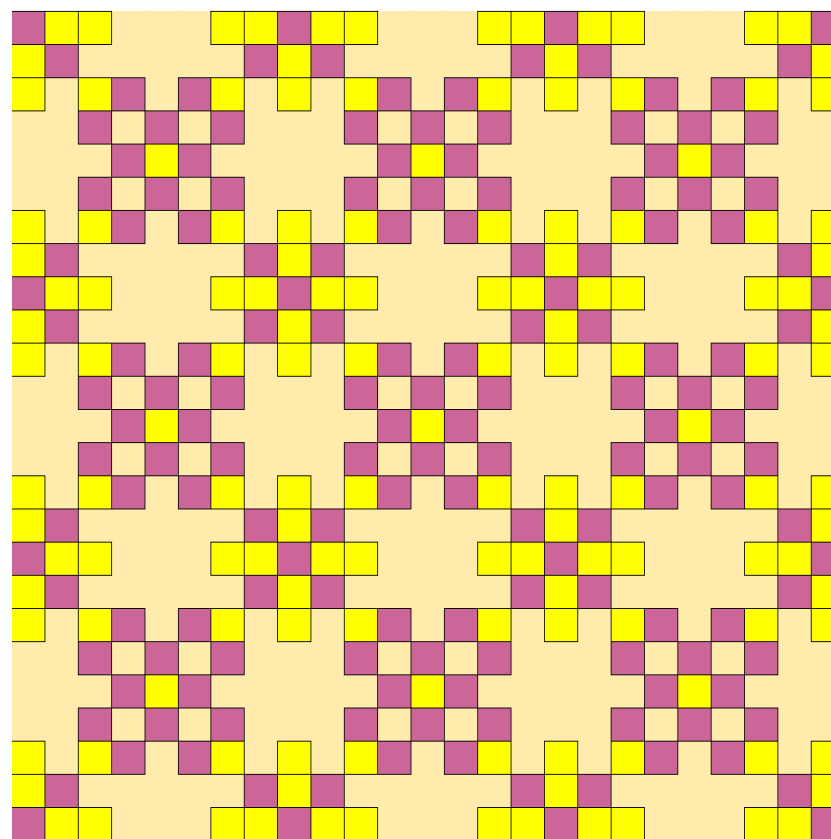
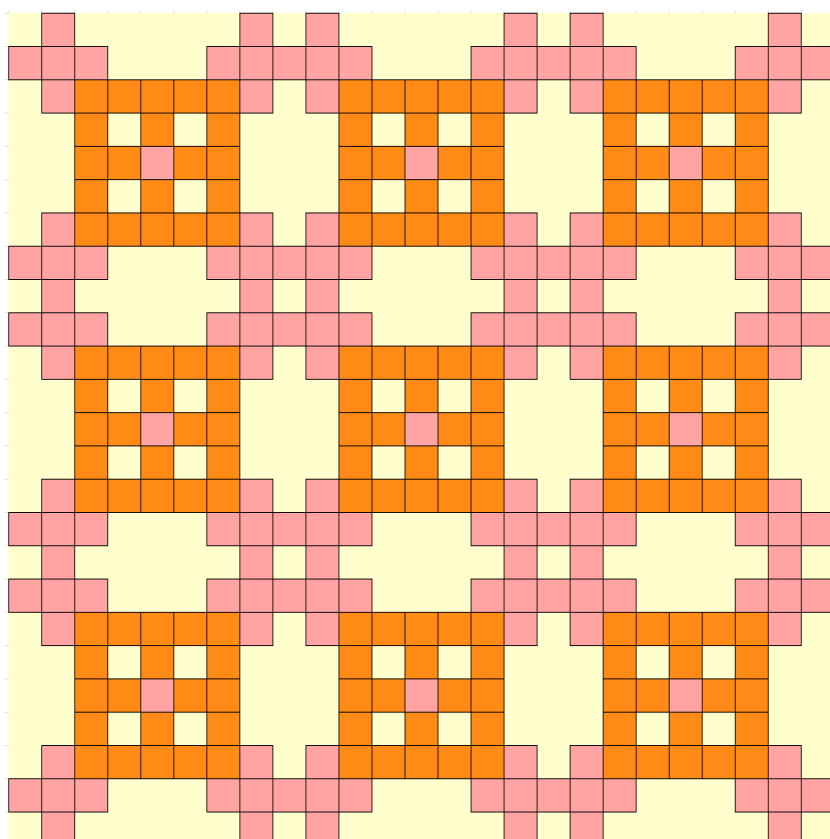


21 Mathematical Highlights for 2021

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W E L C O M E - 2021
Mathematical Style

Life in Colors in 2021



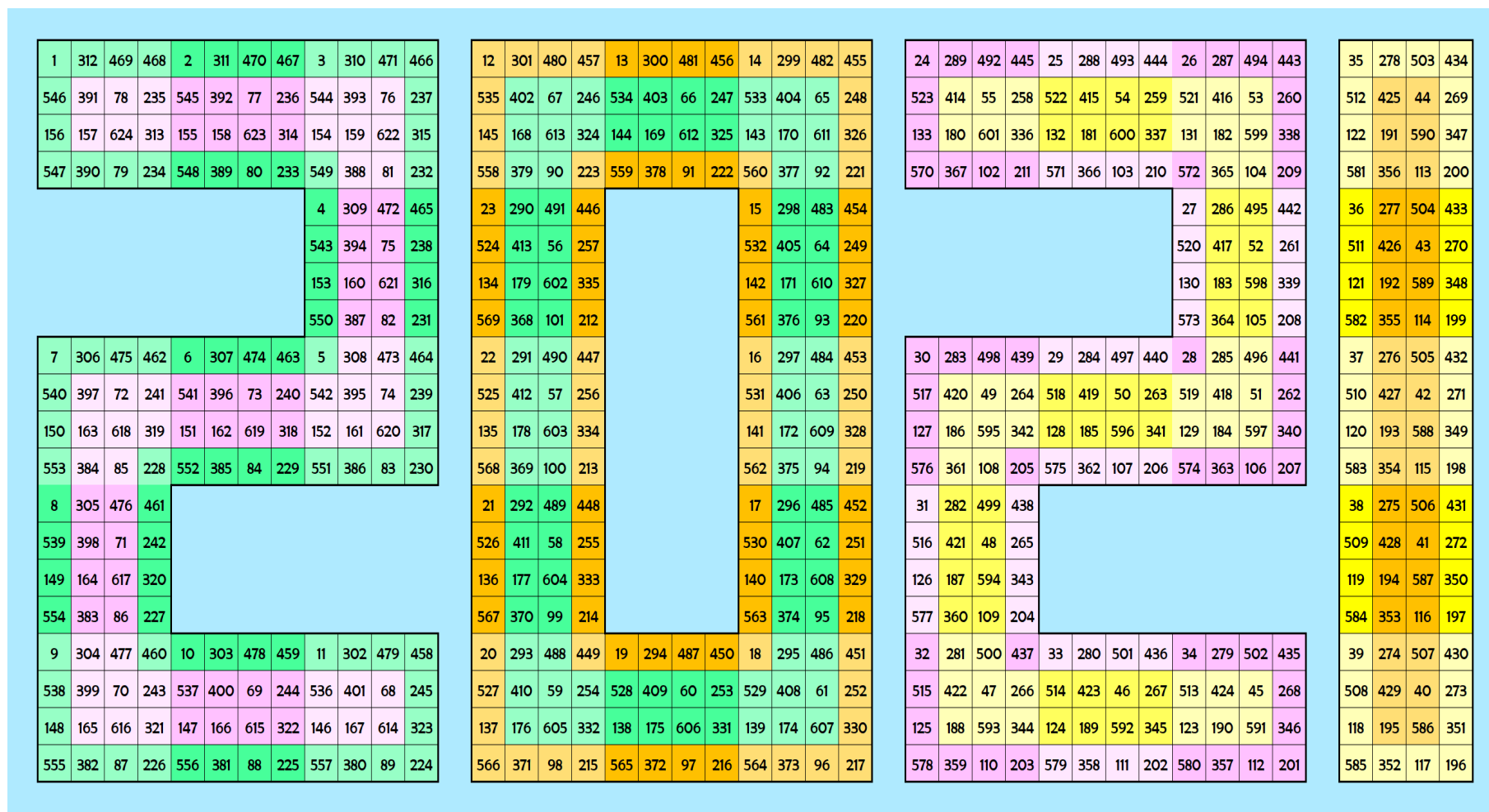
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2021 in Colors



It is made of 39 **pandiagonal** magic squares of order 4 of equal magic sums $S_{4 \times 4} := 1250$, using natural numbers from 1 to 624.

Abstract

This short work brings 21 main representations of 2021 in different ways. These representations are of crazy-type, running numbers, single digit, single letter, Triangular, Fibonacci, palindromic-type, prime numbers, embedded, repeated digits, colored patterns, magic squares, etc..

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1 Crazy Representations

Below are representations of 2021 in terms of 1 to 9 and 9 to 1. These are just with **basic operations** and with **factorial**.

1.1 Basic Operations

$$\begin{aligned} \mathbf{2021} &:= 1 + 2 \times (3^4 \times (5 + 6) + 7 \times (8 + 9)) \\ &:= 12 \times (3 \times 4 + 5 + 6) \times 7 + 89 \\ &:= -1 + 2 \times (3 + 4^5 - 6 + 7 - 8 - 9) \\ &:= (9 \times 8 + 7 + 6) \times 5 \times 4 + 321 \\ &:= (-9 + 8) \times 7 - 6 \times (5 - (4 + 3)^{2+1}) \end{aligned}$$

1.2 Factorial

$$\begin{aligned} \mathbf{2021} &:= -1 - 2 + (3! - 4)^{5+6} - 7 - 8 - 9 \\ &:= 1 - (2 \times 3)! + 4 + (5! - 6) \times (7 + 8 + 9) \\ &:= -9 + 8 + 7! - (6 + 5!) \times 4! + 3 + 2 + 1 \\ &:= 9 - 8 - 7! + 6^5 + 4 - (3 + 2 + 1)! \end{aligned}$$

1.3 Square-Root

$$\begin{aligned} \mathbf{2021} &:= (1 - 2) \times 3 + \sqrt{4^{5+6}} - 7 - 8 - 9 \\ &:= 1 + (-2 + 3) \times \left(4 \times \left(-\sqrt{56 - 7} + 8^{\sqrt{9}} \right) \right) \\ &:= (9 + 8 \times 7) \times \left(6 + 5^{\sqrt{4}} \right) + 3 + 2 + 1 \end{aligned}$$

1.4 Factorial and Square-Root

$$\mathbf{2021} := -9 + 8 \times \left(7 + 6 + 5! \times \sqrt{4} \right) + 3 + 2 + 1$$

1.5 Fibonacci and Triangular Numbers

$$\begin{aligned}2021 &:= 1 + T(23) + T(4) + T(56) + 7 \times F(8) - 9 \\ &:= -9 + 87 + F(6) + T(5) \times (4 \times 32 + 1)\end{aligned}$$

More details on **Fibonacci** and **Triangular** numbers are given in Section 14.

1.6 1 to 10 Numbers: Increasing and Decreasing

$$\begin{aligned}2021 &:= (1 + 2 \times 3 + 45 \times 6) \times 7 + 8 \times 9 + 10 \\ &:= 10 + 9 + 87 + 65 + 43^2 + 1.\end{aligned}$$

2 Single Digit and Single Letter Representations

2.1 Single Digit

$$\begin{aligned}2021 &:= (1 + 1)^{11} - (1 + 1 + 1)^{(1+1+1)} \\ &:= \left(2 \times 22 + \frac{2}{2}\right)^2 - 2 - 2 \\ &:= \left(3 - \frac{3}{3}\right) \binom{33}{3} - 3^3 \\ &:= 4 + (4 + 4) \times (4^4 - 4) + \frac{4}{4} \\ &:= 5 + \frac{(5 + 5)^{(5-\frac{5}{5})} + 55}{5} + 5 \\ &:= 6 + 66 \times (6 \times 6 - 6) + 6 \times 6 - \frac{6}{6} \\ &:= \frac{77 + 7}{7} + 7 \times (7 \times (7 \times 7 - 7) - 7) \\ &:= 8 \times (8 + 8) \times (8 + 8) - 8 - 8 - \frac{88}{8} \\ &:= \left(\frac{9 + 9}{9}\right) \binom{99}{9} - (9 + 9 + 9)\end{aligned}$$

2.1.1 Patterns in Single Digit

$$\begin{aligned} 2021 &:= 6 + 66 \times (6 \times 6 - 6) + 6 \times 6 - \frac{6}{6} \\ 20021 &:= 6 + 666 \times (6 \times 6 - 6) + 6 \times 6 - \frac{6}{6} \\ 200021 &:= 6 + 6666 \times (6 \times 6 - 6) + 6 \times 6 - \frac{6}{6} \\ 2000021 &:= 6 + 66666 \times (6 \times 6 - 6) + 6 \times 6 - \frac{6}{6} \end{aligned}$$

The above pattern is non uniform. This means, it can't be extended to other digits. Let's see below uniform pattern with all the digits:

$$\begin{aligned} 2021 &:= \frac{(11111 - 1) \times (1 + 1)}{1 \times 11} + \frac{1}{1} \\ 20021 &:= \frac{(1111111 - 1) \times (1 + 1)}{1 \times 111} + \frac{1}{1} \\ 200021 &:= \frac{(111111111 - 1) \times (1 + 1)}{1 \times 1111} + \frac{1}{1} \\ 2000021 &:= \frac{(11111111111 - 1) \times (1 + 1)}{1 \times 11111} + \frac{1}{1} \end{aligned}$$

$$\begin{aligned} 2021 &:= \frac{(22222 - 2) \times (2 + 2)}{2 \times 22} + \frac{2}{2} \\ 20021 &:= \frac{(2222222 - 2) \times (2 + 2)}{2 \times 222} + \frac{2}{2} \\ 200021 &:= \frac{(222222222 - 2) \times (2 + 2)}{2 \times 2222} + \frac{2}{2} \\ 2000021 &:= \frac{(22222222222 - 2) \times (2 + 2)}{2 \times 22222} + \frac{2}{2} \end{aligned}$$

$$\begin{aligned} 2021 &:= \frac{(33333 - 3) \times (3 + 3)}{3 \times 33} + \frac{3}{3} \\ 20021 &:= \frac{(3333333 - 3) \times (3 + 3)}{3 \times 333} + \frac{3}{3} \\ 200021 &:= \frac{(333333333 - 3) \times (3 + 3)}{3 \times 3333} + \frac{3}{3} \\ 2000021 &:= \frac{(33333333333 - 3) \times (3 + 3)}{3 \times 33333} + \frac{3}{3} \end{aligned}$$

$$\begin{aligned} 2021 &:= \frac{(44444 - 4) \times (4 + 4)}{4 \times 44} + \frac{4}{4} \\ 20021 &:= \frac{(4444444 - 4) \times (4 + 4)}{4 \times 444} + \frac{4}{4} \\ 200021 &:= \frac{(444444444 - 4) \times (4 + 4)}{4 \times 4444} + \frac{4}{4} \\ 2000021 &:= \frac{(44444444444 - 4) \times (4 + 4)}{4 \times 44444} + \frac{4}{4} \end{aligned}$$

$$\begin{aligned} 2021 &:= \frac{(55555 - 5) \times (5 + 5)}{5 \times 55} + \frac{5}{5} \\ 20021 &:= \frac{(5555555 - 5) \times (5 + 5)}{5 \times 555} + \frac{5}{5} \\ 200021 &:= \frac{(555555555 - 5) \times (5 + 5)}{5 \times 5555} + \frac{5}{5} \\ 2000021 &:= \frac{(55555555555 - 5) \times (5 + 5)}{5 \times 55555} + \frac{5}{5} \end{aligned}$$

$$\begin{aligned} 2021 &:= \frac{(66666 - 6) \times (6 + 6)}{6 \times 66} + \frac{6}{6} \\ 20021 &:= \frac{(6666666 - 6) \times (6 + 6)}{6 \times 666} + \frac{6}{6} \\ 200021 &:= \frac{(666666666 - 6) \times (6 + 6)}{6 \times 6666} + \frac{6}{6} \\ 2000021 &:= \frac{(66666666666 - 6) \times (6 + 6)}{6 \times 66666} + \frac{6}{6} \end{aligned}$$

$$\begin{aligned} 2021 &:= \frac{(77777 - 7) \times (7 + 7)}{7 \times 77} + \frac{7}{7} \\ 20021 &:= \frac{(7777777 - 7) \times (7 + 7)}{7 \times 777} + \frac{7}{7} \\ 200021 &:= \frac{(777777777 - 7) \times (7 + 7)}{7 \times 7777} + \frac{7}{7} \\ 2000021 &:= \frac{(77777777777 - 7) \times (7 + 7)}{7 \times 77777} + \frac{7}{7} \end{aligned}$$

$$\begin{aligned} 2021 &:= \frac{(88888 - 8) \times (8 + 8)}{8 \times 88} + \frac{8}{8} \\ 20021 &:= \frac{(8888888 - 8) \times (8 + 8)}{8 \times 888} + \frac{8}{8} \\ 200021 &:= \frac{(888888888 - 8) \times (8 + 8)}{8 \times 8888} + \frac{8}{8} \\ 2000021 &:= \frac{(88888888888 - 8) \times (8 + 8)}{8 \times 88888} + \frac{8}{8} \end{aligned}$$

$$\begin{aligned} 2021 &:= \frac{(99999 - 9) \times (9 + 9)}{9 \times 99} + \frac{9}{9} \\ 20021 &:= \frac{(9999999 - 9) \times (9 + 9)}{9 \times 999} + \frac{9}{9} \\ 200021 &:= \frac{(999999999 - 9) \times (9 + 9)}{9 \times 9999} + \frac{9}{9} \\ 2000021 &:= \frac{(99999999999 - 9) \times (9 + 9)}{9 \times 99999} + \frac{9}{9} \end{aligned}$$

2.2 Single Letter

$$2021 := \frac{(aaaaa - a) \times (a + a) + a \times aa}{a \times aa}$$

where, $aaaaa = a10^4 + a10^3 + a10^2 + a10 + a$,

$$aaaa = a10^3 + a10^2 + a10 + a,$$

$$aaa = a10^2 + a10 + a,$$

$$aa = a10 + a, \text{ etc.}$$

$$a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

2.2.1 Patterns in Single Letter

$$2021 := \frac{(aaaaa - a) \times (a + a) + a \times aa}{a \times aa}$$

$$20021 := \frac{(aaaaaaaa - a) \times (a + a) + a \times aa}{a \times aaa}$$

$$200021 := \frac{(aaaaaaaaaa - a) \times (a + a) + a \times aa}{a \times aaaa}$$

$$2000021 := \frac{(aaaaaaaaaaaa - a) \times (a + a) + a \times aa}{a \times aaaaa}$$

$$2021 := \frac{(aaaaa - a) \times (a + a) + a \times aa}{a \times aa}$$

$$202021 := \frac{(aaaaaaaa - a) \times (a + a) + a \times aa}{a \times aa}$$

$$20202021 := \frac{(aaaaaaaaaa - a) \times (a + a) + a \times aa}{a \times aa}$$

$$2020202021 := \frac{(aaaaaaaaaaaa - a) \times (a + a) + a \times aa}{a \times aa}$$

where, $aaaaa = a10^4 + a10^3 + a10^2 + a10 + a$,

$aaaa = a10^3 + a10^2 + a10 + a$,

$aaa = a10^2 + a10 + a$,

$aa = a10 + a$, etc.

$a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

3 Selfie Numbers

We don't have representations of only 2021 in terms of its digits. Below are some representations of 2021 included in other numbers. There are in digit's orders and their reverse. These are in terms of **Fibonacci** and **Triangular** numbers.

3.1 Digit's Order

$$42021 := F(4) \times T(T(T(2))) \times (T(T(F(T(T(02)))))) + 1$$

$$20217 := F(20) \times T(2) - T(-1 + F(7))$$

$$372021 := F(3 + 7) \times (F(20) - F(2)) + 1$$

3.2 Reverse Order of Digits

$$\begin{aligned}4 \mathbf{2021} &:= (1 + T(T(F(T(T(2)))))) \times T(02) \times T(T(F(4))) \\ \mathbf{2021} 7 &:= -T(F(7) - 1) + F(20) \times T(2) \\ 37 \mathbf{2021} &:= 1 + (F(20) - F(2)) \times F(7 + 3)\end{aligned}$$

More details on **Fibonacci** and **Triangular** numbers are given in Section 14.

4 Semi-Selfie Expressions

4.1 Semi-Selfies With 2021

$$\begin{aligned}527711 \mathbf{2021} &:= (-5 - 277 + 1 + 1 + \mathbf{2021})^3 \\ 100 \mathbf{2021} 0201 &:= (100202 - 102 + 01)^2 \\ \mathbf{2021} 2559424 &:= (202 + 1 + 2559 - 42 + 4)^3\end{aligned}$$

4.2 Semi-Selfies Powers of 21

$$\begin{aligned}\mathbf{21}^6 &:= 4084101 = (4 - 084 + 101)^5 \\ \mathbf{21}^6 &:= 85766121 = (8 - 5 + 7 + 6 - 6 + 12 - 1)^6 \\ &= (8 - 57 + 66 + 1 + 2 + 1)^6 \\ &= (8 - 5 + 76 - 61 + 2 + 1)^6 \\ &= (8 + 57 - 66 + 1 + 21)^6 \\ \mathbf{21}^7 &:= 1801088541 = (18 + 01 + 08 - 8 + 5 - 4 + 1)^7 \\ &= (1 - 8 + 010 + 8 + 8 + 5 - 4 + 1)^7 \\ &= (1 - 80 + 108 - 8 + 5 - 4 - 1)^7 \\ &= (1 - 80 + 10 + 88 + 5 - 4 + 1)^7 \\ &= (1 - 8 + 0108 - 85 + 4 + 1)^7 \\ \mathbf{21}^8 &:= 37822859361 = (37 - 8 - 2 - 2 - 8 + 5 + 9 - 3 - 6 - 1)^8 \\ &= (37 + 8 - 2 - 28 - 5 + 9 - 3 + 6 - 1)^8\end{aligned}$$

$$\begin{aligned}
 &= (3 - 78 + 2 - 2 + 85 + 9 - 3 + 6 - 1)^8 \\
 &= (3 - 7 + 82 - 2 + 8 - 59 + 3 - 6 - 1)^8 \\
 &= (3 + 7 - 8 - 2 - 28 + 59 - 3 - 6 - 1)^8 \\
 &= (37 - 82 - 2 + 85 - 9 - 3 - 6 + 1)^8 \\
 &= (3 - 7 + 8 - 22 + 85 - 9 - 36 - 1)^8 \\
 &= (378 + 2 - 2 + 8 + 5 - 9 - 361)^8 \\
 &= (37 - 8 - 22 - 8 + 59 - 36 - 1)^8 \\
 &= (37 - 8 - 2 - 28 + 59 - 36 - 1)^8
 \end{aligned}$$

5 Running Equality Expressions

$$\mathbf{2021} := (-1 + 2) \times T(3 \times T(T(F(4)))) + 5 = -T(T(6)) - F(F(7)) + T(T(8) + F(9))$$

The functions F and T are **Fibonacci** and **Triangular** numbers. These explained in more details in Section 14.

Still there are representations of 21 in simplified way. See below

$$9 \times 8 + 7 - 65 + 4 + 3 = 21$$

See more,

$$\begin{aligned}
 \mathbf{21} &:= 1 + (2 + 3) \times 4 = 5 + 6 - 7 + 8 + 9 \\
 &:= 1 \times 23 - \sqrt{4} = 5 + 6 - 7 + 8 + 9 \\
 &:= 12/3 \times 4 + 5 = (6 - 7 + 8) \times \sqrt{9}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{21} &:= (9 + 87)/6 + 5 = (4 - 3) \times 21 \\
 &= 4 - 3 + 2 \times 10 \\
 &:= 98 - 76 - 5 + 4 = 3 \times ((2 + 1)! + 0!) \\
 &:= \sqrt{9} \times (8 - 7 + 6) = 54 - 32 - 1 \\
 &:= (\sqrt{9})! + 8 + 7 = 65 - 43 - 2 + 1 \\
 &= \sqrt{654 - 3 - 210}
 \end{aligned}$$

6 Same Digits Power and Bases

6.1 Pyramid Style

$$\begin{aligned} \mathbf{2021} &:= -1^6 + 2^1 + 3^7 - 6^3 + 7^2 \\ &:= -1^3 - 2^5 + 3^6 + 4^1 + 5^2 + 6^4 \\ &:= 1^8 - 3^5 + 4^9 + 5^6 - 6^7 + 7^3 + 8^4 - 9^1 \\ &:= 0^4 + 1^7 + 2^8 - 3^9 + 4^6 + 5^2 + 6^1 + 7^5 + 8^3 + 9^0 \end{aligned}$$

6.2 Pattern in Power and Bases

This section brings representations of 2021 and some patterns. These representations are in such a way that the bases and powers are of same digits with different permutations.

