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OPTIMAL STABILIZATION ALGORITHM FOR PRODUCTION LINE FLOW PARAMETERS

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ABSTRACT

Context. A method for constructing an algorithm for stabilizing the interoperability of a production line is considered. The object of the study was a model of a multi-operational production line.

Objective. The goal of the work is to develop a method for constructing an optimal algorithm for stabilizing the flow parameters of a production line, which provides asymptotic stability of the state of flow parameters for a given quality of the process.

Method. A method for constructing an algorithm for stabilizing the level of interoperative backlogs of a multi-operational production line is proposed. The stabilization algorithm is based on a two-moment PDE-model of the production line, which made it possible to represent the production line in the form of a complex dynamic distributed system. This representation made it possible to define the stabilizing control in the form of a function that depends not only on time but also on the coordinates characterizing the location of technological equipment along the production line. The use of the method of Lyapunov functions made it possible to synthesize the optimal stabilizing control of the state of interoperation backlogs at technological operations of the production line, which ensures the asymptotic stability of the given unperturbed state of the flow parameters of the production line at the lowest cost of technological resources spent on the formation of the control action. The requirement for the best quality of the transition process from a disturbed state to an unperturbed state is expressed by the quality integral, which depends both on the magnitude of the disturbances that have arisen and on the magnitude of the stabilizing controls aimed at eliminating these disturbances.

Results. On the basis of the developed method for constructing an algorithm for stabilizing the state of flow parameters of a production line, an algorithm for stabilizing the value of interoperation backlogs at technological operations of a production line is synthesized.

Conclusions. The use of the method of Lyapunov functions in the synthesis of optimal stabilizing control of the flow parameters of the production line makes it possible to provide asymptotic damping of the arising disturbances of the flow parameters with the least cost of technological resources spent on the formation of the control action. It is shown that in the problem of stabilizing the state of interoperative backlogs, the stabilizing value of the control is proportional to the value of the arising disturbance. The proportionality coefficient is determined through the coefficients of the quality integral and the Lyapunov function. The prospect of further research is the development of a method for constructing an algorithm for stabilizing the productivity of technological operations of a production line.

KEYWORDS: PDE-model of a production line, multi-moment equations, Lyapunov function, quality integral, optimal control, stabilization problem.

ABBREVIATIONS

PDE is a partial differential equation.

NOMENCLATURE

$[\chi]_0(t,S)$ is a density of inter-operational backlog of parts (WIP) at the time moment t for the position of the production route with the coordinate S;

$[\chi]_1(t,S)$ is a flow of parts at a time t through the position of the technological route with a coordinate S;

$[\chi]_{1\psi}(t,S)$ is a normative productivity of the production equipment at the moment time t for the position of the technological route with the coordinate S;

$\mathrm{Y}_0(t,S)$ is a program control of the state of interoperation backlogs at the time moment t for the position of the technological route with a coordinate S, which ensures the transition of the production system from one state of the production system to another state of the production system;

$\mathrm{Y}_1(t,S)$ is a program control of the normative productivity of production equipment at the time moment t for the position of the technological route with a coordinate S, which ensures the transition of the production system from one state of the production system to another state of the production system;

T_d is a characteristic time of the transfer of the production system from one state of parameters to another state of streaming parameters;

$u_k(t,[y]_0,[y]_1)$ is a control action, providing stabilization of flow parameters $[\chi]_k(t,S)$ relative to the undisturbed state $[\chi]_k^*(t,S)$;

$\omega(t,[y]_0,[y]_1,u_0,u_1)$ is a function that determines the requirements for the lowest possible cost of technological resources (energy, raw materials, labour resources, etc.) spent on the formation of control actions $u_0(t,[y]_0,[y]_1)$, $u_1(t,[y]_0,[y]_1)$.

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INTRODUCTION

The time factor plays a decisive role in planning the operation of a production line and the operational control of its flow parameters [1]. At the same time, the main requirement for the process of planning and operational control of production is to ensure the continuity and rhythm processing of parts along the technological route [2]. In this regard, mathematical methods of planning and operational control of the flow parameters of the production line are exceptional importance [3]. The most important and least studied among them are the dynamic problems of the optimal use of the technological resources of the enterprise to ensure a steady-state of the flow parameters of the production line [4].

The object of study is model of a production multi-operation production line.

