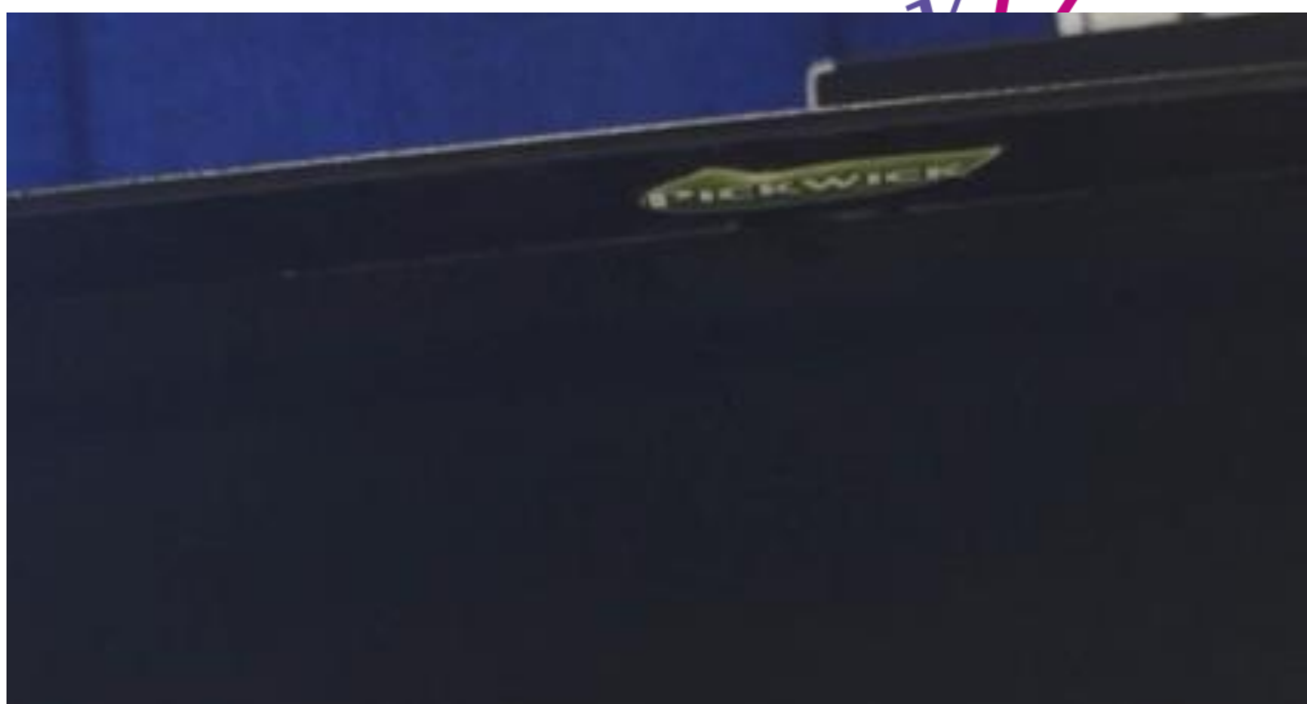
Tesla coil engine

" In this video (https://youtu.be/L5E4NiP4hpM?t=110), one puts a small metal structure on top of a Tesla coil so that the sparks appear in opposite directions. The structure then gets a rotational motion, as if it were "propelled" by the sparks. Explain the phenomenon. Optimise the electrical and mechanical part of the setup to obtain the maximum rotational speed. What is the efficiency of such an engine, compared with conventional electric engines?"

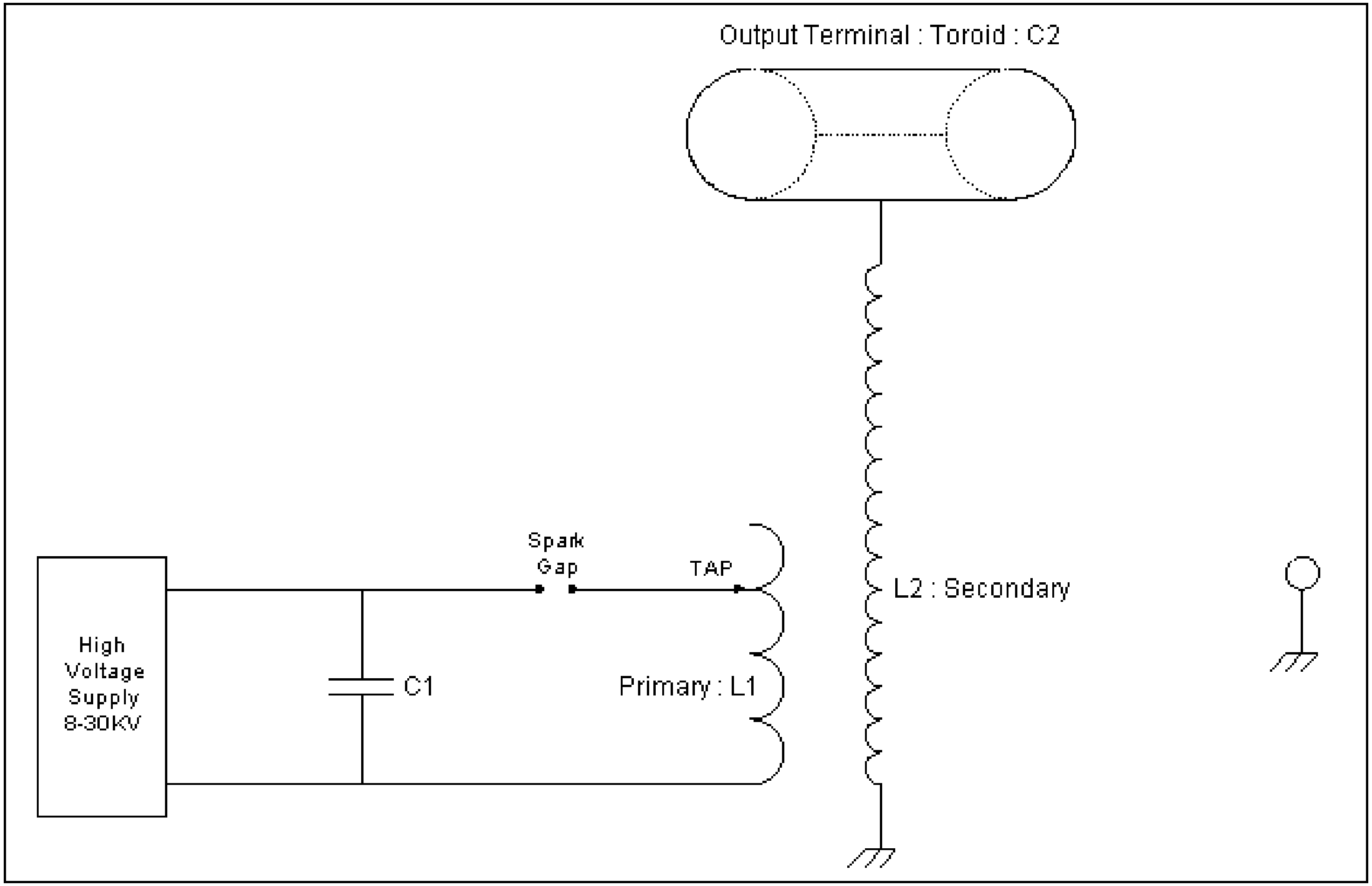
Frederik Laust Durhuus,

Danish Team

DTU Physics
Department of Physics



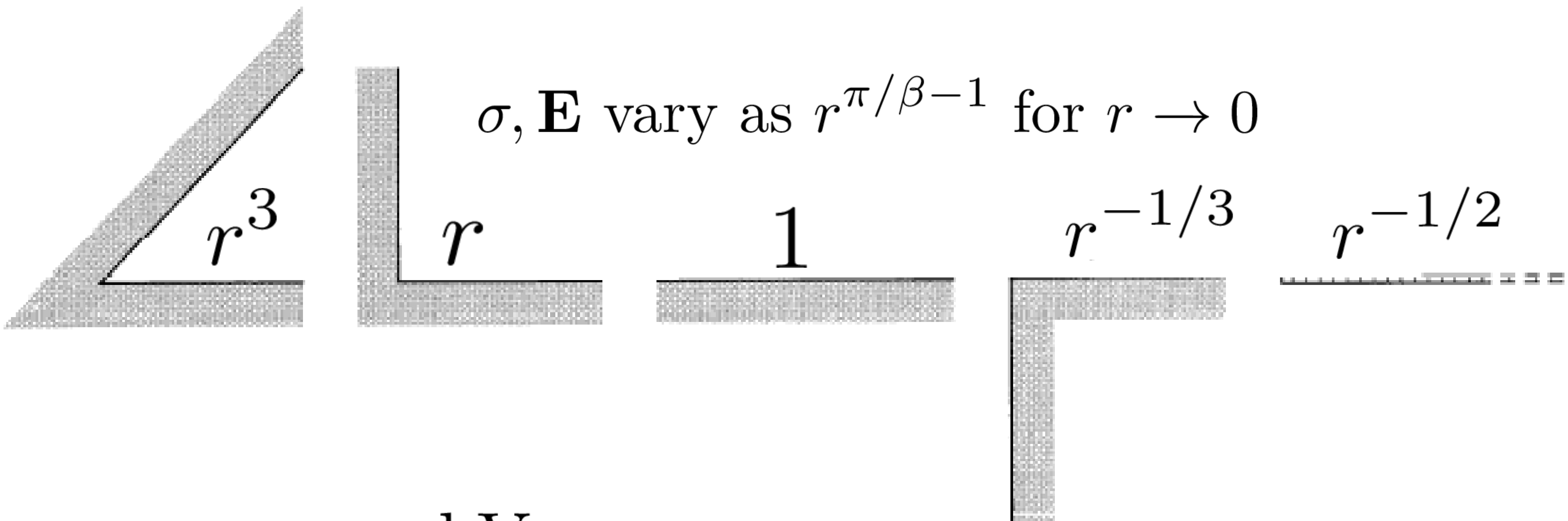
Tesla Coil : Working principle



http://www.tb3.com/tesla/theory.html?fbclid=IwAR2qBJBseGNYjgOwJwii9dlMlg97K7jh6bJ3ToNXnhicB0l6W53Zn24TCPo

Corona discharge

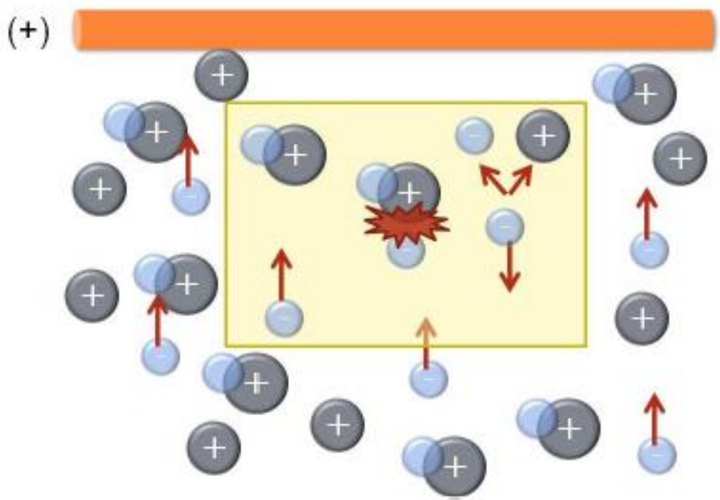
$$\sigma, \mathbf{E} \text{ vary as } r^{\pi/\beta - 1} \text{ for } r \to 0$$

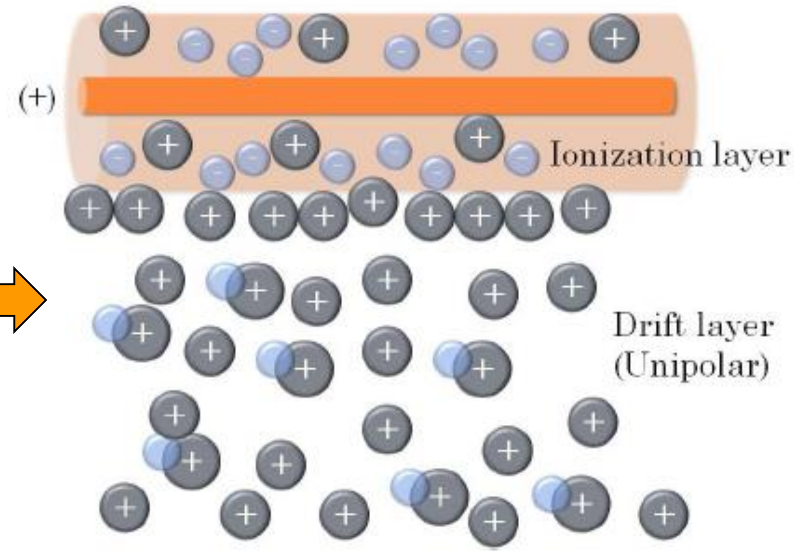


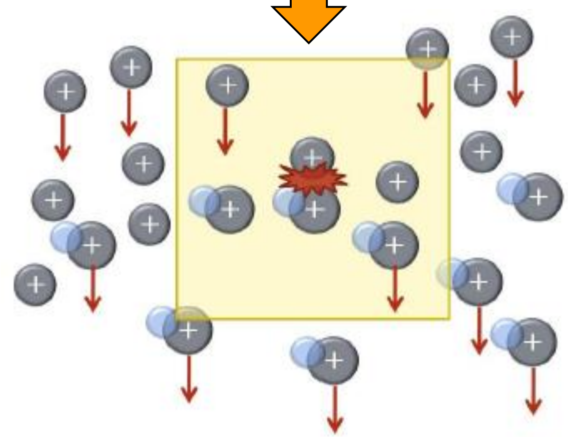
$$|\mathbf{E}| = 30\frac{\text{kV}}{\text{cm}} \Rightarrow \text{dielectric breakdown}$$

- Wan, Clemens., Electro-hydrodynamic (EHD) thruster analysis and optimization
- Jackson, J.D., *Classical electrodynamics*

Force from discharge







$$\mathbf{F}_{\mathrm{rotor}} = -\frac{\mathrm{d}\mathbf{p}_{neutral}}{\mathrm{d}t}$$

- Image credits :
  Wan, Clemens., Electro-hydrodynamic (EHD) thruster analysis and optimization

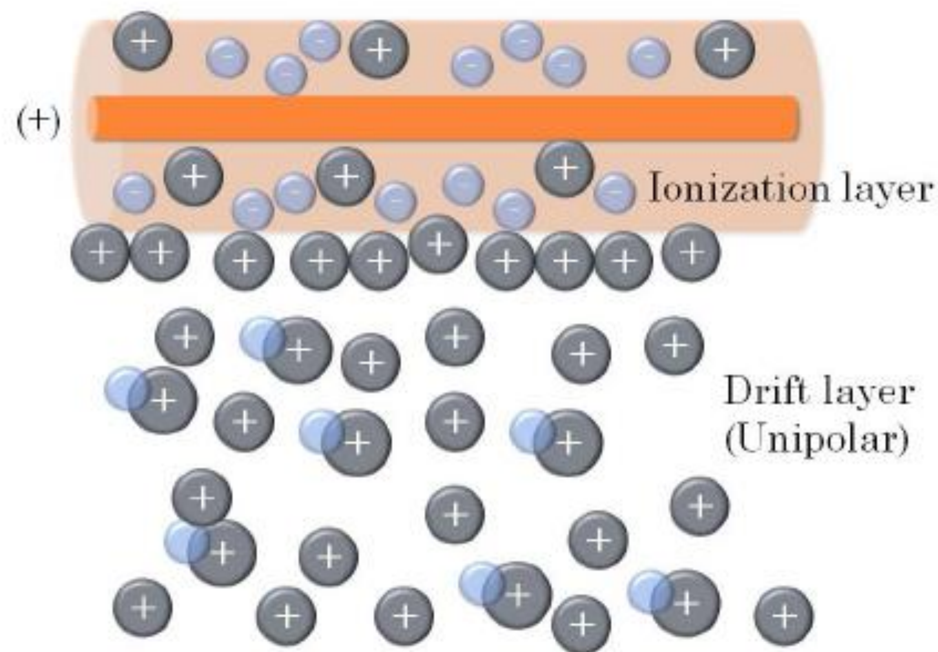
Effect of circuit oscillation

- Polarity change : $f_{sec} \sim 100\text{kHz}$
- Kinetic gas theory : $f_{scattering} \sim 7\text{GHz}$

$$v_{ion} \sim 1.5\frac{\text{km}}{\text{s}} \Rightarrow \Delta x_{cycle} \sim 1.5\text{cm}$$

$$v_{e-} \gg v_{ion}$$

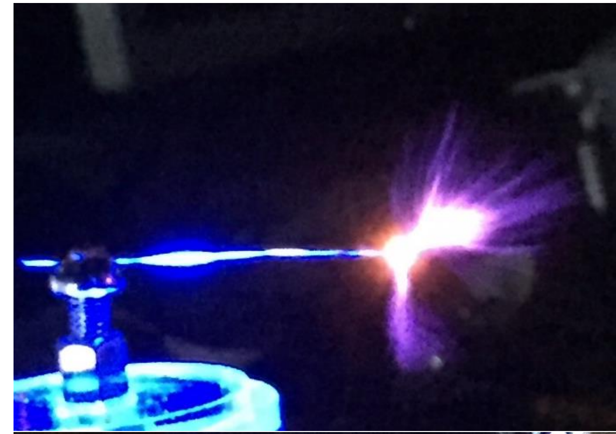
- Drift layer switches polarity rapidly
- E-field is generally same polarity as drift layer

