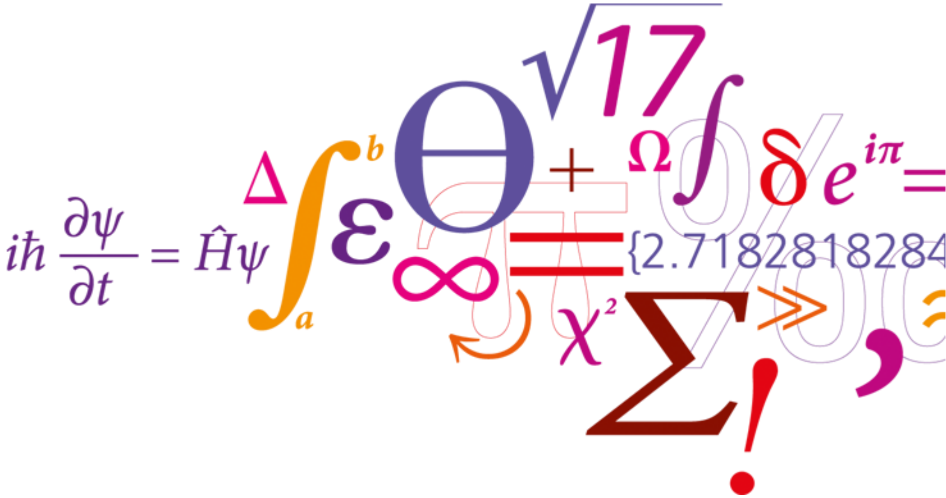
Drunken glass

"Sometimes, when we place a glass upside down on a wet flat table, it starts moving. Investigate its speed dependence on the relevant parameters and try to maximize it."



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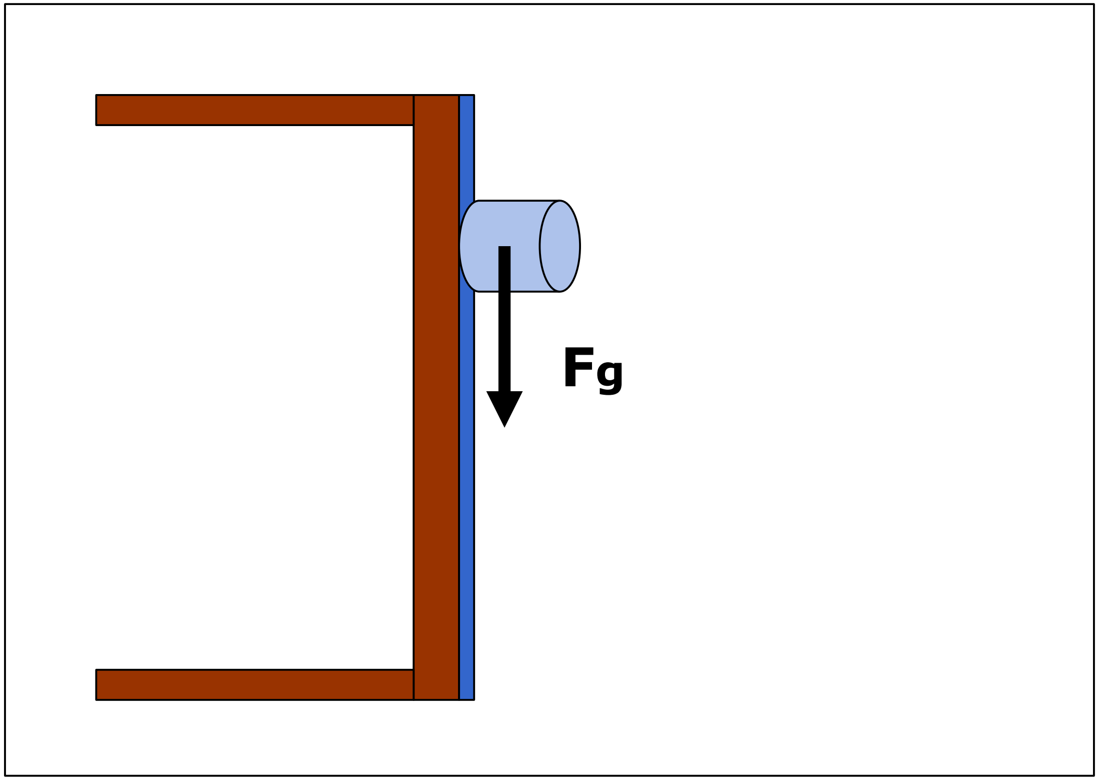
DTU Physics
Department of Physics

The one slide solution

- In the problem text there is nothing indicating the **table could not be tilted:**

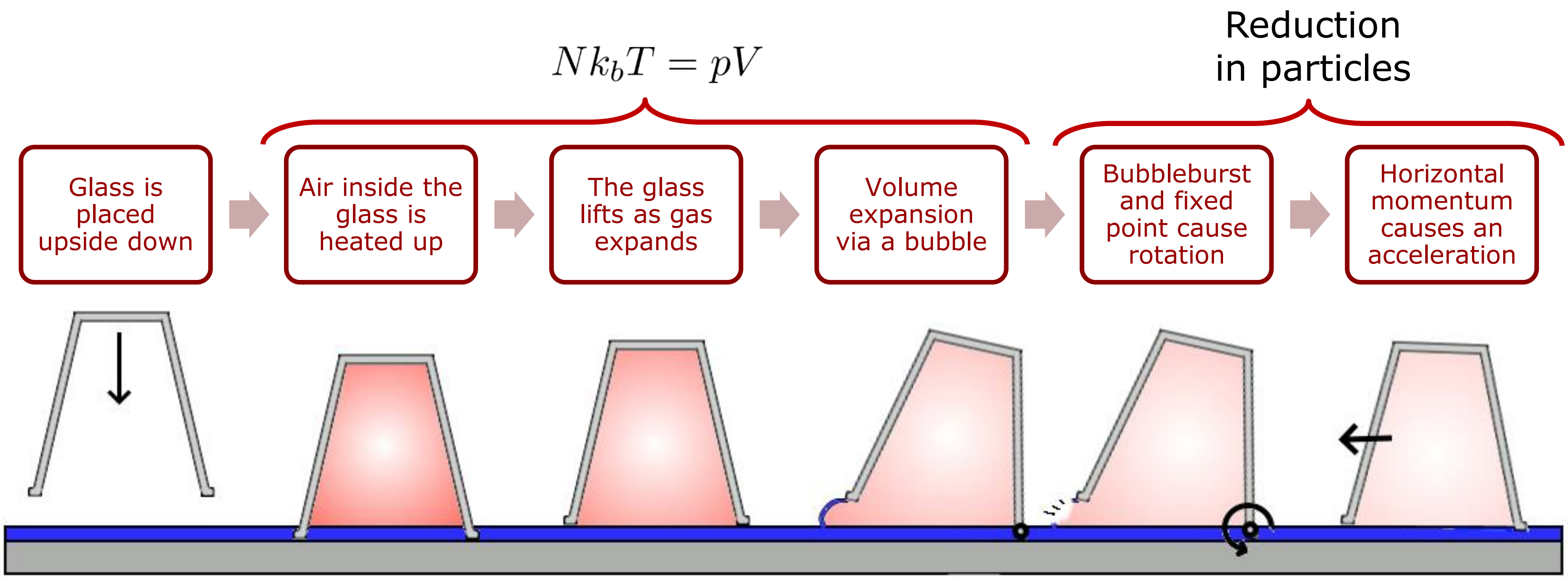


Vacuum camper

Maximize the speed by ma making the **table longer**…

The actually interesting case

- A **semi closed system** forces the gas to **expand into a bubble**.
- High **friction** in case of a contact point with the hard surface
  - Creates a fixed **pivot point** of rotation
- The cup moves in **steps** when the bubble collapses as momentum is conserved.



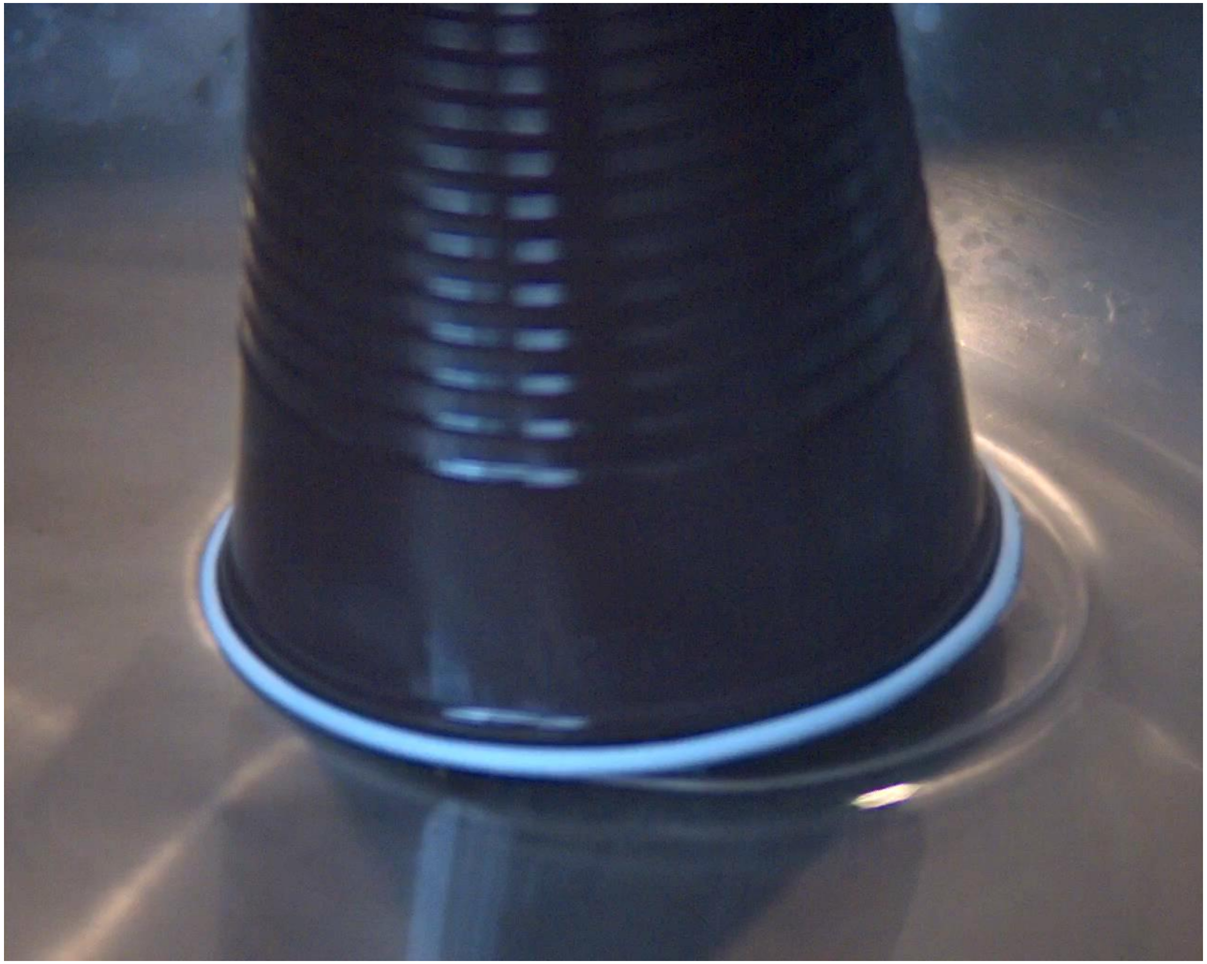
Bursting the bubble

Slow-motion:

