Supporting Information for

¹ Effect of the electric field on the agyrotropic electron 2 distributions

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¹⁷ 1 Contents of this file

¹⁸ 1. Figure S1 - S4

¹⁹ 2. Supplementary Text

²⁰ 2 Description of the test particle method

 The test particle method is applied to generate the electron distributions in the plane perpendicular to the local magnetic field based on the Liouville mapping of the electron 23 phase space density $\left(\frac{df}{dt} = 0\right)$. When the initial conditions are set, the electron phase space density at a specified location are estimated along the electron gyro-orbit. An ex- ample is provided in Fig. S1, and the link for the Matlab code used in the present study can be found in the acknowledgment. Meanwhile, it is noting that the agyrotropic elec-

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Figure S1. Construction of the electron distributions in the test particle simulations. The electron phase space density (a) at a specified location are estimated from the averaging the phase space density along the electron gyro-orbit (b).

 tron distributions generated by the test particle method not is self-consistent, but the similarity between the observations and the test particle results shown in the main text suggests the test particle method can reasonably capture the key process of the gener- ation of the agyrotropic electron distributions. Therefore the test particle method is ba- sically applicable here, but more self-consistent simulations, such as the particle in-cell simulation, are needed if we want to get more accurate results.

33 3 Test particle results from the different settings of the electric field

 In the main text, we have shown a strong local electric field can effectively accel- erate electrons to form agyrotropic electron distributions, but the favorable parameters ³⁶ for this electric field has not been fully addressed. Here we present four different settings of the electric field referred to this problem, which are

(a)
$$
|B| = 30 \text{ nT}
$$
, $n = 6 \text{ cm}^{-3}$, $T_e = 60 \text{ eV}$, and $E(x) = g(x)$ (Fig. S1a1).
\n(b) $|B| = 30 \text{ nT}$, $n = 6 \text{ cm}^{-3}$, $T_e = 60 \text{ eV}$, and $E(x) = g(x)$ (Fig. S1b1).
\n(c) $|B| = 30 \text{ nT}$, $n = 6 \text{ cm}^{-3}$, $T_e = 60 \text{ eV}$, and $E(x) = g(x)$ (Fig. S1c1).

$$
41 \qquad \qquad (d) \ |B| = 10 \ \text{nT}, \, n = 6 \ \text{cm}^{-3}, \, T_e = 60 \ \text{eV}, \, \text{and} \ E(x) = g(x) \ \text{(Fig. S1d1)}.
$$

Figure S2. Test particle results under different settings for the electric field. (a1) The setting of the electric field. Three red circles indicate the location where electron distributions generated in (a2 - a4). (b1 - b4), (c1 - c4), and (d1 - d4) are similar to (a1 - a4), but for the different settings of the electric field.

ω 4 Reproducing the "finger" structure with a time-dependent electric field

 In the main text, we have successfully reproduced the measured electron "finger" structure by assuming a virtual detector that moves towards a static electric field (Fig. S4a1), but the "observed" electric field by the virtual detector during the related time inter- val is different with MMS observations (Fig. S4a2). How can we eliminate this difference? One possible solution is to introduce some temporal variations to this strong electric field, and the parameters of this electric field should be reconsidered due to this time-varying ⁶⁸ effect. The potential drop can be estimated by comparing the decomposed 7.5 ms elec- tron distributions, as the first 7.5 ms electron distribution is roughly gyrotropic (Fig. 4a2). The curve of the phase space density of the agyrotropic distributions (the solid black line τ_1 in Fig. S4b) is shifted to maximize the coefficient of determination (R^2) with the curve of the gyrotropic distributions (the green line in Fig. S3b), and the shifted energy of the τ_3 best fit is taken as the acceleration potential, which is approximately 100 eV. The peak ⁷⁴ magnitude of the electric field (E_0) is a free parameter, which is set to 150 mV/m here,

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Figure S3. The observed electron "finger" structure. (a) the measured 30 ms electron "finger" structure in DBCS coordinates. (b) The electron phase space density measured along the negative y direction at the first 7.5 ms (green) and the rest time interval (black). The dotted black line is the shifted solid black line by ~ 100 eV. (c) The coefficient of determination (R^2) between the green line and the shifted black line with different shift energies.

⁷⁵ so that it is convenient for the virtual detector to "observe" an electric field of ~ 100 mV/m. The electric field function then can be functioned as $E_0 \exp(-x^2/\sigma^2)h(t)$, where $h(t)$ is π a sine-type function to include the temporal variations (Fig. S4b1 and b2).

 Similar to the electron acceleration due to a static electric field, a time-dependent electric field can also successfully accelerate electron to form agyrotropic electron dis- tributions (Fig. S4b3 - b5), as it satisfies the conditions of a strong local electric field. By introducing the time-varying effect, the relative speed of the virtual detector can de- $\frac{1}{82}$ crease to ∼ 9 km/s, which is more consistent with the timing result from the observa- tions on 23 November 2016. The "recorded" electric field from the virtual detector matches better with observations as well (Fig. S4b2). Though the peak amplitude of this elec- tric field is artificially set in test particle runs, we infer that this large-amplitude short- lived localized electric field might be a more physical picture responsible for the gener-⁸⁷ ation measured electron "finger" structures, as it meets the expectations of highly fluc-tuating plasma environments in observations.

Figure S4. The reproducing of the electron "Finger" structure with a static electric field and a time-varying electric field. (a1) The initial setting of the static electric field. The red/green/black circles indicate the central positions for the four intermediate electron distributions at a sampling rate of 30/18/6 ms. (a2) The electric field "observed" by a virtual detector at a sampling rate of 30 ms (red) and MMS observations of the electric field (blue). (a3 - a5) The composite of electron distributions at a sample rate of 30 ms, 18 ms and 6 ms. (b1 - b5) Similar to (a1 - a5), but the electric field is time-dependent.