

# The Euler Product Formula derived from the Sum of the Power of Primes

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**Abstract:** Relate the sum of powers of multiple primes to the sum of powers of natural numbers.

**Key words:** Euler product formula, Riemann conjecture.

If I hadn't calculated the sum of the products of different prime powers, I'm afraid I would never have anything to do with Euler, because knowing how magic works, it would be very simple.

$$\forall m, n, k, d, n_k \in \mathbb{N}, \forall p, p_k \in \text{prime numbers}, \forall p^n = \frac{p^{n+1}-p}{1-p}, \forall \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \frac{p_k^{n+1}-p_k}{1-p_k}$$

$$\text{Divisor of } \forall p^n, \text{ Such as, } p^0, p^1, p^2, p^3, p^4, \dots, p^n, \text{ Sum of divisors of } \forall p^n, S_n = \sum_{m=0}^n p^m = \frac{p^{n+1}-p^0}{p^1-1}$$

The reciprocal of the divisor of  $\forall p^n$ , Such as,  $p^0, p^{-1}, p^{-2}, p^{-3}, p^{-4}, \dots, p^{-n}$ ,

$$\text{The reciprocal sum of the divisor of } \forall p^n, S_{-n} = \sum_{m=0}^n p^{-m} = \frac{p^{-n-1}-p^0}{p^{-1}-1} = \frac{p^{n+1}-p^0}{p^n(p^1-1)}$$

$$\Rightarrow \forall p^n = \frac{S_n}{S_{-n}} = \frac{\sum_{m=0}^n p^m}{\sum_{m=0}^n p^{-m}} = \frac{p^{n+1}-p^0}{p^1-1} / \frac{p^{-n-1}-p^0}{p^{-1}-1}, \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \frac{\sum_{m=0}^n p_k^{m+1}-p_k^0}{\sum_{m=0}^n p_k^{-m}} = \prod_{k=0}^{\infty} \left( \frac{p_k^{n+1}-p_k^0}{p_k^1-1} / \frac{p_k^{-n-1}-p_k^0}{p_k^{-1}-1} \right)$$

$$\left( \sum_{m=0}^2 2^m \right) = 1 + 2 + 4, \left( \sum_{m=0}^2 2^m \right) * \left( \sum_{m=0}^2 3^m \right) = 1 + 2 + 3 + 4 + 6 + 9 + 12 + 18 + 36,$$

$$\left( \sum_{m=0}^2 2^m \right) * \left( \sum_{m=0}^2 3^m \right) * \left( \sum_{m=0}^2 5^m \right) = 1 + 2 + 3 + 4 + 5 + 6 + 9 + 10 + 12 + 15 + 18 + 20 + 25 + 30 +$$

$$36 + 45 + 50 + 60 + 75 + 80 + 100 + 150 + 180 + 225 + 300 + 450 + 900,$$

Therefore, the product of the power sum of all primes, according to the law of multiplicative distribution, their expansions correspond to all natural numbers one by one.

$$n \rightarrow \infty, \Rightarrow \sum_{d=1}^{\infty} d^{\pm 1} = \prod_{k=0}^{\infty} \frac{p_k^{\pm(n+1)-p_k^0}}{p_k^{\pm 1}-1}, \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \left( \frac{p_k^{n+1}-p_k^0}{p_k^1-1} / \frac{p_k^{-n-1}-p_k^0}{p_k^{-1}-1} \right) = \frac{\sum_{d=1}^{\infty} d^{+1}}{\sum_{d=1}^{\infty} d^{-1}}$$

$$n_k \rightarrow \infty, \Rightarrow \sum_{d=1}^{\infty} d^{\pm s} = \prod_{k=0}^{\infty} \frac{p_k^{\pm s(n_k+1)-p_k^0}}{p_k^{\pm s}-1}, \prod_{k=0}^{\infty} p_k^{s*n_k} = \frac{\sum_{d=1}^{\infty} d^{+s}}{\sum_{d=1}^{\infty} d^{-s}}$$

$$\Rightarrow \sum_{d=1}^{\infty} d^{-s} = \prod_{k=0}^{\infty} \frac{p_k^{-s(n_k+1)-p_k^0}}{p_k^{-s}-1} = \left( \sum_{d=1}^{\infty} d^{+s} \right) \prod_{k=0}^{\infty} \frac{1-p_k^{-s*n_k}}{p_k^{s*n_k}-1} = \left( \sum_{d=1}^{\infty} d^{+s} \right) \prod_{k=0}^{\infty} p_k^{-s*n_k}$$

So, the Euler product formula is not accurate, that is,  $\sum_{d=1}^{\infty} d^{-s} \neq \prod_{k=0}^{\infty} \frac{1}{1-p_k^{-s}}$ .

$$\text{If } \left( 1 - \frac{1}{2} \right) \sum_{d=1}^{\infty} d^{-1} = \sum_{d=1}^{\infty} (2d-1)^{-1}, \Rightarrow \left( \sum_{d=1}^{\infty} (2d-1)^{-1} \right) - \frac{1}{2} \sum_{d=1}^{\infty} d^{-1} = 0, \text{ but, } \sum_{d=1}^{\infty} (-1)^{d+1} d^{-1} = \ln 2.$$

Therefore, all papers quoting the Euler product formula are problematic, and the Riemann conjecture related to the Euler product formula is wrong.

**Reference:** none.