IPT train

Problem 9: Rail track divination

April 22, 2019

Benjamin L. Larsen
Danish team

International physicists' tournament

Danmarks
Tekniske
Universitet

Agenda

Problem

The sound of an approaching train, propagating in metals, reaches our ears earlier than the train arrives. Is it possible to estimate the distance to the train and speed of its movement using this phenomenon? Estimate the accuracy and precision of your method.

Sound generation

- Comes from roughness of wheel or track
- Displacement propagates through the rail
- Ground and sleeper damping, rail damping
- $\omega = \frac{2\pi V}{\lambda}$
- λ undulations of the surfaces

Euler-Bernoulli Beam theory

$$EI\frac{\partial}{\partial x^4}^4 u(x) + su(x) + m\frac{\partial}{\partial t^2}^2 u(x) = 0 \quad (1)$$

- Valid for small deflections
- Lateral loads only
- E, Young modulus, I is moment area of cross-section, s is stiffness pr. unit length, m mass pr unit length

Non dimensionilization (resonance frequency, unsupported wavenumber)

$$\omega_0 = \sqrt{\frac{s}{m}} \quad k_b = \left(\frac{m\omega^2}{EI}\right)^{1/4} \quad (2)$$

Wave number

Damping: $\tilde{E} = E(1 + i\eta_E)$, $\tilde{s} = s(1 + i\eta_s)$ Assume solution of form

$$u(x) = U \exp i\omega t \exp -ikx \tag{3}$$

Insert and solve

$$\tilde{E}Ik^4 + \tilde{s} - m\omega^2 = 0 \Rightarrow k^2 = \pm\sqrt{\frac{m\omega^2 - s}{EI}} \tag{4}$$

$$\Rightarrow k(\omega) = k_b(\omega)(1 + i\eta_E)^{-1/4}\left(1 - \frac{\omega_0^2}{\omega^2}(1 + i\eta_s)\right)^{1/4} \tag{5}$$

$$k = k_r + ik_i \tag{6}$$

Sources:
Beams on Elastic Foundation
Terje Haukaas

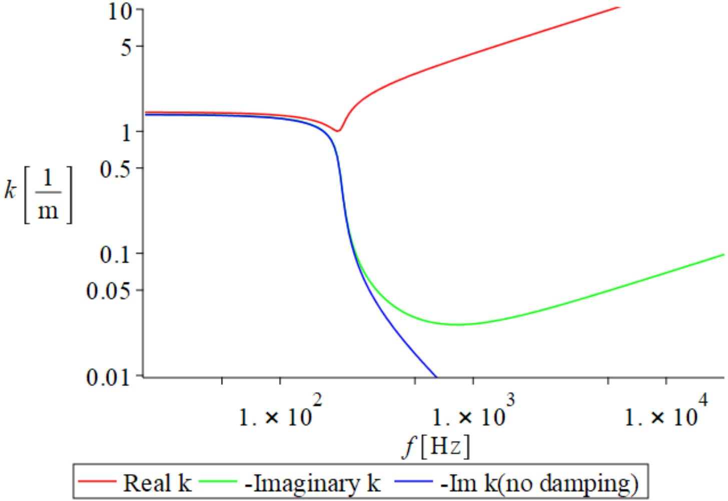
Parameter values

Standard values for UIC60 rail

Variable	Symbol	Value	Unit
Bending stiffness	EI	6.42	MN m^2
Mass pr. length	m	60	kg/m
Foundation stiffness pr. length	s	100	MN / m^2
Damping loss rail	η_E	0.02	
Damping loss foundation	η_s	0.1	

Sources:
Railway Noise and Vibration
D. Thompson

Wave number plot

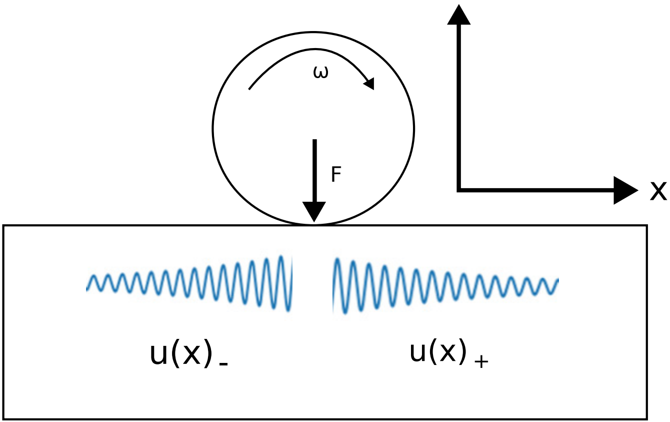


Group and phase velocity

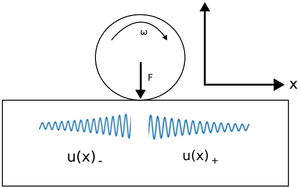
$$v_p = \frac{\omega}{k} = \sqrt{\frac{EI}{m}(1 + i\eta_E)k^2 + \frac{\omega_0^2}{k^2}(1 + i\eta_s)} \tag{7}$$

$$v_g = \frac{d\omega}{dk} = \frac{2}{k + \frac{\omega_0^2(1+i\eta_s)}{EIm(1+i\eta_E)}} \tag{8}$$