=> **18 times slower**

<u>Conclusion:</u>

Each bubble causes momentarily movement due to conservation of **momentum**



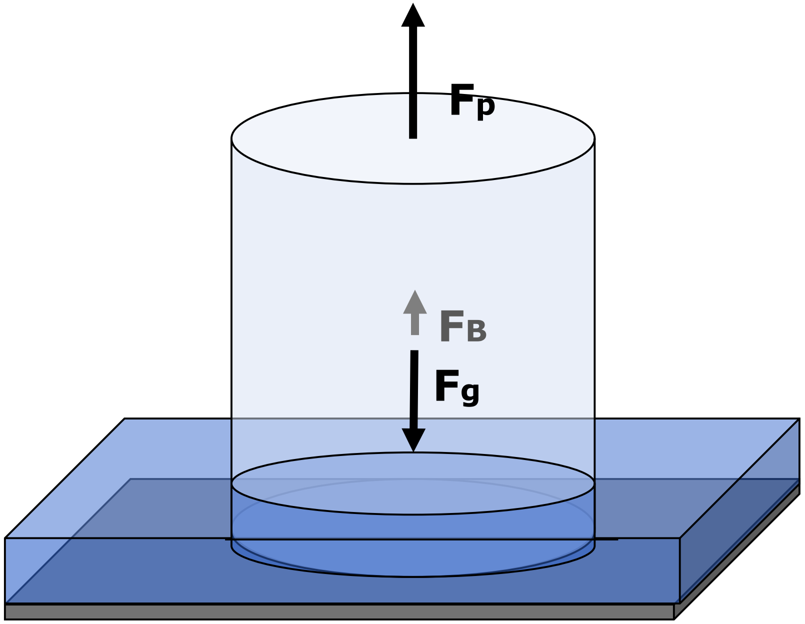
Theory

Force balance to make the glass float

- Pressure: $\mathbf{F_p}$
- Gravity: $\mathbf{F_g}$
- (Buoyancy: $\mathbf{F_B}$)

$$0 = -F_g + F_p$$

Nearly no friction due to **Lubrication**



Lubrication

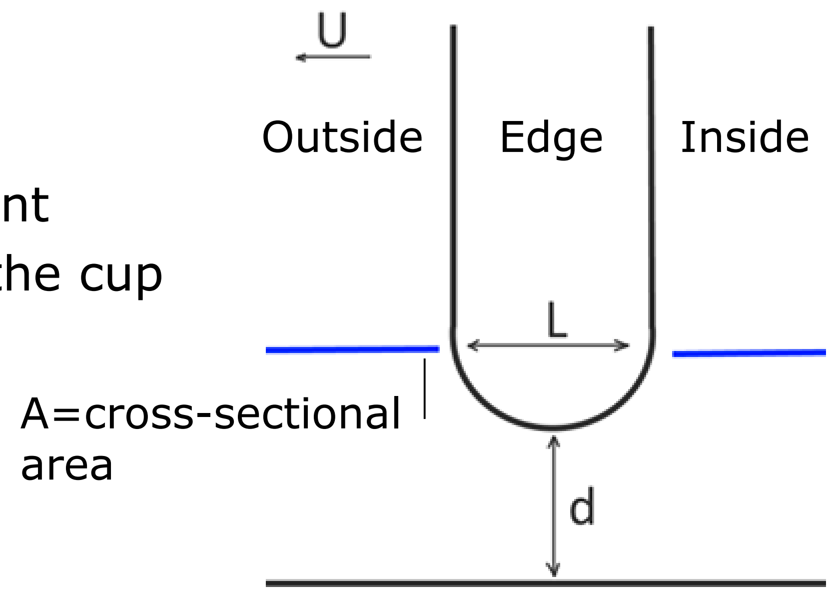
If the cup if **floating**

- **Lift:** Lifted from the water due to the **water flowing under the cup**:

$$Lift \sim c_{lift} \frac{1}{d^2} \eta U L A$$

- **Drag: Displacing the water** in front of the cup creates a drag, slowing the cup down.

$$Drag \sim c_{drag} \frac{1}{d} \eta U A$$



Source:
Physics of Continuous Matter
B.Lautrup, 2.edition

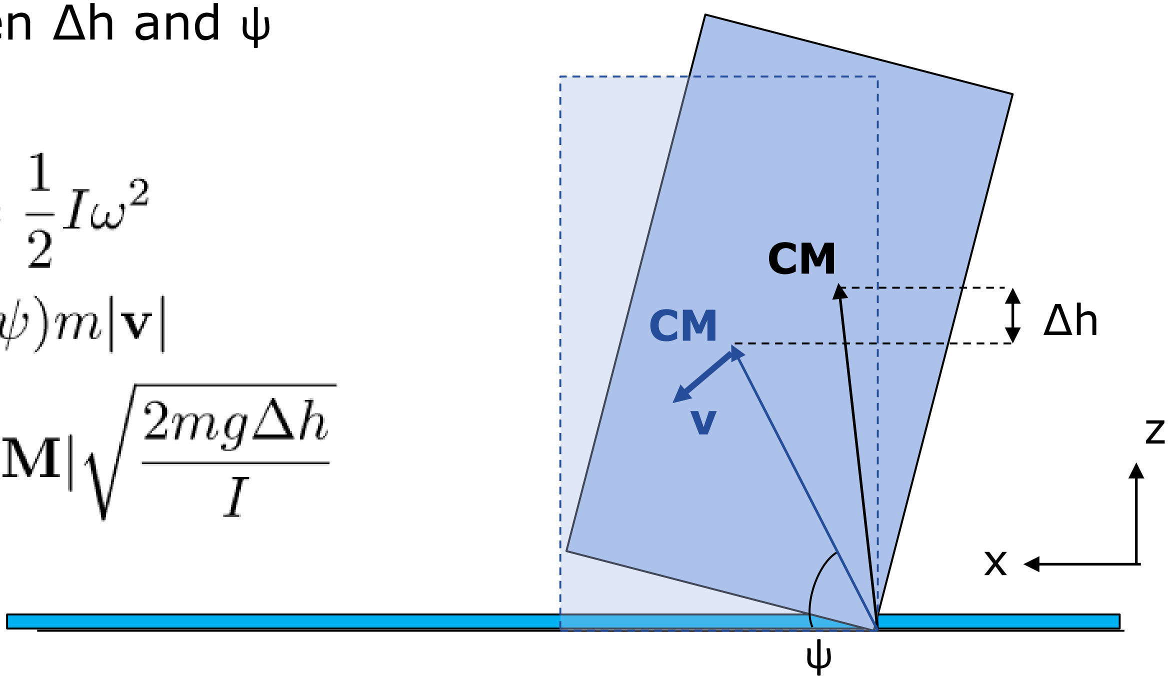
How can the glass start to move?

- Conservation of **energy**
- Conservation of **momentum**
- Maximize vertical momentum component:
  - Tradeoff between Δh and ψ

