Cupflyer

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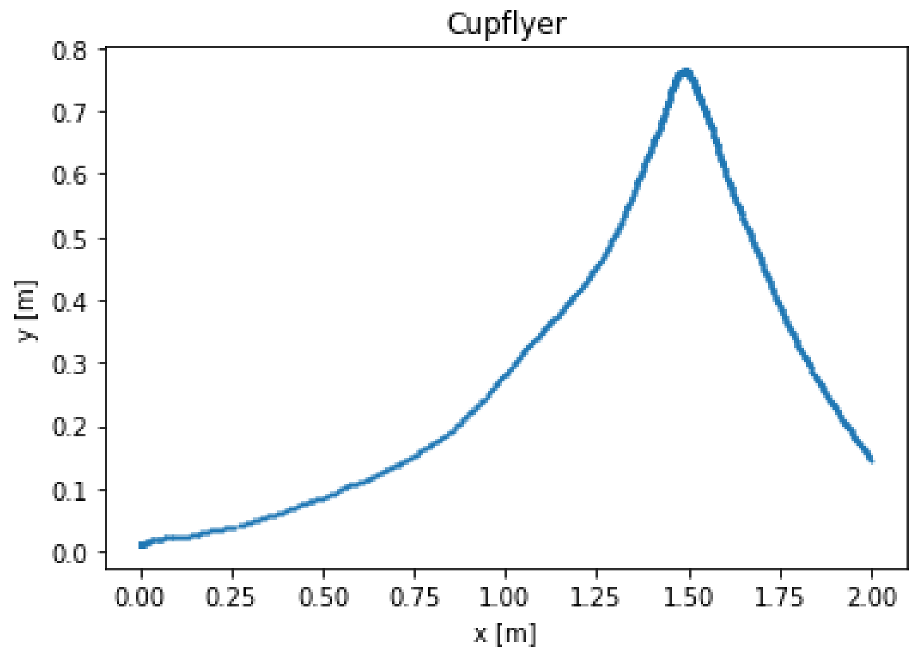
Question:

A lightweight cup flyer thrown horizontally with a high backspin initially rises against gravity. Consider a flyer with a center of mass that is shifted from its geometrical center. Explain the flyer's trajectory and discuss the influence of the center of mass location and other relevant parameters on the maximum height and the stability of the flight.

Why does the cup fly upward



– no shift i center of mass (CM)

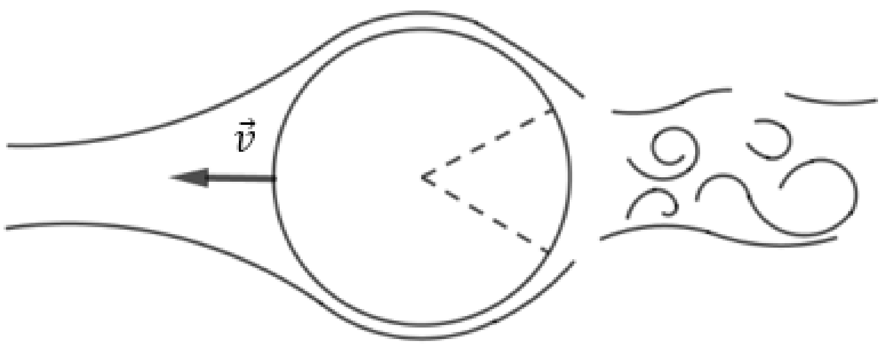
Why does the cupflyer fly upward

– no shift i CM

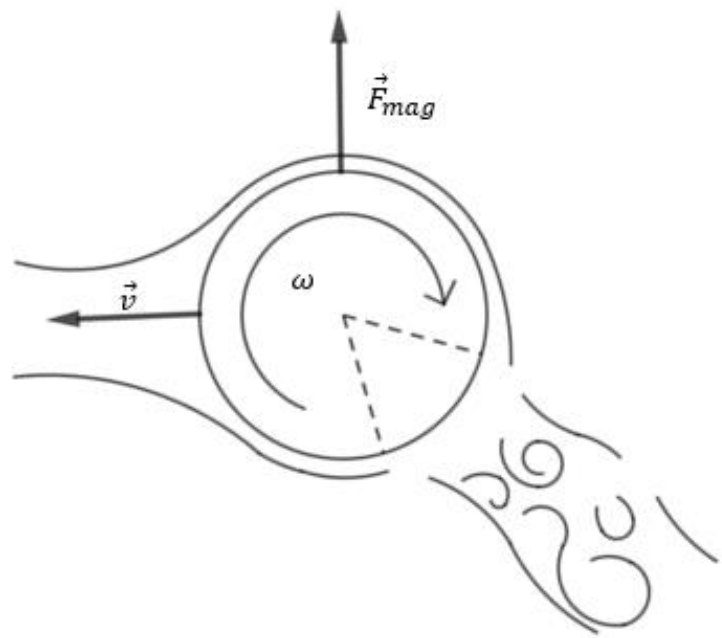
$$Re = \frac{\rho L u}{\mu}$$

Reynolds number between 5000 to 21000.

No rotation



With rotation



Why does the cupflyer fly upward

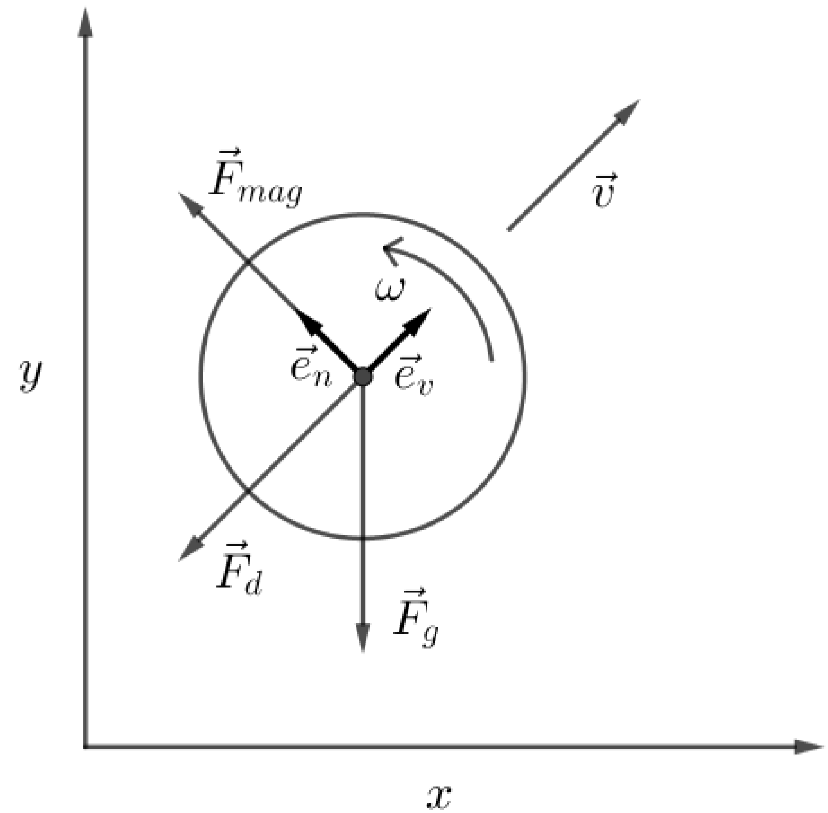
– no shift i CM

- 3 forces, F_d, F_{mag} and F_g.

