the radius of the VS in natural units, R_{VS}, equ. (53) will hold:

$$W_{VS}(r) \approx \frac{b_0 \rho_{VS}^3/2}{(\alpha^{-1}\; r_{VS})^4} \approx \frac{b_0 \alpha_0 \Pi_{VS}^3 (\alpha^{-1} r_e)^3}{(\alpha^{-1} r_{VS})^3 (\alpha^{-1} r_{VS})} \approx \frac{b_0 \alpha_0 \Pi_{VS}^3 (\alpha^{-1} r_e)^3}{(\Pi_{VS} \alpha^{-1} r_e)^3 (\alpha^{-1} r_{VS})} = \frac{b_0 \alpha_0}{(\alpha^{-1} r_{VS})} = \frac{b_0}{R_{VS}} \left(\frac{F_{G,e}}{F_{C,e}}\right)^{0.5} \quad (53)$$

[18]

Considering that the composition of the stress-energy tensor from virtual states is expected to be based on a much more complex mechanism requiring consideration of all possible virtual states at a particular point and appropriate averaging, (53) has to be a first approximation. The crucial factor that turns the r^{-4} dependence of the strong interaction term into r^{-1} of gravitational interaction is the proportionality of ρ_n^3 to the cube of any characteristic particle length, r_n, $\lambda_{C,n}$ etc. which is valid for each particle state subject to the relations of this model.

Equ. (53) is a representation of the gravitational energy of the electron, terms for other particles may be obtained by inserting their energy values relative to the electron according to (41)f in (53) which might be interpreted as the intensity/frequency of the emergence of virtual states being proportional to the energy of the primary particle.

As a consequence of (53) the highest possible particle energy value will be α_0^{-1}, i.e. the value of the Planck energy relative to the electron. This is the fundamental cause for equation (49) to relate W_e and W_{Pl} via an α-term and define the electron as ground state and in turn corroborates the assumption used in the definition of equ. (47)f.

Such a VS-based model implies curvature of space-time to be in general identical to the presence of energy, and spatial coordinate and energy to be intertwined inextricably.

2.12 Applicability to cosmological problems

Chapter 2.11 demonstrates that the results of this particle-based model might be applicable on the scale of cosmology as well, e.g. concerning problems such as dark energy / cosmological constant or dark matter.

2.12.1 Cosmological constant Λ

The 2nd term on the right side of the full 5D equation (5), $\sim 1/\Phi \left(\nabla_\alpha (\partial_\alpha \Phi) - g_{\alpha\beta} \Box \Phi\right)$, might be considered to be a natural candidate for the cosmological constant term, $g_{\alpha\beta}\Lambda$. Its exact expression will depend on the complete 4D or 5D metric used. Nevertheless it will have to contain terms of type $g_{\alpha\beta}\Phi''/\Phi$ such as ρ_n^3/r^5 given e.g. in the bracket part of (65). Referring the resulting expression to the natural unit of length used in this work, i.e. $R_u = e_c/\varepsilon_c$, will yield approximate values in the order of magnitude of critical, vacuum density, ρ_c, ρ_{vac} and Λ ~ 1.11E -52 [m^{-2}] [14][19]. Using (50) will give Λ_{calc} = 4.4E-49 [m^{-2}], the agreement may be improved significantly by dropping the symmetry coefficient σ and the angular limit factor originating from (44), (49). The following equations use the electron coefficient in the exponential, $(3/2\alpha^3)^3$:

$$\frac{\Phi''}{\Phi} \approx \frac{\rho^3}{r^5} \approx \frac{1}{(e_c/\varepsilon_c)^5} \left(\frac{3}{2}\alpha^3 \frac{e_c}{\varepsilon_c}\right)^3 = \left(\frac{3}{2}\alpha^3\right)^3 \left(\frac{\varepsilon_c}{e_c}\right)^2 = 0.228\ [\mathrm{m}^{-2}] \quad (54)$$

multiplied by the conversion factor for the electromagnetic and gravitational equations, equ. (2), $8\pi\varepsilon_c G/c_0^4$ this gives:

$$\left(\frac{3}{2}\alpha^3\right)^3 \frac{\varepsilon_c^3}{e_c^2} \frac{8\pi G}{c_0^4} = 7.60\text{E-}10 \left[\frac{J}{m^3}\right] \frac{8\pi G}{c_0^4} \approx \rho_c \frac{8\pi G}{c_0^4} = 1.58\text{E-}52\ [\mathrm{m}^{-2}] \quad (55)$$

18 The term for gravitational attraction, $F_{m,n;R}$ between two particles, m and n at a distance $r_{m,n}$, would be obtained by using $1/b_0$ as proportionality constant: $F_{m,n;R} \approx W_{VS(m,r)} W_{VS(n,r)}/b_0 \approx b_0\ \alpha_0^2\ \Pi_m \Pi_n R_{m,n}^{-2}$

19 Hubble constant H_0 = 67.66 [km/s/Mpc]

2.12.2 Galactic roatation curves

The terms of this model might provide effects associated with dark matter as well. This model as well as the well known Shapiro delay [15] imply a decrease of c_0 in curved space i.e. near a mass, M, which according to (52) affects the value of the gravitational constant, G. This might result in a lower value of G associated with large mass, M, such as e.g. present in a glactic center, and consequently underestimation of M in the product MG for affected regions.

On the other hand, far from a galactic center a term such as $\Gamma_{22}{}^1 = \Gamma_{\phi\phi}{}^r = -/+ 3/2\ c\ \rho^3/\rho^{*2}\ e^{(c-2b)v}$ in the solution of p = 2, q = 1 in equ. (9) (example 1 in [A4]) provides a contribution for centripetal acceleration, a_c,

$$a_c = \Gamma^r_{\phi\phi}\left(\frac{d\phi}{dt}\right)^2 = -3/2\,c\,\frac{\rho^3}{\rho*^2}e^{(c-2b)v}\omega^2 \approx -3/2\,c\,\frac{\rho^3}{\rho*^2}(1+(c-2b)v)\omega^2 \sim \frac{\rho^3}{\rho*^2}\ \frac{v_\phi^2}{r^2}[\mathrm{m/s^2}] \tag{56}$$

This yields an r-independent rotation velocity, $v_\phi^2 \sim GM_{gal}$, though not the Baryonic-Tully–Fisher relation, $v_\phi^4 \sim GM_{gal}$ [16]. Obtaining the latter requires an expansion of the a_c-term with v_ϕ^2 and inclusion of the v_ϕ^2 of the denominator in the a_0 term [20].

3 Discussion

Theory of everything is a somewhat ironic and pompous term and maybe an unachievable goal. Theodor Kaluza developed a unified field theory of gravitation and electromagnetism that produced the formalism for the field equations of GTR and Maxwell's equations yet met a major obstacle in a mismatch of orders of magnitude of the predicted results with experimental evidence, such as charge-to-mass ratio of elementary particles. The enormous difference in the effects of gravitation and electromagnetism seems to make a consistent unification in this way unfeasible, however, suggests to interpret Kaluza's equations as being entirely based on electromagnetism and therefore to use an appropriate electromagnetic constant in the field equations. This is equivalent to considering curvature of 4D space-time not as an effect induced by some distant source of mass / energy but to be in general identical to (the presence of) energy which may be interpreted as the 5th coordinate in a flat 5D space-time, in line with space-time-matter theory. For gravitation this is implied by the reasoning of chpt. 2.11.2, where virtual particle states provide energy at a distance sufficient to reproduce the effect of gravitational interaction. Whether or not an equivalent mechanism might be considered for electromagnetic fields as well, the electromagnetic field should cause a respectively stronger effect. The concept of curvature of space being strong enough to trap electromagnetic radiation, a black hole, is well studied. Curvature of space-time based on electromagnetic energy will be powerful enough to localize a photon in a self trapping kind of mechanism, yielding energy states in the range of the particle zoo.