$$\mathbf{2021} := 0^4 + 1^7 + 2^8 - 3^9 + 4^6 + 5^2 + 6^1 + 7^5 + 8^3 + 9^0$$

6.2.1 Blocks of 10 Sequential Pattern With 2021

$$\mathbf{20210} := 2^9 + 3^7 - 5^2 + 7^5 + 9^3$$

$$\mathbf{20211} := 1^2 + 2^6 + 3^9 + 4^4 + 6^3 - 9^1$$

$$\mathbf{20212} := -2^4 + 4^8 - 5^7 + 7^2 + 8^5$$

$$\mathbf{20213} := 2^5 + 3^9 - 4^4 + 5^2 + 9^3$$

$$\mathbf{20214} := 1^1 + 2^5 + 3^9 - 4^4 + 5^2 + 9^3$$

$$\mathbf{20215} := 1^2 + 2^6 + 3^9 - 4^4 - 6^1 + 9^3$$

$$\mathbf{20216} := -1^2 - 2^8 + 4^7 - 7^1 + 8^4$$

$$\mathbf{20217} := 1^4 - 2^8 + 3^9 - 4^1 + 8^2 + 9^3$$

$$\mathbf{20218} := 1^2 - 2^8 + 4^7 - 7^1 + 8^4$$

$$\mathbf{20219} := -1^1 - 2^8 + 3^9 + 8^2 + 9^3$$

$$\mathbf{12021} := -1^1 + 2^4 + 3^2 + 4^6 + 5^3 + 6^5$$

$$\mathbf{212021} := 1^4 - 2^1 - 3^8 - 4^6 - 5^3 + 6^2 + 8^5$$

$$\mathbf{32021} := 1^4 - 2^5 + 4^7 + 5^6 + 6^2 + 7^1$$

$$\mathbf{42021} := 1^9 - 3^3 - 4^7 - 5^4 + 7^1 + 9^5$$

$$\mathbf{52021} := -2^6 + 3^8 + 5^7 + 6^3 - 7^2 - 8^5$$

$$\mathbf{62021} := 2^8 - 3^4 + 4^2 + 5^7 - 7^5 + 8^3$$

$$\mathbf{72021} := -1^9 - 2^3 - 3^1 + 4^8 - 8^2 + 9^4$$

$$\mathbf{82021} := 1^9 + 4^8 - 5^1 - 6^6 + 8^4 + 9^5$$

$$\mathbf{92021} := 1^8 + 5^7 + 6^6 + 7^1 - 8^5$$

$$\mathbf{102021} := 2^9 + 4^2 + 5^7 + 7^5 + 9^4$$

$$\mathbf{112021} := 1^3 - 2^5 - 3^4 + 4^8 + 5^1 + 6^6 - 8^2$$

$$\mathbf{122021} := -1^1 - 2^9 - 3^7 + 5^2 + 6^5 + 7^6 - 9^3$$

$$\mathbf{132021} := 1^9 - 3^1 + 4^5 + 5^7 + 6^6 - 7^3 + 9^4$$

$$\mathbf{142021} := -1^3 + 2^9 + 3^1 + 5^7 + 6^6 + 7^5 - 9^2$$

$$\mathbf{162021} := -1^9 - 4^4 + 6^7 - 7^6 - 9^1$$

$$17 \mathbf{2021} := -1^1 - 2^5 - 4^2 - 5^8 + 6^4 + 7^7 - 8^6$$

$$18 \mathbf{2021} := 2^4 - 3^3 - 4^8 - 5^6 + 6^7 - 7^5 + 8^2$$

$$19 \mathbf{2021} := -1^3 - 2^2 - 3^7 - 4^8 + 6^1 - 7^4 + 8^6$$

$$20 \mathbf{2021} := -1^6 + 2^8 + 4^9 - 5^2 - 6^4 - 8^1 - 9^5$$

$$1 \mathbf{2021} 0 := -1^5 + 3^7 + 4^4 + 5^3 - 6^1 + 7^6$$

$$1 \mathbf{2021} 1 := 3^3 - 4^9 + 5^8 + 6^6 + 8^4 - 9^5$$

$$1 \mathbf{2021} 2 := -1^1 + 2^9 + 3^7 - 6^3 + 7^6 + 9^2$$

$$1 \mathbf{2021} 3 := 2^9 + 3^7 - 6^3 + 7^6 + 9^2$$

$$1 \mathbf{2021} 4 := 1^1 + 2^9 + 3^7 - 6^3 + 7^6 + 9^2$$

$$1 \mathbf{2021} 5 := 1^4 + 2^8 + 3^7 + 4^3 - 6^1 + 7^6 + 8^2$$

$$1 \mathbf{2021} 6 := -1^7 - 2^9 + 5^5 - 6^2 + 7^6 - 9^1$$

$$1 \mathbf{2021} 7 := 1^7 + 2^1 + 3^8 + 4^3 + 6^2 + 7^6 - 8^4$$

$$1 \mathbf{2021} 8 := 1^7 - 2^9 + 5^5 - 6^2 + 7^6 - 9^1$$

$$1 \mathbf{2021} 9 := 1^1 - 2^7 + 3^8 + 4^2 + 6^3 + 7^6 - 8^4$$

6.2.2 Blocks of 100 Sequential Pattern With 2021

$$\mathbf{2021} 00 := 1^8 - 3^3 + 4^1 - 5^7 + 6^4 + 7^5 + 8^6$$

$$\mathbf{2021} 01 := 1^7 + 2^1 - 4^8 + 5^5 - 6^2 + 7^4 + 8^6$$

$$\mathbf{2021} 02 := 1^7 - 3^3 - 4^8 + 5^5 - 6^1 + 7^4 + 8^6$$

$$\mathbf{2021} 03 := -1^8 - 2^1 - 4^2 - 5^7 + 6^4 + 7^5 + 8^6$$

$$\mathbf{2021} 04 := -1^1 + 2^6 + 4^9 - 5^7 + 6^4 + 7^5 - 9^2$$

$$\mathbf{2021} 05 := 2^6 + 4^9 - 5^7 + 6^4 + 7^5 - 9^2$$

$$\mathbf{2021} 06 := -1^5 - 2^2 + 3^8 + 5^7 - 6^3 + 7^6 - 8^1$$

$$\mathbf{2021} 07 := -1^8 + 2^1 - 4^2 - 5^7 + 6^4 + 7^5 + 8^6$$

$$\mathbf{2021} 08 := -1^1 - 2^4 - 3^6 + 4^9 - 5^2 - 6^3 - 9^5$$

$$\mathbf{2021} 09 := -2^4 - 3^6 + 4^9 - 5^2 - 6^3 - 9^5$$

$$\mathbf{2021} 10 := -1^5 + 3^8 + 5^7 - 6^3 + 7^6 - 8^1$$

$$\mathbf{2021} 11 := 1^9 - 3^4 + 4^8 + 5^7 - 7^1 - 8^3 + 9^5$$

$$\mathbf{2021} 12 := 1^5 + 3^8 + 5^7 - 6^3 + 7^6 - 8^1$$

$$\mathbf{2021} 13 := -1^8 - 2^2 - 4^1 - 5^7 + 6^4 + 7^5 + 8^6$$

$$\mathbf{2021} 14 := 1^6 + 4^9 - 5^7 + 6^4 + 7^5 - 9^1$$

$$\mathbf{2021} 15 := -1^7 - 2^1 - 3^2 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 16 := 1^5 + 2^2 + 3^8 + 5^7 - 6^3 + 7^6 - 8^1$$

$$\mathbf{2021} 17 := -1^8 - 4^1 - 5^7 + 6^4 + 7^5 + 8^6$$

$$\mathbf{2021} 18 := 1^6 + 2^2 + 4^9 - 5^7 + 6^4 + 7^5 - 9^1$$

$$\mathbf{2021} 19 := 1^8 - 4^1 - 5^7 + 6^4 + 7^5 + 8^6$$

$$\mathbf{2021} 20 := -0^0 + 1^9 - 2^8 - 3^5 + 4^1 + 5^7 + 6^3 + 7^6 + 8^2 + 9^4$$

$$\mathbf{2021} 21 := 1^7 + 2^1 - 3^2 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 22 := -3^6 + 4^9 - 5^7 + 6^4 + 7^5 + 9^3$$

$$\mathbf{2021} 23 := -1^7 - 3^1 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 24 := 1^4 - 2^1 - 3^6 + 4^9 - 5^2 - 6^3 - 9^5$$

$$\mathbf{2021} 25 := 1^7 - 3^1 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 26 := -1^5 + 3^8 + 5^7 - 6^3 + 7^6 + 8^1$$

$$\mathbf{2021} 27 := 1^8 + 4^1 - 5^7 + 6^4 + 7^5 + 8^6$$

$$\mathbf{2021} 28 := 1^5 + 3^8 + 5^7 - 6^3 + 7^6 + 8^1$$

$$\mathbf{2021} 29 := -1^7 + 3^1 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 30 := -1^6 + 4^9 - 5^7 + 6^4 + 7^5 + 9^1$$

$$\mathbf{2021} 31 := 1^7 + 3^1 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 32 := 1^6 + 4^9 - 5^7 + 6^4 + 7^5 + 9^1$$

$$\mathbf{2021} 33 := -1^7 + 2^2 + 3^1 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 34 := -1^6 + 2^2 + 4^9 - 5^7 + 6^4 + 7^5 + 9^1$$

$$\mathbf{2021} 35 := 1^7 + 2^2 + 3^1 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 36 := 2^7 - 4^8 + 5^2 + 6^5 - 7^4 + 8^6$$

$$\mathbf{2021} 37 := -1^7 + 2^1 + 3^2 + 4^9 - 5^4 - 7^3 - 9^5$$

$$\mathbf{2021} 38 := -1^2 - 2^4 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5$$

$$\mathbf{2021} 39 := -1^7 - 4^8 + 5^5 + 6^1 + 7^4 + 8^6$$

$$\mathbf{2021} 40 := 1^2 - 2^4 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5$$

$$\mathbf{2021} 41 := 1^7 - 4^8 + 5^5 + 6^1 + 7^4 + 8^6$$

$$\mathbf{2021} 42 := 1^1 + 2^4 - 3^6 + 4^9 - 5^2 - 6^3 - 9^5$$

$$\begin{aligned} 2021\ 43 &:= -1^7 + 2^2 - 4^8 + 5^5 + 6^1 + 7^4 + 8^6 \\ 2021\ 44 &:= -1^4 - 3^6 + 4^9 - 5^1 - 6^3 - 9^5 \\ 2021\ 45 &:= -2^7 + 4^9 + 5^2 - 6^4 - 7^6 + 9^5 \\ 2021\ 46 &:= 1^4 - 3^6 + 4^9 - 5^1 - 6^3 - 9^5 \\ 2021\ 47 &:= 2^5 + 3^8 + 4^4 + 5^7 + 6^2 + 7^6 - 8^3 \\ 2021\ 48 &:= -1^4 + 2^2 - 3^6 + 4^9 - 5^1 - 6^3 - 9^5 \\ 2021\ 49 &:= -1^1 - 2^8 + 4^9 - 5^4 - 8^2 - 9^5 \\ 2021\ 50 &:= -2^8 + 4^9 - 5^4 - 8^2 - 9^5 \\ 2021\ 51 &:= 1^1 - 2^8 + 4^9 - 5^4 - 8^2 - 9^5 \\ 2021\ 52 &:= 1^4 - 2^2 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5 \\ 2021\ 53 &:= 2^9 + 4^8 + 5^2 + 6^7 - 7^6 - 8^5 + 9^4 \\ 2021\ 54 &:= -1^4 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5 \\ 2021\ 55 &:= -1^6 - 2^8 + 4^9 - 5^4 + 6^1 - 8^2 - 9^5 \\ 2021\ 56 &:= 1^4 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5 \\ 2021\ 57 &:= 1^6 - 2^8 + 4^9 - 5^4 + 6^1 - 8^2 - 9^5 \\ 2021\ 58 &:= -1^4 + 2^2 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5 \\ 2021\ 59 &:= -2^4 - 3^6 + 4^9 + 5^2 - 6^3 - 9^5 \\ 2021\ 60 &:= 1^4 + 2^2 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5 \\ 2021\ 61 &:= -1^9 - 3^5 + 4^3 + 5^7 + 6^1 + 7^6 + 9^4 \\ 2021\ 62 &:= 1^2 + 2^4 - 3^6 + 4^9 - 5^1 - 6^3 - 9^5 \\ 2021\ 63 &:= 1^9 - 3^5 + 4^3 + 5^7 + 6^1 + 7^6 + 9^4 \\ 2021\ 64 &:= -1^6 - 4^7 + 5^9 - 6^8 - 7^1 + 8^4 - 9^5 \\ 2021\ 65 &:= 0^7 + 1^9 - 2^2 + 3^3 - 4^8 + 5^5 + 6^1 + \\ &\quad + 7^4 + 8^6 + 9^0 \\ 2021\ 66 &:= -1^7 + 3^3 - 4^8 + 5^5 + 6^1 + 7^4 + 8^6 \\ 2021\ 67 &:= -1^7 - 2^1 - 4^8 + 5^5 + 6^2 + 7^4 + 8^6 \\ 2021\ 68 &:= 1^7 + 3^3 - 4^8 + 5^5 + 6^1 + 7^4 + 8^6 \\ 2021\ 69 &:= 1^7 - 2^1 - 4^8 + 5^5 + 6^2 + 7^4 + 8^6 \\ 2021\ 70 &:= -1^2 + 2^4 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5 \\ 2021\ 71 &:= -1^7 + 2^1 - 4^8 + 5^5 + 6^2 + 7^4 + 8^6 \end{aligned}$$

$$\begin{aligned} 2021\ 72 &:= 1^2 + 2^4 - 3^6 + 4^9 + 5^1 - 6^3 - 9^5 \\ 2021\ 73 &:= 1^7 + 2^1 - 4^8 + 5^5 + 6^2 + 7^4 + 8^6 \\ 2021\ 74 &:= 1^4 - 2^1 - 3^6 + 4^9 + 5^2 - 6^3 - 9^5 \\ 2021\ 75 &:= 1^9 - 2^8 - 4^1 - 5^4 - 6^2 + 8^6 - 9^5 \\ 2021\ 76 &:= -1^4 + 2^1 - 3^6 + 4^9 + 5^2 - 6^3 - 9^5 \\ 2021\ 77 &:= -2^8 + 3^3 + 4^9 - 5^4 - 8^2 - 9^5 \\ 2021\ 78 &:= 1^4 + 2^1 - 3^6 + 4^9 + 5^2 - 6^3 - 9^5 \\ 2021\ 79 &:= -1^7 + 3^6 + 4^9 - 5^1 - 6^4 - 7^3 - 9^5 \\ 2021\ 80 &:= -1^1 - 2^6 - 3^2 + 4^9 - 5^4 - 6^3 - 9^5 \\ 2021\ 81 &:= -2^6 - 3^2 + 4^9 - 5^4 - 6^3 - 9^5 \\ 2021\ 82 &:= 1^1 - 2^6 - 3^2 + 4^9 - 5^4 - 6^3 - 9^5 \\ 2021\ 83 &:= 1^9 - 2^8 + 4^1 - 5^4 - 6^2 + 8^6 - 9^5 \\ 2021\ 84 &:= -1^5 + 2^1 + 3^8 + 5^7 - 6^3 + 7^6 + 8^2 \\ 2021\ 85 &:= -1^6 - 2^8 + 4^9 - 5^4 - 6^2 + 8^1 - 9^5 \\ 2021\ 86 &:= -1^2 - 2^6 - 3^1 + 4^9 - 5^4 - 6^3 - 9^5 \\ 2021\ 87 &:= 1^6 - 2^8 + 4^9 - 5^4 - 6^2 + 8^1 - 9^5 \\ 2021\ 88 &:= 1^2 - 2^6 - 3^1 + 4^9 - 5^4 - 6^3 - 9^5 \\ 2021\ 89 &:= -2^4 - 3^6 + 4^9 - 5^3 - 6^2 - 9^5 \\ 2021\ 90 &:= -1^1 + 2^4 - 3^6 + 4^9 + 5^2 - 6^3 - 9^5 \\ 2021\ 91 &:= 2^4 - 3^6 + 4^9 + 5^2 - 6^3 - 9^5 \\ 2021\ 92 &:= 1^1 + 2^4 - 3^6 + 4^9 + 5^2 - 6^3 - 9^5 \\ 2021\ 93 &:= 1^9 - 2^8 - 4^2 - 5^4 - 6^1 + 8^6 - 9^5 \\ 2021\ 94 &:= 1^2 - 2^6 + 3^1 + 4^9 - 5^4 - 6^3 - 9^5 \\ 2021\ 95 &:= 0^7 - 1^9 - 2^1 + 3^3 - 4^8 + 5^5 + 6^2 + \\ &\quad + 7^4 + 8^6 + 9^0 \\ 2021\ 96 &:= 1^2 - 2^8 - 3^3 + 4^9 - 5^4 + 8^1 - 9^5 \\ 2021\ 97 &:= -1^1 - 3^7 + 4^9 + 5^4 + 6^3 - 7^6 + 9^5 \\ 2021\ 98 &:= -3^7 + 4^9 + 5^4 + 6^3 - 7^6 + 9^5 \\ 2021\ 99 &:= -2^6 + 3^2 + 4^9 - 5^4 - 6^3 - 9^5 \end{aligned}$$

The three numbers 202120, 202165 and 202195 aren't available for lesser digits. We completed them with bigger lengths.

6.2.3 Blocks of 10 With All Digits 0 to 9

$$\mathbf{20210} := 0^1 - 1^7 - 2^9 + 3^8 + 4^6 + 5^5 + 6^2 + 7^3 + 8^0 + 9^4$$

$$\mathbf{20211} := 0^8 + 1^9 + 2^6 + 3^4 + 4^7 + 5^5 + 6^2 + 7^1 + 8^3 + 9^0$$

$$\mathbf{20212} := 0^1 + 1^7 - 2^9 + 3^8 + 4^6 + 5^5 + 6^2 + 7^3 + 8^0 + 9^4$$

$$\mathbf{20213} := 0^8 + 1^9 + 2^3 + 3^7 + 4^5 + 5^6 + 6^4 + 7^1 + 8^2 + 9^0$$

$$\mathbf{20214} := 0^8 - 1^9 + 2^7 + 3^5 + 4^3 + 5^6 + 6^0 + 7^2 + 8^4 + 9^1$$

$$\mathbf{20215} := 0^8 + 1^9 + 2^3 + 3^7 + 4^5 + 5^6 + 6^4 + 7^0 + 8^2 + 9^1$$

$$\mathbf{20216} := 0^8 + 1^9 + 2^7 + 3^5 + 4^3 + 5^6 + 6^0 + 7^2 + 8^4 + 9^1$$

$$\mathbf{20217} := 0^5 - 1^8 + 2^9 + 3^7 + 4^1 + 5^6 + 6^4 + 7^0 + 8^3 + 9^2$$

$$\mathbf{20218} := -0^0 + 1^7 - 2^9 + 3^8 + 4^6 + 5^5 + 6^2 + 7^3 + 8^1 + 9^4$$

$$\mathbf{20219} := 0^5 + 1^8 + 2^9 + 3^7 + 4^1 + 5^6 + 6^4 + 7^0 + 8^3 + 9^2$$

$$\mathbf{12021} := 0^7 + 1^9 + 2^6 + 3^8 + 4^5 + 5^0 + 6^3 + 7^2 + 8^4 + 9^1$$

$$\mathbf{21201} := 0^6 + 1^8 + 2^9 + 3^5 + 4^7 + 5^0 + 6^1 + 7^2 + 8^4 + 9^3$$

$$\mathbf{31201} := 0^3 + 1^8 + 2^9 + 3^6 + 4^7 + 5^0 + 6^5 + 7^2 + 8^1 + 9^4$$

$$\mathbf{41201} := 0^7 + 1^8 + 2^2 + 3^9 + 4^6 + 5^3 + 6^4 + 7^5 + 8^1 + 9^0$$

$$\mathbf{51201} := 0^8 + 1^9 + 2^7 + 3^3 + 4^5 + 5^0 + 6^6 + 7^1 + 8^4 + 9^2$$

$$\mathbf{61201} := 0^6 - 1^9 + 2^8 + 3^7 + 4^4 + 5^0 + 6^3 + 7^2 + 8^1 + 9^5$$

$$\mathbf{71201} := 0^6 + 1^9 + 2^5 + 3^7 + 4^8 + 5^3 + 6^2 + 7^1 + 8^4 + 9^0$$

$$\mathbf{81201} := 0^4 + 1^9 + 2^8 + 3^7 + 4^3 + 5^0 + 6^6 + 7^1 + 8^5 + 9^2$$

$$\mathbf{91201} := 0^8 + 1^0 + 2^9 + 3^6 + 4^1 + 5^7 + 6^5 + 7^2 + 8^4 + 9^3$$

$$\mathbf{112010} := 0^8 - 1^9 + 2^7 + 3^3 + 4^5 + 5^1 + 6^4 + 7^6 + 8^0 + 9^2$$

$$\mathbf{112011} := 0^7 + 1^0 - 2^9 + 3^2 + 4^8 + 5^6 + 6^3 + 7^1 + 8^5 + 9^4$$

$$\mathbf{112012} := 0^8 + 1^9 + 2^7 + 3^3 + 4^5 + 5^1 + 6^4 + 7^6 + 8^0 + 9^2$$

$$\mathbf{112013} := 0^8 - 1^9 + 2^7 + 3^3 + 4^5 + 5^0 + 6^4 + 7^6 + 8^1 + 9^2$$

$$\mathbf{112014} := 0^7 - 1^8 - 2^9 + 3^1 + 4^5 + 5^2 + 6^4 + 7^6 + 8^0 + 9^3$$

$$\mathbf{112015} := 0^8 + 1^9 + 2^7 + 3^3 + 4^5 + 5^0 + 6^4 + 7^6 + 8^1 + 9^2$$

$$\mathbf{112016} := 0^1 - 1^9 + 2^6 + 3^8 + 4^0 + 5^7 + 6^3 + 7^4 + 8^5 + 9^2$$

$$\mathbf{112017} := 0^5 - 1^8 + 2^9 - 3^7 + 4^2 + 5^3 + 6^1 + 7^6 + 8^4 + 9^0$$

$$\mathbf{112018} := 0^1 + 1^9 + 2^6 + 3^8 + 4^0 + 5^7 + 6^3 + 7^4 + 8^5 + 9^2$$

$$\mathbf{112019} := 0^9 - 1^0 + 2^6 + 3^8 + 4^1 + 5^7 + 6^3 + 7^4 + 8^5 + 9^2$$

$$\begin{aligned}2\mathbf{2021}0 &:= 0^1 - 1^7 + 2^4 + 3^9 - 4^8 + 5^5 + 6^0 + 7^2 + 8^6 + 9^3 \\2\mathbf{2021}1 &:= 0^6 + 1^0 - 2^9 + 3^1 - 4^8 + 5^5 + 6^7 + 7^4 + 8^2 + 9^3 \\2\mathbf{2021}2 &:= 0^1 + 1^7 + 2^4 + 3^9 - 4^8 + 5^5 + 6^0 + 7^2 + 8^6 + 9^3 \\2\mathbf{2021}3 &:= 0^7 - 1^9 - 2^1 - 3^0 - 4^8 + 5^2 + 6^3 + 7^5 + 8^6 + 9^4 \\2\mathbf{2021}4 &:= -0^0 - 1^7 + 2^4 + 3^9 - 4^8 + 5^5 + 6^1 + 7^2 + 8^6 + 9^3 \\2\mathbf{2021}5 &:= 0^7 - 1^0 + 2^4 + 3^9 - 4^8 + 5^5 + 6^1 + 7^2 + 8^6 + 9^3 \\2\mathbf{2021}6 &:= -0^0 + 1^7 + 2^4 + 3^9 - 4^8 + 5^5 + 6^1 + 7^2 + 8^6 + 9^3 \\2\mathbf{2021}7 &:= 0^7 + 1^0 + 2^4 + 3^9 - 4^8 + 5^5 + 6^1 + 7^2 + 8^6 + 9^3 \\2\mathbf{2021}8 &:= 0^0 + 1^7 + 2^4 + 3^9 - 4^8 + 5^5 + 6^1 + 7^2 + 8^6 + 9^3 \\2\mathbf{2021}9 &:= 0^7 - 1^9 + 2^1 + 3^0 - 4^8 + 5^2 + 6^3 + 7^5 + 8^6 + 9^4\end{aligned}$$

$$\begin{aligned}3\mathbf{2021}0 &:= 0^0 + 1^1 - 2^9 + 3^4 + 4^8 + 5^7 + 6^3 + 7^6 + 8^2 + 9^5 \\3\mathbf{2021}1 &:= 0^1 + 1^2 + 2^9 + 3^8 + 4^4 + 5^6 + 6^7 + 7^5 + 8^3 + 9^0 \\3\mathbf{2021}2 &:= -0^0 - 1^9 + 2^8 + 3^1 + 4^6 + 5^2 + 6^7 + 7^4 + 8^5 + 9^3 \\3\mathbf{2021}3 &:= 0^1 + 1^9 + 2^8 + 3^0 + 4^6 + 5^2 + 6^7 + 7^4 + 8^5 + 9^3 \\3\mathbf{2021}4 &:= -0^0 + 1^9 + 2^8 + 3^1 + 4^6 + 5^2 + 6^7 + 7^4 + 8^5 + 9^3 \\3\mathbf{2021}5 &:= 0^5 + 1^7 + 2^1 + 3^8 + 4^9 + 5^2 + 6^6 + 7^0 + 8^4 + 9^3 \\3\mathbf{2021}6 &:= 0^0 + 1^9 + 2^8 + 3^1 + 4^6 + 5^2 + 6^7 + 7^4 + 8^5 + 9^3 \\3\mathbf{2021}7 &:= 0^6 + 1^0 + 2^9 + 3^8 + 4^4 + 5^3 + 6^7 + 7^2 + 8^5 + 9^1 \\3\mathbf{2021}8 &:= 0^0 + 1^6 + 2^9 + 3^8 + 4^4 + 5^3 + 6^7 + 7^2 + 8^5 + 9^1 \\3\mathbf{2021}9 &:= 0^2 + 1^0 + 2^9 + 3^8 + 4^4 + 5^6 + 6^7 + 7^5 + 8^3 + 9^1\end{aligned}$$

$$\begin{aligned}4\mathbf{2021}0 &:= -0^0 + 1^3 + 2^9 + 3^6 + 4^7 + 5^8 + 6^5 + 7^1 + 8^4 + 9^2 \\4\mathbf{2021}1 &:= 0^3 + 1^0 + 2^9 + 3^6 + 4^7 + 5^8 + 6^5 + 7^1 + 8^4 + 9^2 \\4\mathbf{2021}2 &:= 0^0 + 1^3 + 2^9 + 3^6 + 4^7 + 5^8 + 6^5 + 7^1 + 8^4 + 9^2 \\4\mathbf{2021}3 &:= 0^4 + 1^9 + 2^3 + 3^0 + 4^8 + 5^6 + 6^7 + 7^2 + 8^1 + 9^5 \\4\mathbf{2021}4 &:= 0^3 - 1^9 + 2^4 + 3^1 + 4^8 + 5^6 + 6^7 + 7^2 + 8^0 + 9^5 \\4\mathbf{2021}5 &:= 0^9 + 1^0 - 2^4 + 3^3 + 4^8 + 5^6 + 6^7 + 7^2 + 8^1 + 9^5 \\4\mathbf{2021}6 &:= 0^3 + 1^9 + 2^4 + 3^1 + 4^8 + 5^6 + 6^7 + 7^2 + 8^0 + 9^5 \\4\mathbf{2021}7 &:= 0^1 + 1^4 - 2^8 + 3^0 + 4^9 + 5^7 + 6^6 + 7^2 + 8^5 + 9^3 \\4\mathbf{2021}8 &:= 0^1 - 1^9 + 2^0 + 3^3 + 4^8 + 5^7 + 6^5 + 7^2 + 8^6 + 9^4 \\4\mathbf{2021}9 &:= 0^3 - 1^9 + 2^4 + 3^0 + 4^8 + 5^6 + 6^7 + 7^2 + 8^1 + 9^5\end{aligned}$$

$$\begin{aligned}5 \mathbf{2021} 0 &:= 0^7 - 1^9 + 2^0 - 3^2 + 4^3 + 5^8 + 6^5 + 7^6 + 8^4 + 9^1 \\5 \mathbf{2021} 1 &:= 0^7 + 1^9 - 2^2 + 3^1 + 4^3 + 5^8 + 6^5 + 7^6 + 8^4 + 9^0 \\5 \mathbf{2021} 2 &:= 0^7 + 1^9 + 2^0 - 3^2 + 4^3 + 5^8 + 6^5 + 7^6 + 8^4 + 9^1 \\5 \mathbf{2021} 3 &:= 0^7 + 1^9 + 2^2 - 3^1 + 4^3 + 5^8 + 6^5 + 7^6 + 8^4 + 9^0 \\5 \mathbf{2021} 4 &:= 0^0 + 1^5 - 2^7 - 3^8 + 4^9 + 5^3 + 6^1 + 7^4 + 8^6 + 9^2 \\5 \mathbf{2021} 5 &:= 0^7 - 1^9 - 2^2 + 3^0 + 4^3 + 5^8 + 6^5 + 7^6 + 8^4 + 9^1 \\5 \mathbf{2021} 6 &:= 0^7 - 1^9 - 2^3 - 3^1 + 4^0 + 5^8 + 6^5 + 7^6 + 8^4 + 9^2 \\5 \mathbf{2021} 7 &:= 0^7 - 1^9 + 2^2 + 3^1 + 4^3 + 5^8 + 6^5 + 7^6 + 8^4 + 9^0 \\5 \mathbf{2021} 8 &:= -0^0 - 1^7 - 2^1 + 3^9 + 4^3 + 5^8 + 6^6 + 7^2 + 8^4 + 9^5 \\5 \mathbf{2021} 9 &:= 0^7 + 1^9 + 2^2 + 3^1 + 4^3 + 5^8 + 6^5 + 7^6 + 8^4 + 9^0\end{aligned}$$