$$\vec{F}_{mag} = \rho C_l V \omega v \vec{e}_n$$

$$\vec{F}_d = -\frac{1}{2}\rho C_d A v^2 \vec{e}_v$$

$$\vec{F}_g = mg\hat{y}$$



Why does the cupflyer fly upward

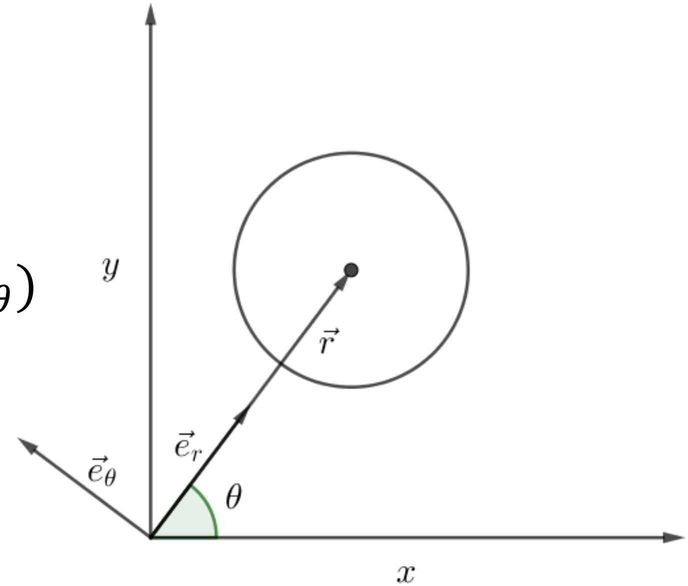
– no shift i CM

- Polar coordinates

$$\vec{F}_{mag} = \frac{1}{2}\rho A C_l\, \omega(t)(-r\,\dot{\theta}\vec{e}_r + \dot{r}\vec{e}_\theta)$$

$$\vec{F}_d = -\frac{1}{2}\rho A C_d \sqrt{\dot{r}^2 + r^2\dot{\theta}^2}\;(\dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta)$$

$$\vec{F}_g = mg(\cos(\theta)\,\vec{e}_r - \sin(\theta)\,\vec{e}_\theta)$$



Why does the cupflyer fly upward

– no shift i CM

$$\sum\vec{F}_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta = \left(\frac{1}{2}\rho A C_d\sqrt{\dot{r}^2 + r^2\dot{\theta}^2}\dot{r} - \frac{1}{2}\rho A C_l \omega(t)\sqrt{\dot{r}^2 + r^2\dot{\theta}^2} r\dot{\theta} - mg\sin(\theta)\right)\vec{e}_\theta$$

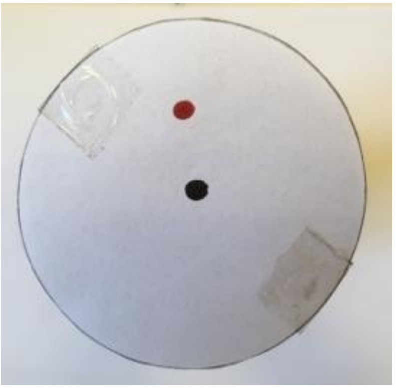
$$\sum\vec{F}_r = m\left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r = \left(-\frac{1}{2}\rho A C_d\sqrt{\dot{r}^2 + r^2\dot{\theta}^2} r\dot{\theta} - \frac{1}{2}\rho A C_l \omega(t)\sqrt{\dot{r}^2 + r^2\dot{\theta}^2}\dot{r} + mg\cos(\theta)\right)\vec{e}_r$$

- Model needs to be calculated numerical.
- With the assumptions:
  - Drag coefficient, C_d and lift coefficient, C_l is constant.
- $\omega(t)$ is found by a fit on experimental data.

Setup of our experiments

Reproducibly

Center of mass displacement



Launch

- Recorded with highspeed camera.
- Tracked in the program tracker.
- Every experiment is repeated 10 times.

Experiment vs. teori – no shift i CM

Parametres:

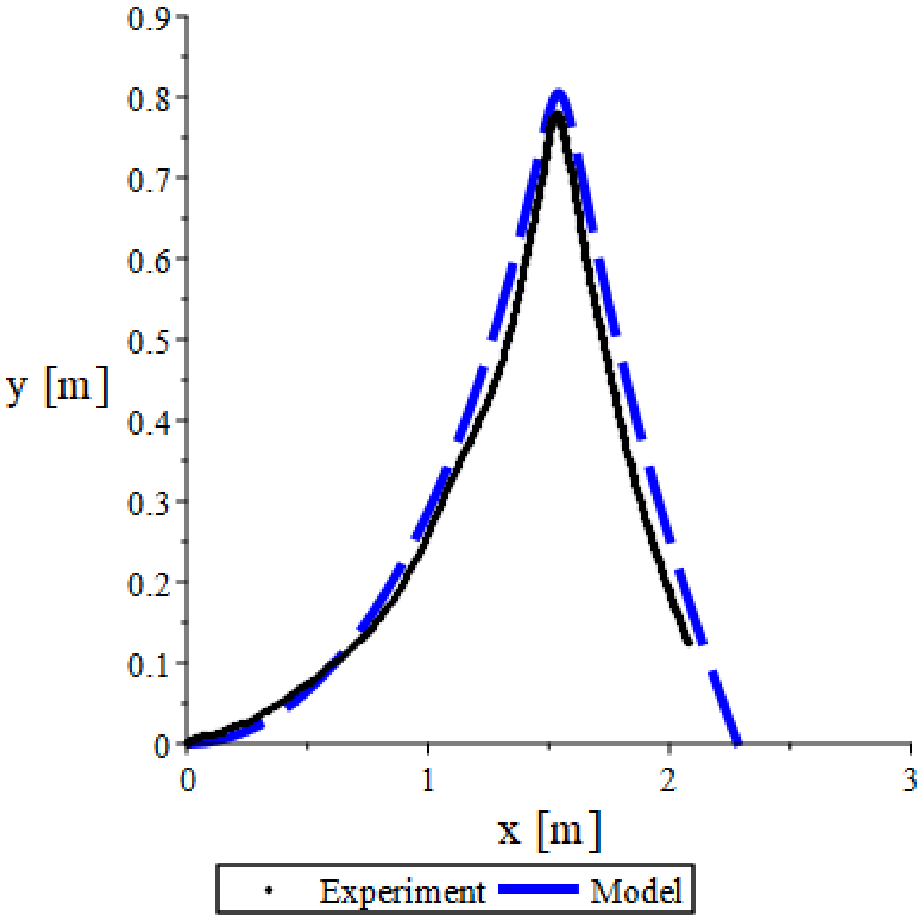
- Mass: $m = 12.20\,\mathrm{kg}$
- Length: $L = 0.198\,\mathrm{m}$
- Avg. radius: $r_{avg} = 0.028\,\mathrm{m}$
- Location of CM: $CM = 0\,\mathrm{mm}$