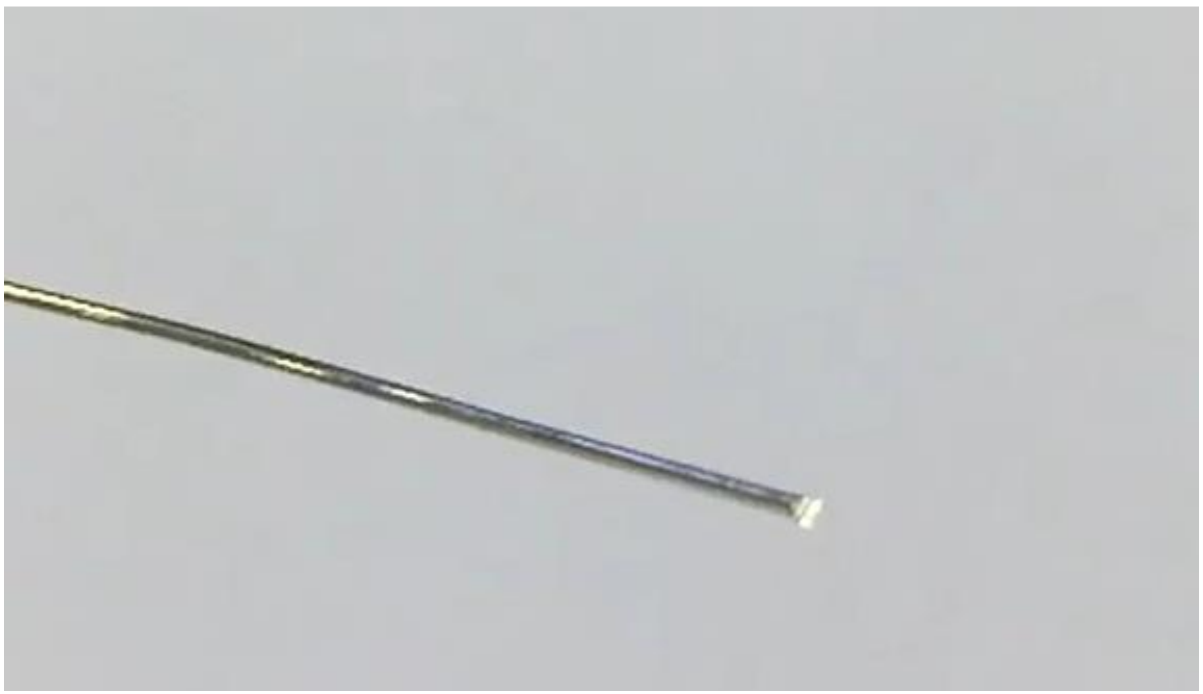


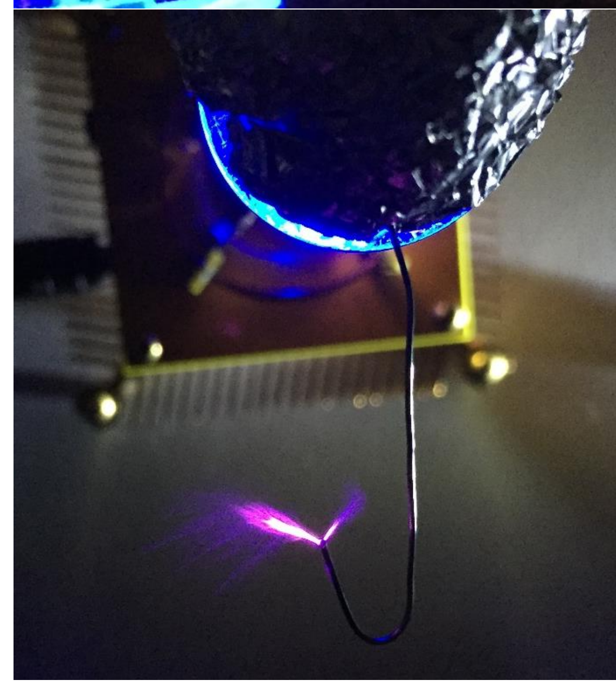
- Image credits : Wan, Clemens., Electro-hydrodynamic (EHD) thruster analysis and optimization

Streamer behaviour

- Emitted at sharpest points
- Follows E-field lines



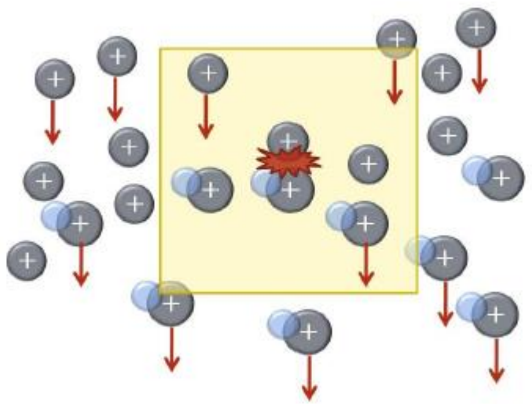




Model : Plasma force

$$\mathbf{F}(t) = \int_{\text{all space}} \rho(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)dV \underbrace{\Rightarrow}_{\text{time average}} \overline{\mathbf{F}} = f_{dis} \underbrace{\int \overline{\rho\mathbf{E}}(\mathbf{r})dV}_{\text{highly non-linear}}$$

$$f_{dis} = \frac{P_{tesla}}{W_{discharge}} = \frac{(LC)^{-1/2}[E_{cycle} - E_{loss}]}{CV_{crit}^2/2}$$



- Adamiak et. Al.

  "Simulation of corona discharge in point–plane configuration"
- Morrow "The theory of positive glow corona"

Model : Plasma force

$$\mathbf{F}(t) = \int_{\text{all space}} \rho(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)dV \underbrace{\Rightarrow}_{\text{time average}} \overline{\mathbf{F}} = f_{dis} \underbrace{\int \overline{\rho\mathbf{E}}(\mathbf{r})dV}_{\text{highly non-linear}}$$

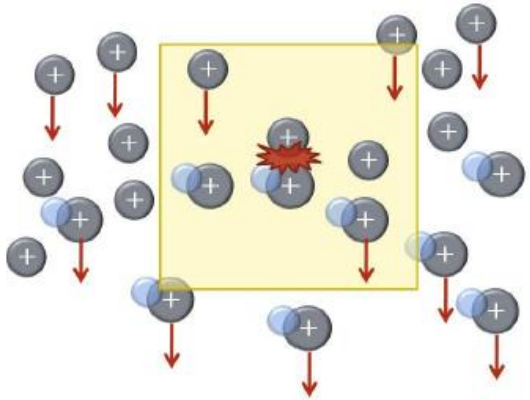
$$f_{dis} = \frac{P_{tesla}}{W_{discharge}} = \frac{(LC)^{-1/2}[E_{cycle} - E_{loss}]}{CV_{crit}^2/2}$$

$$\|\mathbf{E}_{crit}\| = \|\mathbf{E}_{\mathrm{BD}}\|\delta\left(1 + \frac{\gamma}{\sqrt{r_{rotor}}}\right) \qquad V_{crit} \sim r_{rotor}\|\mathbf{E}_{crit}\|$$

$$\|\mathbf{E}_{BD}\|_{air} = 30\frac{\mathrm{kV}}{\mathrm{cm}}$$

Design principles

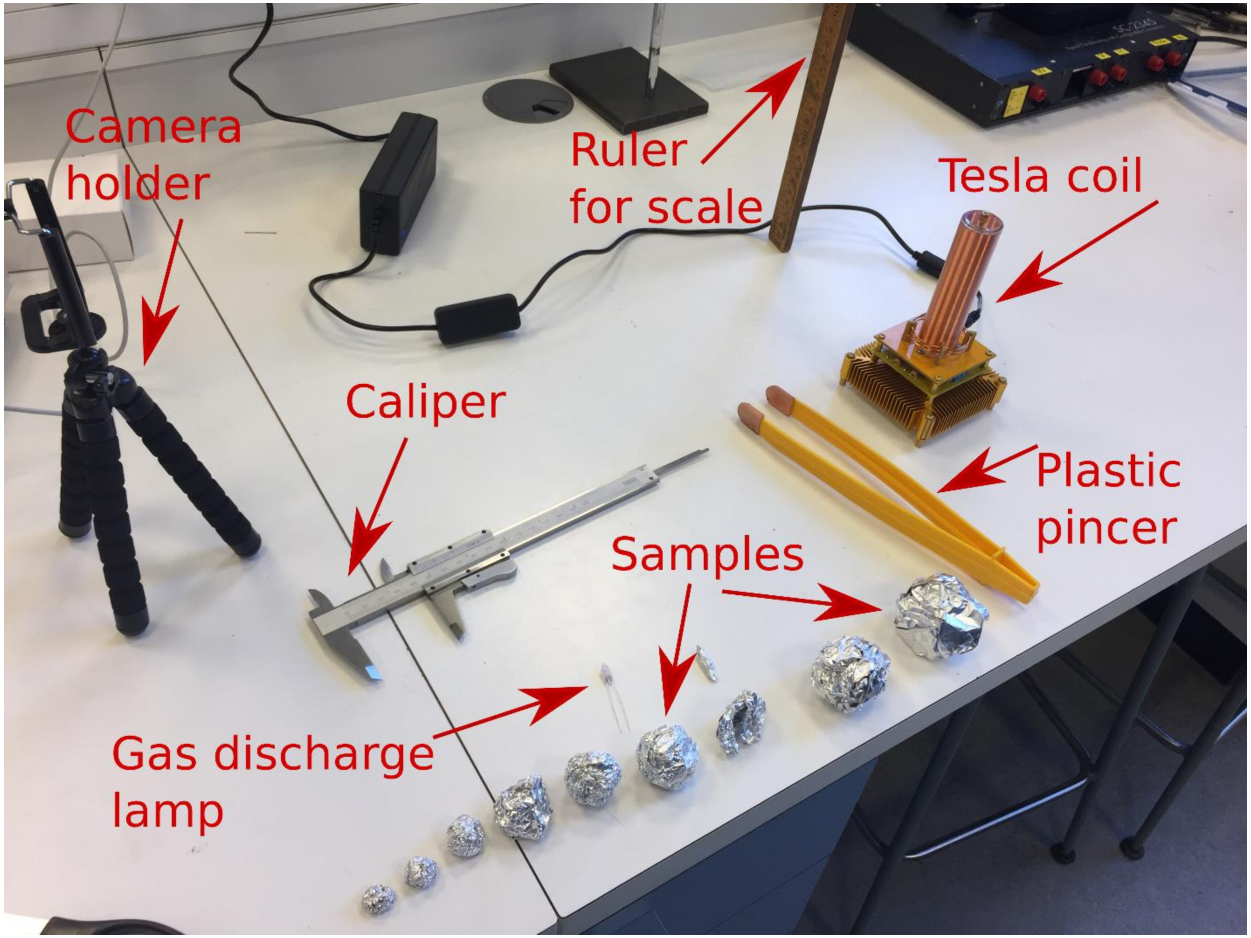
- Minimal circuit resistance
- Low capacitance => small top load
- Low radius of curvature at emission points
- Emission concentrated at few points
- Powerful and efficient tesla coil



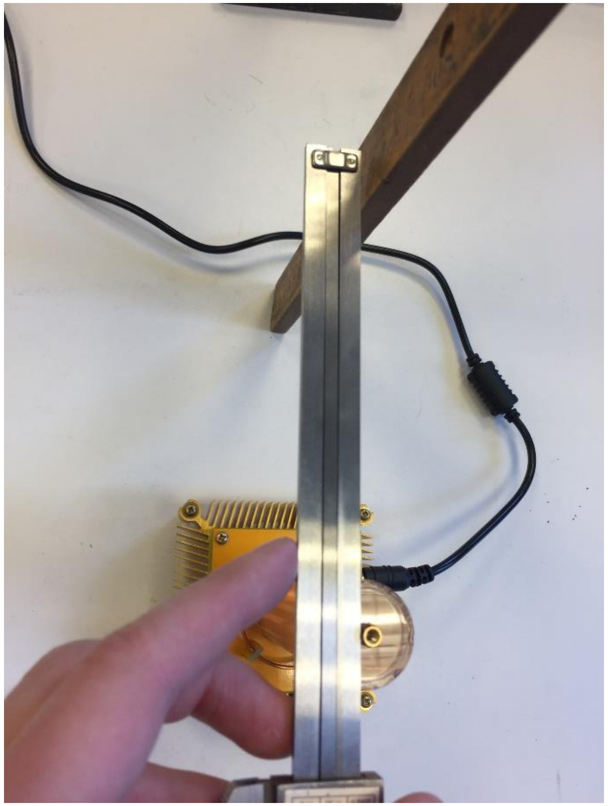
❑ Adamiak et. Al.
"Simulation of corona discharge in point–plane configuration"

❑ Morrow "The theory of positive glow corona"