$$\begin{aligned}6 \mathbf{2021} 0 &:= 0^1 + 1^7 + 2^4 + 3^9 + 4^8 + 5^5 + 6^0 + 7^3 + 8^2 + 9^6 \\6 \mathbf{2021} 1 &:= 0^2 - 1^4 + 2^8 + 3^0 + 4^9 + 5^7 + 6^1 + 7^5 + 8^6 + 9^3 \\6 \mathbf{2021} 2 &:= -0^0 - 1^7 + 2^4 + 3^9 + 4^8 + 5^5 + 6^1 + 7^3 + 8^2 + 9^6 \\6 \mathbf{2021} 3 &:= 0^2 + 1^4 + 2^8 + 3^0 + 4^9 + 5^7 + 6^1 + 7^5 + 8^6 + 9^3 \\6 \mathbf{2021} 4 &:= -0^0 + 1^7 + 2^4 + 3^9 + 4^8 + 5^5 + 6^1 + 7^3 + 8^2 + 9^6 \\6 \mathbf{2021} 5 &:= 0^7 + 1^0 + 2^4 + 3^9 + 4^8 + 5^5 + 6^1 + 7^3 + 8^2 + 9^6 \\6 \mathbf{2021} 6 &:= 0^0 + 1^7 + 2^4 + 3^9 + 4^8 + 5^5 + 6^1 + 7^3 + 8^2 + 9^6 \\6 \mathbf{2021} 7 &:= 0^8 - 1^0 + 2^3 + 3^9 + 4^2 - 5^4 + 6^7 + 7^1 + 8^6 + 9^5 \\6 \mathbf{2021} 8 &:= -0^0 - 1^4 + 2^8 + 3^2 + 4^9 + 5^7 + 6^1 + 7^5 + 8^6 + 9^3 \\6 \mathbf{2021} 9 &:= 0^4 - 1^0 + 2^8 + 3^2 + 4^9 + 5^7 + 6^1 + 7^5 + 8^6 + 9^3\end{aligned}$$

$$\begin{aligned}7 \mathbf{2021} 0 &:= -0^0 + 1^5 + 2^2 + 3^8 + 4^9 - 5^7 - 6^4 - 7^1 - 8^3 + 9^6 \\7 \mathbf{2021} 1 &:= 0^1 + 1^3 - 2^7 + 3^2 + 4^9 + 5^8 + 6^6 + 7^5 + 8^4 + 9^0 \\7 \mathbf{2021} 2 &:= 0^0 + 1^5 + 2^2 + 3^8 + 4^9 - 5^7 - 6^4 - 7^1 - 8^3 + 9^6 \\7 \mathbf{2021} 3 &:= 0^2 + 1^5 - 2^1 + 3^8 + 4^9 - 5^7 - 6^4 + 7^0 - 8^3 + 9^6 \\7 \mathbf{2021} 4 &:= -0^0 - 1^5 - 2^2 + 3^8 + 4^9 - 5^7 - 6^4 + 7^1 - 8^3 + 9^6 \\7 \mathbf{2021} 5 &:= 0^2 - 1^5 + 2^1 + 3^8 + 4^9 - 5^7 - 6^4 + 7^0 - 8^3 + 9^6 \\7 \mathbf{2021} 6 &:= -0^0 - 1^3 - 2^7 + 3^2 + 4^9 + 5^8 + 6^6 + 7^5 + 8^4 + 9^1 \\7 \mathbf{2021} 7 &:= 0^3 - 1^0 - 2^7 + 3^2 + 4^9 + 5^8 + 6^6 + 7^5 + 8^4 + 9^1 \\7 \mathbf{2021} 8 &:= -0^0 + 1^3 - 2^7 + 3^2 + 4^9 + 5^8 + 6^6 + 7^5 + 8^4 + 9^1 \\7 \mathbf{2021} 9 &:= 0^3 + 1^0 - 2^7 + 3^2 + 4^9 + 5^8 + 6^6 + 7^5 + 8^4 + 9^1\end{aligned}$$

$$\begin{aligned}
 8\mathbf{2021}0 &:= 0^5 - 1^8 + 2^9 + 3^4 - 4^6 + 5^3 + 6^2 + 7^7 + 8^0 + 9^1 \\
 8\mathbf{2021}1 &:= 0^1 + 1^2 + 2^8 - 3^9 + 4^6 + 5^3 + 6^5 + 7^7 + 8^4 + 9^0 \\
 8\mathbf{2021}2 &:= 0^5 + 1^8 + 2^9 + 3^4 - 4^6 + 5^3 + 6^2 + 7^7 + 8^0 + 9^1 \\
 8\mathbf{2021}3 &:= 0^2 - 1^3 - 2^9 + 3^1 - 4^4 + 5^8 + 6^7 + 7^6 + 8^5 + 9^0 \\
 8\mathbf{2021}4 &:= 0^1 + 1^9 + 2^6 - 3^8 - 4^4 + 5^5 + 6^3 + 7^7 + 8^0 + 9^2 \\
 8\mathbf{2021}5 &:= 0^2 + 1^3 - 2^9 + 3^1 - 4^4 + 5^8 + 6^7 + 7^6 + 8^5 + 9^0 \\
 8\mathbf{2021}6 &:= -0^0 - 1^2 + 2^8 - 3^9 + 4^6 + 5^3 + 6^5 + 7^7 + 8^4 + 9^1 \\
 8\mathbf{2021}7 &:= 0^2 - 1^0 + 2^8 - 3^9 + 4^6 + 5^3 + 6^5 + 7^7 + 8^4 + 9^1 \\
 8\mathbf{2021}8 &:= -0^0 + 1^2 + 2^8 - 3^9 + 4^6 + 5^3 + 6^5 + 7^7 + 8^4 + 9^1 \\
 8\mathbf{2021}9 &:= 0^2 + 1^0 + 2^8 - 3^9 + 4^6 + 5^3 + 6^5 + 7^7 + 8^4 + 9^1
 \end{aligned}$$

$$\begin{aligned}
 9\mathbf{2021}0 &:= 0^3 - 1^9 + 2^5 - 3^7 + 4^4 + 5^8 + 6^2 + 7^1 + 8^0 + 9^6 \\
 9\mathbf{2021}1 &:= 0^3 - 1^9 + 2^5 - 3^7 + 4^4 + 5^8 + 6^2 + 7^0 + 8^1 + 9^6 \\
 9\mathbf{2021}2 &:= 0^3 + 1^9 + 2^5 - 3^7 + 4^4 + 5^8 + 6^2 + 7^1 + 8^0 + 9^6 \\
 9\mathbf{2021}3 &:= 0^3 + 1^9 + 2^5 - 3^7 + 4^4 + 5^8 + 6^2 + 7^0 + 8^1 + 9^6 \\
 9\mathbf{2021}4 &:= -0^0 + 1^3 + 2^5 - 3^9 + 4^8 + 5^2 + 6^6 + 7^7 + 8^4 + 9^1 \\
 9\mathbf{2021}5 &:= 0^3 + 1^0 + 2^5 - 3^9 + 4^8 + 5^2 + 6^6 + 7^7 + 8^4 + 9^1 \\
 9\mathbf{2021}6 &:= 0^0 + 1^3 + 2^5 - 3^9 + 4^8 + 5^2 + 6^6 + 7^7 + 8^4 + 9^1 \\
 9\mathbf{2021}7 &:= 0^9 + 1^0 + 2^3 - 3^7 - 4^5 + 5^8 + 6^4 + 7^2 + 8^1 + 9^6 \\
 9\mathbf{2021}8 &:= 0^0 + 1^9 + 2^3 - 3^7 - 4^5 + 5^8 + 6^4 + 7^2 + 8^1 + 9^6 \\
 9\mathbf{2021}9 &:= 0^3 + 1^9 + 2^1 - 3^7 - 4^5 + 5^8 + 6^4 + 7^0 + 8^2 + 9^6
 \end{aligned}$$

7 Power Representations

7.1 Powers 2, 3 and 4

$$\begin{array}{lll}
 \mathbf{2021} &:= 1^2 + 16^2 + 42^2 & \mathbf{2021} &:= 6^2 + 31^2 + 32^2 & \mathbf{2021} &:= 14^2 + 15^2 + 40^2 \\
 &:= 1^2 + 24^2 + 38^2 & &:= 7^2 + 26^2 + 36^2 & &:= 14^2 + 23^2 + 36^2 \\
 &:= 2^2 + 9^2 + 44^2 & &:= 9^2 + 28^2 + 34^2 & &:= 16^2 + 26^2 + 33^2 \\
 &:= 4^2 + 18^2 + 41^2 & &:= 10^2 + 20^2 + 39^2 & &:= 17^2 + 24^2 + 34^2 \\
 &:= 4^2 + 22^2 + 39^2 & &:= 10^2 + 25^2 + 36^2 & &:= 22^2 + 24^2 + 31^2 \\
 &:= 6^2 + 7^2 + 44^2 & &:= 12^2 + 14^2 + 41^2 & &:= 2^2 + 21^2 + 26^2 + 30^2
 \end{array}$$

$$\begin{aligned} \mathbf{2021} &:= 11^3 + 7^3 + 7^3 + 1^3 + 1^3 + 1^3 + 1^3 \\ &:= 13^3 - 5^3 - 4^3 + 3^3 - 2^3 - 2^3 + 1^3 + 1^3 \\ &:= 10^3 + 9^3 + 7^3 - 4^3 + 3^3 - 2^3 - 2^3 + 1^3 + 1^3 \\ &:= 12^3 + 7^3 - 4^3 + 3^3 - 2^3 - 2^3 + 1^3 + 1^3 + 1^3 \end{aligned}$$

$$\mathbf{2021} := 6^4 + 5^4 + 3^4 + 2^4 + 1^4 + 1^4 + 1^4$$

7.2 Powers of 2

$$\begin{aligned} \mathbf{2021} &:= 2^{11} - 2^5 + 2^2 + 2^0 \\ &:= 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^2 + 2^0 \\ &:= 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^4 - 2^3 - 2^2 + 2^1 - 2^0 \end{aligned}$$

7.3 Patterns With Power 3

• First Type

$$\begin{aligned} \mathbf{20210} &:= 1^3 + 2^3 + 5^3 + 11^3 + 17^3 + 18^3 + 20^3 \\ &:= 2^3 + 5^3 + 8^3 + 11^3 + 15^3 + 19^3 + 20^3 \\ &:= 2^3 + 5^3 + 8^3 + 12^3 + 14^3 + 18^3 + 21^3 \\ &:= 2^3 + 6^3 + 8^3 + 9^3 + 17^3 + 18^3 + 20^3 \\ &:= 3^3 + 5^3 + 6^3 + 8^3 + 11^3 + 18^3 + 23^3 \end{aligned}$$

$$\begin{aligned} \mathbf{20211} &:= 2^3 + 4^3 + 11^3 + 12^3 + 17^3 + 23^3 \\ &:= 3^3 + 5^3 + 9^3 + 11^3 + 18^3 + 23^3 \end{aligned}$$

$$\mathbf{20212} := 8^3 + 11^3 + 14^3 + 25^3$$

$$\begin{aligned} \mathbf{20213} &:= 1^3 + 8^3 + 11^3 + 14^3 + 25^3 \\ &:= 12^3 + 13^3 + 15^3 + 17^3 + 20^3 \end{aligned}$$

$$\begin{aligned} \mathbf{20214} &:= 3^3 + 9^3 + 13^3 + 20^3 + 21^3 \\ &:= 4^3 + 8^3 + 15^3 + 16^3 + 23^3 \\ &:= 6^3 + 12^3 + 16^3 + 17^3 + 21^3 \end{aligned}$$

$$\begin{aligned} \mathbf{20215} &:= 4^3 + 5^3 + 7^3 + 27^3 \\ &:= 6^3 + 7^3 + 18^3 + 24^3 \end{aligned}$$

$$\mathbf{20216} := 16^3 + 19^3 + 21^3$$

$$\mathbf{20217} := 1^3 + 16^3 + 19^3 + 21^3$$

$$\mathbf{20218} := 1^3 + 4^3 + 10^3 + 11^3 + 13^3 + 25^3$$

$$\begin{aligned} \mathbf{20219} &:= 2^3 + 3^3 + 5^3 + 9^3 + 11^3 + 18^3 + 23^3 \\ &= 5^3 + 8^3 + 12^3 + 13^3 + 14^3 + 17^3 + 20^3 \end{aligned}$$

• Second Type

$$\begin{aligned} 1 \text{ 2021} &:= 1^3 + 2^3 + 5^3 + 8^3 + 15^3 + 20^3 \\ &:= 2^3 + 3^3 + 8^3 + 9^3 + 17^3 + 18^3 \end{aligned}$$

$$2 \text{ 2021} := 13^3 + 14^3 + 17^3 + 23^3$$

$$\begin{aligned} 3 \text{ 2021} &:= 2^3 + 4^3 + 12^3 + 18^3 + 29^3 \\ &:= 2^3 + 5^3 + 16^3 + 23^3 + 25^3 \\ &:= 2^3 + 11^3 + 13^3 + 16^3 + 29^3 \\ &:= 5^3 + 6^3 + 12^3 + 20^3 + 28^3 \\ &:= 5^3 + 9^3 + 15^3 + 23^3 + 25^3 \\ &:= 8^3 + 9^3 + 11^3 + 24^3 + 25^3 \\ &:= 9^3 + 11^3 + 13^3 + 15^3 + 29^3 \end{aligned}$$

$$\begin{aligned} 4 \text{ 2021} &:= 2^3 + 8^3 + 13^3 + 34^3 \\ &:= 5^3 + 19^3 + 22^3 + 29^3 \\ &:= 8^3 + 13^3 + 15^3 + 33^3 \end{aligned}$$

$$\begin{aligned} 5 \text{ 2021} &:= 3^3 + 5^3 + 6^3 + 10^3 + 37^3 \\ &:= 4^3 + 16^3 + 18^3 + 21^3 + 32^3 \\ &:= 5^3 + 6^3 + 12^3 + 22^3 + 34^3 \\ &:= 5^3 + 10^3 + 21^3 + 27^3 + 28^3 \\ &:= 5^3 + 19^3 + 21^3 + 24^3 + 28^3 \\ &:= 6^3 + 7^3 + 12^3 + 19^3 + 35^3 \\ &:= 6^3 + 9^3 + 17^3 + 19^3 + 34^3 \\ &:= 6^3 + 10^3 + 11^3 + 27^3 + 31^3 \\ &:= 6^3 + 11^3 + 19^3 + 24^3 + 31^3 \\ &:= 6^3 + 16^3 + 17^3 + 19^3 + 33^3 \\ &:= 9^3 + 12^3 + 13^3 + 26^3 + 31^3 \\ &:= 10^3 + 13^3 + 19^3 + 26^3 + 29^3 \\ &:= 10^3 + 13^3 + 20^3 + 24^3 + 30^3 \end{aligned}$$

$$\begin{aligned} 6 \text{ 2021} &:= 1^3 + 10^3 + 13^3 + 23^3 + 36^3 \\ &:= 4^3 + 14^3 + 21^3 + 22^3 + 34^3 \end{aligned}$$

$$\begin{aligned} &:= 5^3 + 18^3 + 22^3 + 25^3 + 31^3 \\ &:= 7^3 + 18^3 + 21^3 + 22^3 + 33^3 \end{aligned}$$

$$:= 13^3 + 15^3 + 24^3 + 25^3 + 30^3$$

$$\begin{aligned} 7 \text{ 2021} &:= 9^3 + 13^3 + 31^3 + 34^3 \\ &:= 12^3 + 13^3 + 16^3 + 40^3 \\ &:= 13^3 + 16^3 + 31^3 + 33^3 \end{aligned}$$

$$\begin{aligned} 8 \text{ 2021} &:= 1^3 + 4^3 + 8^3 + 10^3 + 31^3 + 37^3 \\ &:= 1^3 + 4^3 + 10^3 + 11^3 + 25^3 + 40^3 \\ &:= 1^3 + 7^3 + 21^3 + 25^3 + 30^3 + 31^3 \\ &:= 1^3 + 9^3 + 16^3 + 19^3 + 27^3 + 37^3 \\ &:= 4^3 + 7^3 + 16^3 + 17^3 + 28^3 + 37^3 \\ &:= 4^3 + 10^3 + 18^3 + 22^3 + 24^3 + 37^3 \\ &:= 5^3 + 11^3 + 18^3 + 26^3 + 29^3 + 32^3 \end{aligned}$$

$$\begin{aligned} 9 \text{ 2021} &:= 2^3 + 14^3 + 20^3 + 26^3 + 29^3 + 34^3 \\ &:= 3^3 + 20^3 + 21^3 + 26^3 + 29^3 + 32^3 \\ &:= 4^3 + 5^3 + 14^3 + 20^3 + 23^3 + 41^3 \\ &:= 5^3 + 6^3 + 8^3 + 17^3 + 23^3 + 42^3 \\ &:= 5^3 + 6^3 + 11^3 + 21^3 + 23^3 + 41^3 \\ &:= 5^3 + 8^3 + 10^3 + 14^3 + 32^3 + 38^3 \\ &:= 5^3 + 12^3 + 14^3 + 20^3 + 32^3 + 36^3 \\ &:= 5^3 + 14^3 + 17^3 + 23^3 + 32^3 + 34^3 \\ &:= 6^3 + 7^3 + 10^3 + 16^3 + 19^3 + 43^3 \\ &:= 7^3 + 10^3 + 15^3 + 19^3 + 31^3 + 37^3 \\ &:= 10^3 + 19^3 + 22^3 + 25^3 + 28^3 + 33^3 \\ &:= 12^3 + 14^3 + 15^3 + 17^3 + 29^3 + 38^3 \\ &:= 13^3 + 16^3 + 22^3 + 24^3 + 28^3 + 34^3 \\ &:= 14^3 + 15^3 + 20^3 + 26^3 + 29^3 + 33^3 \end{aligned}$$

7.4 Pattern With Power 4

$$0 \mathbf{2021} := 1^4 + 1^4 + 2^4 + 3^4 + 5^4 + 6^4$$

$$1 \mathbf{2021} := 1^4 + 1^4 + 1^4 + 2^4 + 3^4 + 5^4 + 6^4 + 10^4$$

$$2 \mathbf{2021} := 1^4 + 1^4 + 4^4 + 5^4 + 7^4 + 8^4 + 11^4$$

$$3 \mathbf{2021} := 1^4 + 1^4 + 4^4 + 5^4 + 7^4 + 8^4 + 10^4 + 11^4$$

$$4 \mathbf{2021} := 1^4 + 1^4 + 3^4 + 9^4 + 11^4 + 12^4$$

$$5 \mathbf{2021} := 1^4 + 1^4 + 3^4 + 9^4 + 10^4 + 11^4 + 12^4$$

$$6 \mathbf{2021} := 1^4 + 1^4 + 7^4 + 9^4 + 11^4 + 14^4$$

$$7 \mathbf{2021} := 1^4 + 1^4 + 7^4 + 9^4 + 10^4 + 11^4 + 14^4$$

$$8 \mathbf{2021} := 1^4 + 1^4 + 4^4 + 7^4 + 8^4 + 10^4 + 11^4 + 15^4$$

$$9 \mathbf{2021} := 1^4 + 1^4 + 5^4 + 6^4 + 8^4 + 11^4 + 12^4 + 15^4$$

7.5 Irregular Patterns With Power 4

$$05 \mathbf{2021} := 3^4 + 4^4 + 5^4 + 6^4 + 9^4 + 11^4 + 13^4$$

$$06 \mathbf{2021} := 3^4 + 4^4 + 5^4 + 6^4 + 9^4 + 10^4 + 11^4 + 13^4$$

$$09 \mathbf{2021} := 1^4 + 3^4 + 5^4 + 6^4 + 7^4 + 8^4 + 17^4$$

$$:= 1^4 + 4^4 + 5^4 + 6^4 + 8^4 + 9^4 + 13^4 + 15^4$$

$$10 \mathbf{2021} := 1^4 + 4^4 + 5^4 + 6^4 + 8^4 + 9^4 + 10^4 + 13^4 + 15^4$$

$$:= 3^4 + 4^4 + 6^4 + 9^4 + 11^4 + 13^4 + 15^4$$

$$11 \mathbf{2021} := 3^4 + 4^4 + 6^4 + 9^4 + 10^4 + 11^4 + 13^4 + 15^4$$

$$12 \mathbf{2021} := 2^4 + 4^4 + 5^4 + 9^4 + 11^4 + 12^4 + 13^4 + 15^4$$

$$:= 2^4 + 3^4 + 5^4 + 6^4 + 7^4 + 13^4 + 14^4 + 15^4$$

$$13 \mathbf{2021} := 2^4 + 4^4 + 5^4 + 9^4 + 10^4 + 11^4 + 12^4 + 13^4 + 15^4$$

$$:= 2^4 + 3^4 + 5^4 + 6^4 + 7^4 + 10^4 + 13^4 + 14^4 + 15^4$$

$$15 \mathbf{2021} := 5^4 + 7^4 + 8^4 + 9^4 + 12^4 + 13^4 + 14^4 + 15^4$$

$$16 \mathbf{2021} := 5^4 + 7^4 + 8^4 + 9^4 + 10^4 + 12^4 + 13^4 + 14^4 + 15^4$$

$$1 \mathbf{2021} 1 := 4^4 + 6^4 + 8^4 + 11^4 + 12^4 + 13^4 + 15^4$$

$$:= 6^4 + 9^4 + 10^4 + 11^4 + 12^4 + 13^4 + 14^4$$

$$1 \mathbf{2021} 2 := 1^4 + 4^4 + 6^4 + 8^4 + 11^4 + 12^4 + 13^4 + 15^4$$

$$:= 1^4 + 6^4 + 9^4 + 10^4 + 11^4 + 12^4 + 13^4 + 14^4$$

8 Palindromic Days and Time

8.1 Palindromic Days

- Palindromic day on January 12 : 12.1.21 \Rightarrow 12121
- Palindromic day on February 12 : 12.2.21 \Rightarrow 12221
- Palindromic day on March 12 : 12.3.21 \Rightarrow 12321
- Palindromic day on April 12 : 12.4.21 \Rightarrow 12421
- Palindromic day on May 12 : 12.5.21 \Rightarrow 12521
- Palindromic day on June 12 : 12.6.21 \Rightarrow 12621
- Palindromic day on July 12 : 12.7.21 \Rightarrow 12721
- Palindromic day on August 12 : 12.8.21 \Rightarrow 12921
- Palindromic day on September 12 : 12.9.21 \Rightarrow 12921
- Palindromic day on November 12 : 12.11.21 \Rightarrow 121121

Also,

- Palindromic day on February 12 : 12.02.2021 \Rightarrow 12022021

8.2 Palindromic Times

- 12h 02m 11s 2021 \Rightarrow 1202112021
- 12h 02m 22s 2021 \Rightarrow 1202112021
- 12h 02m 33s 2021 \Rightarrow 1202112021
- 12h 02m 44s 2021 \Rightarrow 1202112021
- 12h 02m 55s 2021 \Rightarrow 1202112021

- 12h 02m 0s 2021 \Rightarrow 120202021
- 12h 02m 1s 2021 \Rightarrow 120212021
- 12h 02m 2s 2021 \Rightarrow 120222021
- 12h 02m 3s 2021 \Rightarrow 120232021
- 12h 02m 4s 2021 \Rightarrow 120242021
- 12h 02m 5s 2021 \Rightarrow 120252021
- 12h 02m 6s 2021 \Rightarrow 120262021
- 12h 02m 7s 2021 \Rightarrow 120272021
- 12h 02m 8s 2021 \Rightarrow 120282021
- 12h 02m 9s 2021 \Rightarrow 120292021

It will happen every day in 2021.

9 Upside Down and Mirror Looking

9.1 Upside Down

$$\begin{aligned} 2021 &:= 9 + 1001 + 1 + 1 + 1 + 1 + 1001 + 6 \\ &:= 1 + 1 + 6 + 69 + 609 + 619 + 609 + 96 + 9 + 1 + 1 \\ &:= 1 + 1 + 1 + 1 + 609 + (1 + 1) \times (1 + 6 + 9 + 69 + 619) \end{aligned}$$

See below **upside-down** numbers written with special fonts:

$$\begin{aligned} &9+|00|+|+|+|+|+|00|+6 \\ &|+|+6+69+609+6|9+609+96+9+|+| \\ &|+|+|+|+609+(|+|)x(|+6+9+69+6|9) \end{aligned}$$

9.2 Upside Down and Mirror Looking

$$\begin{aligned} 2021 &:= 8 + 1 + 1001 + 1 + 1001 + 1 + 8 \\ &:= 1 + 502 + 502 + 11 + 502 + 502 + 1 \\ &:= 2 + 1 + 5 + 1 + 1001 + 1 + 1001 + 1 + 5 + 1 + 2 \end{aligned}$$

See below **upside-down** and **mirror looking** numbers written in **digital fonts**:

$$\begin{aligned} &8+1+1001+1+1001+1+8 \\ &1+502+502+11+502+502+1 \\ &2+1+5+1+1001+1+1001+1+5+1+2 \end{aligned}$$

10 Two and Three Digits Representations

Below are representations of 2021 just using either 0, 1 and 2 or 1 and 2.

$$\begin{aligned} 2021 &:= (2 + 1) \times 21 \times 21 + 2 \times (21 \times 21 - 2 \times 2 \times (21 + 2)) \\ &:= 2^2 \times (2 + 2 + 1) \times ((2 + 2 + 1) \times 21 - 2^2) + 1 \\ &:= (2 \times 21 + 1) \times (2 \times 21 + 2^2 + 1) \\ &:= 202 \times 10 + 1 \\ &:= 20 \times 10 \times 10 + 21 \\ &:= 2 \times 2 \times (2 + 2 + 1) \times 101 + 1 \\ &:= 2 \times 2 \times (2 + 2 + 1) \times ((2 + 2 + 1) \times 20 + 1) + 1 \\ &:= (2 + 1) \times 21 \times 21 + 2 \times 21 \times 21 - 201 + 21 - 2 - 2 \end{aligned}$$

All the three digits 0, 1 and 2 can be written as upside-down. The the way the expressions are written above there is only one representation, that give the same sum by making 180° rotation. Other expressions either with negative sign are not symmetric, such as 12. If we write 10 or 20 as 010 and 020, then we can write easier in upside down position and still the sum remains the same. See below some examples wirtten **upside-down** numbers in **digital fonts**:

$$202*010+1$$

$$020*010*010+020+1$$

$$1+101*2*2*(2+2+1)$$

11 Pythagorean Triples and Patterns

11.1 Pythagorean Triples

$$2021^2 + 180^2 := 2029^2$$

$$2021^2 + 43428^2 := 43475^2$$

$$2021^2 + 47472^2 := 47515^2$$

$$2021^2 + 2042220^2 := 2042221^2.$$

11.2 Patterns in Pythagorean Triples

$$2021^2 + 2042220^2 := 2042221^2$$

$$20021^2 + 200420220^2 := 200420221^2$$

$$200021^2 + 20004200220^2 := 20004200221^2$$

$$2000021^2 + 2000042000220^2 := 2000042000221^2.$$

11.2.1 Patterns With 2021

$(20210)^2 + 48504^2 := 52546^2$	$(20211)^2 + 26948^2 := 33685^2$
$(200210)^2 + 480504^2 := 520546^2$	$(200211)^2 + 266948^2 := 333685^2$
$(2000210)^2 + 4800504^2 := 5200546^2$	$(2000211)^2 + 2666948^2 := 3333685^2$
$(20000210)^2 + 48000504^2 := 52000546^2$	$(20000211)^2 + 26666948^2 := 33333685^2$

$$\begin{aligned} (20214)^2 + 26952^2 &:= 33690^2 \\ (200214)^2 + 266952^2 &:= 333690^2 \\ (2000214)^2 + 2666952^2 &:= 3333690^2 \\ (20000214)^2 + 26666952^2 &:= 33333690^2 \end{aligned}$$

$$\begin{aligned} (20215)^2 + 48516^2 &:= 52559^2 \\ (200215)^2 + 480516^2 &:= 520559^2 \\ (2000215)^2 + 4800516^2 &:= 5200559^2 \\ (20000215)^2 + 48000516^2 &:= 52000559^2 \end{aligned}$$

$$\begin{aligned} (20216)^2 + 37905^2 &:= 42959^2 \\ (200216)^2 + 375405^2 &:= 425459^2 \\ (2000216)^2 + 3750405^2 &:= 4250459^2 \\ (20000216)^2 + 37500405^2 &:= 42500459^2 \end{aligned}$$

$$\begin{aligned} (20217)^2 + 26956^2 &:= 33695^2 \\ (200217)^2 + 266956^2 &:= 333695^2 \\ (2000217)^2 + 2666956^2 &:= 3333695^2 \\ (20000217)^2 + 26666956^2 &:= 33333695^2 \end{aligned}$$

