ANTI-SYNCHRONIZING BACKSTEPPING CONTROL DESIGN FOR ARNEODO CHAOTIC SYSTEM

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ABSTRACT

In this paper, we derive new results for backstepping controller design for the anti-synchronization of Arneodo chaotic system (1980). Backstepping control is a recursive procedure that combines the choice of a Lyapunov function with the design of a feedback controller. In anti-synchronization of chaotic systems, the states of the synchronized systems have the same absolute values, but opposite signs. First, we derive an active backstepping controller for the anti-synchronization of identical Arneodo chaotic systems. Next, we derive an adaptive backstepping controller for the anti-synchronization of identical Arneodo chaotic system, when the system parameters are unknown. The anti-synchronization results for Arneodo chaotic systems have been proved using Lyapunov stability theory. Numerical simulations have been shown to illustrate the backstepping controllers derived in this paper for Arneodo chaotic system.

KEYWORDS

Backstepping Control; Chaos; Anti-Synchronization; Arneodo System.

1. INTRODUCTION

Chaos theory deals with the behaviour of nonlinear dynamical systems that are highly sensitive to initial conditions, an effect which is popularly known as the butterfly effect [1]. Small differences in initial conditions result in widely diverging outcomes for chaotic systems, rending long-term prediction impossible in general. The chaos phenomenon was first observed in weather models by the American scientist, Lorenz ([2], 1963). Since then, chaos theory has found applications in a variety of fields in science and engineering [3-9].

The problem of controlling a chaotic system was first introduced by Ott *et al.* ([10], 1990). The problem of chaos synchronization occurs when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator ([11], 1990). The idea of chaos anti-synchronization is to use the output of the master system to control the output of the slave system so that the states of the master and slave systems have the same absolute values, but opposite signs, *i.e.* the sum of the output signals of the master and slave systems can converge to zero asymptotically.

Since the pioneering work by Pecora and Carroll [11], various methods have been developed in the chaos literature for the synchronization of chaotic systems such as active control method [12-15], adaptive control method [16-20], time-delay feedback control method [21], sampled-data control method [22-23], sliding mode control method [24-30], backstepping control method [31-33], etc.

In this paper, we deploy backstepping control method for the anti-synchronization of identical Arneodo chaotic systems ([34], 1980). Backstepping control method is a recursive procedure that combines the choice of a Lyapunov function with the design of a feedback controller.

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The organization of this research paper is as follows. In Section 2, we design an active backstepping controller for the anti-synchronization of identical Arneodo systems when the system parameters are known. In Section 3, we design an adaptive backstepping controller for the anti-synchronization of identical Arneodo systems when the system parameters are unknown. Section 4 contains the conclusions of this work.

2. ACTIVE BACKSTEPPING CONTROLLER DESIGN FOR THE ANTI-SYNCHRONIZATION OF ARNEODO SYSTEMS

2.1 Theoretical Results

Arneodo system ([34], 1980) is one of the classical 3-D chaotic systems as it captures many features of chaotic systems. In this section, we investigate the problem of active backstepping controller design for the anti-synchronization of identical Arneodo chaotic systems, when the system parameters are known.

As the master system, we consider the 3-D Arneodo dynamics

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2, \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b are positive, known parameters of the system.