Experiments : setup

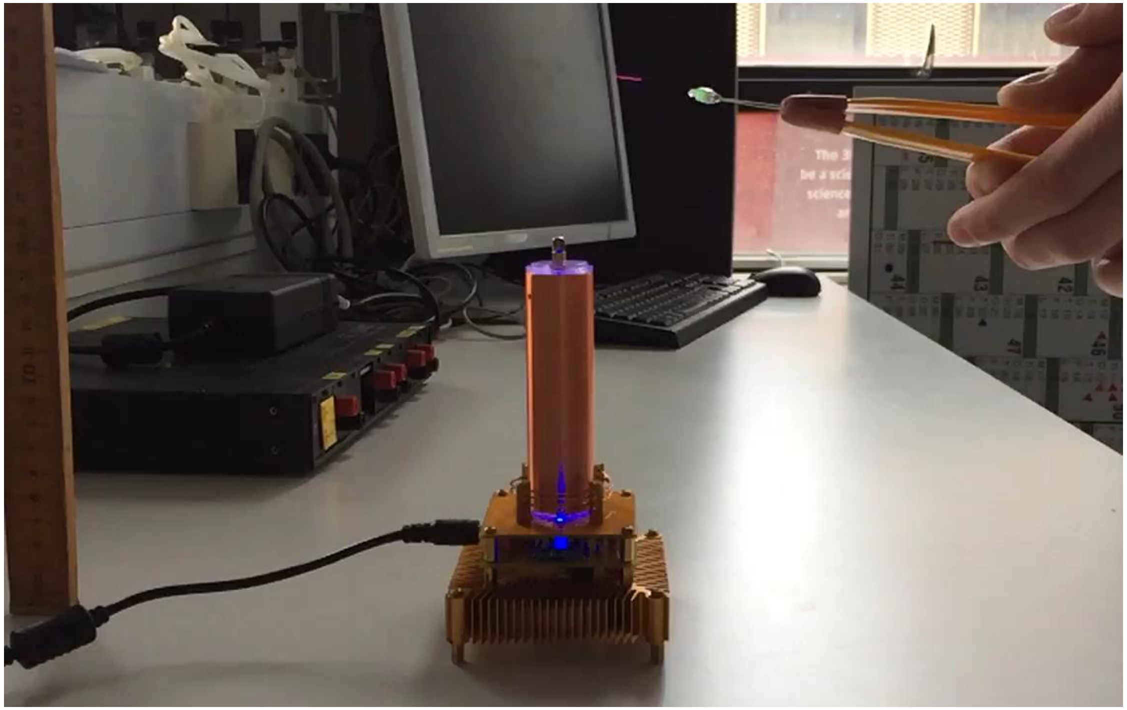


- Alignment



Experiments : method

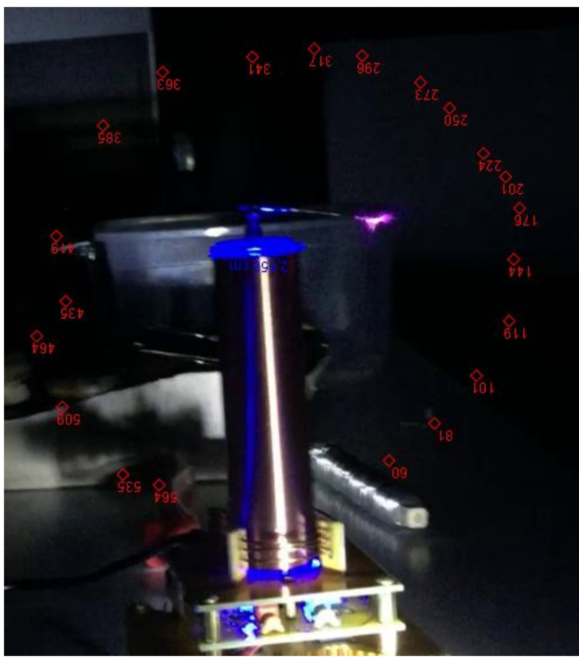
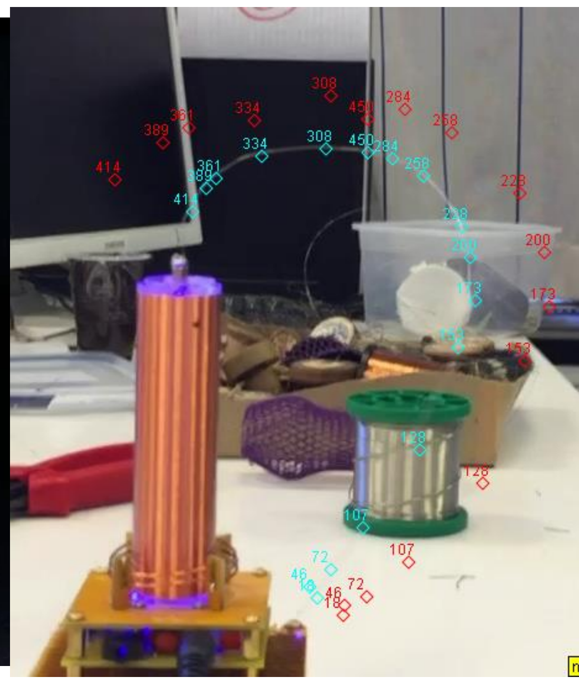
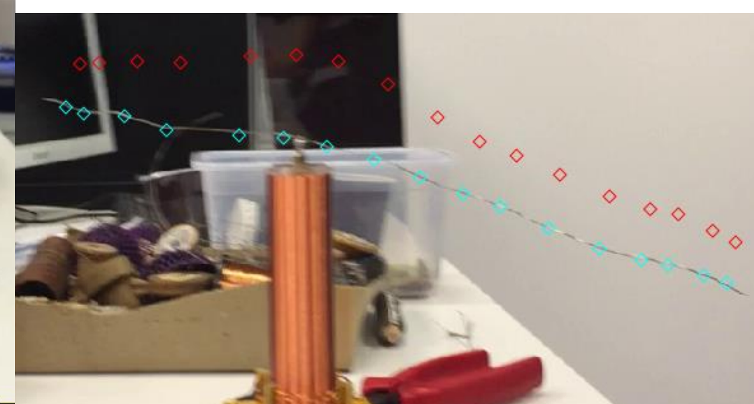
- Automatic luminosity tracking for rotation speed
- Position vs. activation for E-field



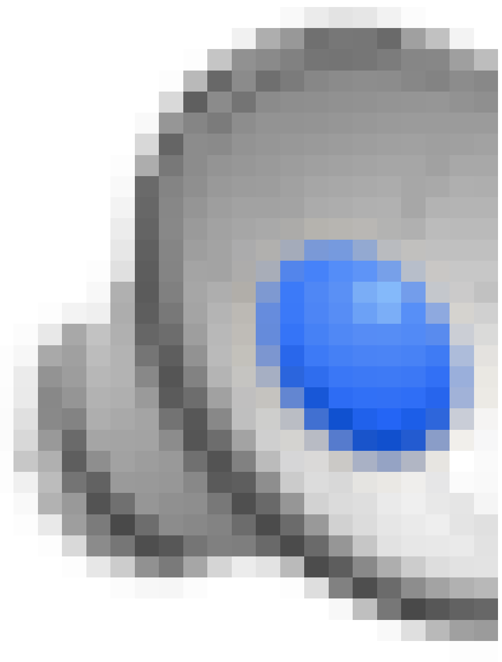
Experiments : Lamp activation

Red = activation points   Blue = closest point on wire

- Matches classical electrostatics

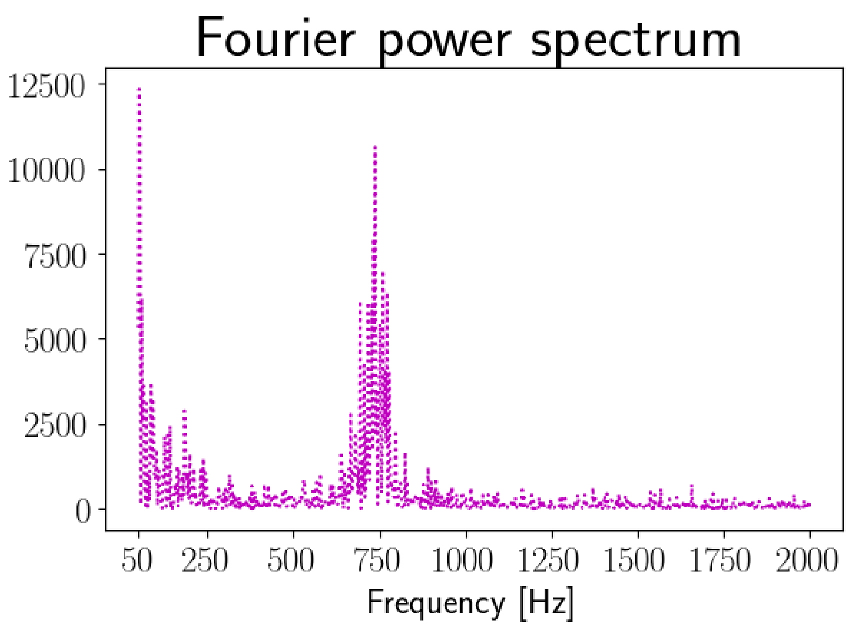




Experiments : High-speed video

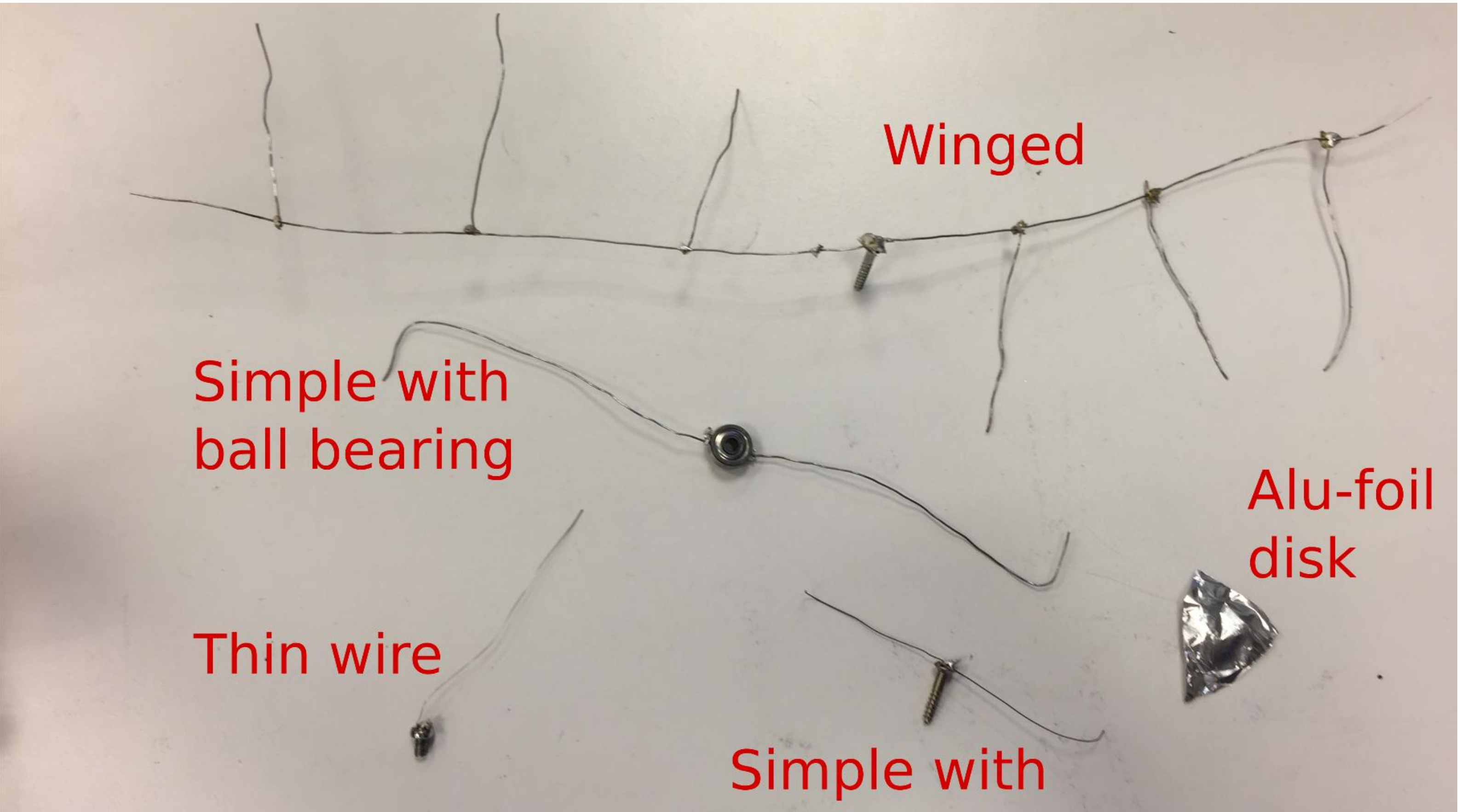


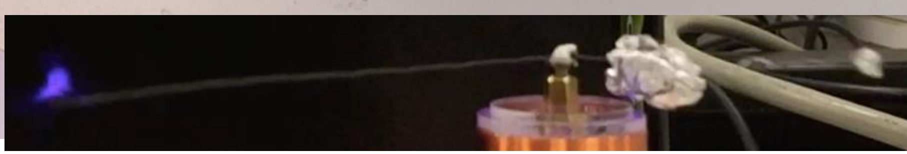
- Tracked colour intensity vs. time
- Numerical fourier transform