11.3 Block of 10 or Less Patterns With 2021

$$\begin{aligned} (20210)^2 + 102111024^2 &:= 102111026^2 \\ (20211)^2 + 204242260^2 &:= 204242261^2 \\ (20212)^2 + 102131235^2 &:= 102131237^2 \\ (20213)^2 + 204282684^2 &:= 204282685^2 \\ (20214)^2 + 102151448^2 &:= 102151450^2 \end{aligned}$$

$$\begin{aligned} (20215)^2 + 204323112^2 &:= 204323113^2 \\ (20216)^2 + 102171663^2 &:= 102171665^2 \\ (20217)^2 + 204363544^2 &:= 204363545^2 \\ (20218)^2 + 102191880^2 &:= 102191882^2 \\ (20219)^2 + 204403980^2 &:= 204403981^2 \end{aligned}$$

Below are some irregular patterns with less number of digits:

$$\begin{aligned} 20210^2 + 44016^2 &:= 48434^2 \\ 20212^2 + 22725^2 &:= 30413^2 \\ 20213^2 + 120684^2 &:= 122365^2 \\ 20214^2 + 26952^2 &:= 33690^2 \\ 20215^2 + 22392^2 &:= 30167^2 \\ 20216^2 + 23712^2 &:= 31160^2 \\ 20217^2 + 26956^2 &:= 33695^2 \\ 20218^2 + 110280^2 &:= 112118^2 \end{aligned}$$

$$\begin{aligned} 1\ 2021^2 + 16028^2 &:= 20035^2 \\ 2\ 2021^2 + 31920^2 &:= 38779^2 \\ 3\ 2021^2 + 55380^2 &:= 63971^2 \\ 4\ 2021^2 + 45472^2 &:= 61915^2 \\ 6\ 2021^2 + 155940^2 &:= 167821^2 \\ 7\ 2021^2 + 96028^2 &:= 120035^2 \\ 9\ 2021^2 + 779472^2 &:= 784885^2 \end{aligned}$$

11.4 Block of 100 or Less Patterns With 2021

$$(2021\ 00)^2 + 10211102499^2 := 10211102501^2$$

$$(2021\ 01)^2 + 20422407100^2 := 20422407101^2$$

$$(2021\ 02)^2 + 10211304600^2 := 10211304602^2$$

$$(2021\ 03)^2 + 20422811304^2 := 20422811305^2$$

$$(2021\ 04)^2 + 10211506703^2 := 10211506705^2$$

$$(2021\ 05)^2 + 20423215512^2 := 20423215513^2$$

$$(2021\ 06)^2 + 10211708808^2 := 10211708810^2$$

$$(2021\ 07)^2 + 20423619724^2 := 20423619725^2$$

$$(2021\ 08)^2 + 10211910915^2 := 10211910917^2$$

$$(2021\ 09)^2 + 20424023940^2 := 20424023941^2$$

$$(2021\ 10)^2 + 10212113024^2 := 10212113026^2$$

$$(2021\ 11)^2 + 20424428160^2 := 20424428161^2$$

$$(2021\ 12)^2 + 10212315135^2 := 10212315137^2$$

$$(2021\ 13)^2 + 20424832384^2 := 20424832385^2$$

$$(2021\ 14)^2 + 10212517248^2 := 10212517250^2$$

$$(2021\ 15)^2 + 20425236612^2 := 20425236613^2$$

$$(2021\ 16)^2 + 10212719363^2 := 10212719365^2$$

$$(2021\ 17)^2 + 20425640844^2 := 20425640845^2$$

$$(2021\ 18)^2 + 10212921480^2 := 10212921482^2$$

$$(2021\ 19)^2 + 20426045080^2 := 20426045081^2$$

$$(2021\ 20)^2 + 10213123599^2 := 10213123601^2$$

$$(2021\ 21)^2 + 20426449320^2 := 20426449321^2$$

$$(2021\ 22)^2 + 10213325720^2 := 10213325722^2$$

$$(2021\ 23)^2 + 20426853564^2 := 20426853565^2$$

$$(2021\ 24)^2 + 10213527843^2 := 10213527845^2$$

$$(2021\ 25)^2 + 20427257812^2 := 20427257813^2$$

$$(2021\ 26)^2 + 10213729968^2 := 10213729970^2$$

$$(2021\ 27)^2 + 20427662064^2 := 20427662065^2$$

$$(2021\ 28)^2 + 10213932095^2 := 10213932097^2$$

$$(2021\ 29)^2 + 20428066320^2 := 20428066321^2$$

$$(2021\ 30)^2 + 10214134224^2 := 10214134226^2$$

$$(2021\ 31)^2 + 20428470580^2 := 20428470581^2$$

$$(2021\ 32)^2 + 10214336355^2 := 10214336357^2$$

$$(2021\ 33)^2 + 20428874844^2 := 20428874845^2$$

$$(2021\ 34)^2 + 10214538488^2 := 10214538490^2$$

$$(2021\ 35)^2 + 20429279112^2 := 20429279113^2$$

$$(2021\ 36)^2 + 10214740623^2 := 10214740625^2$$

$$(2021\ 37)^2 + 20429683384^2 := 20429683385^2$$

$$(2021\ 38)^2 + 10214942760^2 := 10214942762^2$$

$$(2021\ 39)^2 + 20430087660^2 := 20430087661^2$$

$$(2021\ 40)^2 + 10215144899^2 := 10215144901^2$$

$$(2021\ 41)^2 + 20430491940^2 := 20430491941^2$$

$$(2021\ 42)^2 + 10215347040^2 := 10215347042^2$$

$$(2021\ 43)^2 + 20430896224^2 := 20430896225^2$$

$$(2021\ 44)^2 + 10215549183^2 := 10215549185^2$$

$$(2021\ 45)^2 + 20431300512^2 := 20431300513^2$$

$$(2021\ 46)^2 + 10215751328^2 := 10215751330^2$$

$$(2021\ 47)^2 + 20431704804^2 := 20431704805^2$$

$$(2021\ 48)^2 + 10215953475^2 := 10215953477^2$$

$$(2021\ 49)^2 + 20432109100^2 := 20432109101^2$$

$$(2021\ 50)^2 + 10216155624^2 := 10216155626^2$$

$$(2021\ 51)^2 + 20432513400^2 := 20432513401^2$$

$$(2021\ 52)^2 + 10216357775^2 := 10216357777^2$$

$$(2021\ 53)^2 + 20432917704^2 := 20432917705^2$$

$$(2021\ 54)^2 + 10216559928^2 := 10216559930^2$$

$$(2021\ 55)^2 + 20433322012^2 := 20433322013^2$$

$$(2021\ 56)^2 + 10216762083^2 := 10216762085^2$$

$$(2021\ 57)^2 + 20433726324^2 := 20433726325^2$$

$$\begin{aligned} (2021\ 58)^2 + 10216964240^2 &:= 10216964242^2 \\ (2021\ 59)^2 + 20434130640^2 &:= 20434130641^2 \\ (2021\ 60)^2 + 10217166399^2 &:= 10217166401^2 \\ (2021\ 61)^2 + 20434534960^2 &:= 20434534961^2 \\ (2021\ 62)^2 + 10217368560^2 &:= 10217368562^2 \\ (2021\ 63)^2 + 20434939284^2 &:= 20434939285^2 \\ (2021\ 64)^2 + 10217570723^2 &:= 10217570725^2 \\ (2021\ 65)^2 + 20435343612^2 &:= 20435343613^2 \\ (2021\ 66)^2 + 10217772888^2 &:= 10217772890^2 \\ (2021\ 67)^2 + 20435747944^2 &:= 20435747945^2 \\ (2021\ 68)^2 + 10217975055^2 &:= 10217975057^2 \\ (2021\ 69)^2 + 20436152280^2 &:= 20436152281^2 \\ (2021\ 70)^2 + 10218177224^2 &:= 10218177226^2 \\ (2021\ 71)^2 + 20436556620^2 &:= 20436556621^2 \\ (2021\ 72)^2 + 10218379395^2 &:= 10218379397^2 \\ (2021\ 73)^2 + 20436960964^2 &:= 20436960965^2 \\ (2021\ 74)^2 + 10218581568^2 &:= 10218581570^2 \\ (2021\ 75)^2 + 20437365312^2 &:= 20437365313^2 \\ (2021\ 76)^2 + 10218783743^2 &:= 10218783745^2 \\ (2021\ 77)^2 + 20437769664^2 &:= 20437769665^2 \\ (2021\ 78)^2 + 10218985920^2 &:= 10218985922^2 \\ (2021\ 79)^2 + 20438174020^2 &:= 20438174021^2 \end{aligned}$$

$$\begin{aligned} (2021\ 80)^2 + 10219188099^2 &:= 10219188101^2 \\ (2021\ 81)^2 + 20438578380^2 &:= 20438578381^2 \\ (2021\ 82)^2 + 10219390280^2 &:= 10219390282^2 \\ (2021\ 83)^2 + 20438982744^2 &:= 20438982745^2 \\ (2021\ 84)^2 + 10219592463^2 &:= 10219592465^2 \\ (2021\ 85)^2 + 20439387112^2 &:= 20439387113^2 \\ (2021\ 86)^2 + 10219794648^2 &:= 10219794650^2 \\ (2021\ 87)^2 + 20439791484^2 &:= 20439791485^2 \\ (2021\ 88)^2 + 10219996835^2 &:= 10219996837^2 \\ (2021\ 89)^2 + 20440195860^2 &:= 20440195861^2 \\ (2021\ 90)^2 + 10220199024^2 &:= 10220199026^2 \\ (2021\ 91)^2 + 20440600240^2 &:= 20440600241^2 \\ (2021\ 92)^2 + 10220401215^2 &:= 10220401217^2 \\ (2021\ 93)^2 + 20441004624^2 &:= 20441004625^2 \\ (2021\ 94)^2 + 10220603408^2 &:= 10220603410^2 \\ (2021\ 95)^2 + 20441409012^2 &:= 20441409013^2 \\ (2021\ 96)^2 + 10220805603^2 &:= 10220805605^2 \\ (2021\ 97)^2 + 20441813404^2 &:= 20441813405^2 \\ (2021\ 98)^2 + 10221007800^2 &:= 10221007802^2 \\ (2021\ 99)^2 + 20442217800^2 &:= 20442217801^2 \end{aligned}$$

Below are some irregular patterns with less number of digits:

$$\begin{aligned} 2021\ 00^2 + 212205^2 &:= 293045^2 \\ 2021\ 01^2 + 269468^2 &:= 336835^2 \\ 2021\ 03^2 + 445704^2 &:= 489385^2 \\ 2021\ 04^2 + 220550^2 &:= 299146^2 \\ 2021\ 05^2 + 485052^2 &:= 525473^2 \\ 2021\ 06^2 + 509208^2 &:= 547850^2 \\ 2021\ 07^2 + 269476^2 &:= 336845^2 \end{aligned}$$

$$\begin{aligned} 2021\ 10^2 + 269480^2 &:= 336850^2 \\ 2021\ 11^2 + 692952^2 &:= 721825^2 \\ 2021\ 12^2 + 378960^2 &:= 429488^2 \\ 2021\ 13^2 + 269484^2 &:= 336855^2 \\ 2021\ 14^2 + 1102440^2 &:= 1120814^2 \\ 2021\ 15^2 + 485076^2 &:= 525499^2 \\ 2021\ 16^2 + 269488^2 &:= 336860^2 \end{aligned}$$

$$\begin{aligned} 2021 \ 17^2 + 36279720^2 &:= 36280283^2 \\ 2021 \ 18^2 + 692976^2 &:= 721850^2 \\ 2021 \ 19^2 + 250880^2 &:= 322169^2 \\ 2021 \ 20^2 + 212226^2 &:= 293074^2 \\ 2021 \ 22^2 + 224360^2 &:= 301978^2 \\ 2021 \ 23^2 + 1778436^2 &:= 1789885^2 \\ 2021 \ 25^2 + 218196^2 &:= 297429^2 \\ 2021 \ 28^2 + 231605^2 &:= 307403^2 \\ 2021 \ 30^2 + 261744^2 &:= 330706^2 \\ 2021 \ 31^2 + 269508^2 &:= 336885^2 \\ 2021 \ 32^2 + 324855^2 &:= 382607^2 \\ 2021 \ 33^2 + 1181844^2 &:= 1199005^2 \\ 2021 \ 34^2 + 269512^2 &:= 336890^2 \\ 2021 \ 35^2 + 485124^2 &:= 525551^2 \\ 2021 \ 36^2 + 241185^2 &:= 314689^2 \\ 2021 \ 37^2 + 260260^2 &:= 329537^2 \\ 2021 \ 38^2 + 21325080^2 &:= 21326038^2 \\ 2021 \ 39^2 + 634452^2 &:= 665875^2 \\ 2021 \ 40^2 + 212247^2 &:= 293103^2 \\ 2021 \ 41^2 + 1915020^2 &:= 1925659^2 \\ 2021 \ 42^2 + 3633840^2 &:= 3639458^2 \\ 2021 \ 43^2 + 269524^2 &:= 336905^2 \\ 2021 \ 44^2 + 379020^2 &:= 429556^2 \\ 2021 \ 45^2 + 485148^2 &:= 525577^2 \\ 2021 \ 46^2 + 269528^2 &:= 336910^2 \\ 2021 \ 47^2 + 242880^2 &:= 315997^2 \\ 2021 \ 48^2 + 233805^2 &:= 309077^2 \\ 2021 \ 49^2 + 269532^2 &:= 336915^2 \\ 2021 \ 50^2 + 223920^2 &:= 301670^2 \\ 2021 \ 51^2 + 3130080^2 &:= 3136601^2 \\ 2021 \ 52^2 + 269536^2 &:= 336920^2 \\ 2021 \ 53^2 + 693096^2 &:= 721975^2 \end{aligned}$$

$$\begin{aligned} 2021 \ 54^2 + 2741928^2 &:= 2749370^2 \\ 2021 \ 55^2 + 269540^2 &:= 336925^2 \\ 2021 \ 57^2 + 1593876^2 &:= 1606645^2 \\ 2021 \ 58^2 + 269544^2 &:= 336930^2 \\ 2021 \ 59^2 + 2927820^2 &:= 2934791^2 \\ 2021 \ 60^2 + 212268^2 &:= 293132^2 \\ 2021 \ 61^2 + 269548^2 &:= 336935^2 \\ 2021 \ 61^2 + 335720^2 &:= 391889^2 \\ 2021 \ 63^2 + 1306284^2 &:= 1321835^2 \\ 2021 \ 64^2 + 250723^2 &:= 322075^2 \\ 2021 \ 65^2 + 485196^2 &:= 525629^2 \\ 2021 \ 66^2 + 27393120^2 &:= 27393866^2 \\ 2021 \ 67^2 + 269556^2 &:= 336945^2 \\ 2021 \ 68^2 + 379065^2 &:= 429607^2 \\ 2021 \ 69^2 + 1102740^2 &:= 1121119^2 \\ 2021 \ 70^2 + 217872^2 &:= 297222^2 \\ 2021 \ 71^2 + 4142040^2 &:= 4146971^2 \\ 2021 \ 73^2 + 269564^2 &:= 336955^2 \\ 2021 \ 74^2 + 693168^2 &:= 722050^2 \\ 2021 \ 75^2 + 485220^2 &:= 525655^2 \\ 2021 \ 76^2 + 204930^2 &:= 287874^2 \\ 2021 \ 77^2 + 607464^2 &:= 640225^2 \\ 2021 \ 79^2 + 269572^2 &:= 336965^2 \\ 2021 \ 80^2 + 212289^2 &:= 293161^2 \\ 2021 \ 81^2 + 203880^2 &:= 287131^2 \\ 2021 \ 82^2 + 269576^2 &:= 336970^2 \\ 2021 \ 84^2 + 218463^2 &:= 297665^2 \\ 2021 \ 85^2 + 269580^2 &:= 336975^2 \\ 2021 \ 86^2 + 4344648^2 &:= 4349350^2 \\ 2021 \ 88^2 + 205920^2 &:= 288588^2 \\ 2021 \ 89^2 + 788220^2 &:= 813739^2 \\ 2021 \ 90^2 + 485256^2 &:= 525694^2 \end{aligned}$$

$$2021 \ 91^2 + 269588^2 := 336985^2$$

$$2021 \ 92^2 + 379110^2 := 429658^2$$

$$2021 \ 93^2 + 215280^2 := 295343^2$$

$$2021 \ 94^2 + 269592^2 := 336990^2$$

$$2021 \ 95^2 + 204204^2 := 287371^2$$

$$2021 \ 97^2 + 269596^2 := 336995^2$$

$$2021 \ 98^2 + 1659336^2 := 1671610^2$$

$$2021 \ 99^2 + 322920^2 := 381001^2$$

12 Same Digits Equality Expressions

12.1 Powers and Sums

$$2021 := 1^8 + 44^2 + 73^0 + 83^1 = 18 + 442 + 730 + 831$$

$$:= 2^8 + 41^2 + 75^0 + 83^1 = 28 + 412 + 750 + 831$$

$$:= -5^1 + 45^2 + 162^0 = -51 + 452 + 1620$$

$$2703 := 18^0 + 50^2 + 202^1 = 180 + 502 + 2021$$

$$2735 := 33^2 + 38^2 + 202^1 = 332 + 382 + 2021$$

$$3803 := 60^2 + 118^0 + 202^1 = 602 + 1180 + 2021$$

$$5103 := 70^2 + 202^1 + 238^0 = 702 + 2021 + 2380$$

$$6603 := 80^2 + 202^1 + 378^0 = 802 + 2021 + 3780$$

$$8303 := 90^2 + 202^1 + 538^0 = 902 + 2021 + 5380$$

12.2 Factorial and Powers

$$2021 \times 10 + 0 := 1! + 0! + (2! + 5! + 6!) \times 4! = 1^6 + 0^0 + 2^5 \times (5^4 + 6^1) + 4^2$$

$$2021 \times 10 + 4 := 1! \times 4! \times (5! + 2! + 6!) + 3! = 1^1 + 4^5 + 5^6 + (2^3 + 6^2) \times 3^4$$

$$2021 \times 10 + 5 := 1! + (2! + 5! + 6!) \times 4! + 3! = (1^6 \times 2^3 + 5^1) \times (6^4 + 4^2 + 3^5)$$

$$2021 \times 1000 + 760 := 1! \times 8! \times 2! \times 4! + 5! \times 6! = 1^2 \times 8^6 + 2^5 \times 4^1 \times 5^4 + 6^8$$

$$2021 \times 1000 + 761 := 1! + 8! \times 2! \times 4! + 5! \times 6! = 1^2 + 8^6 + 2^5 \times 4^1 \times 5^4 + 6^8$$

12.3 Multiplication

$$2021 \times 2044 = 1022 \times 4042$$

$$2021 \times 2404 = 1202 \times 4042$$

$$2021 \times 3066 = 1022 \times 6063$$

$$2021 \times 3606 = 1202 \times 6063$$

$$2021 \times 4024 = 2012 \times 4042$$

$$2021 \times 4088 = 1022 \times 8084$$

$$2021 \times 4204 = 2102 \times 4042$$

$$2021 \times 4402 = 2201 \times 4042$$

$$2021 \times 4808 = 1202 \times 8084$$

$$2021 \times 6036 = 2012 \times 6063$$

$$2021 \times 6306 = 2102 \times 6063$$

$$2021 \times 6488 = 1622 \times 8084$$

$$2021 \times 6517 = 2107 \times 6251$$

$$2021 \times 6603 = 2201 \times 6063$$

$$2021 \times 8048 = 2012 \times 8084$$

$$2021 \times 8408 = 2102 \times 8084$$

$$2021 \times 8648 = 2162 \times 8084$$

$$2021 \times 8804 = 2201 \times 8084$$

$$2021 \times 8864 = 2216 \times 8084$$

$$202176 := 234 \times 864 = 432 \times 468$$

$$20210428 := 2404 \times 8407 = 4207 \times 4804$$

$$20210796 := 2349 \times 8604 = 4302 \times 4698$$

$$20212832 := 2404 \times 8408 = 4204 \times 4808$$

$$20212956 := 2532 \times 7983 = 3798 \times 5322$$

$$20214480 := 3344 \times 6045 = 4433 \times 4560$$

$$20214972 := 2958 \times 6834 = 3468 \times 5829$$

$$20217180 := 3345 \times 6044 = 4460 \times 4533$$

$$20218572 := 2684 \times 7533 = 3782 \times 5346$$

These numbers sometimes famous as **vamp numbers**.

13 Selfie Fractions

Below are few **selfie fractions** with 2021.

$$\frac{2021}{10105} = \frac{2 \times 0 \times 2 + 1}{1 \times 01 \times 05} = \frac{2 + 0 \times 2 + 1}{1 \times 010 + 5} = \frac{2 + 02 + 1}{10 + 10 + 5} = \frac{2 + 0 \times 21}{(1 + 01) \times 05} = \frac{2 + 021}{10 + 105} = \frac{202 + 1}{1010 + 5}$$

$$\frac{2021}{12126} = \frac{2 \times (02 + 1)}{1 \times 2 \times (12 + 6)} = \frac{2 + 02 + 1}{1 + 2 + 1 + 26} = \frac{2 + 0 \times 21}{1 + 2 + 1 + 2 + 6} = \frac{2 + 021}{1 \times (21 + 2) \times 6} = \frac{202 + 1}{1212 + 6}$$

$$\frac{2021}{14147} := \frac{2 \times 021}{14 \times (14 + 7)} = \frac{2 + 0 \times 2 + 1}{14 + 1^4 \times 7} = \frac{2 + 021}{14 + 147} = \frac{20 \times (2 + 1)}{(14 + 1) \times 4 \times 7} = \frac{20 + 2 \times 1}{14 \times 1 \times (4 + 7)} = \frac{202 + 1}{1414 + 7}$$

$$\frac{2021}{16168} := \frac{2 + 0 \times 2 + 1}{16 + 1^6 \times 8} = \frac{2 + 02 + 1}{16 + 16 + 8} = \frac{2 + 0 \times 21}{1 + 6 + 1^6 + 8} = \frac{2 + 021}{(1 + 6 + 16) \times 8} = \frac{20 + 2 \times 1}{(1 \times 6 + 16) \times 8} = \frac{202 + 1}{1616 + 8}$$

$$\frac{2021}{18189} := \frac{2 + 0 \times 2 + 1}{1 + 8 + 1 + 8 + 9} = \frac{2 + 02 + 1}{18 + 18 + 9} = \frac{2 + 0 \times 21}{1 \times 8 + 1^8 + 9} = \frac{2 + 021}{18 + 189} = \frac{20 + 2 \times 1}{1 + 8 + 189} = \frac{202 + 1}{1818 + 9}$$

$$\frac{2021}{4042} := \frac{2 \times (02 + 1)}{4 + 04 \times 2} = \frac{2^{02} \times 1}{4 \times (0 \times 4 + 2)} = \frac{2 + 0 \times 2 + 1}{4 + 0 \times 4 + 2} = \frac{2 + 02 + 1}{4 + 04 + 2} = \frac{2 + 021}{4 + 042} = \frac{20 + 21}{40 + 42} = \frac{202 + 1}{404 + 2}$$

$$\frac{2021}{6063} := \frac{2 \times (02 + 1)}{6 \times (0 \times 6 + 3)} = \frac{2^{02+1}}{6 + 06 \times 3} = \frac{2 + 0 \times 2 + 1}{6 + 0 \times 6 + 3} = \frac{2 + 02 + 1}{6 + 06 + 3} = \frac{2 + 021}{6 + 063} = \frac{20 + 21}{60 + 63} = \frac{202 + 1}{606 + 3}$$

$$\frac{2021}{8084} := \frac{2 \times 0 \times 2 + 1}{8 \times 0 \times 8 + 4} = \frac{2^{02+1}}{8 \times (0 \times 8 + 4)} = \frac{2 + 0 \times 2 + 1}{8 + 0 \times 8 + 4} = \frac{2 + 02 + 1}{8 + 08 + 4} = \frac{2 + 021}{8 + 084} = \frac{20 + 21}{80 + 84} = \frac{202 + 1}{808 + 4}$$

$$\frac{4007}{1\ 2021} := \frac{4 + 00 \times 7}{12 + 0 \times 21} = \frac{4 \times 00 + 7}{1^2 \times 021} = \frac{4 + 007}{12 + 021} = \frac{40 + 07}{120 + 21}$$

$$\frac{8014}{1\ 2021} := \frac{8 + 0 \times 14}{12 + 0 \times 21} = \frac{8 + 014}{12 + 021} = \frac{8 \times 0 \times 1 + 4}{1 + 2 + 02 + 1} = \frac{8 \times (01 + 4)}{1 \times 20 \times (2 + 1)} = \frac{80 + 14}{120 + 21}$$