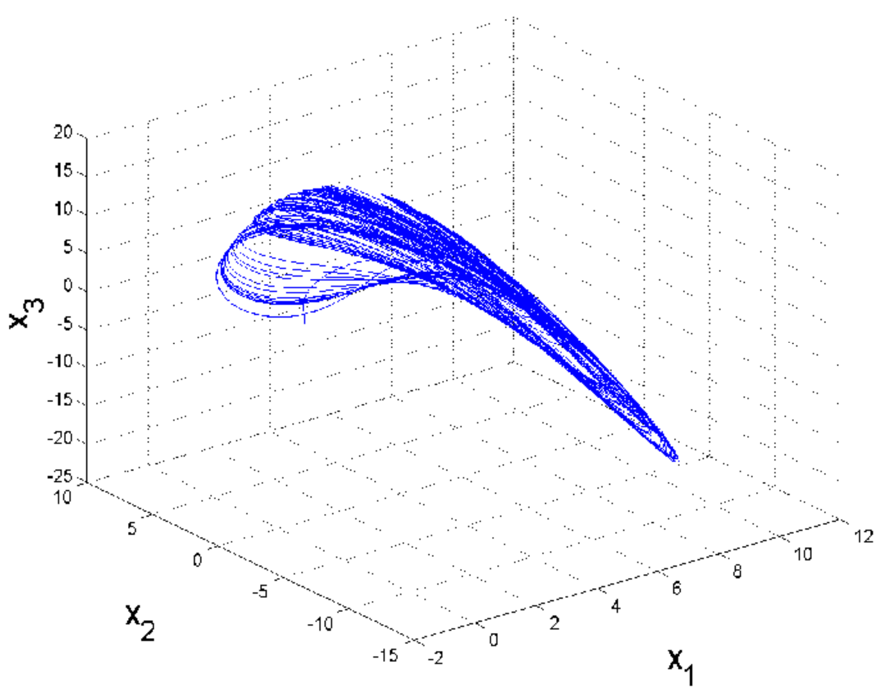

Figure 1. Strange Chaotic Attractor of the Arneodo System

The Arneodo system (1) undergoes chaotic behaviour when the system parameter values are chosen as

$$a = 7.5 \text{ and } b = 3.8.$$

The strange chaotic attractor of the Arneodo system (1) is shown in Figure 1.

As the slave system, we consider the controlled 3-D Arneodo dynamics

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= y_3, \\ \dot{y}_3 &= ay_1 - by_2 - y_3 - y_1^2 + u, \end{aligned} \tag{2}$$

where y_1, y_2, y_3 are the states and u is the active control to be designed.

The anti-synchronization error between the master system (1) and the slave system (2) is defined as

$$\begin{aligned} e_1(t) &= y_1(t) + x_1(t), \\ e_2(t) &= y_2(t) + x_2(t), \\ e_3(t) &= y_3(t) + x_3(t). \end{aligned} \tag{3}$$

The design problem is to find a control $u(t)$ so that the error converges to zero asymptotically, i.e. $e_i(t) \to 0$ as $t \to \infty$ for $i = 1, 2, 3.$

The error dynamics is easily derived as

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= e_3, \\ \dot{e}_3 &= ae_1 - be_2 - e_3 - y_1^2 - x_1^2 + u. \end{aligned} \tag{4}$$

In this section, we apply the active backstepping control method to design a controller $u(t).$

Theorem 1. The identical Arneodo chaotic systems (1) and (2) are globally and exponentially anti-synchronized for all initial conditions by the active backstepping controller

$$u(t) = -(3 + a)e_1 - (5 - b)e_2 - 2e_3 + y_1^2 + x_1^2. \tag{5}$$

Proof. First, we define a Lyapunov function

$$V_1 = \frac{1}{2} z_1^2, \tag{6}$$

where

$$z_1 = e_1. \tag{7}$$

Its time derivative along the solutions of systems (1) and (2) is obtained as

$$\dot{V}_1 = z_1 \dot{z}_1 = e_1 \dot{e}_1 = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2). \tag{8}$$

Next, we define

$$z_2 = e_1 + e_2. \tag{9}$$

From (9), it follows that

$$\dot{V}_1 = -z_1^2 + z_1 z_2. \tag{10}$$

Secondly, we define the Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right). \tag{11}$$

The time derivative of V_2 is given by

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3). \tag{12}$$

Next, we define

$$z_3 = 2e_1 + 2e_2 + e_3. \tag{13}$$

From (13), it follows that

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3. \tag{14}$$

Finally, we define the Lyapunov function

$$V = V_2 + \frac{1}{2} z_3^2 = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2\right). \tag{15}$$

Clearly, V is a positive definite function on R^3.

The time derivative of V is obtained as

$$\dot{V} = -z_1^2 - z_2^2 + z_2 z_3 + z_3\left(2e_2 + 2e_3 + ae_1 - be_2 - e_3 - y_1^2 - x_1^2 + u\right) \tag{16}$$

A simple calculation gives

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3\left[(3+a)e_1 + (5-b)e_2 + 2e_3 - y_1^2 - x_1^2 + u\right]. \tag{17}$$

Substituting the backstepping controller u defined by (5) in (17), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2. \tag{18}$$

Clearly, $\dot{V}$ is a negative definite function on R^3.

Hence, by Lyapunov stability theory [35], the error dynamics (4) is globally exponentially stable.