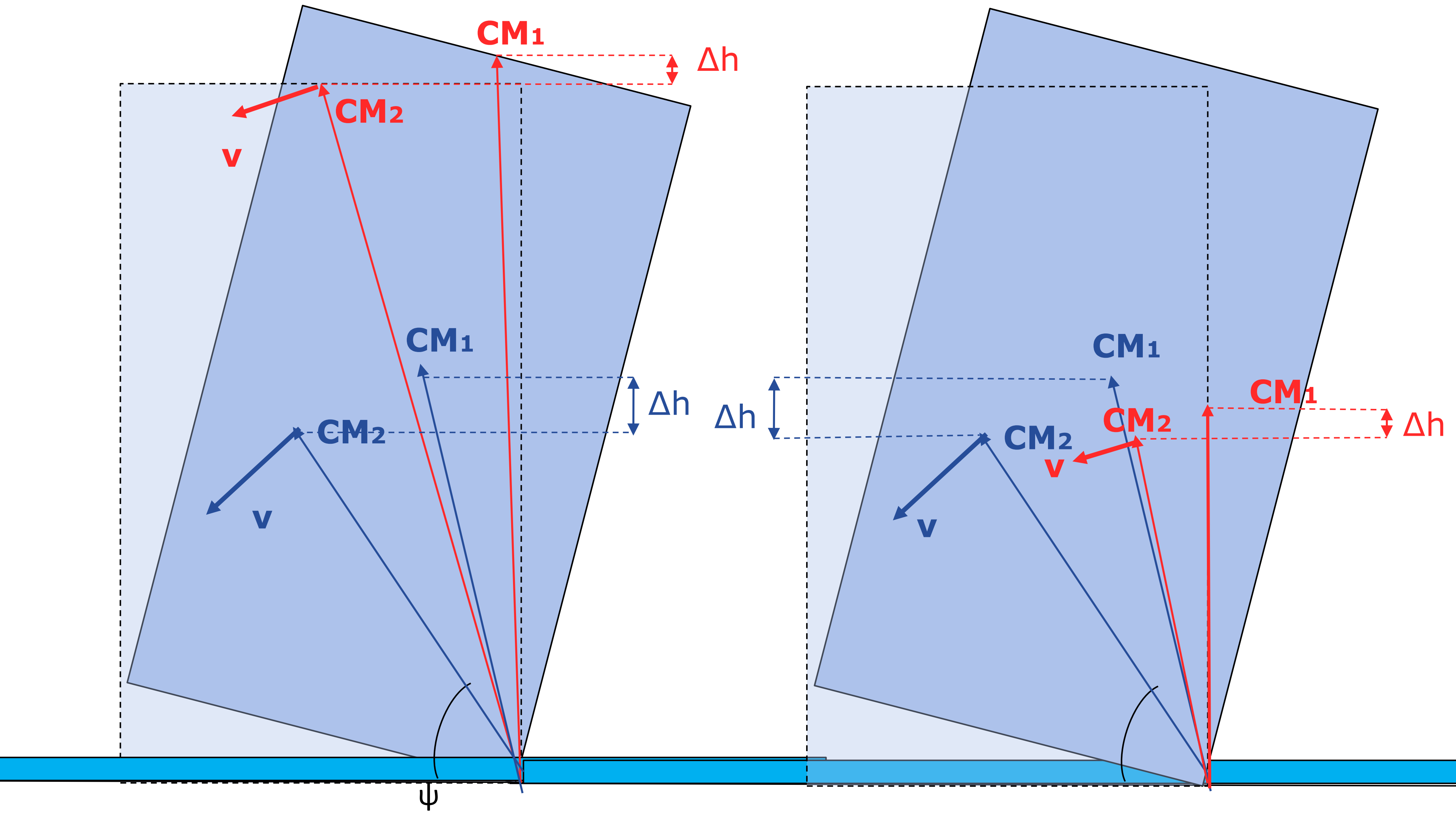
$$mg\Delta h = \frac{1}{2} I\omega^2$$

$$p_x = \sin(\psi) m|\mathbf{v}|$$

$$p_x = \sin(\psi) m|\mathbf{CM}|\sqrt{\frac{2mg\Delta h}{I}}$$

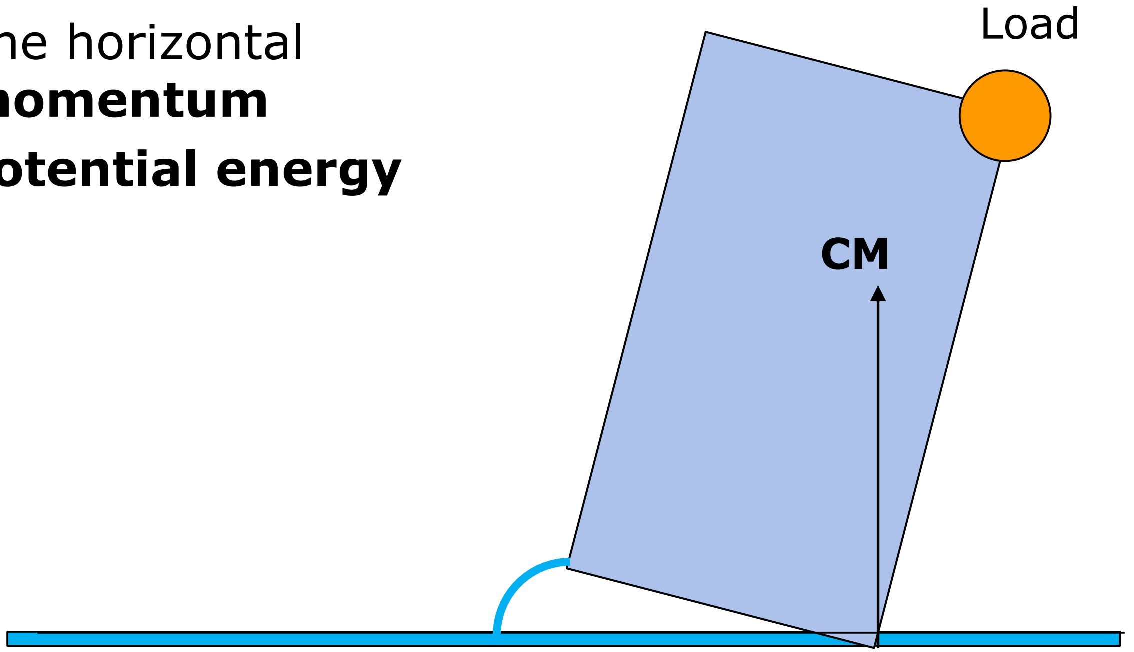


How can the glass start to move?



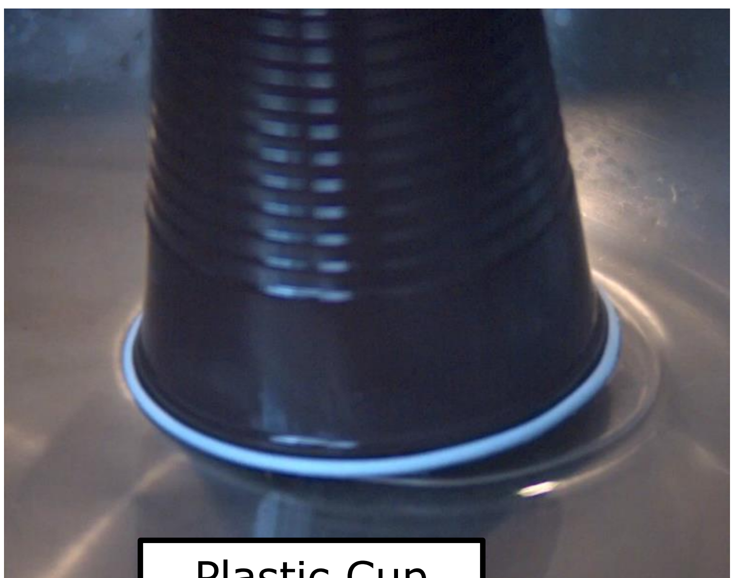
Controlling the bubble.

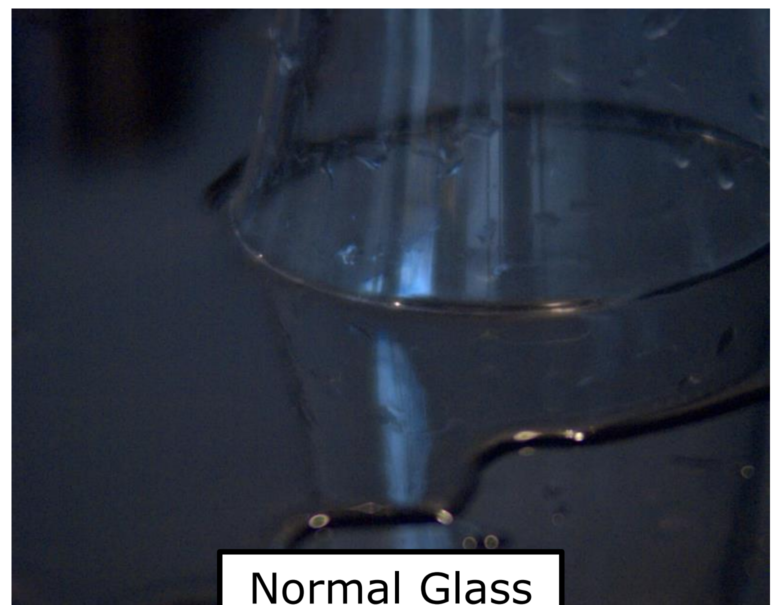
- **Bubble radius** increase Δh
- **Displacement** of the center of mass.
  - Asymmetry **eases bubble** formation
  - Can optimizes the horizontal component of **momentum**
  - Increases the **potential energy**



Advantages of light plastic cups

- **floats** regardless of temperatures
  - Minimum of **friction** from the bottom
- The **center of mass** is manipulated easily
- Light cups generates **larger bubbles**
  - Maximizes the **speed**
  - Better **measurements**


Plastic Cup


Normal Glass

Experimental setup

- plastic cups
- Constant heat from **induction stove**
- Tracking a colored **strip** via python script
- Uncertainties
  - Non uniform heat $\pm 5^{o}C$
  - Parallax errors
  - Evaporation of water