The production lines of modern industrial enterprises contain~10^3 technological operations [5], in the interoperation reserves of which are ~10^5 parts [6]. The structure of the production line can be highly branched [7], consisting of separate sections of the technological route with a sequential position of technological operations [8]. The presence of a branched structure imposes additional difficulties on the construction of a production line model, and, as a consequence, construction of an algorithm for stabilization of the production line flow parameters. For this reason, the construction of an algorithm for stabilizing the flow parameters will be carried out for a production line with a sequential position of technological operations [9] (Fig. 1).

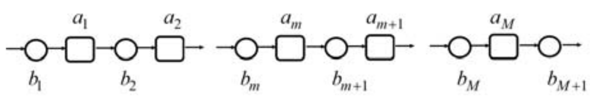

Figure 1 –Serial production line

This simplification will allow us to focus on the key design points of the stabilization algorithm. The obtained results can be used to design control systems for flow parameters of complex branched production lines.

The subject of study is a method for constructing an algorithm for stabilizing the production line flow parameters, based on the PDE model of the production line.

The control problem of the production line flow parameters is often considered as the task of building program control (production schedule of the output plan [11]). Methods for constructing a programmed control using the equations of the fluid–model and the PDE–model are presented in [12, 13]. However, in the presence of disturbances, the programmed control will be implemented without additional stabilizing effects only in the case of asymptotic stability of the production line flow parameters. This circumstance requires the development of methods for designing algorithms for stabilizing the production line flow parameters.

The purpose of the work is to develop a method for constructing an algorithm for optimal stabilizing the production line flow parameters, which ensures the asymptotic stability of the state of flow parameters.

1 PROBLEM STATEMENT

To describe the serial production line (Fig. 1), let's use a two-moment PDE-model, the equations of which have the form [4, 10]:

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=\mathrm{Y}_0(t,S), \tag{1}$$

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\frac{\partial[\chi]_1(t,S)}{\partial S}=\mathrm{Y}_1(t,S). \tag{2}$$

with a control program $\mathrm{Y}_0(t,S)$, $\mathrm{Y}_1(t,S)$ for transferring production systems from a state with parameters $[\chi]_0(0,S)$ to a state with parameters $[\chi]_0(T_d,S)$ for a time $t\in[0,T_d]$.

Let the production line flow parameters $[\chi]_0(t,S)$, $[\chi]_1(t,S)$ receive unknown random small disturbances:

$$[y]_0(t,S)=[\chi]_0(t,S)-[\chi]_0^*(t,S), \tag{3}$$

$$[y]_1(t,S)=[\chi]_1(t,S)-[\chi]_1^*(t,S), \tag{4}$$

with respect to given program control $\mathrm{Y}_0^*(t,S)$, $\mathrm{Y}_1^*(t,S)$ unperturbed parameters $[\chi]_0^*(t,S)$, $[\chi]_1^*(t,S)$. To eliminate the disturbances $[y]_0(t,S)$, $[y]_1(t,S)$ that have arisen, it is required to develop control actions $u_0(t,[y]_0,[y]_1)$, $u_1(t,[y]_0,[y]_1)$, which should ensure the asymptotic stability of a given unperturbed state of flow parameters $[\chi]_0^*(t,S)$, $[\chi]_1^*(t,S)$, satisfying the system of equations (1), (2). It is assumed that the function $u_0(t,[y]_0,[y]_1)$, $u_1(t,[y]_0,[y]_1)$ satisfies the equalities

$$u_0(t,0,0)=0,\ u_1(t,0,0)=0. \tag{5}$$

The given functions are defined and continuous in the area under consideration and are not constrained by any additional inequalities.

When setting the problem of stabilization of production line flow parameters, we supplement the requirement of asymptotic stability of the unperturbed state of flow parameters with the requirement of the best quality of the transient process

$$I=\int_0^\infty \omega(t,[y]_0,[y]_1,u_0,u_1)dt. \tag{6}$$

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2 REVIEW OF THE LITERATURE

Common models that are used for the synthesis of optimal control of the production line flow parameters are Clearing-function models [14, 15], queue theory [16], discrete-event models [17, 18], Fluid– models [12, 13], system dynamics models [3], PDE– models [4, 5].

Clearing-function models and queue theory models are used when the input or output parameters of the production line are controlled. This class of models determines the relationship between input and output parameters using approximate relationships or precise analytical expressions that take into account, with a given accuracy, the state of distributed parameters along the production line. At the same time, there is no control over the state of flow parameters along the production line. Effective use of the Clearing-function model is presented in [19]. However, such cases of effective use of Clearing-function models are very rare.