Apart from the difference in strength of effects, interpreting Kaluza's equations as describing curvature of space-time with the boundary condition that Maxwell's equations have to hold, has a second, equally important consequence, the introduction of phase and eventually spin in the formalism. The static approximation used in this work, focusing on energy levels, does not explicitly implement phase yet it might be considered to be implied by the photon description assumed to be valid for particles [21].

Kaluza obtained Maxwell's equations by assuming the scalar field to be constant, which is in conflict with equ. (6). Using the Φ-terms of this model does not yield standard Maxwell equations but it might be assumed that the equations have to refer to a description of electromagnetism on a photon level and the exponential factors in Φ and the imaginary solution of (19)f might describe the field distribution of a photon. For $r > \rho_n$, $e^v \to 1$, the expressions in equations such as (14) (or (46)) turn into the corresponding Coulomb-term.

Gravitation will reenter the scene via a series expansion of the energy expression, reproducing the effects of the original EFE in the lower order of magnitude of gravitation. The same term in the expansion is supposed to be responsible for strong interaction at short range, suggesting a somewhat unexpected yet consistent unification scheme.

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20 A very rough estimation for a_0 may be given by assuming p, n being the main contributors to mass, giving in the angular limit state $\Gamma_{\phi\phi}{}^r \approx \rho_{p,n}{}^3/\rho^{*2}\ e^{(c-2b)v} \approx$ 1E-40[m]. Multiplied by the number of p,n in a milkyway type galaxy, $N = M_{gal}/m_{p,n} \approx$ 1E+69 gives ≈ 1E+29[m] and with a typical $v_\phi \approx$ 1E+5[m/s] for $a_0 \approx v_\phi^2/(N\Gamma_{\phi\phi}{}^r) \approx$ 1E-19[m/s²], cf a_0 of [16]: a_0 = 1.2E-10[m/s²].

21 The derivation of σ in 2.4 uses J = ħ/2, However, the relation of electroweak coupling constants with σ indicates that simple N-D volumes have their own significance within this model and the relation to J may not be required to define the integration limits.

Weak interaction has no place in the series expansion, yet there is evidence that the model comprises elements of weak / electroweak interaction from the outset. The derivation of α from a photon and a point charge expression implies a rotation of electromagnetic fields and SO(3) symmetry for particles. This link to electroweak phenomena is further backed by the possibility to extend the derivation to 4D-space, giving the weak coupling constant, α(g), and the prominent position of the energy levels corresponding to Higgs boson and vacuum expectation energy at the upper end of the energy series. In a 5D model such as given here or in space-time-matter theory there is no need for an additional mechanism to create mass. However, it may be speculated about a relationship of the Higgs-field with the 5th coordinate and the scalar Φ [5]. In this model SO(3) refers to an actual rotation of a physical object rather than an abstract mathematical group, implying a center of rotation, a rest-frame and consequently rest-energy. Considerations such as given by point b) of chpt. 2.3 require curvature of 4D space-time for an SO(3) object to retain photon properties in a flat 5D space-time.
Concerning the vacuum state, in this model it is the ground state, the electron state, representing the maximal particle volume and the minimal curvature of 4D space, that seems to relate to ρ_c, ρ_{vac} and the cosmological constant, Λ, corresponding to the 4D vacuum as state with minimal spatial curvature. On the other hand, the energy level equivalent to the Higgs VEV refers to a minimal volume and a maximum in curvature of 4D space, corresponding to a maximum in the 5th coordinate.

The formalism of GTR is based on second order differential equations that are a common tool in quantum mechanics as well and some congruence, concerning e.g. Klein-Gordon equations, is elaborated on in [5]. Features of quantum mechanics that are covered in this work include quantization of energy, wave-character of particles and non-locality (cf. 2.11.2). Last not least the pivotal constant of quantum mechanics, Plancks constant, h, may be derived from the electromagnetic constants e_c, ε_c, and geometry as expressed in α and σ. These features do not emerge from classical GTR alone but rely on Kaluza's unification scheme and the photon description of particles.

The model presented here is far from being complete and occasionally requires minor assumptions, yet it provides a coherent, quantitative and parameter-free, Kaluza-based formalism, connecting electromagnetism, strong force and gravitation as well as phenomena on a particle and on a cosmological scale and thus it may serve as a step in the development of a theory of a little bit more [22].

Conclusion

Using a metric based on solutions for the scalar of a 5-dimensional Kaluza model in the Einstein field equations, using an electromagnetic in place of a gravitational constant, yields

- a consistent and coherent relationship between electromagnetic, gravitational and strong interaction,
- a convergent series of quantized particle energies, with electron and the Higgs VEV energy as lower and upper limit,
- a geometric expression for the values of the electroweak coupling constants, including the fine-structure constant, α
- a term for the cosmological constant, Λ, in the correct order of magnitude.

The model works ab initio without free parameters.

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Appendix

[A1] Coefficient σ, differential equation

Analyzing the components of σ_0, in addition to the mandatory term for length, $|\Gamma_{-1/3}|/3$, of the integral (13) for m = -1, r_n and σ_0 contain a factor ≈1.51 α^{-1}, very close to the ratio W_μ/W_e = 206.8 = 1.5088 α^{-1}. The exact value of 1.5133 for ≈1.51 has been chosen due to a geometrical interpretation of the terms in σ_0:

$$1.51\,\alpha^{-1}\,|\Gamma_{-1/3}|/3 \approx |\Gamma_{-1/3}|/\Gamma_{1/3}\;\; 4\pi\,|\Gamma_{-1/3}|\,\Gamma_{1/3}/0.998\;\; |\Gamma_{-1/3}|/3 \;\approx\; \frac{4\pi\left|\Gamma_{-1/3}\right|^3}{3} \;=\; (\sigma_0/8)^{1/3} \quad (57)$$

[23]

and factor 1.5088 of the ratio W_μ/W_e being subject to a 3rd power relationship of the same kind as the α coefficients:

$$\left(\frac{1.5133}{1.5088}\right) = \left(\frac{1.5133}{1.5}\right)^{1/3} \quad (58)$$

indicating that the radial terms of Π_n in ρ_n and the angular components of σ are not correctly separated yet or may not be separable even in the case of spherical symmetric states. Thus it is somewhat ambiguous to include factor 1.5133^3 in σ or the particle coefficient, suggesting to use $\sigma^* = \sigma/1.5133^3$ = 5.112E+7 [-] and α(n)-terms containing factor 1.5133^3 as well. The following relation holds:

$$1.5133 \;=\; 0.998\left|\Gamma_{-1/3}\right|/\Gamma_{1/3} \;=\; 4\pi\left|\Gamma_{-1/3}\right|^2\;\alpha \quad (59)$$