$$f_{discharge} \sim 750\text{Hz}$$



Designs Made

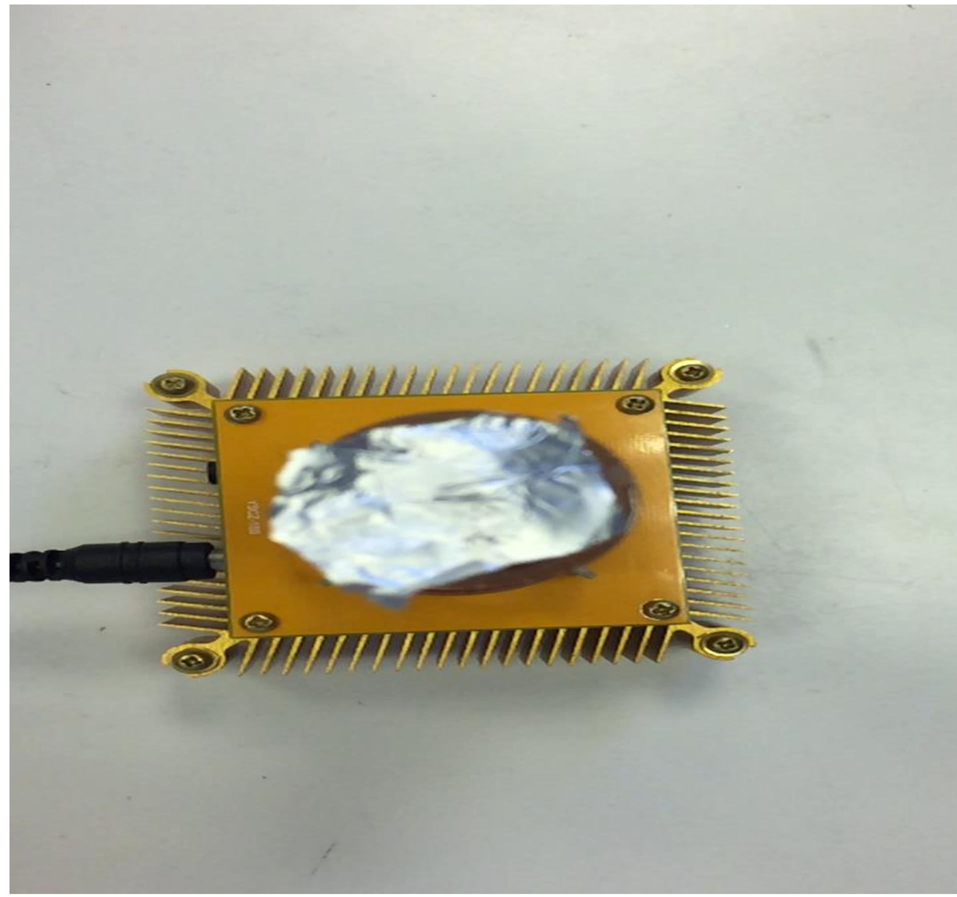




Wires used

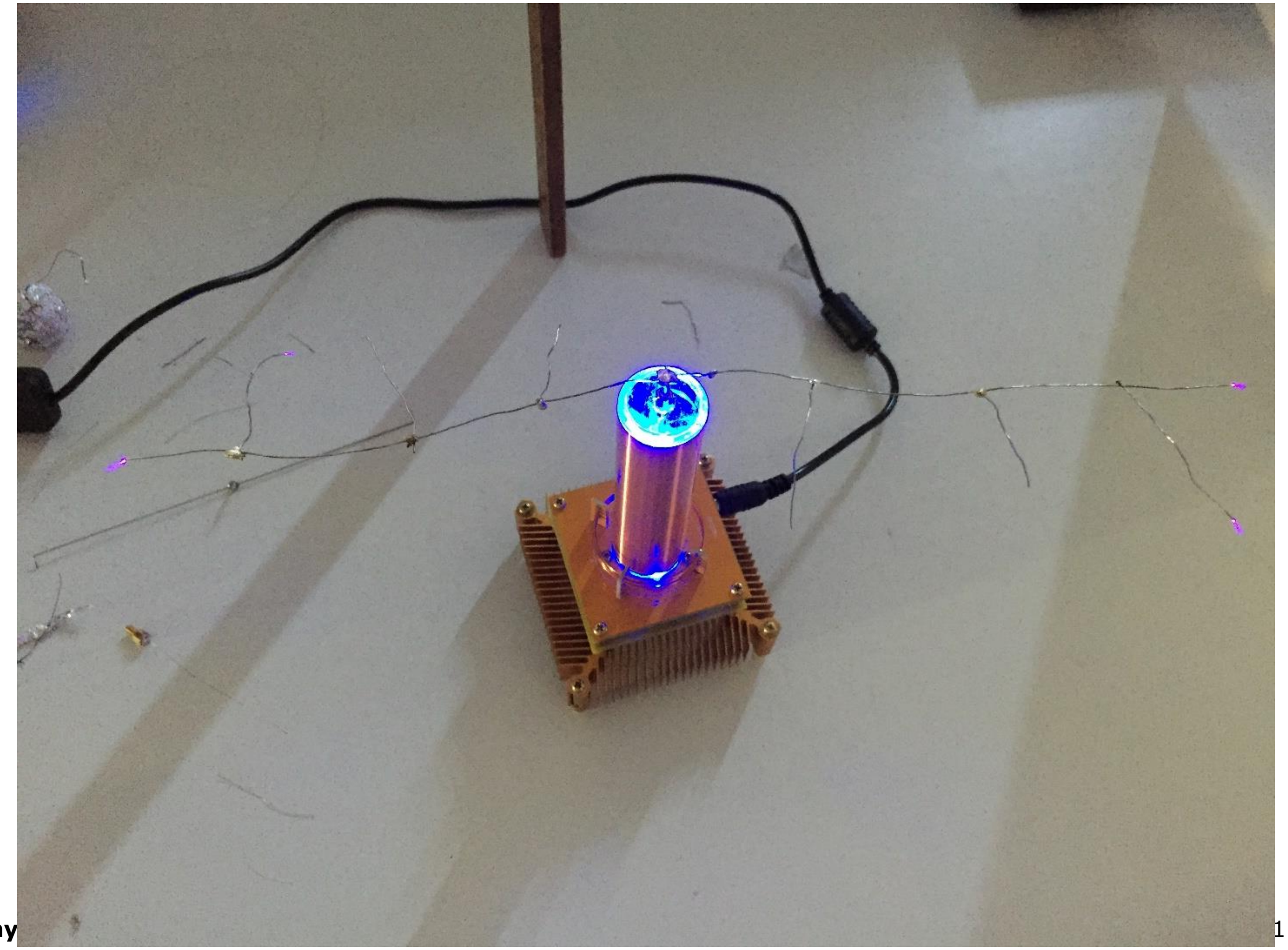
Spinning disk

- Only alu-foil => Light weight
- Stability issues
- Hard to make consistently



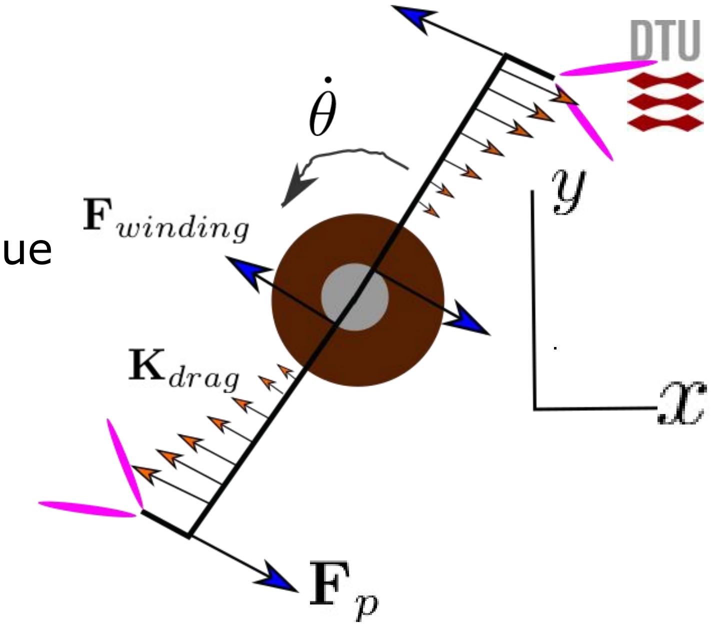
Design : Winged rotor

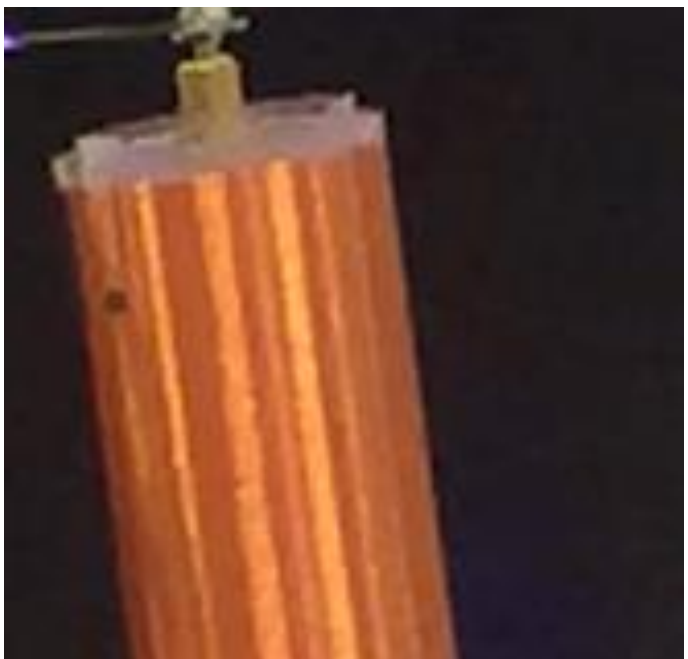
- Multiple arms decrease single plasma stream => needless mass

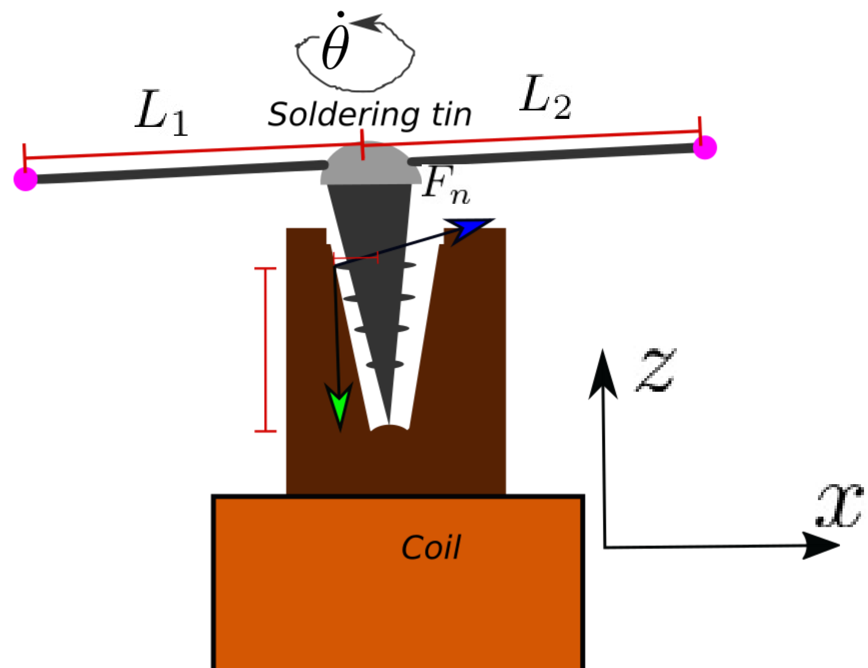


Simple rotor : Design

- Plasma focused at end points => maximal torque
- Friction sensitive to stability