13.1 Patterns in Selfie Fractions

$$\frac{2021}{4042} := \frac{2^{02+1}}{(4 + 04) \times 2}$$

$$\frac{2021}{40420} := \frac{2^{02+1}}{(4 + 04) \times 20}$$

$$\frac{2021}{404200} := \frac{2^{02+1}}{(4 + 04) \times 200}$$

$$\frac{2021}{404200} := \frac{2^{02+1}}{(4 + 04) \times 2000}$$

$$\frac{2021}{12126} := \frac{2 \times 021}{1 \times 2 \times 126}$$

$$\frac{2021}{121260} := \frac{2 \times 021}{1 \times 2 \times 1260}$$

$$\frac{2021}{1212600} := \frac{2 \times 021}{1 \times 2 \times 12600}$$

$$\frac{2021}{12126000} := \frac{2 \times 021}{1 \times 2 \times 126000}$$

$$\frac{2021}{12126} := \frac{2^{02} \times 1}{1 \times 2 \times 1 \times 2 \times 6}$$

$$\frac{2021}{121260} := \frac{2^{02} \times 1}{1 \times 2 \times 1 \times 2 \times 60}$$

$$\frac{2021}{1212600} := \frac{2^{02} \times 1}{1 \times 2 \times 1 \times 2 \times 600}$$

$$\frac{2021}{12126000} := \frac{2^{02} \times 1}{1 \times 2 \times 1 \times 2 \times 6000}$$

$$\frac{2021}{14147} := \frac{2^{02} \times 1}{1 \times 4 \times 1^4 \times 7}$$

$$\frac{2021}{141470} := \frac{2^{02} \times 1}{1 \times 4 \times 1^4 \times 70}$$

$$\frac{2021}{1414700} := \frac{2^{02} \times 1}{1 \times 4 \times 1^4 \times 700}$$

$$\frac{2021}{14147000} := \frac{2^{02} \times 1}{1 \times 4 \times 1^4 \times 7000}$$

$$\frac{2021}{14147} := \frac{2^{02+1}}{(1 \times 4 + 1 \times 4) \times 7}$$

$$\frac{2021}{141470} := \frac{2^{02+1}}{(1 \times 4 + 1 \times 4) \times 70}$$

$$\frac{2021}{1414700} := \frac{2^{02+1}}{(1 \times 4 + 1 \times 4) \times 700}$$

$$\frac{2021}{14147000} := \frac{2^{02+1}}{(1 \times 4 + 1 \times 4) \times 7000}$$

$$\frac{2021}{16168} := \frac{2 + 021}{(1 + 6 + 16) \times 8}$$

$$\frac{2021}{161680} := \frac{2 + 021}{(1 + 6 + 16) \times 80}$$

$$\frac{2021}{1616800} := \frac{2 + 021}{(1 + 6 + 16) \times 800}$$

$$\frac{2021}{16168000} := \frac{2 + 021}{(1 + 6 + 16) \times 8000}$$

$$\frac{2021}{16168} := \frac{2 \times 021}{1 \times 6 \times (1+6) \times 8}$$

$$\frac{2021}{161680} := \frac{2 \times 021}{1 \times 6 \times (1+6) \times 80}$$

$$\frac{2021}{1616800} := \frac{2 \times 021}{1 \times 6 \times (1+6) \times 800}$$

$$\frac{2021}{16168000} := \frac{2 \times 021}{1 \times 6 \times (1+6) \times 8000}$$

$$\frac{2021}{18189} := \frac{2^{02+1}}{1 \times 8 \times 1^8 \times 9}$$

$$\frac{2021}{181890} := \frac{2^{02+1}}{1 \times 8 \times 1^8 \times 90}$$

$$\frac{2021}{1818900} := \frac{2^{02+1}}{1 \times 8 \times 1^8 \times 900}$$

$$\frac{2021}{18189000} := \frac{2^{02+1}}{1 \times 8 \times 1^8 \times 9000}$$

$$\frac{4007}{1 \ 2021} := \frac{4 \times 00 + 7}{1^2 \times 021}$$

$$\frac{4007}{1 \ 20210} := \frac{4 \times 00 + 7}{1^2 \times 0210}$$

$$\frac{4007}{1 \ 202100} := \frac{4 \times 00 + 7}{1^2 \times 02100}$$

$$\frac{4007}{1 \ 2021000} := \frac{4 \times 00 + 7}{1^2 \times 021000}$$

14 Functional Representations

14.1 Fibonacci Sequences

$$F(0) = F(1) = 1, F(n) = F(n-1) + F(n-2), n \geq 2,$$

$$0, 1, 1, 2, 3, 5, 8, 13, ..$$

Then,

$$2021 := F(7) + F(9) + F(14) + F(17)$$

14.2 Fibonacci Sequences Patterns

14.2.1 Blocks of 10

$$\mathbf{2021\ 0} := F(2) + F(4) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 1} := F(5) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 2} := F(2) + F(5) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 3} := F(3) + F(5) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 4} := F(6) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 5} := F(2) + F(6) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 6} := F(3) + F(6) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 7} := F(4) + F(6) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 8} := F(2) + F(4) + F(6) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{2021\ 9} := F(7) + F(10) + F(13) + F(15) + F(17) + F(22)$$

$$\mathbf{1\ 2021} := F(2) + F(4) + F(6) + F(8) + F(10) + F(16) + F(21)$$

$$\mathbf{2\ 2021} := F(2) + F(5) + F(9) + F(11) + F(19) + F(22)$$

$$\mathbf{3\ 2021} := F(5) + F(8) + F(12) + F(15) + F(18) + F(23)$$

$$\mathbf{4\ 2021} := F(2) + F(4) + F(6) + F(10) + F(12) + F(15) + F(17) + F(21) + F(23)$$

$$\mathbf{5\ 2021} := F(2) + F(5) + F(7) + F(11) + F(14) + F(16) + F(19) + F(24)$$

$$\mathbf{6\ 2021} := F(5) + F(12) + F(14) + F(19) + F(21) + F(24)$$

$$\mathbf{7\ 2021} := F(2) + F(4) + F(6) + F(9) + F(12) + F(16) + F(20) + F(22) + F(24)$$

$$\mathbf{8\ 2021} := F(4) + F(6) + F(8) + F(10) + F(12) + F(20) + F(25)$$

$$\mathbf{9\ 2021} := F(5) + F(9) + F(13) + F(17) + F(19) + F(21) + F(25)$$

14.2.2 Block of 100

$$\mathbf{2021\ 00} := F(2) + F(7) + F(9) + F(11) + F(14) + F(16) + F(19) + F(27)$$

$$\mathbf{2021\ 01} := F(3) + F(7) + F(9) + F(11) + F(14) + F(16) + F(19) + F(27)$$

$$\mathbf{2021\ 02} := F(4) + F(7) + F(9) + F(11) + F(14) + F(16) + F(19) + F(27)$$

$$\mathbf{2021\ 03} := F(2) + F(4) + F(7) + F(9) + F(11) + F(14) + F(16) + F(19) + F(27)$$

$$\mathbf{2021\ 04} := F(5) + F(7) + F(9) + F(11) + F(14) + F(16) + F(19) + F(27)$$

$$\mathbf{2021\ 05} := F(2) + F(5) + F(7) + F(9) + F(11) + F(14) + F(16) + F(19) + F(27)$$

$$\mathbf{2021\ 06} := F(3) + F(5) + F(7) + F(9) + F(11) + F(14) + F(16) + F(19) + F(27)$$

$$\mathbf{2021\ 07} := F(12) + F(14) + F(16) + F(19) + F(27)$$

$$\mathbf{2021\ 08} := F(2) + F(12) + F(14) + F(16) + F(19) + F(27)$$

- 2021 09** := $F(3) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 10 := $F(4) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 11 := $F(2) + F(4) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 12 := $F(5) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 13 := $F(2) + F(5) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 14 := $F(3) + F(5) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 15 := $F(6) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 16 := $F(2) + F(6) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 17 := $F(3) + F(6) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 18 := $F(4) + F(6) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 19 := $F(2) + F(4) + F(6) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 20 := $F(7) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 21 := $F(2) + F(7) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 22 := $F(3) + F(7) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 23 := $F(4) + F(7) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 24 := $F(2) + F(4) + F(7) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 25 := $F(5) + F(7) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 26 := $F(2) + F(5) + F(7) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 27 := $F(3) + F(5) + F(7) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 28 := $F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 29 := $F(2) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 30 := $F(3) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 31 := $F(4) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 32 := $F(2) + F(4) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 33 := $F(5) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 34 := $F(2) + F(5) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 35 := $F(3) + F(5) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 36 := $F(6) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 37 := $F(2) + F(6) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 38 := $F(3) + F(6) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 39 := $F(4) + F(6) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 40 := $F(2) + F(4) + F(6) + F(8) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 41 := $F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 42 := $F(2) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 43 := $F(3) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$

- 2021 44** := $F(4) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 45 := $F(2) + F(4) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 46 := $F(5) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 47 := $F(2) + F(5) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 48 := $F(3) + F(5) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 49 := $F(6) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 50 := $F(2) + F(6) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 51 := $F(3) + F(6) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 52 := $F(4) + F(6) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 53 := $F(2) + F(4) + F(6) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 54 := $F(7) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 55 := $F(2) + F(7) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 56 := $F(3) + F(7) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 57 := $F(4) + F(7) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 58 := $F(2) + F(4) + F(7) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 59 := $F(5) + F(7) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 60 := $F(2) + F(5) + F(7) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 61 := $F(3) + F(5) + F(7) + F(9) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 62 := $F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 63 := $F(2) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 64 := $F(3) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 65 := $F(4) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 66 := $F(2) + F(4) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 67 := $F(5) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 68 := $F(2) + F(5) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 69 := $F(3) + F(5) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 70 := $F(6) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 71 := $F(2) + F(6) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 72 := $F(3) + F(6) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 73 := $F(4) + F(6) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 74 := $F(2) + F(4) + F(6) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 75 := $F(7) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 76 := $F(2) + F(7) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 77 := $F(3) + F(7) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$
2021 78 := $F(4) + F(7) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27)$

$$\begin{aligned} \mathbf{2021\ 79} &:= F(2) + F(4) + F(7) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 80} &:= F(5) + F(7) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 81} &:= F(2) + F(5) + F(7) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 82} &:= F(3) + F(5) + F(7) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 83} &:= F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 84} &:= F(2) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 85} &:= F(3) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 86} &:= F(4) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 87} &:= F(2) + F(4) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 88} &:= F(5) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 89} &:= F(2) + F(5) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 90} &:= F(3) + F(5) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 91} &:= F(6) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 92} &:= F(2) + F(6) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 93} &:= F(3) + F(6) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 94} &:= F(4) + F(6) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 95} &:= F(2) + F(4) + F(6) + F(8) + F(10) + F(12) + F(14) + F(16) + F(19) + F(27) \\ \mathbf{2021\ 96} &:= F(17) + F(19) + F(27) \\ \mathbf{2021\ 97} &:= F(2) + F(17) + F(19) + F(27) \\ \mathbf{2021\ 98} &:= F(3) + F(17) + F(19) + F(27) \\ \mathbf{2021\ 99} &:= F(4) + F(17) + F(19) + F(27) \end{aligned}$$

14.3 Triangular Number

$$T(n) := \frac{n \times (n + 1)}{2}, n \geq 0.$$

Then,

$$\mathbf{2021} := T(1) + T(19) + T(60)$$

14.4 Triangular Numbers Patterns

14.5 Blocks of 10

$$2021\ 0 := T(1) + T(42) + T(196)$$

$$2021\ 1 := T(2) + T(77) + T(185)$$

$$2021\ 2 := T(86) + T(181)$$

$$2021\ 3 := T(1) + T(86) + T(181)$$

$$2021\ 4 := T(3) + T(77) + T(185)$$

$$2021\ 5 := T(2) + T(86) + T(181)$$

$$2021\ 6 := T(4) + T(37) + T(197)$$

$$2021\ 7 := T(5) + T(101) + T(173)$$

$$2021\ 8 := T(94) + T(177)$$

$$2021\ 9 := T(1) + T(94) + T(177)$$

$$1\ 2021 := T(2) + T(80) + T(132)$$

$$2\ 2021 := T(2) + T(42) + T(205)$$

$$3\ 2021 := T(4) + T(75) + T(241)$$

$$4\ 2021 := T(80) + T(278)$$

$$5\ 2021 := T(100) + T(306)$$

$$6\ 2021 := T(43) + T(349)$$

$$7\ 2021 := T(1) + T(4) + T(379)$$

$$8\ 2021 := T(114) + T(388)$$

$$9\ 2021 := T(2) + T(244) + T(352)$$

14.6 Block of 100

$$2021\ 00 := T(1) + T(331) + T(542)$$

$$2021\ 01 := T(18) + T(635)$$

$$2021\ 02 := T(1) + T(18) + T(635)$$

$$2021\ 03 := T(3) + T(73) + T(631)$$

$$2021\ 04 := T(276) + T(572)$$

$$2021\ 05 := T(404) + T(490)$$

$$2021\ 06 := T(187) + T(607)$$

$$2021\ 07 := T(321) + T(548)$$

$$2021\ 08 := T(64) + T(632)$$

$$2021\ 09 := T(1) + T(64) + T(632)$$

$$2021\ 10 := T(410) + T(485)$$

$$2021\ 11 := T(1) + T(410) + T(485)$$

$$2021\ 12 := T(3) + T(187) + T(607)$$

$$2021\ 13 := T(170) + T(612)$$

$$2021\ 14 := T(252) + T(583)$$

$$2021\ 15 := T(40) + T(634)$$

$$2021\ 16 := T(1) + T(40) + T(634)$$

$$2021\ 17 := T(2) + T(252) + T(583)$$

$$2021\ 18 := T(2) + T(40) + T(634)$$

$$2021\ 19 := T(3) + T(170) + T(612)$$

$$2021\ 20 := T(19) + T(635)$$

$$2021\ 21 := T(1) + T(19) + T(635)$$

$$2021\ 22 := T(5) + T(321) + T(548)$$

$$2021\ 23 := T(2) + T(19) + T(635)$$

$$2021\ 24 := T(307) + T(556)$$

$$2021\ 25 := T(1) + T(307) + T(556)$$

$$2021\ 26 := T(124) + T(623)$$

$$2021\ 27 := T(1) + T(124) + T(623)$$

$$2021\ 28 := T(5) + T(170) + T(612)$$

$$2021\ 29 := T(151) + T(617)$$

$$2021\ 30 := T(1) + T(151) + T(617)$$

$$2021\ 31 := T(102) + T(627)$$

$$2021\ 32 := T(1) + T(102) + T(627)$$

$$2021\ 33 := T(7) + T(404) + T(490)$$

$$2021\ 34 := T(2) + T(102) + T(627)$$

$$2021\ 35 := T(3) + T(151) + T(617)$$

$$2021\ 36 := T(4) + T(124) + T(623)$$

$$2021\ 37 := T(108) + T(626)$$

$$2021\ 38 := T(129) + T(622)$$

$$2021\ 39 := T(1) + T(129) + T(622)$$

$$2021\ 40 := T(20) + T(635)$$

$$2021\ 41 := T(1) + T(20) + T(635)$$

$$2021\ 42 := T(7) + T(252) + T(583)$$

$$2021\ 43 := T(2) + T(20) + T(635)$$

$$2021\ 44 := T(3) + T(129) + T(622)$$

$$2021\ 45 := T(6) + T(307) + T(556)$$

$$2021\ 46 := T(54) + T(633)$$

$$2021\ 47 := T(356) + T(526)$$

$$2021\ 48 := T(1) + T(356) + T(526)$$

$$2021\ 49 := T(147) + T(618)$$

$$2021\ 50 := T(439) + T(459)$$

$$2021\ 51 := T(1) + T(439) + T(459)$$

$$2021\ 52 := T(2) + T(147) + T(618)$$

$$2021\ 53 := T(272) + T(574)$$

$$2021\ 54 := T(1) + T(272) + T(574)$$

$$2021\ 55 := T(3) + T(147) + T(618)$$

$$2021\ 56 := T(41) + T(634)$$

$$2021\ 57 := T(1) + T(41) + T(634)$$

$$2021\ 58 := T(6) + T(108) + T(626)$$

$$2021\ 59 := T(296) + T(562)$$

$$2021\ 60 := T(259) + T(580)$$

$$2021\ 61 := T(21) + T(635)$$

$$2021\ 62 := T(96) + T(628)$$

$$2021\ 63 := T(1) + T(96) + T(628)$$

$$2021\ 64 := T(2) + T(21) + T(635)$$

$$2021\ 65 := T(2) + T(96) + T(628)$$

$$2021\ 66 := T(3) + T(259) + T(580)$$

$$2021\ 67 := T(3) + T(21) + T(635)$$

$$2021\ 68 := T(82) + T(630)$$

$$2021\ 69 := T(1) + T(82) + T(630)$$

$$2021\ 70 := T(359) + T(524)$$

$$2021\ 71 := T(74) + T(631)$$

$$2021\ 72 := T(1) + T(74) + T(631)$$

$$2021\ 73 := T(65) + T(632)$$

$$2021\ 74 := T(1) + T(65) + T(632)$$

$$2021\ 75 := T(5) + T(259) + T(580)$$

$$2021\ 76 := T(134) + T(621)$$

$$2021\ 77 := T(1) + T(134) + T(621)$$

$$2021\ 78 := T(4) + T(82) + T(630)$$

$$2021\ 79 := T(2) + T(134) + T(621)$$

$$2021\ 80 := T(114) + T(625)$$

$$2021\ 81 := T(1) + T(114) + T(625)$$

$$2021\ 82 := T(3) + T(134) + T(621)$$

$$2021\ 83 := T(22) + T(635)$$

$$2021\ 84 := T(1) + T(22) + T(635)$$

$$2021\ 85 := T(270) + T(575)$$

$$2021\ 86 := T(143) + T(619)$$

$$2021\ 87 := T(1) + T(143) + T(619)$$

$$2021\ 88 := T(2) + T(270) + T(575)$$

$$2021\ 89 := T(2) + T(143) + T(619)$$

$$2021\ 90 := T(4) + T(114) + T(625)$$

$$2021\ 91 := T(174) + T(611)$$

$$2021\ 92 := T(298) + T(561)$$

$$2021\ 93 := T(1) + T(298) + T(561)$$

$$2021\ 94 := T(437) + T(461)$$

$$2021\ 95 := T(250) + T(584)$$

$$2021\ 96 := T(238) + T(589)$$

$$2021\ 97 := T(233) + T(591)$$

$$2021\ 98 := T(42) + T(634)$$

$$2021\ 99 := T(1) + T(42) + T(634)$$

15 Fixed Digits Repetitions Prime Patterns

15.1 Length 9

582 54913	9 5675141
582 21 54913	9 21 5675141
582 21 21 54913	9 21 21 5675141
582 21 21 21 54913	9 21 21 21 5675141
582 21 21 21 21 54913	9 21 21 21 21 5675141
582 21 21 21 21 21 54913	9 21 21 21 21 21 5675141
582 21 21 21 21 21 21 54913	9 21 21 21 21 21 21 5675141
582 21 21 21 21 21 21 21 54913	9 21 21 21 21 21 21 21 5675141
582 21 21 21 21 21 21 21 21 54913	9 21 21 21 21 21 21 21 21 5675141

Remark 15.1. *In this case, we don't have results with 2021, that's why we wrote the results for 21.*

15.2 Length 8

218 2021	11 19 2021
33 218 2021	11 243 19 2021
33 33 218 2021	11 243 243 19 2021
33 33 33 218 2021	11 243 243 243 19 2021
33 33 33 33 218 2021	11 243 243 243 243 19 2021
33 33 33 33 33 218 2021	11 243 243 243 243 243 19 2021
33 33 33 33 33 33 218 2021	11 243 243 243 243 243 243 19 2021
33 33 33 33 33 33 33 218 2021	11 243 243 243 243 243 243 243 19 2021

800 2021 9	2021 5171
800 3 2021 9	2 3 021 5171
800 3 3 2021 9	2 3 3 021 5171
800 3 3 3 2021 9	2 3 3 3 021 5171
800 3 3 3 3 2021 9	2 3 3 3 3 021 5171
800 3 3 3 3 3 2021 9	2 3 3 3 3 3 021 5171
800 3 3 3 3 3 3 2021 9	2 3 3 3 3 3 3 021 5171
800 3 3 3 3 3 3 3 2021 9	2 3 3 3 3 3 3 3 021 5171

15.3 Length 7

5 1 2021	2021143
5 42 1 2021	202187 143
5 42 42 1 2021	202187 87 143
5 42 42 42 1 2021	202187 87 87 143
5 42 42 42 42 1 2021	202187 87 87 87 143
5 42 42 42 42 42 1 2021	202187 87 87 87 87 143
5 42 42 42 42 42 42 1 2021	202187 87 87 87 87 87 143
8 2021 573	3648 2021
21 8 2021 573	3648 202199
21 21 8 2021 573	3648 202199 99
21 21 21 8 2021 573	3648 202199 99 99
21 21 21 21 8 2021 573	3648 202199 99 99 99
21 21 21 21 21 8 2021 573	3648 202199 99 99 99 99
21 21 21 21 21 21 8 2021 573	3648 202199 99 99 99 99 99

15.4 Length 6

2021 9

20213358557 9

20213358557 3358557 9

20213358557 3358557 3358557 9

20213358557 3358557 3358557 3358557 9

20213358557 3358557 3358557 3358557 3358557 9

2021 9

20215649270 9

20215649270 5649270 9

20215649270 5649270 5649270 9

20215649270 5649270 5649270 5649270 9

20215649270 5649270 5649270 5649270 5649270 9

5 **2021**

62040 5 **2021**

62040 62040 5 **2021**

62040 62040 62040 5 **2021**

62040 62040 62040 62040 5 **2021**

62040 62040 62040 62040 62040 5 **2021**

5 **2021**

5 41658 **2021**

5 41658 41658 **2021**

5 41658 41658 41658 **2021**

5 41658 41658 41658 41658 **2021**

5 41658 41658 41658 41658 41658 **2021**

5 **2021**

5 155757 **2021**

5 155757 155757 **2021**

5 155757 155757 155757 **2021**

5 155757 155757 155757 155757 **2021**

5 155757 155757 155757 155757 155757 **2021**

8 **2021**

8 **2021**386703

8 **2021**386703 386703

8 **2021**386703 386703 386703

8 **2021**386703 386703 386703 386703

8 **2021**386703 386703 386703 386703 386703

16 Embedded Prime Numbers Patterns

16.1 Palindromic Prime Numbers: Embedded With Same Digits

1 **12021** 1
1111 **12021** 1111
11111 **12021** 11111
100211111 **12021** 111112001
12212100211111 **12021** 11111200121221
1001212212100211111 **12021** 1111120012122121001
112011001212212100211111 **12021** 111112001212212100110211
12112011001212212100211111 **12021** 11111200121221210011021121
122112112011001212212100211111 **12021** 111112001212212100110211211221
11112122112112011001212212100211111 **12021** 11111200121221210011021121122121111
1211112122112112011001212212100211111 **12021** 1111120012122121001102112112212111121

1000 **2021202** 0001
1111000 **2021202** 0001111
111111000 **2021202** 0001111111
122111111000 **2021202** 0001111111221
12211122111111000 **2021202** 000111111122111221
110112211122111111000 **2021202** 0001111111221112211011
12122110112211122111111000 **2021202** 000111111122111221101122121
121212122110112211122111111000 **2021202** 0001111111221112211011221212121
1221121212122110112211122111111000 **2021202** 00011111112211122110112212121211221

111 **2021** 0 **1202** 111
111111 **2021** 0 **1202** 111111
1111111111 **2021** 0 **1202** 1111111111
1211111111111 **2021** 0 **1202** 1111111111121
122212111111111111 **2021** 0 **1202** 1111111111112122221
1101122212111111111111 **2021** 0 **1202** 11111111111121222211011
1201011101122212111111111111 **2021** 0 **1202** 11111111111121222211011101021
11220112010111011222121111111111111 **2021** 0 **1202** 11111111111121222211011101021102211
102211220112010111011222121111111111111 **2021** 0 **1202** 111111111111212222110111010211022112201

1 2021 1 1202 1
12111 2021 1 1202 11121
1212111 2021 1 1202 1112121
120121212111 2021 1 1202 111212121021
10221120121212111 2021 1 1202 11121212102112201
1112110221120121212111 2021 1 1202 1112121210211220112111
10211112110221120121212111 2021 1 1202 11121212102112201121111201
1020210211112110221120121212111 2021 1 1202 1112121210211220112111120120201
112211020210211112110221120121212111 2021 1 1202 111212121021122011211112012020112211

121 2021 2 1202 121
1002121 2021 2 1202 1212001
101111002121 2021 2 1202 121200111101
121101101111002121 2021 2 1202 121200111101101121
11201121101101111002121 2021 2 1202 12120011110110112110211
110111201121101101111002121 2021 2 1202 121200111101101121102111011
110001110111201121101101111002121 2021 2 1202 121200111101101121102111011100011
111201110001110111201121101101111002121 2021 2 1202 121200111101101121102111011100011102111

16.2 Non Palindromic Prime Numbers: Embedded With Same Digits

111 2021 111
112111 2021 111211
101112111 2021 111211101
11202101112111 2021 11121110120211
100211202101112111 2021 111211101202112001
1101100211202101112111 2021 1112111012021120011011
100021101100211202101112111 2021 111211101202112001101120001
102100021101100211202101112111 2021 111211101202112001101120001201
10011102100021101100211202101112111 2021 11121110120211200110112000120111001

17 More On Prime Numbers

17.1 Magic Square Type Embedded Palindromic Prime Numbers

This section brings magic square type **palindromic prime numbers**, sometimes called as **palprimes**, where rows, columns and principal diagonals are also **palprimes**. The embedded properties are also true. Here we have given three examples of order 7×7 , where there is symmetry in representations

1. Magic Square Type Properties

```
1 3 1 1 1 3 1
3 2 9 1 9 2 3
1 9 5 2 5 9 1
1 1 2 0 2 1 1
1 9 5 2 5 9 1
3 2 9 1 9 2 3
1 3 1 1 1 3 1
```

• Embedded Palprime

1311131 3291923 1952591 11 **2021** 1 1952591 3291923 1311131

2. Magic Square Type Properties

```
1 3 7 1 7 3 1
3 3 3 1 3 3 3
7 3 6 2 6 3 7
1 1 2 0 2 1 1
7 3 6 2 6 3 7
3 3 3 1 3 3 3
1 3 7 1 7 3 1
```

• Embedded Palprime

1371731 3331333 7362637 11 **2021** 1 7362637 3331333 1371731

3. Magic Square Type Properties

7 1 7 7 7 1 7
1 1 2 0 2 1 1
7 2 2 6 2 2 7
7 0 6 9 6 0 7
7 2 2 6 2 2 7
1 1 2 0 2 1 1
7 1 7 7 7 1 7

- **Embedded Palprime**

7177717 11 **20211** 7226227 7069607 7226227 11**2021** 1 7177717

17.2 Primes in Prime

One of **palindromic prime numbers** having **2021** with same digits is **1120211**. Below is a list of prime numbers with same digits, 0, 1 and 2 derived from **1120211**:

2, 11, 101, 211, 10721, 1201, 2011, 2111, 10111, 10211, 12011, 12101, 12211, 21011, 21101,
21121, 21211, 22111, 101221, 102121, 110221, 112121, 120121, 121021, 122011, 201121,
201211, 221101, 1011221, 1112201, 1120121, 1120211, 1210211, 1212011, 2011211, 2121011

18 Amicable Numbers

Amicable numbers with 2021 and 21.

$$\begin{aligned} 2205 &:= 42^2 + 021^2 \Leftrightarrow 4 \mathbf{2021} := -2^2 + 205^2 \\ \mathbf{21384} &:= 147^2 - 015^2 \Leftrightarrow \mathbf{147015} := -\mathbf{21}^2 + 384^2 \end{aligned}$$

$$\begin{aligned} \mathbf{21} &:= \mathbf{2} \times 7 + \mathbf{1} \times 7 \\ \mathbf{216} &:= \mathbf{21} \times 8 + \mathbf{6} \times 8 \end{aligned}$$

The last two expressions are known as self-amicable numbers.

19 Palindromic-Type Expressions and Patterns

19.1 Reverse Square

The year 2021 and 1202 are reverse of each other. The squares, i.e., $2021^2 := 4084441$ and $2021^2 := 1444804$ are also reverse of each other. Below are more properties of similar kinds.