Discrete-event models make it possible to sufficiently accurately simulate the process of processing products at technological operations of a production line. This class of models is used to build control systems of the flow parameters of technological operations of production lines. A significant disadvantage of their application for the design of control systems for the production line parameters is that the construction of effective control systems requires large expenditures of computing resources. When constructing multi-operational models of production lines [5] for batches with a large number of parts [6], the computational time of the model parameters exceeds the time allotted for making a decision on the formation of control actions.

Fluid models, system dynamics models and PDE models are most suitable for describing a production line. Fluid models and models of system dynamics are recommended for describing production lines containing several tens of technological operations [20]. For a large number of technological operations, the most effective way to describe a production line is to use a PDE model. In the paper [19], the limit transition from the PDE-model equation to the Clearing-function equation is provided. The Clearing Equation for a conveyor system determines the relationship between the input and output streaming parameters of the transport system. In the paper [20], the limit transition from the PDE-model equations to the equations of system dynamics is given.

PDE-models are significant practical interest for the control systems design for the parameters of many operational production lines. Further development of the PDE-models class, associated with the development of methods for constructing algorithms for optimal control of production line flow parameters, providing asymptotic stability of the flow parameters state, opens up new prospects for using this class of models in control systems of flow production.

3 MATERIALS AND METHODS

Balance equations (1), (2) determine the state of the multi-operation production line flow parameters $[\chi]_0(t,S)$, $[\chi]_1(t,S)$ (Fig. 1). The main mode of operation of production lines for enterprises with a flow method of organizing production is a synchronized mode, in which the performance of technological equipment is the same for each technological operation [21, 22]. Synchronization of technological equipment is taken into account as an additional condition

$$\frac{\partial[\chi]_{1\psi}(t,S)}{\partial S} \cong 0 . \qquad (6)$$

Then the system of equations for the parameters of the production line corresponds to a linearized system of equations in small perturbations $[y]_0(t,S)$, $[y]_1(t,S)$:

$$\frac{\partial[y]_0(t,S)}{\partial t} + \frac{\partial[y]_1(t,S)}{\partial S} = q_{00}u_0 + q_{01}u_1 , \qquad (7)$$

$$\frac{\partial[y]_1(t,S)}{\partial S} + \frac{\partial[y]_1(t,S)}{\partial S} B + [y]_1(t,S)\frac{\partial B}{\partial S} +$$

$$+ \frac{\partial[y]_0(t,S)}{\partial S} AB + [y]_0(t,S)\frac{\partial(AB)}{\partial S} = q_{10}u_0 + q_{11}u ,$$

$$A = \left.\frac{[\chi]_{1\psi}(t,S) - [\chi]_1(t,S)}{[\chi]_0(t,S)}\right|_0 , \ B = \left.\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\right|_0 ,$$

$$q_{00}u_0 + q_{01}u_1 = \Upsilon_0(t,S) - \Upsilon_0^*(t,S) ,$$

$$q_{10}u_0 + q_{11}u = \Upsilon_1(t,S) - \Upsilon_1^*(t,S) .$$

Functions $q_{nm} = q_{nm}(t)$ are limited and continuous functions of time. The period of existence of the disturbance T_v of the flow parameters ranges from several days to several weeks, while the period of change in the coefficients A and B, determined by the strategic management of the enterprise, ranges from several months to several years [23]. In this regard, let's assume that the coefficients A and B during the period of existence of the disturbance T_v do not explicitly depend on time, and their changes in time are much less than the values of the coefficients themselves:

$$\frac{A}{T_v} >> \frac{\partial A}{\partial t} , \ \frac{B}{T_v} >> \frac{\partial B}{\partial t} . \qquad (8)$$

Inequality (8) allows us to assume that the coefficients A and B in equations (7) depend only on the coordinate S. Let's assume that during control it is possible to measure the current values of the macro parameters $[\chi]_0$ and $[\chi]_1$. On the basis of measurements, the control device develops influences $u_0 = u_0(t,[y]_0,[y]_1)$, $u_1 = u_1(t,[y]_0,[y]_1)$ on the flow parameters of the production line $[\chi]_0(t,S)$, $[\chi]_1(t,S)$. These influences should ensure the asymptotic stability of the given unperturbed

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state of the flow parameters. The problem of the stabilizing production line flow parameters $[\chi]_0(t,S)$, $[\chi]_1(t,S)$ can be reduced to the control problem interoperable stocks or to the control problem the productivity of technological equipment. In this paper, let's consider the control problem of inter-operational backlogs.

