# Block-Wise and Block-Bordered Magic and Bimagic Squares With Magic Sums 21, $21{ }^{2}$ and 2021 

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#### Abstract

We know that we can always write block-wise magic squares of any order except for the orders of type $p$ and $2 p$, where $p$ is a prime number. On the other hand we can always write bordered magic squares of any order. The aims of this work is to bring both bordered and block-wise magic squares. These block-wise and block-bordered magic squares are of orders 8 to 26. All these magic squares are with magic sums as $21,21^{2}$ and 2021. In some cases, such as of orders 8,9, 16 and 25 are also bimagic squares. In case of order 24 with the blocks of orders 3 and 8 the magics squares also turns semi-bimagic squares. This work is based on authors three paper [49, 50, 51]


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## 1 Block-Wise and Block-Bordered Magic Squares

The author [38] wrote bordered magic squares for the general sum as a natural number $n$. Also in 2020, the author [39] wrote bordered magic squares of orders 3 to 25 with magic sum 2020. Based on this idea, recently author worked on bordered magic squares where magic sum is always 2021. The entries of magic squares are either fractional numbers or decimal numbers. These entries are positive and/or negative values to giving sum 2021. Some of the sub-magic squres sums lead us to Pythagorean triples. These are given for the bordered magic squares of even orders. This work brings block-wise and block-bordered magic squares of order 8 to 26 giving final sums of magic squares as $21,21^{2}$ and 2021. The magic squares of orders $8,9,12,15,16,18,20,21,24$ and 25 are written as block-wise. In case of magic squares of orders $8,9,16$ and 25 the magic squares are bimagic. The magic square of order for the blocks of orders 3 and 8 are also semibimagic. The magic squares of orders $10,11,13,17,19,22$ and 26 are written as block-bordred magic squares. These are based on block-wise magic squares written before. For example, the block-bordered magic squares of orders 10 is based on block-wise magic squares of orders 8 . The block-bordred magic squares of orders 11 and 13 are based on block-wise magic square of order 9 . The similar kind of processes goes on for the others orders magic squares. The subsections below give block-wise and block-bordered magic squares of orders 8 to 26 . The whole work is based authors previous three works [49, 50, 51]

### 1.1 Block-Wise Magic Squares of Order 8

Below are block-wise magic squares of order 8 in two different ways giving magic sums as $21,21^{2}$ and 2021. The blocks are of orders 4 . In first case, the blocks of order 4 are pandiagonal with equal magic sums. In the second case the block of order 2 as specified in the figure are of equal sums resulting in a bimagic square of order 8.

### 1.1.1 First Type

Below are block-wise construction of magic square of order 8 resulting in magic sums $21,21^{2}$ and 2021. The blocks of order 4 are pandiagonal magic squares with equal magic sums.

|  | pan | 21 | 21 | , |  |  | 21 | 21 | 21 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -0.875 | 10.125 | -28.875 | 30.125 | -8.875 | 18.125 | -20.875 | 22.125 | 21 |  |
| 21 | -25.875 | 27.125 | 2.125 | 7.125 | -17.875 | 19.125 | -5.875 | 15.125 | 21 |  |
| 21 | 34.125 | -24.875 | 6.125 | -4.875 | 26.125 | -16.875 | 14.125 | -12.875 | 21 |  |
| 21 | 3.125 | -1.875 | 31.125 | -21.875 | 11.125 | -9.875 | 23.125 | -13.875 | 21 |  |
| 21 | 0.125 | 9.125 | -27.875 | 29.125 | -7.875 | 17.125 | -19.875 | 21.125 | 21 |  |
| 21 | -26.875 | 28.125 | 1.125 | 8.125 | -18.875 | 20.125 | -6.875 | 16.125 | 21 |  |
| 21 | 33.125 | -23.875 | 5.125 | -3.875 | 25.125 | -15.875 | 13.125 | -11.875 | 21 |  |
|  | 4.125 | -2.875 | 32.125 | -22.875 | 12.125 | -10.875 | 24.125 | -14.875 | 21 |  |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 1 |


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | 51.625 | 62.625 | 23.625 | 82.625 | 43.625 | 70.625 | 31.625 | 74.625 | 441 |
| 441 | 26.625 | 79.625 | 54.625 | 59.625 | 34.625 | 71.625 | 46.625 | 67.625 | 441 |
| 441 | 86.625 | 27.625 | 58.625 | 47.625 | 78.625 | 35.625 | 66.625 | 39.625 | 441 |
| 441 | 55.625 | 50.625 | 83.625 | 30.625 | 63.625 | 42.625 | 75.625 | 38.625 | 441 |
| 441 | 52.625 | 61.625 | 24.625 | 81.625 | 44.625 | 69.625 | 32.625 | 73.625 | 441 |
| 441 | 25.625 | 80.625 | 53.625 | 60.625 | 33.625 | 72.625 | 45.625 | 68.625 | 441 |
| 441 | 85.625 | 28.625 | 57.625 | 48.625 | 77.625 | 36.625 | 65.625 | 40.625 | 441 |
|  | 56.625 | 49.625 | 84.625 | 29.625 | 64.625 | 41.625 | 76.625 | 37.625 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 249.125 | 260.125 | 221.125 | 280.125 | 241.125 | 268.125 | 229.125 | 272.125 | 2021 |
| 2021 | 224.125 | 277.125 | 252.125 | 257.125 | 232.125 | 269.125 | 244.125 | 265.125 | 2021 |
| 2021 | 284.125 | 225.125 | 256.125 | 245.125 | 276.125 | 233.125 | 264.125 | 237.125 | 2021 |
| 2021 | 253.125 | 248.125 | 281.125 | 228.125 | 261.125 | 240.125 | 273.125 | 236.125 | 2021 |
| 2021 | 250.125 | 259.125 | 222.125 | 279.125 | 242.125 | 267.125 | 230.125 | 271.125 | 2021 |
| 2021 | 223.125 | 278.125 | 251.125 | 258.125 | 231.125 | 270.125 | 243.125 | 266.125 | 2021 |
| 2021 | 283.125 | 226.125 | 255.125 | 246.125 | 275.125 | 234.125 | 263.125 | 238.125 | 2021 |
|  | 254.125 | 247.125 | 282.125 | 227.125 | 262.125 | 239.125 | 274.125 | 235.125 | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the magic sums are given by

$$
\begin{aligned}
{\left[S_{4 \times 4}: S_{8 \times 8}:=21\right] } & :=10.5 \\
{\left[S_{4 \times 4}: S_{8 \times 8}\right.} & :=441] \\
{\left[S_{4 \times 4}: S_{8 \times 8}\right.} & :=220.5 \\
& :=1010.5 .
\end{aligned}
$$

### 1.1.2 Second Type

Below are block-wise construction of magic square of order 8 resulting in magic sums $21,21^{2}$ and 2021. The sum of blocks of $2 \times 4$ with equal sums same as of magic square resulting in bimagic squares of order 8 .


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 38.625 | 63.625 | 58.625 | 27.625 | 49.625 | 84.625 | 77.625 | 40.625 | 441 |
| 441 | 48.625 | 85.625 | 76.625 | 41.625 | 35.625 | 66.625 | 55.625 | 30.625 | 441 |
| 441 | 23.625 | 62.625 | 67.625 | 34.625 | 44.625 | 73.625 | 80.625 | 53.625 | 441 |
| 441 | 45.625 | 72.625 | 81.625 | 52.625 | 26.625 | 59.625 | 70.625 | 31.625 | 441 |
| 441 | 60.625 | 25.625 | 32.625 | 69.625 | 71.625 | 46.625 | 51.625 | 82.625 | 441 |
| 441 | 74.625 | 43.625 | 54.625 | 79.625 | 61.625 | 24.625 | 33.625 | 68.625 | 441 |
| 441 | 65.625 | 36.625 | 29.625 | 56.625 | 86.625 | 47.625 | 42.625 | 75.625 | 441 |
|  | 83.625 | 50.625 | 39.625 | 78.625 | 64.625 | 37.625 | 28.625 | 57.625 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |  |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2021 | 236.125 | 261.125 | 256.125 | 225.125 | 247.125 | 282.125 | 275.125 | 238.125 |
| 2021 | 246.125 | 283.125 | 274.125 | 239.125 | 233.125 | 264.125 | 253.125 | 228.125 | 2021 |
| 2021 | 221.125 | 260.125 | 265.125 | 232.125 | 242.125 | 271.125 | 278.125 | 251.125 | 2021 |
| 2021 | 243.125 | 270.125 | 279.125 | 250.125 | 224.125 | 257.125 | 268.125 | 229.125 | 2021 |
| 2021 | 258.125 | 223.125 | 230.125 | 267.125 | 269.125 | 244.125 | 249.125 | 280.125 | 2021 |
| 2021 | 272.125 | 241.125 | 252.125 | 277.125 | 259.125 | 222.125 | 231.125 | 266.125 | 2021 |
| 2021 | 263.125 | 234.125 | 227.125 | 254.125 | 284.125 | 245.125 | 240.125 | 273.125 | 2021 |
|  | 281.125 | 248.125 | 237.125 | 276.125 | 262.125 | 235.125 | 226.125 | 255.125 | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, magic squares of order 8 are bimagic with bimagic sums given by

$$
\begin{aligned}
{\left[S b_{8 \times 8}: S_{8 \times 8}:=21\right] } & :=2785.125 \\
{\left[S b_{8 \times 8}: S_{8 \times 8}:=441\right] } & =27040.125 \\
{\left[S b_{8 \times 8}: S_{8 \times 8}:=2021\right] } & =513285.125
\end{aligned}
$$

### 1.2 Block-Wise Magic Squares of Order 9

Below are block-wise magic squares of order 9 in two different ways giving magic sums as $21,21^{2}$ and 2021 . The blocks are of orders 3. In first case, the blocks of order 3 are semi-magic squares with equal semi-magic sums. In this case, the magic square of order 9 is pandiagonal In the second case, the sum of the nine entries are the same as of magic square. In this case, the magic square is bimagic square.

### 1.2.1 First Type

Below are block-wise construction of magic square of order 9 resulting in magic sums $21,21^{2}$ and 2021. The blocks of order 3 are semi-magic squares of equal sem-magic sums.

|  | pan | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -16 2/3 | $321 / 3$ | -8 2/3 | -11 2/3 | $251 / 3$ | -6 2/3 | -18 2/3 | $301 / 3$ | -4 2/3 | 21 |
| 21 | -3 2/3 | -172/3 | $281 / 3$ | -10 2/3 | -15 2/3 | $331 / 3$ | -5 2/3 | -13 2/3 | $261 / 3$ | 21 |
| 21 | $271 / 3$ | -72/3 | -12 2/3 | $291 / 3$ | -2 2/3 | -19 2/3 | $311 / 3$ | -9 2/3 | -14 2/3 | 21 |
| 21 | 11/3 | -30 2/3 | $361 / 3$ | $61 / 3$ | -37 2/3 | $381 / 3$ | -2/3 | -32 2/3 | $401 / 3$ | 21 |
| 21 | $411 / 3$ | 1/3 | -34 2/3 | $341 / 3$ | 21/3 | -29 2/3 | $391 / 3$ | $41 / 3$ | -36 2/3 | 21 |
| 21 | -35 2/3 | $371 / 3$ | $51 / 3$ | -33 2/3 | $421 / 3$ | -1 2/3 | -31 2/3 | $351 / 3$ | $31 / 3$ | 21 |
| 21 | $191 / 3$ | $141 / 3$ | -26 2/3 | $241 / 3$ | $71 / 3$ | -24 2/3 | $171 / 3$ | $121 / 3$ | -22 2/3 | 21 |
| 21 | -21 2/3 | $181 / 3$ | $101 / 3$ | -28 2/3 | $201 / 3$ | 151/3 | -23 2/3 | $221 / 3$ | $81 / 3$ | 21 |
|  | $91 / 3$ | -25 2/3 | $231 / 3$ | $111 / 3$ | -20 2/3 | $161 / 3$ | $131 / 3$ | -27 2/3 | $211 / 3$ | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | 30 | 79 | 38 | 35 | 72 | 40 | 28 | 77 | 42 | 441 |
| 441 | 43 | 29 | 75 | 36 | 31 | 80 | 41 | 33 | 73 | 441 |
| 441 | 74 | 39 | 34 | 76 | 44 | 27 | 78 | 37 | 32 | 441 |
| 441 | 48 | 16 | 83 | 53 | 9 | 85 | 46 | 14 | 87 | 441 |
| 441 | 88 | 47 | 12 | 81 | 49 | 17 | 86 | 51 | 10 | 441 |
| 441 | 11 | 84 | 52 | 13 | 89 | 45 | 15 | 82 | 50 | 441 |
| 441 | 66 | 61 | 20 | 71 | 54 | 22 | 64 | 59 | 24 | 441 |
| 441 | 25 | 65 | 57 | 18 | 67 | 62 | 23 | 69 | 55 | 441 |
|  | 56 | 21 | 70 | 58 | 26 | 63 | 60 | 19 | 68 | 441 |
|  | 441441 |  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | 205 5/9 | 254 5/9 | 213 5/9 | 210 5/9 | 247 5/9 | 215 5/9 | 203 5/9 | 252 5/9 | 217 5/9 | 2021 |
| 2021 | 218 5/9 | 204 5/9 | 250 5/9 | $2115 / 9$ | 206 5/9 | 255 5/9 | 216 5/9 | 208 5/9 | 248 5/9 | 2021 |
| 202 | 249 5/9 | 214 5/9 | 209 | 2515 | 219 5/9 | 202 5/9 | 253 5/9 | 212 5/9 | 2075 | 2021 |
| 202 | 2235 | 1915/9 | 258 | 2285 | 184 5/9 | $2605 / 9$ | 221 5/9 | 189 5/9 | 2625 | 2021 |
| 202 | 2635 | 222 5/9 | $1875 / 9$ | 256 | 224 5/9 | 192 5/9 | 261 5/9 | 226 5/9 | 185 | 2021 |
| 202 | 186 5/9 | 259 5/9 | 227 5/9 | 188 5/9 | 264 5/9 | 220 5/9 | 190 5/9 | $2575 / 9$ | 225 5/9 | 2021 |
| 202 | $2415 / 9$ | 236 5/9 | 195 5/9 | 246 5/9 | 229 5/9 | 197 5/9 | 239 5/9 | $2345 / 9$ | 199 5/9 | 2021 |
| 2021 | 200 5/9 | 240 5/9 | 232 5/9 | 193 5/9 | 242 5/9 | $2375 / 9$ | 198 5/9 | $2445 / 9$ | $2305 / 9$ | 2021 |
|  | $2315 / 9$ | 196 5/9 | 245 5/9 | 233 5/9 | 2015/9 | 238 5/9 | 235 5/9 | 1945/9 | 243 5/9 | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the semi-magic sums of order 3 are given by

$$
\begin{aligned}
{\left[S m_{3 \times 3}: S_{9 \times 9}:=21\right] } & :=7 \\
{\left[S m_{3 \times 3}: S_{9 \times 9}:=441\right] } & :=147 \\
{\left[S m_{3 \times 3}: S_{9 \times 9}:=2021\right] } & :=\frac{2021}{3} .
\end{aligned}
$$

### 1.2.2 Second Type

Below are block-wise construction of magic square of order 9 resulting in magic sums $21,21^{2}$ and 2021 . The sums of nine entries in each block of order 3 is equal to as of magic square of order 9 . The magic square is bimagic square

| mgc |  |  |  |  |  |  |  |  | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -29 2/3 | $321 / 3$ | $131 / 3$ | $361 / 3$ | -9 2/3 | -28 2/3 | $31 / 3$ | -15 2/3 | $191 / 3$ | 21 |
| $391 / 3$ | -6 2/3 | -25 2/3 | $61 / 3$ | -12 2/3 | $221 / 3$ | -35 2/3 | $261 / 3$ | $71 / 3$ | 21 |
| 1/3 | -182/3 | $161 / 3$ | -32 2/3 | $291 / 3$ | $101 / 3$ | $421 / 3$ | -3 2/3 | -22 2/3 | 21 |
| $141 / 3$ | -31 2/3 | $331 / 3$ | -27 2/3 | $341 / 3$ | -8 2/3 | $201 / 3$ | 11/3 | -14 2/3 | 21 |
| -24 2/3 | $371 / 3$ | -5 2/3 | $231 / 3$ | $41 / 3$ | $-112 / 3$ | $81 / 3$ | -37 2/3 | $271 / 3$ | 21 |
| $171 / 3$ | -12/3 | -17 2/3 | $111 / 3$ | -34 2/3 | $301 / 3$ | -21 2/3 | $401 / 3$ | -2 2/3 | 21 |
| $311 / 3$ | 151/3 | -30 2/3 | -10 2/3 | -26 2/3 | $351 / 3$ | -16 2/3 | $211 / 3$ | $21 / 3$ | 21 |
| -7 2/3 | -23 2/3 | $381 / 3$ | -13 2/3 | $241 / 3$ | 51/3 | $251 / 3$ | $91 / 3$ | -36 2/3 | 21 |
| -19 2/3 | $181 / 3$ | -2/3 | $281 / 3$ | 121/3 | -33 2/3 | -4 2/3 | -20 2/3 | $411 / 3$ | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |

mgc

| 17 | 79 | 60 | 83 | 37 | 18 | 50 | 31 | 66 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 86 | 40 | 21 | 53 | 34 | 69 | 11 | 73 | 54 | 441 |
| 47 | 28 | 63 | 14 | 76 | 57 | 89 | 43 | 24 | 441 |
| 61 | 15 | $\mathbf{8 0}$ | 19 | 81 | 38 | 67 | 48 | 32 | 441 |
| 22 | 84 | 41 | 70 | 51 | 35 | 55 | 9 | 74 | 441 |
| 64 | 45 | 29 | 58 | 12 | 77 | 25 | 87 | 44 | 441 |
| 78 | 62 | 16 | 36 | 20 | 82 | 30 | 68 | 49 | 441 |
| 39 | 23 | 85 | 33 | 71 | 52 | 72 | 56 | 10 | 441 |
| 27 | 65 | 46 | 75 | 59 | 13 | 42 | 26 | 88 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


| mgc |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 192 5/9 | 254 5/9 | 235 5/9 | 258 5/9 | 212 5/9 | 193 5/9 | 225 5/9 | 206 5/9 | $2415 / 9$ | 2021 |
| $2615 / 9$ | 215 5/9 | 196 5/9 | 228 5/9 | 209 5/9 | $2445 / 9$ | 186 5/9 | 248 5/9 | 229 5/9 | 2021 |
| 222 5/9 | 203 5/9 | 238 5/9 | 189 5/9 | $2515 / 9$ | $2325 / 9$ | 264 5/9 | 218 5/9 | 199 5/9 | 2021 |
| 236 5/9 | 190 5/9 | 255 5/9 | 1945/9 | $2565 / 9$ | 213 5/9 | 242 5/9 | 223 5/9 | 207 5/9 | 2021 |
| 197 5/9 | 259 5/9 | 216 5/9 | 245 5/9 | 226 5/9 | 210 5/9 | $2305 / 9$ | $1845 / 9$ | 249 5/9 | 2021 |
| 239 5/9 | 220 5/9 | $2045 / 9$ | 233 5/9 | 1875/9 | 252 5/9 | $2005 / 9$ | 262 5/9 | 219 5/9 | 2021 |
| 253 5/9 | 237 5/9 | $1915 / 9$ | 211 5/9 | 195 5/9 | $2575 / 9$ | 205 5/9 | 243 5/9 | 224 5/9 | 2021 |
| 214 5/9 | 198 5/9 | $2605 / 9$ | 208 5/9 | 246 5/9 | 227 5/9 | 247 5/9 | $2315 / 9$ | 185 5/9 | 2021 |
| 202 5/9 | 240 5/9 | 221 5/9 | 250 5/9 | $2345 / 9$ | 1885/9 | 217 5/9 | $2015 / 9$ | 263 5/9 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, magic squares of order 9 are bimagic with bimagic sums given by

$$
\begin{aligned}
{\left[S b_{9 \times 9}: S_{9 \times 9}:=21\right] } & :=4969 \\
{\left[S b_{9 \times 9}: S_{9 \times 9}:=441\right] } & :=26529 \\
{\left[S b_{9 \times 9}: S_{9 \times 9}:=2021\right] } & :=\frac{4128721}{9}
\end{aligned}
$$

In this case, the sum of nine entries of blocks of order 3 is the as of magic square of order 9 .

### 1.3 Block-Bordered Magic Square of Order 10

Below are block-bordered magic squares of order 10 in two different ways giving magic sums $21,21^{2}$ and 2021. One is with pandiagonal blocks of order 4 with equal sum magic squares. The second is bimagic squares of order 8 , where sum of eight entries of each block of order $2 \times 4$ are of equal sum as of magic square of order 8 . The magic squares of order 8 are of following sums:

$$
\begin{aligned}
{\left[S_{8 \times 8}: S_{10 \times 10}:=21\right] } & :=16.8 \\
{\left[S_{8 \times 8}: S_{10 \times 10}:=441\right] } & :=352.8 \\
{\left[S_{8 \times 8}: S_{10 \times 10}:=2021\right] } & :=1616.8 .
\end{aligned}
$$

### 1.3.1 First Type

Below are block-bordered of magic squares of order 10 resulting in magic sums $21,21^{2}$ and 2021 . The blocks of order 4 are pandiagonal magic squares with equal magic sums.

| 42.6 | 37.6 | -32.4 | 35.6 | -30.4 | -34.4 | -44.4 | 49.6 | -46.4 | 43.6 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| -35.4 | -1.4 | 9.6 | -29.4 | 29.6 | -9.4 | 17.6 | -21.4 | 21.6 | 39.6 | 21 |
| 40.6 | -26.4 | 26.6 | 1.6 | 6.6 | -18.4 | 18.6 | -6.4 | 14.6 | -36.4 | 21 |
| -37.4 | 33.6 | -25.4 | 5.6 | -5.4 | 25.6 | -17.4 | 13.6 | -13.4 | 41.6 | 21 |
| 47.6 | 2.6 | -2.4 | 30.6 | -22.4 | 10.6 | -10.4 | 22.6 | -14.4 | -43.4 | 21 |
| -47.4 | -0.4 | 8.6 | -28.4 | 28.6 | -8.4 | 16.6 | -20.4 | 20.6 | 51.6 | 21 |
| 44.6 | -27.4 | 27.6 | 0.6 | 7.6 | -19.4 | 19.6 | -7.4 | 15.6 | -40.4 | 21 |
| -41.4 | 32.6 | -24.4 | 4.6 | -4.4 | 24.6 | -16.4 | 12.6 | -12.4 | 45.6 | 21 |
| 46.6 | 3.6 | -3.4 | 31.6 | -23.4 | 11.6 | -11.4 | 23.6 | -15.4 | -42.4 | 21 |
| -39.4 | -33.4 | 36.6 | -31.4 | 34.6 | 38.6 | 48.6 | -45.4 | 50.6 | -38.4 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| 84.6 | 79.6 | 9.6 | 77.6 | 11.6 | 7.6 | -2.4 | 91.6 | -4.4 | 85.6 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 6.6 | 40.6 | 51.6 | 12.6 | 71.6 | 32.6 | 59.6 | 20.6 | 63.6 | 81.6 | 441 |
| 82.6 | 15.6 | 68.6 | 43.6 | 48.6 | 23.6 | 60.6 | 35.6 | 56.6 | 5.6 | 441 |
| 4.6 | 75.6 | 16.6 | 47.6 | 36.6 | 67.6 | 24.6 | 55.6 | 28.6 | 83.6 | 441 |
| 89.6 | 44.6 | 39.6 | 72.6 | 19.6 | 52.6 | 31.6 | 64.6 | 27.6 | -1.4 | 441 |
| -5.4 | 41.6 | 50.6 | 13.6 | 70.6 | 33.6 | 58.6 | 21.6 | 62.6 | 93.6 | 441 |
| 86.6 | 14.6 | 69.6 | 42.6 | 49.6 | 22.6 | 61.6 | 34.6 | 57.6 | 1.6 | 441 |
| 0.6 | 74.6 | 17.6 | 46.6 | 37.6 | 66.6 | 25.6 | 54.6 | 29.6 | 87.6 | 441 |
| 88.6 | 45.6 | 38.6 | 73.6 | 18.6 | 53.6 | 30.6 | 65.6 | 26.6 | -0.4 | 441 |
| 2.6 | 8.6 | 78.6 | 10.6 | 76.6 | 80.6 | 90.6 | -3.4 | 92.6 | 3.6 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  |  | 2021 |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 242.6 | 237.6 | 167.6 | 235.6 | 169.6 | 165.6 | 155.6 | 249.6 | 153.6 | 243.6 | 2021 |
| 164.6 | 198.6 | 209.6 | 170.6 | 229.6 | 190.6 | 217.6 | 178.6 | 221.6 | 239.6 | 2021 |
| 240.6 | 173.6 | 226.6 | 201.6 | 206.6 | 181.6 | 218.6 | 193.6 | 214.6 | 163.6 | 2021 |
| 162.6 | 233.6 | 174.6 | 205.6 | 194.6 | 225.6 | 182.6 | 213.6 | 186.6 | 241.6 | 2021 |
| 247.6 | 202.6 | 197.6 | 230.6 | 177.6 | 210.6 | 189.6 | 222.6 | 185.6 | 156.6 | 2021 |
| 152.6 | 199.6 | 208.6 | 171.6 | 228.6 | 191.6 | 216.6 | 179.6 | 220.6 | 251.6 | 2021 |
| 244.6 | 172.6 | 227.6 | 200.6 | 207.6 | 180.6 | 219.6 | 192.6 | 215.6 | 159.6 | 2021 |
| 158.6 | 232.6 | 175.6 | 204.6 | 195.6 | 224.6 | 183.6 | 212.6 | 187.6 | 245.6 | 2021 |
| 246.6 | 203.6 | 196.6 | 231.6 | 176.6 | 211.6 | 188.6 | 223.6 | 184.6 | 157.6 | 2021 |
| 160.6 | 166.6 | 236.6 | 168.6 | 234.6 | 238.6 | 248.6 | 154.6 | 250.6 | 161.6 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the blocks of order 4 are pandiagonal magic squares with magic sums:

$$
\begin{gathered}
{\left[S_{4 \times 4}: S_{10 \times 10}:=21\right]:=8.4} \\
{\left[S_{4 \times 4}: S_{10 \times 10}:=441\right]:=176.4} \\
{\left[S_{4 \times 4}: S_{10 \times 10}:=2021\right]:=808.4}
\end{gathered}
$$

### 1.3.2 Second Type

Below are block-bordered of magic squares of order 10 resulting in magic sums $21,21^{2}$ and 2021 . The magic square of order 8 is bimagic square.

|  |  |  |  |  |  |  |  |  |  | 21 |  |  |  |  |  |  |  |  |  |  | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 42.6 | 37.6 | -32.4 | 35.6 | -30.4 | -34.4 | -44.4 | 49.6 | -46.4 | 43.6 | 21 | 84.6 | 79.6 | 9.6 | 77.6 | 11.6 | 7.6 | -2.4 | 91.6 | -4.4 | 85.6 | 441 |
| -35.4 | -14.4 | 10.6 | 5.6 | -25.4 | -3.4 | 31.6 | 24.6 | -12.4 | 39.6 | 21 | 6.6 | 27.6 | 52.6 | 47.6 | 16.6 | 38.6 | 73.6 | 66.6 | 29.6 | 81.6 | 441 |
| 40.6 | -4.4 | 32.6 | 23.6 | -11.4 | -17.4 | 13.6 | 2.6 | -22.4 | -36.4 | 21 | 82.6 | 37.6 | 74.6 | 65.6 | 30.6 | 24.6 | 55.6 | 44.6 | 19.6 | 5.6 | 441 |
| -37.4 | -29.4 | 9.6 | 14.6 | -18.4 | -8.4 | 20.6 | 27.6 | 0.6 | 41.6 | 21 | 4.6 | 12.6 | 51.6 | 56.6 | 23.6 | 33.6 | 62.6 | 69.6 | 42.6 | 83.6 | 441 |
| 47.6 | -7.4 | 19.6 | 28.6 | -0.4 | -26.4 | 6.6 | 17.6 | -21.4 | -43.4 | 21 | 89.6 | 34.6 | 61.6 | 70.6 | 41.6 | 15.6 | 48.6 | 59.6 | 20.6 | -1.4 | 441 |
| -47.4 | 7.6 | -27.4 | -20.4 | 16.6 | 18.6 | -6.4 | -1.4 | 29.6 | 51.6 | 21 | -5.4 | 49.6 | 14.6 | 21.6 | 58.6 | 60.6 | 35.6 | 40.6 | 71.6 | 93.6 | 441 |
| 44.6 | 21.6 | -9.4 | 1.6 | 26.6 | 8.6 | -28.4 | -19.4 | 15.6 | 6. -40.4 | 21 | 86.6 | 63.6 | 32.6 | 43.6 | 68.6 | 50.6 | 13.6 | 22.6 | 57.6 | 1.6 | 441 |
| -41.4 | 12.6 | -16.4 | -23.4 | 3.6 | 33.6 | -5.4 | -10.4 | 22.6 | 45.6 | 21 | 0.6 | 54.6 | 25.6 | 18.6 | 45.6 | 75.6 | 36.6 | 31.6 | 64.6 | 87.6 | 441 |
| 46.6 | 30.6 | -2.4 | -13.4 | 25.6 | 11.6 | -15.4 | -24.4 | 4.6 | - -42.4 | 21 | 88.6 | 72.6 | 39.6 | 28.6 | 67.6 | 53.6 | 26.6 | 17.6 | 46.6 | -0.4 | 441 |
| -39.4 | -33.4 | 36.6 | -31.4 | 34.6 | 38.6 | 48.6 | -45.4 | 50.6 | -38.4 | 21 | 2.6 | 8.6 | 78.6 | 10.6 | 76.6 | 80.6 | 90.6 | -3.4 | 92.6 | 3.6 | 441 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 242.6 | 237.6 | 167.6 | 235.6 | 169.6 | 165.6 | 155.6 | 249.6 | 153.6 | 243.6 | 2021 |  |
| 164.6 | 185.6 | 210.6 | 205.6 | 174.6 | 196.6 | 231.6 | 224.6 | 187.6 | 239.6 | 2021 |  |
| 240.6 | 195.6 | 232.6 | 223.6 | 188.6 | 182.6 | 213.6 | 202.6 | 177.6 | 163.6 | 2021 |  |
| 162.6 | 170.6 | 209.6 | 214.6 | 181.6 | 191.6 | 220.6 | 227.6 | 200.6 | 241.6 | 2021 |  |
| 247.6 | 192.6 | 219.6 | 228.6 | 199.6 | 173.6 | 206.6 | 217.6 | 178.6 | 156.6 | 2021 |  |
| 152.6 | 207.6 | 172.6 | 179.6 | 216.6 | 218.6 | 193.6 | 198.6 | 229.6 | 251.6 | 2021 |  |
| 244.6 | 221.6 | 190.6 | 201.6 | 226.6 | 208.6 | 171.6 | 180.6 | 215.6 | 159.6 | 2021 |  |
| 158.6 | 212.6 | 183.6 | 176.6 | 203.6 | 233.6 | 194.6 | 189.6 | 222.6 | 245.6 | 2021 |  |
| 246.6 | 230.6 | 197.6 | 186.6 | 225.6 | 211.6 | 184.6 | 175.6 | 204.6 | 157.6 | 2021 |  |
| 160.6 | 166.6 | 236.6 | 168.6 | 234.6 | 238.6 | 248.6 | 154.6 | 250.6 | 161.6 | 2021 |  |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |  |

In this case, the sum of eight entries in each block of order $2 \times 4$ is the same as of magic square. Moreover, the magic square of order 8 is bimagic.