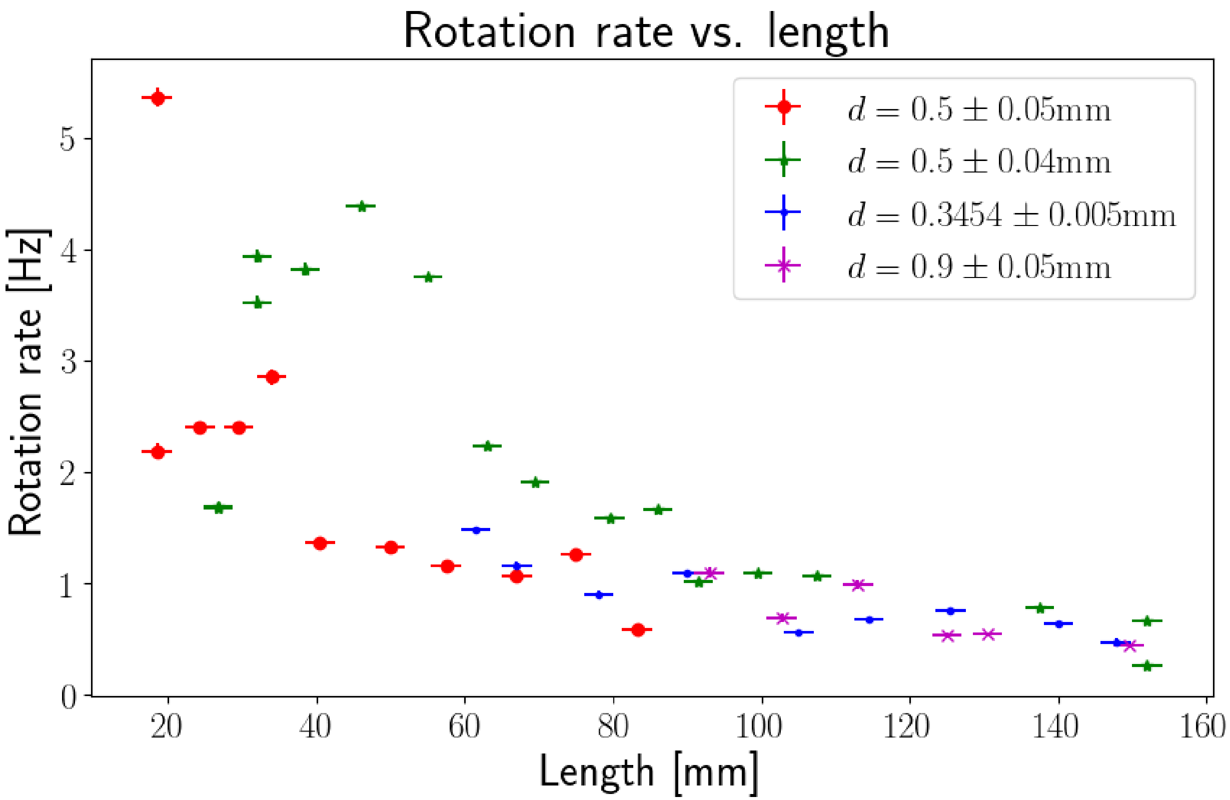






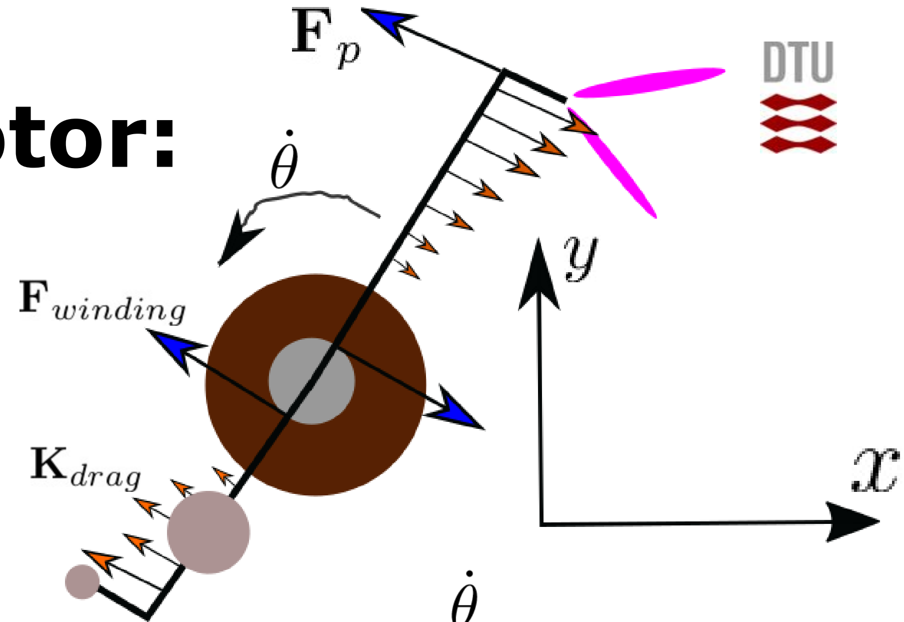
Simple rotor : Optimisation

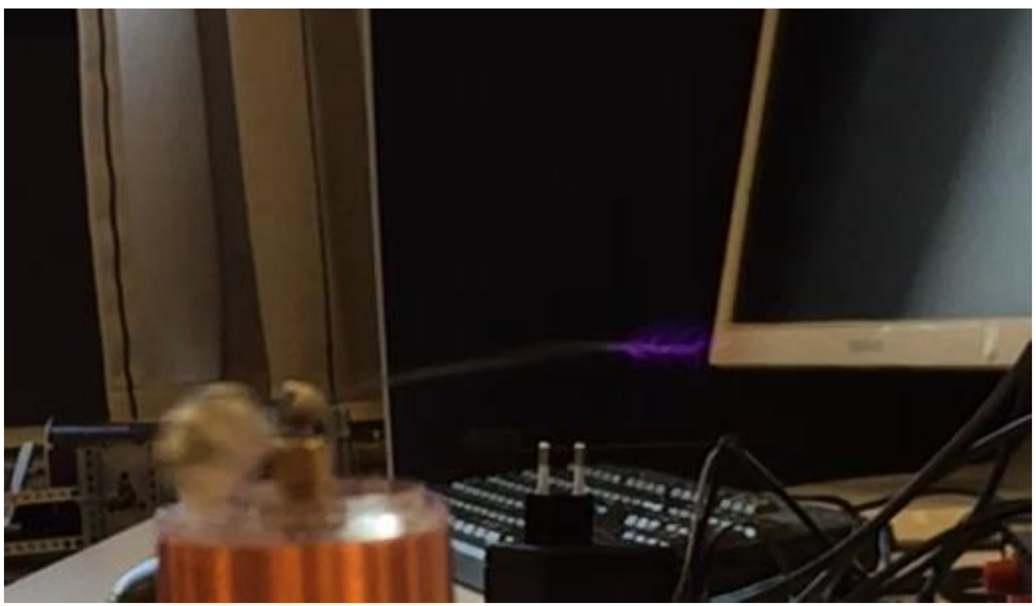
$$f_{max} = 5.37 \pm 0.08\text{Hz}$$

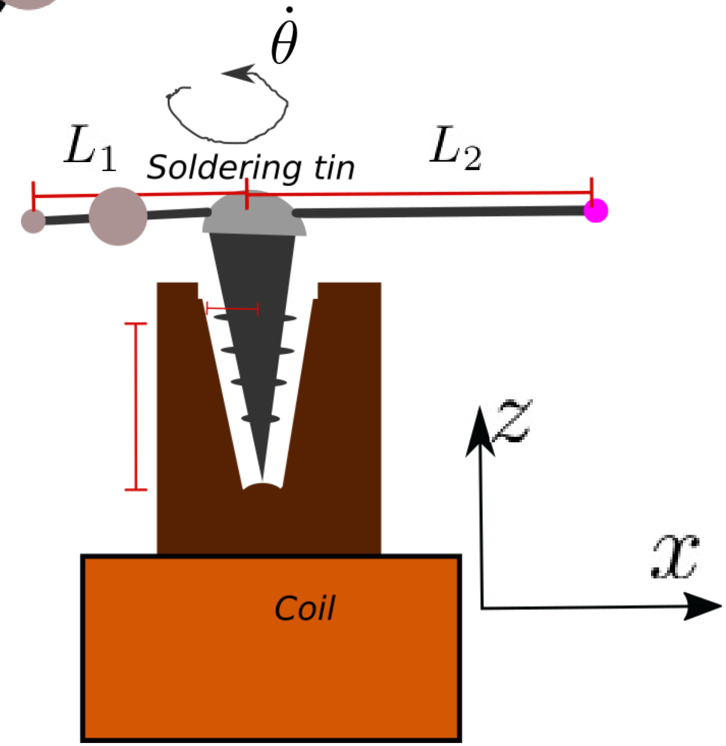


Counterbalance rotor: Design

- Counterbalance for stability
- Capped to direct plasma

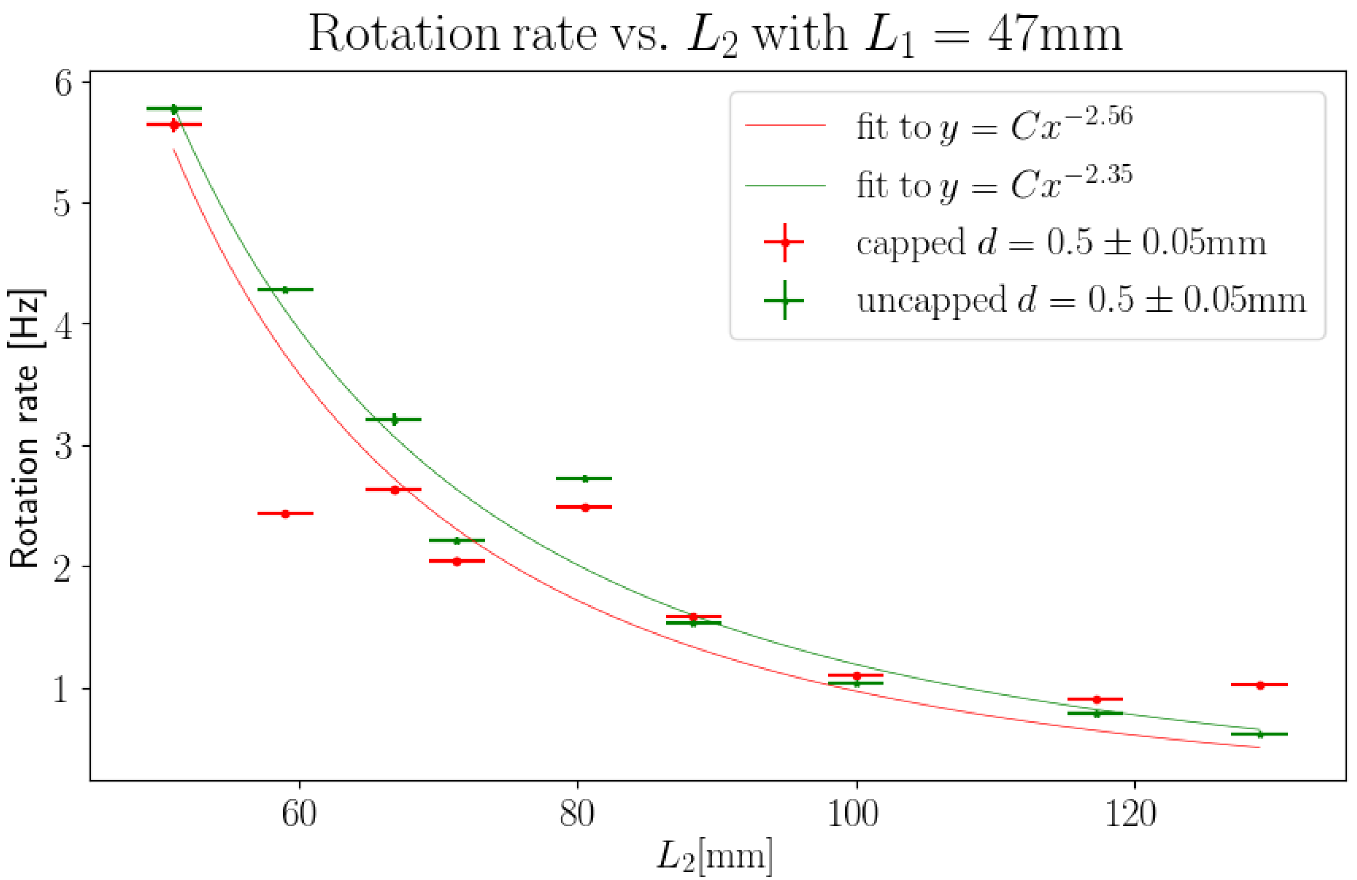






Counterbalance rotor : Optimisation

$$f_{max} = 5.77 \pm 0.03\text{Hz}$$



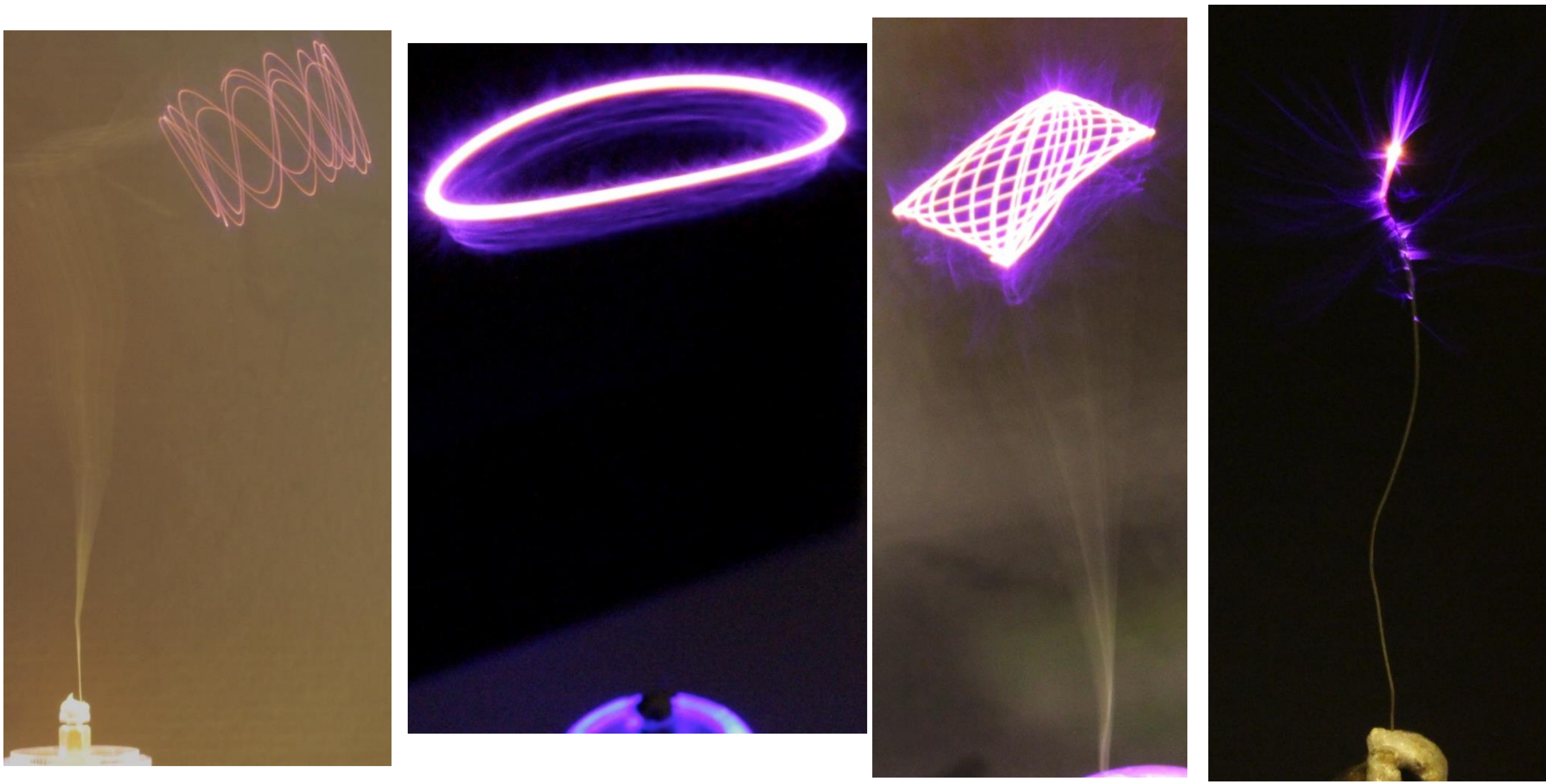
Thin wire : Design

- No extra mass
- Minimal friction
- Thin
- Reproducible

- Parameters :
  - Length L
  - Thickness r_w
  - Mass per length μ
  - Young modulus E
  - Electrical resistance R

Thin wire : rotation regimes

$L \sim 200\text{mm}$ → $L \sim 20\text{mm}$

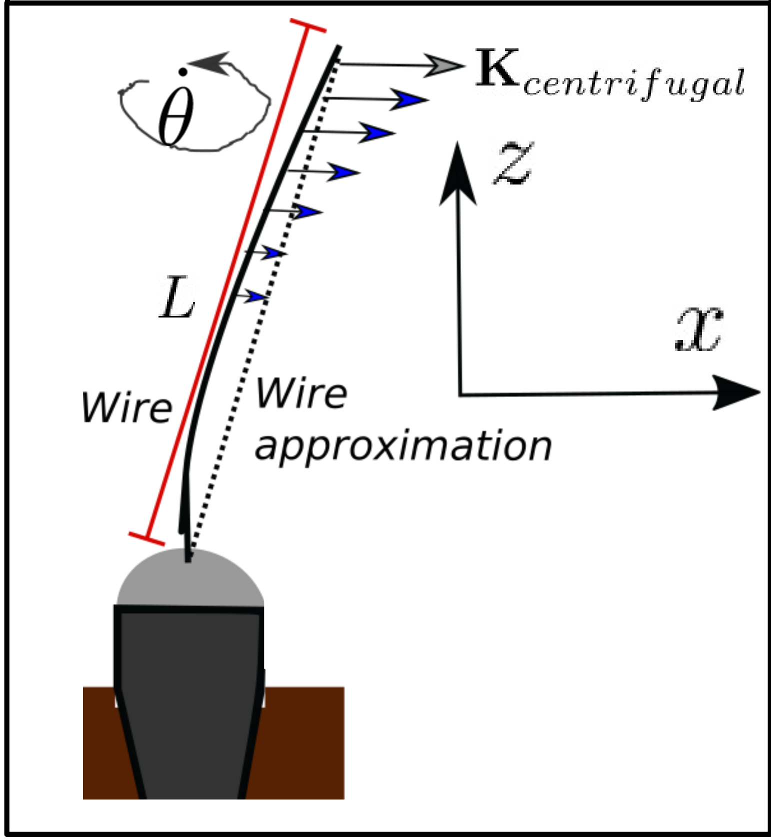


Thin wire : Theory

$$\underbrace{EI\frac{\mathrm{d}^4 y}{\mathrm{d}z^4} = \mu\dot{\theta}^2 y}_{\text{slender rod + centrifugal force}} \quad \Rightarrow \quad \underbrace{\tilde{y}'''' = \tilde{y}}_{\text{non-dimensional}}$$

$$\tilde{y} = \frac{y}{y_c} \qquad y_c = \sqrt[4]{\frac{EI}{\mu\dot{\theta}^2}}$$

$$I = \frac{\pi}{4} r_w^4$$



Thin wire : Theory

$$\underbrace{EI\frac{\mathrm{d}^4y}{\mathrm{d}z^4} = \mu\dot{\theta}^2 y}_{\text{slender rod + centrifugal force}} \quad \Rightarrow \quad \underbrace{\tilde{y}'''' = \tilde{y}}_{\text{non-dimensional}}$$

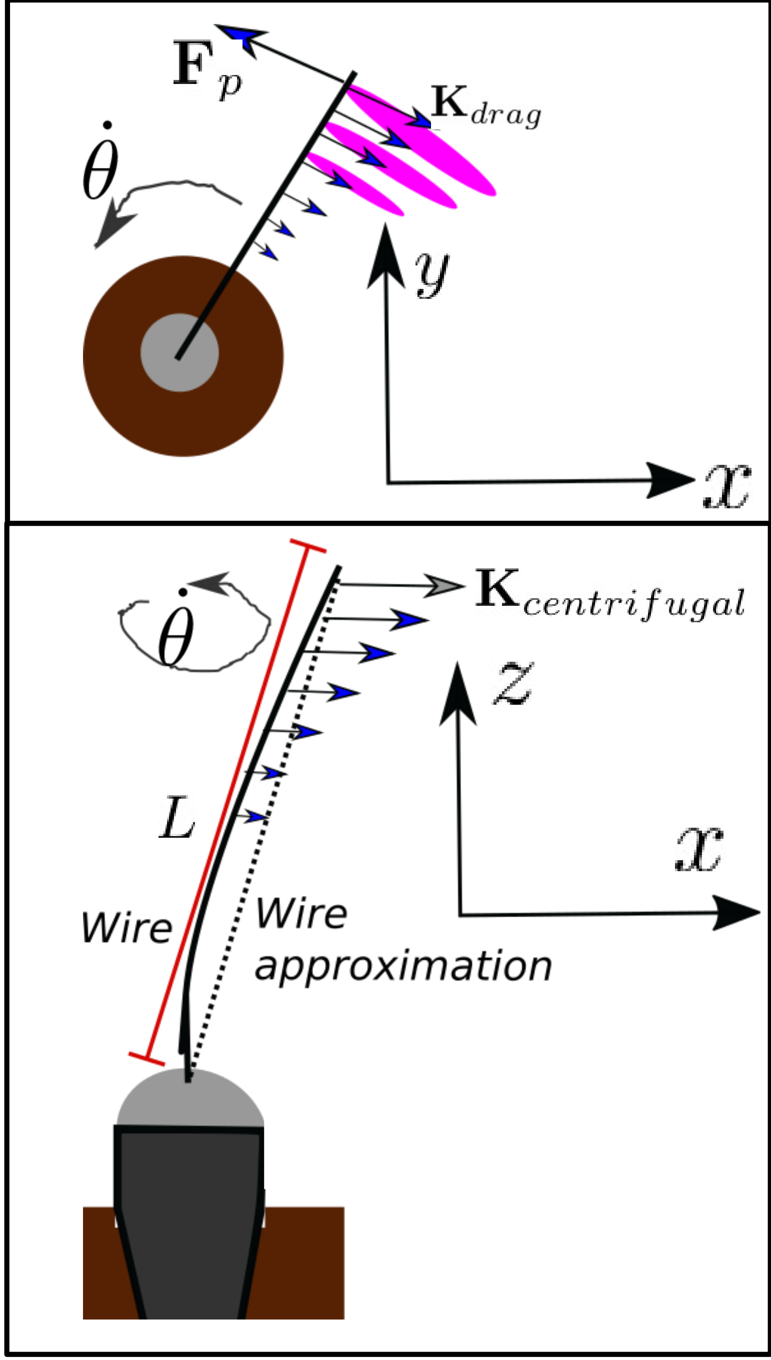
$$\tilde{y} = \frac{y}{y_c} \qquad y_c = \sqrt[4]{\frac{EI}{\mu\dot{\theta}^2}}$$

Simplification : $y(z) = \frac{y_c}{L}z$

$$\tau_{drag} \sim C_D L\dot{\theta}y_c^2$$

$$I = \frac{\pi}{4}r_w^4$$

$$C_D = 6\pi r_w \eta_{air}$$



Thin wire : Theory

$$\underbrace{EI\frac{\mathrm{d}^4 y}{\mathrm{d}z^4} = \mu\dot{\theta}^2 y}_{\text{slender rod + centrifugal force}} \quad \Rightarrow \quad \underbrace{\tilde{y}'''' = \tilde{y}}_{\text{non-dimensional}}$$

$$\tilde{y} = \frac{y}{y_c} \qquad y_c = \sqrt[4]{\frac{EI}{\mu\dot{\theta}^2}}$$

Simplification : $y(z) = \frac{y_c}{L} z$

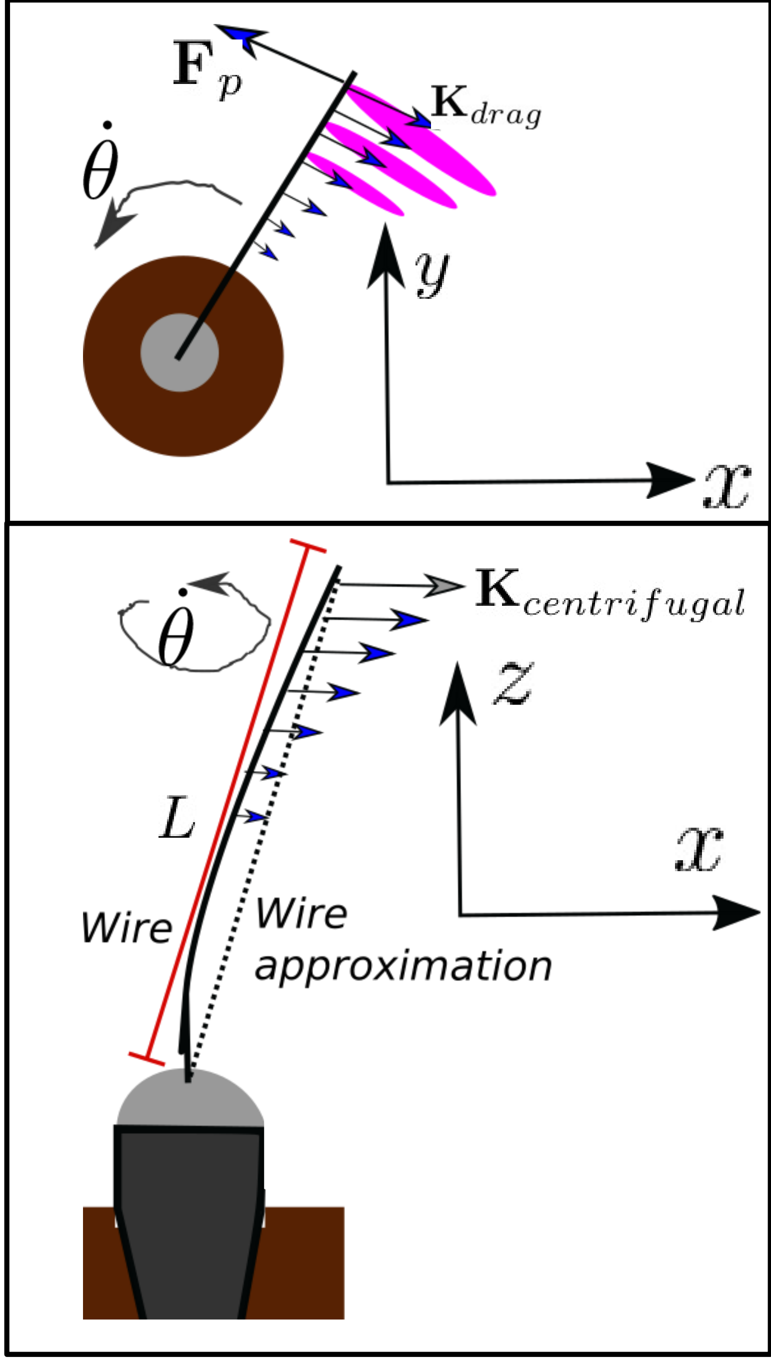
$$\tau_{drag} \sim C_D L \dot{\theta} y_c^2 \qquad \tau_p \approx F_p y(L) \sim y_c F_p$$

$$\tau_p = \tau_{drag}$$

$$\dot{\theta} \sim \sqrt{\frac{\mu}{E}\frac{F_p^2}{\eta_{air}^2 r_w^4}\frac{1}{L^2}}$$