Correction, coordination and approval of the production plan require accurate information about the distribution of parts by technological operations along the technological route. The problem of tactical production planning is closely related to the problem of reserve placement of stocks. For a production line consisting of two technological operations in the paper [24], a solution to the problem of tactical planning at the level of individual elements is proposed using linear programming methods. In paper [8], the application of a fluid model for tactical production planning is considered, which optimizes the amount of safety stock to ensure the normative functioning of a multi-flow line. A nonlinear problem of planning the safety stock is formulated, which can be used to stabilize the flow parameters of a multi-flow production line with an objective function that minimizes the total cost of the safety stock.

Along with the planning problem, the value of the optimal safety stock, the stabilizing the parameters by a multi-flow line is no less urgent. The solution of the problem makes it possible to find the optimal control actions that ensure the asymptotic stability of the planned given unperturbed state of flow parameters with the least cost of technological resources spent on the formation of the control action $u_0 = u_0(t,[y]_0,[y]_1)$. Suppose that in a synchronized operation mode of the production line, the productivity of the technological equipment is equal to the standard value

$$[\chi]_{1\psi}(t,S) \cong [\chi]_1(t,S),$$

and the parts are evenly distributed over the technological operations

$$\frac{\partial[\chi]_0(t,S)}{\partial S} \cong 0.$$

Then, when using only the management of inter-operational reserve $u_0 = u_0(t,[y]_0,[y]_1)$ the system of two-moment balance equations (7) can be represented in the form

$$\frac{\partial[y]_0(t,S)}{\partial t} + \frac{\partial[y]_1(t,S)}{\partial S} = q_{00}u_0, \quad \frac{\partial[y]_1(t,S)}{\partial t} + \frac{\partial[y]_1(t,S)}{\partial S}B = 0. \tag{9}$$

It is assumed that if the amount of inter-operative backlogs deviates from the undisturbed state $[\chi]_n = [\chi]_n^*(t,S)$ it is possible to compensate for this deviation with intensity $q_{00}\cdot u_0$ at the expense of external sources of replenishment or from insurance backlogs. The absence of a control action $u_0 = u_0(t,[y]_0,[y]_1)$ in the equation for the productivity of technological equipment позволяет утверждать, allows us to assert that there is no source of change in the productivity of technological equipment, $q_{10} \equiv 0$. The change in the value of inter-operative backlogs occurs due to desynchronization in the performance of technological equipmen $\frac{\partial[y]_1(t,S)}{\partial S}$, which is ultimately compensated by the source of insurance backlogs $q_{00}\cdot u_0$. The negative value of the receipt of parts from the source of insurance reserves $q_{00}\cdot u_0$ indicates the formation of an excessive inter-operational reserve, which is taken from the technological operation reserve. When $[y]_0(t,S) < 0$, additional costs are required to replenish the deviations of the inter-operative backlog. Since the system of equations (9) contains only one multiplier q_{00}, so let's put $q_{00} = 1$, thereby taking into account the considered coefficient directly in the unknown function u_0.

As a criterion evaluating the quality of the flow parameters operational control (6), let's choose the condition that determines the minimum of the integral:

$$I = \int_0^{\infty}\int_0^{S_d} \left(\alpha([y]_0)^2 + \beta(u_0)^2\right) dSdt. \tag{10}$$

where α, β are the coefficients characterizing the costs associated with the deviations of the flow parameters and the costs associated with the control actions necessary to eliminate these deviations do not depend on S. Taking into account the expansion of small disturbances $[y]_0(t,S)$, $[y]_1(t,S)$ of the flow parameters $[\chi]_0(t,S)$, $[\chi]_1(t,S)$ and control actions $u_0(t,[y]_0,[y]_1)$ in a Fourier series, the integrand ω for the quality integral takes the form

$$\omega = \\ = \alpha\left(\{y_n\}_0 + \sum_{j=1}^{\infty}\{y_n\}_j \sin[k_j S] + \sum_{j=1}^{\infty}[y_n]_j \cos[k_j S]\right)^2 + \\ + \beta\left(\{u_n\}_0 + \sum_{j=1}^{\infty}\{u_n\}_j \sin[k_j S] + \sum_{j=1}^{\infty}[u_n]_j \cos[k_j S]\right)^2. \tag{11}$$