Assumptions:

- The cupflyer is a rotation cylinder: $C_d = 1.1$

Initial conditions:

- $x(0) = 0\,\mathrm{m},\ \dot{x}(0) = 8.63\,\frac{\mathrm{m}}{\mathrm{s}}$
- $y(0) = 0,\ \dot{y}(0) = 0.0026\,\frac{\mathrm{m}}{\mathrm{s}}$



C_d: http://brennen.caltech.edu/fluidbook/externalflows/drag/dragonasphere.pdf

What happens when we shift the center of mass?

Parametres:

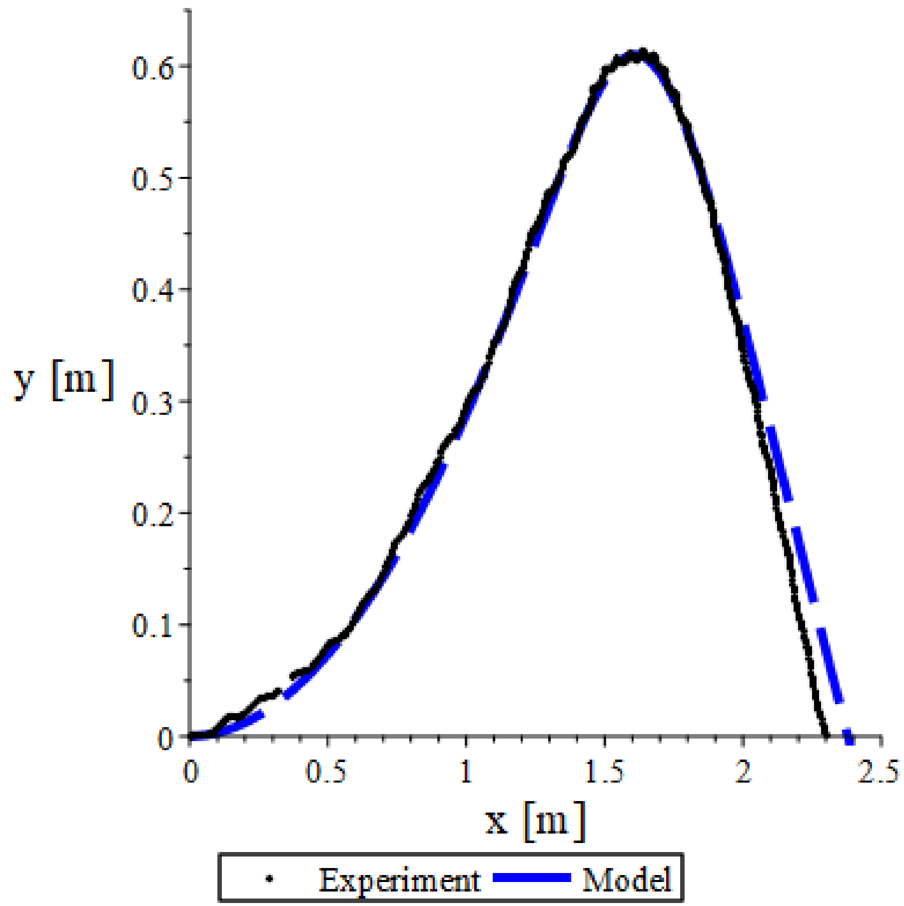
- Mass: $m = 18.98\,\mathrm{g}$
- Length: $L = 0.198\,\mathrm{m}$
- Avg. radius: $r_{avg} = 0.028\,\mathrm{m}$
- Location of CM: $CM = 12.32$

Assumptions:

- The cupflyer is a rotation cylinder: $C_d = 1.1$

Initial conditions:

- $x(0) = 0\,\mathrm{m}$, $\dot{x}(0) = 5.79\,\frac{\mathrm{m}}{\mathrm{s}}$
- $y(0) = 0$, $\dot{y}(0) = 0.014\,\frac{\mathrm{m}}{\mathrm{s}}$

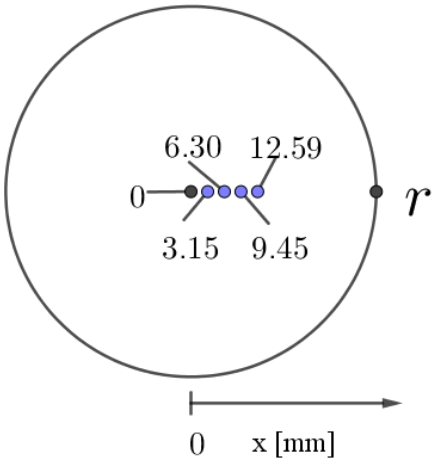


Moving the center of mass:

- Constant mass: $m_{total} = 17.27\text{g}$
- x_{cm} is the location of the CM:

$$x_{cm} = r \cdot \frac{m_r}{m_{total}}$$

- r is the location where the mass is shifted to.
- m_r is the mass in position r.
- m_{total} is the total mass.