Scale

Camera

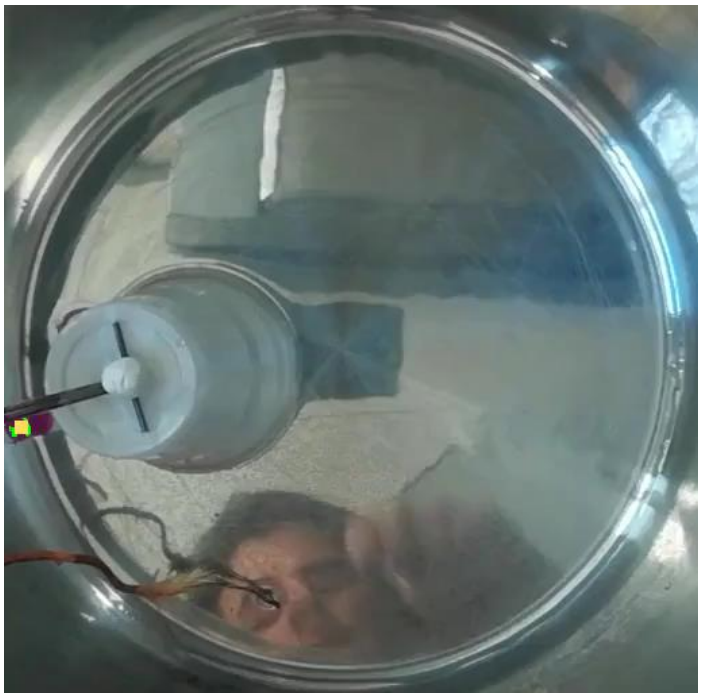
Cups

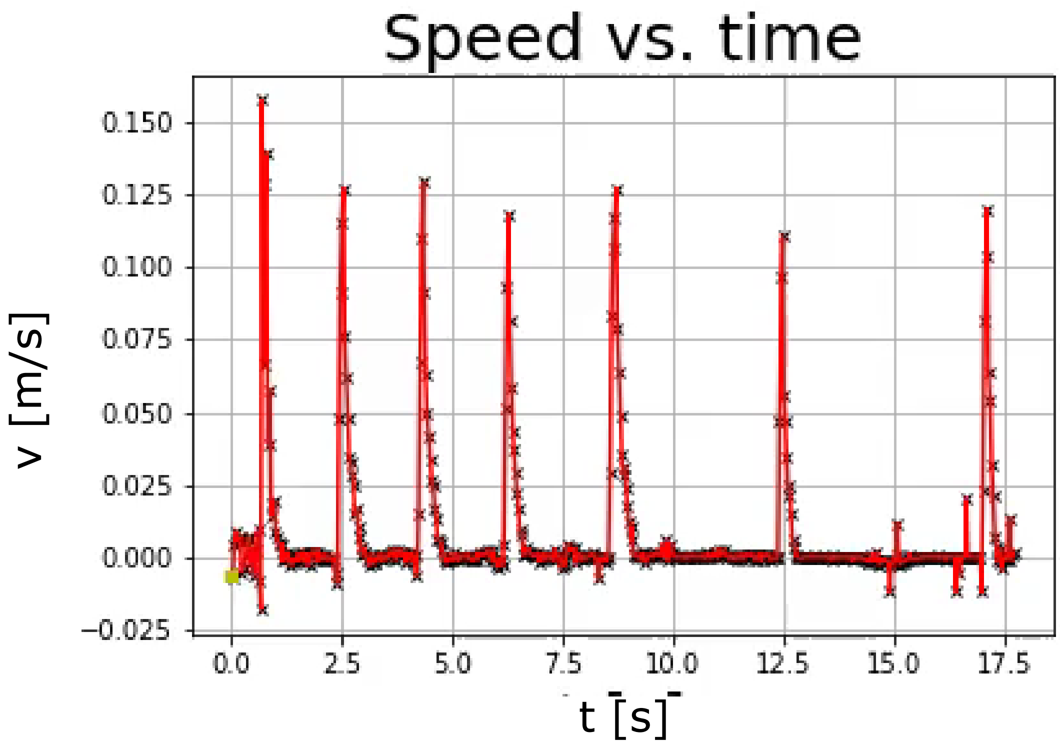
Thermometer

Pot with water

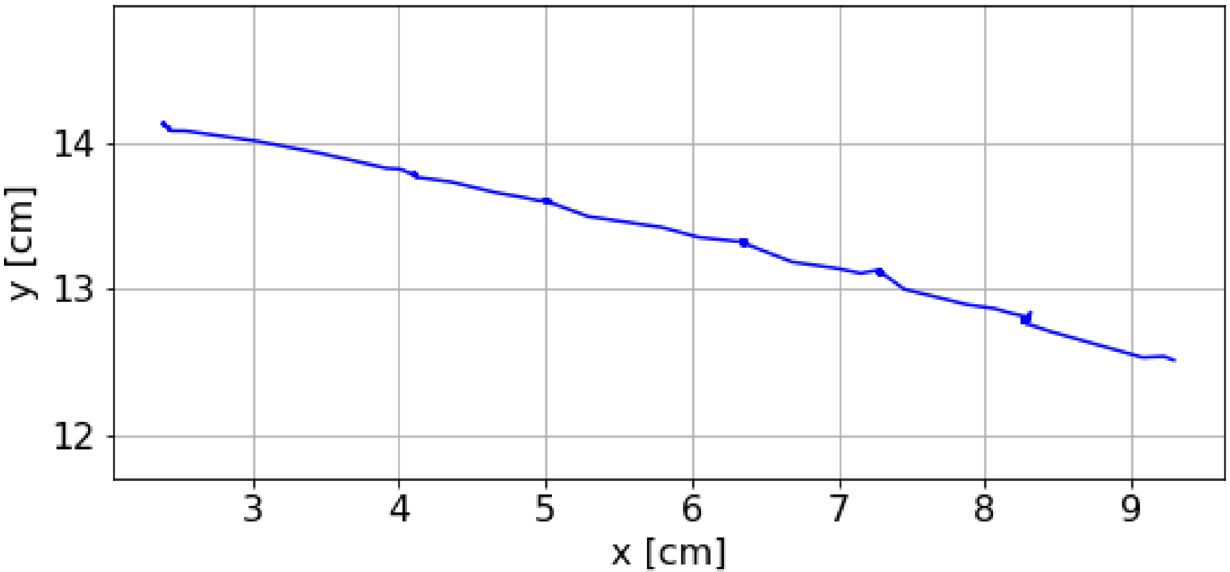
Spirit level

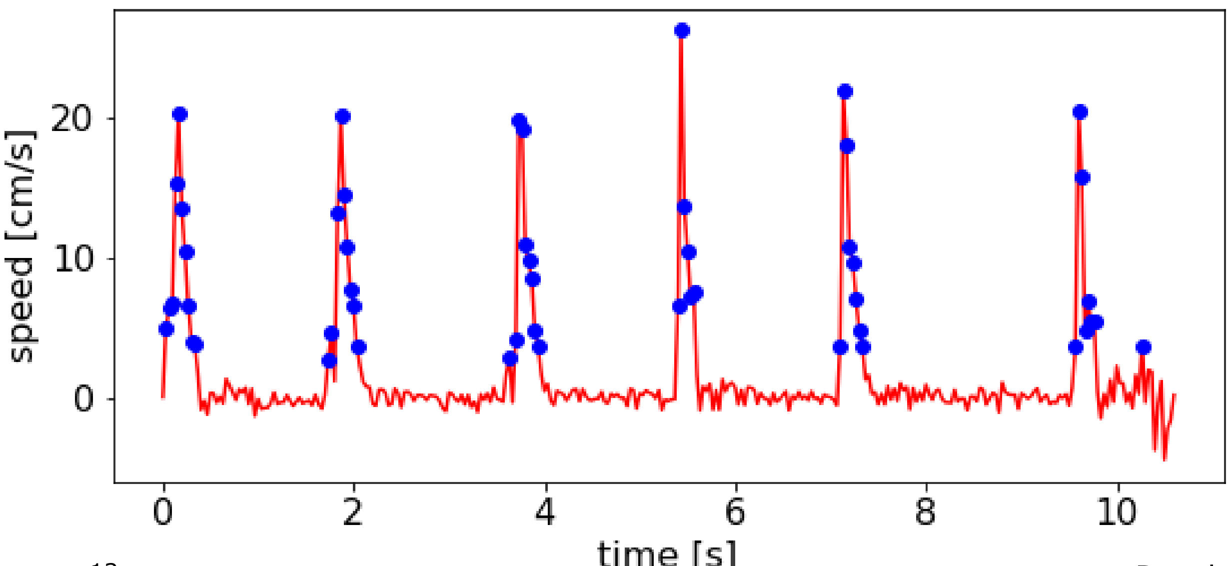
Tracking strip





Data analysis



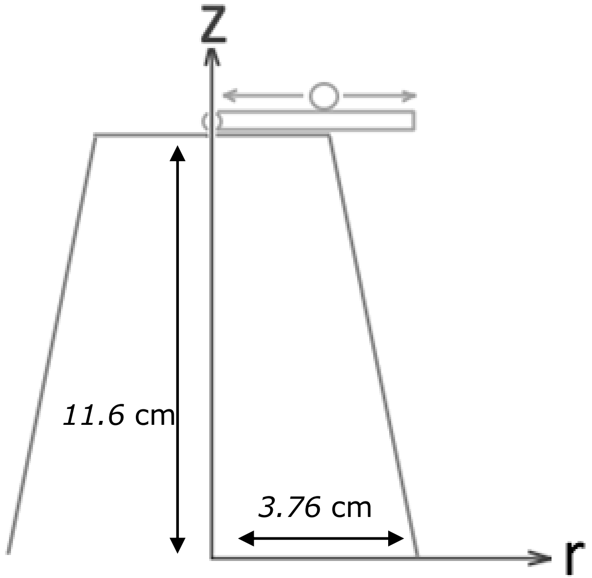


Peak speed: Top blue point in each peak

Floating time: Time in which each peak exists over a given threshold. (~0.1 max peak speed)

Alteration of Center of mass along r

Displacement along r	Displacement in Center of Mass along r
-0.50 cm	0.42 cm
0 cm	0.51 cm
1.5 cm	0.79 cm
3.0 cm	1.07 cm
Displacement in z: 8.80 cm	



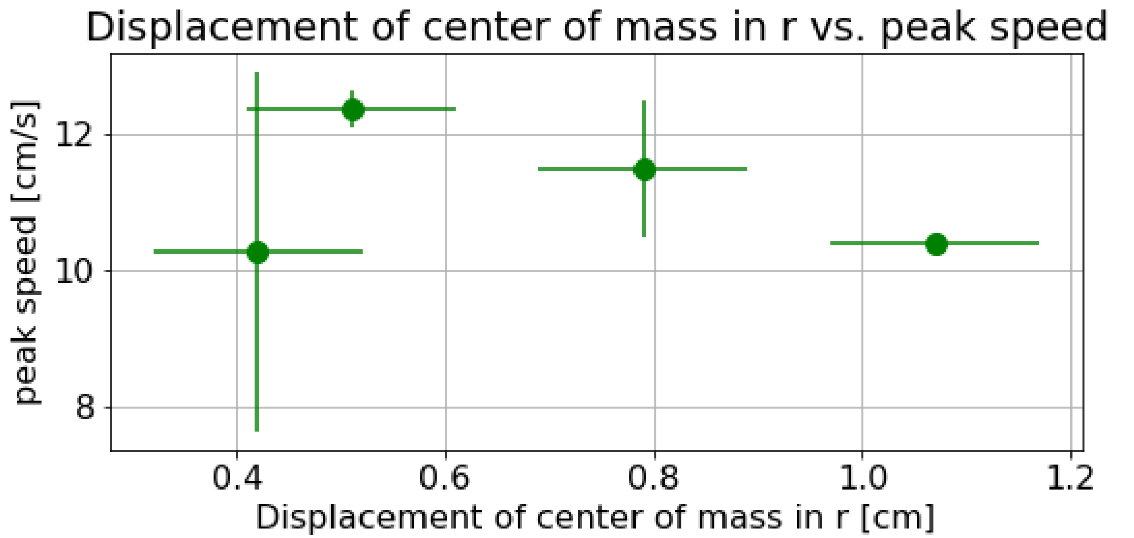
Alteration of Center of mass along r

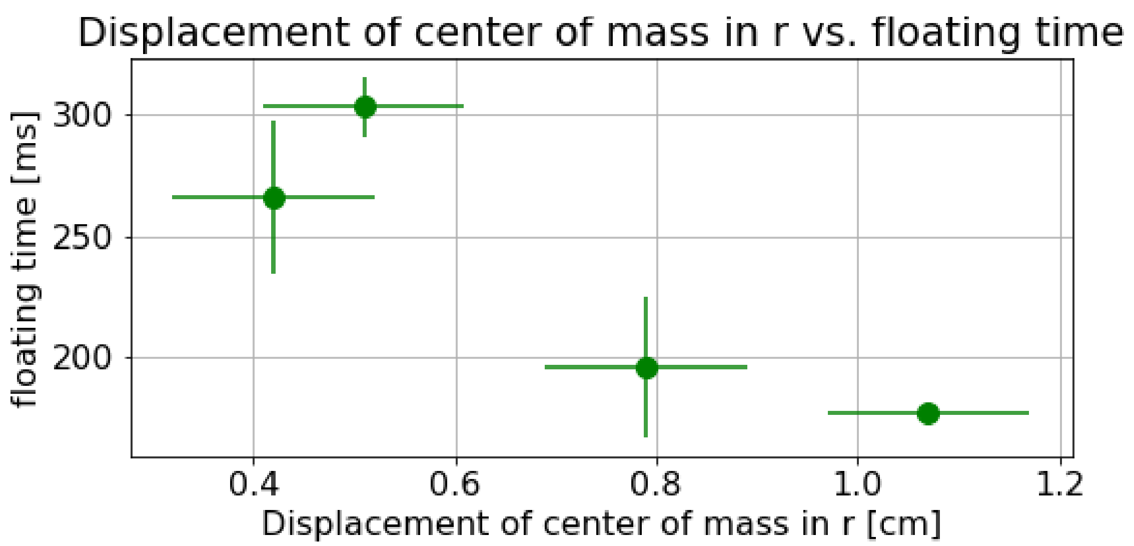
Error bar in y: actual normal distribution

Initial increase: displacement eases **bubble formation** => larger angle. (lifetime)

Later decrease: **Δh** becomes too small to be optimal with $\mathbf{v_x}$

Peak speed and floating time decreases with the displacement due to **friction** with the table