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Let us perform integration the quality criterion (10) by the coordinate S

$$I=\int_0^\infty \alpha\left(\{y_0\}_0^2+\frac{1}{2}\sum_{j=1}^\infty\left(\{y_0\}_j^2+[y_0]_j^2\right)\right)dt+ \\ +\int_0^\infty \beta\left(\{u_0\}_0^2+\frac{1}{2}\sum_{j=1}^\infty\left(\{u_0\}_j^2+[u_0]_j^2\right)\right)dt. \quad (12)$$

Substituting into the system of equations (9) the expansion of small perturbations $[y]_0(t,S)$, $[y]_1(t,S)$ of the flow parameters and control action $u_0(t,[y]_0,[y]_1)$ in the Fourier series, the system of equations in small perturbations is obtained:

$$\frac{d\{y_0\}_0}{dt}=\{u_0\}_0,\ \frac{d\{y_1\}_0}{dt}=0, \\ \frac{d\{y_0\}_1}{dt}+k[y_1]_1=\{u_0\}_1,\ \frac{d[y_0]_1}{dt}+k\{y_0\}_1=[u_0]_1, \quad (13) \\ \frac{d\{y_1\}_1}{dt}-[y_1]_1 kB=0,\ \frac{d[y_1]_1}{dt}+\{y_1\}_1 kB=0.$$

When constructing the system of equations (13), for clarity of demonstrating the method of constructing the stabilization algorithm, the expansion in the Fourier series is limited by the first two terms. Let's seek the Lyapunov function in the quadratic form:

$$V^0=\int_0^{S_d}\left(c_0([y]_0)^2+c_1([y]_1)^2\right)dS,\ \frac{\partial V^0}{\partial t}=0 \quad (14)$$

with constant coefficients c_0, c_1. Then Lyapunov function V^0 can be written in terms of the coefficients $\{y_n\}_0$, $\{y_n\}_j$, $[y_n]_j$

$$V^0=c_0\left(\{y_0\}_0^2+\frac{1}{2}\sum_{j=1}^\infty\left(\{y_0\}_j^2+[y_0]_j^2\right)\right)+ \\ +c_1\left(\{y_1\}_0^2+\frac{1}{2}\sum_{j=1}^\infty\left(\{y_1\}_j^2+[y_0]_j^2\right)\right). \quad (15)$$

Let us define the Hamiltonian $B[V^0]$ for the system under study in the following form:

$$B[V^0]=\sum_{n=0}^{1}\frac{\partial V^0}{\partial\{y_n\}_0}\frac{d\{y_n\}_0}{dt}+ \\ +\sum_{j=1}^\infty\left(\sum_{n=0}^{1}\frac{\partial V^0}{\partial\{y_n\}_j}\frac{d\{y_n\}_j}{dt}+\sum_{n=0}^{1}\frac{\partial V^0}{\partial[y_n]_j}\frac{d[y_n]_j}{dt}\right)+\omega. \quad (16)$$

Under optimal control $u_0=u_0^*(t,[y]_0,[y]_1)$ the value $B[V^0]$ should have a minimum and thus become zero. Hence the first equation for determining the form of the Lyapunov function V^0 and the optimal control action $u_0=u_0^*(t,[y]_0,[y]_1)$:

$$B[V^0]=0. \quad (17)$$

4 RESULTS

Differentiating $B[V^0]$ by $\{u_0\}_0$, $\{u_0\}_j$, $[u_1]_j$ and equating the results to zero, we obtain the missing equations for determining the form of the Lyapunov function V^0 and the optimal control action $u_0=u_0^*(t,[y]_0,[y]_1)$:

$$\frac{\partial B[V^0]}{\partial\{u_0\}_0}=\sum_{n=0}^{1}\frac{\partial V^0}{\partial\{y_n\}_0}\frac{\partial}{\partial\{u_0\}_0}\left(\frac{d\{y_n\}_0}{dt}\right)+ \\ +\sum_{j=1}^\infty\sum_{n=0}^{1}\frac{\partial V^0}{\partial\{y_n\}_j}\frac{\partial}{\partial\{u_0\}_0}\left(\frac{d\{y_n\}_j}{dt}\right)+ \quad (18) \\ +\sum_{j=1}^\infty\sum_{n=0}^{1}\frac{\partial V^0}{\partial[y_n]_j}\frac{\partial}{\partial\{u_0\}_0}\left(\frac{d[y_n]_j}{dt}\right)+2\beta\{u_0\}_0=0.$$