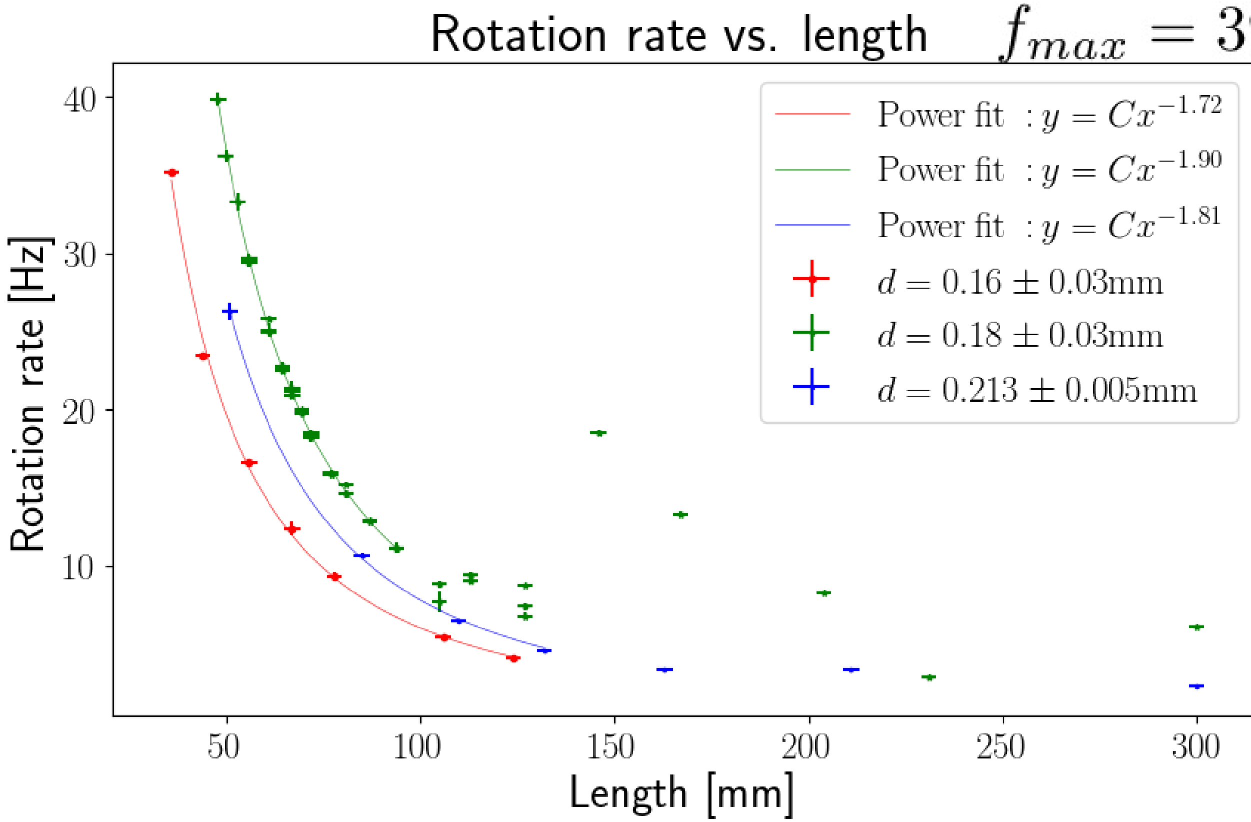
$$I = \frac{\pi}{4} r_w^4$$

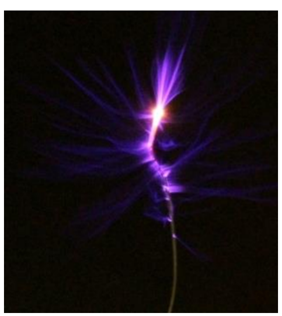
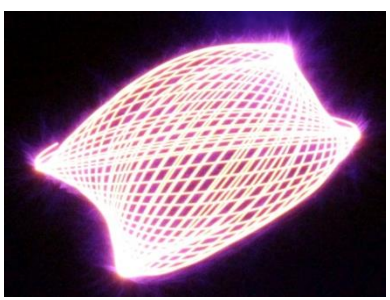
$$C_D = 6\pi r_w \eta_{air}$$

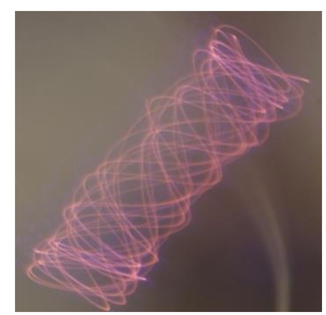


Thin wire : Optimisation

$$f_{max} = 39.9 \pm 0.4\text{Hz}$$





Efficiency

- Conventional electric engine according to US department of energy : $\eta_{\mathrm{electric}} = 59\% - 62\%$

  https://www.fueleconomy.gov/feg/evtech.shtml

- Own ”engine” : $P_{in} \sim 40\mathrm{W}$ $P_{out} \sim 10^{-7}\mathrm{W}$

$$\eta_{\mathrm{plasma\ rotor}} = 2 \times 10^{-9} \approx 4 \times 10^{-9} \eta_{\mathrm{electric}}$$

Conclusion

- High voltage => plasma => momentum transfer to neutral particles
- Single thin wire is optimal
- Rotation speed scales as : $\dot{\theta} \sim \sqrt{\frac{\mu}{E}} \frac{F_p^2}{\eta_{air}^2 r_w^4} \frac{1}{L^2}$
- Maximal achieved frequency : $f_{max} = 39.9 \pm 0.4\mathrm{Hz}$
- Efficiency is virtually nonexistent : $\eta_{\text{plasma rotor}} = 2 \times 10^{-9}$

$$\approx 4 \times 10^{-9} \eta_{\text{electric}}$$

Bibliography

- K. Adamiak, P. Atten,

  Simulation of corona discharge in point–plane configuration,

  Journal of Electrostatics,

  Volume 61, Issue 2, 2004,

  https://doi.org/10.1016/j.elstat.2004.01.021.

- Wan, C.L.E.M.E.N.S., 2009. Electro-hydrodynamic (EHD) thruster analysis and optimization. *Master of Engineering thesis (The Cooper Union for the Advancement of Science and Art, Albert Nerken School of Engineering, Department of electrical Engineering, 2009)*.

- Griffiths, D.J., 2005. Introduction to electrodynamics.

- Jackson, J.D., 2007. *Classical electrodynamics*. John Wiley & Sons.

- Morrow, R.
  The theory of positive glow corona
  Journal of Physics D: Applied Physics
  http://dx.doi.org/10.1088/0022-3727/30/22/008

- Nae Cho, S., 2012. Physics of self-sustained oscillations in the positive glow corona. *Physics of Plasmas*, *19*(7), p.072113.

- Liu, Y., Huang, S., Liu, S. and Liu, D., 2018. A helical charge simulation based 3-D calculation model for corona loss of AC stranded conductors in the corona cage. *Aip Advances*, *8*(1), p.015303.

- Meroth, A.M., Gerber, T., Munz, C.D., Levin, P.L. and Schwab, A.J., 1999. Numerical solution of nonstationary charge coupled problems. *Journal of Electrostatics*, *45*(3), pp.177-198.

- http://hazardousphysics.christophergerekos.eu/main/zeus/The_Zeus_Tesla_Coil_2.html

Model : Plasma force

$$\mathbf{F}(t) = \int_{\text{all space}} \rho(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)dV \underbrace{\Rightarrow}_{\text{time average}} \overline{\mathbf{F}} = f_{dis} \underbrace{\int \overline{\rho\mathbf{E}}(\mathbf{r})dV}_{\text{highly non-linear}}$$

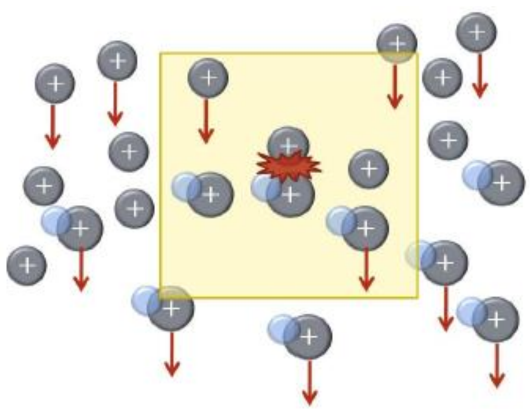
$$f_{dis} = \frac{P_{tesla}}{W_{discharge}} = \frac{(LC)^{-1/2}[E_{cycle} - E_{loss}]}{CV_{crit}^2/2}$$

$$\|\mathbf{E}_{crit}\| = \|\mathbf{E}_{\text{BD}}\|\delta\left(1 + \frac{\gamma}{\sqrt{r_{rotor}}}\right) \qquad V_{crit} \sim r_{rotor}\|\mathbf{E}_{crit}\|$$

$$\|\mathbf{E}_{BD}\|_{air} = 30\frac{\text{kV}}{\text{cm}}$$

Design principles

- Minimal circuit resistance
- Low capacitance => small top load
- Low radius of curvature at emission points
- Emission concentrated at few points
- Powerful and efficient tesla coil



- ❑ Adamiak et. Al. “Simulation of corona discharge in point–plane configuration”
- ❑ Morrow ”The theory of positive glow corona”

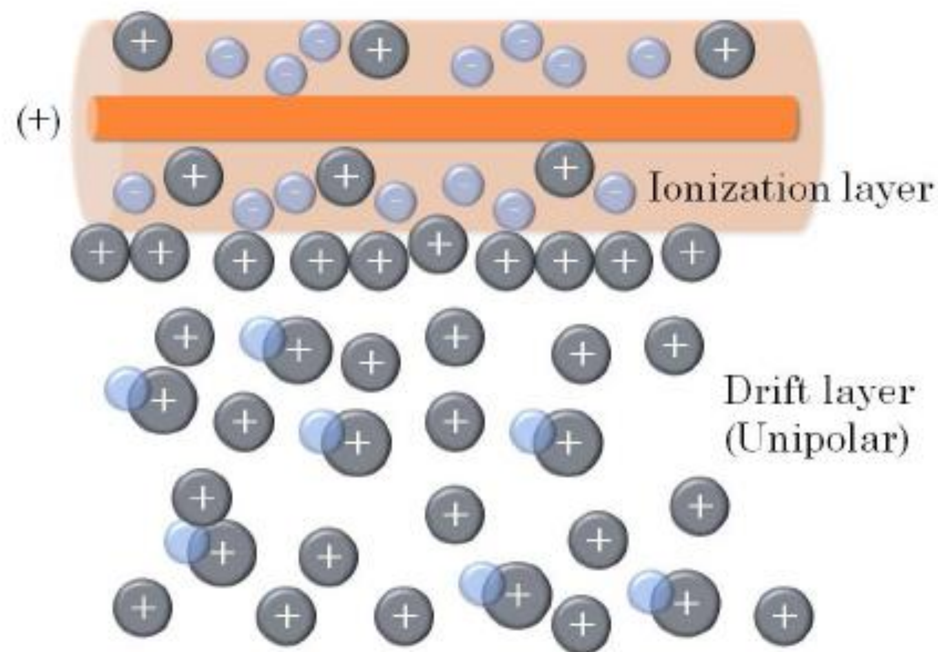
Appendix : Effect of circuit oscillation

- Polarity change : $f_{sec} \sim 100\mathrm{kHz}$
- Kinetic gas theory : $f_{scattering} \sim 7\mathrm{GHz}$

$$v_{ion} \sim 1.5\frac{\mathrm{km}}{\mathrm{s}} \Rightarrow \Delta x_{cycle} \sim 1.5\mathrm{cm}$$

$$v_{e^-} \gg v_{ion}$$

- Drift layer switches polarity rapidly
- E-field is generally same polarity as drift layer



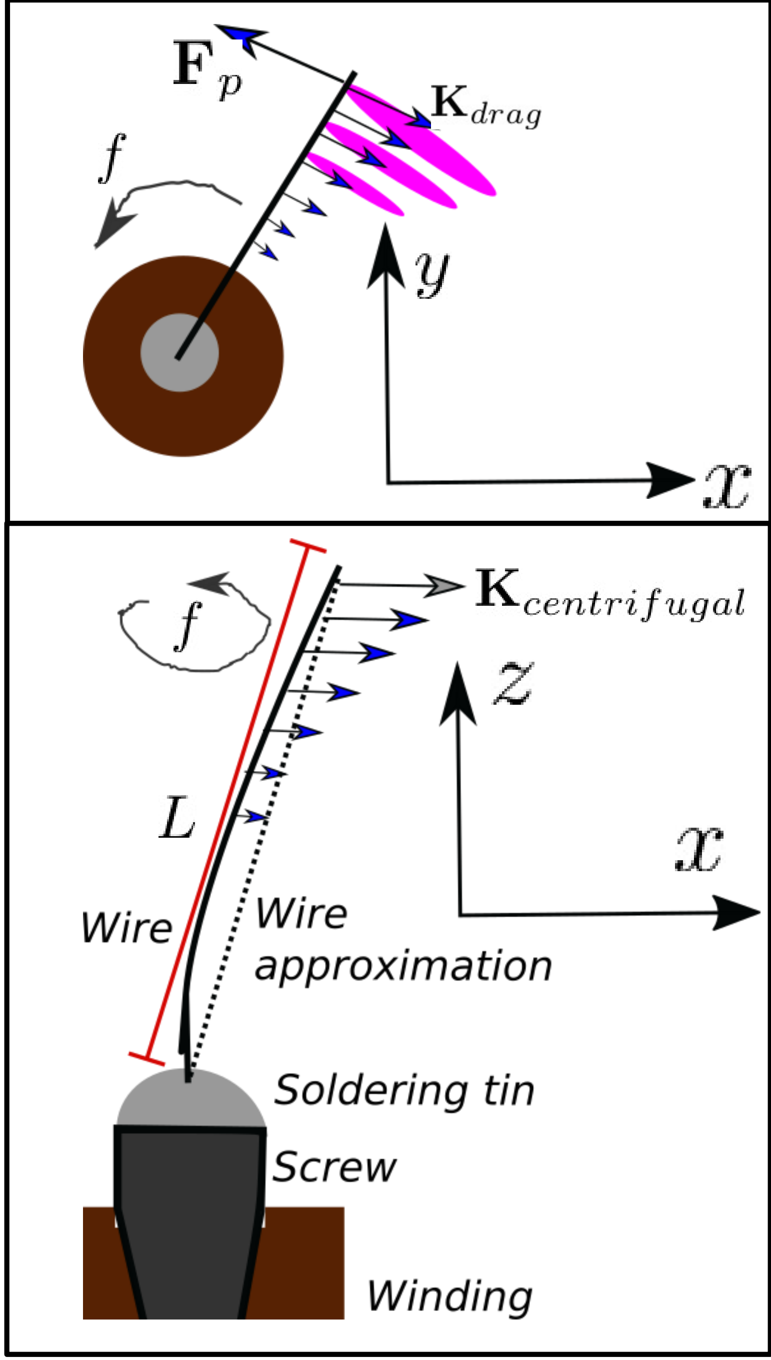
❑ Image credits : Wan, Clemens., Electro-hydrodynamic (EHD) thruster analysis and optimization