Sound generation



Harmonic force



Harmonic force $F\exp(i\omega)$

$$u_-(x) = A_1 e^{kx} + A_2 e^{ikx} \quad (9)$$

$$u_+(x) = A_1 e^{-kx} + A_2 e^{-ikx} \quad (10)$$

Solving with boundary cconditions:

$$u(x) = \frac{-iF}{4EIk^3}\left(e^{-ik|x|} - ie^{-k|x|}\right) \quad (11)$$

Velocity amplitude

$$v(x) = i\omega u(x) \quad (12)$$

Problem

The sound of an approaching train, propagating in metals, reaches **our ears** earlier than the train arrives. Is it possible to estimate the distance to the train and speed of its movement using this phenomenon? Estimate the accuracy and precision of your method.

Sound power

$$W = \rho_0 c_0 S \langle \bar{v}^2 \rangle \sigma = \frac{1}{2} \rho_0 c_0 \sigma \int_{-\infty}^{\infty} |v(x)|^2 dx \qquad (13)$$

- Density and speed of sound, S surface perimeter ($\approx 2\pi a$), velocity normal to surface $v(x) = i\omega u(x)$
- Radiation ratio σ, actual sound power to idealized case (plane waves)
- Cylinder

  $\sigma = \left(\pi |ka H_0^2(ka) - H_1^2(ka)|^2\right)^{-1} \approx \frac{\pi}{2}(ka)^3 (ka << 1)$

Sources:
Railway Noise and Vibration
D. Thompson

Sound power

Insert

$$W = \frac{\rho_0 c_0 \pi (ka)^3 S \omega F}{16 E I k^3} \int_{-\infty}^{\infty} |\exp - ikx|^2 dx \tag{14}$$

Now at a distance $x = x + d$ symmetric around $x = 0$

$$W(d) = \frac{\rho_0 c_0 \pi^2 a^4 \omega F}{8 E I} \exp(2k_i d) \int_0^{\infty} \exp(2k_i x) dx \tag{15}$$

$$W(d) = \frac{\rho_0 c_0 \pi^2 a^4 \omega F}{8 E I} \exp(2k_i d) \left[\frac{\exp(2k_i x)}{2k_i} \right]_0^{\infty} \tag{16}$$

$$W(f, d) = -\frac{\rho_0 c_0 \pi^3 a^4 f F}{8 k_i(f) E I} \exp(2k_i(f) d) \tag{17}$$

IPT train

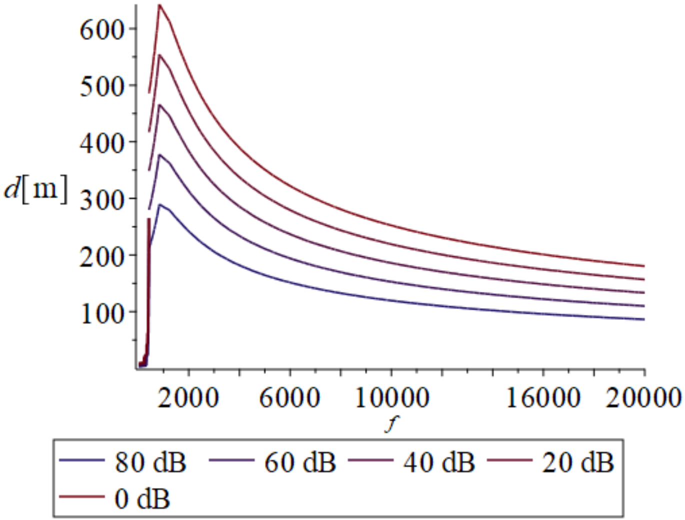
B. Larsen

Sound power level

$$L(f,d) = 10\log_{10}\left(\frac{W(f,d)}{W_0}\right) \tag{18}$$

- Sound power reference level (0 dB)
- $L > 0$ can be heard by humans, 40 dB whisper

Sound power level plot



TRAIN TIME!