$$\frac{\partial B[V^0]}{\partial\{u_0\}_m}=\sum_{n=0}^{1}\frac{\partial V^0}{\partial\{y_n\}_0}\frac{\partial}{\partial\{u_0\}_m}\left(\frac{d\{y_n\}_0}{dt}\right)+ \\ +\sum_{j=1}^\infty\sum_{n=0}^{1}\frac{\partial V^0}{\partial\{y_n\}_j}\frac{\partial}{\partial\{u_0\}_m}\left(\frac{d\{y_n\}_j}{dt}\right)+ \quad (19) \\ +\sum_{j=1}^\infty\sum_{n=0}^{1}\frac{\partial V^0}{\partial[y_n]_j}\frac{\partial}{\partial\{u_0\}_m}\left(\frac{d[y_n]_j}{dt}\right)+\beta\{u_0\}_m=0,$$

$$\frac{\partial B[V^0]}{\partial[u_0]_m}=\sum_{n=0}^{1}\frac{\partial V^0}{\partial\{y_n\}_0}\frac{\partial}{\partial[u_0]_m}\left(\frac{d\{y_n\}_0}{dt}\right)+ \\ +\sum_{j=1}^\infty\sum_{n=0}^{1}\frac{\partial V^0}{\partial\{y_n\}_j}\frac{\partial}{\partial[u_0]_m}\left(\frac{d\{y_n\}_j}{dt}\right)+ \quad (20) \\ +\sum_{j=1}^\infty\sum_{n=0}^{1}\frac{\partial V^0}{\partial[y_n]_j}\frac{\partial}{\partial[u_0]_m}\left(\frac{d[y_n]_j}{dt}\right)+\beta[u_0]_m=0.$$

Substituting (13) into the above equations (18)–(20), the following equations are obtained:

$$c_0\{y_0\}_0+c_0\{y_0\}_0+\beta\{u_0\}_0=0, \quad (21) \\ c_0\{y_0\}_m+\beta\{u_0\}_m=0, \\ c_0[y_0]_m+[\beta]_0[u_0]_m=0.$$

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The system of equations (21) is solvable with respect to coefficients $\{u_0\}_0$, $\{u_0\}_m$, $[u_0]_m$:

$$\{u_0\}_0 = -\frac{c_0}{\beta}\{y_0\}_0, \qquad (22)$$

$$\{u_0\}_m = -\frac{c_0}{\beta}\{y_0\}_m,$$

$$[u_0]_m = -\frac{c_0}{\beta}[y_0]_m.$$

The coefficient c_0 can be determined from equation (17). If it is possible to find a limited particular solution of the equation for determining the coefficient c_0, such that form V^0 (15) turns out to be definitely positive, then the control actions (22) will ensure the asymptotic stability of the given unperturbed state of the flow parameters due to equations (9).

5 DISCUSSION

The system of equations (22) determines the algorithm for the optimal stabilization of the inter-operational backlogs of the production line. Taking into account the relation for the expansion coefficients of functions $[y]_0(t,S)$, $u_0(t,S)$ (22), the stabilization algorithm can be represented in the form

$$u_0(t,S) = -\frac{c_0}{\beta}[y]_0(t,S). \qquad (23)$$

The stabilizing action is proportional to the disturbance that has arisen and is opposite to it in sign. The optimal control $u_0 = u_0^*(t,[y]_0,[y]_1)$ is set by the value of the coefficient c_0, which is calculated taking into account the given form of the function of the quality criterion of the process of stabilization of the flow parameters (6). Substitution of function (23) into equation (9)

$$\frac{\partial[y]_0(t,S)}{\partial t} + \frac{\partial[y]_1(t,S)}{\partial S} = -\frac{c_0}{\beta}[y]_0(t,S). \qquad (24)$$

allows a qualitative analysis of the obtained solution. The stabilizing action (23) compensates for the desynchronization of technological equipment The minimum value of the constant c_0 in the Lyapunov function can be determined from the condition

$$\frac{c_0}{\beta}[y]_0(t,S) + \frac{\partial[y]_1(t,S)}{\partial S} \geq 0. \qquad (25)$$