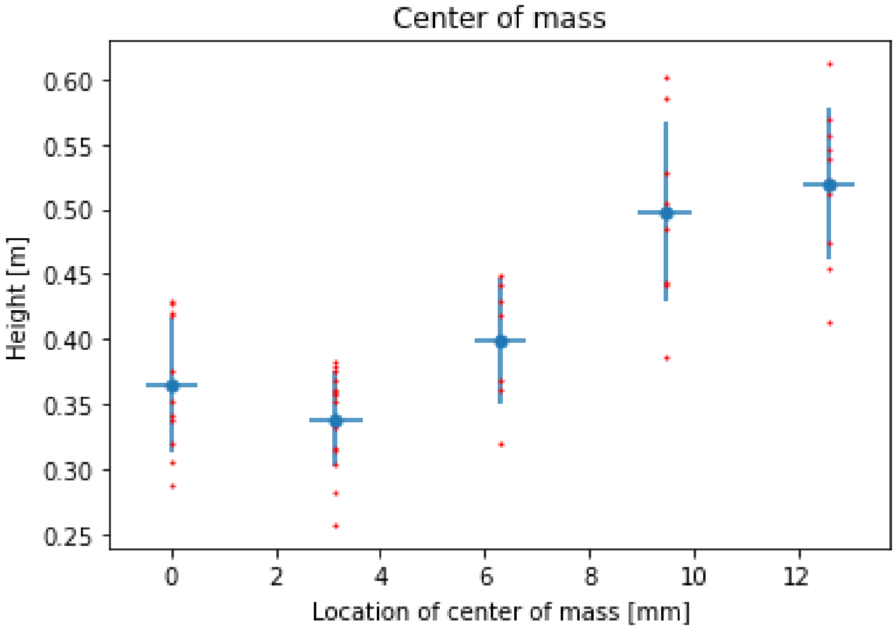


Moving the center of mass:

Parameters:

- Length: $L = 0.198\ \mathrm{m}$
- Avg. radius: $r_{avg} = 0.028\ \mathrm{m}$
- Mass: $m_{total} = 17.29\mathrm{g}$

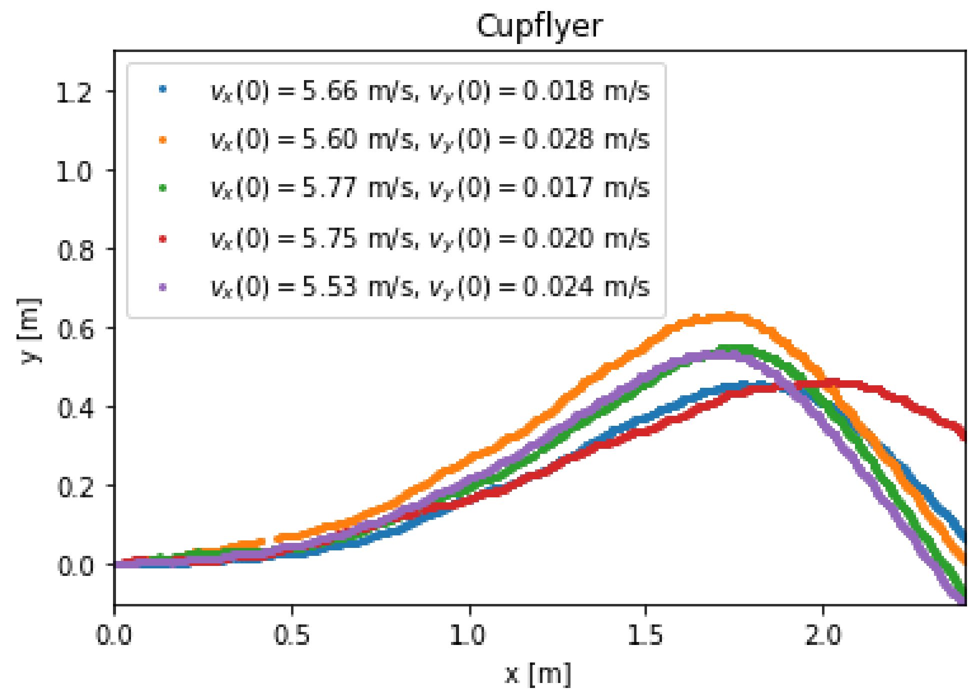
$m_{gm} \pm 0.01\mathbf{g}$	$m_r \pm 0.01\mathbf{g}$	x_{cm}
17.29	0	0
14.73	2.56	3.15
12.17	5.12	6.30
9.61	7.68	9.45
7.05	10.24	12.59



Repetitions:

Parametres:

- Mass: $m = 0.01898\,\mathrm{kg}$
- Length: $L = 0.198\,\mathrm{m}$
- Avg. radius: $r_{avg} = 0.028\,\mathrm{m}$
- Location of CM: $x_{CM} = 12.32$



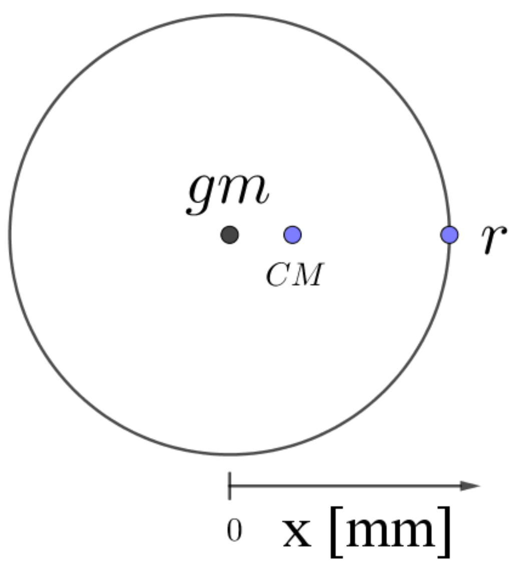
Mass:

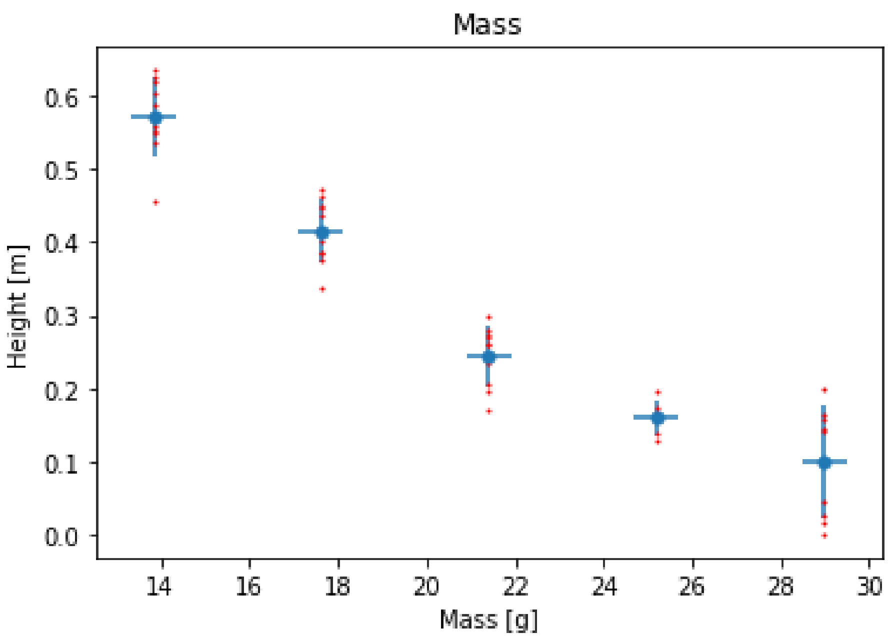
- Constant center of mass: $x_{cm} = 10\text{mm}$
- We varies the mass at the gm center, m_{gm}