The limit of a corresponding partial product in the energy expression is given by 1.5133 Π_1^∞ (1.5/1.533)^1/3^k ≈ 1.5066. The corresponding term in ρ_n^3 will be: 1.5133^{-3} Π_1^n(1.533/1.5)^3/3^k, n={1;2;..}, for particles above the electron, see [A2]. The value of σ is related to angular momentum / spin and thus may not be representable with a conventional metric. This might hint at a relationship with quantum mechanics. With the 3rd term in (46) used for potential energy, V:

$$V(r) = b_0\,\rho_e^3/(2\,r^4) = b_0\,[\,\sigma^*\,\alpha_0\,(e_c/\varepsilon_c)^3/(4\pi)^2]\,/(2r^4) \quad (60)$$

and a corresponding expansion by $(\hbar c_0)^2\alpha^{-2}/b_0^2$ for the 2nd order term of (20), an approximate differential equation for this model, including σ, may be given that resembles quantum mechanical terms (with $e^v = \Psi$):

$$-\frac{(\hbar c_0)^2 r}{\alpha^{-2} b_0}\;\frac{d^2\Psi(r)}{dr^2} \;+\; r\,V(r)\;\frac{d\Psi(r)}{dr} \;-\; \frac{V(r)}{\sigma}\Psi(r) \;=\; 0 \quad (61)$$

[A2] Particle parameter ρ_n

A more detailed expression for ρ than given in (50) will be attempted in the following.
The term (59) will be used within the particle specific factor (square brackets), thus coefficient 1.5133 of σ will be placed there, giving for the general term (i.e. excluding the electron):

$$\rho_n^3 \;=\; \sigma^*\;\frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3\;\frac{2}{(2\pi)^3}\;\;1.5133^{-3}\,\Pi_{k=0}^{n}\left[\alpha^3\left(\frac{1.5133}{1.5}\right)\right]^\wedge\left(\frac{3}{3^k}\right) \qquad n = \{0;1;2;...\} \quad (62)$$

23 The term $4\pi\,|\Gamma_{-1/3}|^3/3$ is used for σ_0 in all calculations.

factor 1.5133^{-3} represents ≈3/2 for the ratio of W_μ/W_e, to be omitted in the term for the electron:

$$\rho_e^3 = \sigma * \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \frac{\mathbf{2}}{(\mathbf{2\pi})^{\mathbf{3}}} \left[\boldsymbol{\alpha}^{\mathbf{3}}\left(\frac{\mathbf{1.5133}}{\mathbf{1.5}}\right)\right]^{\mathbf{3}} \approx \sigma * \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \boldsymbol{\alpha_0} \quad ^{24} \tag{63}$$

the particle specific factor is given in square brackets (α_0 in bold). The other factors are due to
- factor 2: from e^{2v},
- factor $1/(2\pi)^3$: representing 2π of the integral limit in (17),
- factor 1.5133^{-3}: due to anomalous factor 2/3 in W_e/W_μ,
- $1/(4\pi)^2$: it is unclear why this term appears with power of 2 instead of the power of 3 as for the other components. It might be related to b_0 appearing squared in (61) (where $V\sim b_0$) or might be related to the term $\rho*^2$ in 2.3.

Using (63) W_e may be given as:

$$W_e = 2b_0\frac{\Gamma_{+1/3}}{3}\left(\frac{9\pi^{5/3}\alpha}{|\Gamma_{-1/3}|}\left(\frac{\varepsilon_c}{e_c}\right)\left[\frac{\alpha^{-3}}{1.5133}\right]\right) = \frac{1.5\pi^{2/3}}{1.5133}\frac{\Gamma_{+1/3}}{|\Gamma_{-1/3}|}\frac{e_c}{\alpha^2} = 1.0001\ W_{e,exp} \tag{64}$$

[A3] Additional particle states

Assignment of more particle states will not be obvious. The following gives some possible approaches.

[A3.1] Partial products

Additional partial product series will have to start with higher exponents n in α^($-1/3^n$) giving smaller differences in energy while density of experimentally detected states is high. There might be a tendency of particles to exhibit a lower mean lifetime (MLT), making experimental detection of particles difficult [25]. To determine the factor y_l^m requires an appropriate ansatz for the differential equation, including angular terms, yet to be found.

One more partial product might be inferred from considering d-like-orbital equivalents with a factor of $5^{1/3}$ as energy ratio relative to η giving the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV = 0.98 W(η'), i.e. close to energy values of the first particles available as starting point, η', Φ^0. However, in general it is not expected that partial products can explain all values of particle energies.

[A3.2] Linear combinations

The first particle family that does not fit to the partial product series scheme are the kaons at ~ 495MeV. They might be considered to be linear combination states of π-states. The π-states of the y_1^0 series are assumed to exhibit one angular node, giving a charge distribution of +|+, -|- and +|-. A linear combination of two π-states would yield the basic symmetry properties of the 4 kaons as:

		+			-			-			+	
K^+	+		+	K^-	-	-	K_S^o	+	+	K_L^o	+	-	(+/- = charge)
		+			-			-			-	

providing two neutral kaons of different structure and parity, implying a decay with different parity and MLT values. For the charged Kaons, K^+, K^-, a configuration for wave function sign equal to the configuration for charge of K_S^o and K_L^o might be possible, giving two versions of P+ and P- parity of otherwise identical particles and corresponding decay modes not violating parity conservation.

[A4] Metric

(For both examples $v = -(\rho/r)^3$; +/- signs in ϑ, φ provisional for option to choose sign of R ~ $+/-1/r^2$, see 2.3 b).)

Example 1

The following gives an example with Φ squared for the t and r-part of the metric, $\Phi^2 \sim [(\rho*/r)^2 e^v]^2$ The application of a $(\rho/r)^2$ term in the angular terms as well will cancel their r^2-dependence, implying the same effect as discussed in b) of 2.3.

$$g_{\mu\nu} = \left[\left(\frac{\rho*}{r}\right)^2 \exp\left(-a\left(\frac{\rho}{r}\right)^3\right)\right]^{\mathbf{2}},\ -\left[\left(\frac{\rho*}{r}\right)^2 \exp\left(-b\left(\frac{\rho}{r}\right)^3\right)\right]^{\mathbf{2}},\ -/+\boldsymbol{\rho*^2}\exp\left(-c\left(\frac{\rho}{r}\right)^3\right),\ -/+\boldsymbol{\rho*^2}\exp\left(-c\left(\frac{\rho}{r}\right)^3\right)\sin^2\theta$$

$\Gamma_{01}^0 = \Gamma_{10}^0 = -2/r^1 + 3a\rho^3/r^4$ $\quad\Gamma_{00}^1 = -2/r^1 e^{2(a-b)v} + 3a\rho^3/r^4 e^{2(a-b)v}$

$\Gamma_{11}^1 = -2/r^1 + 3b\rho^3/r^4$

$\Gamma_{12}^2 = \Gamma_{21}^2 = \Gamma_{13}^3 = \Gamma_{31}^3 = +3/2\, c\rho^3/r^4$ $\quad\Gamma_{22}^1 = -/+ 3/2\, c\rho^3/\rho*^2 e^{(c-2b)v} = \Gamma_{33}^1/\sin^2\vartheta$

$\Gamma_{23}^3 = \Gamma_{32}^3 = \cot\vartheta$ $\quad\Gamma_{33}^2 = -\sin\vartheta\cos\vartheta$

$R_{00} = e^{2(a-b)v}[(-2/r^2 + 12(a-b)\rho^3/r^5 + 12a\rho^3/r^5 - 18a(a-b)\rho^6/r^8) + 2(\Gamma_{01}^0\Gamma_{00}^1) - \Gamma_{00}^1(\Gamma_{10}^0 + \Gamma_{11}^1 + 2\Gamma_{12}^2)]$