This completes the proof. ■

2.2 Numerical Results

For numerical simulations using MATLAB, the fourth order Runge-Kutta method with initial step $h = 10^{-8}$ is used to solve the Arneodo systems (1) and (2) with the backstepping controller u defined by (5). The parameters of the Arneodo chaotic systems are selected as $a = 7.5$ and $b = 3.8.$

The initial values of the master system (1) are chosen as

$$x_1(0) = 14, \quad x_2(0) = -5, \quad x_3(0) = 6$$

The initial values of the slave system (2) are chosen as

$$y_1(0) = 18, \quad y_2(0) = 12, \quad y_3(0) = -16$$

Figure 2 depicts the anti-synchronization of Arneodo chaotic systems (1) and (2).

Figure 3 depicts the time-history of the anti-synchronization errors $e_1, e_2, e_3.$

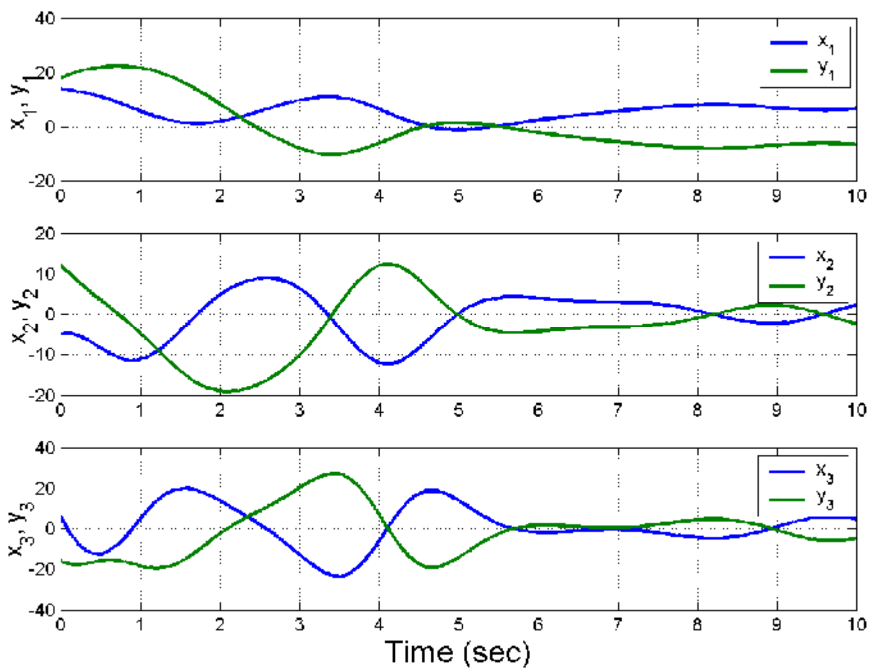

Figure 2. Anti-Synchronization of Arneodo Chaotic Systems

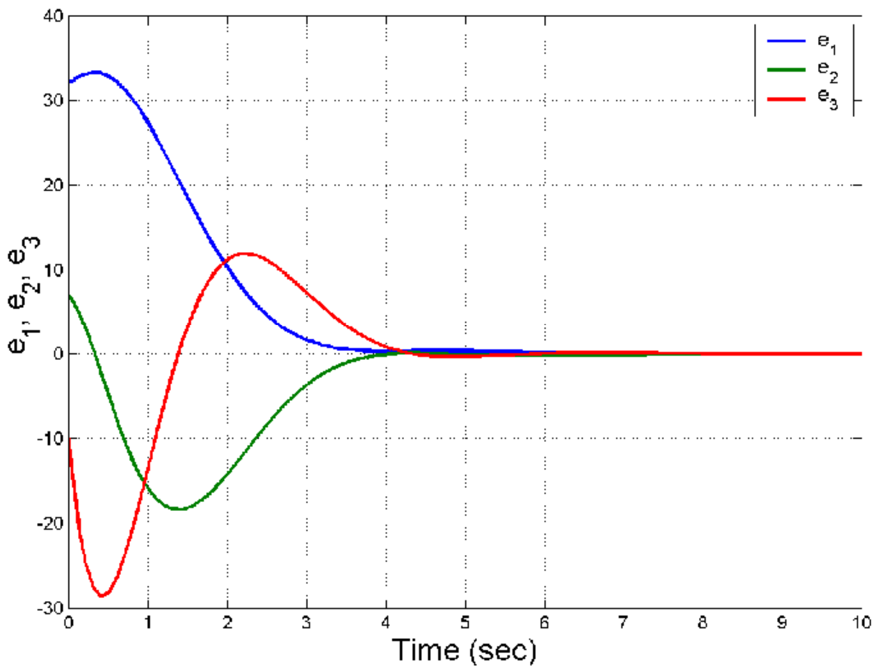

Figure 3. Time-History of the Anti-Synchronizing Errors e_1, e_2, e_3

3. Regulating Active Backstepping Controller Design for the Anti-Synchronization of Arneodo Systems

3.1 Theoretical Results

In this section, we derive new results for the adaptive backstepping controller design for anti-synchronization of Arneodo systems when the parameters a and b are unknown.

As the master system, we consider the 3-D Arneodo dynamics

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2, \end{aligned} \tag{19}$$

where x_1, x_2, x_3 are the states and a, b are unknown parameters of the system.

As the slave system, we consider the controlled 3-D Arneodo dynamics

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= y_3, \\ \dot{y}_3 &= ay_1 - by_2 - y_3 - y_1^2 + u, \end{aligned} \tag{20}$$

where y_1, y_2, y_3 are the states and u is the adaptive control to be designed.

The anti-synchronization error between the master system (19) and the slave system (20) is defined as

$$\begin{aligned} e_1(t) &= y_1(t) + x_1(t), \\ e_2(t) &= y_2(t) + x_2(t), \\ e_3(t) &= y_3(t) + x_3(t). \end{aligned} \tag{21}$$

The design problem is to find a control $u(t)$ so that the error converges to zero asymptotically, i.e. $e_i(t) \to 0$ as $t \to \infty$ for $i = 1, 2, 3.$