Inequality (25) must hold for any values of t and S. If the condition is satisfied

$$\left|\frac{c_0}{\beta}[y]_0(t,S)\right| >> \left|\frac{\partial[y]_1(t,S)}{\partial S}\right|,$$

the solution of equation (24) takes the form

$$[y]_0(t,S) \sim \exp\left(-\frac{c_0}{\beta}t\right).$$

The resulting deviation of inter-operative backlogs decreases exponentially with characteristic decay time $t_d = \beta / c_0$. An increase in the value of the coefficient c_0 leads to an increase in the rate of the damping process of the arisen disturbance and, accordingly, decreases the characteristic time of the process of stabilization of the arisen disturbances. Condition (25), which determines the minimum value of the coefficient c_0, which the asymptotic damping of the arising disturbances of the inter-operational reserve is ensured, can be represented in the following form

$$\frac{c_0}{\beta}[y]_0(t,S) - \frac{1}{B}\frac{\partial[y]_1(t,S)}{\partial t} \geq 0. \qquad (26)$$

Inequality (26) must be satisfied for any values of t and S. The reasons for the disturbance can be different. The main cause of disturbances is desynchronization of technological equipment. The technological time for processing a part with technological equipment is not a deterministic value. The processing time is a random variable, which is determined by a given law of distribution of a random variable with a mathematical expectation inversely proportional to the standard productivity of technological equipment $[\chi]_{1\psi}(t,S)$ [25]. Thus, a stochastic process of processing parts with an average processing time $[\chi]_{1\psi}^{-1}(t,S)$ leads to the emergence of a function gradient $[y]_1(t,S)$, and, accordingly, to desynchronization of technological equipment along the production line. Also, a significant source of deviations in the value $[y]_0(t,S)$ is the presence of a source of defective products. The occurrence of this situation can be caused by both random factors and systemic factors associated, for example, with the wear of a tool or technological equipment. Such a source leads both to the loss of parts due to the impossibility of their further processing and to a decrease in the productivity of technological equipment for processing modes of illiquid parts.

CONCLUSIONS

A method for constructing an algorithm for stabilizing the production line flow parameters was proposed.

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The scientific novelty of obtained results is that for the first time a method for constructing an algorithm for stabilizing the production line flow parameters was proposed. The stabilization algorithm makes it possible to provide asymptotic damping of the arising disturbances of inter-operative reserves, provided that the stabilization proceeds in accordance with a given quality criterion. This makes it possible to ensure a given production output in accordance with planned indicators with a minimum expenditure of technological resources. The algorithm for stabilizing the flow parameters allows you to avoid equipment downtime due to the lack of parts in the inter-operational bunkers or due to their overflow.

The practical significance lies in the use of the developed methodology for the design of highly efficient systems for stabilizing the production line flow parameters of industrial enterprises with the flow method of organizing production.

Prospects for further research are the development of a method for constructing an algorithm for stabilizing the productivity of technological operations of a production line.

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АЛГОРИТМ ОПТИМАЛЬНОЇ СТАБІЛІЗАЦІЇ ПОТОКОВИХ ПАРАМЕТРІВ ВИРОБНИЧОЇ ЛІНІЇ РОЗМІРНОСТІ

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АНОТАЦІЯ

Актуальність. Розглянуто метод побудови алгоритму стабілізації меж-операційних заділів виробничої лінії. Об'єктом дослідження була модель виробничої багато-операційної потокової лінії.

Мета роботи. Метою роботи є розробка методу побудови оптимального алгоритму стабілізації потокових параметрів виробничої лінії, при якому забезпечується асимптотична стійкість стану потокових параметрів при заданому якості процесу ліквідації виниклих збурень..

Метод.. Запропоновано метод побудови алгоритму стабілізації рівня міжопераційних заділів багатоопераційної виробничої лінії В основу побудови алгоритму стабілізація покладена двох моментная PDE-модель виробничої лінії, що дозволило представити виробничу лінію у вигляді складної динамічної розподіленої системи. Таке уявлення дало можливість визначити стабілізуючий управління у вигляді функції, яка залежить не тільки від часу, але і координати, що характеризує місце розташування технологічного обладнання уздовж виробничої лінії. Використання методу функцій Ляпунова дозволило синтезувати оптимальне стабілізуючий управління станом меж-операційних заділів на технологічних операціях виробничої лінії, яка забезпечує асимптотичну стійкість заданого невозмущенного стану потокових параметрів виробничої лінії при найменших витратах технологічних ресурсів, що витрачаються на формування керуючого впливу. Вимога про найкращій якості перехідного процесу від обуреного стану до незбурених виражено інтегралом якості, який залежить як від величини виникли збурень, так і від величини стабілізуючих управлінь, націлених на ліквідацію даних збурень.