### 1.4 Block-Bordered Magic Squares of Order 11

Below are block-bordered magic squares of order 11 in two different ways giving magic sums $21,21^{2}$ and 2021. The blocks are of orders 3. In the first case, the blocks of order 3 are semi-magic squares with equal semi-magic sums. In this case,
the magic square of order 9 is pandiagonal. In the second case, the sum of the nine entries are the same as of magic square of order 9 . In this case, the magic square is bimagic square. The magic sums of magic squares of order 9 are given by

$$
\begin{aligned}
{\left[S_{9 \times 9}: S_{11 \times 11}:=21\right] } & :=21 \times \frac{9}{11}=\frac{189}{11} \\
{\left[S_{9 \times 9}: S_{11 \times 11}:=441\right] } & :=441 \times \frac{9}{11}=\frac{3969}{11} \\
{\left[S_{9 \times 9}: S_{11 \times 11}:=2021\right] } & :=2021 \times \frac{9}{11}=\frac{18189}{11}
\end{aligned}
$$

### 1.4.1 First Type

| -471/11 | -39 1/11 | -411/11 | -431/11 | -451/11 | 53 10/11 | 5410/11 | 56 10/11 | $5810 / 11$ | $6010 / 11$ | -49 1/11 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 61 10/11 | -17 1/11 | $3110 / 11$ | -9 1/11 | -12 1/11 | $2410 / 11$ | -7 1/11 | -19 1/11 | $2910 / 11$ | -51/11 | -58 1/11 | 21 |
| $5910 / 11$ | -4 1/11 | -18 1/11 | 27 10/11 | -11 1/11 | -16 1/11 | 32 10/11 | -6 1/11 | -14 1/11 | $2510 / 11$ | -56 1/11 | 21 |
| 57 | 26 | -8 | -13 1/11 | 28 10/11 | -31/11 | -20 1/11 | $3010 / 11$ | -10 1/11 | -15 1/11 | -54 1/11 | 21 |
| 5510/11 | 10/11 | -31 1/11 | $3510 / 11$ | $510 / 11$ | -38 1/11 | 37 10/11 | -1 1/11 | -33 1/11 | $3910 / 11$ | -52 1/11 | 21 |
| -48 1/11 | $4010 / 1$ | -1/11 | -35 1/11 | $3310 / 11$ | 110/11 | -30 1/11 | $3810 / 11$ | $310 / 11$ | -371/11 | 51 10/11 | 1 |
| -46 1/11 | -36 1/11 | 3610/11 | 410/11 | -34 1/11 | 4110/11 | -2 1/11 | -32 1/11 | 3410/11 | 210/11 | $4910 / 11$ | 21 |
| -44 1/11 | $1810 / 11$ | $1310 / 11$ | -271/11 | 23 10/11 | 610/11 | -25 $1 / 11$ | 1610/11 | 11 10/11 | -23 1/11 | $4710 / 11$ | 21 |
| -42 1/11 | -22 1/11 | 17 10/11 | 910/11 | -29 1/11 | $1910 / 11$ | $1410 / 11$ | -24 1/11 | 21 10/11 | $710 / 11$ | $4510 / 11$ | 21 |
| -40 1/11 | $810 / 11$ | $-261 / 11$ | 22 10/11 | $1010 / 11$ | -21 1/11 | 1510/11 | 1210/11 | -28 1/11 | $2010 / 11$ | $4310 / 11$ | 21 |
| 52 10/11 | $4210 / 11$ | $4410 / 11$ | $4610 / 11$ | $4810 / 11$ | -50 1/11 | -511/11 | -53 1/11 | -55 1/11 | -57 1/11 | $5010 / 11$ | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| -8 | - 10/11 | -210 | -4 10/1 | -6 10/11 | 921 | $931 / 11$ | $951 / 11$ | $971 / 11$ | $991 / 11$ | -10 10/11 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1001 | 211 | 701 | $291 /$ | $261 / 11$ | 63 | 31 | $191 / 11$ | 68 | $331 / 11$ | 1 |  |
| $981 / 11$ | 34 | 201 | 66 | 271 | 22 | 71 | 32 | 24 | $641 / 11$ | -1710/11 |  |
| 96 | 65 | 301 | 25 | 67 | 351 | 18 1/1 | 69 | 28 | $231 / 11$ | -15 |  |
| 94 | 391 | 71 | 74 | 44 | 1/11 | 761 | 37 | 5 | $781 / 11$ | -13 10/11 |  |
| -9 | 79 | 381 | 31 | 721 | 401 | 8 | 771 | 42 | 1 | 90 |  |
| -7 | 21 | 75 | 431 | 4 | 801 | 36 | 6 | 73 | 41 | 88 1/11 |  |
| -5 | 57 | 52 | 11 | $621 / 11$ | 45 | 13 | 55 |  | 15 | 86 |  |
| -3 | 161 | 561 | 48 | $91 / 11$ | 58 | 53 | 14 | 60 | 46 | 84 |  |
| -1 | 47 | 121 | 61 1/1 | $491 / 11$ | 171 | 541 | $511 / 11$ | $101 / 11$ | $591 / 11$ | 82 |  |
| $911 / 1$ | 811 | 831 | 851 | $871 / 11$ | -11 10/11 | -12 10 | -1410 | -16 10/11 | -1810 | 891 |  |
| 441 | 441 | 44 | 441 | 44 | 441 | 441 | 441 | 441 | 44 | 44 |  |


|  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1348 / 11$ | 142 8/11 | $1408 / 11$ | $1388 / 11$ | $1368 / 11$ | $2358 / 11$ | $2368 / 11$ | $2388 / 11$ | $2408 / 11$ | 242 8/11 | 132 8/11 | 2021 |
| $2438 / 11$ | 164 8/11 | $2138 / 11$ | 172 8/11 | 169 8/11 | $2068 / 11$ | $1748 / 11$ | 162 8/11 | $2118 / 11$ | $1768 / 11$ | $1238 / 11$ | 2021 |
| $2418 / 11$ | 177 8/11 | 163 8/11 | $2098 / 11$ | 170 8/11 | $1658 / 11$ | $2148 / 11$ | $1758 / 11$ | 167 8/11 | $2078 / 11$ | $1258 / 11$ | 2021 |
| $2398 / 11$ | $2088 / 11$ | 173 8/11 | 168 8/11 | $2108 / 11$ | 178 8/11 | $1618 / 11$ | 212 8/11 | $1718 / 11$ | $1668 / 11$ | $1278 / 11$ | 2021 |
| $2378 / 11$ | $1828 / 11$ | 150 8/11 | 217 8/11 | 187 8/11 | $1438 / 11$ | 219 8/11 | $1808 / 11$ | 148 8/11 | $2218 / 11$ | 129 8/11 | 2021 |
| $1338 / 11$ | 222 8/11 | $1818 / 11$ | $1468 / 11$ | 215 8/11 | $1838 / 11$ | 151 8/11 | $2208 / 11$ | $1858 / 11$ | $1448 / 11$ | $2338 / 11$ | 2021 |
| $1358 / 11$ | $1458 / 11$ | 218 8/11 | $1868 / 11$ | $1478 / 11$ | $2238 / 11$ | 179 8/11 | $1498 / 11$ | $2168 / 11$ | $1848 / 11$ | $2318 / 11$ | 2021 |
| $1378 / 11$ | $2008 / 11$ | 195 8/11 | 154 8/11 | $2058 / 11$ | $1888 / 11$ | $1568 / 11$ | $1988 / 11$ | 193 8/11 | 158 8/11 | $2298 / 11$ | 2021 |
| $1398 / 11$ | 159 8/11 | $1998 / 11$ | $1918 / 11$ | 152 8/11 | 201 8/11 | $1968 / 11$ | $1578 / 11$ | 203 8/11 | 1898111 | 227 8/11 | 2021 |
| $1418 / 11$ | $1908 / 11$ | 155 8/11 | $2048 / 11$ | 192 8/11 | $1608 / 11$ | 197 8/11 | $1948 / 11$ | 153 8/11 | $2028 / 11$ | $2258 / 11$ | 2021 |
| 234 8/ | 224 8/11 | $2268 / 11$ | $2288 / 11$ | $2308 / 11$ | $1318 / 11$ | $1308 / 11$ | 128 8/11 | $1268 / 11$ | 124 8/11 | $2328 / 11$ | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the blocks of order 3 are semi-magic squares with sums given by

$$
\begin{gathered}
{\left[\operatorname{Sm}_{3 \times 3}: S_{11 \times 11}:=21\right]:=\frac{63}{11}} \\
{\left[\operatorname{Sm}_{3 \times 3}: S_{11 \times 11}:=441\right]:=\frac{1323}{11}} \\
{\left[\operatorname{Sm}_{3 \times 3}: S_{11 \times 11}:=2021\right]:=\frac{6063}{11}}
\end{gathered}
$$

### 1.4.2 Second Type

21

| -47 | -39 | -41 | -43 |  | 53 | $5410 / 11$ | 56 10/11 | 58 10/11 | $6010 / 11$ | 11 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $6110 / 11$ | -30 1/11 | $3110 / 11$ | $1210 / 11$ | 3510/11 | -10 1/11 | -291/11 | 210/11 | $-161 / 11$ | $1810 / 11$ | -581/11 | 21 |
| 59 | $3810 /$ | -71/11 | -26 | 510 | -13 1/11 | $2110 / 11$ | -36 1/11 | 25 10/11 | $610 / 11$ | -56 | 21 |
| 57 | -1/11 | -19 | 1510 | -33 1 | 28 | 910/11 | 4110 | -4 1/11 | -23 1/11 | -54 | 21 |
| 55 | 1310 | -32 1 | 3210 | -28 | 3310 | -9 | 19 | 1 | -15 1/11 | -5 | 21 |
| -48 | -25 1 | 3610 | -61/1/ | 2210 | $310 / 1$ | -12 1 | $710 / 11$ | -38 1/11 | $2610 / 11$ | 51 | 2 |
| -46 | 1610 | -2 1/1 | -18 1 | 1010 | -35 | 2910 | -22 | $3910 / 11$ | -3 1/11 | 49 | 21 |
| -44 | $3010 /$ | 1410 | -31 | $-111 /$ | -27 1/11 | $3410 / 11$ | -171/11 | $2010 / 11$ | 110/11 | $4710 / 11$ | 21 |
| -42 | -8 1/11 | -24 1 | 37 10/1 | -14 1/11 | $2310 / 11$ | 410 | $2410 / 11$ | $810 / 11$ | -37 | $4510 / 11$ | 21 |
| -40 | -20 1 | 1710 | -1 1/11 | $2710 / 1$ | $1110 / 11$ | -34 1 | -5 1/11 | -21 1 | 4010 | $4310 / 11$ | 21 |
| $5210 / 11$ | 42 10/11 | $4410 / 11$ | $4610 / 11$ | $4810 / 11$ | -50 1/11 | -51 1/11 | -53 1 | -55 | -57 | $5010 / 11$ | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 2 |


| -810/11 | - 10/11 | -2 10/11 | -4 10/11 | -610/11 | $921 / 11$ | 931/11 | $951 / 11$ | $971 / 11$ | $991 / 11$ | -10 10/11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1001 / 11$ | $81 / 11$ | $701 / 11$ | $511 / 11$ | $741 / 11$ | $281 / 11$ | $91 / 11$ | $411 / 11$ | $221 / 11$ | $571 / 11$ | -19 10 |
| $981 / 11$ | $771 / 11$ | $311 / 11$ | $121 / 11$ | $441 / 11$ | $251 / 11$ | 60 1/11 | $21 / 11$ | $641 / 11$ | $451 / 11$ | -17 |
| $961 / 11$ | $381 / 11$ | 19 1/11 | $541 / 11$ | $51 / 11$ | $671 / 11$ | $481 / 11$ | $801 / 11$ | $341 / 11$ | $151 / 11$ | -151011 |
| $941 / 11$ | $521 / 11$ | $61 / 11$ | $711 / 11$ | $101 / 11$ | $721 / 11$ | $291 / 11$ | 581 | $391 / 11$ | $231 / 11$ | - |
| -9 10/11 | $131 / 1$ | 751 | 321 | $611 / 11$ | 421 | $261 / 11$ | 46 | 1/11 | $651 / 11$ | $901 / 1$ |
| -7 10/11 | $551 / 11$ | $361 / 11$ | $201 / 11$ | 49 1/11 | $31 / 11$ | 68 1/11 | $161 / 11$ | $781 / 11$ | $351 / 11$ | 881 |
| -510/11 | $691 / 11$ | $531 / 11$ | $71 / 11$ | $271 / 11$ | $111 / 11$ | $731 / 11$ | 211 | $591 / 11$ | $401 / 11$ | 86 |
| -310/11 | $301 / 11$ | $141 / 11$ | $761 / 11$ | 24 1/11 | 62 1/11 | $431 / 11$ | $631 / 11$ | $471 / 11$ | $11 / 11$ | $841 /$ |
| -110/11 | $181 / 11$ | $561 / 11$ | $371 / 11$ | $661 / 11$ | $501 / 11$ | $41 / 11$ | $331 / 11$ | $171 / 11$ | $791 / 11$ | $821 /$ |
| $911 / 11$ | $811 / 11$ | $831 / 11$ | $851 / 11$ | $871 / 11$ | -11 10/11 | -12 10/11 | -1410/11 | -16 10/11 | -1810/11 | $891 / 11$ |


|  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1348 / 11$ | 142 8/11 | $1408 / 11$ | $1388 / 11$ | $1368 / 11$ | $2358 / 11$ | $2368 / 11$ | $2388 / 11$ | 240 8/11 | 242 8/11 | 132 8/11 | 2021 |
| 243 8/11 | $1518 / 11$ | $2138 / 11$ | 194 8/11 | 217 8/11 | 171 8/11 | 152 8/11 | $1848 / 11$ | $1658 / 11$ | $2008 / 11$ | 123 8/11 | 2021 |
| $2418 / 11$ | $2208 / 11$ | 174 8/11 | $1558 / 11$ | $1878 / 11$ | 168 8/11 | 203 8/11 | $1458 / 11$ | 207 8/11 | $1888 / 11$ | 125 8/11 | 2021 |
| 239 8/11 | $1818 / 11$ | 162 8/11 | 197 8/11 | 148 8/11 | 210 8/11 | 191 8/11 | $2238 / 11$ | 177 8/11 | 158 8/11 | 127 8/11 | 2021 |
| 237 8/11 | 195 8/11 | $1498 / 11$ | 214 8/11 | 153 8/11 | 215 8/11 | 172 8/11 | $2018 / 11$ | $1828 / 11$ | 166 8/11 | 129 8/11 | 2021 |
| $1338 / 11$ | 156 8/11 | 218 8/11 | 175 8/11 | 204 8/11 | 185 8/11 | 169 8/11 | $1898 / 11$ | $1438 / 11$ | 208 8/11 | $2338 / 11$ | 2021 |
| $1358 / 11$ | 198 8/11 | 179 8/11 | 163 8/11 | 192 8/11 | 146 8/11 | 211 8/11 | $1598 / 11$ | $2218 / 11$ | 178 8/11 | $2318 / 11$ | 2021 |
| $1378 / 11$ | 212 8/11 | $1968 / 11$ | 150 8/11 | $1708 / 11$ | 154 8/11 | $2168 / 11$ | 164 8/11 | $2028 / 11$ | $1838 / 11$ | $2298 / 11$ | 2021 |
| $1398 / 11$ | 173 8/11 | 157 8/11 | 219 8/11 | 167 8/11 | $2058 / 11$ | $1868 / 11$ | $2068 / 11$ | $1908 / 11$ | 144 8/11 | 227 8/11 | 2021 |
| $1418 / 11$ | $1618 / 11$ | $1998 / 11$ | $1808 / 11$ | $2098 / 11$ | 193 8/11 | $1478 / 11$ | $1768 / 11$ | $1608 / 11$ | 222 8/11 | 225 8/11 | 2021 |
| 234 8/11 | $2248 / 11$ | 226 8/11 | $2288 / 11$ | $2308 / 11$ | $1318 / 11$ | $1308 / 11$ | $1288 / 11$ | $1268 / 11$ | 124 8/11 | $2328 / 11$ | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the sum of nine entries of blocks of order 3 is the as of magic square of order 9 .

### 1.5 Block-Wise Magic Squares of Order 12

Below are block-wise magic squares of order 12 in three different ways giving magic sums as $21,21^{2}$ and 2021. The blocks are of orders 3,4 and 6 . The blocks of order are magic squares of order 3 with different magic sums. The blocks of order 4 are pandiagonal magic squares with equal magic sums. The blocks of order 6 are magic squares with equal magic sums.

### 1.5.1 First Type

|  | pan | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 27.25 | -69.75 | -19.75 | 1.25 | -48.75 | 48.25 | 7.25 | -42.75 | 54.25 | 69.25 | -27.75 | 22.25 | 21 |
| 21 | -67.75 | -20.75 | 26.25 | 47.25 | 0.25 | -46.75 | 53.25 | 6.25 | -40.75 | -25.75 | 21.25 | 68.25 | 21 |
| 21 | -21.75 | 28.25 | -68.75 | -47.75 | 49.25 | -0.75 | -41.75 | 55.25 | 5.25 | 20.25 | 70.25 | -26.75 | 21 |
| 21 | 60.25 | -36.75 | 13.25 | 16.25 | -33.75 | 63.25 | -13.75 | -63.75 | 33.25 | 42.25 | -54.75 | -4.75 | 21 |
| 21 | -34.75 | 12.25 | 59.25 | 62.25 | 15.25 | -31.75 | 32.25 | -14.75 | -61.75 | -52.75 | -5.75 | 41.25 | 21 |
| 21 | 11.25 | 61.25 | -35.75 | -32.75 | 64.25 | 14.25 | -62.75 | 34.25 | -15.75 | -6.75 | 43.25 | -53.75 | 21 |
| 21 | -3.75 | 46.25 | -50.75 | -65.75 | 31.25 | -18.75 | -23.75 | 73.25 | 23.25 | 2.25 | 52.25 | -44.75 | 21 |
| 21 | -49.75 | -2.75 | 44.25 | 29.25 | -17.75 | -64.75 | 71.25 | 24.25 | -22.75 | -43.75 | 3.25 | 50.25 | 21 |
| 21 | 45.25 | -51.75 | -1.75 | -16.75 | -66.75 | 30.25 | 25.25 | -24.75 | 72.25 | 51.25 | -45.75 | 4.25 | 21 |
| 21 | 17.25 | 67.25 | -29.75 | -38.75 | 58.25 | 8.25 | -56.75 | 40.25 | -9.75 | -12.75 | 37.25 | -59.75 | 21 |
| 21 | -28.75 | 18.25 | 65.25 | 56.25 | 9.25 | -37.75 | 38.25 | -8.75 | -55.75 | -58.75 | -11.75 | 35.25 | 21 |
|  | 66.25 | -30.75 | 19.25 | 10.25 | -39.75 | 57.25 | -7.75 | -57.75 | 39.25 | 36.25 | -60.75 | -10.75 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | 62.25 | -34.75 | 15.25 | 36.25 | -13.75 | 83.25 | 42.25 | -7.75 | 89.25 | 104.25 | 7.25 | 57.25 | 441 |
| 441 | -32.75 | 14.25 | 61.25 | 82.25 | 35.25 | -11.75 | 88.25 | 41.25 | -5.75 | 9.25 | 56.25 | 103.25 | 441 |
| 441 | 13.25 | 63.25 | -33.75 | -12.75 | 84.25 | 34.25 | -6.75 | 90.25 | 40.25 | 55.25 | 105.25 | 8.25 | 441 |
| 441 | 95.25 | -1.75 | 48.25 | 51.25 | 1.25 | 98.25 | 21.25 | -28.75 | 68.25 | 77.25 | -19.75 | 30.25 | 441 |
| 441 | 0.25 | 47.25 | 94.25 | 97.25 | 50.25 | 3.25 | 67.25 | 20.25 | -26.75 | -17.75 | 29.25 | 76.25 | 441 |
| 441 | 46.25 | 96.25 | -0.75 | 2.25 | 99.25 | 49.25 | -27.75 | 69.25 | 19.25 | 28.25 | 78.25 | -18.75 | 441 |
| 441 | 31.25 | 81.25 | -15.75 | -30.75 | 66.25 | 16.25 | 11.25 | 108.25 | 58.25 | 37.25 | 87.25 | -9.75 | 441 |
| 441 | -14.75 | 32.25 | 79.25 | 64.25 | 17.25 | -29.75 | 106.25 | 59.25 | 12.25 | -8.75 | 38.25 | 85.25 | 441 |
| 441 | 80.25 | -16.75 | 33.25 | 18.25 | -31.75 | 65.25 | 60.25 | 10.25 | 107.25 | 86.25 | -10.75 | 39.25 | 441 |
| 441 | 52.25 | 102.25 | 5.25 | -3.75 | 93.25 | 43.25 | -21.75 | 75.25 | 25.25 | 22.25 | 72.25 | -24.75 | 441 |
| 441 | 6.25 | 53.25 | 100.25 | 91.25 | 44.25 | -2.75 | 73.25 | 26.25 | -20.75 | -23.75 | 23.25 | 70.25 | 441 |
|  | 101.25 | 4.25 | 54.25 | 45.25 | -4.75 | 92.25 | 27.25 | -22.75 | 74.25 | 71.25 | -25.75 | 24.25 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | $19311 / 12$ | 9611/12 | $14611 / 12$ | $16711 / 12$ | $11711 / 12$ | 21411/12 | $17311 / 12$ | 123 11/12 | $22011 / 12$ | 235 11/12 | $13811 / 12$ | 188 11/12 | 2021 |
| 2021 | 9811/12 | 14511/12 | 192 11/12 | $21311 / 12$ | 166 11/12 | $11911 / 12$ | $21911 / 12$ | 172 11/12 | $12511 / 12$ | 140 11/12 | $18711 / 12$ | 234 11/12 | 2021 |
| 2021 | 144 11/12 | 194 11/12 | 9711/12 | $11811 / 12$ | 215 11/12 | 165 11/12 | $12411 / 12$ | $22111 / 12$ | 17111/12 | 186 11/12 | 236 11/12 | $13911 / 12$ | 2021 |
| 2021 | $22611 / 12$ | 129 11/12 | 179 11/12 | 182 11/12 | 132 11/12 | $22911 / 12$ | 152 11/12 | 102 11/12 | $19911 / 12$ | $20811 / 12$ | $11111 / 12$ | $16111 / 12$ | 2021 |
| 2021 | $13111 / 12$ | $17811 / 12$ | $22511 / 12$ | $22811 / 12$ | $18111 / 12$ | 134 11/12 | $19811 / 12$ | $15111 / 12$ | $10411 / 12$ | $11311 / 12$ | 160 11/12 | $20711 / 12$ | 2021 |
| 2021 | $17711 / 12$ | $22711 / 12$ | $13011 / 12$ | $13311 / 12$ | $23011 / 12$ | 180 11/12 | $10311 / 12$ | 200 11/12 | $15011 / 12$ | 159 11/12 | $20911 / 12$ | 112 11/12 | 2021 |
| 20 | 162 11/12 | 212 11/12 | $11511 / 12$ | $10011 / 12$ | $19711 / 12$ | $14711 / 12$ | 142 11/12 | $23911 / 12$ | $18911 / 12$ | 168 11/12 | $21811 / 12$ | $12111 / 12$ | 2021 |
| 2021 | $11611 / 12$ | 163 11/12 | $21011 / 12$ | $19511 / 12$ | 14811/12 | $10111 / 12$ | $23711 / 12$ | $19011 / 12$ | $143111 / 12$ | 122 11/12 | 169 11/12 | 216 11/12 | 2021 |
| 2021 | $21111 / 12$ | 11411/12 | $16411 / 12$ | $14911 / 12$ | $9911 / 12$ | 196 11/12 | $19111 / 12$ | 14111/12 | $23811 / 12$ | $21711 / 12$ | $12011 / 12$ | $17011 / 12$ | 2021 |
| 2021 | 183 11/12 | $23311 / 12$ | $13611 / 12$ | $12711 / 12$ | $22411 / 12$ | 174 11/12 | 109 11/12 | 206 11/12 | 156 11/12 | $15311 / 12$ | $20311 / 12$ | $10611 / 12$ | 2021 |
| 2021 | $13711 / 12$ | 184 11/12 | $23111 / 12$ | 222 11/12 | 175 11/12 | 128 11/12 | 204 11/12 | $15711 / 12$ | $11011 / 12$ | $10711 / 12$ | 154 11/12 | $20111 / 12$ | 2021 |
|  | $23211 / 12$ | $13511 / 12$ | $18511 / 12$ | $17611 / 12$ | $12611 / 12$ | 223 11/12 | $15811 / 12$ | 108 11/12 | $20511 / 12$ | 202 11/12 | $10511 / 12$ | $15511 / 12$ | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, all the blocks of order 3 are magic squares with different magic sums.