$= e^{2(a-b)v}[(-2/r^2 + 12(2a-b)\rho^3/r^5 - 18a(a-b)\rho^6/r^8) - \Gamma_{00}^1(-\Gamma_{10}^0 + \Gamma_{11}^1 + 2\Gamma_{12}^2)]$

$= e^{2(a-b)v}[(-2/r^2 + 12(2a-b)\rho^3/r^5 - 18a(a-b)\rho^6/r^8) + (+2/r^1 - 3a\rho^3/r^4)(+3(-a+b+c)\rho^3/r^4)]$

$= e^{2(a-b)v}[(-2/r^2 + 12(2a-b)\rho^3/r^5 - 18a(a-b)\rho^6/r^8) + 6(-a+b+c)\rho^3/r^5 - 9a(-a+b+c)\rho^6/r^8)]$

24 Note: $2(2/3)^3/(2\pi)^3 \approx (1.5133\,\alpha^{-1}\,2)^{-1}$, i.e. indicating a relation to the angular limit factor of chpt. 2.8.

25 Which might explain missing particles of higher n in the y_0^0 and y_1^0 series as well.

$R_{00} = e^{2(a\text{-}b)v}$ [$-2/r^2 + 6(+3a -b +c)\ \rho^3/r^5 - 9(+a^2 -ab +ac)\rho^6/r^8$)]

R_{11} = [$+2/r^2$ -12a ρ^3/r^5 $+2/r^2$ -12b ρ^3/r^5 -12 c ρ^3/r^5 $-2/r^2$ +12bρ^3/r^5 +$\Gamma_{10}^0\ \Gamma_{01}^0$ +$\Gamma_{11}^1\Gamma_{11}^1$ +2$\Gamma_{12}^2\ \Gamma_{21}^2$ - Γ_{11}^1 (Γ_{10}^0 +Γ_{11}^1 + 2Γ_{12}^2)]
= [$+2/r^2$ - 12a ρ^3/r^5 - 12 c ρ^3/r^5 + $\Gamma_{10}^0\ \Gamma_{01}^0$ + 2$\Gamma_{12}^2\ \Gamma_{21}^2$ - Γ_{11}^1 (Γ_{10}^0 + 2Γ_{12}^2)]
= [$+2/r^2$ - 12(a + c) ρ^3/r^5 + $4/r^2$ + 9a^2 ρ^6/r^8 - 12a ρ^3/r^5 + 9/2 c^2 ρ^6/r^8 + ($+2/r^1$ - 3b ρ^3/r^4) ($-2/r^1$ + 3(+a + c) ρ^3/r^4)]
= [$+6/r^2$ - 12(2a + c) ρ^3/r^5 + 9/2(+2a^2 + c^2)ρ^6/r^8 $-4/r^2$ + 6bρ^3/r^5 + 6(a + c) ρ^3/r^5 - 9b(a +c) ρ^6/r^8]
R_{11} = [$+2/r^2$ - 6(3a - b + c) ρ^3/r^5 + 9/2(+2a^2+ c^2 - 2ab - 2bc) ρ^6/r^8]

R_{22} = - 1+ $e^{(c-2b)v}$ [+/− 9/2 c (c-2b) $\rho^6/(\rho^{*2} r^4)$ +2($\Gamma_{21}^2\ \Gamma_{22}^1$) - Γ_{22}^1 (Γ_{10}^0 + Γ_{11}^1+ 2Γ_{12}^2)]
= - 1+ $e^{(c-2b)v}$ [+/− 9/2 c (c-2b) $\rho^6/(\rho^{*2} r^4)$ - Γ_{22}^1 (Γ_{10}^0 + Γ_{11}^1- Γ_{12}^2 + Γ_{13}^3)]
= - 1+ $e^{(c-2b)v}$ [+/− 9/2 c (c-2b) $\rho^6/(\rho^{*2} r^4)$ +/− 3/2 c ρ^3/ρ^{*2} ($-4/r^1$ + 3(a + b)ρ^3/r^4)]
= - 1+ $e^{(c-2b)v}$ [+/− 9/2 c (c-2b) $\rho^6/(\rho^{*2} r^4)$ −/+ 6 c $\rho^3/(\rho^{*2}r^1)$ +/− 9/2c(a + b)$\rho^6/(\rho^{*2}r^4)$)]
R_{22} = - 1+ $e^{(c-2b)v}$ [−/+ 6 c$\rho^3/(\rho^{*2}r^1)$ +/− 9/2(ac -bc +c^2)$\rho^6/(\rho^{*2}r^4)$]

$g^{00}R_{00}$ = e^{-2bv} [$-2r^2/\rho^{*4}$ + 6(+3a -b +c)$\rho^3/(r\rho^{*4})$ – 9(+a^2 -ab +ac)$\rho^6/(r^4\rho^{*4})$)]
$g^{11}R_{11}$ = - e^{-2bv} [$+2r^2/\rho^{*4}$ - 6(3a - b + c)$\rho^3/(r\rho^{*4})$ + 9/2(+2a^2+ c^2 - 2ab - 2bc) $\rho^6/(r^4\rho^{*4})$)]
$g^{22}R_{22}$ + $g^{33}R_{33}$ = +/−2e^{-cv} /ρ^{*2} −/+ e^{-2bv} [−/+ 12 c $\rho^3/(r\rho^{*4})$ +/− 9(ac -bc +c^2)$\rho^6/(r^4\rho^{*4})$)]

R = +/− 2e^{-cv}/ρ^{*2} + e^{-2bv} [$-4r^2/\rho^{*4}$ + 6(6a -2b + 4c)$\rho^3/(r\rho^{*4})$ - 9/2(+4a^2 + 3c^2 -4ab +4ac -4bc) $\rho^6/(r^4\rho^{*4})$)]
G_{00} will be:
G_{00} = $e^{2(a\text{-}b)v}$ [$-2/r^2$ + 6(+3a -b +c) ρ^3/r^5 - 9(+a^2 -ab +ac)ρ^6/r^8)] −/+ ρ^{*2}/r^4 $e^{(2a-c)v}$ + $e^{2(a\text{-}b)v}$ [$+2/r^2$ - 3(+6a -2b + 4c) ρ^3/r^5 + 9/4(+4a^2 + 3c^2 -4ab +4ac -4bc) ρ^6/r^8]

$$G_{00} = = \frac{-/+\rho^{*2}}{r^4} e^{(2a-c)v} + e^{(2a-b)v}\left[-6\frac{c\rho^3}{r^5} + \frac{9}{4}\left(+3c^2-4bc\right)\frac{\rho^6}{r^8}\right] \qquad (65)$$

giving a trivial solution for c = 0

G_{00} = −/+ $\rho^{*2}/r^4\, e^{2av}$

For chpt. 2.2, 2.3 a = 1 is chosen, giving e^{2v} as exponential term.
The condition c = 0 is not necessary to give solutions for the integral of (12) since $\int\rho^{n-2}/r^n\, d^3r \approx \Gamma[(n-1)/3]\ \rho^{n-2}/\ \rho^{n-3} \approx \rho \ll \rho^{*2}/\rho$ and the integral over the second, square bracket part of (65) may be set to zero by appropriate choice of parameters a, b and c. Moreover, while the ρ^3/r^5-term might be related to Λ, the ρ^6/r^8-term may be eliminated by setting c = 3/4 b.