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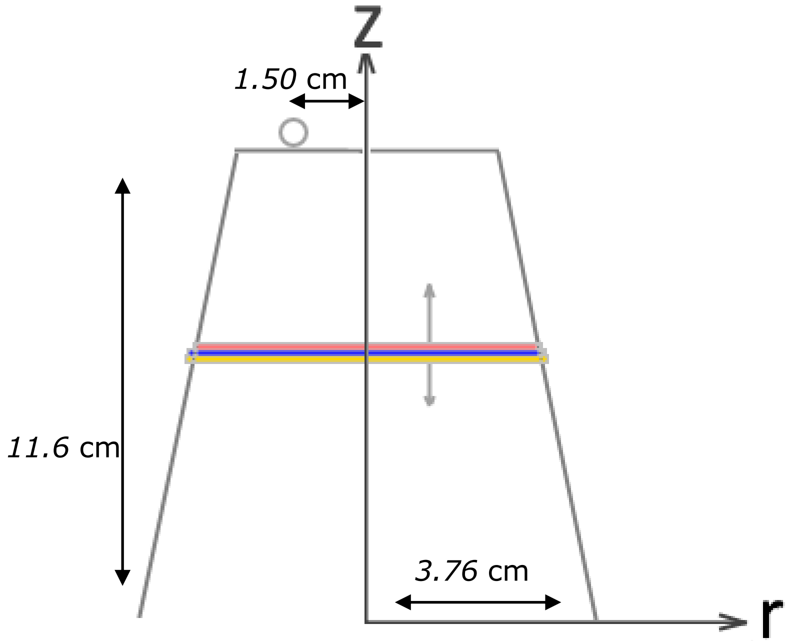
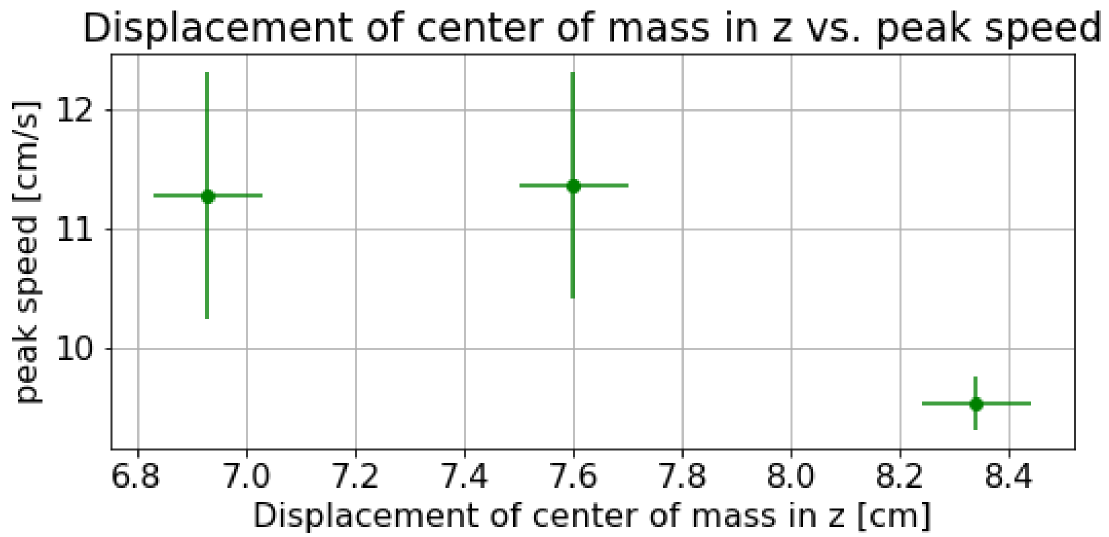
Alteration of Center of mass along z

- Vertical displacement
  - With rubber bands
  - With load for direction control

Displacement along z	Displacement in Center of Mass along z
4.9 cm	6.93 cm
7.8 cm	7.60 cm
11.0 cm	8.34 cm
Displacement in r: 0.31 cm	



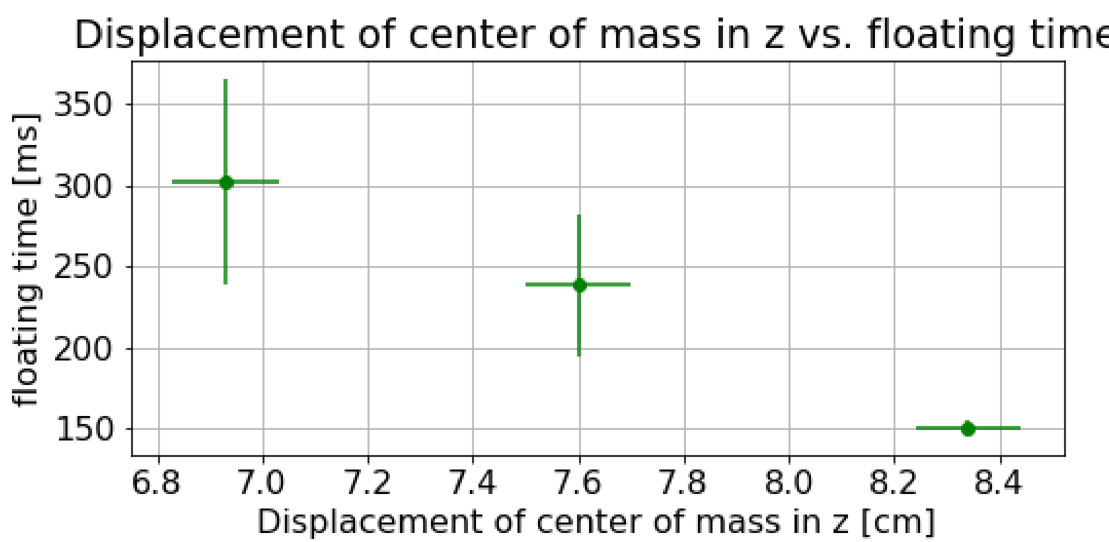
Alteration of Center of mass: along z



High center of mass: more pressure on the pivot point for same angle resulting in more **friction**

Low center of mass results in larger **Δh** but smaller $\mathbf{v_x}$

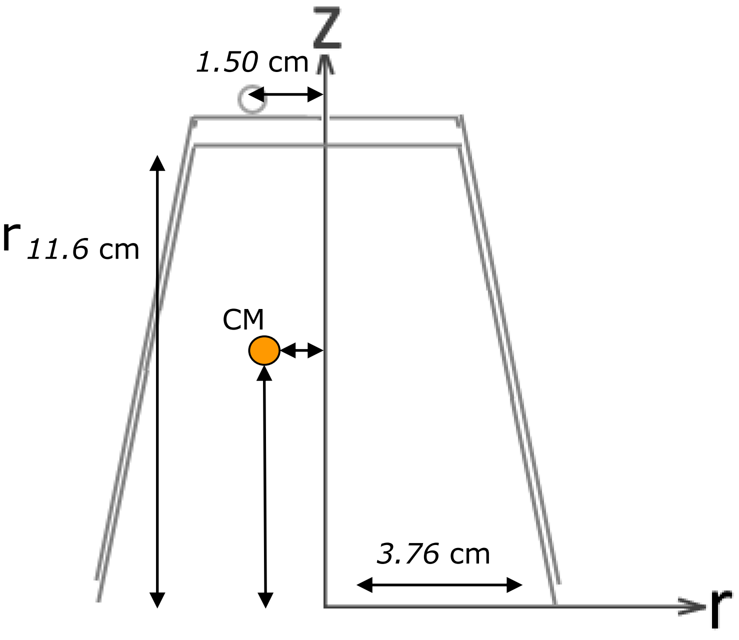
The easier bubble formation does not seem to increase the speed significantly



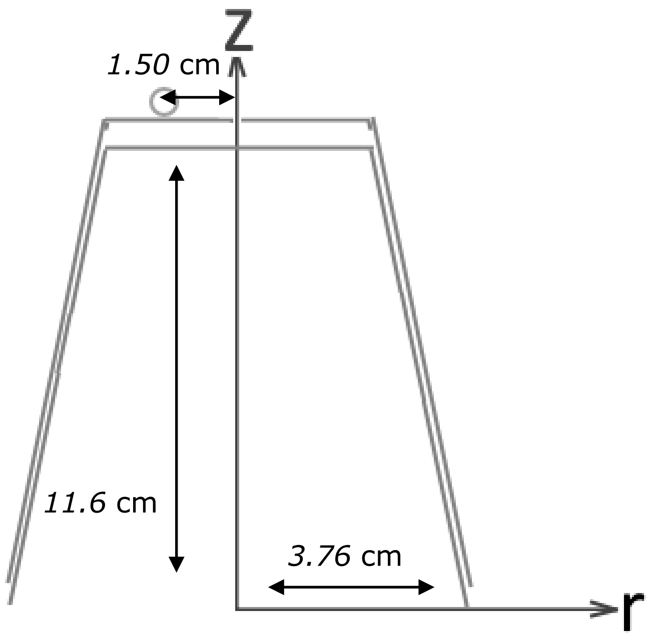
Alteration of mass

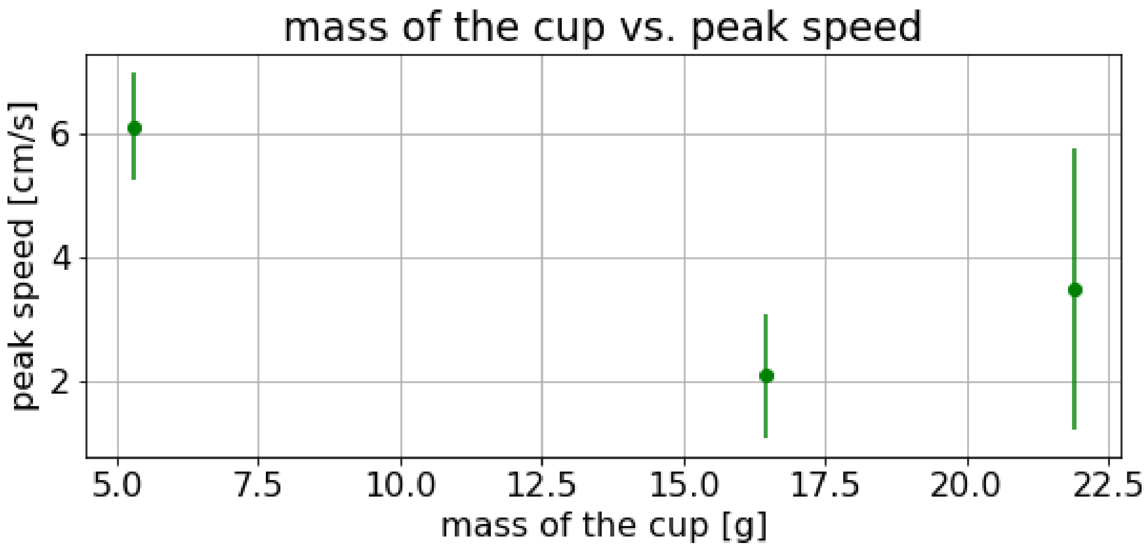
- Stacking several cups for increased mass
  - The little alteration in center of mass Is ignored
- Load
  - Load is added for direction control
  - Load mass is varied to make center of mass constant.