### 1.5.2 Second Type

|  | pan | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -15.75 | 37.25 | -69.75 | 55.25 | -14.75 | 36.25 | -68.75 | 54.25 | -13.75 | 35.25 | -67.75 | 53.25 | 21 |
| 21 | -52.75 | 38.25 | 1.25 | 20.25 | -53.75 | 39.25 | 0.25 | 21.25 | -54.75 | 40.25 | -0.75 | 22.25 | 21 |
| 21 | 73.25 | -51.75 | 19.25 | -33.75 | 72.25 | -50.75 | 18.25 | -32.75 | 71.25 | -49.75 | 17.25 | -31.75 | 21 |
| 21 | 2.25 | -16.75 | 56.25 | -34.75 | 3.25 | -17.75 | 57.25 | -35.75 | 4.25 | -18.75 | 58.25 | -36.75 | 21 |
| 21 | -12.75 | 34.25 | -66.75 | 52.25 | -11.75 | 33.25 | -65.75 | 51.25 | -10.75 | 32.25 | -64.75 | 50.25 | 21 |
| 21 | -55.75 | 41.25 | -1.75 | 23.25 | -56.75 | 42.25 | -2.75 | 24.25 | -57.75 | 43.25 | -3.75 | 25.25 | 21 |
| 21 | 70.25 | -48.75 | 16.25 | -30.75 | 69.25 | -47.75 | 15.25 | -29.75 | 68.25 | -46.75 | 14.25 | -28.75 | 21 |
| 21 | 5.25 | -19.75 | 59.25 | -37.75 | 6.25 | -20.75 | 60.25 | -38.75 | 7.25 | -21.75 | 61.25 | -39.75 | 21 |
| 21 | -9.75 | 31.25 | -63.75 | 49.25 | -8.75 | 30.25 | -62.75 | 48.25 | -7.75 | 29.25 | -61.75 | 47.25 | 21 |
| 21 | -58.75 | 44.25 | -4.75 | 26.25 | -59.75 | 45.25 | -5.75 | 27.25 | -60.75 | 46.25 | -6.75 | 28.25 | 21 |
| 21 | 67.25 | -45.75 | 13.25 | -27.75 | 66.25 | -44.75 | 12.25 | -26.75 | 65.25 | -43.75 | 11.25 | -25.75 | 21 |
|  | 8.25 | -22.75 | 62.25 | -40.75 | 9.25 | -23.75 | 63.25 | -41.75 | 10.25 | -24.75 | 64.25 | -42.75 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | 19.25 | 72.25 | -34.75 | 90.25 | 20.25 | 71.25 | -33.75 | 89.25 | 21.25 | 70.25 | -32.75 | 88.25 | 441 |
| 441 | -17.75 | 73.25 | 36.25 | 55.25 | -18.75 | 74.25 | 35.25 | 56.25 | -19.75 | 75.25 | 34.25 | 57.25 | 441 |
| 441 | 108.25 | -16.75 | 54.25 | 1.25 | 107.25 | -15.75 | 53.25 | 2.25 | 106.25 | -14.75 | 52.25 | 3.25 | 441 |
| 441 | 37.25 | 18.25 | 91.25 | 0.25 | 38.25 | 17.25 | 92.25 | -0.75 | 39.25 | 16.25 | 93.25 | -1.75 | 441 |
| 441 | 22.25 | 69.25 | -31.75 | 87.25 | 23.25 | 68.25 | -30.75 | 86.25 | 24.25 | 67.25 | -29.75 | 85.25 | 441 |
| 441 | -20.75 | 76.25 | 33.25 | 58.25 | -21.75 | 77.25 | 32.25 | 59.25 | -22.75 | 78.25 | 31.25 | 60.25 | 441 |
| 441 | 105.25 | -13.75 | 51.25 | 4.25 | 104.25 | -12.75 | 50.25 | 5.25 | 103.25 | -11.75 | 49.25 | 6.25 | 441 |
| 441 | 40.25 | 15.25 | 94.25 | -2.75 | 41.25 | 14.25 | 95.25 | -3.75 | 42.25 | 13.25 | 96.25 | -4.75 | 441 |
| 441 | 25.25 | 66.25 | -28.75 | 84.25 | 26.25 | 65.25 | -27.75 | 83.25 | 27.25 | 64.25 | -26.75 | 82.25 | 441 |
| 441 | -23.75 | 79.25 | 30.25 | 61.25 | -24.75 | 80.25 | 29.25 | 62.25 | -25.75 | 81.25 | 28.25 | 63.25 | 441 |
| 441 | 102.25 | -10.75 | 48.25 | 7.25 | 101.25 | -9.75 | 47.25 | 8.25 | 100.25 | -8.75 | 46.25 | 9.25 | 441 |
|  | 43.25 | 12.25 | 97.25 | -5.75 | 44.25 | 11.25 | 98.25 | -6.75 | 45.25 | 10.25 | 99.25 | -7.75 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | $15011 / 12$ | $20311 / 12$ | $9611 / 12$ | $22111 / 12$ | $15111 / 12$ | $20211 / 12$ | $9711 / 12$ | $22011 / 12$ | $15211 / 12$ | $20111 / 12$ | $9811 / 12$ | $21911 / 12$ | 2021 |
| 2021 | $11311 / 12$ | 204 11/12 | $16711 / 12$ | $18611 / 12$ | $11211 / 12$ | $20511 / 12$ | $16611 / 12$ | 18711/12 | $11111 / 12$ | $20611 / 12$ | 165 11/12 | 188 11/12 | 2021 |
| 2021 | $23911 / 12$ | 114 11/12 | 185 11/12 | $13211 / 12$ | 238 11/12 | $11511 / 12$ | $18411 / 12$ | 133 11/12 | $23711 / 12$ | $11611 / 12$ | $183111 / 12$ | $13411 / 12$ | 2021 |
| 2021 | 168 11/12 | $14911 / 12$ | $22211 / 12$ | $13111 / 12$ | $16911 / 12$ | $14811 / 12$ | $22311 / 12$ | $13011 / 12$ | $17011 / 12$ | $14711 / 12$ | $22411 / 12$ | $12911 / 12$ | 2021 |
| 2021 | $15311 / 12$ | $20011 / 12$ | $9911 / 12$ | $21811 / 12$ | $15411 / 12$ | $19911 / 12$ | $10011 / 12$ | $21711 / 12$ | $15511 / 12$ | $19811 / 12$ | $10111 / 12$ | $21611 / 12$ | 2021 |
| 2021 | $11011 / 12$ | $20711 / 12$ | 164 11/12 | $18911 / 12$ | 109 11/12 | $20811 / 12$ | $16311 / 12$ | $19011 / 12$ | 108 11/12 | $20911 / 12$ | $16211 / 12$ | 19111/12 | 2021 |
| 2021 | $23611 / 12$ | $11711 / 12$ | 182 11/12 | 135 11/12 | $23511 / 12$ | $11811 / 12$ | $18111 / 12$ | 136 11/12 | 234 11/12 | $11911 / 12$ | $18011 / 12$ | 137 11/12 | 2021 |
| 2021 | $17111 / 12$ | $14611 / 12$ | $22511 / 12$ | 128 11/12 | 172 11/12 | $14511 / 12$ | $22611 / 12$ | $12711 / 12$ | $17311 / 12$ | $14411 / 12$ | $22711 / 12$ | $12611 / 12$ | 2021 |
| 2021 | $15611 / 12$ | $19711 / 12$ | $10211 / 12$ | $21511 / 12$ | $15711 / 12$ | $19611 / 12$ | $10311 / 12$ | 21411/12 | 158 11/12 | $19511 / 12$ | $10411 / 12$ | $213111 / 12$ | 2021 |
| 2021 | $10711 / 12$ | $21011 / 12$ | $16111 / 12$ | $19211 / 12$ | $10611 / 12$ | $21111 / 12$ | $16011 / 12$ | $19311 / 12$ | $10511 / 12$ | $21211 / 12$ | $15911 / 12$ | $194111 / 12$ | 2021 |
| 2021 | $23311 / 12$ | $12011 / 12$ | $17911 / 12$ | $13811 / 12$ | $23211 / 12$ | $12111 / 12$ | $17811 / 12$ | $13911 / 12$ | $23111 / 12$ | $12211 / 12$ | $17711 / 12$ | $14011 / 12$ | 2021 |
|  | $17411 / 12$ | $14311 / 12$ | $22811 / 12$ | $12511 / 12$ | $17511 / 12$ | $14211 / 12$ | $22911 / 12$ | $12411 / 12$ | $17611 / 12$ | $14111 / 12$ | $23011 / 12$ | $12311 / 12$ | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case all the blocks of order 4 are pandiagonal magic squares with equal magic sums given by

$$
\begin{gathered}
{\left[S_{4 \times 4}: S_{12 \times 12}:=21\right]:=7} \\
{\left[S_{4 \times 4}: S_{12 \times 12}:=441\right]:=147} \\
{\left[S_{4 \times 4}: S_{12 \times 12}:=2021\right]:=\frac{2021}{3} .}
\end{gathered}
$$

### 1.5.3 Third Type

|  | mgc |  |  |  |  |  |  |  |  |  |  |  | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -69.75 | 66.25 | 65.25 | 58.25 | -62.75 | -46.75 | -68.75 | 67.25 | 64.25 | 59.25 | -63.75 | -47.75 | 21 |
|  | 49.25 | -38.75 | 41.25 | -37.75 | -29.75 | 26.25 | 48.25 | -39.75 | 40.25 | -36.75 | -28.75 | 27.25 | 21 |
|  | 25.25 | 18.25 | -13.75 | -6.75 | 9.25 | -21.75 | 24.25 | 19.25 | -12.75 | -7.75 | 8.25 | -20.75 | 21 |
|  | 1.25 | -14.75 | 10.25 | 17.25 | -5.75 | 2.25 | 0.25 | -15.75 | 11.25 | 16.25 | -4.75 | 3.25 | 21 |
|  | -45.75 | 33.25 | -30.75 | 34.25 | 42.25 | -22.75 | -44.75 | 32.25 | -31.75 | 35.25 | 43.25 | -23.75 | 21 |
|  | 50.25 | -53.75 | -61.75 | -54.75 | 57.25 | 73.25 | 51.25 | -52.75 | -60.75 | -55.75 | 56.25 | 72.25 | 21 |
|  | -67.75 | 68.25 | 63.25 | 60.25 | -64.75 | -48.75 | -66.75 | 69.25 | 62.25 | 61.25 | -65.75 | -49.75 | 21 |
|  | 47.25 | -40.75 | 39.25 | -35.75 | -27.75 | 28.25 | 46.25 | -41.75 | 38.25 | -34.75 | -26.75 | 29.25 | 21 |
|  | 23.25 | 20.25 | -11.75 | -8.75 | 7.25 | -19.75 | 22.25 | 21.25 | -10.75 | -9.75 | 6.25 | -18.75 | 21 |
|  | -0.75 | -16.75 | 12.25 | 15.25 | -3.75 | 4.25 | -1.75 | -17.75 | 13.25 | 14.25 | -2.75 | 5.25 | 21 |
|  | -43.75 | 31.25 | -32.75 | 36.25 | 44.25 | -24.75 | -42.75 | 30.25 | -33.75 | 37.25 | 45.25 | -25.75 | 21 |
|  | 52.25 | -51.75 | -59.75 | -56.75 | 55.25 | 71.25 | 53.25 | -50.75 | -58.75 | -57.75 | 54.25 | 70.25 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | mgc |  |  |  |  |  |  |  |  |  |  |  | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -34.75 | 101.25 | 100.25 | 93.25 | -27.75 | -11.75 | -33.75 | 102.25 | 99.25 | 94.25 | -28.75 | -12.75 | 441 |
|  | 84.25 | -3.75 | 76.25 | -2.75 | 5.25 | 61.25 | 83.25 | -4.75 | 75.25 | -1.75 | 6.25 | 62.25 | 441 |
|  | 60.25 | 53.25 | 21.25 | 28.25 | 44.25 | 13.25 | 59.25 | 54.25 | 22.25 | 27.25 | 43.25 | 14.25 | 441 |
|  | 36.25 | 20.25 | 45.25 | 52.25 | 29.25 | 37.25 | 35.25 | 19.25 | 46.25 | 51.25 | 30.25 | 38.25 | 441 |
|  | -10.75 | 68.25 | 4.25 | 69.25 | 77.25 | 12.25 | -9.75 | 67.25 | 3.25 | 70.25 | 78.25 | 11.25 | 441 |
|  | 85.25 | -18.75 | -26.75 | -19.75 | 92.25 | 108.25 | 86.25 | -17.75 | -25.75 | -20.75 | 91.25 | 107.25 | 441 |
|  | -32.75 | 103.25 | 98.25 | 95.25 | -29.75 | -13.75 | -31.75 | 104.25 | 97.25 | 96.25 | -30.75 | -14.75 | 441 |
|  | 82.25 | -5.75 | 74.25 | -0.75 | 7.25 | 63.25 | 81.25 | -6.75 | 73.25 | 0.25 | 8.25 | 64.25 | 441 |
|  | 58.25 | 55.25 | 23.25 | 26.25 | 42.25 | 15.25 | 57.25 | 56.25 | 24.25 | 25.25 | 41.25 | 16.25 | 441 |
|  | 34.25 | 18.25 | 47.25 | 50.25 | 31.25 | 39.25 | 33.25 | 17.25 | 48.25 | 49.25 | 32.25 | 40.25 | 441 |
|  | -8.75 | 66.25 | 2.25 | 71.25 | 79.25 | 10.25 | -7.75 | 65.25 | 1.25 | 72.25 | 80.25 | 9.25 | 441 |
|  | 87.25 | -16.75 | -24.75 | -21.75 | 90.25 | 106.25 | 88.25 | -15.75 | -23.75 | -22.75 | 89.25 | 105.25 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


| mgc |  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $9611 / 12$ | $23211 / 12$ | $23111 / 12$ | 224 11/12 | 103 11/12 | $11911 / 12$ | $9711 / 12$ | 233 11/12 | $23011 / 12$ | $22511 / 12$ | 102 11/12 | $11811 / 12$ | 2021 |
| $21511 / 12$ | $12711 / 12$ | $20711 / 12$ | 128 11/12 | 136 11/12 | $19211 / 12$ | 21411/12 | 126 11/12 | $20611 / 12$ | $12911 / 12$ | $13711 / 12$ | $19311 / 12$ | 2021 |
| $19111 / 12$ | $18411 / 12$ | $15211 / 12$ | $15911 / 12$ | 17511/12 | $14411 / 12$ | $19011 / 12$ | $18511 / 12$ | $15311 / 12$ | 158 11/12 | 17411/12 | 145 11/12 | 2021 |
| 167 11/12 | $15111 / 12$ | $17611 / 12$ | $18311 / 12$ | 160 11/12 | $16811 / 12$ | $16611 / 12$ | $15011 / 12$ | $17711 / 12$ | $18211 / 12$ | $16111 / 12$ | $16911 / 12$ | 2021 |
| 120 11/12 | $19911 / 12$ | $13511 / 12$ | $20011 / 12$ | $20811 / 12$ | $14311 / 12$ | $12111 / 12$ | 198 11/12 | $13411 / 12$ | $20111 / 12$ | 20911/12 | 142 11/12 | 2021 |
| 216 11/12 | $11211 / 12$ | $10411 / 12$ | 111 11/12 | $22311 / 12$ | $23911 / 12$ | $21711 / 12$ | $11311 / 12$ | $10511 / 12$ | $11011 / 12$ | 222 11/12 | $23811 / 12$ | 2021 |
| $9811 / 12$ | $23411 / 12$ | $22911 / 12$ | $22611 / 12$ | $10111 / 12$ | $11711 / 12$ | $9911 / 12$ | $23511 / 12$ | $22811 / 12$ | $22711 / 12$ | $10011 / 12$ | $11611 / 12$ | 2021 |
| $21311 / 12$ | $12511 / 12$ | $20511 / 12$ | $13011 / 12$ | 138 11/12 | $19411 / 12$ | 212 11/12 | 124 11/12 | 20411/12 | $13111 / 12$ | 139 11/12 | $19511 / 12$ | 2021 |
| 189 11/12 | $18611 / 12$ | $15411 / 12$ | $15711 / 12$ | 17311/12 | $14611 / 12$ | 188 11/12 | $18711 / 12$ | $15511 / 12$ | 156 11/12 | 172 11/12 | $14711 / 12$ | 2021 |
| 165 11/12 | $14911 / 12$ | $17811 / 12$ | 18111/12 | 162 11/12 | $17011 / 12$ | 164 11/12 | 14811/12 | $17911 / 12$ | $18011 / 12$ | 163 11/12 | $17111 / 12$ | 2021 |
| 122 11/12 | $19711 / 12$ | $13311 / 12$ | $20211 / 12$ | $21011 / 12$ | $14111 / 12$ | $12311 / 12$ | $19611 / 12$ | $13211 / 12$ | $20311 / 12$ | $21111 / 12$ | $14011 / 12$ | 2021 |
| 218 11/12 | $11411 / 12$ | $10611 / 12$ | $10911 / 12$ | $22111 / 12$ | $23711 / 12$ | $21911 / 12$ | 115 11/12 | $10711 / 12$ | $10811 / 12$ | $22011 / 12$ | $23611 / 12$ | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case all the blocks of order 6 are magic squares with equal magic sums given by

$$
\begin{aligned}
{\left[S_{6 \times 6}: S_{12 \times 12}:=21\right] } & :=10.5 \\
{\left[S_{6 \times 6}: S_{12 \times 12}:=441\right] } & :=220.5 \\
{\left[S_{6 \times 6}: S_{12 \times 12}:=2021\right] } & :=1010.5 .
\end{aligned}
$$

### 1.6 Block-Bordered Magic Squares of Order 13

Below are block-bordered magic squares of order 13 in two different ways giving magic sums $21,21^{2}$ and 2021. The blocks are of orders 3. In the first case, the blocks of order 3 are semi-magic squares with equal semi-magic sums. In this case, the magic square of order 9 is pandiagonal. In the second case, the sum of the nine entries are the same as of magic square of order 9. In this case, the magic square is bimagic square. The magic sums of magic squares of order 9 are given by

$$
\begin{aligned}
{\left[S_{9 \times 9}: S_{13 \times 13}:=21\right]:=21 \times \frac{9}{13} } & =\frac{189}{13} \\
{\left[S_{9 \times 9}: S_{13 \times 13}:=441\right]:=441 \times \frac{9}{13} } & =\frac{3969}{13} \\
{\left[S_{9 \times 9}: S_{13 \times 13}:=2021\right]:=2021 \times \frac{9}{13} } & =\frac{18189}{13}
\end{aligned}
$$

### 1.6.1 First Type

| 72 8/13 | 63 8/13 | 65 8/13 | 67 8/13 | 69 8/13 | 71 8/13 | 73 8/13 | -74 5/13 | -76 5/13 | -78 5/13 | -80 5/13 | -82 5/13 | -71 5/13 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -81 5/13 | -475/13 | -39 5/13 | $-415 / 13$ | -43 5/13 | -45 5/13 | $538 / 13$ | $548 / 13$ | $568 / 13$ | 58 8/13 | 60 8/13 | -49 5/13 | 84 8/13 | 21 |
| -79 5/13 | 61 8/13 | -175/13 | $318 / 13$ | -9 5/13 | -12 5/13 | $248 / 13$ | -75/13 | -19 5/13 | $298 / 13$ | -5 5/13 | -58 5/13 | $828 / 13$ | 21 |
| -77 5/13 | $598 / 13$ | $-45 / 13$ | -18 5/13 | 27 8/13 | -11 5/13 | -16 5/13 | $328 / 13$ | -65/13 | -14 5/13 | $258 / 13$ | -56 5/13 | $808 / 13$ | 21 |
| -75 5/13 | 57 8/13 | $268 / 13$ | -8 5/13 | -135/13 | $288 / 13$ | -3 5/13 | -20 5/13 | $308 / 13$ | -10 5/13 | -15 5/13 | -54 5/13 | 78 8/13 | 21 |
| -73 5/13 | 55 8/13 | 8/13 | -31 5/13 | $358 / 13$ | $58 / 13$ | -38 5/13 | $378 / 13$ | -15/13 | -33 5/13 | $398 / 13$ | -52 5/13 | 76 8/13 | 21 |
| -72 5/13 | -48 5/13 | $408 / 13$ | - 5/13 | -35 5/13 | 33 8/13 | $18 / 13$ | -30 5/13 | $388 / 13$ | $38 / 13$ | -37 5/13 | $518 / 13$ | 75 8/13 | 21 |
| 70 8/13 | -46 5/13 | -36 5/13 | $368 / 13$ | 48/13 | -34 5/13 | $418 / 13$ | -2 5/13 | -32 5/13 | $348 / 13$ | $28 / 13$ | $498 / 13$ | -67 5/13 | 21 |
| 68 8/13 | -44 5/13 | 18 8/13 | $138 / 13$ | -27 5/13 | $238 / 13$ | $68 / 13$ | -25 5/13 | $168 / 13$ | $118 / 13$ | -23 5/13 | 47 8/13 | -65 5/13 | 21 |
| 66 8/13 | -42 5/13 | -22 5/13 | 17 8/13 | $98 / 13$ | -29 5/13 | 19 8/13 | 14 8/13 | -24 5/13 | $218 / 13$ | $78 / 13$ | $458 / 13$ | -63 5/13 | 21 |
| 64 8/13 | -40 5/13 | $88 / 13$ | -26 5/13 | $228 / 13$ | 10 8/13 | -21 5/13 | 15 8/13 | 12 8/13 | -28 5/13 | $208 / 13$ | $438 / 13$ | -61 5/13 | 21 |
| 62 8/13 | $528 / 13$ | 42 8/13 | $448 / 13$ | $468 / 13$ | 48 8/13 | -50 5/13 | -51 5/13 | -53 5/13 | -55 5/13 | -57 5/13 | $508 / 13$ | -59 5/13 | 21 |
| 74 8/13 | -60 5/13 | -62 5/13 | -64 5/13 | -66 5/13 | -68 5/13 | -70 5/13 | 77 8/13 | 79 8/13 | $818 / 13$ | $838 / 13$ | 85 8/13 | -69 5/13 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  |  |  |  |  |  |  |  |  |  |  |  |  | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 104 12/13 | 9512/13 | 9712/13 | 9912/13 | 101 12/13 | 103 12/13 | 10512/13 | -42 1/13 | -44 1/13 | -46 1/13 | -48 1/13 | -50 1/13 | -39 1/13 | 441 |
| -49 1/13 | -15 1/13 | -71/13 | -9 1/13 | -11 1/13 | -13 1/13 | 8512/13 | $8612 / 13$ | $8812 / 13$ | 90 12/13 | 92 12/13 | -17 1/13 | 116 12/13 | 441 |
| -471/13 | 9312/13 | 1412/13 | $6312 / 13$ | $2212 / 13$ | 1912/13 | 5612/13 | 2412/13 | 1212/13 | 61 12/13 | 2612/13 | -26 1/13 | $11412 / 13$ | 441 |
| -45 1/13 | 9112/13 | $2712 / 13$ | 1312/13 | $5912 / 13$ | $2012 / 13$ | 1512/13 | 6412/13 | 2512/13 | 1712/13 | 5712/13 | -24 1/13 | $11212 / 13$ | 441 |
| -43 1/13 | 89 12/13 | $5812 / 13$ | $2312 / 13$ | 1812/13 | 60 12/13 | 28 12/13 | $1112 / 13$ | $6212 / 13$ | 21 12/13 | 1612/13 | -22 1/13 | $11012 / 13$ | 441 |
| -411/13 | 8712/13 | 32 12/13 | 12/13 | $6712 / 13$ | 37 12/13 | -6 1/13 | 6912/13 | $3012 / 13$ | -1 1/13 | 7112/13 | -20 1/13 | 108 12/13 | 441 |
| -401/13 | -16 1/13 | $7212 / 13$ | 3112/13 | -3 1/13 | 6512/13 | $3312 / 13$ | 112/13 | 70 12/13 | 3512/13 | -5 1/13 | 8312/13 | 107 12/13 | 441 |
| 102 12/13 | -14 1/13 | $-41 / 13$ | $6812 / 13$ | $3612 / 13$ | -2 1/13 | 7312/13 | 2912/13 | - 1/13 | 6612/13 | 3412/13 | 8112/13 | -35 1/13 | 441 |
| $10012 / 13$ | -12 1/13 | $5012 / 13$ | 4512/13 | 412/13 | 5512/13 | 38 12/13 | 612/13 | 4812/13 | 4312/13 | 812/13 | 7912/13 | -33 1/13 | 441 |
| 9812/13 | -10 1/13 | 912/13 | 4912/13 | 4112/13 | 212/13 | 51 12/13 | 4612/13 | $712 / 13$ | 5312/13 | 39 12/13 | $7712 / 13$ | -31 1/13 | 441 |
| 9612/13 | -8 1/13 | $4012 / 13$ | 512/13 | $5412 / 13$ | 42 12/13 | 10 12/13 | $4712 / 13$ | 4412/13 | 312/13 | 52 12/13 | 7512/13 | -29 1/13 | 441 |
| 9412/13 | 8412/13 | 7412/13 | 7612/13 | $7812 / 13$ | 8012/13 | -18 1/13 | -19 1/13 | -21 1/13 | -23 1/13 | -25 1/13 | 8212/13 | -27 1/13 | 441 |
| $10612 / 13$ | -28 1/13 | -30 1/13 | -32 1/13 | -34 1/13 | -36 1/13 | -38 1/13 | $10912 / 13$ | 11112/13 | 113 12/13 | 115 12/13 | $11712 / 13$ | -37 1/13 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


| 226 6/13 | 217 6/13 | 219 6/13 | 221 6/13 | 223 6/13 | 225 6/13 | 227 6/13 | 79 6/13 | 77 6/13 | 75 6/13 | 73 6/13 | 71 6/13 | 82 6/13 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 72 6/13 | 106 6/13 | 114 6/13 | 112 6/13 | $1106 / 13$ | $1086 / 13$ | 207 6/13 | $2086 / 13$ | 210 6/13 | 212 6/13 | 214 6/13 | 104 6/13 | $2386 / 13$ | 20 |
| 74 6/13 | 215 6/13 | 136 6/13 | 185 6/13 | 144 6/13 | 141 6/13 | 178 6/13 | 146 6/13 | 134 6/13 | $1836 / 13$ | 148 6/13 | 95 6/13 | $2366 / 13$ | 2021 |
| 76 6/13 | 213 6/13 | 149 6/13 | 135 6/13 | 181 6/13 | 142 6/13 | 137 6/13 | 186 6/13 | 147 6/13 | 139 6/13 | 179 6/13 | 97 6/13 | 234 6/13 | 2021 |
| 78 6/13 | 211 6/13 | 180 6/13 | $1456 / 13$ | 140 6/13 | 182 6/13 | 150 6/13 | 133 6/13 | 184 6/13 | 143 6/13 | 138 6/13 | 99 6/13 | 232 6/13 | 2021 |
| 80 6/13 | 209 6/13 | 154 6/13 | 122 6/13 | 189 6/13 | 159 6/13 | 115 6/13 | 191 6/13 | 152 6/13 | 120 6/13 | 193 6/13 | 101 6/13 | 230 6/13 | 2021 |
| 81 6/13 | 105 6/13 | 194 6/13 | 153 6/13 | 118 6/13 | 187 6/13 | 155 6/13 | 123 6/13 | 192 6/13 | 157 6/13 | 116 6/13 | 205 6/13 | 229 6/13 | 2021 |
| 224 6/13 | 107 6/13 | 117 6/13 | 190 6/13 | 158 6/13 | 119 6/13 | 195 6/13 | 151 6/13 | 121 6/13 | 188 6/13 | 156 6/13 | 203 6/13 | 86 6/13 | 20 |
| 222 6/13 | 109 6/13 | 172 6/13 | 167 6/13 | 126 6/13 | 177 6/13 | 160 6/13 | 128 6/13 | 170 6/13 | 165 6/13 | 130 6/13 | 201 6/13 | 88 6/13 | 202 |
| 220 6/13 | 111 6/13 | 131 6/13 | 171 6/13 | 163 6/13 | 124 6/13 | 173 6/13 | 168 6/13 | 129 6/13 | 175 6/13 | 161 6/13 | 199 6/13 | $906 / 13$ | 20 |
| 218 6/13 | 113 6/13 | 162 6/13 | 127 6/13 | 176 6/13 | 164 6/13 | 132 6/13 | 169 6/13 | 166 6/13 | 125 6/13 | 174 6/13 | 197 6/13 | 92 6/13 | 202 |
| 216 6/13 | 206 6/13 | 196 6/13 | 198 6/13 | 200 6/13 | 202 6/13 | 103 6/13 | 102 6/13 | 100 6/13 | 98 6/13 | 96 6/13 | 204 6/13 | 94 6/13 | 202 |
| 228 6/13 | $936 / 13$ | 91 6/13 | 89 6/13 | 87 6/13 | 85 6/13 | 83 6/13 | 231 6/13 | 233 6/13 | 235 6/13 | 237 6/13 | 239 6/13 | 84 6/13 | 202 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the blocks of order 3 are semi-magic squares with sums given by

$$
\begin{gathered}
{\left[\operatorname{Sm}_{3 \times 3}: S_{13 \times 13}:=21\right]:=\frac{63}{13}} \\
{\left[{S m_{3 \times 3}}: S_{13 \times 13}:=441\right]:=\frac{1323}{13}} \\
{\left[\operatorname{Sm}_{3 \times 3}: S_{13 \times 13}:=2021\right]:=\frac{6063}{13}}
\end{gathered}
$$

### 1.6.2 Second Type

| 72 8/13 | 63 8/13 | 65 8/13 | 67 8/13 | 69 8/13 | $718 / 13$ | 73 8/13 | -74 5/13 | -76 5/13 | -78 5/13 | -80 5/13 | -82 5/13 | -71 5/13 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -81 5/13 | -475/13 | -39 5/13 | -41 5/13 | -43 5/13 | -45 5/13 | 53 8/13 | 54 8/13 | $568 / 13$ | 58 8/13 | 60 8/13 | -49 5/13 | $848 / 13$ | 21 |
| -79 5/13 | 61 8/13 | -30 5/13 | $318 / 13$ | 12 8/13 | 35 8/13 | -10 5/13 | -29 5/13 | $28 / 13$ | -16 5/13 | 18 8/13 | -58 5/13 | 828/13 | 21 |
| -77 5/13 | 59 8/13 | $388 / 13$ | -75/13 | -26 5/13 | $58 / 13$ | -13 5/13 | $218 / 13$ | -36 5/13 | $258 / 13$ | $68 / 13$ | -56 5/13 | $808 / 13$ | 21 |
| -75 5/13 | 57 8/13 | - 5/13 | -19 5/13 | 15 8/13 | -33 5/13 | $288 / 13$ | 98/13 | $418 / 13$ | $-45 / 13$ | -23 5/13 | -54 5/13 | 78 8/13 | 21 |
| -73 5/13 | 55 8/13 | $138 / 13$ | -32 5/13 | 32 8/13 | -28 5/13 | $338 / 13$ | -9 5/13 | $198 / 13$ | 8/13 | -15 5/13 | -52 5/13 | 76 8/13 | 21 |
| -72 5/13 | -485/13 | -25 5/13 | $368 / 13$ | -6 5/13 | 22 8/13 | $38 / 13$ | -12 5/13 | $78 / 13$ | -38 5/13 | 26 8/13 | 51 8/13 | 75 8/13 | 21 |
| 70 8/13 | -46 5/13 | 16 8/13 | -2 5/13 | -18 5/13 | 10 8/13 | -35 5/13 | 29 8/13 | -22 5/13 | 39 8/13 | -3 5/13 | $498 / 13$ | -67 5/13 | 21 |
| 68 8/13 | -44 5/13 | $308 / 13$ | $148 / 13$ | -31 5/13 | -11 5/13 | -27 5/13 | $348 / 13$ | -17 5/13 | $208 / 13$ | $18 / 13$ | $478 / 13$ | -65 5/13 | 21 |
| 66 8/13 | -42 5/13 | -8 5/13 | -24 5/13 | 37 8/13 | -14 5/13 | 23 8/13 | $48 / 13$ | $248 / 13$ | 8 8/13 | -37 5/13 | $458 / 13$ | -63 5/13 | 21 |
| 64 8/13 | -40 5/13 | -20 5/13 | $178 / 13$ | -15/13 | 27 8/13 | $118 / 13$ | -345/13 | -5 5/13 | -21 5/13 | $408 / 13$ | $438 / 13$ | -61 5/13 | 21 |
| 62 8/13 | 52 8/13 | 42 8/13 | 44 8/13 | 46 8/13 | 48 8/13 | -50 5/13 | -51 5/13 | -53 5/13 | -55 5/13 | -57 5/13 | 50 8/13 | -59 5/13 | 21 |
| 74 8/13 | -60 5/13 | -62 5/13 | -64 5/13 | -66 5/13 | -68 5/13 | -70 5/13 | 77 8/13 | 79 8/13 | 81 8/13 | 83 8/13 | 85 8/13 | -69 5/13 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |



In this case, the sum of nine entries of blocks of order 3 is the as of magic square of order 9 .