$$x_{cm} = r\frac{m_r}{m_r + m_{gm}}$$

$$m_r = \left(\frac{r}{x_{cm}m_{gm}} - \frac{1}{m_{gm}}\right)^{-1}$$

$m_{gm} \pm 0.01\text{g}$	$m_r \pm 0.01\text{g}$	$m_{total} \pm 0.01\text{g}$
7.33	6.52	13.85
9.33	8.29	17.62
11.33	10.07	21.40
13.33	11.85	25.18
15.33	13.63	28.96

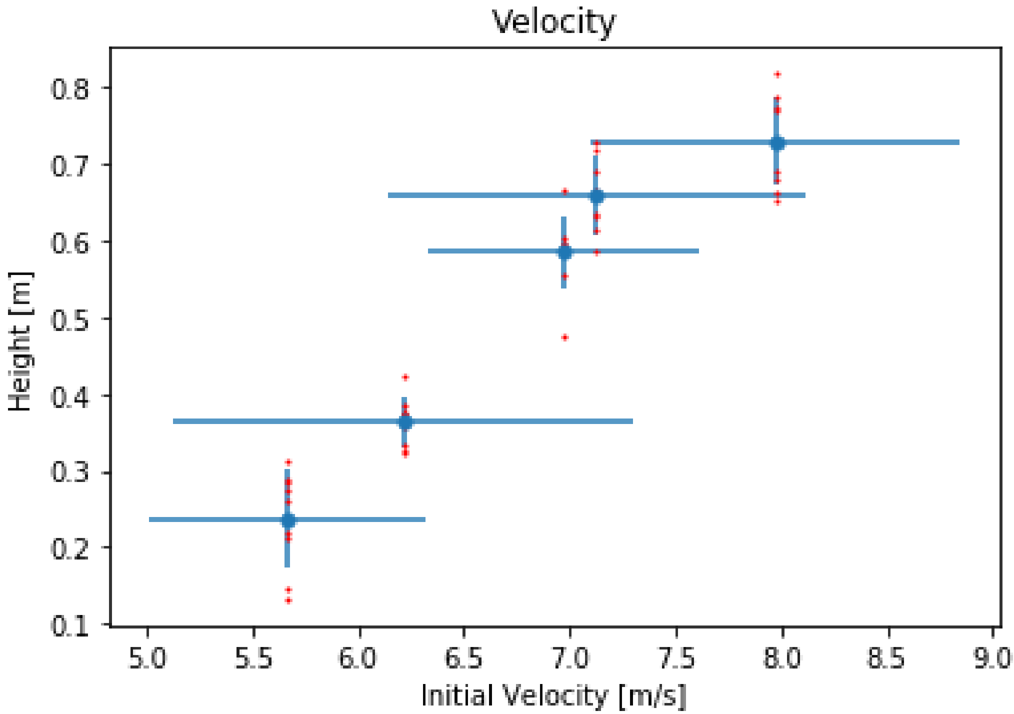




Initial velocity:

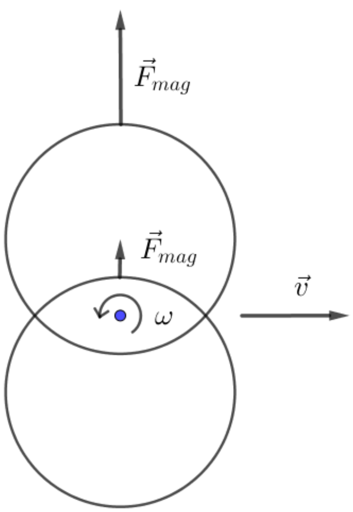
Parameters:

- Length: $L = 0.198\ \mathrm{m}$
- Avg. radius: $r_{avg} = 0.028\ \mathrm{m}$
- Mass: $m_{total} = 12.20\mathrm{g}$

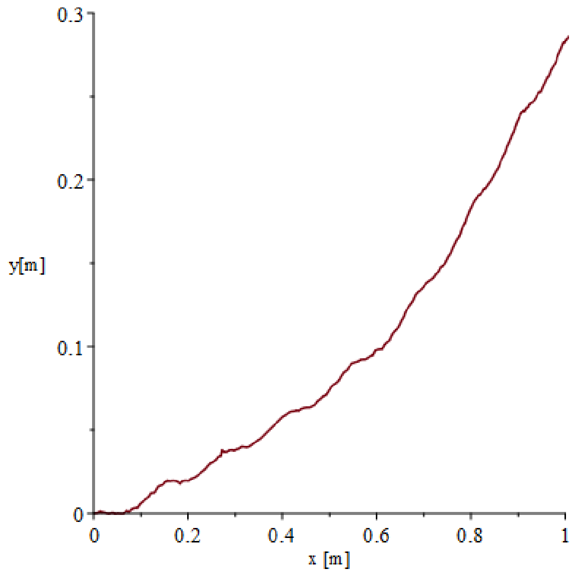


Stability of cupflyer :