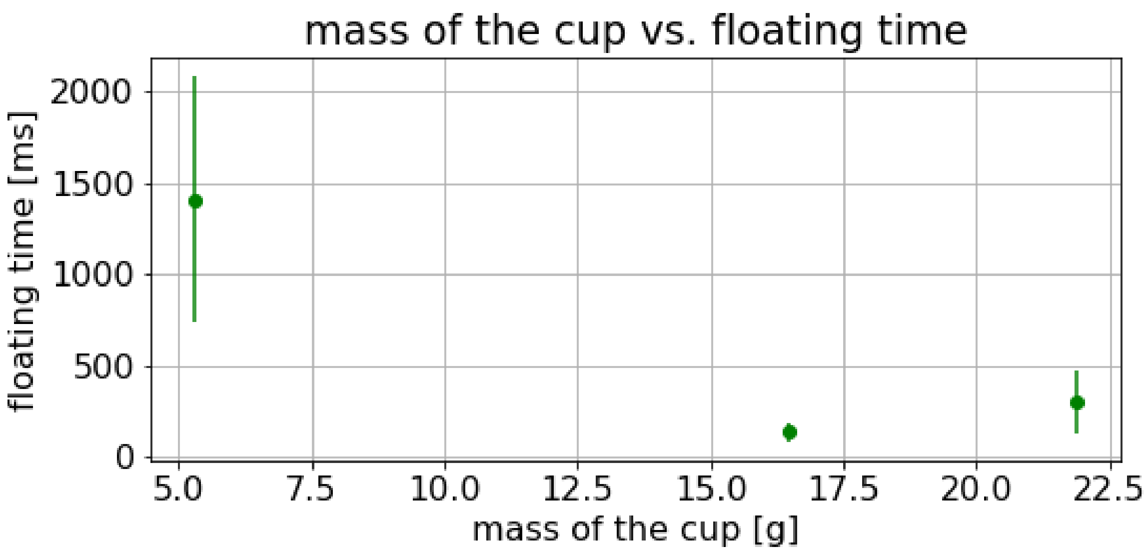
Number of cups	Mass of the system
1	5.3 g
3	16.45 g
4	21.88 g



Alteration of mass







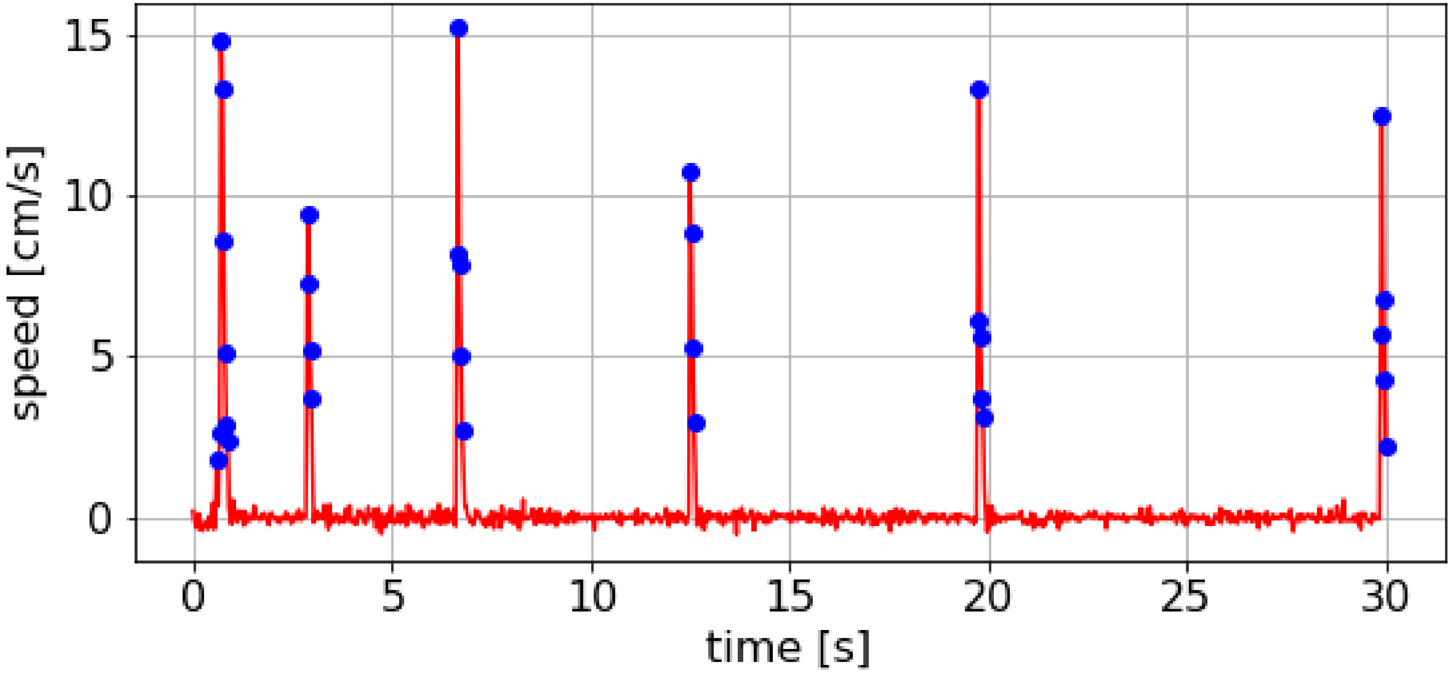
A larger mass presses the cup **deeper into the water** while floating

More **pressure on the pivoting point**

Optimizing the temperature
Peak speed

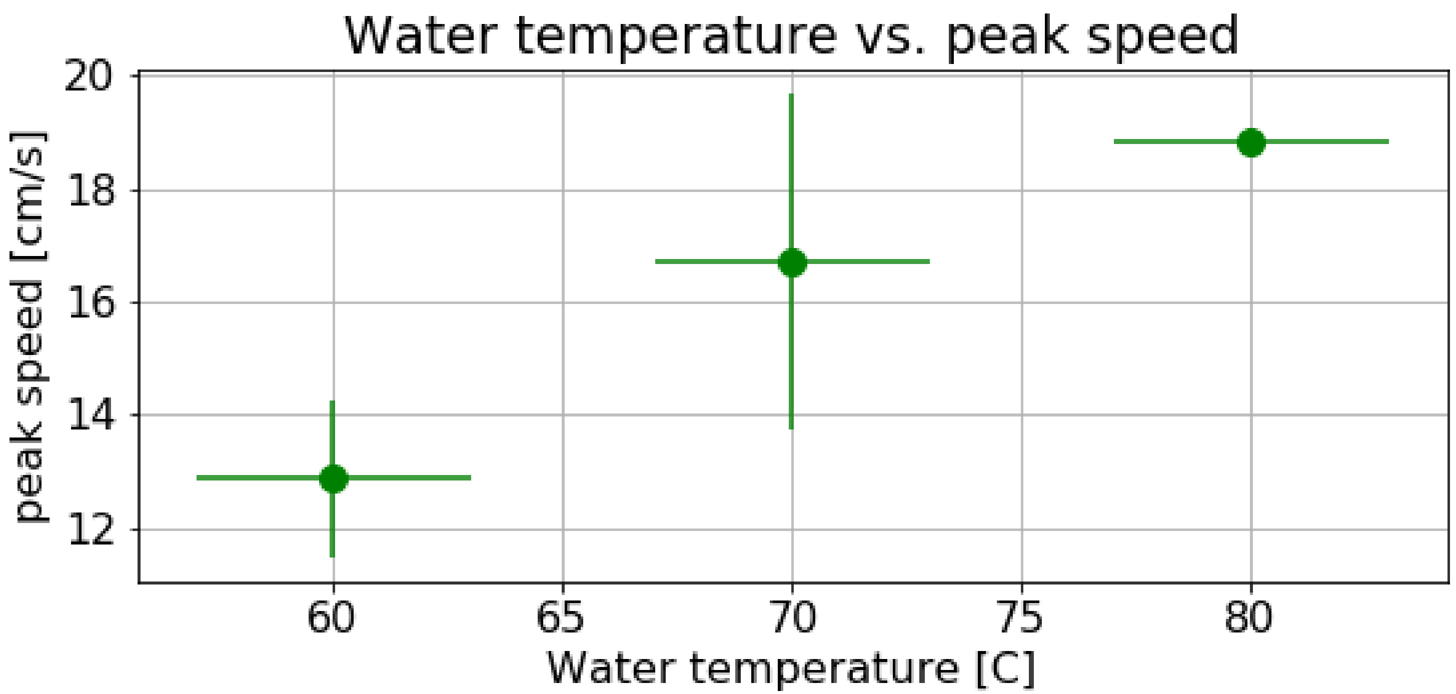
- Increases the initial **frequency of bubbles**
- Surface tension varies with temperature a lot

How much?



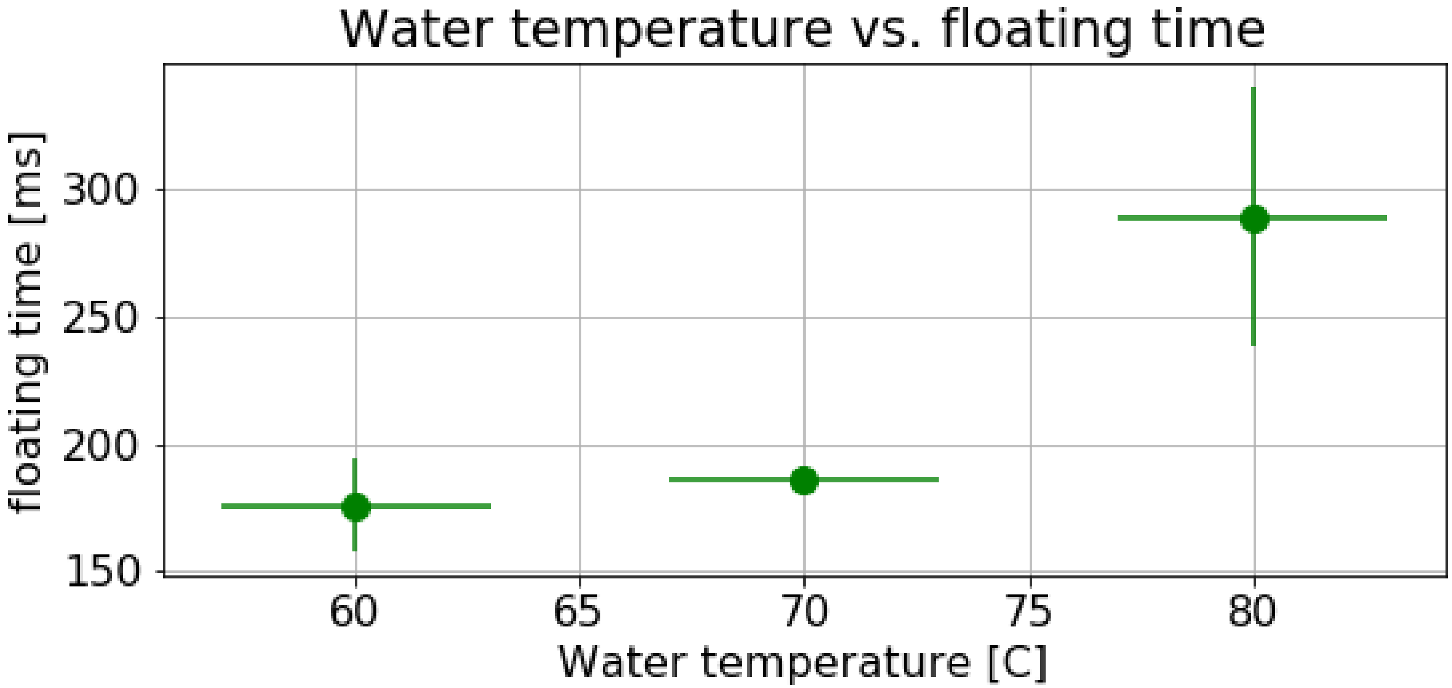
Optimizing the temperature peak speed

- Increases the frequency of bubbles.
- Allow for **larger bubbles** due to life time of bubbles are the same