19.2 Palindromic-Type Square Expressions

$$1202^2 + 2021^2 = 1444804 + 4084441$$

$$11202^2 + 20211^2 = 125484804 + 408484521$$

$$102021^2 + 120201^2 := 10408284441 + 14448280401$$

$$101202^2 + 202101^2 := 10241844804 + 40844814201$$

$$111202^2 + 202111^2 := 12365884804 + 40848856321$$

19.3 Palindromic-Type Expressions

$$1202 \times 11 + 11 \times 2021 := 13222 + 22231$$

$$1202 \times 12 + 21 \times 2021 := 14424 + 42441$$

$$1202 \times 13 + 31 \times 2021 := 15626 + 62651$$

$$1202 \times 14 + 41 \times 2021 := 16828 + 82861$$

$$1202 \times 21 + 12 \times 2021 := 25242 + 24252$$

$$1202 \times 22 + 22 \times 2021 := 26444 + 44462$$

$$1202 \times 23 + 32 \times 2021 := 27646 + 64672$$

$$1202 \times 24 + 42 \times 2021 := 28848 + 84882$$

$$1202 \times 31 + 13 \times 2021 := 37262 + 26273$$

$$1202 \times 32 + 23 \times 2021 := 38464 + 46483$$

$$1202 \times 33 + 33 \times 2021 := 39666 + 66693$$

$$1202 \times 41 + 14 \times 2021 := 49282 + 28294$$

$$1202 \times 101 + 101 \times 2021 := 121402 + 204121$$

$$1202 \times 102 + 201 \times 2021 := 122604 + 406221$$

$$1202 \times 103 + 301 \times 2021 := 123806 + 608321$$

$$1202 \times 111 + 111 \times 2021 := 133422 + 224331$$

$$1202 \times 112 + 211 \times 2021 := 134624 + 426431$$

$$1202 \times 113 + 311 \times 2021 := 135826 + 628531$$

$$1202 \times 121 + 121 \times 2021 := 145442 + 244541$$

$$1202 \times 122 + 221 \times 2021 := 146644 + 446641$$

$$1202 \times 123 + 321 \times 2021 := 147846 + 648741$$

$$1202 \times 131 + 131 \times 2021 := 157462 + 264751$$

$$1202 \times 132 + 231 \times 2021 := 158664 + 466851$$

$$1202 \times 133 + 331 \times 2021 := 159866 + 668951$$

$$1202 \times 141 + 141 \times 2021 := 169482 + 284961$$

$$1202 \times 201 + 102 \times 2021 := 241602 + 206142$$

$$1202 \times 202 + 202 \times 2021 := 242804 + 408242$$

$$1202 \times 211 + 112 \times 2021 := 253622 + 226352$$

$$1202 \times 212 + 212 \times 2021 := 254824 + 428452$$

$$1202 \times 221 + 122 \times 2021 := 265642 + 246562$$

$$1202 \times 222 + 222 \times 2021 := 266844 + 448662$$

$$1202 \times 231 + 132 \times 2021 := 277662 + 266772$$

$$1202 \times 232 + 232 \times 2021 := 278864 + 468872$$

$$1202 \times 241 + 142 \times 2021 := 289682 + 286982$$

$$1202 \times 301 + 103 \times 2021 := 361802 + 208163$$

$$1202 \times 311 + 113 \times 2021 := 373822 + 228373$$

$$1202 \times 321 + 123 \times 2021 := 385842 + 248583$$

$$1202 \times 331 + 133 \times 2021 := 397862 + 268793$$

$$1202 \times 1001 + 1001 \times 2021 := 1203202 + 2023021$$

$$1202 \times 1002 + 2001 \times 2021 := 1204404 + 4044021$$

$$1202 \times 1003 + 3001 \times 2021 := 1205606 + 6065021$$

$$1202 \times 1004 + 4001 \times 2021 := 1206808 + 8086021$$

$$1202 \times 1011 + 1101 \times 2021 := 1215222 + 2225121$$

$$1202 \times 1012 + 2101 \times 2021 := 1216424 + 4246121$$

$$1202 \times 1013 + 3101 \times 2021 := 1217626 + 6267121$$

$$1202 \times 1014 + 4101 \times 2021 := 1218828 + 8288121$$

$$1202 \times 1021 + 1201 \times 2021 := 1227242 + 2427221$$

$$1202 \times 1022 + 2201 \times 2021 := 1228444 + 4448221$$

$$1202 \times 1023 + 3201 \times 2021 := 1229646 + 6469221$$

$$\begin{aligned}
 1202 \times 1031 + 1301 \times 2021 &:= 1239262 + 2629321 \\
 1202 \times 1101 + 1011 \times 2021 &:= 1323402 + 2043231 \\
 1202 \times 1102 + 2011 \times 2021 &:= 1324604 + 4064231 \\
 1202 \times 1103 + 3011 \times 2021 &:= 1325806 + 6085231 \\
 1202 \times 1111 + 1111 \times 2021 &:= 1335422 + 2245331 \\
 1202 \times 1112 + 2111 \times 2021 &:= 1336624 + 4266331 \\
 1202 \times 1113 + 3111 \times 2021 &:= 1337826 + 6287331 \\
 1202 \times 1121 + 1211 \times 2021 &:= 1347442 + 2447431 \\
 1202 \times 1122 + 2211 \times 2021 &:= 1348644 + 4468431 \\
 1202 \times 1123 + 3211 \times 2021 &:= 1349846 + 6489431 \\
 1202 \times 1131 + 1311 \times 2021 &:= 1359462 + 2649531 \\
 1202 \times 1201 + 1021 \times 2021 &:= 1443602 + 2063441 \\
 1202 \times 1202 + 2021 \times 2021 &:= 1444804 + 4084441
 \end{aligned}$$

The last expression is the same the first given in the beginning of the subsection, i.e.,

$$1202^2 + 2021^2 = 1444804 + 4084441$$

19.4 Palindromic-Type Patterns

$$\begin{aligned}
 12021 \times 11 + 11 \times 12021 &:= 132231 + 132231 \\
 12021 \times 111 + 111 \times 12021 &:= 1334331 + 1334331 \\
 12021 \times 1111 + 1111 \times 12021 &:= 13355331 + 13355331 \\
 12021 \times 11111 + 11111 \times 12021 &:= 133565331 + 133565331
 \end{aligned}$$

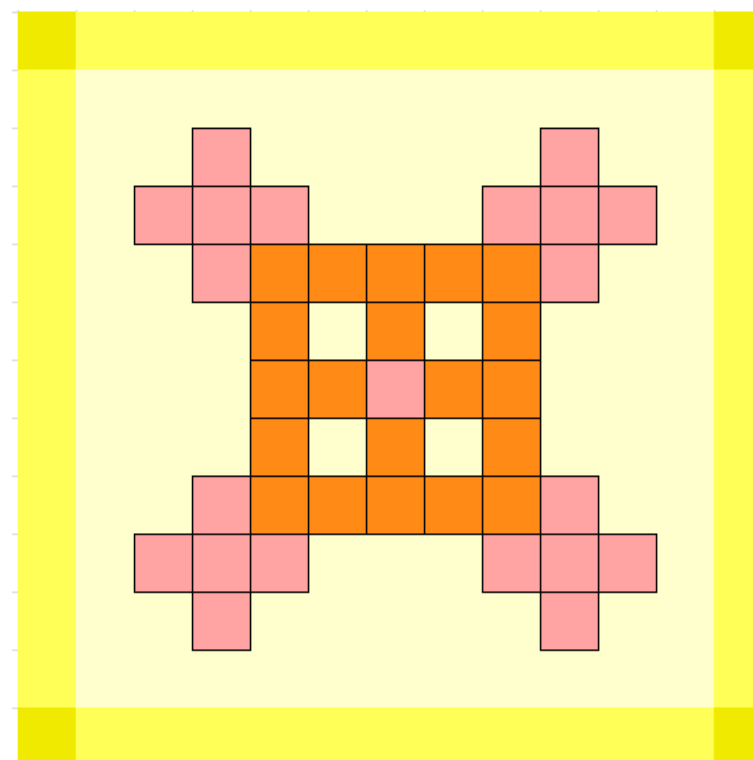
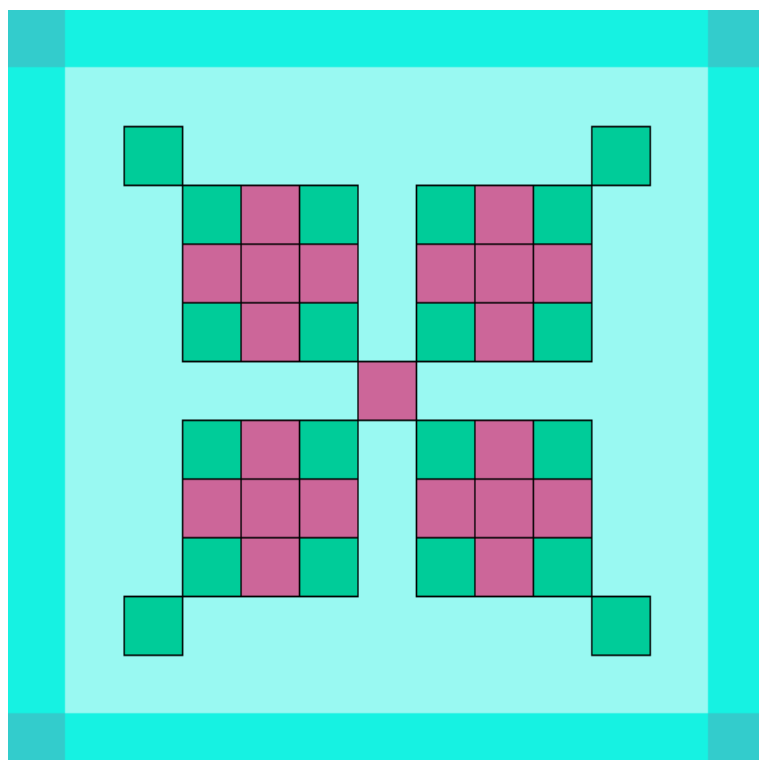
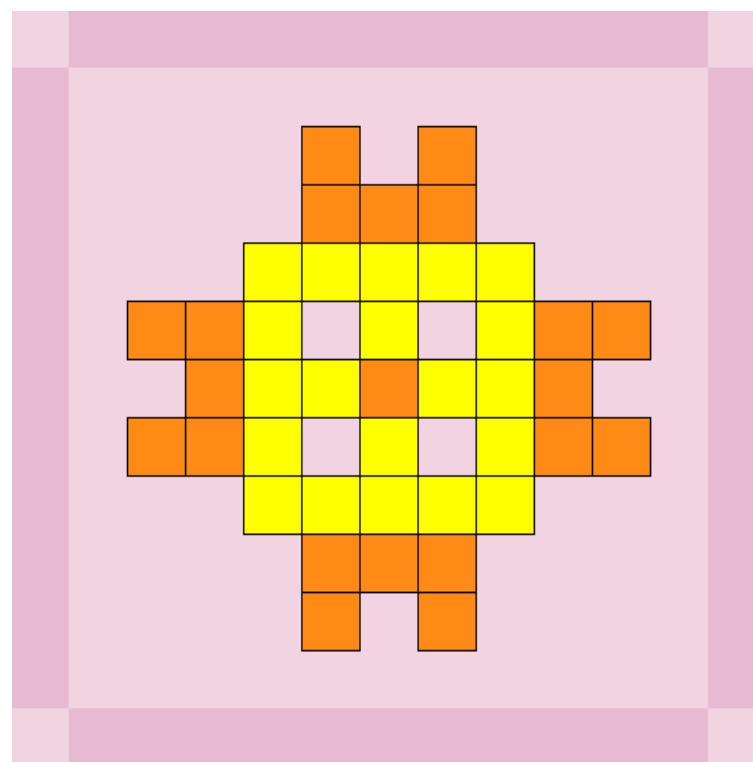
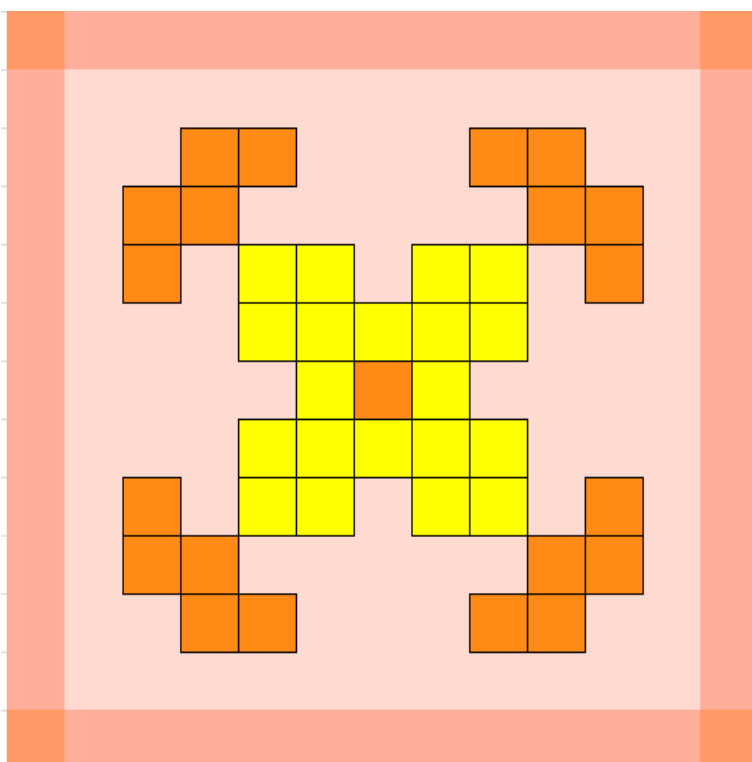
$$\begin{aligned}
 2021 \times 10001 + 10001 \times 2021 &:= 20212021 + 20212021 \\
 2021 \times 100001 + 100001 \times 2021 &:= 202102021 + 202102021 \\
 2021 \times 1000001 + 1000001 \times 2021 &:= 2021002021 + 2021002021 \\
 2021 \times 10000001 + 10000001 \times 2021 &:= 20210002021 + 20210002021
 \end{aligned}$$

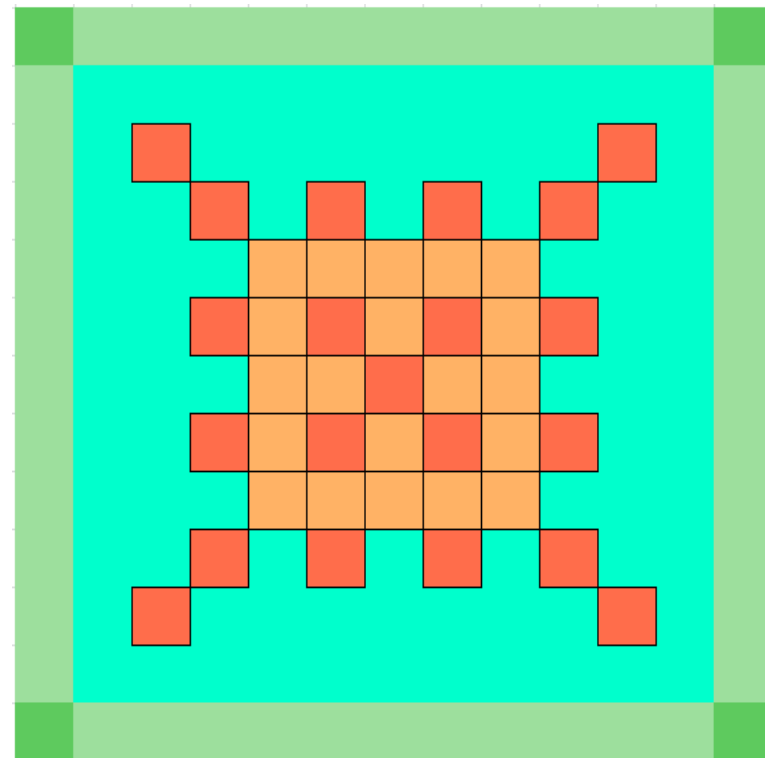
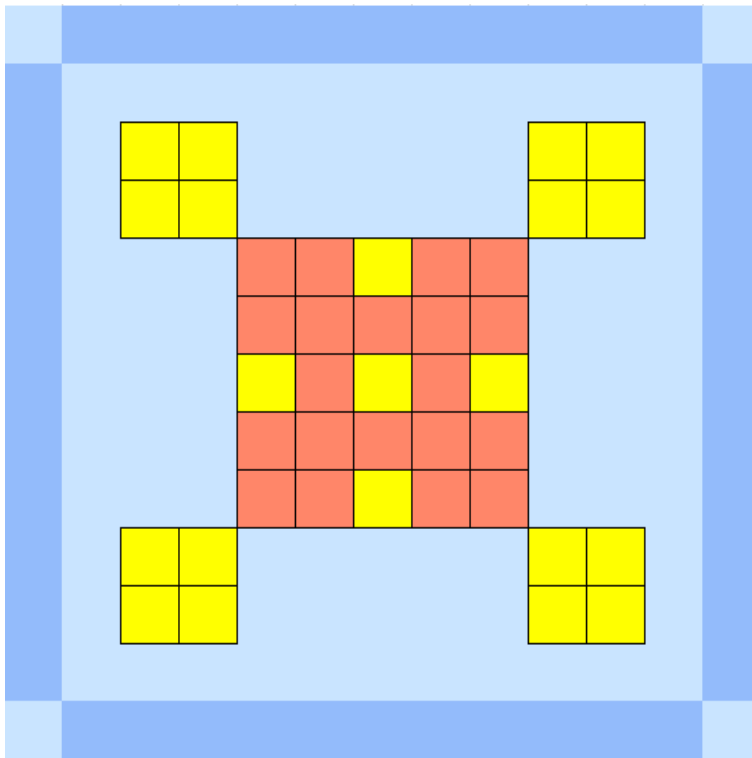
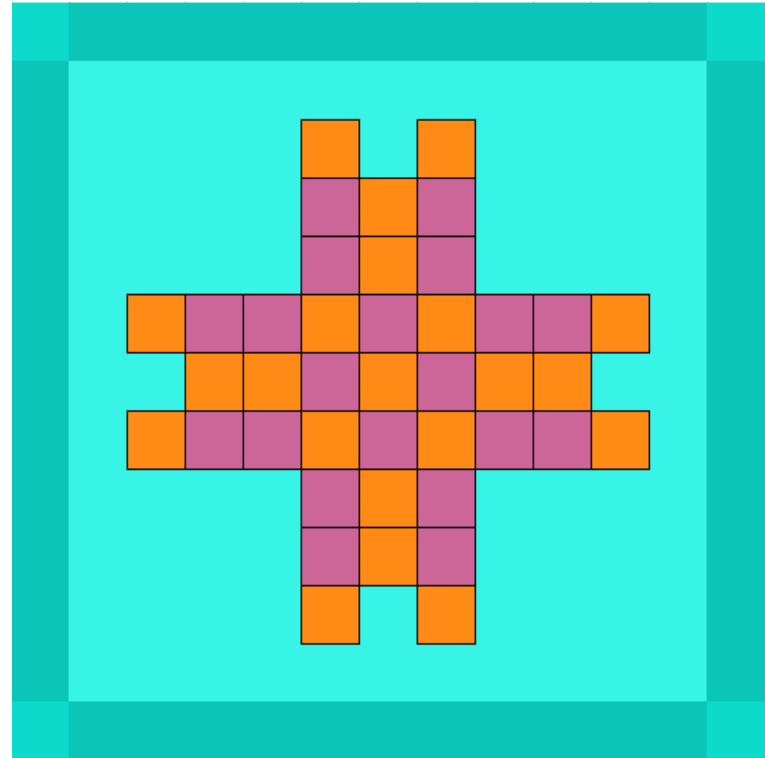
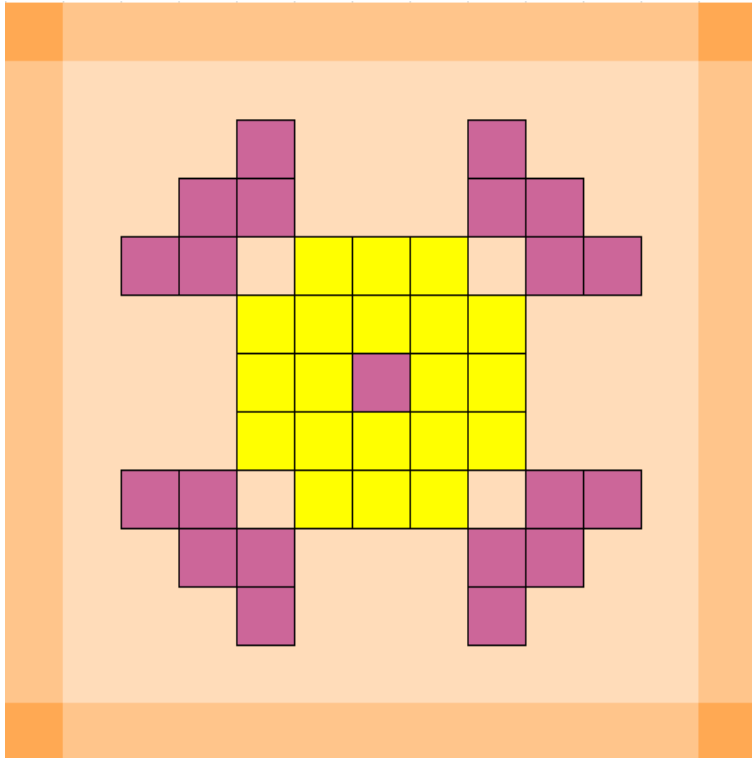
$$\begin{aligned}
 1002 \times 12 + 21 \times 2001 &:= 12024 + 42021 &:= 54045 \\
 10002 \times 112 + 211 \times 20001 &:= 1120224 + 4220211 &:= 5340435 \\
 100002 \times 1112 + 2111 \times 200001 &:= 111202224 + 422202111 &:= 533404335 \\
 1000002 \times 11112 + 21111 \times 2000001 &:= 11112022224 + 42222021111 &:= 53334043335
 \end{aligned}$$

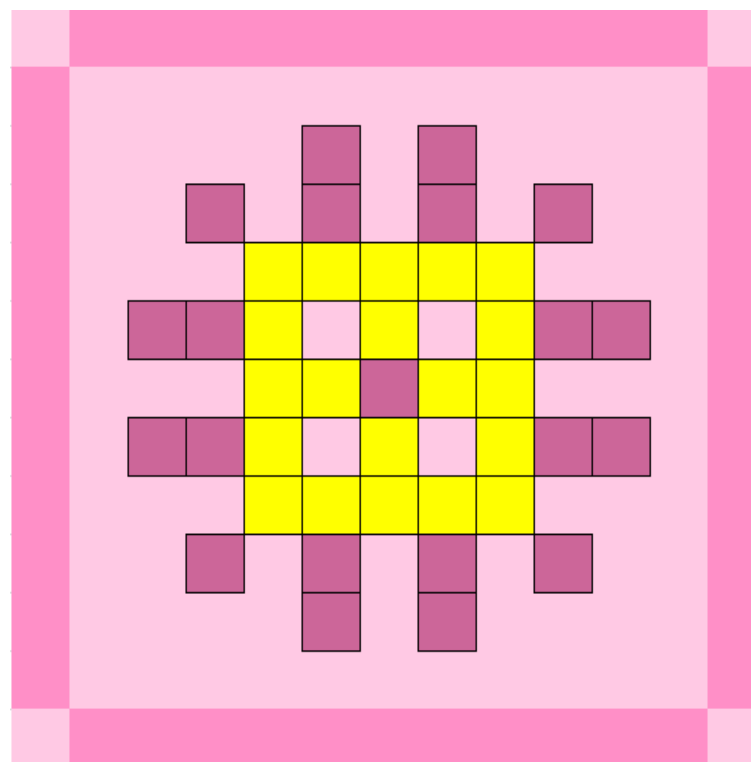
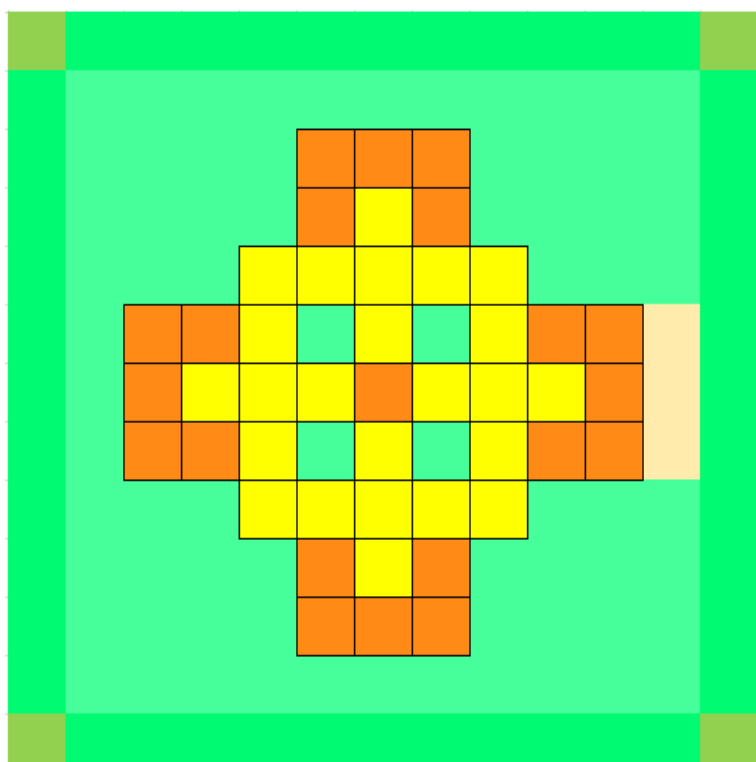
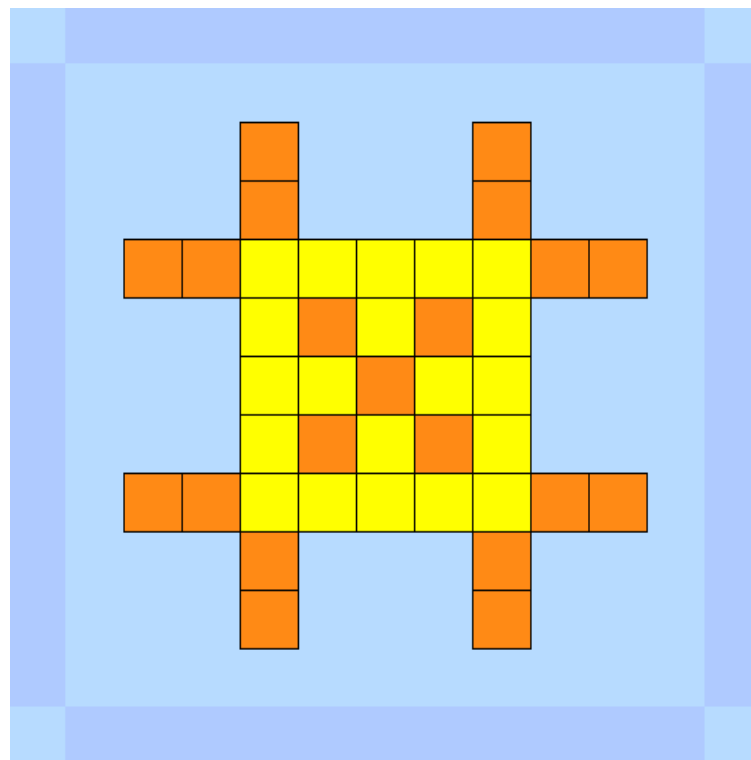
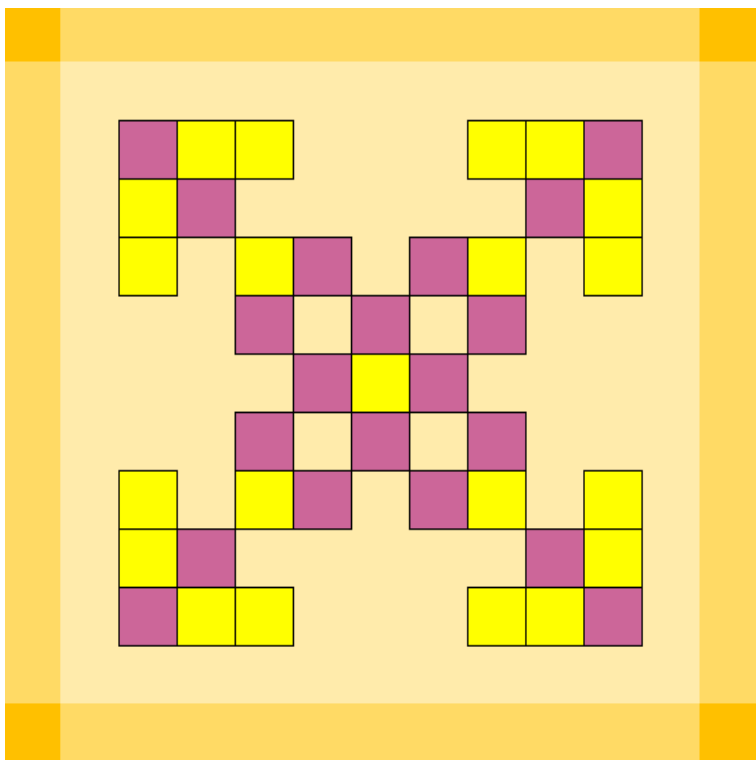
20 Pattern Designs With 2021