Example 2
Example 1 is not an exclusive solution for this model, an alternate example is given by e.g.:

$$g_{\mu\nu} = \left(\frac{\rho^*}{r}\right)^2 \exp\left(-a\left(\frac{\rho}{r}\right)^3\right),\ -\left(\frac{\rho^*}{r}\right)^2 \exp\left(-b\left(\frac{\rho}{r}\right)^3\right),\ -/+\ r^2,\ -/+\ r^2\sin^2\vartheta \qquad (66)$$

$\Gamma_{01}^0 = \Gamma_{10}^0$	$= -1/r^1 + 3/2\ a\ \rho^3/r^4$	$\Gamma_{00}^1 = -1/r^1\, e^{(a-b)v} + 3/2\ a\ \rho^3/r^4 e^{(a-b)v}$
Γ_{11}^1	$= -1/r^1 + 3/2\ b\ \rho^3/r^4$	
$\Gamma_{12}^2 = \Gamma_{21}^2 = \Gamma_{13}^3 = \Gamma_{31}^3$	$= +1/r^1$	$\Gamma_{22}^1 = -/+\ r^3/\rho^{*2}\, e^{(c-b)v} = \Gamma_{33}^1/\sin^2\vartheta$
$\Gamma_{23}^3 = \Gamma_{32}^3 = \cot\vartheta$		$\Gamma_{33}^2 = -\sin\vartheta\cos\vartheta$

R_{00} = $e^{(a-b)v}$ [($-1/r^2$ + 3 (a-b) ρ^3/r^5 + 6 a ρ^3/r^5 - 9/2 a (a-b) ρ^6/r^8) | 2($\Gamma_{01}^0\ \Gamma_{00}^1$) - Γ_{00}^1 (Γ_{10}^0 + Γ_{11}^1+ 2Γ_{12}^2)]
= $e^{(a-b)v}$ [($-1/r^2$ + (9a -3b) ρ^3/r^5 - 9/2a(a-b) ρ^6/r^8 - Γ_{00}^1 (-Γ_{10}^0 + Γ_{11}^1+ 2Γ_{12}^2)]
= $e^{(a-b)v}$ [($-1/r^2$ + (9a -3b) ρ^3/r^5 - 9/2a(a-b) ρ^6/r^8 + ($+1/r^1$ - 3/2 a ρ^3/r^4) ($+2/r^1$)]
= $e^{(a-b)v}$ [($-1/r^2$ + (9a -3b) ρ^3/r^5 - 9/2 a(a-b) ρ^6/r^8 + $2/r^2$ - 3aρ^3/r^5]
R_{00} = $e^{(a-b)v}$ [$+1/r^2$ + (6a -3b)ρ^3/r^5 - 9/2a(a-b)ρ^6/r^8]

R_{11} = [+ $1/r^2$ - 6a ρ^3/r^5 + $1/r^2$ - 6b ρ^3/r^5 - $2/r^2$ - $1/r^2$ + 6bρ^3/r^5 + $\Gamma_{10}^0\ \Gamma_{01}^0$ + $\Gamma_{11}^1\Gamma_{11}^1$ + 2$\Gamma_{12}^2\ \Gamma_{21}^2$ - Γ_{11}^1 (Γ_{10}^0 + Γ_{11}^1+ 2Γ_{12}^2)]
= [$-1/r^2$ - 6 a ρ^3/r^5 + $\Gamma_{10}^0\ \Gamma_{01}^0$ + 2$\Gamma_{12}^2\ \Gamma_{21}^2$ - Γ_{11}^1 (Γ_{10}^0 + 2Γ_{12}^2)]
= [$-1/r^2$ - 6a ρ^3/r^5 + $1/r^2$ + 9/4 a^2 ρ^6/r^8 - 3 a ρ^3/r^5 + $2/r^2$ + (+ $1/r^1$ - 3/2 b ρ^3/r^4) ($+1/r^1$ + 3/2 a ρ^3/r^4]
= [$+2/r^2$ - 9a ρ^3/r^5 + 9/4 a^2 ρ^6/r^8 $+1/r^2$ + 3/2aρ^3/r^5 - 3/2bρ^3/r^5 - 9/4 ab ρ^6/r^8]
R_{11} = [$+3/r^2$ - (15/2a + 3/2b)ρ^3/r^5 + 9/4(+a^2 -ab)ρ^6/r^8]

R_{22} = - 1+ $e^{(c-b)v}$ [(+/−3 r^2/ρ^{*2} +/− 3(c-b) $\rho^3/(r\rho^{*2})$ + 2($\Gamma_{21}^2\ \Gamma_{22}^1$) - Γ_{22}^1 (Γ_{10}^0 + Γ_{11}^1+ Γ_{12}^2 + Γ_{13}^3)]
= - 1+ $e^{(c-b)v}$ [(+/−3 r^2/ρ^{*2} +/− 3(c-b) $\rho^3/(r\rho^{*2})$ - Γ_{22}^1 (Γ_{10}^0 + Γ_{11}^1- Γ_{12}^2 + Γ_{13}^3)]
= - 1+ $e^{(c-b)v}$ [(+/−3 r^2/ρ^{*2} +/− 3(c-b) $\rho^3/(r\rho^{*2})$ +/− r^3/ρ^{*2} ($-2/r^1$ + 3/2(a + b)ρ^3/r^4)]
= - 1+ $e^{(c-b)v}$ [(+/−3 r^2/ρ^{*2} +/− 3(c-b) $\rho^3/(r\rho^{*2})$ −/+ 2r^2/ρ^{*2} +/− 3/2(a + b)$\rho^3/(r\rho^{*2})$]

$R_{22} = - 1+ e^{(c-b)v}$ [(+/− $1r^2/\rho*^2$ +/− $3/2(+a -b + 2c)\rho^3/(r\rho*^2)$]
$g^{00}R_{00} = e^{-bv}$ [$+1/\rho*^2 + (6a -3b)\rho^3/(r^3\rho*^2) - 9/2a(a-b)\rho^6/(r^6\rho*^2)$]
$g^{11}R_{11} = - e^{-bv}$ [$+3/\rho*^2 - (15/2a + 3/2b)\rho^3/(r^3\rho*^2) + 9/4(+a^2 -ab)\rho^6/(r^6\rho*^2)$]
$g^{22}R_{22} + g^{33}R_{33}$ = +/− $2/r^2$ −/+ e^{-bv} [(+/− $2/\rho*^2$ +/− $3(+a -b + 2c)\rho^3/(r^3\rho*^2)$]
The two solutions for R with different sign of $R_{22,33}$ will be:
R = +/− $2/r^2 + e^{-bv}$ [(− $4/\rho*^2 + (+21/2a + 3/2b - 6c)\rho^3/(r^3\rho*^2) - 9/4(+3a^2 - 3ab)\rho^6/(r^6\rho*^2)$]
G_{00} will be:
$G_{00} = e^{(a-b)v}$ [$+1/r^2 + (6a -3b)\rho^3/r^5 - 9/8(4a^2 -4ab)\rho^6/r^8$] −/+ $\rho*^2/r^4 e^{av}$ + $e^{(a-b)v}$ [$(+ 2/r^2 + (-21/4a - 3/4b + 3c)\rho^3/r^5 - 9/8(-3a^2 + 3ab)\rho^6/r^8$] = −/+ $\rho*^2/r^4 e^{av}$ + $e^{(a-b)v}$ [$(+ 3/r^2 + (+3/4a - 15/4b + 3c)\rho^3/r^5 - 9/8(+a^2 - ab)\rho^6/r^8$]
giving a solution
G_{00} = −/+ $\rho*^2/r^4 e^{av} + 3/r^2 \approx$ −/+ $\rho*^2/r^4 e^{av}$
for a = b = c. For chpt. 2.2, 2.3 a = 2 is chosen, giving e^{2v} as exponential term.
While higher orders of ρ^n-terms in G_{00} are in general easy to eliminate by appropriate choice of the factors in the exponents, a,b..., the lowest order term, i.e. in the metric of example 2: ~$1/r^2$, lacks these factors and needs a metric of the type of example 1 to be eliminated. However, with the integral limits for the particles discussed here volume integrals over the second term will give negligible contributions to particle energy < 10^{-6} and might still be considered a valid solution.