### 1.7 Block-Bordered Magic Squares of Order 14

Below are block-bordered magic squares of order 14 in three different ways giving magic sums $21,21^{2}$ and 2021. The block-wise magic square considered is of order 12. The block are of orders 3, 4 and 6 . In each case, the sum of magic squares of order 12 are given by

$$
\begin{gathered}
{\left[S_{12 \times 12}: S_{14 \times 14}:=21\right]:=18} \\
{\left[S_{12 \times 12}: S_{14 \times 14}:=441\right]:=378} \\
{\left[S_{12 \times 12}: S_{14 \times 14}:=2021\right]:=\frac{12126}{7} .}
\end{gathered}
$$

### 1.7.1 First Type

| -83 | -89 | 91 | -87 | 89 | -85 | 99 | -90 | 85 | -81 | 83 | -79 | 81 | 87 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 98 | 27 | -70 | -20 | 1 | -49 | 48 | 7 | -43 | 54 | 69 | -28 | 22 | -95 | 21 |
| -94 | -68 | -21 | 26 | 47 | 0 | -47 | 53 | 6 | -41 | -26 | 21 | 68 | 97 | 21 |
| 96 | -22 | 28 | -69 | -48 | 49 | -1 | -42 | 55 | 5 | 20 | 70 | -27 | -93 | 21 |
| -92 | 60 | -37 | 13 | 16 | -34 | 63 | -14 | -64 | 33 | 42 | -55 | -5 | 95 | 21 |
| 94 | -35 | 12 | 59 | 62 | 15 | -32 | 32 | -15 | -62 | -53 | -6 | 41 | -91 | 21 |
| 80 | 11 | 61 | -36 | -33 | 64 | 14 | -63 | 34 | -16 | -7 | 43 | -54 | -77 | 21 |
| 74 | -4 | 46 | -51 | -66 | 31 | -19 | -24 | 73 | 23 | 2 | 52 | -45 | -71 | 21 |
| -72 | -50 | -3 | 44 | 29 | -18 | -65 | 71 | 24 | -23 | -44 | 3 | 50 | 75 | 21 |
| 76 | 45 | -52 | -2 | -17 | -67 | 30 | 25 | -25 | 72 | 51 | -46 | 4 | -73 | 21 |
| -74 | 17 | 67 | -30 | -39 | 58 | 8 | -57 | 40 | -10 | -13 | 37 | -60 | 77 | 21 |
| 78 | -29 | 18 | 65 | 56 | 9 | -38 | 38 | -9 | -56 | -59 | -12 | 35 | -75 | 21 |
| -76 | 66 | -31 | 19 | 10 | -40 | 57 | -8 | -58 | 39 | 36 | -61 | -11 | 79 | 21 |
| -84 | 92 | -88 | 90 | -86 | 88 | -96 | 93 | -82 | 84 | -80 | 82 | -78 | 86 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| -53 | -59 | 121 | -57 | 119 | -55 | 129 | -60 | 115 | -51 | 113 | -49 | 111 | 117 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- | :--- |
| 128 | 57 | -40 | 10 | 31 | -19 | 78 | 37 | -13 | 84 | 99 | 2 | 52 | -65 | 441 |
| -64 | -38 | 9 | 56 | 77 | 30 | -17 | 83 | 36 | -11 | 4 | 51 | 98 | 127 | 441 |
| 126 | 8 | 58 | -39 | -18 | 79 | 29 | -12 | 85 | 35 | 50 | 100 | 3 | -63 | 441 |
| -62 | 90 | -7 | 43 | 46 | -4 | 93 | 16 | -34 | 63 | 72 | -25 | 25 | 125 | 441 |
| 124 | -5 | 42 | 89 | 92 | 45 | -2 | 62 | 15 | -32 | -23 | 24 | 71 | -61 | 441 |
| 110 | 41 | 91 | -6 | -3 | 94 | 44 | -33 | 64 | 14 | 23 | 73 | -24 | -47 | 441 |
| 104 | 26 | 76 | -21 | -36 | 61 | 11 | 6 | 103 | 53 | 32 | 82 | -15 | -41 | 441 |
| -42 | -20 | 27 | 74 | 59 | 12 | -35 | 101 | 54 | 7 | -14 | 33 | 80 | 105 | 441 |
| 106 | 75 | -22 | 28 | 13 | -37 | 60 | 55 | 5 | 102 | 81 | -16 | 34 | -43 | 441 |
| -44 | 47 | 97 | 0 | -9 | 88 | 38 | -27 | 70 | 20 | 17 | 67 | -30 | 107 | 441 |
| 108 | 1 | 48 | 95 | 86 | 39 | -8 | 68 | 21 | -26 | -29 | 18 | 65 | -45 | 441 |
| -46 | 96 | -1 | 49 | 40 | -10 | 87 | 22 | -28 | 69 | 66 | -31 | 19 | 109 | 441 |
| -54 | 122 | -58 | 120 | -56 | 118 | -66 | 123 | -52 | 114 | -50 | 112 | -48 | 116 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


| 596 | $536 / 7$ | 23367 | 55 6/7 | 2316 | $576 / 7$ | 2416 | 526/7 | 2276 | 616/7 | 225 6/7 | $636 / 7$ | 223 6/7 | 229 6/7 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 240 6/7 | 169 6/7 | 72 6/7 | 122 6/7 | 143 | $936 / 7$ | 1906 | 1496 | $996 / 7$ | $1966 / 7$ | $2116 / 7$ | $1146 / 7$ | 164 6/7 | $476 / 7$ |  |
|  | 74 |  |  |  |  |  |  |  |  |  |  | 210 6/7 |  | 202 |
| 238 6/7 | 120 | 170 | 73 | 94 | 19 | 14 | 10 | 19 | 7 | 16 | 212 6/7 | 115 6/7 | 49 | 2021 |
|  |  |  |  |  |  |  |  |  |  |  | $876 / 7$ |  |  |  |
| 236 | 107 | 1546 | 20 | 2046/7 | 15 | 11 | 17 | 12 | $806 / 7$ | 896 | 136 6/7 | 183 | 51 | 202 |
|  |  |  |  |  |  |  |  |  |  |  |  | 88 6/7 |  |  |
| 21 | 138 6/ | 18 | $916 / 7$ |  | 173 6/7 | 12 |  | 21 |  | TH | $1946 / 7$ | $976 / 7$ | $716 / 7$ | 202 |
| 70617 | 92 |  | 18 |  |  |  |  |  | $1196 / 7$ | 98 | 17 | 192 6/7 | 217 6/7 | 202 |
| 21 |  | 90 |  |  |  |  |  |  | 7 | 193 6/7 |  | 146 6/7 |  |  |
| 68 | $1596 /$ | 2096 | 112 6/ | 103 | 200 6/7 | 150 | $856 / 7$ | 182 6/7 | 132 6/7 | 129 6/7 | 1796 | $826 / 7$ | 219 | 2021 |
| 220 6/7 | 113 6/7 | 160 | 20 | 198 | 151 | 10 | 180 | 1336 | 86 | $836 / 7$ | 130 6/7 | 177 6/7 | 67 6/7 |  |
| 666/7 | 208 6/' | 1116 | $1616 / 7$ | 1526 | 1026/7 | 1996 | 1346 | 8461 | $1816 / 7$ | 178 6/7 | $816 / 7$ | $1316 / 7$ | 221 | 2021 |
| 58 6/7 | 234 | 546 | 232 | 566 | 2306 | 466 | 2356/7 | 6061 | 226 6/7 | $626 / 7$ | 2246/7 | $646 / 7$ | 228 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 2021 | 202 | 2021 |

In this case, all the blocks of order 3 are magic squares with different magic sums.

### 1.7.2 Second Type

| -83 | -89 | 91 | -87 | 89 | -85 | 99 | -90 | 85 | -81 | 83 | -79 | 81 | 87 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 98 | -16 | 37 | -70 | 55 | -15 | 36 | -69 | 54 | -14 | 35 | -68 | 53 | -95 | 21 |
| -94 | -53 | 38 | 1 | 20 | -54 | 39 | 0 | 21 | -55 | 40 | -1 | 22 | 97 | 21 |
| 96 | 73 | -52 | 19 | -34 | 72 | -51 | 18 | -33 | 71 | -50 | 17 | -32 | -93 | 21 |
| -92 | 2 | -17 | 56 | -35 | 3 | -18 | 57 | -36 | 4 | -19 | 58 | -37 | 95 | 21 |
| 94 | -13 | 34 | -67 | 52 | -12 | 33 | -66 | 51 | -11 | 32 | -65 | 50 | -91 | 21 |
| 80 | -56 | 41 | -2 | 23 | -57 | 42 | -3 | 24 | -58 | 43 | -4 | 25 | -77 | 21 |
| 74 | 70 | -49 | 16 | -31 | 69 | -48 | 15 | -30 | 68 | -47 | 14 | -29 | -71 | 21 |
| -72 | 5 | -20 | 59 | -38 | 6 | -21 | 60 | -39 | 7 | -22 | 61 | -40 | 75 | 21 |
| 76 | -10 | 31 | -64 | 49 | -9 | 30 | -63 | 48 | -8 | 29 | -62 | 47 | -73 | 21 |
| -74 | -59 | 44 | -5 | 26 | -60 | 45 | -6 | 27 | -61 | 46 | -7 | 28 | 77 | 21 |
| 78 | 67 | -46 | 13 | -28 | 66 | -45 | 12 | -27 | 65 | -44 | 11 | -26 | -75 | 21 |
| -76 | 8 | -23 | 62 | -41 | 9 | -24 | 63 | -42 | 10 | -25 | 64 | -43 | 79 | 21 |
| -84 | 92 | -88 | 90 | -86 | 88 | -96 | 93 | -82 | 84 | -80 | 82 | -78 | 86 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| -53 | -59 | 121 | -57 | 119 | -55 | 129 | -60 | 115 | -51 | 113 | -49 | 111 | 117 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 128 | 14 | 67 | -40 | 85 | 15 | 66 | -39 | 84 | 16 | 65 | -38 | 83 | -65 | 441 |
| -64 | -23 | 68 | 31 | 50 | -24 | 69 | 30 | 51 | -25 | 70 | 29 | 52 | 127 | 441 |
| 126 | 103 | -22 | 49 | -4 | 102 | -21 | 48 | -3 | 101 | -20 | 47 | -2 | -63 | 441 |
| -62 | 32 | 13 | 86 | -5 | 33 | 12 | 87 | -6 | 34 | 11 | 88 | -7 | 125 | 441 |
| 124 | 17 | 64 | -37 | 82 | 18 | 63 | -36 | 81 | 19 | 62 | -35 | 80 | -61 | 441 |
| 110 | -26 | 71 | 28 | 53 | -27 | 72 | 27 | 54 | -28 | 73 | 26 | 55 | -47 | 441 |
| 104 | 100 | -19 | 46 | -1 | 99 | -18 | 45 | 0 | 98 | -17 | 44 | 1 | -41 | 441 |
| -42 | 35 | 10 | 89 | -8 | 36 | 9 | 90 | -9 | 37 | 8 | 91 | -10 | 105 | 441 |
| 106 | 20 | 61 | -34 | 79 | 21 | 60 | -33 | 78 | 22 | 59 | -32 | 77 | -43 | 441 |
| -44 | -29 | 74 | 25 | 56 | -30 | 75 | 24 | 57 | -31 | 76 | 23 | 58 | 107 | 441 |
| 108 | 97 | -16 | 43 | 2 | 96 | -15 | 42 | 3 | 95 | -14 | 41 | 4 | -45 | 441 |
| -46 | 38 | 7 | 92 | -11 | 39 | 6 | 93 | -12 | 40 | 5 | 94 | -13 | 109 | 441 |
| -54 | 122 | -58 | 120 | -56 | 118 | -66 | 123 | -52 | 114 | -50 | 112 | -48 | 116 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $596 / 7$ | $536 / 7$ | 233 6/7 | $556 / 7$ | $2316 / 7$ | $576 / 7$ | 2416/7 | 52 6/7 | 227 6/7 | 616/7 | $2256 / 7$ | $636 / 7$ | 2236/7 | $2296 / 7$ | 2021 |
| 240 6/7 | 126 6/7 | 179 6/7 | 72 6/7 | 197 6/7 | 127 | 178 6/7 | $736 / 7$ | $1966 / 7$ | 128 6/7 | 177 6/7 | 746/7 | 195 6/7 | $476 / 7$ | 2021 |
| 48617 | $896 / 7$ | $1806 / 7$ | $1436 / 7$ | 162 6/7 | 88 | 181 | $1426 / 7$ | 163 | 87617 | 182 6/7 | 141 6/7 | 164 6/7 | /7 | 2021 |
| 238 6/7 | 215 6/7 | 906/7 | 161 | 108 6/7 | 2146 | 916/7 | 160 | 109 6/7 | 213 6/7 | 926/7 | 159 6/7 | $1106 / 7$ | $496 / 7$ | 2021 |
| 50 | 1446/7 | 125 | 198 | 10 | 145 | $1246 / 7$ | 1996 | 106 6/7 | 146 6/7 | 123 6/7 | 200 6/7 | $1056 / 7$ | /7 | 2021 |
| $2366 / 7$ | $1296 / 7$ | $1766 / 7$ | $756 / 7$ | 1946/7 | 130 6/7 | 175 6/7 | 766/7 | 1936/7 | 1316/7 | 174 6/7 | 77 6/7 | 192 6/7 | $516 / 7$ | 202 |
| 222 | 86 | 183 | 140 | 16 | 85 | 18 | 13 | 16 | $846 / 7$ | $1856 / 7$ | 138 6/7 | 7 | 17 | 21 |
| 21661 | 212 6/7 | 93 6/7 | 1586 | 1116/7 | 2116 | 946/7 | 1576 | 112 6/7 | 210 6/7 | 956/7 | $1566 / 7$ | $1136 / 7$ | $716 / 7$ | 202 |
| 70 6/7 | 1476/7 | 122 6/7 | 2016/7 | 1046/7 | 148 6/7 | 121 6/7 | $2026 / 7$ | 103 /7 | $1496 / 7$ | 120 6/7 | $2036 / 7$ | 102 6/7 | 217 6/7 | 2021 |
| 218 6/7 | 13261 | 173 6/7 | 78 | $1916 / 7$ | 1336 | 172 6/7 | $796 / 7$ | $1906 / 7$ | $1346 / 7$ | 1716/7 | $806 / 7$ | 1896/7 | $696 / 7$ | 2021 |
| 68 6/7 | $836 / 7$ | 186 6/7 | 137 6/7 | 168 6/7 | 82 6/7 | 187 6/7 | 136 6/7 | 169 6/7 | 816/7 | 188 6/7 | 135 6/7 | 170 6/7 | 219 6/7 | 2021 |
| 220 6/7 | 2096/7 | 966/7 | $1556 /$ | $1146 / 7$ | 208 6/7 | $976 / 7$ | 154 6/7 | 115 6/7 | 207 6/7 | 986/7 | 153 6/7 | $1166 / 7$ | $676 / 7$ | 2021 |
| 66 6/7 | 150 6/7 | $1196 / 7$ | 2046/7 | 1016/7 | $1516 / 7$ | 118 6/7 | 205 6/7 | $1006 / 7$ | 152 6/7 | 117 6/7 | $2066 / 7$ | $996 / 7$ | 2216/7 | 2021 |
| 58 6/7 | 2346/7 | $546 / 7$ | 232 6/7 | $566 / 7$ | 230 6/7 | $466 / 7$ | $2356 / 7$ | $606 / 7$ | 226 /7 | $626 / 7$ | $2246 / 7$ | $646 / 7$ | 228 6/7 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case all the blocks of order 4 are pandiagonal magic squares with equal magic sums are:

$$
\begin{gathered}
{\left[S_{4 \times 4}: S_{14 \times 14}:=21\right]:=6} \\
{\left[S_{4 \times 4}: S_{14 \times 14}:=441\right]:=126} \\
{\left[S_{4 \times 4}: S_{14 \times 14}:=2021\right]:=\frac{4042}{7} .}
\end{gathered}
$$

### 1.7.3 Third Type

| -83 | -89 | 91 | -87 | 89 | -85 | 99 | -90 | 85 | -81 | 83 | -79 | 81 | 87 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 98 | -70 | 66 | 65 | 58 | -63 | -47 | -69 | 67 | 64 | 59 | -64 | -48 | -95 | 21 |
| -94 | 49 | -39 | 41 | -38 | -30 | 26 | 48 | -40 | 40 | -37 | -29 | 27 | 97 | 21 |
| 96 | 25 | 18 | -14 | -7 | 9 | -22 | 24 | 19 | -13 | -8 | 8 | -21 | -93 | 2 |
| -92 | 1 | -15 | 10 | 17 | -6 | 2 | 0 | -16 | 11 | 16 | -5 | 3 | 95 | 21 |
| 94 | -46 | 33 | -31 | 34 | 42 | -23 | -45 | 32 | -32 | 35 | 43 | -24 | -91 | 2 |
| 80 | 50 | -54 | -62 | -55 | 57 | 73 | 51 | -53 | -61 | -56 | 56 | 72 | -77 | 21 |
| 74 | -68 | 68 | 63 | 60 | -65 | -49 | -67 | 69 | 62 | 61 | -66 | -50 | -71 | 2 |
| -72 | 47 | -41 | 39 | -36 | -28 | 28 | 46 | -42 | 38 | -35 | -27 | 29 | 75 | 21 |
| 76 | 23 | 20 | -12 | -9 | 7 | -20 | 22 | 21 | -11 | -10 | 6 | -19 | -73 | 21 |
| -74 | -1 | -17 | 12 | 15 | -4 | 4 | -2 | -18 | 13 | 14 | -3 | 5 | 77 | 2 |
| 78 | -44 | 31 | -33 | 36 | 44 | -25 | -43 | 30 | -34 | 37 | 45 | -26 | -75 | 21 |
| -76 | 52 | -52 | -60 | -57 | 55 | 71 | 53 | -51 | -59 | -58 | 54 | 70 | 79 | 21 |
| -84 | 92 | -88 | 90 | -86 | 88 | -96 | 93 | -82 | 84 | -80 | 82 | -78 | 86 | 2 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |

441

| -53 | -59 | 121 | -57 | 119 | -55 | 129 | -60 | 115 | -51 | 113 | -49 | 111 | 117 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 128 | -40 | 96 | 95 | 88 | -33 | -17 | -39 | 97 | 94 | 89 | -34 | -18 | -65 | 441 |
| -64 | 79 | -9 | 71 | -8 | 0 | 56 | 78 | -10 | 70 | -7 | 1 | 57 | 127 | 441 |
| 126 | 55 | 48 | 16 | 23 | 39 | 8 | 54 | 49 | 17 | 22 | 38 | 9 | -63 | 441 |
| -62 | 31 | 15 | 40 | 47 | 24 | 32 | 30 | 14 | 41 | 46 | 25 | 33 | 125 | 441 |
| 124 | -16 | 63 | -1 | 64 | 72 | 7 | -15 | 62 | -2 | 65 | 73 | 6 | -61 | 441 |
| 110 | 80 | -24 | -32 | -25 | 87 | 103 | 81 | -23 | -31 | -26 | 86 | 102 | -47 | 441 |
| 104 | -38 | 98 | 93 | 90 | -35 | -19 | -37 | 99 | 92 | 91 | -36 | -20 | -41 | 441 |
| -42 | 77 | -11 | 69 | -6 | 2 | 58 | 76 | -12 | 68 | -5 | 3 | 59 | 105 | 441 |
| 106 | 53 | 50 | 18 | 21 | 37 | 10 | 52 | 51 | 19 | 20 | 36 | 11 | -43 | 441 |
| -44 | 29 | 13 | 42 | 45 | 26 | 34 | 28 | 12 | 43 | 44 | 27 | 35 | 107 | 441 |
| 108 | -14 | 61 | -3 | 66 | 74 | 5 | -13 | 60 | -4 | 67 | 75 | 4 | -45 | 441 |
| -46 | 82 | -22 | -30 | -27 | 85 | 101 | 83 | -21 | -29 | -28 | 84 | 100 | 109 | 441 |
| -54 | 122 | -58 | 120 | -56 | 118 | -66 | 123 | -52 | 114 | -50 | 112 | -48 | 116 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


| $596 / 7$ | 536/7 | 233 6/7 | 556/7 | $2316 / 7$ | $576 / 7$ | 2416/7 | 52 6/7 | 227 6/7 | 616/7 | 225 /7 | 636/7 | $2236 / 7$ | $2296 / 7$ | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2406 / 7$ | 72 6/7 | 208 6/7 | 2076/7 | 200 6/7 | $796 / 7$ | $956 / 7$ | $736 / 7$ | 2096/7 | 2066/7 | $2016 / 7$ | $786 / 7$ | 946/7 | $476 / 7$ | 2021 |
| 48617 | 191 6/7 | $1036 / 7$ | $1836 / 7$ | $1046 / 7$ | 112 6/7 | 168 6/7 | $1906 / 7$ | $1026 / 7$ | $1826 / 7$ | $1056 / 7$ | $1136 / 7$ | 169 6/7 | 239 6/7 | 2021 |
| 238 6/7 | 167 6/7 | 160 /7 | 128 6/7 | $1356 / 7$ | 1516/7 | $1206 / 7$ | 166 6/7 | 1616/7 | $1296 / 7$ | 1346/7 | 150 6/7 | 1216/7 | 17 | 2021 |
| $506 / 7$ | $1436 / 7$ | 127 6/7 | 152 6/7 | $1596 / 7$ | $1366 / 7$ | 1446/7 | $1426 / 7$ | 126 6/7 | $1536 / 7$ | 158 6/7 | 137 6/7 | $1456 / 7$ | 237 6/7 | 202 |
| $2366 / 7$ | $966 / 7$ | $1756 / 7$ | 111 | $1766 / 7$ | 1846/7 | $1196 / 7$ | 97 | 174 | $1106 / 7$ | 177 6/7 | $1856 / 7$ | 118 6/7 | 51 | 2021 |
| $2226 / 7$ | 192 6/7 | $886 / 7$ | $806 / 7$ | $876 / 7$ | $1996 / 7$ | $2156 / 7$ | $1936 / 7$ | 896/7 | 816/7 | 866/7 | $1986 / 7$ | $2146 / 7$ | $656 / 7$ | 2021 |
| 216 6/7 | 746/7 | 210 6/7 | 2056/7 | 2026/7 | $776 / 7$ | $936 / 7$ | $756 / 7$ | $2116 / 7$ | 2046/7 | 203 6/7 | 766/7 | $926 / 7$ | 716/7 | 202 |
| 70 6/7 | 189 6/7 | $1016 / 7$ | 1816/7 | $1066 / 7$ | 1146/7 | 170 6/7 | 188 6/7 | $1006 / 7$ | $1806 / 7$ | 107 6/7 | 1156/7 | 1716/7 | $2176 / 7$ | 2021 |
| 218 6/7 | $1656 / 7$ | $1626 / 7$ | $1306 / 7$ | $1336 / 7$ | $1496 / 7$ | $1226 / 7$ | $1646 / 7$ | $1636 / 7$ | $1316 / 7$ | $1326 / 7$ | 148 6/7 | $1236 / 7$ | $696 / 7$ | 202 |
| $686 / 7$ | $1416 / 7$ | $1256 / 7$ | $1546 / 7$ | 157 6/7 | 138 6/7 | $1466 / 7$ | $1406 / 7$ | $1246 / 7$ | $1556 / 7$ | $1566 / 7$ | $1396 / 7$ | $1476 / 7$ | 219 6/7 | 2021 |
| $2206 / 7$ | $986 / 7$ | $1736 / 7$ | $1096 / 7$ | 178 6/7 | $1866 / 7$ | $1176 / 7$ | $996 / 7$ | 172 6/7 | 108 6/7 | $1796 / 7$ | $1876 / 7$ | $1166 / 7$ | $676 / 7$ | 2021 |
| $666 / 7$ | $1946 / 7$ | 906/7 | $826 / 7$ | $856 / 7$ | 197 6/7 | $2136 / 7$ | $1956 / 7$ | 916/7 | $836 / 7$ | 846/7 | $1966 / 7$ | 212 6/7 | $2216 / 7$ | 2021 |
| 58 6/7 | 2346/7 | 546/7 | $2326 / 7$ | 566/7 | $2306 / 7$ | $466 / 7$ | $2356 / 7$ | $606 / 7$ | 226 /7 | $626 / 7$ | $2246 / 7$ | $646 / 7$ | 228 6/7 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, all the blocks of order 6 are magic squares with equal magic sums are:

$$
\begin{gathered}
{\left[S_{6 \times 6}: S_{14 \times 14}:=21\right]:=9} \\
{\left[S_{6 \times 6}: S_{14 \times 14}:=441\right]:=189} \\
{\left[S_{6 \times 6}: S_{14 \times 14}:=2021\right]:=\frac{6063}{7} .}
\end{gathered}
$$

### 1.8 Block-Wise Magic Squares of Order 15

Below are block-wise magic squares of order 15 in two different ways giving magic sums $21,21^{2}$ and 2021. One is where blocks of order 3 are semi-magic with equal semi-magic sums. The second, where blocks of order 5 are pandiagonal magic squares with equal magic sums.