Appendix : Thin wire calculations

$$\tau_D = \int_0^{h\approx L} \alpha z \underbrace{\sqrt{1+\alpha^2}}_{\approx 1} C_D \dot{\theta} \alpha z dz$$

$$\approx C_D L \dot{\theta} \alpha^2 \frac{L^3}{3}$$

$$\sim C_D L \dot{\theta} y_c^2$$



$$y(z) = \alpha z \quad \text{where} \quad \alpha \propto \frac{y_c}{L}$$

Appendix : Efficiency

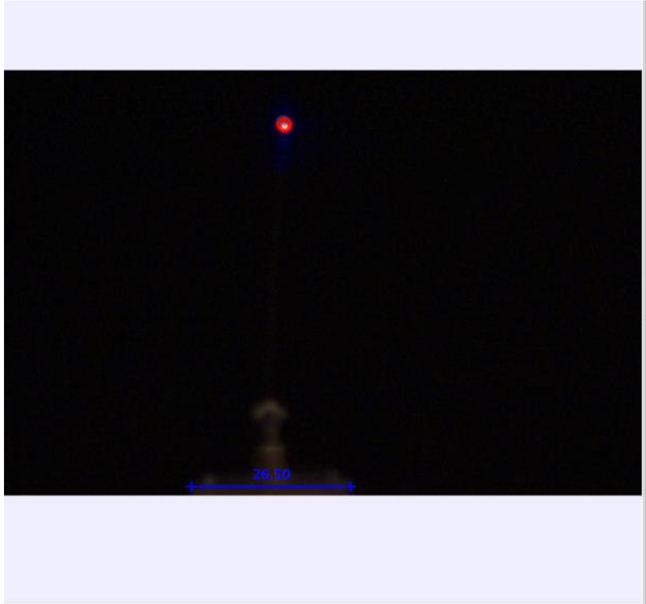
- Conventional electric engine according to US department of energy : $\eta_{\text{electric}} = 59\% - 62\%$

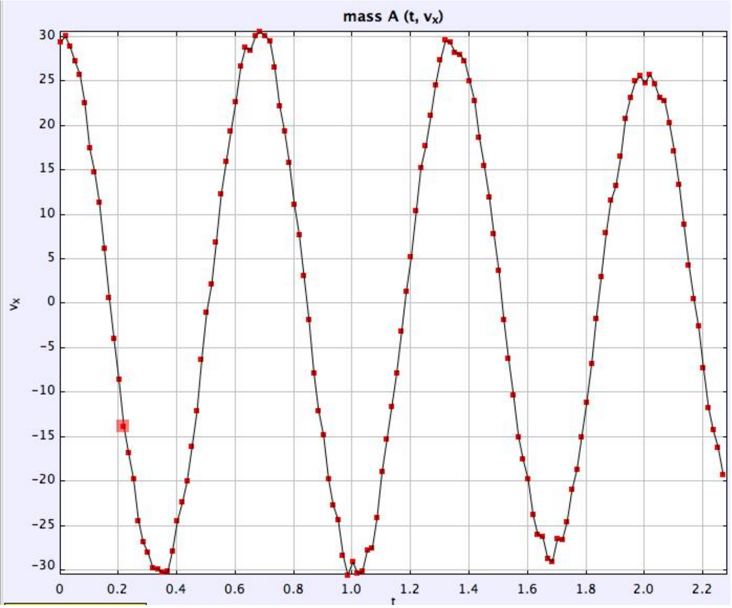
  https://www.fueleconomy.gov/feg/evtech.shtml

- Own ”engine” : $P_{out} \sim 10^{-7}\text{W}$ $\quad \eta_{\text{plasma rotor}} = 2 \times 10^{-9} \approx 4 \times 10^{-9} \eta_{\text{electric}}$

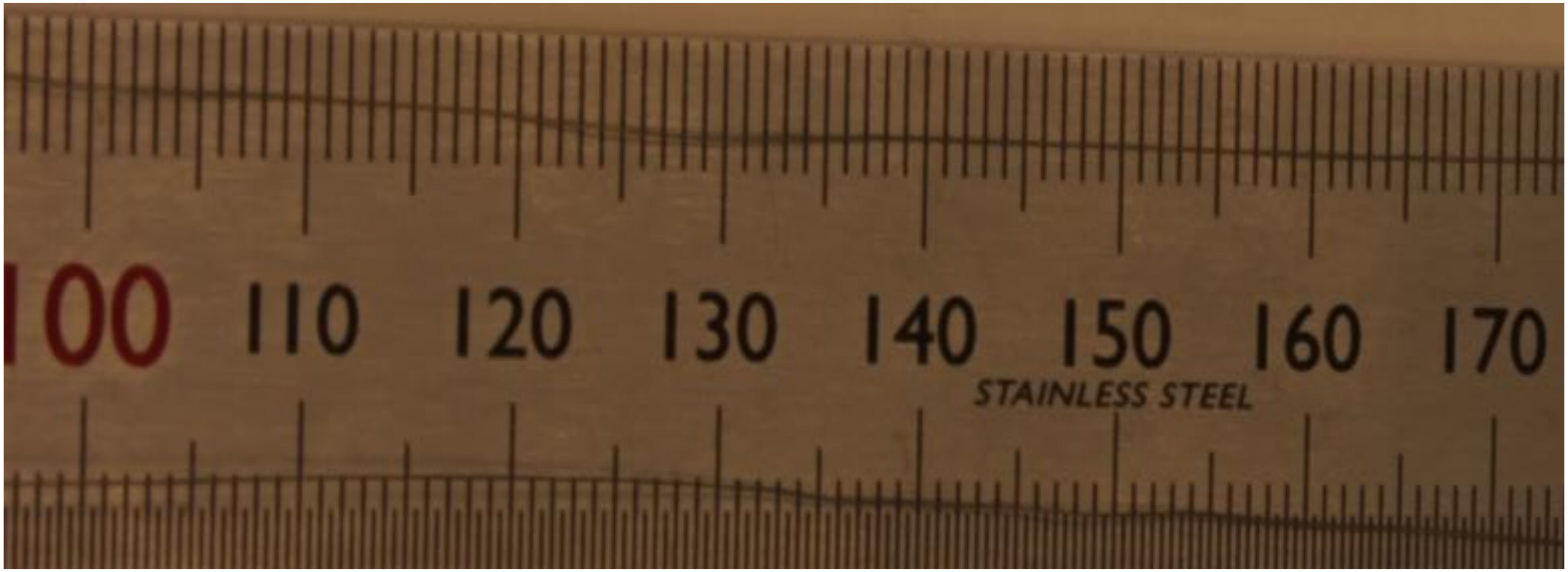
- 2111fps video
- Track change in peak speed for $t_{trk} = 19\text{ms}$

$$P_{out} = \frac{\Delta E_{kin}}{t_{trk}}$$

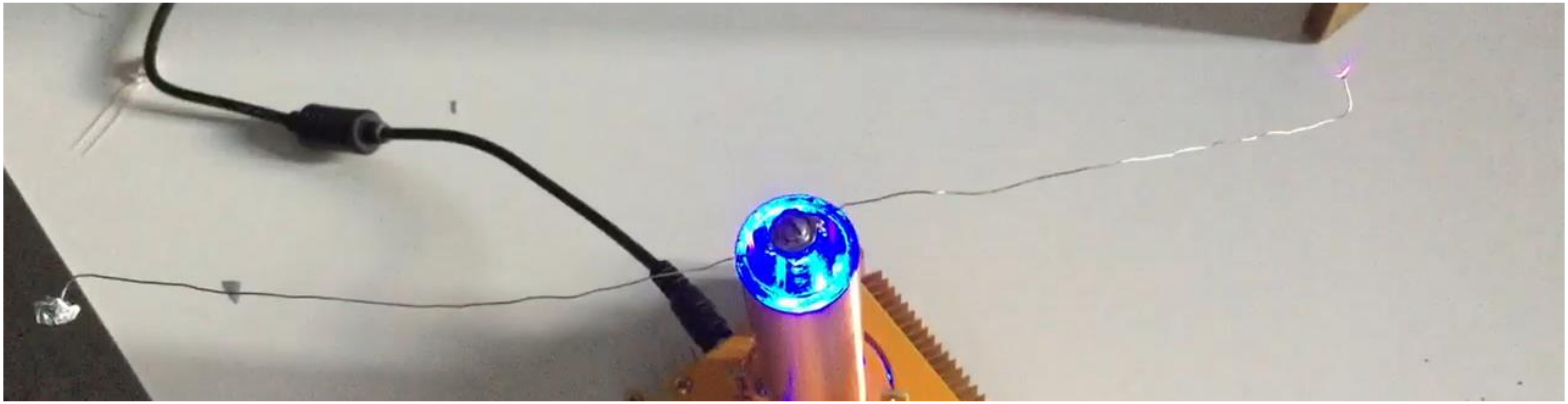




Appendix:
Thickness measurement



Appendix :
Effect of fewer discharge points



Peeks law

- Cylindrical geometry : $$E_{crit} = 3.1 \times 10^4 \delta \left(1 + \frac{0.308}{\sqrt{\delta r}}\right)$$
- Spherical gemoetry : $$E_{crit} = 3.1 \times 10^4 \delta \left(1 + \frac{0.308}{\sqrt{0.5\delta r}}\right)$$

$$\delta = \frac{T_0 p}{T p_0}$$

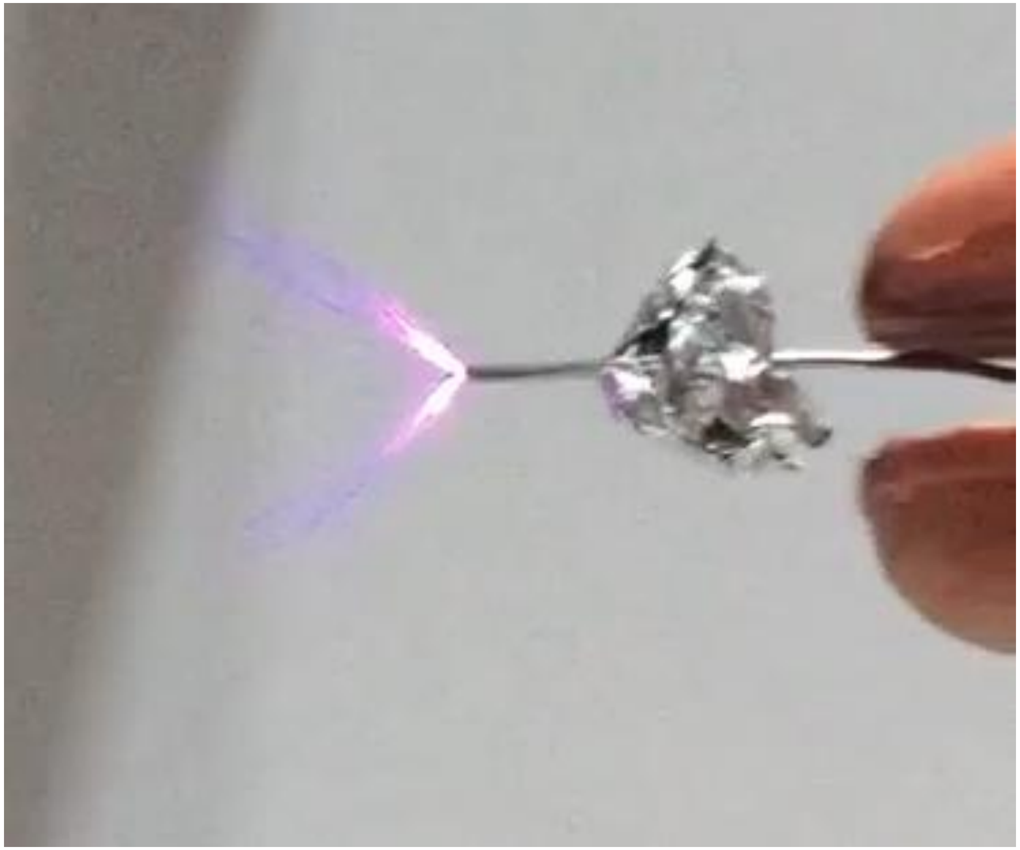
$$[E_{crit}] = \frac{\mathrm{V}}{\mathrm{cm}}$$

$$[r] = \mathrm{cm}$$

Adamiak, P. Atten,

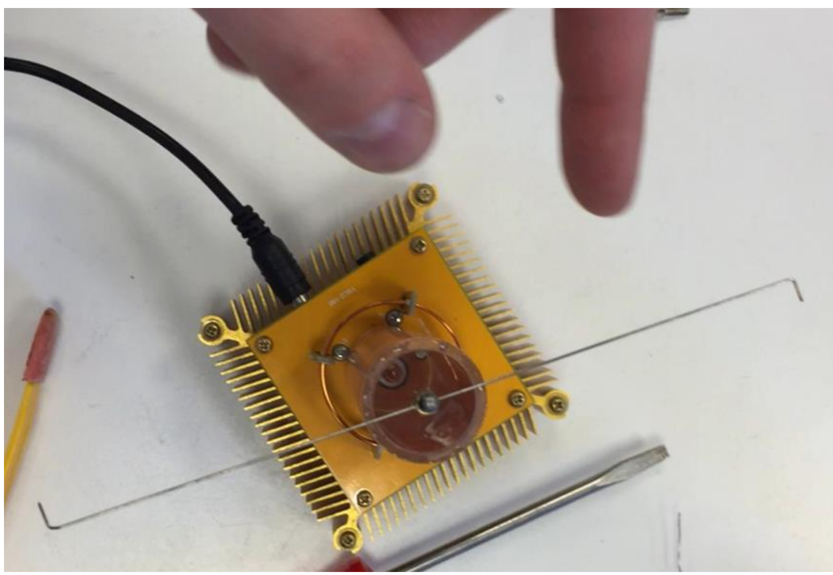
Simulation of corona discharge in point–plane configuration

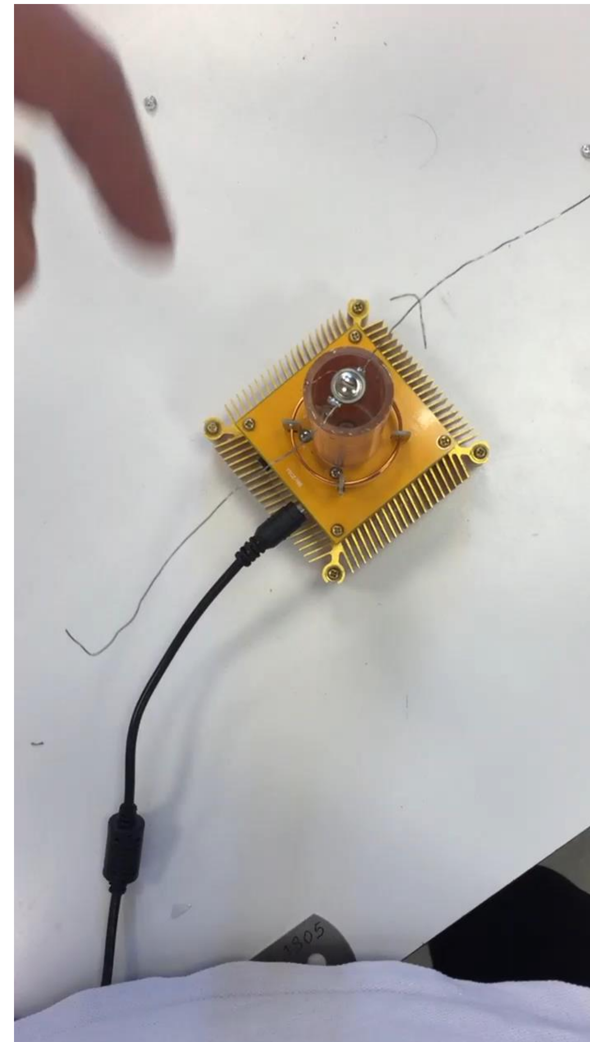
Appendix : Rotor with attachments



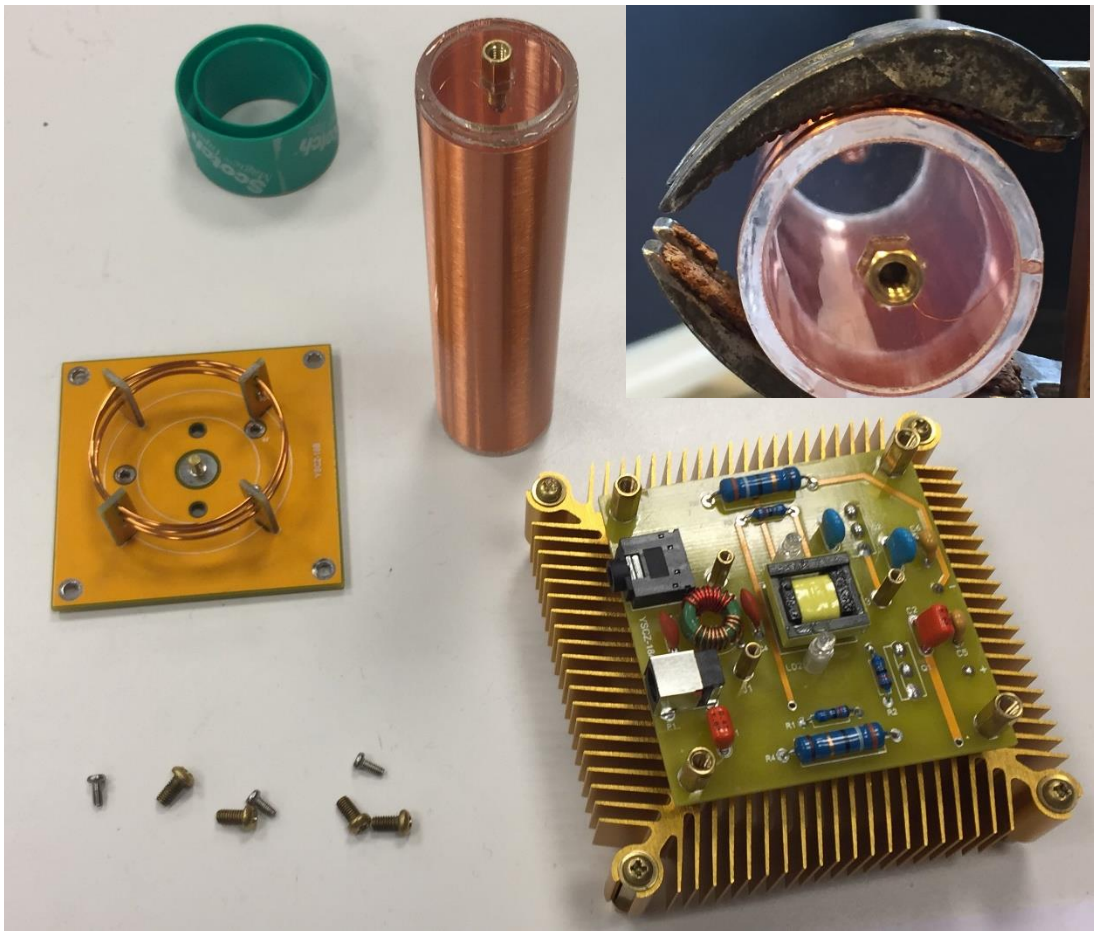
Friction issue

- Ball bearing has way too high friction
- Best solution seems a screw slightly too small for winding