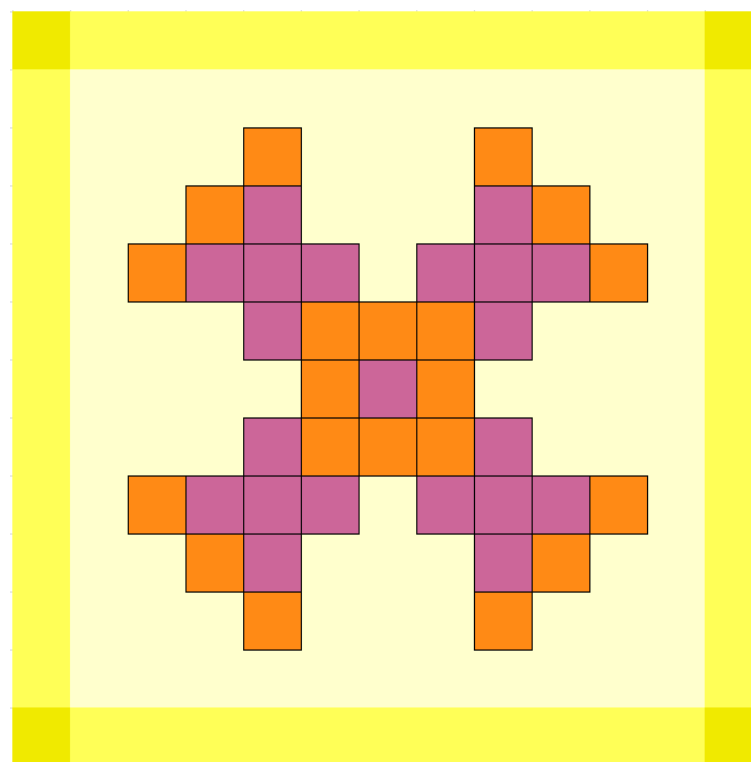
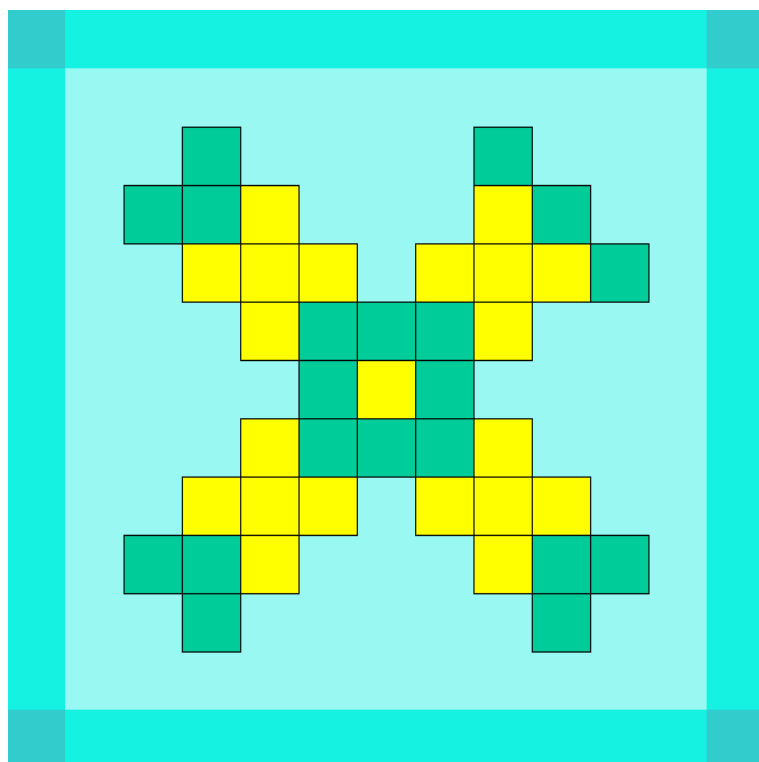
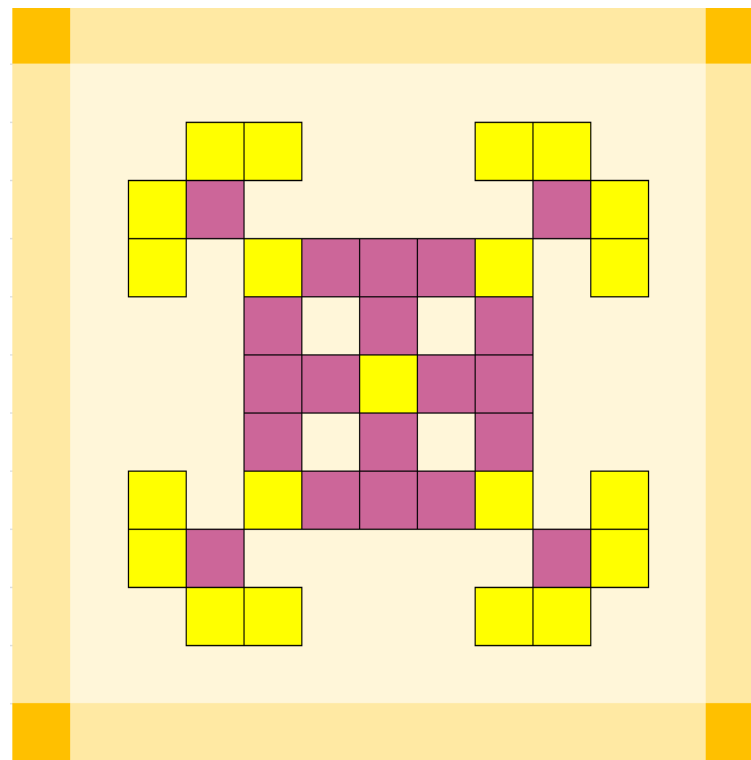
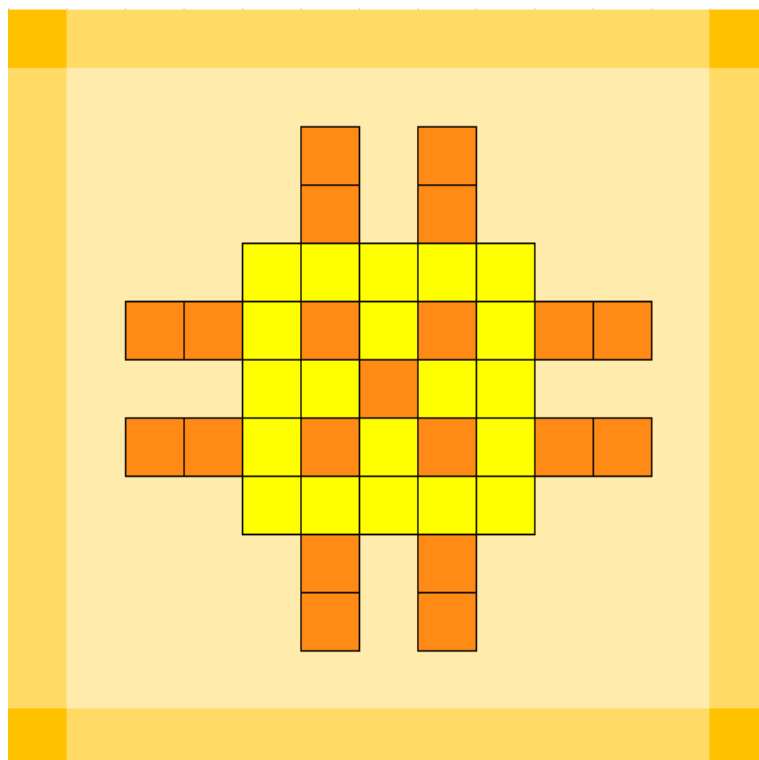
20.1 Colored Patterns With 2021

Below are few patterns on 2021 with any conditions on colors. Again we are using two colors on a board of 9×9 .



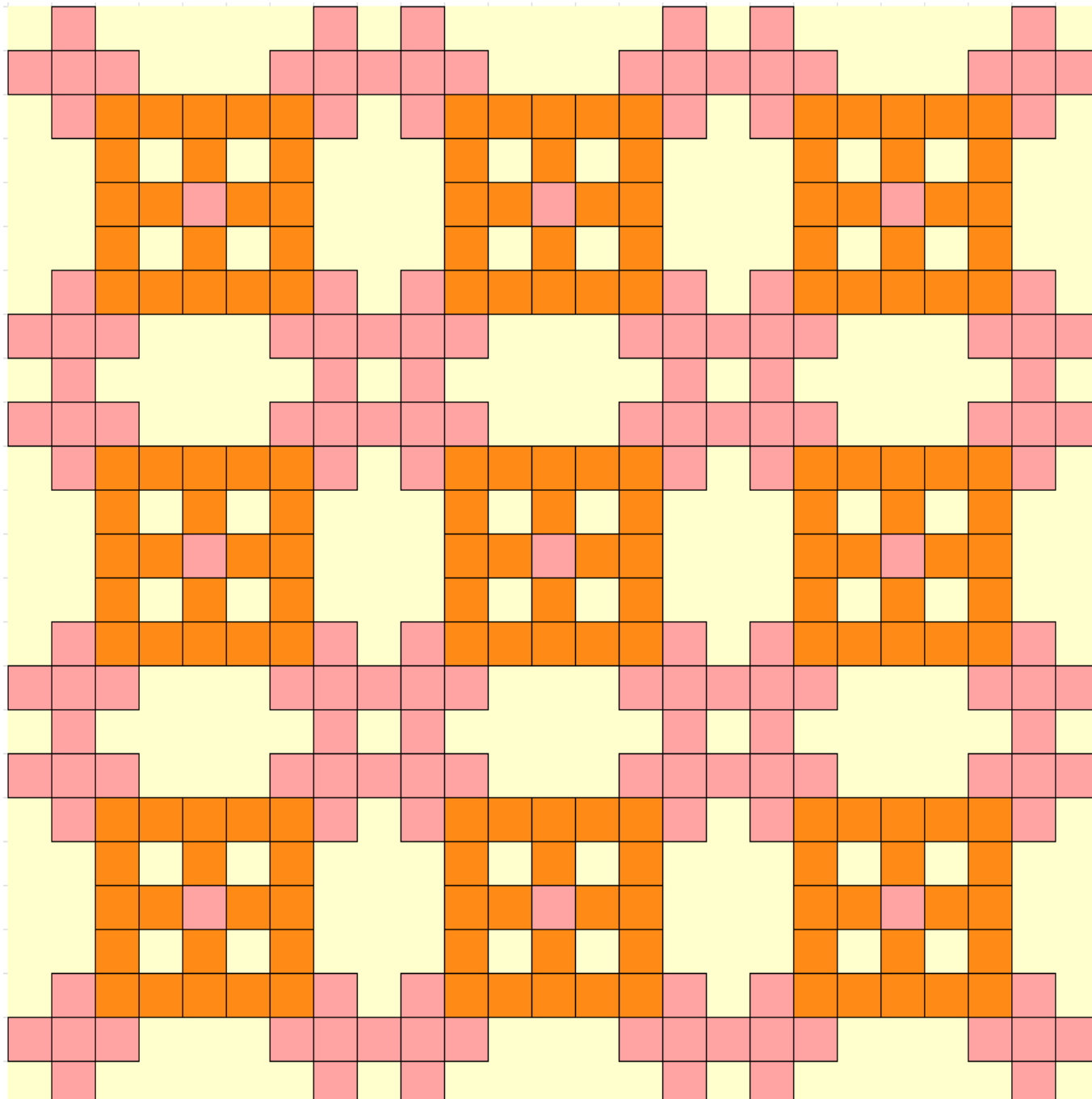


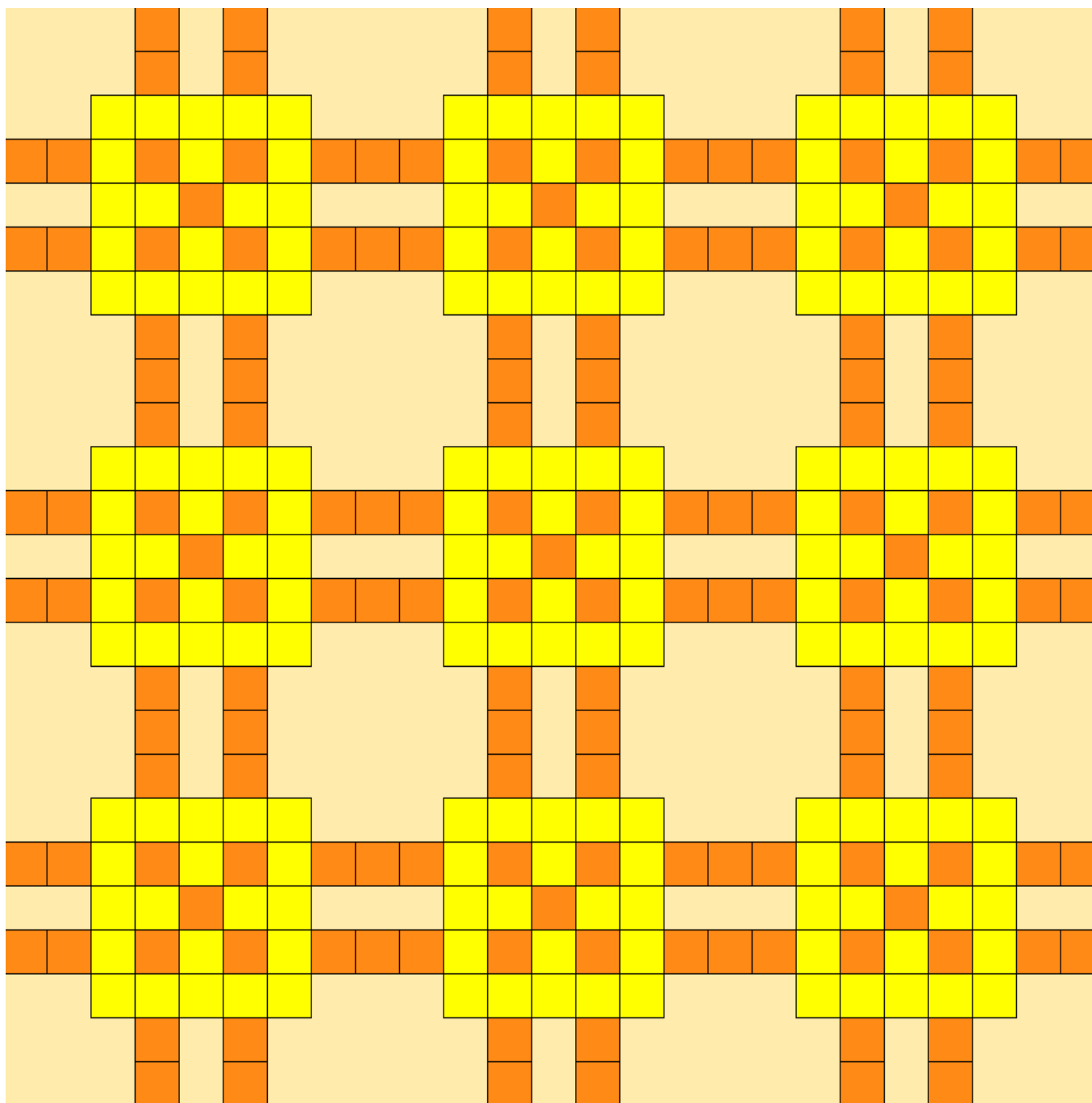


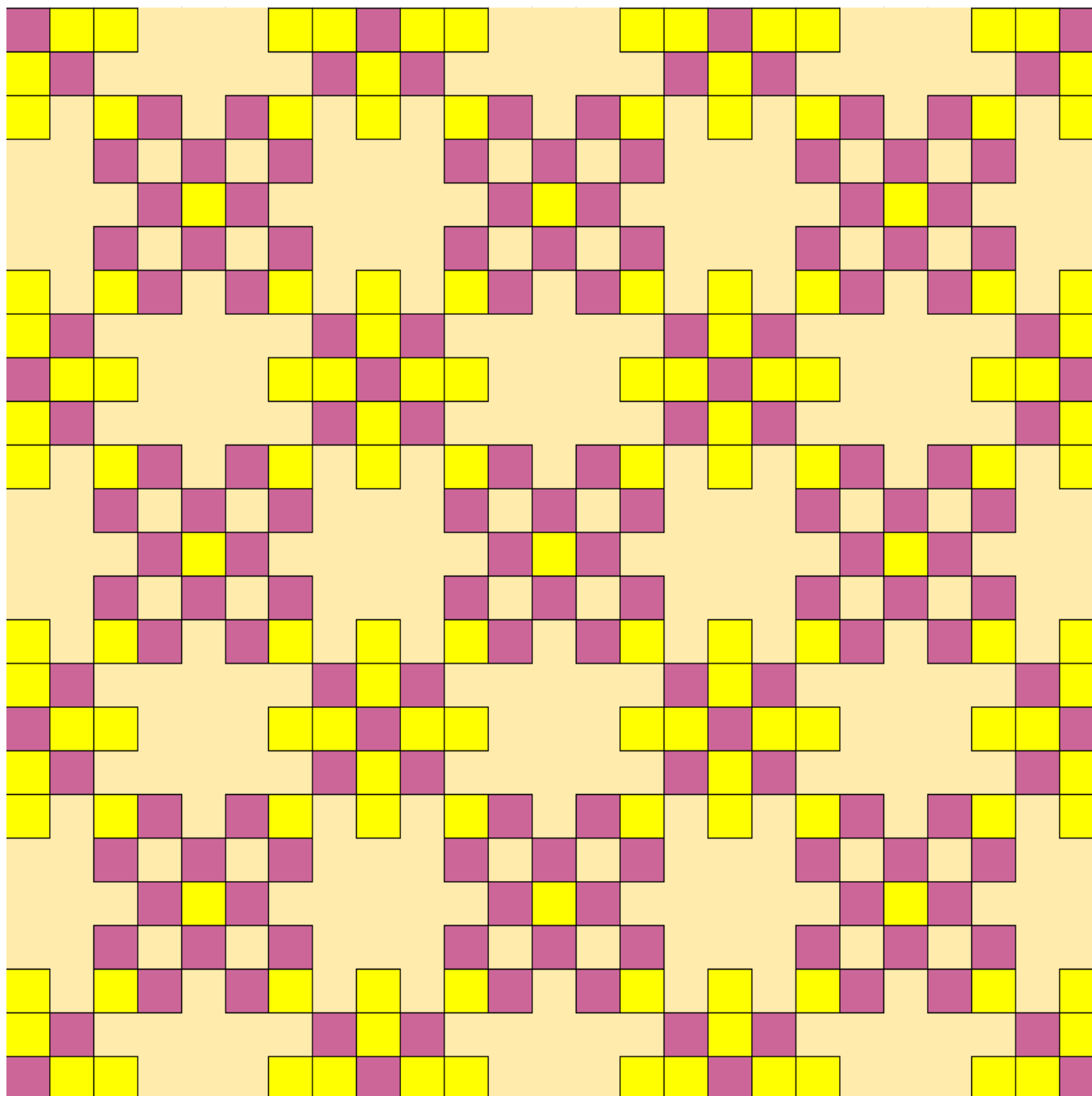


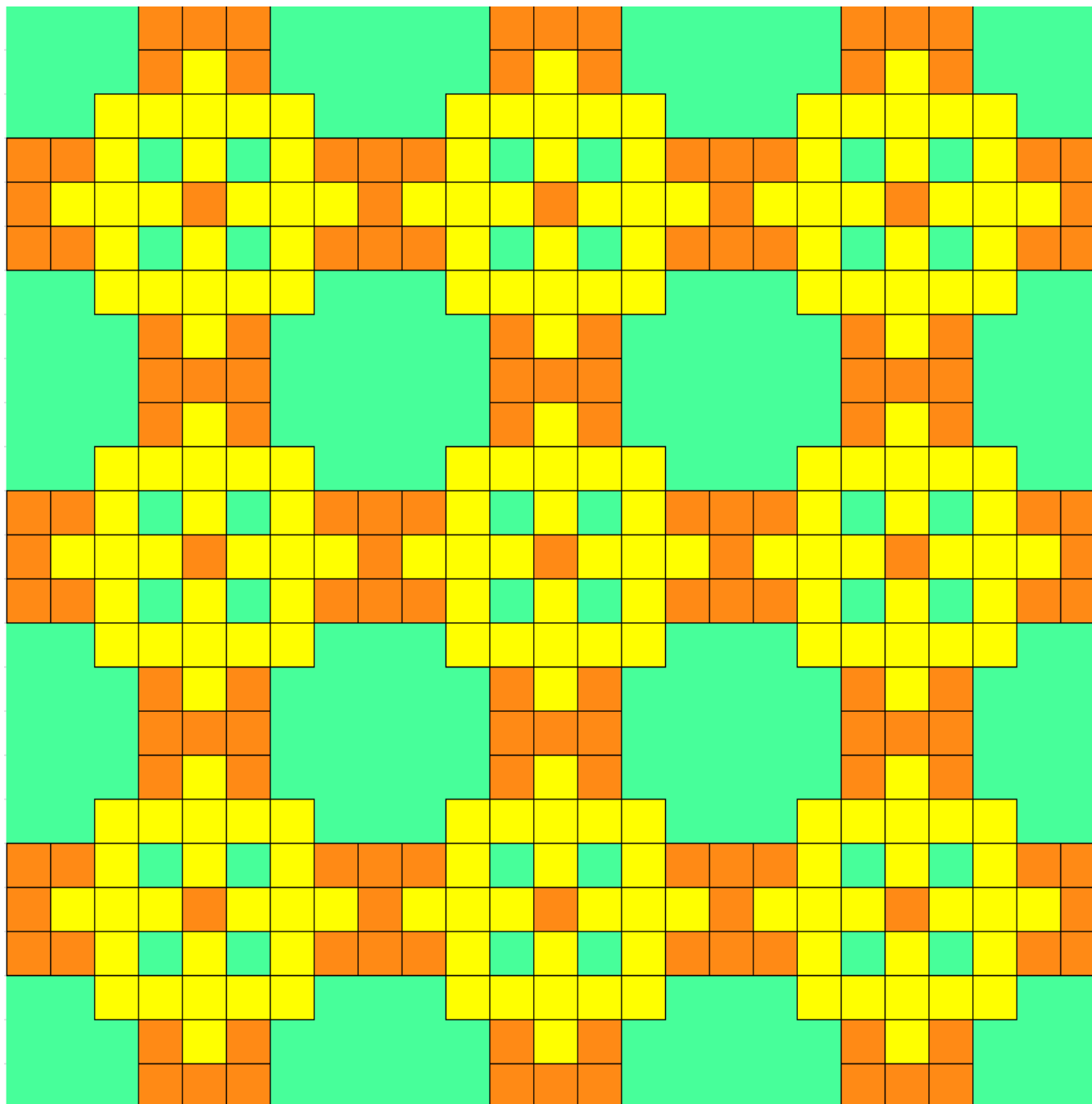
20.2 Superposition Designs With 2021

Similar to above subsection, here also we can make continuous designs. Below are few examples:





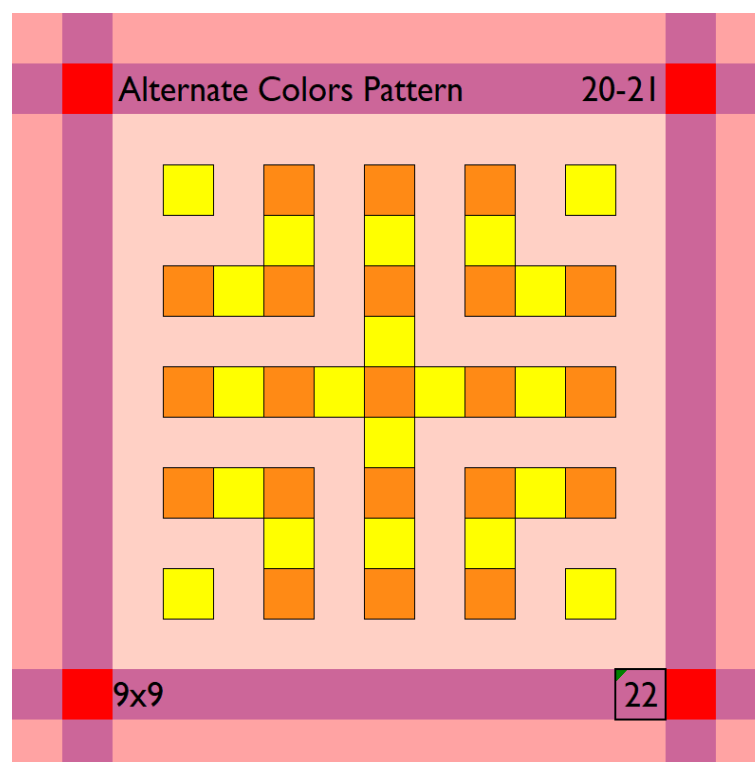
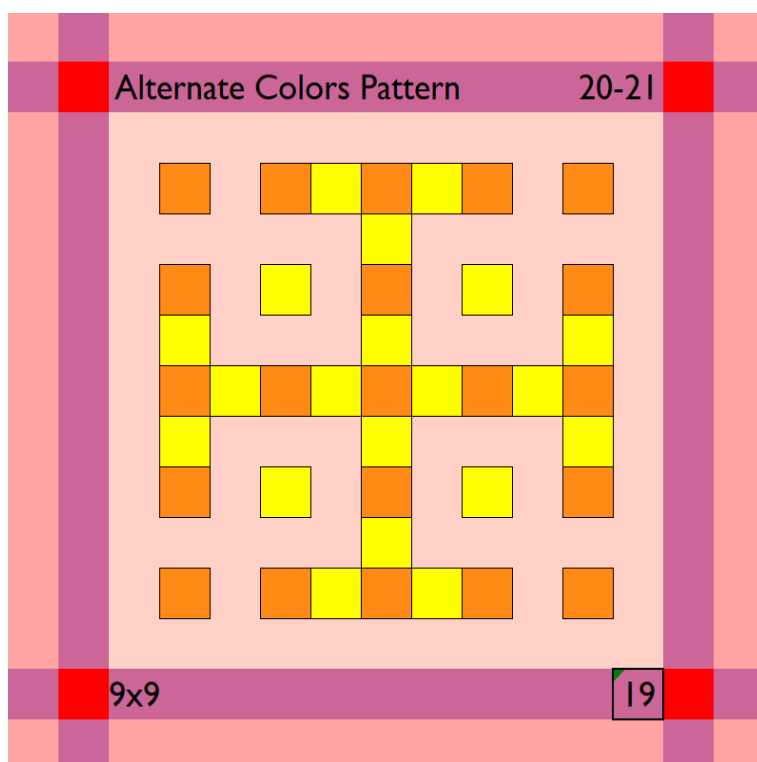
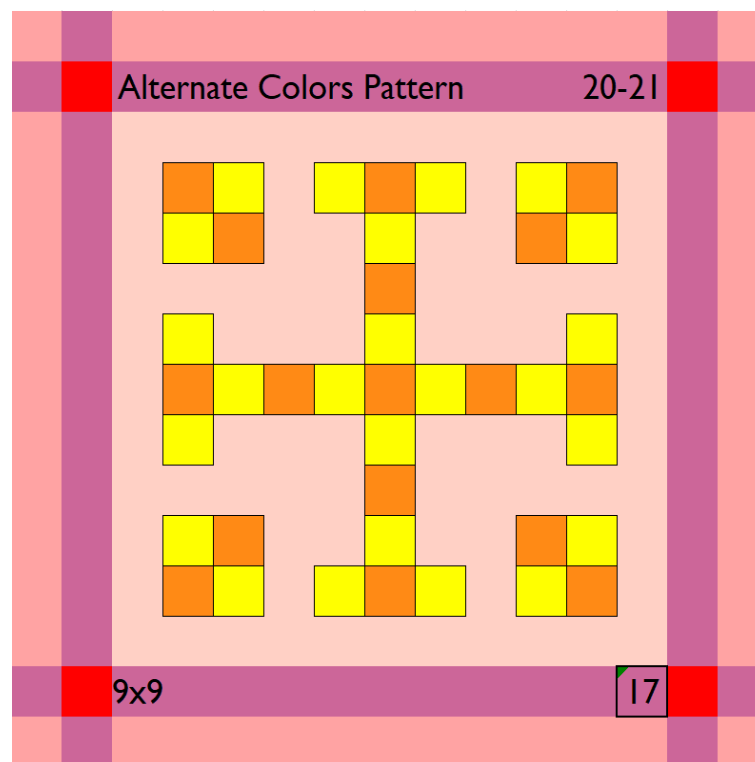
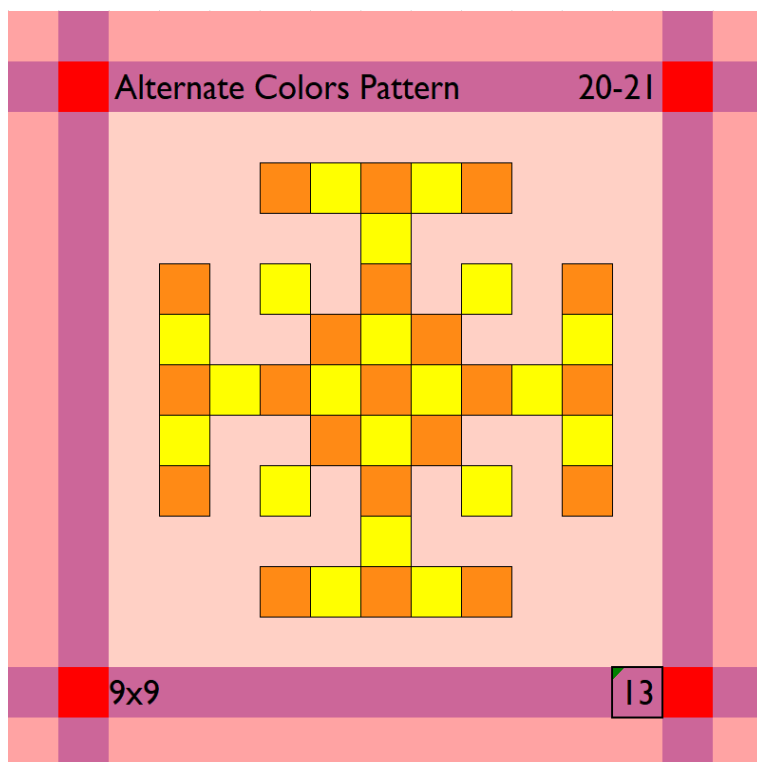




Based on above designs we can make many more.