### 1.8.1 First Type

|  | pan | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 2 | 21 | 21 | 21 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 74.4 | -46.6 | -23.6 | 75.4 | -37.6 | -33.6 | 76.4 | -36.6 | -35.6 | 77.4 | -47.6 | -25.6 | 78.4 | -49.6 | -24.6 |  |
| 21 | -31.6 | 81.4 | -45.6 | -22.6 | 71.4 | -44.6 | -21.6 | 69.4 | -43.6 | -32.6 | 79.4 | -42.6 | -34.6 | 80.4 | -41.6 | 21 |
|  | -38.6 | -30.6 | 73.4 | -48.6 | -29.6 | 82.4 | -50.6 | -28.6 | 83.4 | -40.6 | -27.6 | 72.4 | -39.6 | -26.6 | 70.4 | 21 |
|  | -75.6 | 88.4 | -8.6 | -74.6 | 97.4 | -18.6 | -73.6 | 98.4 | -20.6 | -72.6 | 87.4 | -10.6 | -71.6 | 85.4 | -9.6 | 21 |
|  | -16.6 | -68.6 | 89.4 | -7.6 | -78.6 | 90.4 | -6.6 | -80.6 | 91.4 | -17.6 | -70.6 | 92.4 | -19.6 | -69.6 | 93.4 | 21 |
| 2 | 96.4 | -15.6 | $-76.6$ | 86.4 | -14.6 | -67.6 | 84.4 | -13.6 | -66.6 | 94.4 | -12.6 | -77.6 | 95.4 | -11.6 | -79.6 | 21 |
|  | -105.6 | 103.4 | 6.4 | -104.6 | 112.4 | -3.6 | -103.6 | 113.4 | -5.6 | -102.6 | 102.4 | 4.4 | -101.6 | 100.4 | 5.4 | 21 |
|  | -1.6 | -98.6 | 104.4 | 7.4 | -108.6 | 105.4 | 8.4 | -110.6 | 106.4 | -2.6 | -100.6 | 107.4 | -4.6 | -99.6 | 108.4 | 21 |
| 21 | 111.4 | -0.6 | -106.6 | 101.4 | 0.4 | -97.6 | 99.4 | 1.4 | -96.6 | 109.4 | 2.4 | -107.6 | 110.4 | 3.4 | -109.6 | 21 |
|  | 44.4 | -61.6 | 21.4 | 45.4 | -52.6 | 11.4 | 46.4 | -51.6 | 9.4 | 47.4 | -62.6 | 19.4 | 48.4 | -64.6 | 20.4 | 21 |
|  | 13.4 | 51.4 | -60.6 | 22.4 | 41.4 | -59.6 | 23.4 | 39.4 | -58.6 | 12.4 | 49.4 | -57.6 | 10.4 | 50.4 | -56.6 | 21 |
| 21 | -53.6 | 14.4 | 43.4 | -63.6 | 15.4 | 52.4 | -65.6 | 16.4 | 53.4 | -55.6 | 17.4 | 42.4 | -54.6 | 18.4 | 40.4 | 21 |
|  | 59.4 | -91.6 | 36.4 | 60.4 | -82.6 | 26.4 | 61.4 | -81.6 | 24.4 | 62.4 | -92.6 | 34.4 | 63.4 | -94.6 | 35.4 | 21 |
| 21 | 28.4 | 66.4 | -90.6 | 37.4 | 56.4 | -89.6 | 38.4 | 54.4 | -88.6 | 27.4 | 64.4 | -87.6 | 25.4 | 65.4 | -86.6 | 21 |
|  | -83.6 | 29.4 | 58.4 | -93.6 | 30.4 | 67.4 | -95.6 | 31.4 | 68.4 | -85.6 | 32.4 | 57.4 | -84.6 | 33.4 | 55.4 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 44 | 4 | 441 | 441 | 441 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | 102.4 | -18.6 | 4.4 | 103.4 | -9.6 | -5.6 | 104.4 | -8.6 | -7.6 | 105.4 | -19.6 | 2.4 | 106.4 | -21.6 | 3.4 | 441 |
|  | -3. | 109.4 | -17.6 | 5.4 | 99.4 | -16.6 | 6.4 | 97.4 | -15.6 | -4.6 | 107.4 | -14.6 | -6.6 | 108.4 | -13.6 | 1 |
|  | -10.6 | -2.6 | 101.4 | -20. | -1 | 110.4 | -22 | -0 | 111.4 | -12.6 | 0.4 | 100.4 | -11.6 | 1.4 | 98.4 | 1 |
| 441 | -47.6 | 116.4 | 19.4 | -46.6 | 125.4 | 9.4 | -45.6 | 126.4 | 7.4 | -44.6 | 115.4 | 17.4 | -43.6 | 113.4 | 18.4 | 441 |
|  | 11 | -40.6 | 11 | 20 | -5 | 118. | 21 | -52.6 | 119.4 | 10.4 | -42.6 | 120.4 | 8.4 | -41.6 | 121.4 | 1 |
|  | 124.4 | 12.4 | -48 | 114.4 | 13.4 | -39 | 112.4 | 14.4 | -38.6 | 122.4 | 15.4 | -49.6 | 123.4 | 16.4 | -51.6 | 1 |
| 441 | -7 | 131. | 34.4 | -76.6 | 140 | 24.4 | -7 | 141 | 22.4 | -74.6 | 130.4 | 32.4 | -73.6 | 128.4 | 33.4 | 441 |
|  | 26 | -70.6 | 13 | 35 | -8 | 133 | 36 | -8 | 13 | 25.4 | -72.6 | 135.4 | 23.4 | -71.6 | 136.4 | 441 |
| 441 | 139.4 | 27.4 | -78.6 | 129.4 | 28.4 | -69.6 | 127.4 | 29.4 | -68.6 | 137.4 | 30.4 | -79.6 | 138.4 | 31.4 | -81.6 | 441 |
| 4 | 72.4 | -33.6 | 49 | 73.4 | -24. | 39. | 74 | -23.6 | 37.4 | 75.4 | -34.6 | 47.4 | 76.4 | -36.6 | 48.4 | 441 |
| 441 | 41 | 79.4 | -32.6 | 50.4 | 69.4 | -31.6 | 51.4 | 67.4 | -30. | 40.4 | 77.4 | -29.6 | 38.4 | 78.4 | -28.6 | 441 |
| 441 | -25.6 | 42.4 | 71.4 | -35.6 | 43.4 | 80.4 | -37.6 | 44.4 | 81.4 | -27.6 | 45.4 | 70.4 | -26.6 | 46.4 | 68.4 | 441 |
| 441 | 87.4 | -63.6 | 64.4 | 88.4 | -54.6 | 54.4 | 89.4 | -53.6 | 52.4 | 90.4 | -64.6 | 62.4 | 91.4 | -66.6 | 63.4 | 441 |
| 441 | 56.4 | 94.4 | -62.6 | 65.4 | 84.4 | -61.6 | 66.4 | 82.4 | -60.6 | 55.4 | 92.4 | -59.6 | 53.4 | 93.4 | -58.6 | 441 |
|  | -55.6 | 57.4 | 86.4 | -65.6 | 58.4 | 95.4 | -67.6 | 59.4 | 96.4 | -57.6 | 60.4 | 85.4 | -56.6 | 61.4 | 83.4 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | $20711 / 15$ | $86111 / 15$ | $10911 / 15$ | 208 11/15 | $9511 / 15$ | 9911/15 | $20911 / 15$ | 9611/15 | $9711 / 15$ | $21011 / 15$ | $8511 / 15$ | $10711 / 15$ | $21111 / 15$ | $8311 / 15$ | $10811 / 15$ | 2021 |
| 2021 | 10111/15 | 21411/15 | $8711 / 15$ | $11011 / 15$ | 204 11/15 | $8811 / 15$ | 11111/15 | 202 11/15 | $8911 / 15$ | $10011 / 15$ | $21211 / 15$ | $9011 / 15$ | 9811/15 | $21311 / 15$ | 9111/15 | 021 |
| 2021 | 9411/15 | 102 11/15 | $20611 / 15$ | $8411 / 15$ | $10311 / 15$ | $21511 / 15$ | 82 11/15 | 104 11/15 | 216 11/15 | 92 11/15 | $10511 / 15$ | $20511 / 15$ | $93111 / 15$ | $10611 / 15$ | $20311 / 15$ | 2021 |
| 2021 | $5711 / 15$ | $22111 / 15$ | 124 11/15 | $5811 / 15$ | $23011 / 15$ | 11411/15 | $5911 / 15$ | $23111 / 15$ | $11211 / 15$ | $6011 / 15$ | $22011 / 15$ | 122 11/15 | $6111 / 15$ | $21811 / 15$ | $12311 / 15$ | 2021 |
| 20 | 116 11/15 | $6411 / 15$ | 222 11/15 | 125 11/15 | 5411/15 | $22311 / 15$ | 126 11/15 | $5211 / 15$ | $22411 / 15$ | 115 11/15 | $6211 / 15$ | 225 11/15 | 113 11/15 | $6311 / 15$ | 22611 | 2021 |
| 2021 | $22911 /$ | $11711 / 15$ | $5611 / 15$ | 21911 | 11811 | $6511 / 15$ | $21711 / 15$ | $11911 / 15$ | $6611 / 15$ | $22711 / 15$ | $12011 / 15$ | $5511 / 15$ | $22811 / 15$ | 12111/15 | 5311 | 2021 |
| 20 | $2711 / 15$ | $23611 /$ | 13911 | $2811 / 15$ | 24511 | 12911 | $2911 / 15$ | 24611 | 127 11/15 | $3011 / 15$ | $23511 / 15$ | $13711 / 15$ | $3111 / 15$ | $23311 / 15$ | 138 | 2021 |
| 20 | 131 11/15 | $3411 / 15$ | $23711 / 15$ | 14011/15 | 2411/15 | $23811 / 15$ | 14111/15 | $2211 / 15$ | $23911 / 15$ | $13011 / 15$ | $3211 / 15$ | $24011 / 15$ | 128 11/15 | $3311 / 15$ | 24111/15 | 2021 |
| 2021 | 24 | 132 | $2611 / 15$ | 234 | 133 11/15 | $3511 / 15$ | $23211 /$ | 134 11/15 | $3611 / 15$ | $24211 / 15$ | 135 11/15 | $2511 / 15$ | $243111 / 5$ | $136111 / 15$ | 23 11/15 | 2021 |
| 2021 | $17711 / 15$ | 7111/15 | 154 11/15 | 178 11/15 | $8011 / 15$ | $14411 / 15$ | $17911 / 15$ | $8111 / 15$ | 142 11/15 | 180 11/15 | 7011/15 | 152 11/15 | $18111 / 15$ | $6811 / 15$ | $15311 /$ | 2021 |
| 202 | $14611 / 15$ | $18411 / 15$ | $7211 / 15$ | 155 11/15 | 17411/15 | $7311 / 15$ | 156 11/15 | 172 11/15 | $7411 / 15$ | 145 11/15 | $18211 / 15$ | $7511 / 15$ | 14311/15 | 183 11/15 | $7611 / 15$ | 2021 |
| 202 | 7911/15 | $14711 / 15$ | 176 11/15 | 6911/15 | $14811 / 15$ | $18511 / 15$ | $6711 / 15$ | 14911/15 | $18611 / 15$ | $7711 / 15$ | $15011 / 15$ | 175 11/15 | 7811/15 | 15111/15 | 17311 | 2021 |
| 202 | $19211 / 15$ | $4111 / 15$ | 169 11/15 | $193111 / 15$ | $5011 / 15$ | $15911 / 15$ | 194411/15 | $5111 / 15$ | 157 11/15 | 195 11/15 | $4011 / 15$ | 16711/15 | 19611/15 | $3811 / 15$ | $16811 /$ | 2021 |
| 2021 | 16111/15 | $19911 / 15$ | $4211 / 15$ | $17011 / 15$ | 189 11/15 | $4311 / 15$ | 17111/15 | $18711 / 15$ | 4411/15 | 160 11/15 | $19711 / 15$ | $4511 / 15$ | 158 11/15 | $19811 / 15$ | 4611/15 | 2021 |
|  | $4911 / 15$ | $16211 / 15$ | $19111 / 15$ | $3911 / 15$ | $16311 /$ | $20011 / 15$ | $3711 / 15$ | 164 11/15 | $20111 / 15$ | 4711/15 | 165 11/15 | $19011 / 15$ | 4811/15 | $16611 / 15$ | $18811 / 15$ | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 021 | 021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 |

In this case, the blocks of order 3 are semi-magic squares, with following semi-magic sums:

$$
\begin{gathered}
{\left[\operatorname{Sm}_{3 \times 3}: S_{15 \times 15}:=21\right]:=4.2} \\
{\left[\operatorname{Sm}_{3 \times 3}: S_{15 \times 15}:=441\right]:=88.2} \\
{\left[\operatorname{Sm}_{3 \times 3}: S_{15 \times 15}:=2021\right]:=404.2}
\end{gathered}
$$

### 1.8.2 Second Type

|  | pan | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -110.6 | 106.4 | -81.6 | 7.4 | 85.4 | -80.6 | 76.4 | -36.6 | -7.6 | 55.4 | -65.6 | 46.4 | -21.6 | 22.4 | 25.4 | 21 |
| 21 | 8.4 | 97.4 | -109.6 | 99.4 | -88.6 | -6.6 | 67.4 | -79.6 | 69.4 | -43.6 | 23.4 | 37.4 | -64.6 | 39.4 | -28.6 | 21 |
| 21 | 100.4 | -95.6 | 1.4 | 98.4 | -97.6 | 70.4 | -50.6 | -13.6 | 68.4 | -67.6 | 40.4 | -35.6 | 16.4 | 38.4 | -52.6 | 21 |
| 21 | 91.4 | -96.6 | 112.4 | -94.6 | -5.6 | 61.4 | -66.6 | 82.4 | -49.6 | -20.6 | 31.4 | -51.6 | 52.4 | -34.6 | 9.4 | 21 |
| 21 | -82.6 | -4.6 | 84.4 | -103.6 | 113.4 | -37.6 | -19.6 | 54.4 | -73.6 | 83.4 | -22.6 | 10.4 | 24.4 | -58.6 | 53.4 | 21 |
| 21 | -108.6 | 105.4 | -83.6 | 5.4 | 88.4 | -78.6 | 75.4 | -38.6 | -9.6 | 58.4 | -63.6 | 45.4 | -23.6 | 20.4 | 28.4 | 21 |
| 21 | 6.4 | 95.4 | -106.6 | 101.4 | -89.6 | -8.6 | 65.4 | -76.6 | 71.4 | -44.6 | 21.4 | 35.4 | -61.6 | 41.4 | -29.6 | 21 |
| 21 | 103.4 | -93.6 | 0.4 | 96.4 | -99.6 | 73.4 | -48.6 | -14.6 | 66.4 | -69.6 | 43.4 | -33.6 | 15.4 | 36.4 | -54.6 | 21 |
| 21 | 90.4 | -98.6 | 110.4 | -91.6 | -3.6 | 60.4 | -68.6 | 80.4 | -46.6 | -18.6 | 30.4 | -53.6 | 50.4 | -31.6 | 11.4 | 21 |
| 21 | -84.6 | -1.6 | 86.4 | -104.6 | 111.4 | -39.6 | -16.6 | 56.4 | -74.6 | 81.4 | -24.6 | 13.4 | 26.4 | -59.6 | 51.4 | 21 |
| 21 | -107.6 | 107.4 | -85.6 | 3.4 | 89.4 | -77.6 | 77.4 | -40.6 | -11.6 | 59.4 | -62.6 | 47.4 | -25.6 | 18.4 | 29.4 | 21 |
| 21 | 4.4 | 93.4 | -105.6 | 102.4 | -87.6 | -10.6 | 63.4 | -75.6 | 72.4 | -42.6 | 19.4 | 33.4 | -60.6 | 42.4 | -27.6 | 21 |
| 21 | 104.4 | -92.6 | 2.4 | 94.4 | -101.6 | 74.4 | -47.6 | -12.6 | 64.4 | -71.6 | 44.4 | -32.6 | 17.4 | 34.4 | -56.6 | 21 |
| 21 | 92.4 | -100.6 | 108.4 | -90.6 | -2.6 | 62.4 | -70.6 | 78.4 | -45.6 | -17.6 | 32.4 | -55.6 | 48.4 | -30.6 | 12.4 | 21 |
|  | -86.6 | -0.6 | 87.4 | -102.6 | 109.4 | -41.6 | -15.6 | 57.4 | -72.6 | 79.4 | -26.6 | 14.4 | 27.4 | -57.6 | 49.4 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | -82.6 | 134.4 | -53.6 | 35.4 | 113.4 | -52.6 | 104.4 | -8.6 | 20.4 | 83.4 | -37.6 | 74.4 | 6.4 | 50.4 | 53.4 | 441 |
| 441 | 36.4 | 125.4 | -81.6 | 127.4 | -60.6 | 21.4 | 95.4 | -51.6 | 97.4 | -15.6 | 51.4 | 65.4 | -36.6 | 67.4 | -0.6 | 441 |
| 441 | 128.4 | -67.6 | 29.4 | 126.4 | -69.6 | 98.4 | -22.6 | 14.4 | 96.4 | -39.6 | 68.4 | -7.6 | 44.4 | 66.4 | -24.6 | 441 |
| 441 | 119.4 | -68.6 | 140.4 | -66.6 | 22.4 | 89.4 | -38.6 | 110.4 | -21.6 | 7.4 | 59.4 | -23.6 | 80.4 | -6.6 | 37.4 | 441 |
| 441 | -54.6 | 23.4 | 112.4 | -75.6 | 141.4 | -9.6 | 8.4 | 82.4 | -45.6 | 111.4 | 5.4 | 38.4 | 52.4 | -30.6 | 81.4 | 441 |
| 441 | -80.6 | 133.4 | -55.6 | 33.4 | 116.4 | -50.6 | 103.4 | -10.6 | 18.4 | 86.4 | -35.6 | 73.4 | 4.4 | 48.4 | 56.4 | 441 |
| 441 | 34.4 | 123.4 | -78.6 | 129.4 | -61.6 | 19.4 | 93.4 | -48.6 | 99.4 | -16.6 | 49.4 | 63.4 | -33.6 | 69.4 | -1.6 | 441 |
| 441 | 131.4 | -65.6 | 28.4 | 124.4 | -71.6 | 101.4 | -20.6 | 13.4 | 94.4 | -41.6 | 71.4 | -5.6 | 43.4 | 64.4 | -26.6 | 441 |
| 441 | 118.4 | -70.6 | 138.4 | -63.6 | 24.4 | 88.4 | -40.6 | 108.4 | -18.6 | 9.4 | 58.4 | -25.6 | 78.4 | -3.6 | 39.4 | 441 |
| 441 | -56.6 | 26.4 | 114.4 | -76.6 | 139.4 | -11.6 | 11.4 | 84.4 | -46.6 | 109.4 | 3.4 | 41.4 | 54.4 | -31.6 | 79.4 | 441 |
| 441 | -79.6 | 135.4 | -57.6 | 31.4 | 117.4 | -49.6 | 105.4 | -12.6 | 16.4 | 87.4 | -34.6 | 75.4 | 2.4 | 46.4 | 57.4 | 441 |
| 441 | 32.4 | 121.4 | -77.6 | 130.4 | -59.6 | 17.4 | 91.4 | -47.6 | 100.4 | -14.6 | 47.4 | 61.4 | -32.6 | 70.4 | 0.4 | 441 |
| 441 | 132.4 | -64.6 | 30.4 | 122.4 | -73.6 | 102.4 | -19.6 | 15.4 | 92.4 | -43.6 | 72.4 | -4.6 | 45.4 | 62.4 | -28.6 | 441 |
| 441 | 120.4 | -72.6 | 136.4 | -62.6 | 25.4 | 90.4 | -42.6 | 106.4 | -17.6 | 10.4 | 60.4 | -27.6 | 76.4 | -2.6 | 40.4 | 441 |
|  | -58.6 | 27.4 | 115.4 | -74.6 | 137.4 | -13.6 | 12.4 | 85.4 | -44.6 | 107.4 | 1.4 | 42.4 | 55.4 | -29.6 | 77.4 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | $2211 / 15$ | $23911 / 15$ | $5111 / 15$ | $14011 / 15$ | $21811 / 15$ | $5211 / 15$ | 20911/15 | 9611/15 | 125 11/15 | 188 11/15 | $6711 / 15$ | $17911 / 15$ | $11111 / 15$ | 155 11/15 | 158 11/15 | 2021 |
| 2021 | 14111/15 | $23011 / 15$ | $2311 / 15$ | $23211 / 15$ | $4411 / 15$ | $12611 / 15$ | 200 11/15 | $5311 / 15$ | 202 11/15 | $8911 / 15$ | 156 11/15 | 170 11/15 | $6811 / 15$ | 172 11/15 | 104 11/15 | 2021 |
| 2021 | $233111 / 5$ | 3711/15 | $134111 / 15$ | $23111 / 15$ | $3511 / 15$ | 203 11/15 | $8211 / 15$ | 119 11/15 | 20111/15 | $6511 / 15$ | 173 11/15 | $9711 / 15$ | 149 11/15 | $17111 / 15$ | $8011 / 15$ | 2021 |
| 2021 | $22411 / 15$ | $3611 / 15$ | $24511 / 15$ | $3811 / 15$ | $12711 / 15$ | 194 11/15 | $6611 / 15$ | 215 11/15 | $8311 / 15$ | 112 11/15 | 164 11/15 | $8111 / 15$ | $18511 / 15$ | $9811 / 15$ | 142 11/15 | 2021 |
| 2021 | $5011 / 15$ | $12811 / 15$ | $21711 / 15$ | $2911 / 15$ | $24611 / 15$ | $9511 / 15$ | 113 11/15 | 18711/15 | $5911 / 15$ | 216 11/15 | 110 11/15 | 143 11/15 | $15711 / 15$ | 7411/15 | 186 11/15 | 2021 |
| 2021 | $2411 / 15$ | $23811 / 15$ | $4911 / 15$ | $13811 / 15$ | $22111 / 15$ | $5411 / 15$ | $20811 / 15$ | 9411/15 | 123 11/15 | 19111/15 | $6911 / 15$ | $17811 / 15$ | $10911 / 15$ | 153 11/15 | 161 11/15 | 2021 |
| 2021 | $13911 / 15$ | $22811 / 15$ | $2611 / 15$ | $23411 / 15$ | $4311 / 15$ | 124 11/15 | 198 11/15 | $5611 / 15$ | 204 11/15 | $8811 / 15$ | 154 11/15 | 168 11/15 | 7111/15 | 174 11/15 | 103 11/15 | 221 |
| 2021 | $23611 / 15$ | $3911 / 15$ | $13311 / 15$ | $22911 / 15$ | $3311 / 15$ | 206 11/15 | $8411 / 15$ | 118 11/15 | 199 11/15 | $6311 / 15$ | 176 11/15 | $9911 / 15$ | 14811/15 | 169 11/15 | $7811 / 15$ | 2021 |
| 2021 | $22311 / 15$ | $3411 / 15$ | $24311 / 15$ | $4111 / 15$ | $12911 / 15$ | $19311 / 15$ | $6411 / 15$ | $21311 / 15$ | $8611 / 15$ | 114 11/15 | 163 11/15 | $7911 / 15$ | 183 11/15 | $10111 / 15$ | $14411 / 15$ | 2021 |
| 2021 | 4811/15 | $13111 / 15$ | 219 11/15 | $2811 / 15$ | $24411 / 15$ | $9311 / 15$ | 116 11/15 | 189 11/15 | $5811 / 15$ | 214 11/15 | 10811 | 146 11/15 | 159 11/15 | $7311 / 15$ | 184 11/15 | 2021 |
| 2021 | $2511 / 15$ | $24011 / 15$ | $4711 / 15$ | $13611 / 15$ | 222 11/15 | $5511 / 15$ | $21011 / 15$ | $92111 / 15$ | $12111 / 15$ | 192 11/15 | $7011 / 15$ | $18011 / 15$ | $10711 / 15$ | $15111 / 15$ | 162 11/15 | 2021 |
| 2021 | $13711 / 15$ | $22611 / 15$ | $2711 / 15$ | $23511 / 15$ | $4511 / 15$ | 122 11/15 | 196 11/15 | $5711 / 15$ | 205 11/15 | $9011 / 15$ | 152 11/15 | 166 11/15 | $7211 / 15$ | 175 11/15 | $10511 / 15$ | 2021 |
| 2021 | $23711 / 15$ | $4011 / 15$ | 135 11/15 | $22711 / 15$ | $3111 / 15$ | 20711/15 | $8511 / 15$ | 120 11/15 | 19711/15 | $6111 / 15$ | 177 11/15 | 100 11/15 | 150 11/15 | 167 11/15 | $7611 / 15$ | 2021 |
| 2021 | $22511 / 15$ | $3211 / 15$ | $24111 / 15$ | 4211/15 | $13011 / 15$ | 195 11/15 | $6211 / 15$ | $21111 / 15$ | $8711 / 15$ | 115 11/15 | 165 11/15 | $7711 / 15$ | $18111 / 15$ | 102 11/15 | 145 11/15 | 2021 |
|  | $4611 / 15$ | $13211 / 15$ | $22011 / 15$ | $3011 / 15$ | $24211 / 15$ | 9111/15 | $11711 / 15$ | 190 11/15 | $6011 / 15$ | 212 11/15 | 106 11/15 | $14711 / 15$ | $16011 / 15$ | $7511 / 15$ | 182 11/15 | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, blocks of order 5 are pandiagonal with equal magic square sums. These sums are given by

$$
\begin{gathered}
{\left[S_{5 \times 5}: S_{15 \times 15}:=21\right]:=7} \\
{\left[S_{5 \times 5}: S_{15 \times 15}:=441\right]:=147} \\
{\left[S_{5 \times 5}: S_{15 \times 15}:=2021\right]:=\frac{2021}{3} .}
\end{gathered}
$$

### 1.9 Block-Wise Magic Squares of Order 16

Below are block-wise magic squares of order 16 in two different ways giving magic sums $21,21^{2}$ and 2021. One is where blocks of order 4 are pandiagonal magic squares of equal magic sums. The second, where blocks of order 4 are magic squares resulting in bimagic square of order 16 .

### 1.9.1 First Type



|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | -3.9375 | 91.0625 | -99.9375 | 123.0625 | $-2.9375$ | 90.0625 | -98.9375 | 122.0625 | $-1.9375$ | 89.0625 | -97.9375 | 121.0625 | $-0.9375$ | 88.0625 | -96.9375 | 120.0625 | 441 |
| 441 | -68.9375 | 92.0625 | 27.0625 | 60.0625 | -69.9375 | 93.0625 | 26.0625 | 61.0625 | -70.9375 | 94.0625 | 25.0625 | 62.0625 | -71.9375 | 95.0625 | 24.0625 | 63.0625 | 441 |
| 441 | 155.0625 | -67.9375 | 59.0625 | -35.9375 | 154.0625 | -66.9375 | 58.0625 | -34.9375 | 153.0625 | -65.9375 | 57.0625 | -33.9375 | 152.0625 | -64.9375 | 56.0625 | -32.9375 | 441 |
| 441 | 28.0625 | -4.9375 | 124.0625 | -36.9375 | 29.0625 | $-5.9375$ | 125.0625 | -37.9375 | 30.0625 | -6.9375 | 126.0625 | -38.9375 | 31.0625 | -7.9375 | 127.0625 | -39.9375 | 441 |
| 441 | 0.0625 | 87.0625 | -95.9375 | 119.0625 | 1.0625 | 86.0625 | -94.9375 | 118.0625 | 2.0625 | 85.0625 | -93.9375 | 117.0625 | 3.0625 | 84.0625 | -92.9375 | 116.0625 | 441 |
| 441 | -72.9375 | 96.0625 | 23.0625 | 64.0625 | -73.9375 | 97.0625 | 22.0625 | 65.0625 | -74.9375 | 98.0625 | 21.0625 | 66.0625 | -75.9375 | 99.0625 | 20.0625 | 67.0625 | 441 |
| 441 | 151.0625 | -63.9375 | 55.0625 | -31.9375 | 150.0625 | -62.9375 | 54.0625 | -30.9375 | 149.0625 | -61.9375 | 53.0625 | -29.9375 | 148.0625 | -60.9375 | 52.0625 | -28.9375 | 441 |
| 441 | 32.0625 | -8.9375 | 128.0625 | -40.9375 | 33.0625 | -9.9375 | 129.0625 | -41.9375 | 34.0625 | -10.9375 | 130.0625 | -42.9375 | 35.0625 | -11.9375 | 131.0625 | -43.93 | 441 |
| 441 | 4.0625 | 83.0625 | -91.9375 | 115.0625 | 5.0625 | 82.0625 | -90.9375 | 114.0625 | 6.0625 | 81.0625 | -89.9375 | 113.0625 | 7.0625 | 80.0625 | -88.9375 | 112.0625 | 441 |
| 441 | -76.9375 | 100.0625 | 19.0625 | 68.0625 | -77.9375 | 101.0625 | 18.0625 | 69.0625 | -78.9375 | 102.0625 | 17.0625 | 70.0625 | -79.9375 | 103.0625 | 16.0625 | 71.0625 | 441 |
| 441 | 147.0625 | -59.9375 | 51.0625 | -27.9375 | 146.0625 | -58.9375 | 50.0625 | -26.9375 | 145.0625 | -57.9375 | 49.0625 | -25.9375 | 144.0625 | -56.9375 | 48.0625 | -24.9375 | 441 |
| 441 | 36.0625 | -12.9375 | 132.0625 | -44.9375 | 37.0625 | -13.9375 | 133.0625 | -45.9375 | 38.0625 | -14.9375 | 134.0625 | -46.9375 | 39.0625 | -15.9375 | 135.0625 | -47.9375 | 441 |
| 441 | 8.0625 | 79.0625 | -87.9375 | 11.0625 | 9.0625 | 78.0625 | -86.9375 | 110.0625 | 10.0625 | 77.0625 | -85.9375 | 109.0625 | 11.0625 | 76.0625 | -84.9375 | 108.0625 | 441 |
| 441 | -80.9375 | 104.0625 | 15.0625 | 72.0625 | -81.9375 | 105.0625 | 14.0625 | 73.0625 | -82.9375 | 106.0625 | 13.0625 | 74.0625 | -83.9375 | 107.0625 | 12.0625 | 75.0625 | 441 |
| 441 | 14.0625 | -55.9375 | 47.0625 | -23.9375 | 142.0625 | -54.9375 | 46.0625 | -22.9375 | 141.0625 | -53.9375 | 45.0625 | -21.9375 | 140.0625 | -52.9375 | 44.0625 | -20.9375 | 441 |
|  | 40.0625 | -16.9375 | 136.0625 | -48.9375 | 41.0625 | -17.9375 | 137.0625 | -49.9375 | 42.0625 | -18.9375 | 138.0625 | -50.9375 | 43.0625 | -19.9375 | 139.0625 | -51.9375 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | 94.8125 | 189.8125 | -1.1875 | 221.8125 | 95.8125 | 188.8125 | -0.1875 | 220.8125 | 96.8125 | 187.8125 | 0.8125 | 29.8125 | 97.8125 | 186.8125 | 1.8125 | 218.8125 |  |
| 2021 | 29.8125 | 190.8125 | 125.8125 | 158.8125 | 28.8125 | 199.8125 | 124.8125 | 159.8125 | 27.8125 | 192.8125 | 123.8125 | 160.8125 | 26.8125 | 199.8125 | 122.8125 | 161.8125 | 2021 |
| 2021 | 253.8125 | 30.8125 | 157.8125 | 1235 | 252.8125 | 8125 | 156.8125 | 63.8125 | 251.8125 | 32.8125 | 155.8125 | 64.8125 | 250.8125 | 33.8125 | 154.8125 | 65.8125 | 2021 |
| 2021 | 126.8125 | 93.8125 | 222.8125 | 61.8125 | 127.8125 | 92.8125 | 223.8125 | 60.8125 | 128.8125 | 91.8125 | 224.8125 | 59.8125 | 129.8125 | 90.8125 | 225.8125 | 58.8125 | 2021 |
| 2021 | 98.8125 | 185.8125 | 2.8125 | 27.8125 | 99.8125 | 184.8125 | 3.8125 | 216.8125 | 100.8125 | 183.8125 | 4.8125 | 215.8125 | 101.8125 | 182.8125 | 5.8125 | 214.8125 | 2021 |
| 2021 | 25.8125 | 194.8125 | 121.8125 | 162.8125 | 24.8125 | 195.8125 | 120.8125 | 163.8125 | 23.8125 | 196.8125 | 119.8125 | 164.8125 | 22.8125 | 197.8125 | 118.8125 | 165.8125 | 2021 |
| 2021 | 249.8125 | 34.8125 | 155.8125 | 6.81215 | 248.8125 | 35.8125 | 152.8125 | 67.8125 | 247.8125 | 36.8125 | 151.8125 | 68.8125 | 246.8125 | 37.8125 | 150.8125 | 69.8125 | 2021 |
| 2021 | 130.8125 | 89.8125 | 226.8125 | 57.8125 | 131.8125 | 88.8125 | 227.8125 | 56.8125 | 132.8125 | 87.8125 | 228.8125 | 55.8125 | 133.8125 | 86.8125 | 229.8125 | 54.8125 | 2021 |
| 2021 | 102.8125 | 181.8125 | 6.8125 | 213.8125 | 103.8125 | 180.8125 | 7.8125 | 212.8125 | 104.8125 | 179.8125 | 8.8125 | 21.8125 | 105.8125 | 178.8125 | 9.8125 | 210.8125 | 2021 |
| 2021 | 21.8125 | 198.8125 | 117.8125 | 166.8125 | 20.8125 | 199.8125 | 116.8125 | 167.8125 | 19.8125 | 200.8125 | 115.8125 | 168.8125 | 18.8125 | 201.8125 | 114.8125 | 169.8125 | 2021 |
| 2021 | 245.8125 | 38.8125 | 149.8125 | 70.8125 | 244.8125 | 39.8125 | 148.8125 | 71.8125 | 244.8125 | 40.8125 | 147.8125 | 72.8125 | 242.8125 | 4.8125 | 146.8125 | 73.8125 | 2021 |
| 2021 | 134.8125 | 85.8125 | 230.8125 | 53.8125 | 135.8125 | 84.8125 | 231.8125 | 52.8125 | 136.8125 | 83.8125 | 232.8125 | 51.8125 | 137.8125 | 82.8125 | 233.8125 | 50.8125 | 2021 |
| 2021 | 106.8125 | 177.8125 | 10.8125 | 209.8125 | 107.8125 | 176.8125 | 11.8125 | 208.8125 | 108.8125 | 175.8125 | 12.8125 | 207.8125 | 109.8125 | 174.8125 | 13.8125 | 206.8125 | 2021 |
| 2021 | 17.8125 | 202.8125 | 113.8125 | 170.8125 | 16.8125 | 203.8125 | 112.8125 | 17.17125 | 15.8125 | 204.8125 | 11.18125 | 172.8125 | 14.8125 | 205.8125 | 110.8125 | 173.1215 | 2021 |
| 2021 | 24.818125 | 42.8125 | 145.8125 | 74.8125 | 240.8125 | 43.8125 | 144.8125 | 75.8125 | 239.8125 | 44.8125 | 14.8 .8125 | 76.8125 | 238.8125 | 45.8125 | 142.8125 | 77.8125 | 2021 |
|  | 138.8125 | 81.8125 | 234.8125 | 49.8125 | 139.8125 | 80.8125 | 235.8125 | 48.8125 | 140.8125 | 79.8125 | 236.8125 | 47.8125 | 14.8125 | 78.8125 | 237.8125 | 4.88125 | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the block of order 4 are pandiagonal magic squares with equal magic sums given by