My tesla coil



Specs

$$d_{p,wire} = 1.1 \pm 0.05\text{mm}$$

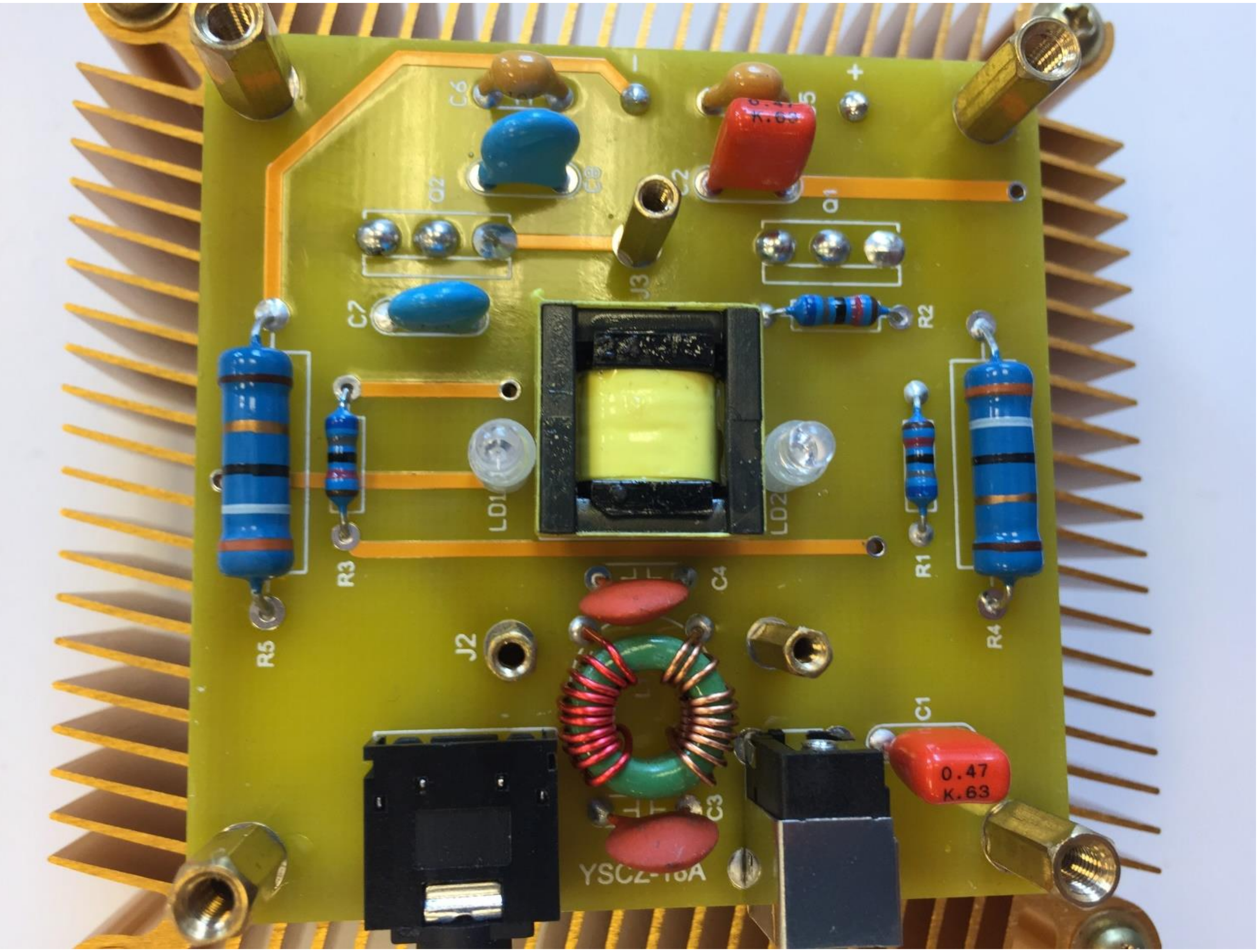
$$spacing_p = 2.40 \pm 0.05\text{mm}$$

$$D_p = 37.9 \pm 0.05\text{mm}$$

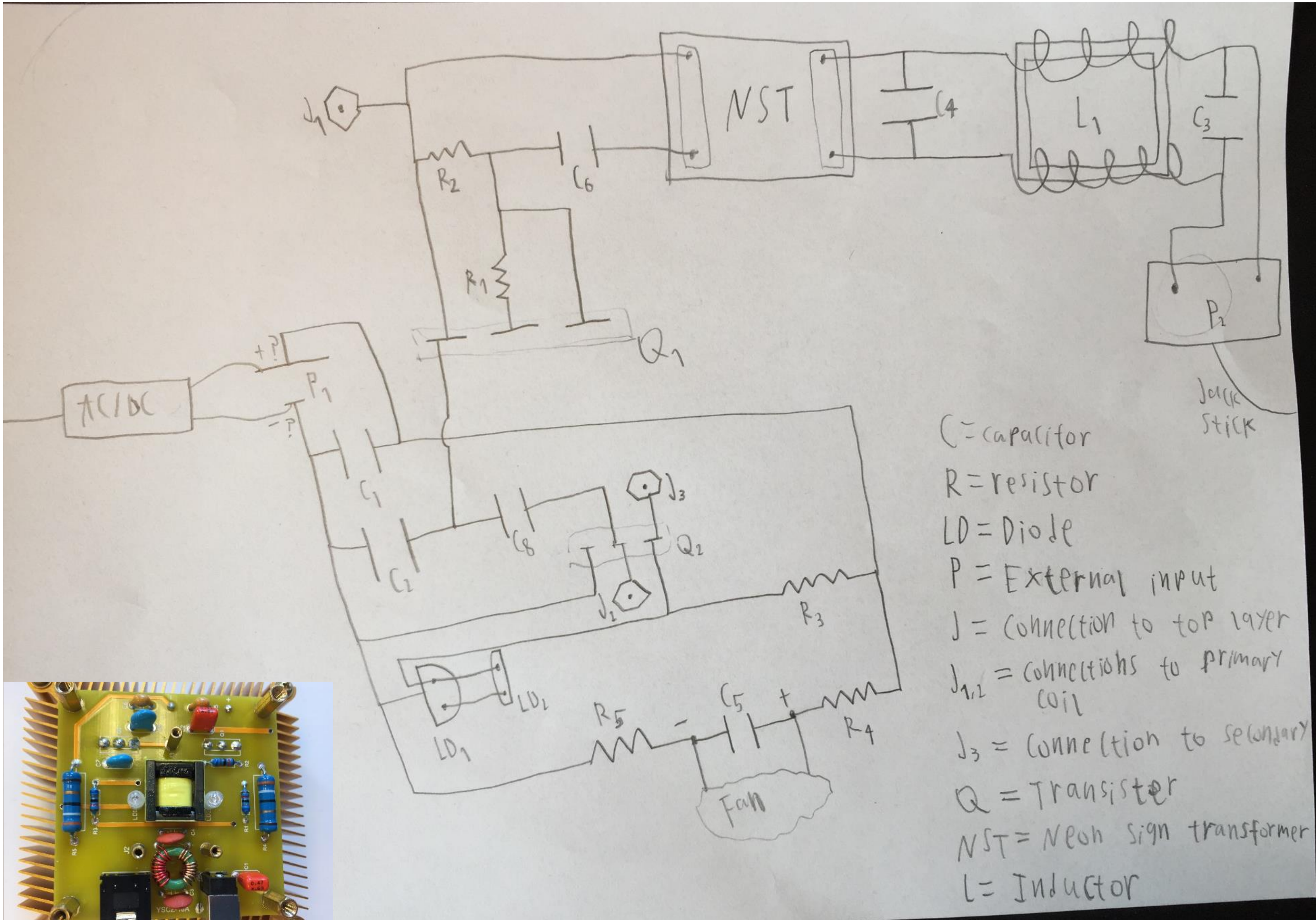
$$D_s = 26.2 \pm 0.05\text{mm}$$

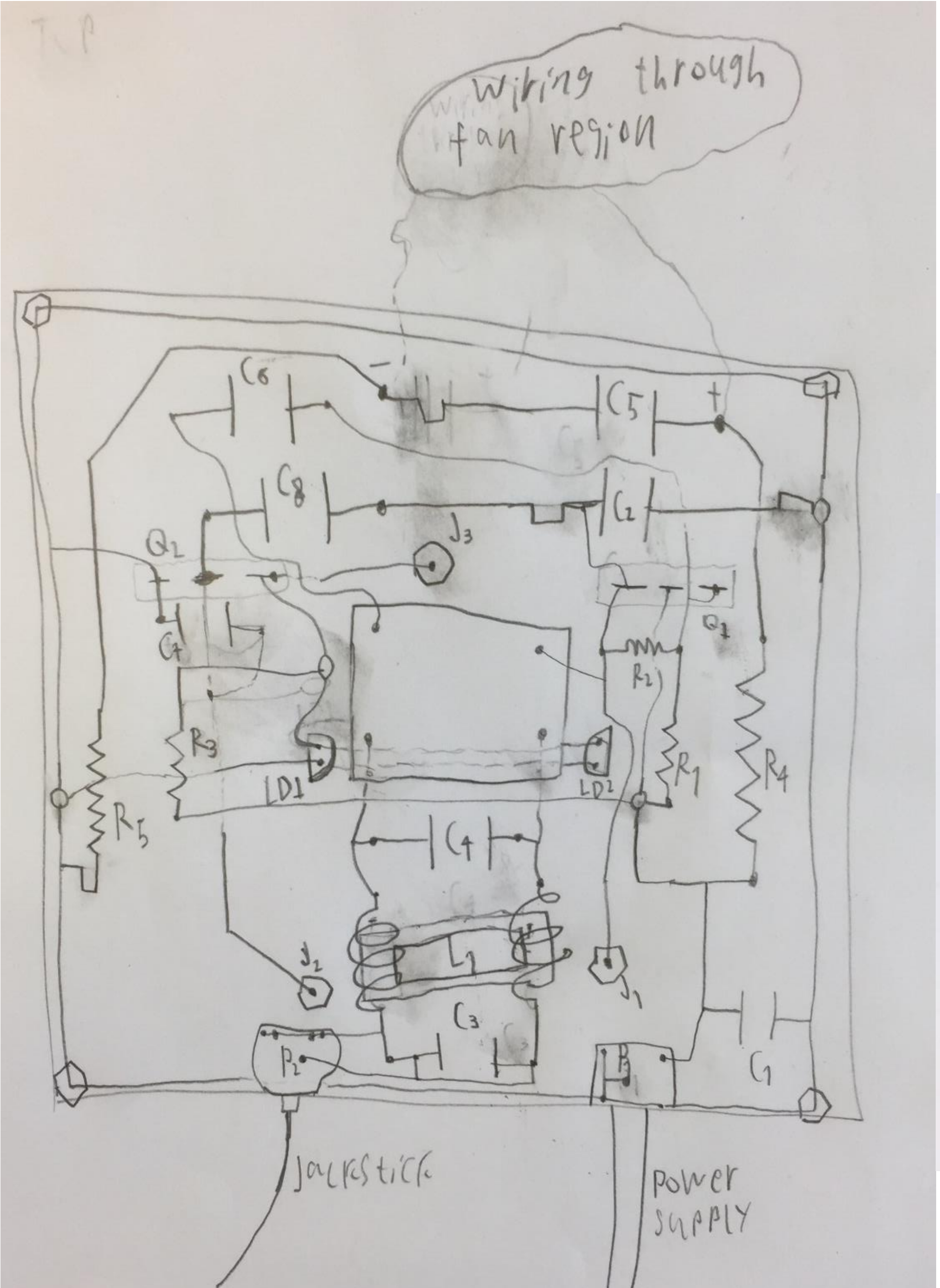
$$h_s = 93.85 \pm 0.05\text{mm}$$

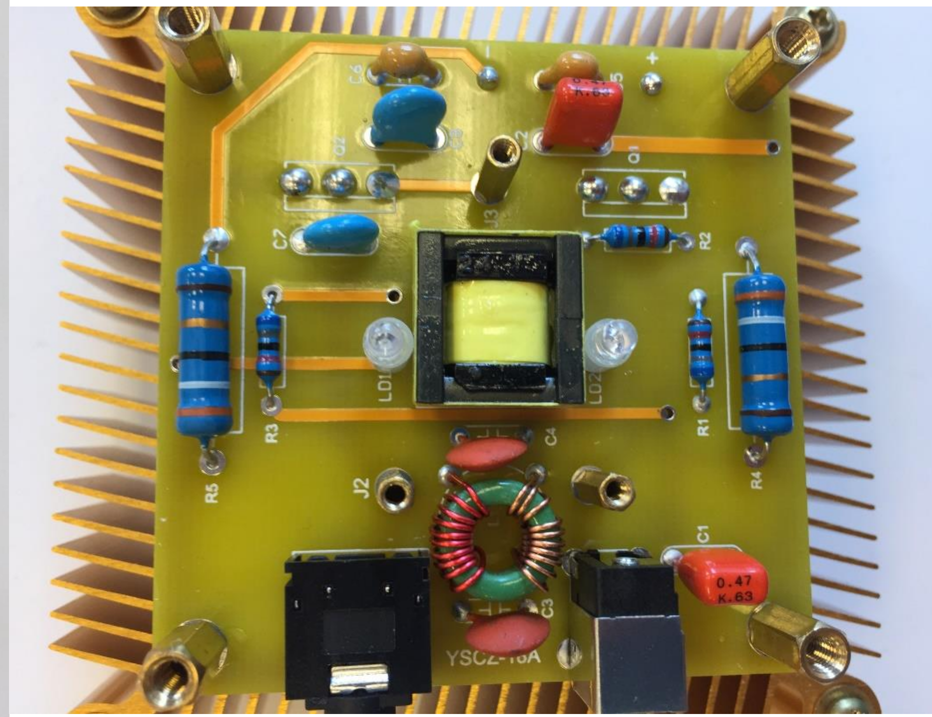
Circuit board



Circuit board diagram




wiring through fan region
C6
C5
C8
C2
J3
Q2
Q1
C7
R2
R3
R1
R4
LD1
LD2
R5
C4
L1
J2
J1
C3
C1
P2
jackstick
power supply


0.47
K.63
YSCZ-16A