[A5] Scalar potential Φ

The solutions for the scalar Φ depend on the complete metric used. As in [A4] the main problem to obtain $R_{44} = 0$ is to eliminate the terms of lowest order in ρ, which lack coefficients in their terms enabling an easy cancellation of them. As in [A4] solutions can be given by using a metric with squared terms, i.e. p = 2, for either g_{00} or g_{11} e.g.:

$$g_{\mu\nu} = \left[\left(\frac{\rho}{r}\right)^2 \exp\left(-a\left(\frac{\rho}{r}\right)^3\right)\right]^{\mathbf{2}},\ -\left(\frac{\rho}{r}\right)^2 \exp\left(-b\left(\frac{\rho}{r}\right)^3\right),\ -r^2\left(-c\left(\frac{\rho}{r}\right)^3\right),\ -r^2\left(-c\left(\frac{\rho}{r}\right)^3\right)\sin^2\theta, -\left[\left(\frac{\rho}{r}\right)^2 \exp\left(-a\left(\frac{\rho}{r}\right)^3\right)\right]^{\mathbf{2}} \tag{67}$$

Using hyperspherical coordinates in a 5D metric with the line element

$$ds^2 = e^{av}dt^2 - e^{bv}dr^2 - r^2\left(d\psi^2 + \sin^2\psi\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right) \tag{68}$$

gives a formal solution as well yet r would be supposed to represent the 5th coordinate.

[A6] Coupling constant in 5D

3D case:

Equations (32)f have their origin in the integrals over $e^v = \Psi_N$, to be recapped and examined in more depth for the 3D case:
omitting the dimensioned constants in (14)f and (27), α may be expressed directly via the integral over $1/r^2$ representing a point source in 3D times a complementary 2nd integral symmetric in the Γ-function to give a dimensionless term:

$$2\int_0^r \Psi_3(r)^2 r^{-2}\,dr \int_0^r \Psi_3(r)^2 dr = 2\left[\frac{\Gamma_{1/3}}{3}\right]\left[2\pi\, 2\pi\, 9\,\frac{|\Gamma_{-1/3}|}{3}\right] = 4\pi\Gamma_{1/3}|\Gamma_{-1/3}|\ \mathbf{2\pi} = \mathbf{2\pi}\ \alpha_e^{-1} \tag{69}$$

The term of 2*2π may indicate that the volume integral over the square of $1/r^2$ is involved, as actually used in the derivation of chpt. 2.6, $\int \Psi_3(r)^2 r^{-4}\ d^3r = \int \Psi_3(r)^2 r^{-4}\,\mathbf{4\pi} r^2\ dr$. One of the 2π terms originating from the second integral of equation (69) is required for turning h into ħ. Otherwise this would give a dimensionless constant α(e)' = h c_0 4π ε/e^2 and it is a matter of choice to include 2π in the dimensionless coupling constant [26].
The exact value of (69) depends on the integration limit of the second integral, i.e. the lower integration limit of the corresponding Euler integral which can be expressed as 3D volume with $|\Gamma_{-1/3}|$ as radius (see 2.4):

$$\rho_n^3/\lambda_{C,n}^3 = 8/\left(3^{1.5}\sigma\right) = \left(3^{0.5}\frac{4\pi}{3}\ |\Gamma_{-1/3}|^3\right)^{-3} \tag{70}$$

in the limit $\rho_n^3/\lambda_{C,n}^3$ -> 0 to be multiplied by |1/arg(Γ(x))| = 3 according to equ. (25). The additional factor $3^{0.5}$ gives the ratio between r_n of equ. (21) and $\lambda_{C,n}$ [27] as required in the expression for photon *energy*.
This limit yields the result of the second integral of (69) as: $\int\Psi_3(r)^2 dr \sim \Gamma(-1/3, 8/(3^{1.5}\sigma_3)) = 3^{0.5}4\pi|\Gamma_{-1/3}|^3 \approx 36\pi^2|\Gamma_{-1/3}|$.
The general N-dimensional version of (70) will be:

$$8/\sigma_N = \left(3^{0.5\delta} V_N\ |\Gamma(-\mathrm{N})|^N\right)^{-N} \tag{71}$$

V_N is the coefficent for volume in N-D, coefficient $3^{0.5}$ will be omitted in 4D where coordinate r is considered to be directly related to energy via $r_n \sim 1/W_n$ and r_n might be directly identified with $\lambda_{C,n}$.

26 The term 2π may be traced back to the more detailed expression for ρ_n, equ. (62)f, including the cube of 2π.
27 $\lambda_{C,n} = 3\beta^{1/3}hc_0/(2b_0\Gamma_{1/3}) = 3\pi\beta^{1/3}/(\alpha\Gamma_{1/3})$, $r_n = \Gamma_{-1/3}\beta^{1/3}/(2\alpha)$ => $\lambda_{C,n}/r_n = 6\pi/(\Gamma_{1/3}\Gamma_{-1/3}) = 3^{0.5}$.

4D case:
Using Ψ_4 according to the definition (29) and (71) for 4D:

$$\rho_n^4/\lambda_{C,n}^4 \;=\; 8/\sigma_4 \;=\left(\frac{\pi^2}{2}\;|\Gamma_{-1/4}|^4\right)^{-4} \tag{72}$$

as integration limit the non-point charge integral in 4D will be given by (with factor 4 according to equ. (25)):

$$\int_0^r \Psi_4(r)^2 r\,dr \;\sim\; \Gamma(-1/4\,,8/\sigma_4) \;=\; \int_{8/\sigma_4}^{\infty} t^{-1.25} e^{-t}\,dt \;\approx\; 4(\pi^2/2\;|\Gamma_{-1/4}|^4) \approx 32\pi^4|\Gamma_{-1/2}| \approx\; 1/11390 \quad ^{28} \tag{73}$$

The 4D equivalent of (69) will be:

$$\int_0^r \Psi_4(r)^2 r^{-3} dr \int_0^r \Psi_4(r)^2 r\,dr = \left[\frac{\Gamma_{1/2}}{4}\right]\left[2\pi^4 16\,\frac{|\Gamma_{-1/2}|}{4}\right] = \frac{\pi^2}{2}\Gamma_{1/2}|\Gamma_{-1/2}|\;\mathbf{4\pi^2} \;=\; \pi^3\mathbf{4\pi^2} \;=\; \alpha_g^{-1}\mathbf{4\pi^2} \tag{74}$$

The term $4\pi^2$ is the square of the 2π term in the last expression of (69) since the integrals in (74) refer to ρ_n^2 and thus to the square of energy and h, ħ.