$$
\begin{gathered}
{\left[S_{4 \times 4}: S_{16 \times 16}:=21\right]:=\frac{21}{4}=5.25} \\
{\left[S_{4 \times 4}: S_{16 \times 16}:=441\right]:=\frac{441}{4}=110.25} \\
{\left[S_{4 \times 4}: S_{16 \times 16}:=2021\right]:=\frac{2021}{4}=505.25}
\end{gathered}
$$

### 1.9.2 Second Type

| mgc |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2121 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -126.1875 | 26.8125 | 11.8125 | $-7.1875$ | -104.1875 | 16.8125 | 121.8125 | -29.1875 | -83.1875 | 51.8125 | 70.8125 | $-34.1875$ | -65.1875 | 37.8125 | 84.8125 | -52.1875 |  |
| 104.8125 | -0.1875 | -117.1875 | 17.8125 | 14.8125 | -22.1875 | -95.1875 | 7.8125 | 77.8125 | -41.1875 | -92.1875 | 60.8125 | 91.8125 | -59.1875 | -74.1875 | 46.8125 | 21 |
| -5.1875 | 97.8125 | 24.8125 | -112.1875 | -15.1875 | 119.8125 | 2.8125 | -102.1875 | -44.1875 | 76.8125 | 61.8125 | -89.1875 | -58.877 | 94.8125 | 43.8125 | -75.1875 | 21 |
| 31.8125 | -119.1875 | -14.1875 | 106.8125 | 9.8125 | -109.1875 | -24.1875 | 128.8125 | 54.8125 | -82.1875 | -35.1875 | 67.8125 | 36.8125 | -68.1875 | -49.1875 | 85.8125 | 21 |
| -81.1875 | 53.8125 | 68.8125 | -36.1875 | -67.1875 | 35.8125 | 86.8125 | $-50.1875$ | -120.1875 | 32.8125 | 105.8125 | $-13.1875$ | -110.1875 | 10.8125 | 127.8125 | -23.1875 | 21 |
| 75.8125 | -43.1875 | -90.1875 | 62.8125 | 93.8125 | -57.1875 | -76.1875 | 44.8125 | 98.8125 | $-6.1875$ | -111.1875 | 23.8125 | 120.8125 | $-16.1875$ | -101.1875 | 1.8125 | 21 |
| -42.1875 | 78.8125 | 59.8125 | -91.1875 | -60.1875 | 92.8125 | 45.8125 | -73.1875 | 0.8125 | 103.8125 | 18.8125 | -118.1875 | -21.1875 | 113.8125 | 8.8125 | -96.1875 | 21 |
| 52.8125 | -84.1875 | -33.1875 | 69.8125 | 38.8125 | -66.1875 | -51.1875 | 83.8125 | 25.8125 | -125.1875 | -8.875 | 112.8125 | 15.8125 | -103.1875 | -30.1875 | 122.8125 | 21 |
| -72.1875 | 48.8125 | 89.8125 | -61.1875 | -94.1875 | 58.8125 | 79.8125 | -39.875 | -97.1875 | 5.8125 | 116.8125 | -20.1875 | -115.1875 | 19.8125 | 102.8125 | -2.1875 | 21 |
| 82.8125 | -54.1875 | -63.1875 | 39.8125 | 72.8125 | -32.1875 | -85.1875 | 49.8125 | 123.8125 | $-27.1875$ | -106.1875 | 14.8125 | 109.8125 | -9.1875 | -124.1875 | 28.8125 | 21 |
| -47.1875 | 87.8125 | 34.8125 | $-70.1875$ | -37.1875 | 65.8125 | 56.8125 | -80.1875 | -26.1875 | 126.8125 | 11.8125 | -107.1875 | -12.1875 | 108.8125 | 29.8125 | -121.1875 | 21 |
| 41.8125 | -77.1875 | -56.1875 | 96.8125 | 63.8125 | -87.1875 | -46.1875 | 74.8125 | 4.8125 | -100.1875 | -17.1875 | 117.8125 | 22.8125 | -14.1875 | $-3.1875$ | 99.8125 | 21 |
| -99.1875 | 3.8125 | 118.8125 | -18.1875 | -113.1875 | 21.8125 | 100.8125 | -4.1875 | -78.1875 | 42.8125 | 95.8125 | -55.1875 | -88.1875 | 64.8125 | 73.8125 | -45.1875 | 21 |
| 125.8125 | -25.1875 | -108.1875 | 12.8125 | 107.8125 | -11.1875 | -122.1875 | 30.8125 | 88.8125 | -48.1875 | -69.1875 | 33.8125 | 66.8125 | -38.1875 | -79.1875 | 55.8125 | 21 |
| -28.1875 | 124.8125 | 13.8125 | -105.1875 | -10.1875 | 110.8125 | 27.8125 | -123.1875 | $-53.1875$ | 81.8125 | 40.8125 | -64.1875 | -31.1875 | 71.8125 | 50.8125 | -86.1875 | 21 |
| 6.8125 | -98.1875 | -19.1875 | 115.8125 | 20.8125 | -116.1875 | $-1.1875$ | 101.8125 | 47.8125 | -71.1875 | -62.1875 | 90.8125 | 57.8125 | -93.1875 | -40.1875 | 80.8125 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| mgc |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 441 <br> 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -99.9375 | 53.0625 | 138.0625 | 19.0625 | -77.9375 | 43.0625 | 148.0625 | $-2.9375$ | -56.9375 | 78.0625 | 97.0625 | -7.9375 | -38.9375 | 64.0625 | 111.0625 | -25.9375 |  |
| 131.0625 | 26.0625 | -90.9375 | 44.0625 | 141.0625 | 4.0625 | -68.9375 | 34.0625 | 104.0625 | -14.9375 | -65.9375 | 87.0625 | 118.0625 | -32.9375 | -47.9375 | 73.0625 | 441 |
| 21.0625 | 124.0625 | 51.0625 | -85.9375 | 11.0625 | 146.0625 | 29.0625 | -75.9375 | -17.9375 | 103.0625 | 88.0625 | -62.9375 | -31.9375 | 121.0625 | 70.0625 | -48.9375 | 441 |
| 58.0625 | -92.9375 | 12.0625 | 133.0625 | 36.0625 | -82.9375 | 2.0625 | 155.0625 | 81.0625 | -55.9375 | -8.9375 | 94.0625 | 63.0625 | -41.9375 | -22.9375 | 112.0625 | 441 |
| -54.9375 | 80.0625 | 95.0625 | $-9.9375$ | -40.9375 | 62.0625 | 113.0625 | -23.9375 | -93.9375 | 59.0625 | 132.0625 | 13.0625 | -83.9375 | 37.0625 | 154.0625 | 3.0625 | 441 |
| 102.0625 | -16.9375 | -63.9375 | 89.0625 | 120.0625 | -30.9375 | -49.9375 | 71.0625 | 125.0625 | 20.0625 | -84.9375 | 50.0625 | 147.0625 | 10.0625 | -74.9375 | 28.0625 | 441 |
| -15.9375 | 105.0625 | 86.0625 | -64.9375 | -33.9375 | 119.0625 | 72.0625 | -46.9375 | 27.0625 | 130.0625 | 45.0625 | -91.9375 | 5.0625 | 140.0625 | 35.0625 | -69.9375 | 441 |
| 79.0625 | -57.9375 | -6.9375 | 96.0625 | 65.0625 | -39.9375 | -24.9375 | 110.0625 | 52.0625 | -98.9375 | 18.0625 | 139.0625 | 42.0625 | -76.9375 | -3.9375 | 149.0625 | 441 |
| -45.9375 | 75.0625 | 116.0625 | -34.9375 | -67.9375 | 85.0625 | 106.0625 | -12.9375 | -70.9375 | 32.0625 | 143.0625 | 6.0625 | -88.9375 | 46.0625 | 129.0625 | 24.0625 | 441 |
| 109.0625 | -27.9375 | -36.9375 | 66.0625 | 99.0625 | -5.9375 | -58.9375 | 76.0625 | 150.0625 | $-0.9375$ | -79.9375 | 41.0625 | 136.0625 | 17.0625 | -97.9375 | 55.0625 | 441 |
| -20.9375 | 114.0625 | 61.0625 | -43.9375 | -10.9375 | 92.0625 | 83.0625 | -53.9375 | 0.0625 | 153.0625 | 38.0625 | -80.9375 | 14.0625 | 135.0625 | 56.0625 | -94.9375 | 441 |
| 68.0625 | -50.9375 | -29.9375 | 123.0625 | 90.0625 | -60.9375 | -19.9375 | 101.0625 | 31.0625 | -73.9375 | 9.0625 | 144.0625 | 49.0625 | -87.9375 | 23.0625 | 126.0625 | 441 |
| -72.9375 | 30.0625 | 145.0625 | 8.0625 | -86.9375 | 48.0625 | 127.0625 | 22.0625 | -51.9375 | 69.0625 | 122.0625 | -28.9375 | -61.9375 | 91.0625 | 100.0625 | -18.9375 | 441 |
| 152.0625 | 1.0625 | -81.9375 | 39.0625 | 134.0625 | 15.0625 | -95.9375 | 57.0625 | 115.0625 | -21.9375 | -42.9375 | 60.0625 | 93.0625 | -11.9375 | -52.9375 | 82.0625 | 441 |
| -1.9375 | 151.0625 | 40.0625 | -78.9375 | 16.0625 | 137.0625 | 54.0625 | -96.9375 | -26.9375 | 108.0625 | 67.0625 | -37.9375 | -4.9375 | 98.0625 | 77.0625 | -59.9375 | 441 |
| 33.0625 | -71.9375 | 7.0625 | 142.0625 | 47.0625 | -89.9375 | 25.0625 | 128.0625 | 74.0625 | -44.9375 | -35.9375 | 117.0625 | 84.0625 | -66.9375 | -13.9375 | 107.0625 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


| mgc |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1.1875 | 151.8125 | 236.8125 | 117.8125 | 20.8125 | 14.81812 | 246.8125 | 95.8125 | 41.8125 | 176.8125 | 195.8125 | 90.8125 | 59.8125 | 162.8125 | 209.8125 | 72.8125 |  |
| 229.8125 | 124.8125 | 7.8125 | 142.8125 | 239.8125 | 102.8125 | 29.8125 | 132.8125 | 202.8125 | 83.8125 | 32.8125 | 185.8125 | 216.8125 | 65.8125 | 50.8125 | 171.8125 | 2021 |
| 119.8125 | 222.8125 | 149.8125 | 12.8125 | 109.8125 | 244.8125 | 127.8125 | 22.8125 | 80.8125 | 201.8125 | 186.8125 | 35.8125 | 66.8125 | 219.8125 | 168.8125 | 49.8125 | 2021 |
| 15.86125 | 5.8125 | 110.8125 | 231.8125 | 134.8125 | 15.8125 | 100.8125 | 253.8125 | 179.8125 | 42.8125 | 89.8125 | 192.8125 | 171.8125 | 56.8125 | 75.8125 | 210.8125 | 2021 |
| 43.8125 | 178.8125 | 193.8125 | 88.8125 | 57.8125 | 160.8125 | 21.81815 | 74.8125 | 4.8125 | 157.8125 | 230.8125 | 11.81215 | 14.8125 | 135.8125 | 252.8125 | 10.8125 | 2021 |
| 200.8125 | 81.8125 | 34.8125 | 187.8125 | 218.8125 | 67.8125 | 48.8125 | 169.8125 | 223.8125 | 118.8125 | 13.8125 | 1488.125 | 245.8125 | 108.8125 | 23.8125 | 126.8125 | 2021 |
| 82.8125 | 203.8125 | 184.8125 | 33.8125 | 64.8125 | 217.125 | 170.8125 | 51.8125 | 125.8125 | 228.8125 | 14.8 .815 | 6.8125 | 103.8125 | 238.8125 | 133.8125 | 28.8125 | 2021 |
| 177.8125 | 40.8125 | 9.8125 | 194.8125 | 163.8125 | 58.8125 | 73.8125 | 208.8125 | 150.8125 | -0.1875 | 11.8 .8125 | 237.8125 | 140.8125 | 21.8125 | 94.8125 | 247.8125 | 2021 |
| 52.8125 | 173.8125 | 214.8125 | 63.8125 | 30.8125 | 183.8125 | 204.8125 | 85.8125 | 27.8125 | 130.8125 | 241.8125 | 104.8125 | 9.8125 | 144.8125 | 227.812 | 122.8125 | 2021 |
| 207.8125 | 70.8125 | 61.8125 | 164.8125 | 197.812 | 92.8125 | 39.8125 | 174.8125 | 2488.812 | 97.8125 | 18.8125 | 139.8125 | 234.8125 | 115.812 | 0.8125 | 155.8125 | 2021 |
| 77.8125 | 212.8125 | 159.8125 | 54.8125 | 87.8125 | 190.8125 | 181.8125 | 44.8125 | 98.8125 | 251.8125 | 136.8125 | 17.8125 | 112.8125 | 233.8125 | 154.8125 | 3.8125 | 2021 |
| 166.8125 | 47.8125 | 68.8125 | 221.8125 | 188.8125 | 37.8125 | 78.8125 | 199.8125 | 129.8125 | 24.8125 | 107.8125 | 242.8125 | 147.8125 | 10.8125 | 121.8125 | 224.8125 | 2021 |
| 25.8125 | 128.8125 | 243.8125 | 106.8125 | 11.8125 | 146.8125 | 225.8125 | 120.8125 | 46.8125 | 167.8125 | 220.8125 | 69.8125 | 36.8125 | 189.8125 | 198.8125 | 79.8125 | 2021 |
| 250.8125 | 99.8125 | 16.8125 | 137.8125 | 232.8125 | 11.81215 | 2.8125 | 155.8125 | 213.8125 | 76.8125 | 55.8125 | 158.8125 | 199.8125 | 86.8125 | 4.85125 | 188.8125 | 2021 |
| 96.8125 | 249.8125 | 138.8125 | 19.8125 | 14.8125 | 235.8125 | 152.8125 | 1.8125 | 71.8125 | 206.8125 | 165.8125 | 60.8125 | 93.8125 | 196.8125 | 175.8125 | 38.8125 | 2021 |
| 131.8125 | 26.8125 | 105.8125 | 240.8125 | 145.8125 | 8.8125 | 123.8125 | 226.8125 | 172.812 | 53.8125 | 62.8125 | 215.8125 | 182.8125 | 318182 | 84.8125 | 205.8125 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the block of order 4 are magic squares with equal magic sums. The sums are same given above. Moreover, the magic square of order 16 is bimagic square with bimagic sums given by

$$
\begin{aligned}
{\left[S b_{16 \times 16}: S_{16 \times 16}:=21\right] } & :=87407.5625 \\
{\left[S b_{16 \times 16}: S_{16 \times 16}:=441\right] } & :=99535.0625 \\
{\left[S b_{16 \times 16}: S_{16 \times 16}:=2021\right] } & :=342657.5625
\end{aligned}
$$

### 1.10 Block-Bordered Magic Squares of Order 17

Below are block-bordered magic squares of order 17 in two different ways giving magic sums $21,21^{2}$ and 2021. The blockwise magic square considered is of order 15 with blocks of order 3 and 5 . The blocks of order 3 are semi-magic with equal semi-magic sums. The blocks of order 5 are pandiagonal with equal magic sums. In both the cases the magic square of
order 15 is pandiagonal as given in Subsection 1.8. The magic sums of order 15 are given by

$$
\begin{gathered}
{\left[S_{15 \times 15}: S_{17 \times 17}:=21\right]:=\frac{315}{17}} \\
{\left[S_{15 \times 15}: S_{17 \times 17}:=441\right]:=\frac{6615}{17}} \\
{\left[S_{15 \times 15}: S_{17 \times 17}:=2021\right]:=\frac{30315}{17} .}
\end{gathered}
$$

### 1.10.1 First Type

| -127 13/17 | 1444177 | $1424 / 17$ | 1404117 | $1384 / 17$ | $1364 / 17$ | 1344177 | $1324 / 17$ | $1314 / 17$ | -123 13/17 | -12113/17 | -119 13/17 | -117 13/17 | -11513/17 | -113 13/17 | $-11113 / 17$ | $12513 / 17$ | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -142 13/17 | 74417 | $-4613 / 17$ | $-2313 / 17$ | $754 / 17$ | -37 13/17 | $-3313 / 17$ | $764 / 17$ | -3613/17 | -3513/17 | 77417 | -4713/17 | $-2513 / 17$ | $784 / 17$ | -4913/17 | $-2413 / 17$ | $1454 / 17$ | 21 |
| -140 13/17 | -3113/17 | $814 / 17$ | -4513/17 | -22 13/17 | 71417 | $-4413 / 17$ | -2113/17 | 69417 | $-4313 / 17$ | -32 13/17 | 17 | $-4213 / 17$ | -34 13/17 | 804117 | $-4113 / 17$ | 17 | 21 |
| -138 13/17 | -38 13/17 | -30 13/17 | 73417 | -48 13/17 | -29 13/17 | $824 / 17$ | -50 13/17 | -28 13/17 | 834117 | -4013/17 | $-2713 / 17$ | 724177 | -39 13/17 | -2613/17 | 70417 | $1414 / 17$ | 21 |
| -136 13/17 | -75 13/17 | 88 | -8 | -7413/17 | $974 / 17$ | -18 13/17 | -73 | $984 / 17$ | -20 13/17 | -72 13/17 | $874 / 17$ | -10 13/17 | -7113/17 | $854 / 17$ | -913/17 | 17 | 21 |
| -134 13/1 | -1613/17 | -68 13/17 | 89417 | $-713 / 17$ | -78 13/17 | $904 / 17$ | -613/17 | -80 13/17 | 91417 | -17 13/17 | -70 13/17 | 17 | -19 13/17 | -69 13/17 | 93417 | 17 | 21 |
| -132 13/17 | 96417 | -15 13/17 | -7613/17 | $864 / 17$ | -14 13/17 | -67 13/17 | $844 / 17$ | -13 $13 / 17$ | -6613/17 | 94417 | $-1213 / 17$ | -77 13/17 | $954 / 17$ | -1113/17 | -79 13/17 | $1354 / 17$ | 21 |
| -130 13/17 | -105 13/17 | 1034 | 6 | -104 13/17 | $1124 / 17$ | $-313 / 17$ | -103 13/17 | 113 | $-513 / 17$ | -102 13/17 | $1024 / 17$ | 17 | -101 13/17 | $1004 / 17$ | 17 | 17 | 21 |
| $1294 / 17$ | -113/17 | -9813/17 | $1044 / 17$ | $74 / 17$ | -108 13/17 | $1054 / 17$ | $84 / 17$ | -110 13/17 | $1064 / 17$ | -213/17 | -100 13/17 | $1074 / 17$ | $-413 / 17$ | -9913/17 | $1084 / 17$ | -126 13/17 | 21 |
| $1274 / 17$ | 111417 | -13/17 | -10613/17 | $1014 / 17$ | $4 / 17$ | -9713/17 | 994177 | $14 / 17$ | -9613/17 | $1094 / 17$ | $24 / 17$ | -107 13/17 | $1104 / 17$ | $34 / 17$ | -109 13/17 | -124 13/17 | 21 |
| $1254 / 17$ | 444177 | $-6113 / 17$ | $214 / 17$ | $454 / 17$ | -52 13/17 | $114 / 17$ | $464 / 17$ | -51 13/17 | 94117 | $474 / 17$ | -62 13/17 | 19 4/17 | $484 / 17$ | -64 13/17 | 20417 | -122 13/17 | 21 |
| 1234177 | $134 / 17$ | $514 / 17$ | -60 13/17 | $224 / 17$ | $414 / 17$ | -59 13/17 | 23417 | 39417 | -5813/17 | $124 / 17$ | 494177 | -57 13/17 | $104 / 17$ | $504 / 17$ | -56 13/17 | -120 13/17 | 21 |
| $1214 / 17$ | -5313/17 | $144 / 17$ | 434117 | -63 13/17 | $154 / 17$ | $524 / 17$ | -65 13/17 | $164 / 17$ | 53417 | -5513/17 | $174 / 17$ | 424177 | -54 13/17 | $184 / 17$ | $404 / 17$ | -118 13/17 | 21 |
| $1194 / 17$ | $594 / 17$ | $-9113 / 17$ | $364 / 17$ | $604 / 17$ | -82 13/17 | $264 / 17$ | $614 / 17$ | -8113/17 | 24417 | 62417 | -92 13/17 | $344 / 17$ | $634 / 17$ | -9413/17 | 35417 | -116 13/17 | 21 |
| $1174 / 17$ | 28417 | $664 / 17$ | -90 13/17 | $374 / 17$ | $564 / 17$ | -8913/17 | $384 / 17$ | $544 / 17$ | -88 13/17 | 27417 | $644 / 17$ | -87 13/17 | $254 / 17$ | $654 / 17$ | -8613/17 | -114 13/17 | 21 |
| $1154 / 17$ | -83 13/17 | $294 / 17$ | $584 / 17$ | -93 13/17 | $304 / 17$ | 67417 | -9513/17 | 314177 | 684117 | -8513/17 | 324117 | $574 / 17$ | -84 13/17 | 334117 | $554 / 17$ | -112 13/17 | 21 |
| $1284 / 17$ | -141 13/17 | -139 13/17 | -137 13/17 | -135 13/17 | -133 13/17 | -13113/17 | -129 13/17 | -128 13/17 | $11264 / 17$ | $1244 / 17$ | $1224 / 17$ | $1204 / 17$ | $1184 / 17$ | $1164 / 17$ | $1144 / 17$ | $1304 / 17$ | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |



| -10 2/17 | 26115/17 | 259 15/17 | $25715 / 17$ | 255 15/17 | 253 15/17 | 251 15/17 | $24915 / 17$ | $24815 / 17$ | -6 2/17 | -4 2/17 | -2 2/17 | -2/17 | 115/17 | 315/17 | 515/17 | -8 2/17 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -25 2/17 | 19115/17 | $7015 / 17$ | 9315/17 | 192 15/17 | $7915 / 17$ | $8315 / 17$ | $19315 / 17$ | 80 15/17 | 8115/17 | 19415/17 | 6915/17 | 9115/17 | 19515/17 | $6715 / 17$ | 92 15/17 | 262 15/17 | 2021 |
| -23 2/17 | 85 15/17 | 198 | 71 | 9415/17 | 188 15/17 | $7215 / 17$ | 9515/17 | 18615/17 | $7315 / 17$ | 84 15/17 | 19615/17 | 7415/17 | 82 15/17 | $19715 / 17$ | $7515 / 17$ | 17 | 2021 |
| -21 2/17 | $7815 / 17$ | $8615 / 17$ | $19015 / 17$ | 6815/17 | 8715/17 | 19915/17 | 6615/17 | 8815/17 | 20015/17 | 7615/17 | 8915/17 | 189 15/17 | 77 15/17 | 9015/17 | 187 15/17 | 25815/17 |  |
| -19 2/17 | $4115 / 17$ | $20515 / 17$ | $10815 / 17$ | 42 15/17 | 21415/17 | 9815/17 | $4315 / 17$ | $21515 / 17$ | 9615/17 | 4415/17 | 20415/17 | 10615/17 | $4515 / 17$ | 202 15/17 | $10715 / 17$ | 256 15/17 | 2021 |
| -172/17 | 100 15/17 | 4815/17 | 20615/ | $10915 / 17$ | $3815 / 17$ | 20715/17 | $11015 / 17$ | $3615 / 17$ | 20815/17 | 9915/17 | 4615/17 | 20915/17 | 9715/17 | $4715 / 17$ | $21015 / 17$ | 25415 | 2021 |
| -15 2/17 | 213 15/17 | $10115 / 17$ | 4015/17 | 20315 | 102 15/17 | $4915 / 17$ | $20115 / 17$ | $10315 / 17$ | $5015 / 17$ | 21115/17 | 104 15/17 | $3915 / 17$ | 212 15/17 | 105 15/17 | $3715 / 17$ | $25215 / 17$ |  |
| -13 2/17 | $1115 / 17$ | $22015 / 17$ | 123 15/17 | 12 15/17 | $22915 / 17$ | 113 15/17 | $1315 / 17$ | $23015 / 17$ | 11115/17 | $1415 / 17$ | $21915 / 17$ | $12115 / 17$ | 1515/17 | 217 15/17 | 122 15/17 | 25015/17 | 2021 |
| $24615 / 17$ | 115 15/17 | 1815/17 | $22115 / 17$ | 124 15/ | $815 / 17$ | $22215 / 17$ | $12515 / 17$ | 615/17 | 22315/17 | 114 15/17 | 1615/17 | 22415/17 | $11215 / 17$ | $1715 / 17$ | 22515/17 | -9 2/17 | 2021 |
| $24415 / 17$ | $22815 / 17$ | 116 15/17 | $1015 / 17$ | $21815 / 17$ | 117 15/17 | $1915 / 17$ | 216 15/17 | 11815/17 | $2015 / 17$ | $22615 / 17$ | $11915 / 17$ | 915/17 | $22715 / 17$ | $12015 / 17$ | $715 / 17$ | -7 2/17 |  |
| $24215 / 17$ | 161 15/17 | 55 15/17 | 138 15/17 | 162 15/ | $6415 / 17$ | 128 15/17 | 163 15/17 | 6515/17 | 12615/17 | 164 15/17 | 5415/17 | 13615/17 | 165 15/17 | 52 15/17 | 137 15/17 | -5 21 | 2021 |
| $24015 / 17$ | $13015 / 17$ | 168 15/17 | $5615 / 17$ | 139 15/17 | 158 15/17 | $5715 / 17$ | $14015 / 17$ | 156 15/17 | $5815 / 17$ | $12915 / 17$ | 16615/17 | $5915 / 17$ | $12715 / 17$ | 16715/17 | $6015 / 17$ | -3 2/17 | 2021 |
| $23815 / 17$ | $6315 / 17$ | 131 15/17 | 160 15/17 | $5315 / 17$ | 132 15/17 | 169 15/17 | $5115 / 17$ | $13315 / 17$ | 170 15/17 | $6115 / 17$ | 134 15/17 | 15915/17 | $6215 / 17$ | 135 15/17 | 157 15/17 | -1 2/17 | 2021 |
| $23615 / 17$ | $17615 / 17$ | 2515/17 | 153 15/17 | $17715 / 17$ | $3415 / 17$ | 143 15/17 | 178 15/17 | 3515/17 | 141 15/17 | $17915 / 17$ | 2415/17 | $15115 / 17$ | 180 15/17 | $2215 / 17$ | 15215 | 15/17 | 2021 |
| $23415 / 17$ | $14515 / 17$ | 183 15/17 | 2615/17 | $15415 / 17$ | $17315 / 17$ | $2715 / 17$ | 155 15/17 | 17115/17 | 2815/17 | $14415 / 17$ | $18115 / 17$ | $2915 / 17$ | 142 15/17 | 182 15/17 | $3015 / 17$ | 215/17 | 2021 |
| $23215 / 17$ | $3315 / 17$ | 14615/17 | 175 15/17 | $2315 / 17$ | $14715 / 17$ | 184 15/17 | $2115 / 17$ | $14815 / 17$ | 185 15/17 | $3115 / 17$ | 14915/17 | 17415/17 | $3215 / 17$ | 150 15/17 | 172 15/17 | 415/17 | 2021 |
| $24515 / 17$ | -24 2/17 | -22 2/17 | -20 2/17 | -182/17 | -16 2/17 | -14 2/17 | -12 2/17 | -112/17 | 24315/17 | 24115/17 | 23915/17 | $23715 / 17$ | 235 15/17 | $23315 / 17$ | $23115 / 17$ | $24715 / 17$ | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the blocks of order 3 are semi-magic squares with equal magic sums. In each case the semi-magic sums are given by

$$
\begin{gathered}
{\left[\operatorname{Sm}_{3 \times 3}: S_{17 \times 17}:=21\right]:=\frac{63}{17}} \\
{\left[{\left.S m_{3 \times 3}: S_{17 \times 17}:=441\right]:=\frac{1323}{17}}_{\left[S m_{3 \times 3}: S_{17 \times 17}:=2021\right]:=\frac{6063}{17}}\right.}
\end{gathered}
$$