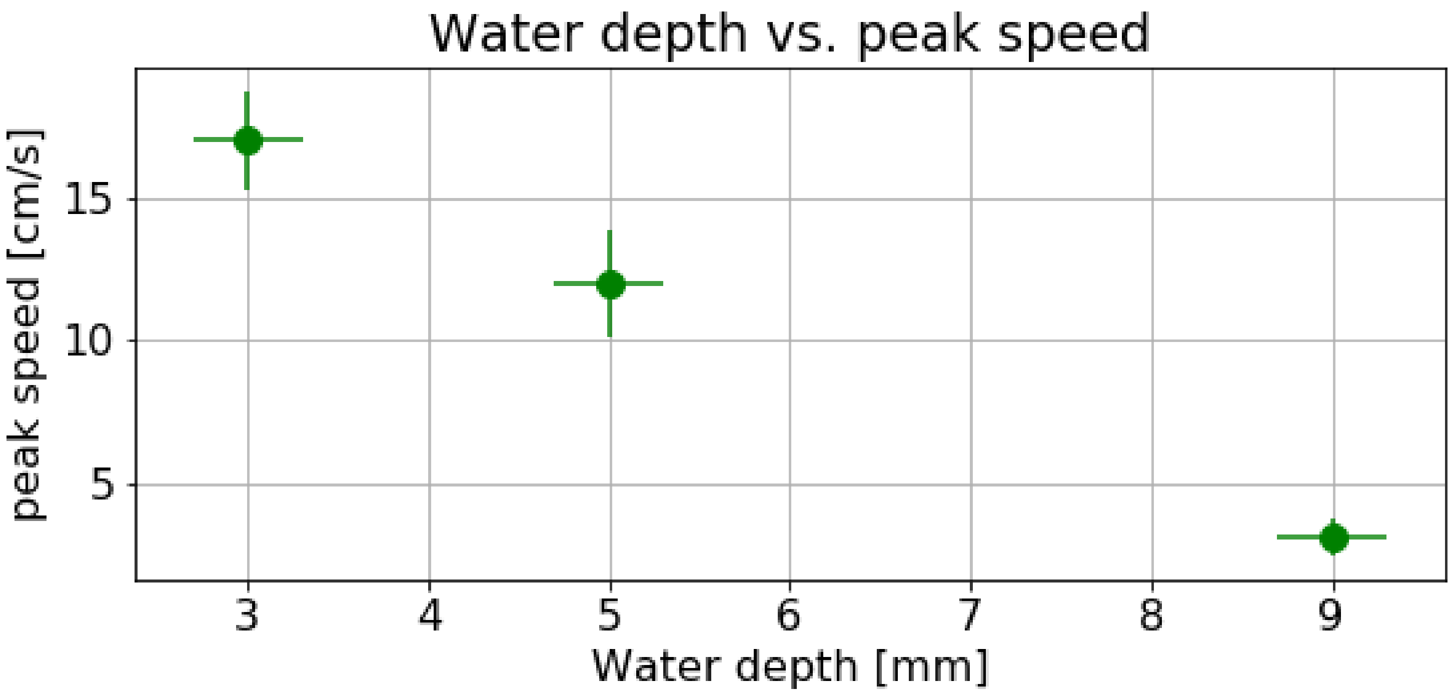
Optimizing the temperature floating time

- Increases the frequency of bubbles.
- Allow for larger bubbles due to life time of bubbles are the same
- **Larger peak speed** gives a slightly larger floating time and **lift** is increased



Thickness of water layer
Peak speed

- Displacement in mass causes the **pivot point to rest** on the bottom =>**Larger drag**
- A larger depth makes it **harder generate bubbles**.



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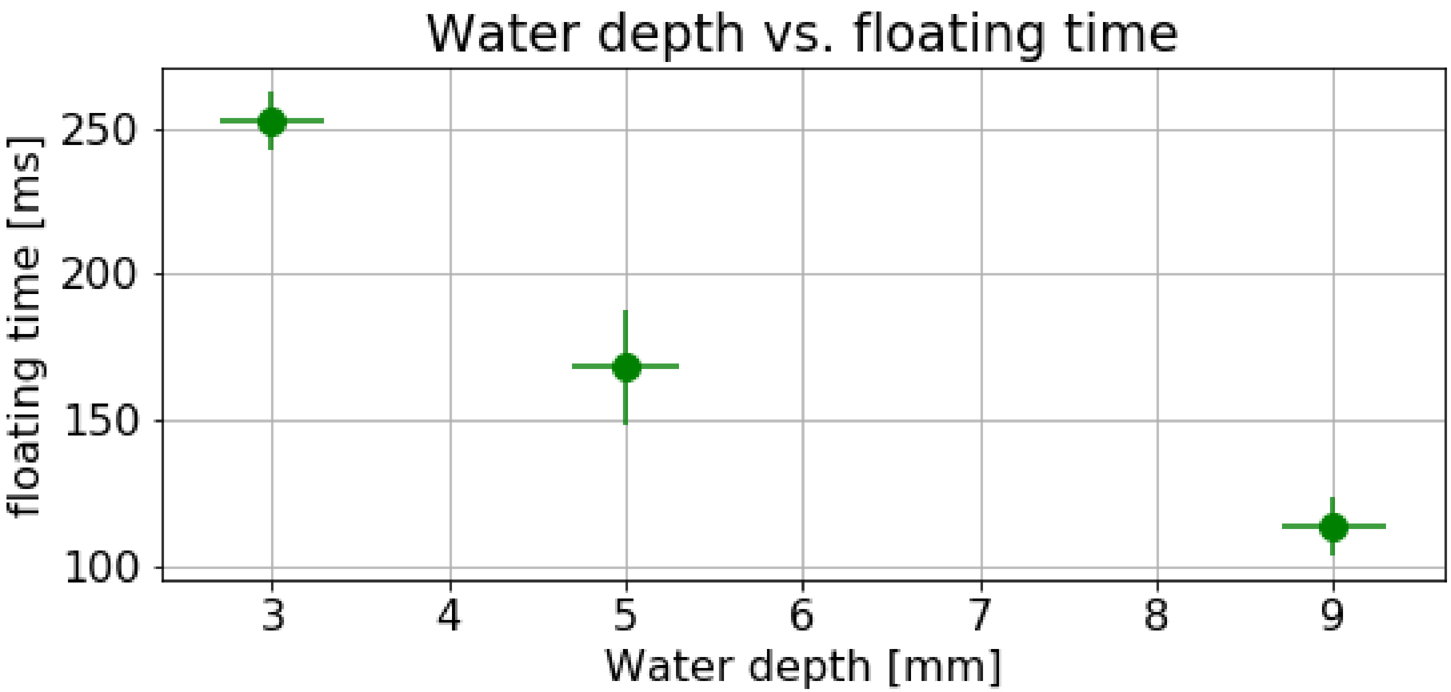


Conclusion

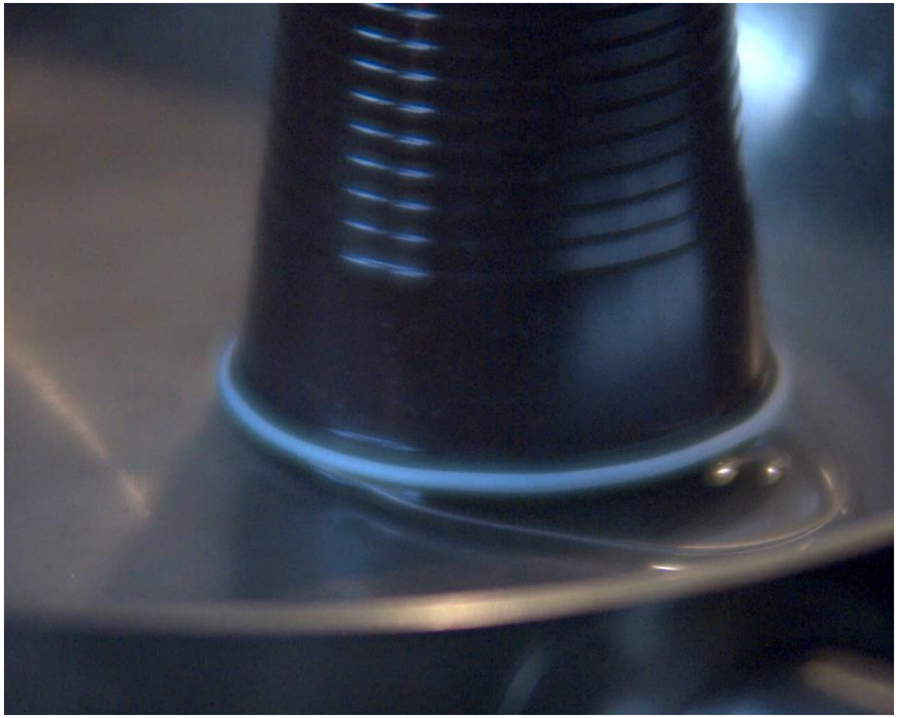
- Optimizes relevant parameters
  - **Center of mass:** shifted slightly in r to control bubbles and maximize Δh and v_x => tradeoff
  - **Mass of the glass:** as light as possible
  - **Temperature:** The higher the better
  - **Water depth:** Shallow but sufficient for the cup to float

  - **Theoretically:** Width of the cup edge

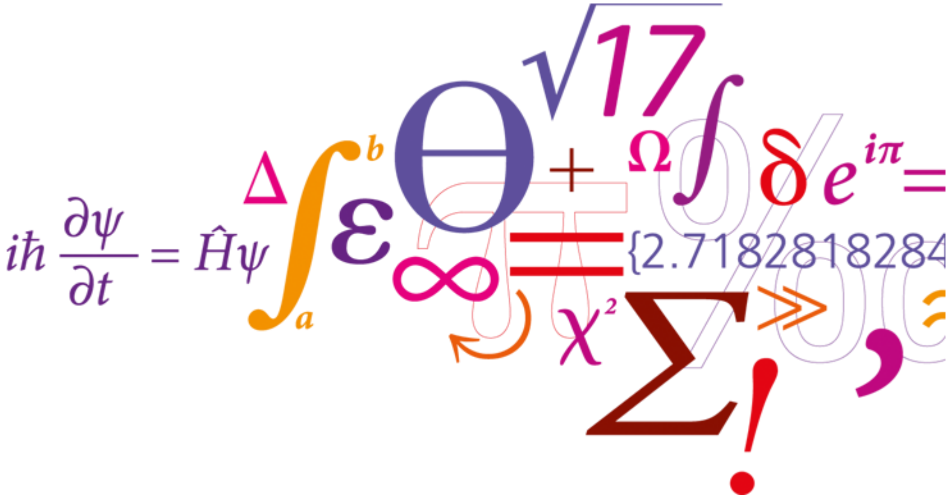
- Maximum speed obtained was 19 cm/s by having T=80° C