- Diffrent magnus force



Zoom of CM shifted

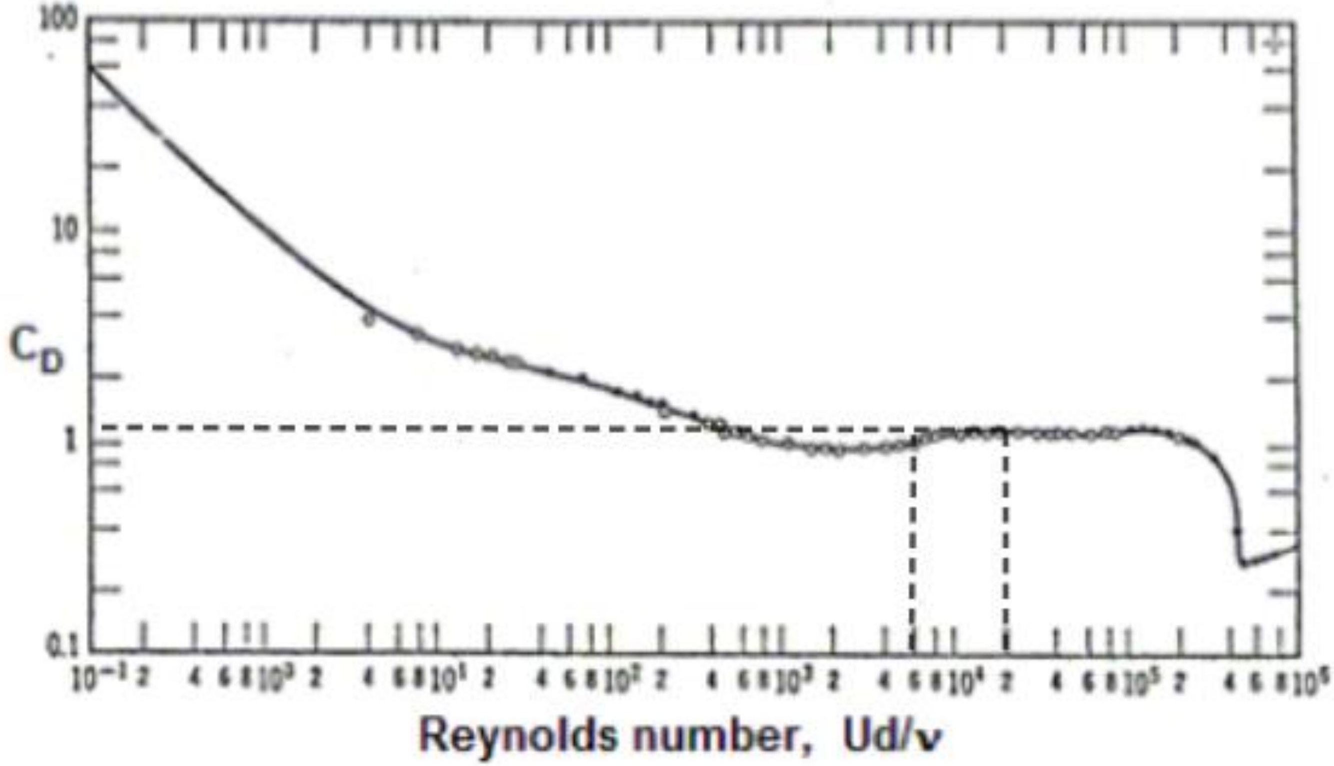


Conclusion

- The forces on the cup flyer are: $\vec{F}_d, \vec{F}_{mag}$ og $\vec{F}_g$.
- The CM has an effect on the height
- When the mass increases the height decreases.
- The stabability is effected by the CM.

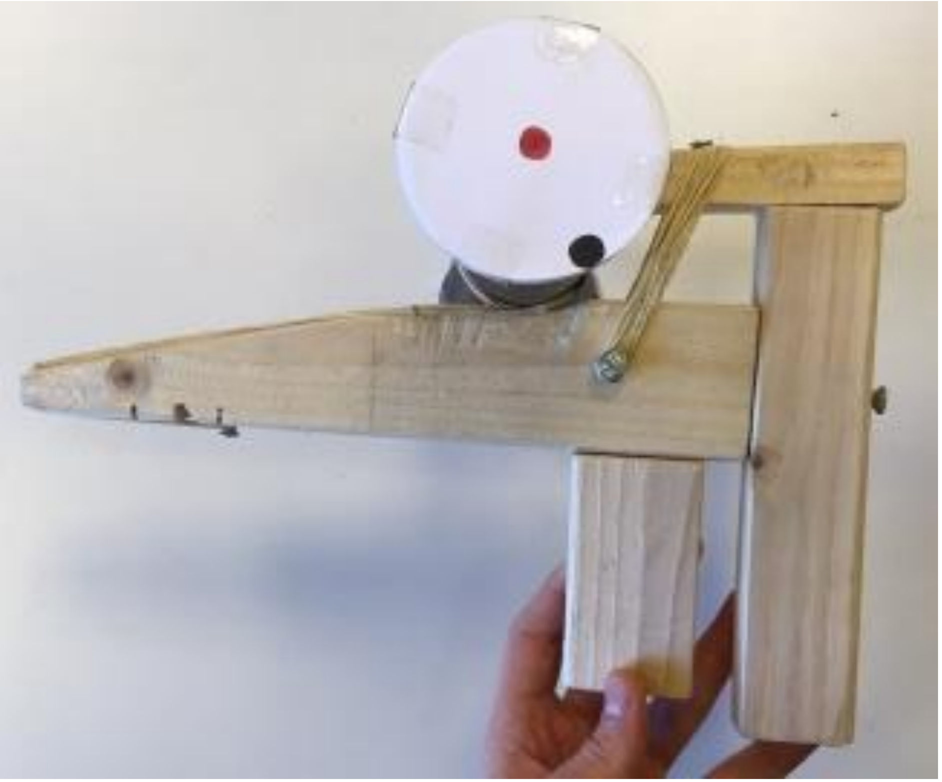
End

Drag coefficient for rotation cylinder:

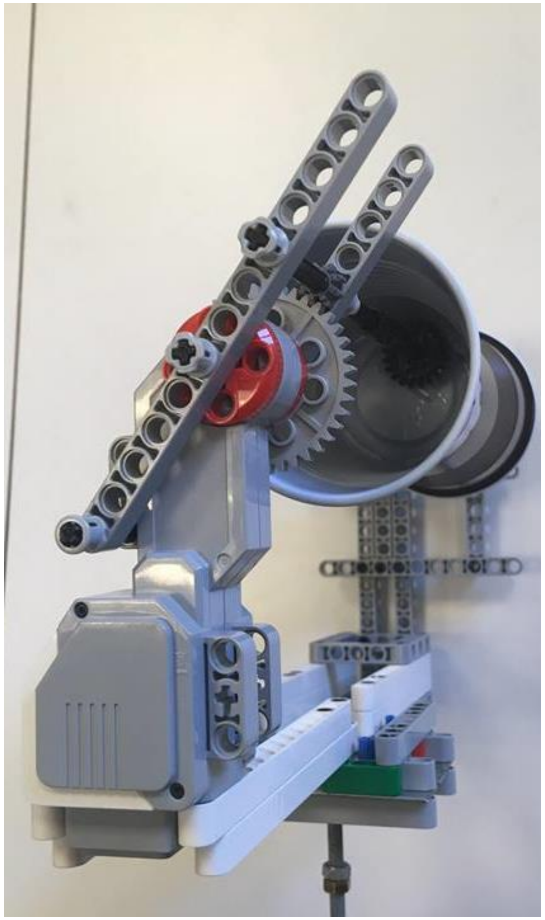


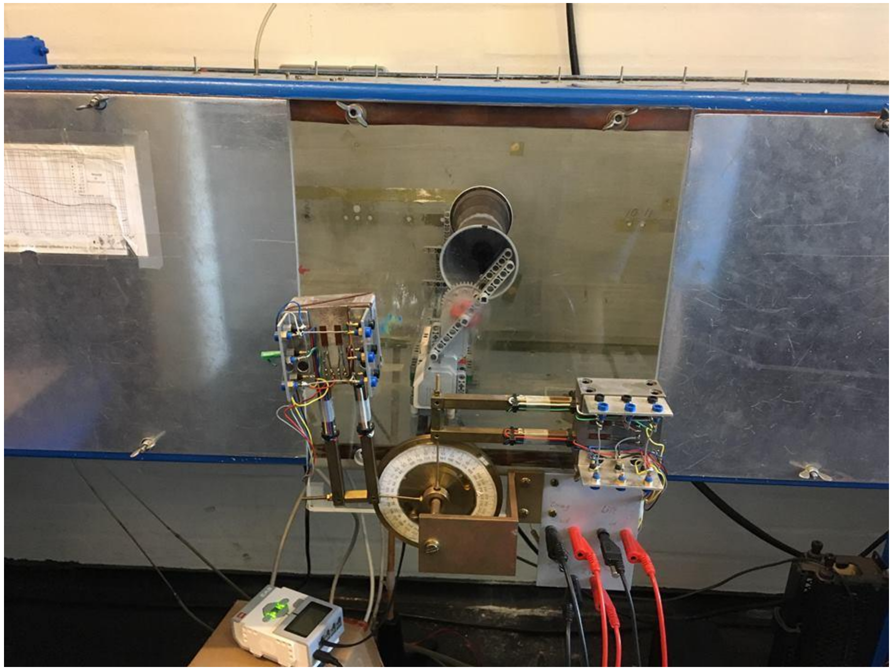
http://brennen.caltech.edu/fluidbook/externalflows/drag/dragonasphere.pdf

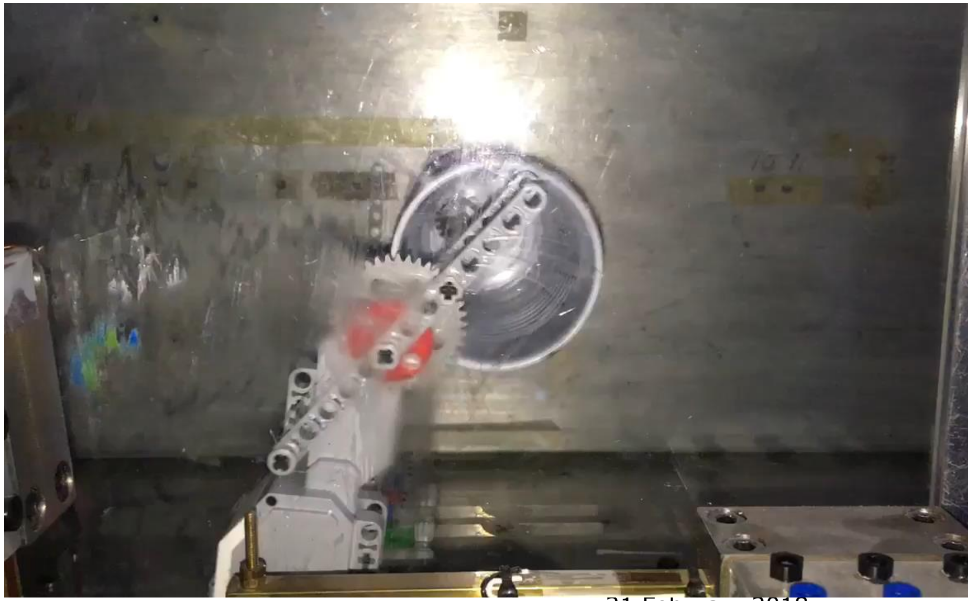
Initial conditions:



Wind tunnel:





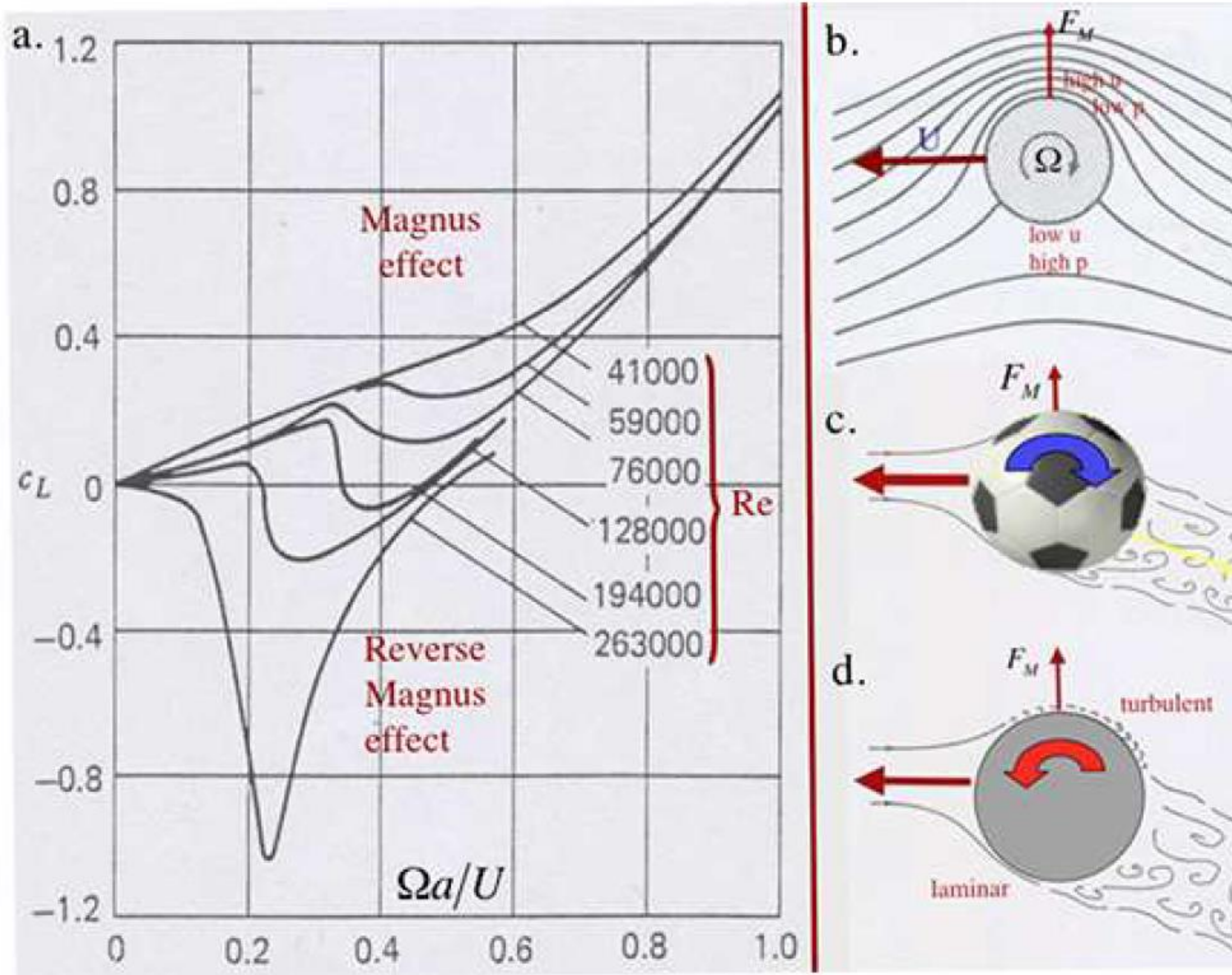


How do we maximize the stability

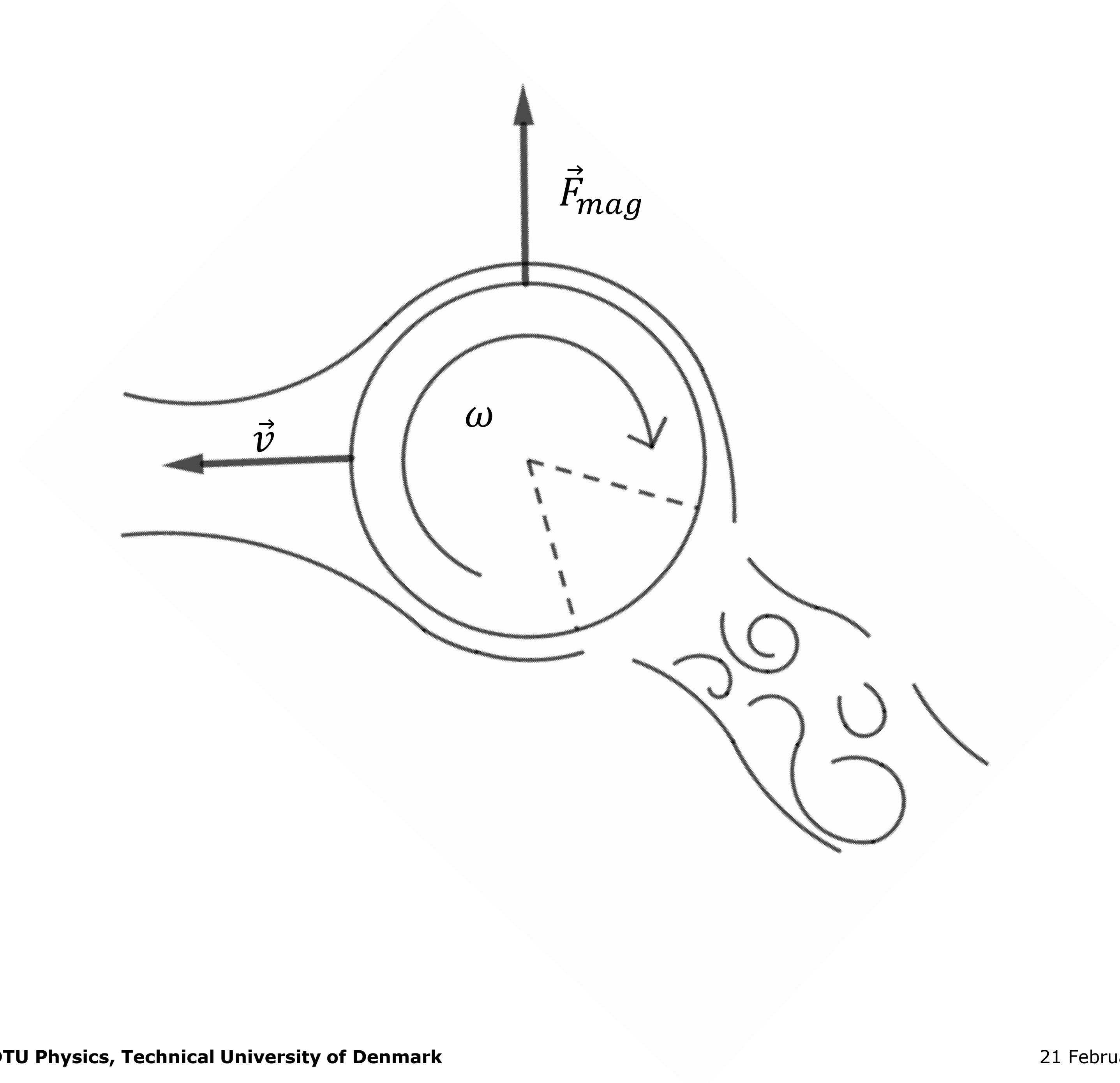
- Increased rotation speed and moment of inertia creates high angular momentum. Which in turn creates stability of flight

How do we maximize the height

- We need a good ratio between speed and rotation to optimize the height, to ensure that the magnus force is as large as posible without doing loops.
- The change in center of mass does not effect the center of masses tragectory (for small speeds)
- The radius and length of the cup will increase the lift and drag, so an optimal area could be found (experiments needed).
- Ripple on the cup increase the reynolds number and therefore change the lift coeftionen (experiments needed)



a.
1.2
0.8
0.4
0
−0.4
−0.8
−1.2
c_L
Magnus effect
Reverse Magnus effect
41000
59000
76000
128000
194000
263000
Re
$\Omega a/U$
0
0.2
0.4
0.6
0.8
1.0
b.
F_M
high u
low p
U
Ω
low u
high p
c.
F_M
d.
F_M
turbulent
laminar


$\vec{F}_{mag}$
ω
$\vec{v}$