### 1.10.2 Second Type

| -127 13/17 | 1444117 | $1424 / 17$ | $1404 / 17$ | 1384117 | $1364 / 17$ | $1344 / 17$ | $1324 / 17$ | $1314 / 17$ | $-12313 / 17$ | -12113/17 | -119 13/17 | -117 13/17 | -115 $13 / 17$ | -113 $13 / 17$ | -11113/17 | 3/17 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -142 13/17 | -110 13/17 | $1064 / 17$ | -81 13/17 | 7 | $854 / 17$ | -80 13/17 | $764 / 17$ | -3613/17 | -713/17 | $554 / 17$ | $-6513 / 17$ | $464 / 17$ | $-2113 / 17$ | 224117 | $254 / 17$ | $1454 / 17$ |  |
|  | 8 | 97 | -109 | 99 | -88 |  |  | -79 |  | -4 |  | $374 / 17$ | 117 | 17 | $-2813 / 17$ | 1434117 |  |
| -138 | $1004 / 17$ | -95 $13 / 17$ | 14 | 98 | -97 13/17 | 70 417 | -50 13 | -1313/17 | 68 |  | 40 | 17 |  | 17 | 17 |  |  |
| -136 | 91 | -96 | 112 | -94 | -5 |  | -66 | 82 | -49 | -20 | 314177 | $-5113 / 17$ | 17 | 17 | 9 | $1394 / 17$ |  |
|  | -821 | -41317109 |  |  |  |  |  |  | -73 |  | -22 13/17 |  | 24.17 | -5813/17 | 5 | $1374 / 17$ |  |
| -132 13/1 | -108 $13 / 17$ | $1054 / 17$ | -83 | $54 / 17$ | 884117 | -78 13 | $754 / 17$ | -38 13/17 | -913/17 | $584 / 17$ | -63 13/17 | 454117 | $-2313 / 17$ | $204 / 17$ | $284 / 17$ | $1354 / 17$ |  |
|  | 6 | 95 | -10 |  |  |  |  |  |  | -4 | 21 | $354 / 17$ | 17 | 17 | 17 | 1334177 |  |
|  | 103 |  | 4/17 |  |  |  |  |  |  |  |  |  | $154 / 17$ | $364 / 17$ | -54 13/17 | 126 |  |
| 127 | 90 | -98 | 1104 |  | -3 | 60 | -68 | 80417 | -46 | -18 | 30 | -5313/17 | $504 / 17$ | -3113/17 | $4 / 17$ | -124 13/17 |  |
| 125 | -84 | -11 | 86 | -10 | $1114 / 17$ | -39 |  | 564177 | -74 | 814177 | $-2413 / 17$ | 13 | 26417 | -5913/17 | $514 / 17$ | 13/ |  |
| 123 | -107 | 107 |  |  |  | -77 | 77 | $-4013 / 17$ | -1113/17 | $594 / 17$ | $-6213 / 17$ | 17 | -25 13/17 | $184 / 17$ | 294117 | -120 13/17 |  |
| 121 | 44 | $934 / 17$ | -105 131 | 1024 | -87 13/17 | -10 | 63 | -75 | $724 / 17$ | -42 | $194 / 17$ | 334117 | -60 13/17 | $424 / 17$ | -27 13/17 | -118 13 |  |
| 1194177 | $1044 / 17$ | -92 | 24 | 94 | -101 13/17 | $744 / 17$ | -47 | -1213/17 | 64417 | $-7113 / 17$ | $444 / 17$ | -32 13/17 | $174 / 17$ | 344117 | -5613/17 | -116 13/11 |  |
| 117 | $924 / 17$ | -100 13/17 | $1084 / 17$ | -90 | -213/17 | 62411 | -70 13/17 | 78417 | $-4513 / 17$ | -17 13/17 | 3241 | -55 13/17 | 484117 | -30 13/17 | 12417 | -114 13/2 |  |
| $1154 / 17$ | -8613/17 | -13/17 | $874 / 17$ | -102 13/17 | $1094 / 17$ | $-4113 / 17$ | -15 13/17 | $574 / 17$ | -72 13/17 | 79417 | $-2613 / 17$ | 144117 | $274 / 17$ | -57 13/17 | $494 / 17$ | -112 $13 / 17$ |  |
| 1284117 | -14113/17 | -139 13/17 | -137 13/17 | -135 13/17 | -133 13/17 | -13113/17 | -129 13/17 | -128 13/17 | $1264 / 17$ | $1244 / 17$ | $1224 / 17$ | $1204 / 17$ | $1184 / 17$ | 1164117 | $1144 / 17$ | $1304 / 17$ |  |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |  |


| -103 1/17 | $16816 / 17$ | $16616 / 17$ | 164 16/17 | $16216 / 17$ | 160 16/17 | $15816 / 17$ | $15616 / 17$ | $15516 / 17$ | -99 1/17 | -971/17 | -95 1/17 | -93 1/17 | -91 1/17 | -89 1/17 | -87 1/17 | -101 1/17 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -118 1/17 | -86 1/17 | $13016 / 17$ | -57 1/17 | $3116 / 17$ | $10916 / 17$ | -56 1/17 | $10016 / 17$ | -12 1/17 | 1616/17 | $7916 / 17$ | $-411 / 17$ | $7016 / 17$ | 216/17 | $4616 / 17$ | $4916 / 17$ | 6916/17 | 441 |
| -116 1/17 | 32 16/17 | 121 16/17 | -85 1/17 | $12316 / 17$ | -64 1/17 | $1716 / 17$ | 91 | -55 1/17 | 9316/17 | -19 1/17 | $4716 / 17$ | $6116 / 17$ | -40 1/17 | $6316 / 17$ | $-41 / 17$ | 17 | 441 |
| -114 1/17 | $12416 / 17$ | -71 1/17 | $2516 / 17$ | $12216 / 17$ | -73 1/17 | 9416/17 | -261/17 | $1016 / 17$ | 92 16/17 | -43 1/17 | $6416 / 17$ | -11 1/17 | $4016 / 17$ | $6216 / 17$ | -28 1/17 | 165 16/17 | 441 |
| -112 1/17 | $11516 / 17$ | -72 1/17 | $13616 / 17$ | -70 1/17 | 1816/17 | $8516 / 17$ | -42 1/17 | $10616 / 17$ | -25 1/17 | 316/17 | 5516/17 | -271/17 | $7616 / 17$ | -10 1/17 | $3316 / 17$ | 17 | 441 |
| -11 | -58 | 1916 | $10816 / 17$ | -79 1/17 | $13716 / 17$ | -13 | $416 / 17$ | 7816/17 | -49 1/17 | $10716 / 17$ | 116/17 | $3416 / 17$ | $4816 / 17$ | -34 1/17 | 17 | 161 16/17 | 441 |
| -108 1/17 | -84 1/17 | $12916 / 17$ | -59 1/17 | $2916 / 17$ | $11216 / 17$ | -54 1/17 | $9916 / 17$ | -14 1/17 | 1416/17 | 8216/17 | -39 1/17 | $6916 / 17$ | 16/17 | $4416 / 17$ | $5216 / 17$ | $15916 / 17$ | 441 |
| -106 1/17 | $3016 / 17$ | $11916 / 17$ | -82 1/17 | $12516 / 17$ | -65 1/17 | 1516/17 | $8916 / 17$ | -52 1/17 | 9516/17 | -20 1/17 | 4516/17 | $5916 / 17$ | -37 1/17 | $6516 / 17$ | -5 1/17 | 15716 | 441 |
| $15316 / 17$ | $12716 / 17$ | -69 1/17 | $2416 / 17$ | $12016 / 17$ | -75 1/17 | $9716 / 17$ | -24 | 91 | $9016 / 17$ | -45 1/17 | $6716 / 17$ | -9 1/17 | $3916 / 17$ | $6016 / 17$ | -30 1/17 | -102 1/17 | 441 |
| $15116 / 17$ | $11416 / 17$ | -74 1/17 | 13416/17 | -67 1/17 | $2016 / 17$ | 8416/17 | -44 1/17 | $10416 / 17$ | -22 1/17 | $516 / 17$ | $5416 / 17$ | -29 1/17 | 7416/17 | -71/17 | $3516 / 17$ | -100 1/17 | 441 |
| $14916 / 17$ | -60 1/17 | $2216 / 17$ | $11016 / 17$ | -80 1/17 | $13516 / 17$ | -15 1/17 | $716 / 17$ | 8016/17 | -50 1/17 | $10516 / 17$ | -1/17 | $3716 / 17$ | $5016 / 17$ | -35 1/17 | $7516 / 17$ | -981/1/ | 441 |
| $14716 / 17$ | -83 1/17 | 131 16/17 | -61 1/17 | $2716 / 17$ | $11316 / 17$ | -53 1/17 | $10116 / 17$ | -16 1/17 | 1276/17 | $8316 / 17$ | -38 1/17 | $7116 / 17$ | $-11 / 17$ | $4216 / 17$ | $5316 / 17$ | -961/17 | 441 |
| $14516 / 17$ | $2816 / 17$ | $11716 / 17$ | -81 1/17 | $12616 / 17$ | -63 1/17 | 1316/17 | $8716 / 17$ | -51 1/17 | 9616/17 | -181/17 | $4316 / 17$ | $5716 / 17$ | -36 1/17 | $6616 / 17$ | -3 1/17 | -94 $1 / 17$ | 441 |
| $14316 / 17$ | $12816 / 17$ | -68 1/17 | $2616 / 17$ | $11816 / 17$ | -77 1/17 | $9816 / 17$ | -23 1/17 | $1116 / 17$ | $8816 / 17$ | -47 1/17 | $6816 / 17$ | -81/17 | $4116 / 17$ | $5816 / 17$ | -32 1/17 | -92 $1 / 17$ |  |
| $14116 / 17$ | 11616/17 | -76 1/17 | 132 16/17 | -66 1/17 | 2116/17 | 8616/17 | -46 1/17 | 102 16/17 | -21 1/17 | $616 / 17$ | $5616 / 17$ | -31 1/17 | $7216 / 17$ | -6 1/17 | $3616 / 17$ | -90 1/17 |  |
| $13916 / 17$ | -62 1/17 | $2316 / 17$ | $11116 / 17$ | -78 1/17 | $13316 / 17$ | -17 1/17 | $816 / 17$ | $8116 / 17$ | -48 1/17 | $10316 / 17$ | -2 1/17 | $3816 / 17$ | $5116 / 17$ | -33 1/17 | $7316 / 17$ | -88 1/17 | 441 |
| $15216 / 17$ | -117 1/17 | -115 1/17 | -113 1/17 | -1111/17 | -1091/17 | -1071/17 | -105 1/17 | -104 1/17 | $15016 / 17$ | $14816 / 17$ | $14616 / 17$ | $14416 / 17$ | $14216 / 17$ | $14016 / 17$ | $13816 / 17$ | 154 16/17 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


| -10 2/17 | 261 15/17 | 259151 | $25715 / 17$ | 25515 | $25315 / 17$ | 251 15/17 |  | $24815 / 17$ | -6 2/17 | -4 2/17 | -2 2/17 | -2/17 | 115/17 | 315/17 | 515/17 | -8/117 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -25 | 615 | 22315 | 35 | 124 | 20215/17 | 3615/17 | 193 15/17 | $8015 / 17$ | 10915/17 | 172 | $5115 / 17$ | 163 15/17 | 9515/17 | 139 15/17 | $14215 / 17$ | 17 |  |
| -23 | $12515 /$ | 21415 | $715 / 17$ | 21615/17 | $2815 / 17$ | $11015 / 17$ | 184 | $3715 / 17$ | 18615/17 | 73 15/17 | 14015 | 154 15/17 | 52 15/17 | 156 15/17 | 88 15/17 |  |  |
| -21 2/17 | 217 15/1 | $2115 / 17$ | 118 15/17 | $21515 / 17$ | 1915/17 | $18715 / 17$ | $6615 / 17$ | 103 15/17 | 185 15/17 | $4915 / 17$ | 157 15/17 | $8115 / 17$ | 133 15/17 | 155 15/17 | 6415/17 | $25815 / 17$ |  |
| -19 | 208 | 2015 | 22 | 22 | 11115/17 | 178 | $5015 / 17$ | 199 | 67 | 96 | 148 | 6515/17 | 16915/17 | 82 15/17 | 5/17 | 7 |  |
| -17 | 3415/1 | 11215 | 20115/17 | 1315 | 23015 | 7915/17 | 9715/17 | 17115/17 | 4315/17 | 20015 | 9415/17 | 127 15/17 | 141 $15 / 17$ | $5815 / 17$ | 17015 |  |  |
| -15 2 | $815 / 1$ | 22215 | $3315 / 17$ | 12215 | 20515/17 | $3815 / 17$ | 192 15/17 | $7815 / 17$ | $10715 / 17$ | $17515 / 17$ | 53 15/17 | 162 15/17 | 9315/17 | 137 15/17 | 14515/17 | $25215 / 17$ |  |
| -1321 | 123 15/ | 21215 | $1015 / 17$ | 218 | 27 | 108 | 182 | $4015 / 17$ | 188 | 72 | 138 | 152 15/17 | $5515 / 17$ | 158 15/17 | 87 15/17 | 250 15/17 |  |
| 246 | 22015 | 2315 | $11715 / 17$ | 213 | 1715/17 | $19015 / 17$ | 68 15/17 | 102 15/17 | 183 | 4715/17 | 160 15/17 | $8315 / 17$ | 132 15/17 | $15315 / 17$ | 62 15/17 | -9 |  |
| 244 | 207 15/1 | 1815/17 | $22715 / 17$ | $2515 /$ | 113 15/ | $17715 / 17$ | $4815 / 17$ | 19715 | 70 15/1 | 9815 | 147 | $6315 / 17$ | 16715/17 | $8515 / 17$ | $12815 / 17$ | -7 2/17 |  |
| 242 15/1 | $3215 / 17$ | $11515 /$ | 203 15/17 | 1215/17 | $22815 / 17$ | $7715 / 17$ | 100 15/17 | 173 15/17 | 4215/17 | 19815/17 | 92 15/17 | 130 15/17 | 14315/17 | 5715/17 | 168 15/17 | -5 |  |
| $24015 /$ | 915 | 224 | 3115 | 120 | 20615/17 | 39 15/17 | 19415/1 | 7615/17 | 105 15/17 | $17615 / 17$ | 54 15/17 | 164 15/17 | 9115/17 | 135 15/17 | 14615/17 | -3 |  |
| 238 | 121 | 210 | 111 | 219 | $2915 / 17$ | 106 | 180 | 4115/17 | 18915/17 | 74 | 136 | 150 15/17 | 5615/17 | 15915/17 | 89 | -1 2/17 |  |
| $23615 / 17$ | 22115/17 | 2415/17 | $11915 / 17$ | 21115/17 | 1515/17 | 19115/17 | $6915 / 17$ | 104 15/17 | 181 15/17 | 4515/17 | 161 15/17 | 8415/17 | 13415/17 | 151 15/17 | $6015 / 17$ | 15/17 |  |
| 234 15/ | $20915 / 17$ | 1615/17 | 225 15/17 | $2615 / 17$ | 114 15/17 | $17915 / 17$ | 4615/17 | 195 15/17 | 7115/17 | 9915/17 | $14915 / 17$ | $6115 / 17$ | 165 15/17 | 8615/17 | $12915 / 17$ | 215/17 |  |
| $23215 / 17$ | 3015/17 | 116 15/17 | 20415/17 | 1415/17 | 22615/17 | 7515/17 | $10115 / 17$ | 17415/17 | 4415/17 | 19615/17 | 9015/17 | 131 15/17 | 14415/17 | 5915/17 | 16615 | 415/17 |  |
| $24515 / 17$ | -24 2/17 | -22 2/17 | -20 2/17 | -18 2/17 | -16 2/17 | -14 2/17 | -12 2/17 | -11 2/17 | 24315/17 | 241 15/17 | 23915 | 23715 | 23515 | 23315 | 23115 | 24715/17 |  |
| 2021 | 202 | 021 | 021 | 202 | 2021 | 2021 | 2021 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 202 |  |

In this case, the blocks of order 5 are pandiagonal with equal magic sums. In each case the magic sums order 5 are given by

$$
\begin{aligned}
{\left[S_{5 \times 5}: S_{17 \times 17}:=21\right] } & :=\frac{105}{17} \\
{\left[S_{5 \times 5}: S_{17 \times 17}:=441\right] } & :=\frac{2205}{17} \\
{\left[S_{5 \times 5}: S_{17 \times 17}:=2021\right] } & :=\frac{10105}{17} .
\end{aligned}
$$

### 1.11 Block-Wise Magic Squares of Order 18

Below are block-wise magic squares of order 18 in two different ways giving magic sums $21,21^{2}$ and 2021 . One is where blocks of order 3 are magic squares with different magic sums. The second, where blocks of order 6 are magic squares with equal magic sums.

### 1.11.1 First Type





In this case, the blocks of order 3 are magic squares with different magic sums.

### 1.11.2 Second Type



| -134 | 172 | 165 | 154 | -123 | -87 | -129 | 177 | 160 | 159 | -128 | -92 | -136 | 170 | 167 | 152 | -121 | -85 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 129 | -69 | 111 | -62 | -44 | 82 | 124 | -74 | 106 | -57 | -39 | 87 | 131 | -67 | 113 | -64 | -46 | 80 | 441 |
| 75 | 64 | -8 | 3 | 39 | -26 | 70 | 69 | -3 | -2 | 34 | -21 | 77 | 62 | -10 | 5 | 41 | -28 | 441 |
| 21 | -15 | 46 | 57 | 10 | 28 | 16 | -20 | 51 | 52 | 15 | 33 | 23 | -13 | 44 | 59 | 8 | 26 | 441 |
| -80 | 93 | -51 | 100 | 118 | -33 | -75 | 88 | -56 | 105 | 123 | -38 | -82 | 95 | -49 | 98 | 116 | -31 | 441 |
| 136 | -98 | -116 | -105 | 147 | 183 | 141 | -93 | -111 | -110 | 142 | 178 | 134 | -100 | -118 | -103 | 149 | 185 | 441 |
| -135 | 171 | 166 | 153 | -122 | -86 | -133 | 173 | 164 | 155 | -124 | -88 | -131 | 175 | 162 | 157 | -126 | -90 | 441 |
| 130 | -68 | 112 | -63 | -45 | 81 | 128 | -70 | 110 | -61 | -43 | 83 | 126 | -72 | 108 | -59 | -41 | 85 | 441 |
| 76 | 63 | -9 | 4 | 40 | -27 | 74 | 65 | -7 | 2 | 38 | -25 | 72 | 67 | -5 | 0 | 36 | -23 | 441 |
| 22 | -14 | 45 | 58 | 9 | 27 | 20 | -16 | 47 | 56 | 11 | 29 | 18 | -18 | 49 | 54 | 13 | 31 | 441 |
| -81 | 94 | -50 | 99 | 117 | -32 | -79 | 92 | -52 | 101 | 119 | -34 | -77 | 90 | -54 | 103 | 121 | -36 | 441 |
| 135 | -99 | -117 | -104 | 148 | 184 | 137 | -97 | -115 | -106 | 146 | 182 | 139 | -95 | -113 | -108 | 144 | 180 | 441 |
| -130 | 176 | 161 | 158 | -127 | -91 | -137 | 169 | 168 | 151 | -120 | -84 | -132 | 174 | 163 | 156 | -125 | -89 | 441 |
| 125 | -73 | 107 | -58 | -40 | 86 | 132 | -66 | 114 | -65 | -47 | 79 | 127 | -71 | 109 | -60 | -42 | 84 | 441 |
| 71 | 68 | -4 | -1 | 35 | -22 | 78 | 61 | -11 | 6 | 42 | -29 | 73 | 66 | -6 | 1 | 37 | -24 | 441 |
| 17 | -19 | 50 | 53 | 14 | 32 | 24 | -12 | 43 | 60 | 7 | 25 | 19 | -17 | 48 | 55 | 12 | 30 | 441 |
| -76 | 89 | -55 | 104 | 122 | -37 | -83 | 96 | -48 | 97 | 115 | -30 | -78 | 91 | -53 | 102 | 120 | -35 | 441 |
| 140 | -94 | -112 | -109 | 143 | 179 | 133 | -101 | -119 | -102 | 150 | 186 | 138 | -96 | -114 | -107 | 145 | 181 |  |
| 441 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


| -462/9 | 2597/9 | 2527/9 | 2417/9 | -352/9 | 7/9 | -412/9 | 2647/9 | 2477/9 | 2467/9 | $-402 / 9$ | -42/9 | -482/9 | 2577/9 | 2547/9 | 2397/9 | $-332 / 9$ | 27/9 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2167/9 | 187/9 | 1987/9 | 257/9 | 437/9 | 1697/9 | 2117/9 | 137/9 | 1937/9 | $307 / 9$ | 487/9 | 1747/9 | 2187/9 | $207 / 9$ | 200 7/9 | 237/9 | 417/9 | 1677/9 | 2021 |
| 162 7/9 | 1517/9 | 797/9 | 907/9 | 1267/9 | 617/9 | 157 7/9 | 1567/9 | 847/9 | 857/9 | 1217/9 | $667 / 9$ | 1647/9 | 1497/9 | 77719 | 927/9 | 1287/9 | 597/9 | 2021 |
| 108719 | 727/9 | 1337/9 | 1447/9 | 97719 | 1157/9 | 1037/9 | 677/9 | 1387/9 | 1397/9 | 1027/9 | $1207 / 9$ | 110 7/9 | 747/9 | 1317/9 | 1467/9 | 957/9 | 7/9 | 2021 |
| 77 | 1807/9 | 367/9 | 1877/9 | 2057/9 | 547/9 | 127/9 | 1757/9 | $317 / 9$ | 1927/9 | 210 7/9 | 497/9 | 57/9 | 1827/9 | $387 / 9$ | 1857/9 | 2037/9 | 567/9 | 2021 |
| 2237/9 | -102/9 | -282/9 | -172/9 | 2347/9 | $2707 / 9$ | $2287 / 9$ | $-52 / 9$ | -23/9 | -22 2/9 | $2297 / 9$ | 2657/9 | 2217/9 | $-122 / 9$ | -302/9 | -152/9 | 2367/9 | 2727/9 | 2021 |
| -472/9 | $2587 / 9$ | 2537/9 | $2407 / 9$ | $-342 / 9$ | 1719 | $-452 / 9$ | 260 7/9 | 2517/9 | 2427/9 | $-362 / 9$ | -2/9 | $-432 / 9$ | 2627/9 | $2497 / 9$ | 2447/9 | $-382 / 9$ | $-22 / 9$ | 2021 |
| $2177 / 9$ | 197/9 | 1997/9 | 247/9 | 427/9 | 1687/9 | 2157/9 | $177 / 9$ | 1977/9 | $267 / 9$ | 447/9 | $1707 / 9$ | 2137/9 | 157/9 | 1957/9 | $287 / 9$ | $467 / 9$ | 1727/9 | 2021 |
| 1637/9 | 150 7/9 | 787/9 | 917/9 | 127 7/9 | $607 / 9$ | 1617/9 | 152 7/9 | 807/9 | 897/9 | 1257/9 | $627 / 9$ | 1597/9 | 1547/9 | 827/9 | $877 / 9$ | 1237/9 | 647/9 | 2021 |
| 1097/9 | 737/9 | 1327/9 | 1457/9 | 967/9 | 1147/9 | 1077/9 | 717/9 | 1347/9 | 1437/9 | 987/9 | 1167/9 | 1057/9 | 69719 | 1367/9 | 1417/9 | 100 7/9 | 1187/9 | 2021 |
| 67/9 | 1817/9 | $377 / 9$ | 1867/9 | 2047/9 | 557/9 | 87/9 | 1797/9 | 357/9 | 1887/9 | 2067/9 | 537/9 | $107 / 9$ | 1777/9 | 337/9 | 1907/9 | 2087/9 | 517/9 | 2021 |
| 2227/9 | $-112 / 9$ | -292/9 | $-162 / 9$ | 2357/9 | 2717/9 | 2247/9 | -92/9 | $-272 / 9$ | -182/9 | 2337/9 | $2697 / 9$ | 2267/9 | -72/9 | -25 2/9 | $-202 / 9$ | 2317/9 | 2677/9 | 2021 |
| -42 2/9 | 2637/9 | 2487/9 | 2457/9 | -39 2/9 | $-32 / 9$ | -492/9 | 2567/9 | 2557/9 | $2387 / 9$ | -32 /9 | 37/9 | -442/9 | 2617/9 | 250 7/9 | 2437/9 | $-372 / 9$ | -12/9 | 2021 |
| 2127/9 | 147/9 | 1947/9 | 297/9 | 477/9 | 1737/9 | 2197/9 | 217/9 | 2017/9 | $227 / 9$ | 407/9 | 1667/9 | 2147/9 | $167 / 9$ | 1967/9 | 27719 | 457/9 | 1717/9 | 2021 |
| 158 7/9 | 1557/9 | 837/9 | 867/9 | 122 7/9 | 657/9 | 1657/9 | 1487/9 | 767/9 | 937/9 | 1297/9 | 587/9 | 160 7/9 | 1537/9 | 817/9 | $887 / 9$ | 1247/9 | 63719 | 2021 |
| 1047/9 | 687/9 | 1377/9 | 1407/9 | 1017/9 | 1197/9 | 1117/9 | 757/9 | $1307 / 9$ | 1477/9 | 947/9 | 1127/9 | 1067/9 | 707/9 | 1357/9 | 1427/9 | 997/9 | 117 7/9 | 2021 |
| 117/9 | 1767/9 | 327/9 | 1917/9 | $2097 / 9$ | $507 / 9$ | 47/9 | 1837/9 | 397/9 | 1847/9 | 2027/9 | $577 / 9$ | 97/9 | 1787/9 | 347/9 | 1897/9 | $2077 / 9$ | 527/9 | 2021 |
| 227 7/9 | -62/9 | -242/9 | $-212 / 9$ | $2307 / 9$ | 2667/9 | $2207 / 9$ | $-132 / 9$ | $-312 / 9$ | -142/9 | $2377 / 9$ | 2737/9 | 2257/9 | -82/9 | $-262 / 9$ | -192/9 | $2327 / 9$ | 2687/9 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case all the blocks of order 6 are magic squares with equal magic sums given by

$$
\begin{gathered}
{\left[S_{6 \times 6}: S_{18 \times 18}:=21\right]:=7} \\
{\left[S_{6 \times 6}: S_{18 \times 18}:=441\right]:=147} \\
{\left[S_{6 \times 6}: S_{18 \times 18}:=2021\right]:=\frac{2021}{3} .}
\end{gathered}
$$