Результати. На основі розробленого методу побудови алгоритму стабілізації стану потокових параметрів виробничої лінії синтезований алгоритм стабілізації величини меж-операційних заділів на технологічних операціях виробничої лінії.

Висновки. Використання методу функцій Ляпунова при синтезі оптимального стабілізуючого управління потоковими параметрами виробничої лінії дозволяє забезпечити асимптотическое загасання виникають збурень потокових параметрів при найменших витратах технологічних ресурсів, що витрачаються на формування керуючого впливу. Показано, що в задачі стабілізації стану міжопераційних заділів стабілізуючий управління по величині пропорційно величині виникає обурення. Коефіцієнт пропорційності визначається через коефіцієнти інтеграла якості і функції Ляпунова. Перспективою подальших досліджень є розробка методу побудови алгоритму стабілізації продуктивності технологічних операцій виробничої лінії

КЛЮЧОВІ СЛОВА: PDE-модель виробничої лінії, багато-моментні рівняння, функція Ляпунова, інтеграл якості, оптимальне управління, стабілізація..

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АЛГОРИТМ ОПТИМАЛЬНОЙ СТАБИЛИЗАЦИИ ПОТОКОВЫХ ПАРАМЕТРОВ ПРОИЗВОДСТВЕННОЙ ЛИНИИ

Пигнастый О. М. – д-р техн. наук, профессор, профессор кафедры распределенных информационных систем и облачных технологий Национального технического университета «Харьковский политехнический институт», г. Харьков, Украина.

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АННОТАЦИЯ

Актуальность. Рассмотрен метод построения алгоритма стабилизации межоперационных заделов производственной линии. Объектом исследования являлась модель производственной многооперационной поточной линии.

Цель работы. Целью работы является разработка метода построения оптимального алгоритма стабилизации потоковых параметров производственной линии, при котором обеспечивается асимптотическая устойчивость состояния потоковых параметров при заданном качестве процесса ликвидации возникших возмущений.

Метод. Предложен метод построения алгоритма стабилизации уровня межоперационных заделов многооперационной производственной линии. В основу построения алгоритма стабилизация положена двух моментная PDE–модель производственной линии, что позволило представить производственную линию в виде сложной динамической распределенной системы. Такое представление дало возможность определить стабилизирующее управление в виде функции, которая зависит не только от времени, но и координаты, характеризующей место расположения технологического оборудования вдоль производственной линии. Использование метода функций Ляпунова позволило синтезировать оптимальное стабилизирующее управление состоянием межоперационных заделов на технологических операциях производственной линии, которое обеспечивает асимптотическую устойчивость планового заданного невозмущенного состояния потоковых параметров производственной линии при наименьших затратах технологических ресурсов, расходуемых на формирование управляющего воздействия. Требование о наилучшем качестве переходного процесса от возмущенного состояния к невозмущенному выражено интегралом качества, который зависит как от величины возникших возмущений, так и от величины стабилизирующих управлений, нацеленных на ликвидацию данных возмущений.

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Результаты. На основе разработанного метода построения алгоритма стабилизации состояния потоковых параметров производственной линии синтезирован алгоритм стабилизации величины межоперационных заделов на технологических операциях производственной линии.

Выводы. Использование метода функций Ляпунова при синтезе оптимального стабилизирующего управления потоковыми параметрами производственной линии позволяет обеспечить асимптотическое затухание возникающих возмущений потоковых параметров при наименьших затратах технологических ресурсов, расходуемых на формирование управляющего воздействия. Показано, что в задаче стабилизации состояния межоперационных заделов стабилизирующее управление по величине пропорционально величине возникающего возмущения. Коэффициент пропорциональности определяется через коэффициенты интеграла качества и функции Ляпунова. Перспективой дальнейших исследований является разработка метода построения алгоритма стабилизации производительности технологических операций производственной линии.

КЛЮЧЕВЫЕ СЛОВА: PDE-модель производственной линии, много-моментные уравнения, функция Ляпунова, интеграл качества, оптимальное управление, стабилизация.

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