While the integral $\int\Psi_3(r)^2dr$ in 3D yields the wavelength of *one* photon, $\int\Psi_4(r)^2\,r\,dr$ may be considered as an integration over 1/W of *all* photons within the integration limits, giving a term $\int\Psi_4(\lambda)^2\,\lambda\,d\lambda \sim 1/W^2$.

2D case:
the 2D case is not as straightforward as the 4D case. The integral over the 1D point charge

$$\int_0^r \Psi_2(r)^2 r^{-1} dr \;=\; \Gamma(0\,,\rho_n^2/r_2^2)\;/\;2 \tag{75}$$

features Γ(0, x) and with Γ(0, x) -> ∞ for x -> 0 the simple relation between integral limit and integral value according to (25) is not valid. Using nevertheless the 2D equivalent of the integration limit

$$\rho_n^2/\lambda_{C,n}^2 \;=\; 8/(3\sigma_2) = \left(3^{0.5}\pi\;|\Gamma_{-1/2}|^2\right)^{-2} \approx 1/4676 \tag{76}$$

and calculating $\Gamma(0, \rho_2^2/r_2^2)$ numerically gives $\int\Psi_2(r)^2 r^{-1}\,dr \approx \Gamma(0, \rho_2^2/r_2^2)/2 = 7.872/2$. In the 2D case the complementary integral would be identical to the point charge integral, giving $(\int\Psi_2(r)^2 r^{-1}\,dr)^2 \approx 2\pi^3/4$. This will give the expected value of $\alpha_{g'} \approx \pi^4$ *if multiplied by a factor 2π*. Unlike to the 3D, 4D case 2π will not appear in the denominator of the expression for α, since the 2D integrals yield dimensionless terms and refer to angular momentum rather than energy. Though the reason for the appearance of 2π in the nominator of the integral term is not obvious it is possible to include the 2D case in the unified expressions given by equations (32)f. [29]

[A7] Magnetic moment [30]
Within this model particles are treated as electromagnetic objects principally enabling a direct calculation of the magnetic moment, ***M*** from the electromagnetic fields.
The magnetic moment ***M(***e) of the electron is given as product of the anomalous g-factor, g_a = 1.00116, Dirac-g-factor, g_D = 2, and the Bohr magneton, $\mu_B = e\,\hbar/(2m_e)$, times the quantum number for angular momentum J = 1/2:

$$\boldsymbol{M}(e) \;=\; g_a g_D \mu_B/2 \;=\; g_a\frac{2e\,c_0^2}{2W_e}\;\frac{\hbar}{2} \;=\; g_a 9.274\text{E-}24[\mathrm{Am}^2] \tag{77}$$

The factor g_a arises from the interaction of the electron with virtual photons as calculated in quantum electrodynamics and should not be part of a calculation of the magnetic moment from the field of the electron itself. Within this model the factor 2 of g_D originates from the fact that particle energy is supposed to be equally divided into contributions of the electric and magnetic field, $W_{el} = W_{mag} = W_n/2$ and only the magnetic field, i.e. W_{mag} contributes to the magnetic moment.
Inserting the term for particle energy of (15) in (77) gives:

$$\frac{\boldsymbol{M}(e)}{g_a} = \frac{e\hbar c_0^2}{2W_e} = \frac{e\hbar c_0^2}{2}\;\frac{3\beta_e^{1/3}}{2b_0\Gamma_{1/3}} = e\,c_0\beta_e^{1/3}\left(\frac{\Gamma_{-1/3}}{3}\;\frac{3}{\Gamma_{-1/3}}\right)\frac{3[\hbar c_0/b_0]}{4\Gamma_{1/3}} = e\,c_0\beta_e^{1/3}\frac{\Gamma_{-1/3}}{3}\left[\frac{9[\alpha^{-1}]}{4\Gamma_{1/3}\Gamma_{-1/3}}\right] \tag{78}$$

The term on the right is expanded by $\Gamma_{-1/3}/3$ and turned into a form that will be needed for comparison with a calculation starting directly from the fields as explained in the following.
The relation of the values of E and B in an electromagnetic wave is given by $B = E/c_0$. This gives as first approximation for the value of $\boldsymbol{M}_n$ of a particle n:

---

28 Factor 2 representing electric and magnetic contributions in the 3D equations will be dropped in the 4D case.
29 Inserting a factor 2π in one of the two integrals $\int\Psi_2(r)^2 r^{-1}$ dr would turn this integral into the volume integral over the square of $1/r^1$ in analogy to the derivation of the 3D term.
30 Note: to allow for comparison with tabulated values of ***M*** in units of [Am²] the calculations in this chapter use
<u>e [C] not e_c [J]</u>, conversion factor: [m²C/s] /[m² J/s] = e/e_c = 1/19.4 [C/J].

$$\boldsymbol{M_n} \approx \frac{1}{\mu} \int_0^{\lambda_{C,n}/2} B(r) \Psi_n(r)^2 d^3 r = \varepsilon c_0 \int_0^{\lambda_{C,n}/2} E(r) \Psi_n(r)^2 d^3 r = e\, c_0\, \beta_n^{1/3} \frac{\Gamma_{-1/3}}{3} \left[\frac{3^{0.5}}{2} \frac{3}{2} [\alpha]^{-1} \right] = e\, c_0 \frac{\lambda_{C,n}}{2} \tag{79}$$

As integral limit $\lambda_{C,n}/2$ is chosen with $\lambda_{C,n}$ given by $\lambda_{C,n} \approx 3^{0.5} r_n$ [27]. The expression linear in the fields B, E, equ. (79), is in its major terms identical to equ. (78) originating from W ~ E^2, the difference of the bracket terms is exactly 2π due to the general relation Γ(x) Γ(-x) = π/(x sin(πx) [13], giving Γ(+1/3) Γ(-1/3) = $6\pi/3^{0.5}$.

Equation (79) will give a rough approximation for magnetic moment, see tab. 4 col. 4, the term in brackets of (78)f contains integral terms over $\Psi(r)^2$ and suggests appropriate corrections using such integrals.

In the following, however, a toy model will be used to calculate the magnetic moments, in particular of the pair p and n. The energy of the particles p and n differs by only ~ 1E-3 i.e. in the order of QED correction effects. Within this model this implies an essentially identical distribution of the absolute value of the E-field. Since E appears squared in the energy equation, this poses no problem for the difference in charge. However, the difference in magnetic moment rules out an expression of type (78) and requires a solution of type (79) to be based on an identical distribution of absolute values of the fields of p and n which sets a very severe restriction for possible solutions.

The toy model uses quaternions to describe the rotation of a set of vectors E, B, V under the conditions[31]:

- E, B, V being orthogonal
- continuous rotation
- in phase, constructive interference
- in each step rotating angles for E and B are equal and some multiple of V: n ω_E = n ω_B = ω_V

The average of the B-orientation for a full period, i.e. up to the angle where all E, B, V-orientations are identical to the starting condition, is examined. The first elements in the series of solutions are given in tab. 3.