The error dynamics is easily derived as

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= e_3, \\ \dot{e}_3 &= ae_1 - be_2 - e_3 - y_1^2 - x_1^2 + u. \end{aligned} \tag{22}$$

In this section, we apply the adaptive backstepping control method to design a controller $u(t).$

Inspired by the control law defined by Eq. (5) in the active backstepping controller design, we may consider the adaptive backstepping controller design law given by

$$u(t) = -(3 + \hat{a})e_1 - (5 - \hat{b})e_2 - 2e_3 + y_1^2 + x_1^2, \tag{23}$$

where $\hat{a}(t)$ and $\hat{b}(t)$ are estimates of the unknown parameters a and $b,$ respectively.

We define the parameter estimation errors as

$$e_a(t) = a - \hat{a}(t) \quad \text{and} \quad e_b(t) = b - \hat{b}(t) \tag{24}$$

Note that

$$\dot{e}_a(t) = -\dot{\hat{a}}(t) \quad \text{and} \quad \dot{e}_b(t) = -\dot{\hat{b}}(t) \tag{25}$$

Next, we shall state and prove the second main result of this paper.

Theorem 2. The identical Arneodo chaotic systems (19) and (20) with unknown parameters a and b are globally and exponentially anti-synchronized for all initial conditions by the adaptive backstepping controller

$$u(t) = -(3 + \hat{a})e_1 - (5 - \hat{b})e_2 - 2e_3 + y_1^2 + x_1^2, \tag{26}$$

where $\hat{a}(t)$ and $\hat{b}(t)$ are estimates of a and $b,$ respectively, and the parameter update law is given by

$$\begin{aligned} \dot{\hat{a}}(t) &= (2e_1 + 2e_2 + e_3)e_1 + k_a e_a, \\ \dot{\hat{b}}(t) &= -(2e_1 + 2e_2 + e_3)e_2 + k_b e_b, \end{aligned} \tag{27}$$

with positive control gains k_a and k_b.

Proof. First, we define the Lyapunov function

$$V_1 = \frac{1}{2} z_1^2, \tag{28}$$

where

$$z_1 = e_1. \tag{29}$$

The time derivative of V_1 is given by

$$\dot{V}_1 = z_1 \dot{z}_1 = e_1 \dot{e}_1 = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2). \tag{30}$$

Next, we define

$$z_2 = e_1 + e_2. \tag{31}$$

From (30), it follows that

$$\dot{V}_1 = -z_1^2 + z_1 z_2. \tag{32}$$

Secondly, we define the Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2 \right). \tag{33}$$

