The case of two complex Lorentzians has a power spectrum given by:

$$
\begin{aligned}
PSD(\omega) &= \sqrt{\frac{2}{\pi}}\left(\frac{\omega^2(ar_1br_1 - ai_1bi_1) + (ai_1bi_1 + ar_1br_1)(bi_1^2 + br_1^2)}{\omega^4 + 2(br_1^2 - bi_1^2)\omega^2 + (bi_1^2 + br_1^2)^2}\right. && (1)\\
&+ \left.\frac{\omega^2(ar_2br_2 - ai_2bi_2) + (ai_2bi_2 + ar_2br_2)(bi_2^2 + br_2^2)}{\omega^4 + 2(br_2^2 - bi_2^2)\omega^2 + (bi_2^2 + br_2^2)^2}\right) && (2)
\end{aligned}
$$

The corresponding kernel is:

$$
\begin{aligned}
K(\tau) &= e^{-br_1\tau}(ar_1\cos(bi_1\tau) + ai_1\sin(bi_1\tau)) && (3)\\
&+ e^{-br_2\tau}(ar_2\cos(bi_2\tau) + ai_2\sin(bi_2\tau)). && (4)
\end{aligned}
$$

Each of the denominators is positive, so we just need to check that the numerator of the combined fraction is positive.

This gives a cubic equation in $z = \omega^2$:

$$z^3 + az^2 + bz + c > 0 \quad (5)$$

where

$$
\begin{aligned}
d &= ar_1br_1 - ai_1bi_1 + ar_2br_2 - ai_2bi_2 && (6)\\
a\cdot d &= 2(br_2^2 - bi_2^2)(ar_1br_1 - ai_1bi_1) + 2(br_1^2 - bi_1^2)(ar_2br_2 - ai_2bi_2) && (7)\\
&+ (bi_1^2 + br_1^2)(ai_1bi_1 + ar_1br_1) + (bi_2^2 + br_2^2)(ai_2bi_2 + ar_2br_2) && (8)\\
b\cdot d &= 2(br_2^2 - bi_2^2)(bi_1^2 + br_1^2)(ai_1bi_1 + ar_1br_1) && (9)\\
&+ 2(br_1^2 - bi_1^2)(bi_2^2 + br_2^2)(ai_2bi_2 + ar_2br_2) && (10)\\
&+ (bi_2^2 + br_2^2)^2(ar_1br_1 - ai_1bi_1) + (bi_1^2 + br_1^2)^2(ar_2br_2 - ai_2bi_2) && (11)\\
c\cdot d &= (bi_2^2 + br_2^2)^2(bi_1^2 + br_1^2)(ai_1bi_1 + ar_1br_1) && (12)\\
&+ (bi_1^2 + br_1^2)^2(bi_2^2 + br_2^2)(ai_2bi_2 + ar_2br_2). && (13)
\end{aligned}
$$

Note that if $d = 0$, then we get a quadratic with the requirement $az^2 + bz + c > 0$, where $a\cdot d \rightarrow a$, $b\cdot d \rightarrow b$, and $c\cdot d \rightarrow c$ in the equations for the coefficients.

The number of real roots of the cubic is three if

$$\Delta = 18abc - 4a^3c + a^2b^2 - 4b^3 - 27c^2 \geq 0, \quad (14)$$

and only one otherwise (see NR 5.6 and Wikipedia).

If $\Delta \geq 0$, then compute:

$$\theta = \arccos\left(\frac{2a^3 - 9ab + 27c}{2\left(a^2 - 3b\right)^{3/2}}\right). \quad (15)$$

Note: $a^2 - 3b > 0$ if $\Delta > 0$, and $\theta = 0$ if $\Delta = 0$.

Then, the maximum root, z_{max}, is:

$$z_{max} = -\frac{2}{3}\sqrt{a^2 - 3b}\cos\left(\frac{((\theta + \pi) \bmod 2\pi) + \pi}{3}\right) - \frac{a}{3}. \quad (16)$$

When $\Delta < 0$, then compute:

$$R = \frac{2a^3 - 9ab + 27c}{54} \tag{17}$$

$$Q = \frac{a^2 - 3b}{9} \tag{18}$$

$$A = -sgn(R)\left[|R| + \sqrt{R^2 - Q^3}\right]^{1/3} \tag{19}$$

$$B = \begin{cases} Q/A & \text{if } A \neq 0; \\ 0 & \text{if } A = 0. \end{cases} \tag{20}$$

$$z_{max} = A + B - \frac{a}{3}. \tag{21}$$

In the quadratic case ($d = 0$), then if $b^2 - 4ac < 0$, then there is no solution. If $b^2 - 4ac \geq 0$, then $z_{max} = max\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$.

If $z_{max} \geq 0$, then there is a real solution for $\omega = \sqrt{z_{max}}$, and hence the power spectrum goes non-positive for real $\omega \geq 0$. If $z_{max} < 0$, then the power spectrum is everywhere positive.