The solution for $2\omega_E = 2\omega_B = 1\omega_V$ which yields the same phase for all 3 components at an angle of 120° with respect to V might be considered the best suited version for particles considered to be composed of J = 1/2 spin components since it requires two (E, B) rotation cycles for the V-component to complete full 360 degrees. The volume covered by such a solution is mainly restricted to an octant sector of a three-dimensional coordinate system suggesting to examine a combination of orthogonal solutions. Since the coefficient of 2/3 for B_avg is composed of components 4/9, 4/9 and 2/9 in x,y,z-direction, combinations featuring a coefficient $(2/3)^n$ for B_avg can be constructed easily, e.g. $(((4/9 + 4/9)/2)^2 + 0 + 0)^{0.5} = (2/3)^2$.

n	n^2	ω_V	B_avg
1/∞	1/∞	360	-
1.000	1	270	0.591
1.225	1.5	180	0.612
2.000	4	120	0.667
2.739	7.5	90	0.684

Table 3: Average values calculated for magnetic moment, B_{avg}, for varying rotation angle ratios.

Factor 2/3 appears in the equations for ρ as well, see [A1], [A2, (62)f] and the start of the series of magnetic moments as indicated in tab 4, col 6 gives some indication to apply $(2/3)^n$ with the μ having $(2/3)^1$ [32] and $(2/3)^3$ giving a precise value for ***M***(p) .

While a particle such as μ at the beginning of the series might feature some additional degree of freedom in trajectories for such a 120° object and thus an additional term of 2π (cf. (78)f), for analyzing the pair p, n orthogonal combinations of 120° solutions will be used.

Neutral particles may be constructed from two combinations with opposite direction of the E-field with respect to the origin, experimental data suggest to use $((2/3)^n)^2$. Based on a relation such as $(((4/9 + 4/9)/2)^2 + 0 + 0)^{0.5}$ this would imply the coefficient 1/2 to be given with an exponent of n, to reduce to $(1/2)^1$ necessary for two states (of different E-orientation) this requires a correction term of (n-1).

The difference of 2π in (78)f for e, μ indicates that some additional factor for geometry might be required in the expression for ***M***.

The following modification for equ. (79) will be applied for particles starting with the μ (see 35ff, 49f):

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31 Basic algorithm of type:

```
dee = dee + de;    uu = Sqr(ex ^ 2 + ey ^ 2 + ez ^ 2);    sih = Sin(de / 2);    qw = Cos(de / 2);    qx = (ex / uu) * sih    qy = (ey / uu) * sih;    qz = (ez / uu) * sih;   bx = bbx;    by = bby;    bz = bbz;
bxx = bx * (qx * qx + qw * qw - qy * qy - qz * qz)  + by * (2 * qx * qy - 2 * qw * qz) + bz * (2 * qx * qz + 2 * qw * qy);
byy = bx * (2 * qw * qz + 2 * qx * qy)   + by * (qw * qw - qx * qx + qy * qy - qz * qz) + bz * (-2 * qw * qx + 2 * qy * qz);
bzz = bx * (-2 * qw * qy + 2 * qx * qz)  + by * (2 * qw * qx + 2 * qy * qz) + bz * (qw * qw - qx * qx - qy * qy + qz * qz);
bx = bxx;  by = byy;  bz = bzz;  bbx = bx;  bby = by;  bbz = bz;
```

The model sets starting values for E, B, V without discerning polarity of E, B; the point of origin is the same for all vectors, giving an approximation for r >> r_n, see [17] for more details.

32 The identical value of μ with respect to e may be somehow due to the 3/2 anomaly of the latter, cf. chpt. 2.7

$$\boldsymbol{M_n} \approx e c_0 \lambda^*_{C,n} \frac{3}{2}\left(\frac{2}{3}\right)^n [n-1] X_{geo} \quad [30] \qquad (80)$$

with 3/2 representing the additional anomalous electron coefficient included and $\lambda_{C,n}$* refering to a length ~ $\Pi_{k=o}^{n}$ α^(9/3^k) i.e. without the extra 3/2 term of e, the term in square brackets, n-1, refering to the correction of the combination states and X_{geo} to some remaining coefficient for geometry.
For the pair p, n this would give coefficients $(2/3)^3$ for p and 4 $((2/3)^3)^2$ for n plus an additional geometry coefficient that can be given as $3^{0.5}/3$, i.e. cos(104,5/2°) of a tetrahedron, as indicated in fig. 1. A tetrahedral orientation might be somewhat unexpected, some 3-fold symmetry and the relation to the orthogonal combinations of above may be obtained by constructing the tetrahedral branches by 3 combinations along the orthogonal axes as indicated in fig. 1 as well (and giving an alternate interpretation of the term $3^{0.5}/3$).

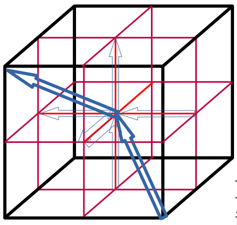

Figure 1: possible combination of 120° solutions for magnetic moment of n, foreground octants indicated only;

As required it is thus possible to explain the different magnetic moments of p, n, based on the same (120°) constituent, with a specific weighted geometric distribution [33] of inversely charged components in case of the n.
Tab. 4 demonstrates that the parameters used to obtain this result are not completely arbitrary but seem to exhibit a simple pattern, though a more refined quaternion model and analysis will be needed to obtain a more unambiguous solution.

	M_lit [Am^2]	$\lambda_{C,n}$	$ec_0\ \lambda_{C,n}/2$	$3/2ec_0\ \lambda^*_{C,n}/2$	Coeff. (2/3)	(n-1)	X_{geo}	M_calc [Am^2]	M_calc/M_lit
e^{+-}	9.285E-24	2.426E-12	5.827E-23				1/(2π)	9.274E-24	0.999
μ^{+-}	4.490E-26	1.170E-14	2.810E-25	4.215E-25	(2/3)^1		1/(2π)	4.472E-26	0.996
η^0	0.000E+00				(2/3)^2				
p^{+-}	1.411E-26	1.321E-15	3.174E-26	4.760E-26	(2/3)^3			1.410E-26	1.000
n	9.662E-27	1.321E-15	3.174E-26	4.760E-26	((2/3)^3)^2	4	√3/3	9.651E-27	0.999
Λ^0	3.096E-27	1.101E-15	2.645E-26	3.967E-26	((2/3)^4)^2	8	0.25	3.096E-27	1.000
Σ^-	5.859E-27	1.040E-15	2.497E-26	3.745E-26	(2/5)^1			4.932E-27	0.842
Σ^+	1.241E-26	1.040E-15	2.497E-26	3.745E-26	((2/3)^5)^2	16	1	1.234E-26*	0.994*

Table 4: Absolute values calculated for magnetic moment; * relative to Σ^- with factor $(2/3)^5$16.

[A8] Values used

π = 3.141592654
$\Gamma_{1/3}$ = 2.678938535
$|\Gamma_{-1/3}|$ = 4.062353818
α^{-1} = 137.035999084
c_0 = 2.99792458 [m/s]
e = 1.602176634 E-019 [C]
ε = 8.854187813 E-12 [F/m]
b_0 = 2.307077552 E-28 [Jm]
G = 6.67430 E-11 [$m^5/(Js^4)$]
$W_{e,\,exp}$ = 8.187105777 [J]
$\lambda_{C,e}$ = 2.426310239 E-12 [m]
e_c = 3.109751438 E-18 [J]
β_{dim} = 5.131205555 E-30 [m^3]
$\sigma = 8(4\pi\ |\Gamma_{-1/3}|\ ^3/3)^3$ = 177155864 [-]
r_e = 1.413269970 E-12 [m]

33 which might represent a time average