The time derivative of V_2 is given by

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3). \tag{34}$$

Next, we define

$$z_3 = 2e_1 + 2e_2 + e_3. \tag{35}$$

From (34), it follows that

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3. \tag{35}$$

Finally, we define the Lyapunov function

$$V = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2}\left(e_a^2 + e_b^2 \right) = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2 + e_a^2 + e_b^2 \right). \tag{36}$$

The time derivative of V is obtained as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3\left[(3+a)e_1 + (5-b)e_2 + 2e_3 - y_1^2 - x_1^2 + u \right] - e_a \dot{\hat{a}} - e_b \dot{\hat{b}}. \tag{37}$$

Substituting the backstepping controller u defined by (26) in (37), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + e_a\left(e_1 z_3 - \dot{\hat{a}} \right) + e_b\left(-e_2 z_3 - \dot{\hat{b}} \right). \tag{38}$$

Substituting the parameter law (27) in (38) and noting that $z_3 = 2e_1 + 2e_2 + e_3$, we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - k_a e_a^2 - k_b e_b^2, \tag{39}$$

which is a negative definite function on R^5.

Thus, by Lyapunov stability theory [35], the proof is complete.■

3.2 Numerical Results

For numerical simulations with MATLAB, the fourth-order Runge-Kutta method with initial step $h = 10^{-8}$ is used to solve the Arneodo systems (19) and (20) with the backstepping controller u defined by (26) and the parameter update law defined by (27).

The parameters of the Arneodo chaotic systems are chosen as $a = 7.5$ and $b = 3.8$.

The initial values of the parameter estimates are chosen as $\hat{a}(0) = 16$ and $\hat{b}(0) = 9$.

The control gains are chosen as $k_a = 6$ and $k_b = 6$.

The initial values of the master system (19) are chosen as

$$x_1(0) = 4, \quad x_2(0) = 5, \quad x_3(0) = -8$$

The initial values of the slave system (20) are chosen as

$$y_1(0) = 2, \quad y_2(0) = 6, \quad y_3(0) = -5$$

Figure 4 depicts the anti-synchronization of Arneodo chaotic systems. Figure 5 depicts the time-history of the anti-synchronization errors e_1, e_2, e_3. Figure 6 depicts the time-history of the parameter estimation errors e_a, e_b.