20.3 Alternate Colors Patterns

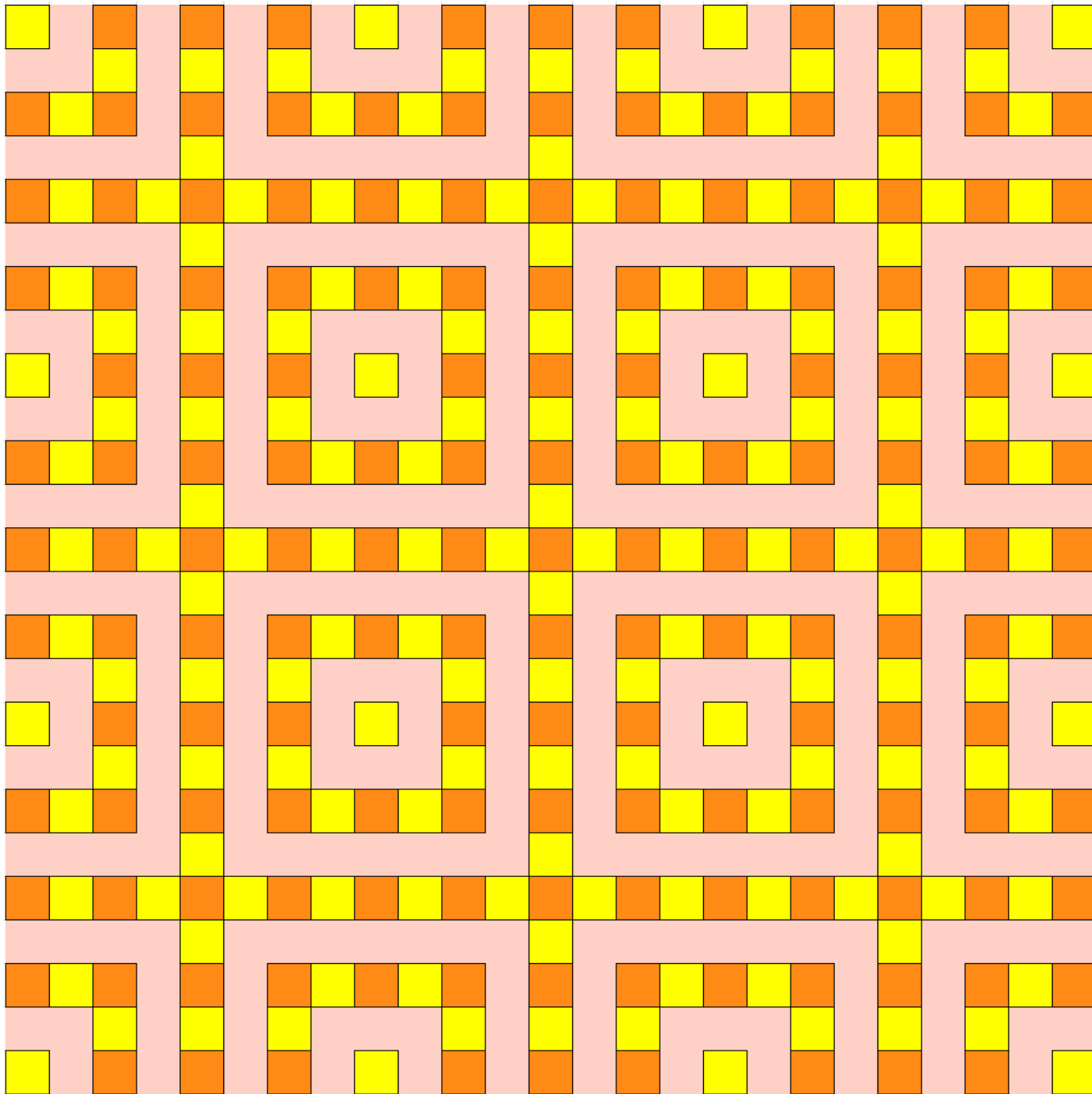
Below are few patterns on 2021 with alternate colors done on the board of 9×9 .

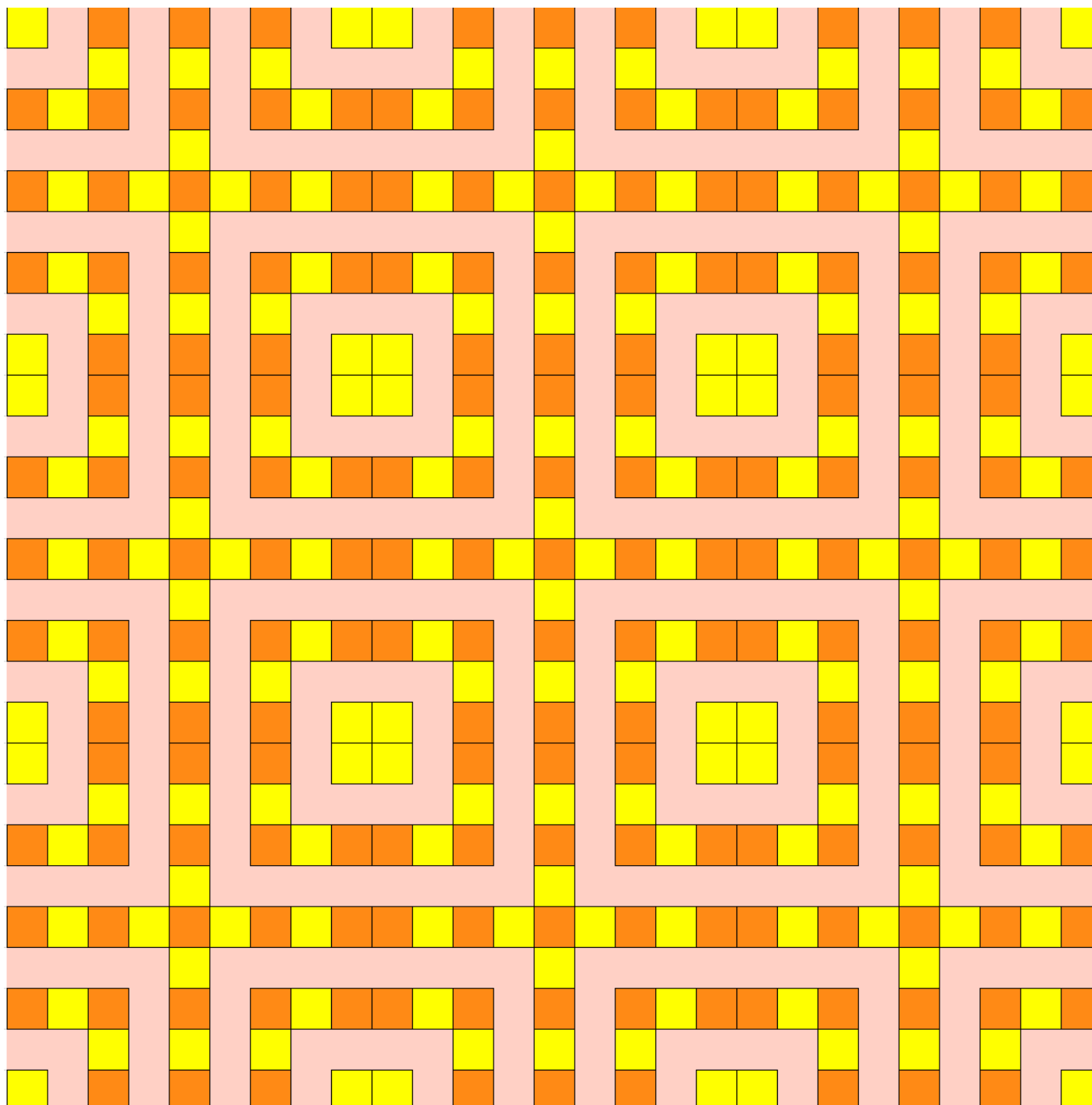


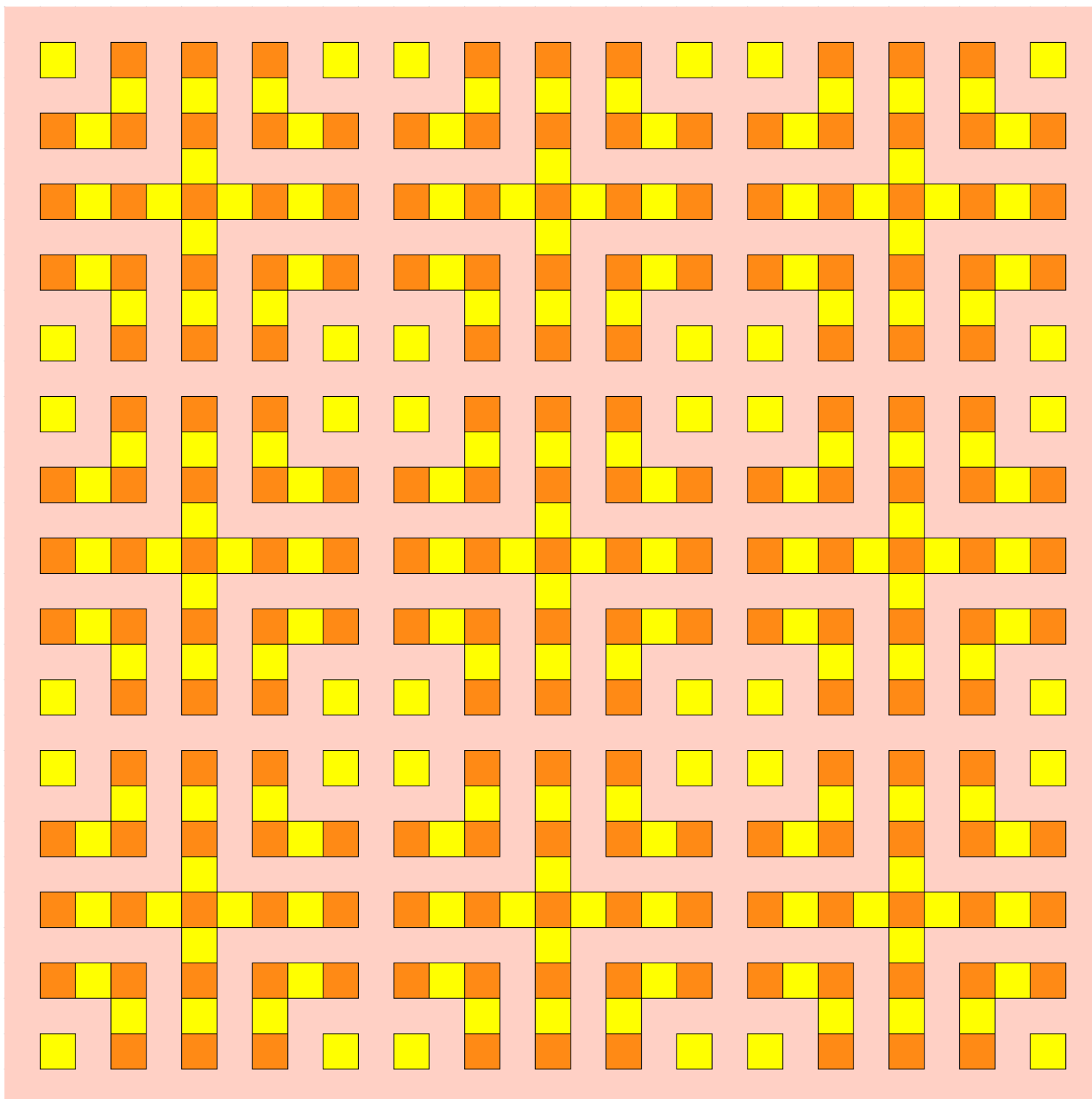
There are two colors one with 20 and another 21 small squares making 2021. More of similar kinds can be seen at author's site written at the beginning of the work.

20.4 Continuous Designs With 2021

Below are three different continuous designs made from the forth figure:







In a similar way there are much more possibilities of making more continuous designs from other figures.

21 Magic Squares

In this section there are few magic squares connected with 2021. These are of type block-wised, borderd and with only three digits 0, 1 and 2.

21.1 Upside-Down Magic Square With 0, 1 and 2

Below is magic square of order 9 with same digits are of 2021, i.e., 0, 1 and 2. It is written in such a way that it can be seen upside-down with the same sum.

1111	1022	1200	0120	0001	0212	2102	2010	2221
0102	0010	0221	0211	2022	2200	1120	1001	1212
2120	2001	2212	1102	1010	1221	0111	0022	2000
1222	1100	1011	0201	0112	0020	2210	2121	2002
0210	0121	0002	2222	2100	2011	1201	1112	1020
2201	2112	2020	1210	1121	1002	0222	0100	0011
1000	1211	1122	0012	0220	0101	2021	2202	2110
0021	0202	0110	2000	2211	2122	1012	1220	1101
2012	2220	2101	1021	1202	1110	0000	0211	0122

The above magic square is bimagic too. Its sums are as given below:

$$S_{9 \times 9} := 9999 \text{ and } S_{b_{9 \times 9}} := 17169495$$

The above magic is also mirror looking. In this the magic sum is different as in this case 2 becomes 5.

21.2 Block-Bordered Magic Squares of Order 14 with Magic Sum 2021

Below are **block-bordered magic squares** of order 14 with magic sums 2021. The **block-wise** magic square considered is of order 12. The block are of orders 3, 4 and 6. In each case, the magic sum of magic squares of order 12 are the same given by

$$S_{12 \times 12} := \frac{12126}{7}$$

21.2.1 First Type

														2021
59 6/7	53 6/7	233 6/7	55 6/7	231 6/7	57 6/7	241 6/7	52 6/7	227 6/7	61 6/7	225 6/7	63 6/7	223 6/7	229 6/7	2021
240 6/7	169 6/7	72 6/7	122 6/7	143 6/7	93 6/7	190 6/7	149 6/7	99 6/7	196 6/7	211 6/7	114 6/7	164 6/7	47 6/7	2021
48 6/7	74 6/7	121 6/7	168 6/7	189 6/7	142 6/7	95 6/7	195 6/7	148 6/7	101 6/7	116 6/7	163 6/7	210 6/7	239 6/7	2021
238 6/7	120 6/7	170 6/7	73 6/7	94 6/7	191 6/7	141 6/7	100 6/7	197 6/7	147 6/7	162 6/7	212 6/7	115 6/7	49 6/7	2021
50 6/7	202 6/7	105 6/7	155 6/7	158 6/7	108 6/7	205 6/7	128 6/7	78 6/7	175 6/7	184 6/7	87 6/7	137 6/7	237 6/7	2021
236 6/7	107 6/7	154 6/7	201 6/7	204 6/7	157 6/7	110 6/7	174 6/7	127 6/7	80 6/7	89 6/7	136 6/7	183 6/7	51 6/7	2021
222 6/7	153 6/7	203 6/7	106 6/7	109 6/7	206 6/7	156 6/7	79 6/7	176 6/7	126 6/7	135 6/7	185 6/7	88 6/7	65 6/7	2021
216 6/7	138 6/7	188 6/7	91 6/7	76 6/7	173 6/7	123 6/7	118 6/7	215 6/7	165 6/7	144 6/7	194 6/7	97 6/7	71 6/7	2021
70 6/7	92 6/7	139 6/7	186 6/7	171 6/7	124 6/7	77 6/7	213 6/7	166 6/7	119 6/7	98 6/7	145 6/7	192 6/7	217 6/7	2021
218 6/7	187 6/7	90 6/7	140 6/7	125 6/7	75 6/7	172 6/7	167 6/7	117 6/7	214 6/7	193 6/7	96 6/7	146 6/7	69 6/7	2021
68 6/7	159 6/7	209 6/7	112 6/7	103 6/7	200 6/7	150 6/7	85 6/7	182 6/7	132 6/7	129 6/7	179 6/7	82 6/7	219 6/7	2021
220 6/7	113 6/7	160 6/7	207 6/7	198 6/7	151 6/7	104 6/7	180 6/7	133 6/7	86 6/7	83 6/7	130 6/7	177 6/7	67 6/7	2021
66 6/7	208 6/7	111 6/7	161 6/7	152 6/7	102 6/7	199 6/7	134 6/7	84 6/7	181 6/7	178 6/7	81 6/7	131 6/7	221 6/7	2021
58 6/7	234 6/7	54 6/7	232 6/7	56 6/7	230 6/7	46 6/7	235 6/7	60 6/7	226 6/7	62 6/7	224 6/7	64 6/7	228 6/7	2021
2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021

In this case, all the blocks of order 3 are magic squares with different magic sums.

21.2.2 Second Type

														2021
59 6/7	53 6/7	233 6/7	55 6/7	231 6/7	57 6/7	241 6/7	52 6/7	227 6/7	61 6/7	225 6/7	63 6/7	223 6/7	229 6/7	2021
240 6/7	126 6/7	179 6/7	72 6/7	197 6/7	127 6/7	178 6/7	73 6/7	196 6/7	128 6/7	177 6/7	74 6/7	195 6/7	47 6/7	2021
48 6/7	89 6/7	180 6/7	143 6/7	162 6/7	88 6/7	181 6/7	142 6/7	163 6/7	87 6/7	182 6/7	141 6/7	164 6/7	239 6/7	2021
238 6/7	215 6/7	90 6/7	161 6/7	108 6/7	214 6/7	91 6/7	160 6/7	109 6/7	213 6/7	92 6/7	159 6/7	110 6/7	49 6/7	2021
50 6/7	144 6/7	125 6/7	198 6/7	107 6/7	145 6/7	124 6/7	199 6/7	106 6/7	146 6/7	123 6/7	200 6/7	105 6/7	237 6/7	2021
236 6/7	129 6/7	176 6/7	75 6/7	194 6/7	130 6/7	175 6/7	76 6/7	193 6/7	131 6/7	174 6/7	77 6/7	192 6/7	51 6/7	2021
222 6/7	86 6/7	183 6/7	140 6/7	165 6/7	85 6/7	184 6/7	139 6/7	166 6/7	84 6/7	185 6/7	138 6/7	167 6/7	65 6/7	2021
216 6/7	212 6/7	93 6/7	158 6/7	111 6/7	211 6/7	94 6/7	157 6/7	112 6/7	210 6/7	95 6/7	156 6/7	113 6/7	71 6/7	2021
70 6/7	147 6/7	122 6/7	201 6/7	104 6/7	148 6/7	121 6/7	202 6/7	103 6/7	149 6/7	120 6/7	203 6/7	102 6/7	217 6/7	2021
218 6/7	132 6/7	173 6/7	78 6/7	191 6/7	133 6/7	172 6/7	79 6/7	190 6/7	134 6/7	171 6/7	80 6/7	189 6/7	69 6/7	2021
68 6/7	83 6/7	186 6/7	137 6/7	168 6/7	82 6/7	187 6/7	136 6/7	169 6/7	81 6/7	188 6/7	135 6/7	170 6/7	219 6/7	2021
220 6/7	209 6/7	96 6/7	155 6/7	114 6/7	208 6/7	97 6/7	154 6/7	115 6/7	207 6/7	98 6/7	153 6/7	116 6/7	67 6/7	2021
66 6/7	150 6/7	119 6/7	204 6/7	101 6/7	151 6/7	118 6/7	205 6/7	100 6/7	152 6/7	117 6/7	206 6/7	99 6/7	221 6/7	2021
58 6/7	234 6/7	54 6/7	232 6/7	56 6/7	230 6/7	46 6/7	235 6/7	60 6/7	226 6/7	62 6/7	224 6/7	64 6/7	228 6/7	2021
2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021

In this case all the blocks of order 4 are **pandiagonal** magic squares with equal magic sums given by

$$S_{4 \times 4} := \frac{4042}{7}$$

21.2.3 Third Type

														2021
59 6/7	53 6/7	233 6/7	55 6/7	231 6/7	57 6/7	241 6/7	52 6/7	227 6/7	61 6/7	225 6/7	63 6/7	223 6/7	229 6/7	2021
240 6/7	72 6/7	208 6/7	207 6/7	200 6/7	79 6/7	95 6/7	73 6/7	209 6/7	206 6/7	201 6/7	78 6/7	94 6/7	47 6/7	2021
48 6/7	191 6/7	103 6/7	183 6/7	104 6/7	112 6/7	168 6/7	190 6/7	102 6/7	182 6/7	105 6/7	113 6/7	169 6/7	239 6/7	2021
238 6/7	167 6/7	160 6/7	128 6/7	135 6/7	151 6/7	120 6/7	166 6/7	161 6/7	129 6/7	134 6/7	150 6/7	121 6/7	49 6/7	2021
50 6/7	143 6/7	127 6/7	152 6/7	159 6/7	136 6/7	144 6/7	142 6/7	126 6/7	153 6/7	158 6/7	137 6/7	145 6/7	237 6/7	2021
236 6/7	96 6/7	175 6/7	111 6/7	176 6/7	184 6/7	119 6/7	97 6/7	174 6/7	110 6/7	177 6/7	185 6/7	118 6/7	51 6/7	2021
222 6/7	192 6/7	88 6/7	80 6/7	87 6/7	199 6/7	215 6/7	193 6/7	89 6/7	81 6/7	86 6/7	198 6/7	214 6/7	65 6/7	2021
216 6/7	74 6/7	210 6/7	205 6/7	202 6/7	77 6/7	93 6/7	75 6/7	211 6/7	204 6/7	203 6/7	76 6/7	92 6/7	71 6/7	2021
70 6/7	189 6/7	101 6/7	181 6/7	106 6/7	114 6/7	170 6/7	188 6/7	100 6/7	180 6/7	107 6/7	115 6/7	171 6/7	217 6/7	2021
218 6/7	165 6/7	162 6/7	130 6/7	133 6/7	149 6/7	122 6/7	164 6/7	163 6/7	131 6/7	132 6/7	148 6/7	123 6/7	69 6/7	2021
68 6/7	141 6/7	125 6/7	154 6/7	157 6/7	138 6/7	146 6/7	140 6/7	124 6/7	155 6/7	156 6/7	139 6/7	147 6/7	219 6/7	2021
220 6/7	98 6/7	173 6/7	109 6/7	178 6/7	186 6/7	117 6/7	99 6/7	172 6/7	108 6/7	179 6/7	187 6/7	116 6/7	67 6/7	2021
66 6/7	194 6/7	90 6/7	82 6/7	85 6/7	197 6/7	213 6/7	195 6/7	91 6/7	83 6/7	84 6/7	196 6/7	212 6/7	221 6/7	2021
58 6/7	234 6/7	54 6/7	232 6/7	56 6/7	230 6/7	46 6/7	235 6/7	60 6/7	226 6/7	62 6/7	224 6/7	64 6/7	228 6/7	2021
2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021

In this case, all the blocks of order 6 are magic squares with equal magic sums given by

$$S_{6 \times 6} := \frac{6063}{7}$$

21.3 Bordered Magic Square of Order 14

The bordered magic square of order 14 for the magic sum 2021 is given by

															2021
59 6/7	53 6/7	233 6/7	55 6/7	231 6/7	57 6/7	241 6/7	52 6/7	227 6/7	61 6/7	225 6/7	63 6/7	223 6/7	229 6/7	2021	
240 6/7	205 6/7	214 6/7	74 6/7	212 6/7	76 6/7	72 6/7	93 6/7	195 6/7	91 6/7	197 6/7	89 6/7	204 6/7	47 6/7	2021	
48 6/7	87 6/7	184 6/7	179 6/7	109 6/7	177 6/7	111 6/7	107 6/7	97 6/7	191 6/7	95 6/7	185 6/7	200 6/7	239 6/7	2021	
238 6/7	86 6/7	106 6/7	119 6/7	113 6/7	173 6/7	175 6/7	162 6/7	124 6/7	164 6/7	118 6/7	181 6/7	201 6/7	49 6/7	2021	
50 6/7	202 6/7	182 6/7	116 6/7	157 6/7	155 6/7	128 6/7	161 6/7	129 6/7	131 6/7	171 6/7	105 6/7	85 6/7	237 6/7	2021	
236 6/7	203 6/7	104 6/7	117 6/7	127 6/7	149 6/7	136 6/7	139 6/7	150 6/7	160 6/7	170 6/7	183 6/7	84 6/7	51 6/7	2021	
222 6/7	210 6/7	189 6/7	122 6/7	133 6/7	142 6/7	147 6/7	144 6/7	141 6/7	154 6/7	165 6/7	98 6/7	77 6/7	65 6/7	2021	
216 6/7	199 6/7	94 6/7	172 6/7	135 6/7	146 6/7	143 6/7	140 6/7	145 6/7	152 6/7	115 6/7	193 6/7	88 6/7	71 6/7	2021	
70 6/7	81 6/7	186 6/7	167 6/7	153 6/7	137 6/7	148 6/7	151 6/7	138 6/7	134 6/7	120 6/7	101 6/7	206 6/7	217 6/7	2021	
218 6/7	80 6/7	100 6/7	166 6/7	156 6/7	132 6/7	159 6/7	126 6/7	158 6/7	130 6/7	121 6/7	187 6/7	207 6/7	69 6/7	2021	
68 6/7	208 6/7	188 6/7	169 6/7	174 6/7	114 6/7	112 6/7	125 6/7	163 6/7	123 6/7	168 6/7	99 6/7	79 6/7	219 6/7	2021	
220 6/7	78 6/7	102 6/7	108 6/7	178 6/7	110 6/7	176 6/7	180 6/7	190 6/7	96 6/7	192 6/7	103 6/7	209 6/7	67 6/7	2021	
66 6/7	83 6/7	73 6/7	213 6/7	75 6/7	211 6/7	215 6/7	194 6/7	92 6/7	196 6/7	90 6/7	198 6/7	82 6/7	221 6/7	2021	
58 6/7	234 6/7	54 6/7	232 6/7	56 6/7	230 6/7	46 6/7	235 6/7	60 6/7	226 6/7	62 6/7	224 6/7	64 6/7	228 6/7	2021	
2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	2021	

The sub-magic squares sums are as given by

$$S_{4 \times 4} := 2021 \times \frac{4}{14} = \frac{4042}{7}$$

$$S_{6 \times 6} := 2021 \times \frac{6}{14} = \frac{6063}{7}$$

$$S_{8 \times 8} := 2021 \times \frac{8}{14} = \frac{8084}{7}$$

$$S_{10 \times 10} := 2021 \times \frac{10}{14} = \frac{10105}{7}$$

$$S_{12 \times 12} := 2021 \times \frac{12}{14} = \frac{12126}{7}$$

$$S_{14 \times 14} := 2021 \times \frac{14}{14} = 2021.$$

In this case, there is a **Pythagorean triples** with magic sums:

$$S_{6 \times 6}^2 + S_{8 \times 8}^2 := S_{10 \times 10}^2$$

21.4 Numbers With Magic Squares

Below is 2021 written pandiagonal magic squares of order 4 with equal magic sums, i.e., $S_{4 \times 4} := 1250$. Interestingly, sum of 16 entries of block of order 4 is 5000. These magic squares are constructed using natural numbers from 1 to 624.

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