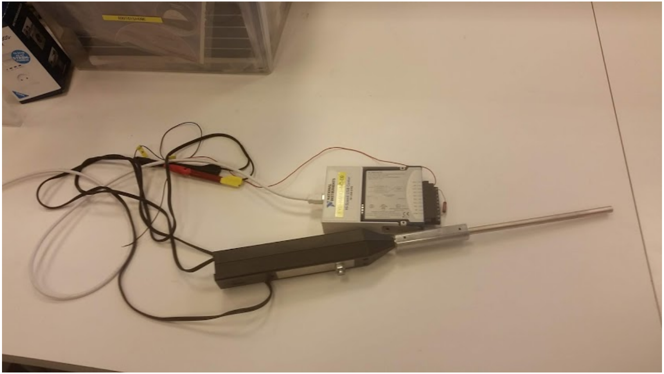
Sources:
Thomas the tank engine

Nærum regional train

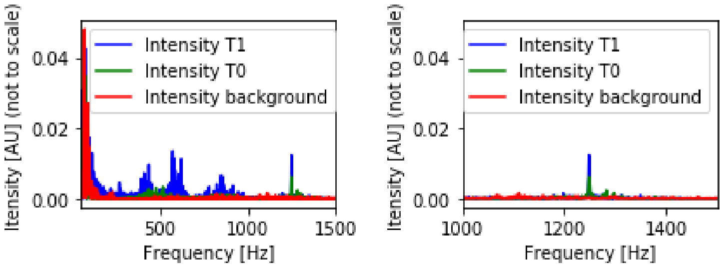
- RegioSprinter by Siemens-Duewag

Sources:
Kurt Rasmussen

Microphone



Spectrum


Intensity T1
Intensity T0
Intensity background
Itensity [AU] (not to scale)
0.04
0.02
0.00
500
1000
1500
Frequency [Hz]
Intensity T1
Intensity T0
Intensity background
Itensity [AU] (not to scale)
0.04
0.02
0.00
1000
1200
1400
Frequency [Hz]

Determining speed of train

$$W(f_1, d) = -\frac{\rho_0 c_0 \pi^3 a^4 f_1 F}{8k_i(f_1)EI} \exp\left(2k_i(f_1)d\right) \quad (19)$$

$$W(f_1, d - VT_1) = -\frac{\rho_0 c_0 \pi^3 a^4 f_1 F}{8k_i(f_1)EI} \exp\left(2k_i(f_1)(d - VT_1)\right) \quad (20)$$

$$\Rightarrow V = \frac{\ln\left(\frac{W(f_1, d - VT_1)}{W(f_1, d)}\right)}{2k_i(f_1)T_1} \approx 10.8 \pm 0.5 \frac{m}{s} \quad (21)$$

- Or around 40 km/h (reasonably close to the operating speed at this route)
- This gives $\lambda = \frac{V}{f_1} \approx 8$ mm

Determining distance to train

- Calibration of microphone:
- $L_{meas} = 10\log_{10}\left(\frac{CW_{meas}}{W_0}\right)$

$$W_{meas} = W(f_1, d) \Rightarrow d = 327m \quad (22)$$

- with $T_{arrive} = \frac{327m}{10.8m/s} = 31s$
- However the measured time was 26 s

Group and phase velocity

$$v_p(\omega) = \frac{\omega}{k} = \sqrt{\frac{EI}{m}(1 + i\eta_E)k^2 + \frac{\omega_0^2}{k^2}(1 + i\eta_s)} \tag{23}$$

$$v_g = \frac{d\omega}{dk} = \frac{2}{k + \frac{\omega_0^2(1+i\eta_s)}{EIm(1+i\eta_E)}} \tag{24}$$

Phase velocity is the speed of sound. For $f = 1249$ Hz we get:

$$v_p(2\pi f) = 1613\mathrm{m/s} \tag{25}$$

Which is a lot higher than the speed of sound in air $c_0 = 343$ m/s

Summary

- The method seems to get a realistic speed and distance
- Difficult to check the roughness of the tracks/wheel
- Speed of sound a lot higher than the speed of sound in air which explain why the sound arrives in the track before the air.
- Hard to actually hear with the human ear.

Issues with the model

- Effect of sleepers/other damping types
- Two rails/ sound wave interference
- Train kinematic (constant speed)
- "Correct" values of constants
- Rail junctions
- And much more

Thanks for listening! / Any questions?

Appedix A: Wave number

Damping: $\tilde{E} = E(1 + i\eta_E)$, $\tilde{s} = s(1 + i\eta_s)$ Assume solution of form

$$u(x) = U \exp i\omega t \exp -ikx \tag{26}$$

Insert and solve

$$\tilde{E}Ik^4 + \tilde{s} - m\omega^2 = 0 \Rightarrow k^2 = \pm\sqrt{\frac{m\omega^2 - s}{EI}} \tag{27}$$

$$\Rightarrow k(\omega) = k_b(\omega)(1 + i\eta_E)^{-1/4}\left(1 - \frac{\omega_0^2}{\omega^2}(1 + i\eta_s)\right)^{1/4} \tag{28}$$

$$k = k_r + ik_i \tag{29}$$

Sources:
Beams on Elastic Foundation
Terje Haukaas