Bubble that do not burst

- Still causes an acceleration



Thanks for your attention



The tracker strip

- The tracker strip decreases the floating time of the cup

Size of the bubbles

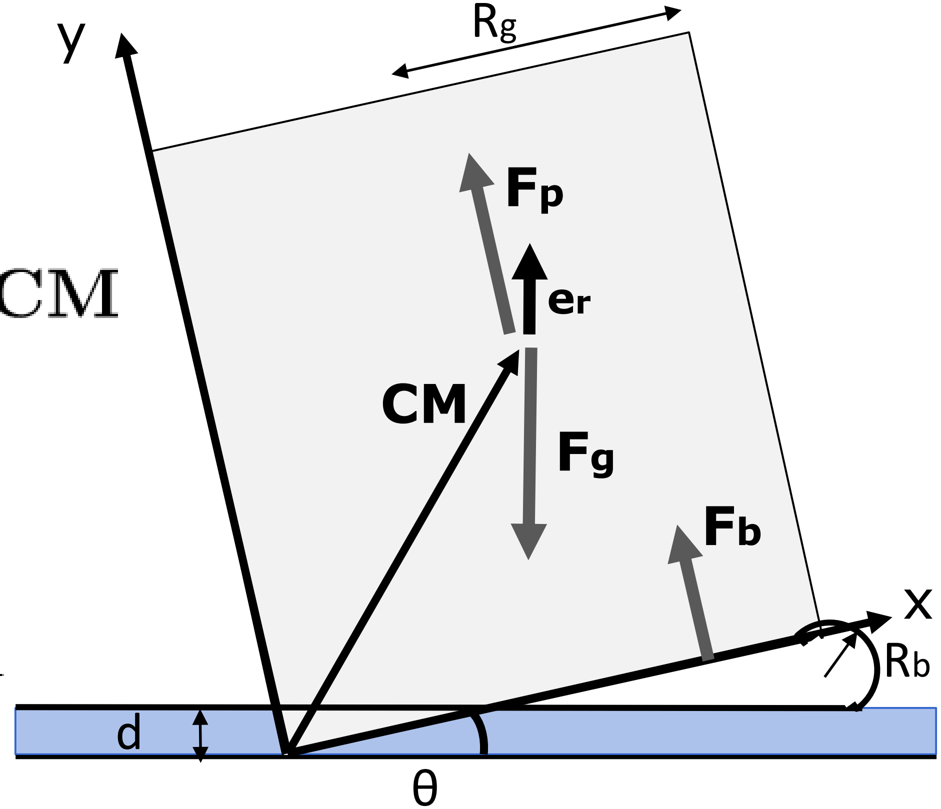
Balancing torque

$$\tau_p - \tau_g + \tau_b = 0$$

$$\tau_p = \underbrace{\Delta p}_{\alpha\left(\frac{1}{R_b}+\frac{1}{R_g}\right)} \pi R_g^2 \mathbf{e_r} \times \mathbf{CM}$$

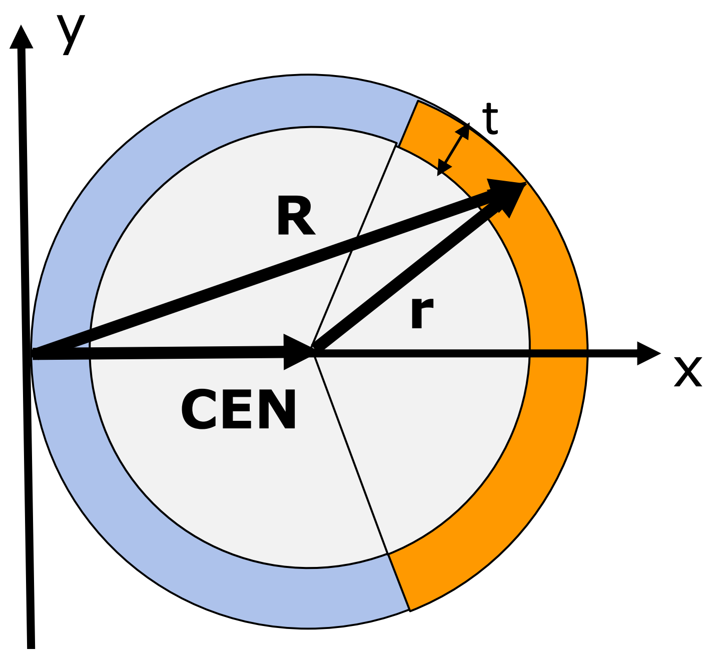
$$\tau_g = mg\mathbf{e_r} \times \mathbf{CM}$$

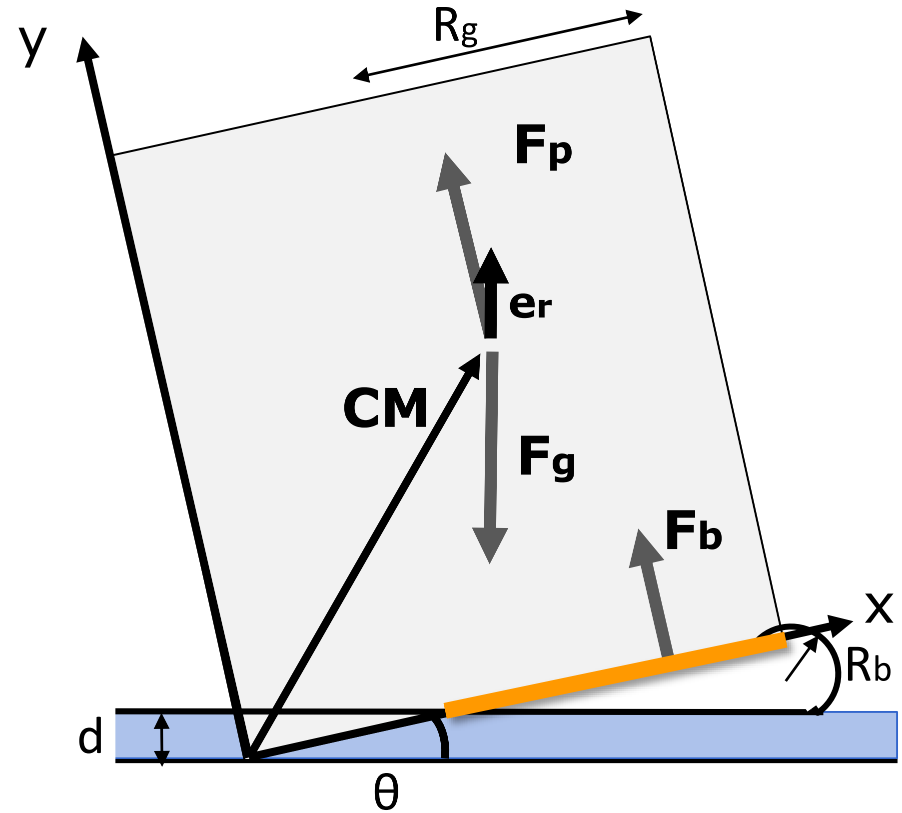
$$\alpha = \text{surface tension}$$



Size of the bubbles

Bubble torque





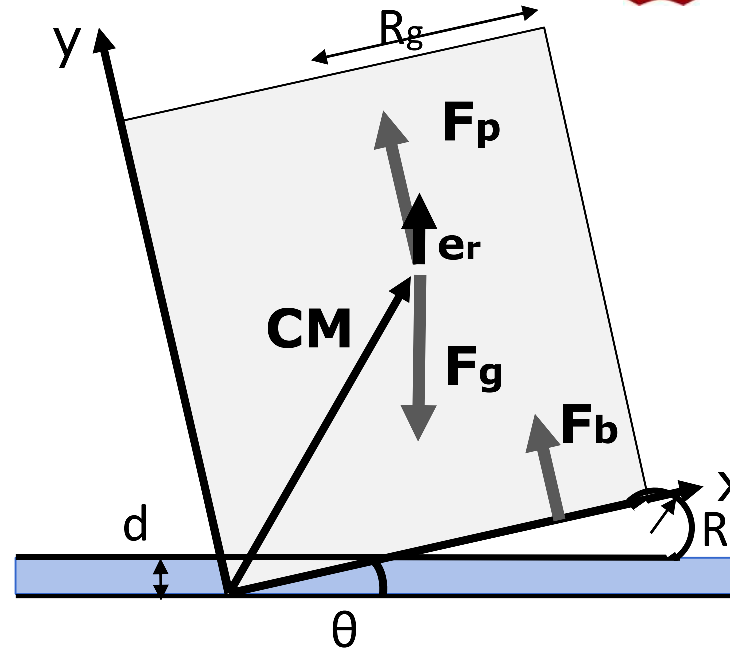
$$\tau_b = \int \Delta p \, \mathbf{R} \times d\mathbf{A}$$

$$= \Delta p \int (\mathbf{CEN} + \mathbf{r}) \times d\mathbf{A}$$

Size of the bubbles

Maximal angle

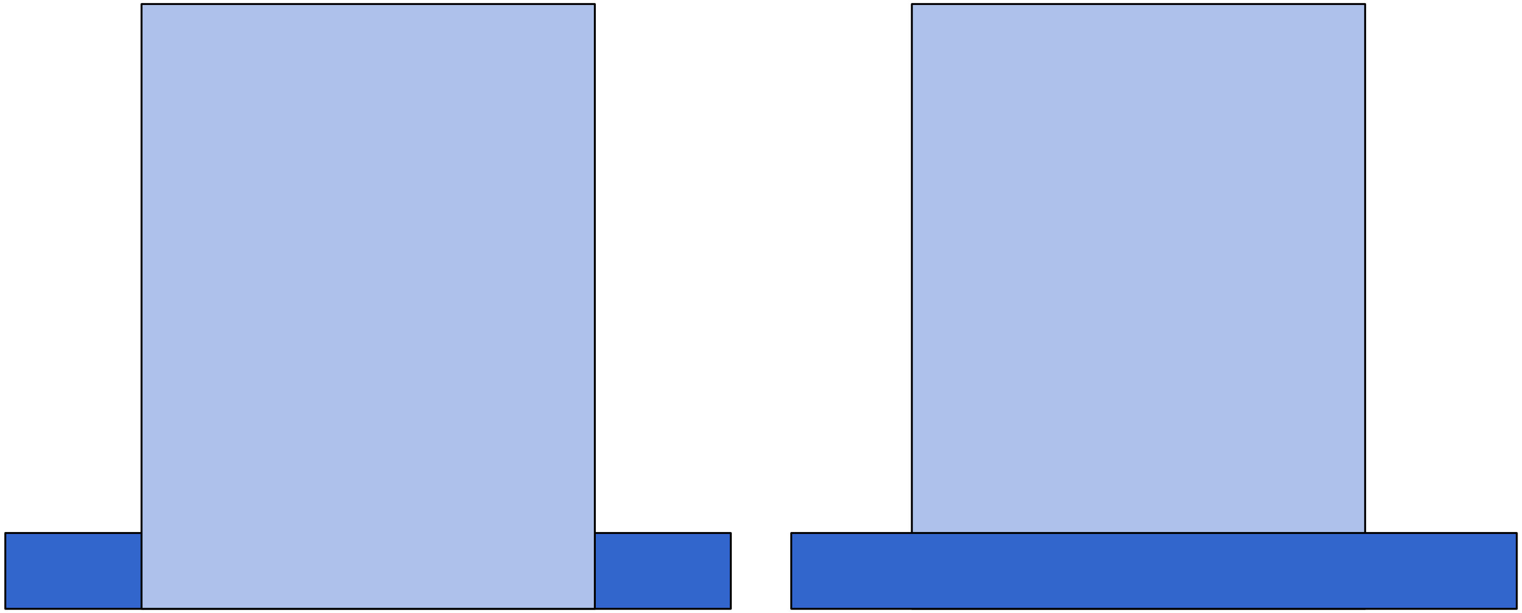
After some math and geometry



$$0 = \pi R_g^2 \Delta p \mathbf{CM} \times \mathbf{e_v} - mg\mathbf{MC} \times \mathbf{e_v} - \Delta p R_g^2 t(\phi - 2\sin(\phi/2)$$

$$\text{where:} \quad \phi = 2\cos^{-1}\left(\frac{1}{R_g}\left[Rg - \frac{2}{\theta}\left(\sin(\theta)R_g - \frac{d}{2}\right)\right]\right)$$

Flat surface material



Hydrophobic surface like Teflon

Hydrophilic surface like steel

Long floating time