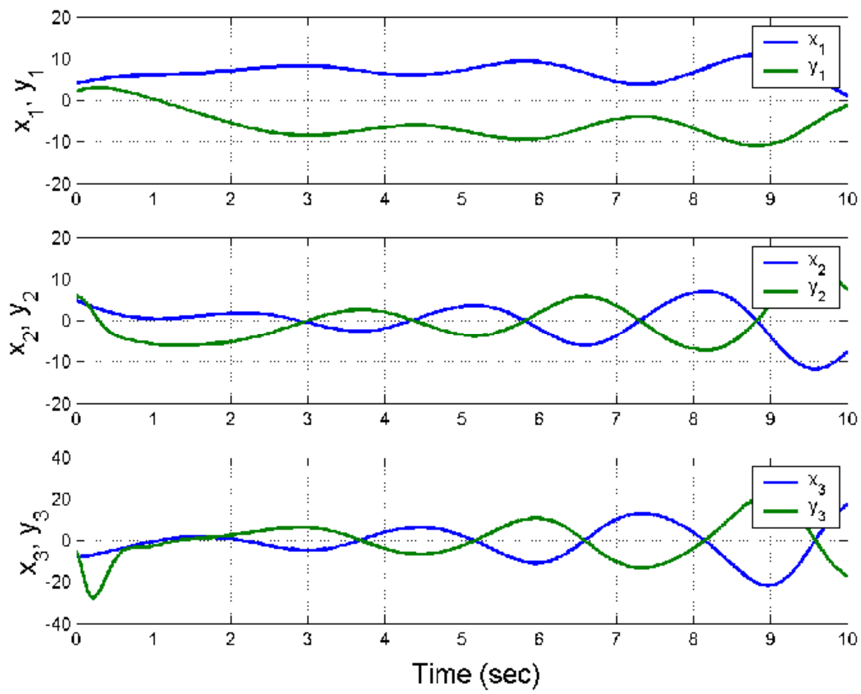

Figure 4. Anti-Synchronization of Arneodo Chaotic Systems

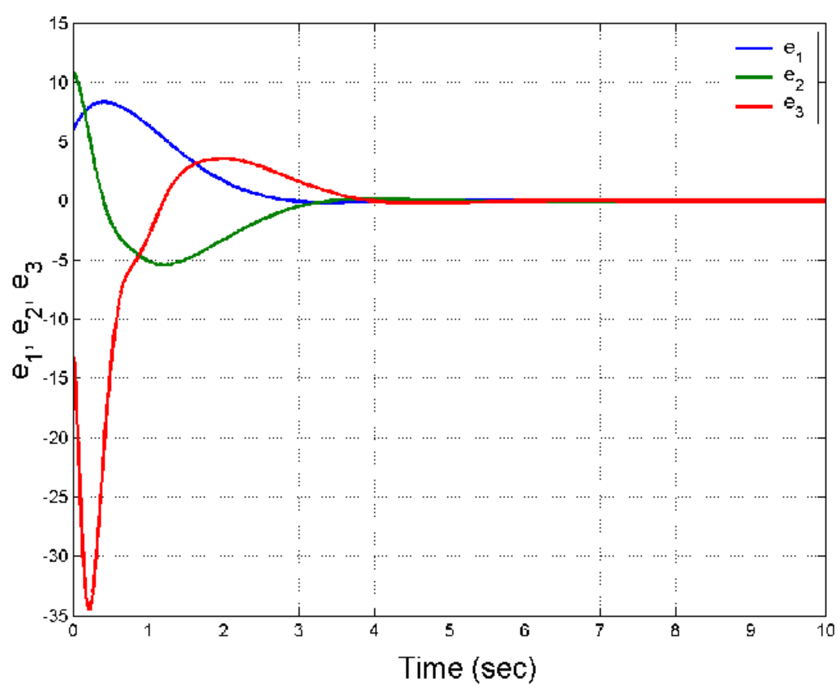

Figure 5. Time-History of the Anti-Synchronization Errors e_1, e_2, e_3

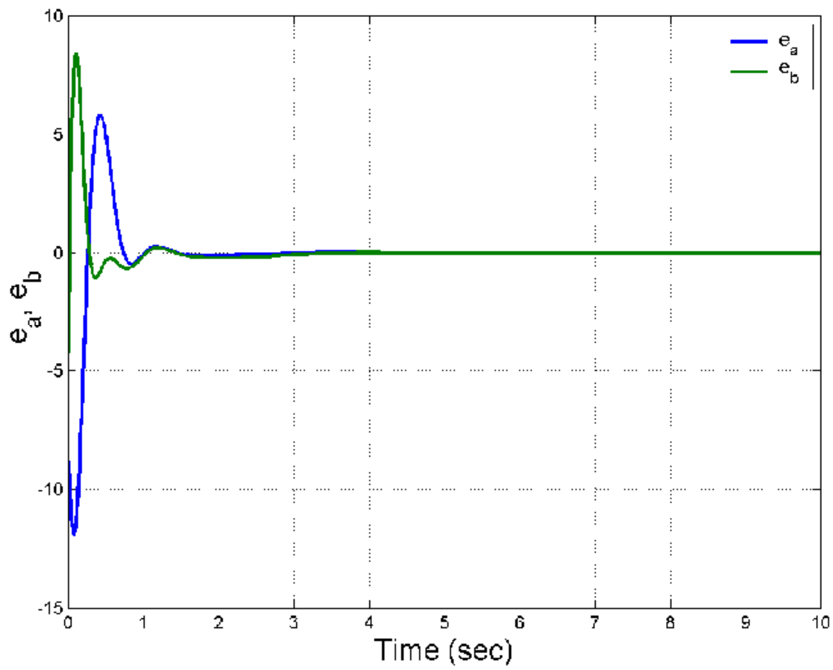

Figure 6. Time-History of the Parameter Estimation Errors e_a, e_n

4. CONCLUSIONS

In this paper, we derived new results for the anti-synchronization of identical Arneodo chaotic systems (1980) via backstepping control method. First, active backstepping controller was designed for the anti-synchronization of identical Arneodo chaotic systems with known system parameters. Next, adaptive backstepping controller was designed for the anti-synchronization of identical Arneodo chaotic systems with unknown system parameters. All the stability results in this paper were established using Lyapunov stability theory. Numerical figures using MATLAB were shown to illustrate the validity and effectiveness of the backstepping controller design for the anti-synchronization of identical chaotic systems for both the cases of known and unknown system parameters.

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