### 1.12 Block-Bordered Magic Squares of Order 19

Below are block-bordered magic squares of order 19 in two different ways giving magic sums $21,21^{2}$ and 2021. The blockwise magic square considered is of order 15 with blocks of order 3 and 5 . The blocks of order 3 are semi-magic with equal semi-magic sums. The blocks of order 5 are pandiagonal with equal magic sums. In both the cases the magic square of order 15 is pandiagonal as given in Subsection 1.8. The magic sums of order 15 are given by

$$
\begin{gathered}
{\left[S_{15 \times 15}: S_{19 \times 19}:=21\right]:=\frac{315}{19}} \\
{\left[S_{15 \times 15}: S_{19 \times 19}:=441\right]:=\frac{6615}{19}} \\
{\left[S_{15 \times 15}: S_{19 \times 19}:=2021\right]:=\frac{30315}{19} .}
\end{gathered}
$$

### 1.12.1 First Type

| -15917/19 | -14317/19 | $-14517 / 19$ | -14717/19 | -149 17/19 | -1517/19 | -15317/19 | -15517/19 | 9-157 17/19 | $1652 / 19$ | $1662 / 19$ | 168 2/19 | $1702 / 19$ | 172 2/19 | $1742 / 19$ | $1762 / 19$ | $1782 / 19$ | 1802119 | $-16177 / 19$ | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1812119 | -127 17 | 1442119 | 1422119 | $1402 / 19$ | $1382 / 19$ | $1362 / 19$ | $1342 / 19$ | 1322119 | $1312 / 19$ | -12317/19 | 9-121 $17 / 19$ | -11917/19 | -11717/19 | $-11517 / 19$ | $-11317 / 19$ | -11177/19 | -12517/19 | 19 | 21 |
| 179219 | -14217 | 742119 | $-4617 / 19$ | -2317/ | 75 2/19 | $-3717 / 19$ | -3317/19 | $762 / 19$ | $-3617 / 19$ | -3517/19 | 712 | $-4717 / 19$ | $-2517 / 19$ | 78 2/19 | $-4917119$ | $-2417 / 19$ | 1452119 | 119 | 21 |
| 177219 | -140 17/19 | $-3117 / 19$ | $812 / 19$ | -45 | -22 | $712 / 19$ | -44 | -21 | 692119 | $-4317 / 19$ | -3217/19 | 79 2/19 | $-4217 / 19$ | -3417/19 | 80219 | $-4117 / 19$ | 1432119 | 9 | 21 |
| 175 | -138 | -38 | -30 | 73 |  | -29 | 82 | 5 | -28 | $832 / 19$ | $-4017 / 19$ | -27 | 72 2/19 | -3917/19 | $-2617 / 19$ | $702 / 19$ | $1412 / 19$ |  | 21 |
| 1732119 | -13617/1 | $-7517 / 19$ | 88 2/19 | -817/19 | -74 | $972 / 19$ | -1817/19 | $-7317 / 19$ | $982 / 19$ | $-2017 / 19$ | $-7217 / 19$ | $872 / 19$ | -1017/19 | $-7117 / 19$ | 85 2/19 | -917/19 | $1392 / 19$ |  | 21 |
| $1712 / 19$ | -13417/19 | -1617/19 | -68 | 89 | -7 | -78 | 902 | -6 | -80 | 912119 | -17 | -70 | 92 2/19 | -1917/19 | -6917/19 | 932119 | $1372 / 19$ | -168 17/19 | 21 |
|  | -132 17/ | 962119 |  |  |  |  |  | 8421 | -1317/19 | -661719 |  | -1217/19 | -7119 | $952 / 19$ | $-1117 / 19$ | -7917/19 | $1352 / 19$ | -16617/19 | 21 |
| 167 2/19 | -130 17/19 | -10517/19 | 103 2/19 | $62 / 19$ | -10417/19 | 112 2/19 | $-317 / 19$ | -103 | $1132 / 19$ | -517/19 | -102 | 102 2/19 | $42 / 19$ | -10117/19 | $1002 / 19$ | $52 / 19$ | $1332 / 19$ | -164 17/19 | 21 |
| -16 | 129 2/1 | -117 | -98 | 104 | 72 | , | 105 | $82 / 19$ | -110 17/19 | $1062 / 19$ | -21711 | -100 17 | 107 2/19 | $-417 / 19$ | -9917/19 | 1082119 | -12617/19 | 163 2/19 | 21 |
| -158 17/19 | 127 2/19 | $1112 / 19$ |  | -106 | 1012119 | 2/19 | -9717 | 992/19 | 12/19 | -9617/19 | $1092 / 19$ | $22 / 19$ | -10717\| | $1102 / 19$ | $32 / 19$ | -10917/19 | $-12417 / 19$ | $1612 / 19$ | 21 |
| -15617/19 | 1252119 | 442119 | -61 | 21 | 45 | -5217\| | 112 | 46 | -51 | 9 2/19 | 47 | -6217 | $192 / 19$ | 482119 | $-6417 / 19$ | 202119 | -122 17/19 | 2/19 | 21 |
| -154 17/19 | 1232119 | $132 / 19$ | 51211 | -60 | 22 211 | $412 / 19$ | -59 | 232 | $392 / 19$ | -5817\| | 12 21 | 49 2/19 | -5717/19 | $102 / 19$ | 50 2/19 | -5617/19 | -120 17/19 | $1572 / 19$ | 21 |
| -152 17/19 | 1212119 | -5317/19 | 142 | 43219 | -63 | $152 / 19$ | 522 | -6517 | $162 / 19$ | $532 / 19$ | -5517 | $172 / 19$ | 42 2/19 | -5417/19 | $182 / 19$ | 40219 | -11817/19 | 155 2/19 | 21 |
| -150 17/19 | $1192 / 19$ | 59219 | -9117/1 | 36211 | 60 2/19 | -82 17/19 | 262119 | 61 | -8117/19 | $242 / 19$ | 622 | -9217/19 | 34219 | $632 / 19$ | $-9417 / 19$ | $352 / 19$ | -11617/19 | 1532 | 21 |
| -14817/19 | $1172 / 19$ | 282119 | $662 / 19$ | -9017\| | $372 / 19$ | 56 2/19 | -8917/19 | $382 / 19$ | $542 / 19$ | -8817/19 | 272119 | $642 / 19$ | -8717/19 | 25 2/19 | 65 2/19 | -8617/19 | -11417/19 | $1512 / 19$ | 21 |
| -14617/19 | $1152 / 19$ | -8317/19 | 29219 | 58219 | -9317/19 | 302119 | 67 2/19 | -9517/19 | $312 / 19$ | 68219 | -8517/19 | 32 2/19 | $572 / 19$ | -8417/19 | $332 / 19$ | 55 2/19 | -11217/19 | $1492 / 19$ | 21 |
| -14417/19 | $1282 / 19$ | $-14117 / 19$ | -13917/19 | -137 $17 / 19$ | -13517/19 | -13317/19 | $-13117 / 19$ | -129 17/19 | -1281719 | $1262 / 19$ | $1242 / 19$ | 122 2/19 | $1202 / 19$ | $1182 / 19$ | $1162 / 19$ | $1142 / 19$ | 1302119 | $1471 / 19$ | 21 |
| 1642119 | $1462 / 19$ | $1482 / 19$ | $1502 / 19$ | 152 2/19 | $1542 / 19$ | $1562 / 19$ | 158 2/19 | 160 2/19 | -162 17/19 | -16317/19 | -165 17/19 | -16717/19 | -169 17/19 | -17117/19 | -17317/19 | -17517/19 | -17717/19 | 162 2/19 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 2 |


| -137 15/19 | -121 15/19 | -123 15/19 | -125 15/19 | -127 15/19 | -129 15/19 | -131 15/19 | -133 15/19 | -135 15/19 | 187 4/19 | 188 4/19 | $1904 / 19$ | $1924 / 19$ | 194 4/19 | $1964 / 19$ | $1984 / 19$ | $2004 / 19$ | $2024 / 19$ | -139 15/19 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 203 4/19 | -105 15/19 | $1664 / 19$ | 164 4/19 | 162 4/19 | 160 4/19 | $1584 / 19$ | $1564 / 19$ | 154 4/19 | 153 4/19 | -10115/19 | -9915/19 | -9715/19 | -9515/19 | -9315/19 | -9115/19 | -8915/19 | -103 15/19 | -156 15/19 | 441 |
| 201 4/19 | -120 15/19 | $964 / 19$ | $-2415 / 19$ | -1 15/19 | 974/19 | -15 15/19 | $-1115 / 19$ | $984 / 19$ | $-1415 / 19$ | $-1315 / 19$ | 99 4/19 | $-2515 / 19$ | $-315 / 19$ | 100 4/19 | $-2715 / 19$ | -2 15/19 | $1674 / 19$ | -154 15/19 | 441 |
| 199 4/19 | -118 15/19 | -915/19 | 103 4/19 | $-2315 / 19$ | -15/19 | 93 4/19 | $-2215 / 19$ | 4/19 | $914 / 19$ | $-2115 / 19$ | -10 15/19 | $1014 / 19$ | $-2015 / 19$ | $-1215 / 19$ | 102 4/19 | -1915/19 | 165 4/19 | -152 15/19 | 441 |
| 197 4/19 | -116 15/19 | -16 15/19 | -815/19 | $954 / 19$ | $-2615 / 19$ | -715/19 | 104 4/19 | $-2815 / 19$ | -615/19 | 105 4/19 | -1815/19 | $-515 / 19$ | 944/19 | -17 15/19 | $-415 / 19$ | $924 / 19$ | 163 4/19 | -150 15/19 | 441 |
| 195 4/19 | -114 15/19 | -53 15/19 | 110 4/19 | 13 4/19 | -52 15/19 | $1194 / 19$ | 3 4/19 | -51 15/19 | 120 4/19 | $14 / 19$ | -50 15/19 | 109 4/19 | $114 / 19$ | -4915/19 | 107 4/19 | 12 4/19 | $1614 / 19$ | -148 15/19 | 441 |
| 193 4/19 | -112 15/19 | 5 | -4615/19 | 111 | 14 | $-5615 / 19$ | $1124 / 19$ | $154 / 19$ | -58 15/19 | $1134 / 19$ | 4 | -4815/19 | 114 4/19 | $24 / 19$ | -4715/19 | $1154 / 19$ | 19 | -146 15/19 | 441 |
| 191 | -110 15/19 | $1184 / 19$ | 6 4/19 | -54 15/19 | 108 4/19 | $74 / 19$ | $-4515 / 19$ | 106 4/19 | $84 / 19$ | -4415/19 | $1164 / 19$ | $94 / 19$ | -55 15/19 | $1174 / 19$ | $104 / 19$ | -57 15/19 | $1574 / 19$ | -144 15/19 | 441 |
| 189 4/19 | -108 15/19 | -83 15/19 | $1254 / 19$ | $284 / 19$ | -82 15/19 | $1344 / 19$ | $184 / 19$ | -81 15/19 | $1354 / 19$ | 16 4/19 | -80 15/19 | 124 4/19 | 26 4/19 | -79 15/19 | 122 4/19 | $274 / 19$ | $1554 / 19$ | -142 15/19 | 441 |
| -138 15/19 | 151 | $204 / 19$ | -7615/19 | $1264 / 19$ | $294 / 19$ | $-8615 / 19$ | 127 4/19 | $304 / 19$ | -8815/19 | $1284 / 19$ | 19 4/19 | -7815/19 | $1294 / 19$ | $174 / 19$ | -7715/19 | $1304 / 19$ | -104 15/19 | $1854 / 19$ | 441 |
| -136 15/19 | 149411 | 133 4/19 | $214 / 19$ | -84 15/19 | 123 4/19 | 22 4/19 | -75 15/19 | $1214 / 19$ | 23 4/19 | -74 15/19 | $1314 / 19$ | 24 4/19 | -85 15/19 | $1324 / 19$ | 25 4/19 | -87 15/19 | -102 15/19 | 183 4/19 | 441 |
| -134 15/19 | $1474 / 19$ | 66 4/19 | -3915/19 | $434 / 19$ | 67 4/19 | $-3015 / 19$ | 33 4/19 | 68 4/19 | -29 15/19 | $314 / 19$ | 69 4/19 | -40 15/19 | 41419 | $704 / 19$ | -42 15/19 | 42 4/19 | -100 15/19 | $1814 / 19$ | 441 |
| -132 15/19 | $1454 / 19$ | 35 4/19 | 73 4/19 | -3815/19 | 44 4/19 | 63 4/19 | -3715/19 | $454 / 19$ | 61 4/19 | -3615/19 | 34 4/19 | $714 / 19$ | -35 15/19 | $324 / 19$ | $724 / 19$ | -3415/19 | -9815/19 | 179 4/19 | 441 |
| -130 15/19 | $1434 / 19$ | -31 15/19 | 36 4/19 | $654 / 19$ | $-4115 / 19$ | $374 / 19$ | 74 4/19 | $-4315 / 19$ | 38 4/19 | $754 / 19$ | -3315/19 | 39 4/19 | 64 4/19 | -32 15/19 | $404 / 19$ | 62 4/19 | -96 $15 / 19$ | $1774 / 19$ | 441 |
| -128 15/19 | $1414 / 19$ | 81 4/19 | -69 15/19 | 58 4/19 | $824 / 19$ | -60 15/19 | 48 4/19 | 83 4/19 | -59 15/19 | $464 / 19$ | 84 4/19 | -70 15/19 | 56 4/19 | $854 / 19$ | -72 15/19 | 57 4/19 | -9415/19 | $1754 / 19$ | 441 |
| -126 15/19 | $1394 / 19$ | 50 4/19 | $884 / 19$ | -68 15/19 | 59 4/19 | 78 4/19 | $-6715 / 19$ | 60 4/19 | $764 / 19$ | -66 15/19 | 49 4/19 | 86 4/19 | -65 15/19 | 47 4/19 | $874 / 19$ | -6415/19 | -92 15/19 | $1734 / 19$ | 441 |
| -124 15/19 | $1374 / 19$ | -61 15/19 | 51 4/19 | 80 4/19 | -71 15/19 | 52 4/19 | 89 4/19 | -73 15/19 | 53 4/19 | 90 4/19 | -63 15/19 | 54 4/19 | 79 4/19 | -62 15/19 | $554 / 19$ | 77 4/19 | -90 15/19 | 171 4/19 | 441 |
| -122 15/19 | 150 4/19 | -119 15/19 | -117 15/19 | -115 15/19 | -113 15/19 | -111 15/19 | -109 15/19 | -107 15/19 | -106 15/19 | 148 4/19 | 146 4/19 | 144 4/19 | 142 4/19 | 140 4/19 | $1384 / 19$ | $1364 / 19$ | 152 4/19 | 169 4/19 | 441 |
| 186 4/19 | $1684 / 19$ | $1704 / 19$ | $1724 / 19$ | 174419 | $1764 / 19$ | $1784 / 19$ | 180 4/19 | 182 4/19 | -140 15/19 | -141 15/19 | -143 15/19 | -145 15/19 | -147 15/19 | -149 15/19 | -151 15/19 | -153 15/19 | -155 15/19 | 184 4/19 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -5412/19 | -38 12/19 | $-4012 / 19$ | $-4212 / 19$ | -44 12/19 | -4612/19 | -4812/19 | -50 12/19 | -52 |  | $2717 / 19$ | 273 7/19 | 275 7/19 | 277 7/19 | 279 7/19 | $2817 / 19$ | 7/19 | $857 / 19$ | 5612/19 |  |
| 2867 | -22 12/19 | 249 | 247 | 245 | 2437 | $2417 / 19$ | 2397119 | 237 | 236 | -1812/19 | -1612/19 | -1412/19 | -1212/19 | -10 12/19 | -812/19 | -612/19 | 2012/19 | 7312/19 | 2021 |
| 284 | -37 12/19 | 179 7/19 | $587 / 19$ | $817 / 19$ | 180 7/19 | 67 7/19 | $717 / 19$ | $1817 / 19$ | 68 7/19 | $697 / 19$ | 182 7/19 | 577119 | $797 / 19$ | 1837/19 | $557 / 19$ | $807 / 19$ | $2507 / 19$ | 19 | 2021 |
| $2827 / 19$ | -3512/19 | 73 7/19 | $1867 / 19$ | 597/19 | $827 / 19$ | 1767/19 | $607 / 19$ | $837 / 19$ | 1747/19 | $617 / 19$ | $727 / 19$ | 1847119 | $627 / 19$ | $707 / 19$ | 185 7/19 | 63 7/19 | 2487119 | -69 12/19 | 2021 |
| 280 | -33 | 66 | 74 | 178 | 56 | 75 | 187 | 54 |  | 188 | 64 7/19 | $717 / 19$ | $1777 / 19$ | 19 | 78 7/19 | 1757/19 | 2467119 | 19 | 2021 |
| 278 | -31 | 297 | $1937 / 19$ | 96 |  | 202 | 86 | 317 | 203 7/19 |  | $327 / 19$ | 192 7/19 | 947/19 | 19 | 190 7/19 | 119 | 19 | -6512/19 | 2021 |
| 276 | -29 121 | 88 7/19 | $367 / 19$ | 194 | 977/19 | $267 / 19$ | $1957 / 19$ | $987 / 19$ | $247 / 19$ | $1967 / 19$ | 87 7/19 | $347 / 19$ | 197 7/19 | $857 / 19$ | $357 / 19$ | 198 7/19 | $2427 / 19$ | -6312/19 | 2021 |
| 2747119 | -27 | 201 | 89 | 28 | 191 | 90 | 37 | 189 | 91 | 38 | 199 | $927 / 19$ | 277119 | 2007119 | 19 | 19 | 119 | -61 12/19 |  |
| 272 | -25 |  | 208 |  | 7119 | 217 7/19 | $1017 / 19$ |  | 2187/19 | $997 / 19$ |  | 207 7/19 | $1097 / 19$ | $37 / 19$ | $2057 / 19$ | $1107 / 19$ | 119 | 19 |  |
| -5512/19 | 234 | 103 | 6 | 20 | 112 | $-312 / 19$ | 210 7/19 | 113 | $-512 / 19$ | $2117 / 19$ | 102 | $47 / 19$ | 212 7/19 | 100 7/19 | $57 / 19$ | $2137 / 19$ | -2112/19 | 268 7/19 | 2021 |
| -5312/19 | $2327 / 19$ | 216 | 104 7/19 | -1 | 206 | 105 | 7 | 204 | 106 | 8 | 214 7/19 | 107 7/19 | -2 12/19 | 215 7/19 | 108 7/19 | $-412 / 19$ | -1912/19 | $2667 / 19$ |  |
| -51 12/19 | 230 7/19 | 1497/19 | $437 / 19$ | 1267 | 150 7/19 | $527 / 19$ | $1167 / 19$ | $1517 / 19$ | $537 / 19$ | 114 7/19 | 152 7/19 | $427 / 19$ | 124 7/19 | 153 7/19 | $407 / 19$ | 125 7/19 | -1712/19 | 264 7/19 | 2021 |
|  | 228 | 118 7/19 | 156 7/19 | 44 | 127 | 146 7/19 | $457 / 19$ | 128 | 144 7/19 | $467 / 19$ | $1177 / 19$ | 154 7/19 | $477 / 19$ | 1155 | $1557 / 19$ | 487/19 | -1512/19 | 262 7/19 | 2021 |
| $-4712 / 19$ | 226 | $517 / 19$ | $1197 / 19$ | 148 | $417 / 19$ | 120 7/19 | 157 7/19 | 39 | 1217/19 | 158 7/19 | $497 / 19$ | 122 7/19 | 1477/19 | 50 7/19 | 123 7/19 | 1457/19 | -1312/19 | 260 7/19 | 2021 |
| -4512/19 | 224 7/19 | 164 7/19 | $137 / 19$ | $1417 / 19$ | 165 7/19 | $227 / 19$ | 131 7/19 | 166 7/19 | $237 / 19$ | 129 7/19 | 167 7/19 | $127 / 19$ | $1397 / 19$ | 168 7/19 | $107 / 19$ | $1407 / 19$ | -1112/19 | $2587 / 19$ | 2021 |
| -4312/19 | 222 71 | $1337 / 1$ | $1717 / 19$ | 14 7/19 | 142 | 1617 | $157 / 19$ | 14371 | $1597 / 19$ | $167 / 19$ | 132 7/19 | $1697 / 19$ | $177 / 19$ | $1307 / 19$ | $1707 / 19$ | 18 7/19 | -912/19 | $2567 / 19$ | 2021 |
| -4112/19 | 2207 | 21719 | 134 7/19 | 16371 | $117 / 19$ | 135 7/19 | 172 7/19 | 97/19 | 136 7/19 | 173 7/19 | $197 / 19$ | 137 7/19 | 162 7/19 | $207 / 19$ | $13887 / 19$ | 160 7/19 | $-712 / 19$ | 254 7/19 | 2021 |
| -3912/19 | 233 7/19 | -3612/19 | -34 12/19 | -32 12/19 | -3012/19 | -28 12/19 | $-2612 / 19$ | -24 12/19 | -2312/19 | $2317 / 19$ | $2297 / 19$ | 227 7/19 | 225 7/19 | 223 7/19 | $2217 / 19$ | $2197 / 19$ | 2357119 | 252 7/19 | 2021 |
| $2697 / 19$ | 2517119 | 253 7/19 | 255 7/19 | $2577 / 19$ | $2597 / 19$ | 2617119 | 263 7/19 | 265 7/19 | -57 12/19 | -58 12/19 | -60 12 | -62 12/19 | -64 12/19 | -6612/19 | -68 12/19 | -70 12/19 | -72 12/19 | 267 7/19 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the blocks of order 3 are semi-magic squares with equal magic sums. In each case the semi-magic sums are given by

$$
\begin{gathered}
{\left[\operatorname{Sm}_{3 \times 3}: S_{19 \times 19}:=21\right]:=\frac{63}{19}} \\
{\left[\operatorname{Sm}_{3 \times 3}: S_{19 \times 19}:=441\right]:=\frac{1323}{19}} \\
{\left[\operatorname{Sm}_{3 \times 3}: S_{19 \times 19}:=2021\right]:=\frac{6063}{19}}
\end{gathered}
$$

### 1.12.2 Second Type

| -15917/19 | -14317/19 | -14517/19 | $-14717 / 19$ | -149 17/19 | $-15177 / 19$ | -15317/19 | $-15517 / 19$ | -15717/19 | 165 2/19 | 1662119 | $1682 / 19$ | 170 2/19 | 172 2/19 | $1742 / 19$ | $1762 / 19$ | 178 2/19 | $1802 / 19$ | -16117/19 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1812 / 19$ | -127 17/19 | 1442119 | 142 2/19 | $1402 / 19$ | $1382 / 19$ | 1362119 | $1342 / 19$ | 132 2/19 | $1312 / 19$ | -123 17/19 | -12117/19 | -119 17/19 | -11717/19 | $-11517 / 19$ | $-11317 / 19$ | $-11117 / 19$ | -12517/19 | -178 17/19 | 21 |
| 179 2/19 | -14217/19 | -110 17/19 | $1062 / 19$ | -8177/19 | $72 / 19$ | $852 / 19$ | -8017/19 | $762 / 19$ | $-3617 / 19$ | $-717 / 19$ | $552 / 19$ | -6517/19 | $462 / 19$ | $-2117 / 19$ | 222119 | 25 2/19 | $1452 / 19$ | $-17617 / 19$ | 21 |
| 177 2/19 | -14017/19 | $82 / 19$ | 972/19 | -10917/19 | $992 / 19$ | -88 17/19 | -617/19 | $672 / 19$ | -7917/19 | 69 2/19 | $-4317119$ | $232 / 19$ | $372 / 19$ | -6417/19 | 392119 | $-2817 / 19$ | 1432119 | $-17417 / 19$ | 21 |
| $1752 / 19$ | -13817/19 | $1002 / 19$ | -9517/19 | $12 / 19$ | 982119 | -9717/19 | 70 2/19 | -50 17/19 | -1317/19 | $682 / 19$ | -6717/19 | 402119 | -3517/19 | $162 / 19$ | 382119 | -5217/19 | 1412119 | 析 | 21 |
| 1732119 | -13617/19 | 912119 | -9617/19 | $1122 / 19$ | -9417/19 | $-517 / 19$ | 612119 | -6617/19 | 82 2/19 | -4917/19 | -201719 | 312119 | -5117/19 | 52 2/19 | $-3417119$ | 92119 | 139219 | -170 17/19 | 1 |
| $1712 / 19$ | -13417/19 | -8217/19 | $-417 / 19$ | 842119 | -10317/19 | $1132 / 19$ | -3717/19 | -1917/19 | $542 / 19$ | $-7317 / 19$ | $832 / 19$ | $-2217 / 19$ | 10 2/19 | $242 / 19$ | -581719 | $53 / 19$ | $1372 / 19$ | -168 17/19 | 21 |
| 169 2/19 | -132 17/19 | -108 17/ | 105 2/19 | -8317/19 | $52 / 19$ | $882 / 19$ | $-7817 / 19$ | $752 / 19$ | -38 | -91 | $582 / 19$ | -6317/19 | $452 / 19$ | $-2317 / 19$ | $202 / 19$ | $282 / 19$ | $1352 / 19$ | 171 | 21 |
| 1672119 | -13017/19 | 62119 | 952119 | -10617/19 | $1012 / 19$ | -8917/19 | -817/19 | $652 / 19$ | $-7617 / 19$ | $712 / 19$ | $-44171 / 19$ | 212119 | $352 / 19$ | $-6117 / 19$ | 412119 | $-2917 / 19$ | $1332 / 19$ | -164 17/1 | 21 |
| -160 171/19 | 1292119 | 1032119 | -9317/19 | 2/19 | $962 / 19$ | -9917/19 | 73219 | $-4817119$ | $-1417 / 19$ | 662119 | -6917/19 | $432 / 19$ | -3317/19 | $152 / 19$ | $362 / 19$ | -5417/19 | -12617119 | 163 2/19 |  |
| -158 171/19 | 127 2/19 | $902 / 19$ | -9817/19 | $1102 / 19$ | -9117/19 | $-317 / 19$ | 60 2/19 | -6817/19 | $802 / 19$ | -4617/19 | -1817/19 | $302 / 19$ | -5317/19 | 50 2/19 | -3117/19 | 112119 | -124 17/19 | 161211 | 21 |
| -156171/19 | $1252 / 19$ | -8417/19 | $-117 / 19$ | $862 / 19$ | -10417/19 | $1112 / 19$ | -3917/19 | $-1617 / 19$ | $562 / 19$ | $-7417 / 19$ | $812 / 19$ | $-2417 / 19$ | 13 2/19 | $262 / 19$ | -5917/19 | $512 / 19$ | -12217119 | 159 2/1 | 1 |
| -154171/19 | 1232119 | -10717/19 | $1072 / 19$ | -8517/19 | $32 / 19$ | $892 / 19$ | $-7717 / 19$ | $772 / 19$ | $-4017119$ | -1117/19 | 59219 | -62 17/19 | $472 / 19$ | -2517/19 | 18 2/19 | 29219 | -120 17/19 | 157 2/19 | 21 |
| -152 171/19 | 1212119 | $42 / 19$ | 932/19 | -10517/19 | 102 2/19 | -8717/19 | -1017/19 | 63 2/19 | $-7517 / 19$ | 72 2/19 | $-4217 / 19$ | 19 2/19 | $332 / 19$ | -6017/19 | 422119 | $-2717119$ | -118 171/19 | 155 2/19 | 21 |
| -150 171/19 | 1192119 | $1042 / 19$ | -921719 | 2 2/19 | 942119 | -10117/19 | 742119 | -4717/19 | -1217/19 | 642119 | $-7117 / 19$ | 442119 | -3217/19 | $172 / 19$ | 342119 | -561719 | -116171/19 | 15321 | 21 |
| -148 17/19 | $1172 / 19$ | $922 / 19$ | -10017/19 | $1082 / 19$ | -9017/19 | $-217 / 19$ | 62 2/19 | -70 17/19 | 78 2/19 | -4517/19 | -1717/19 | 32 2/19 | -5517/19 | $482 / 19$ | -3017/19 | 12 2/19 | -114 171/19 | $1512 / 19$ | 21 |
| -146171/9 | $1152 / 19$ | -8617/19 | -17/19 | $872 / 19$ | -102 17/19 | 109 2/19 | -4117119 | -1517/19 | $572 / 19$ | -7217/19 | 79 2/19 | $-2617 / 19$ | $142 / 19$ | 272119 | -5717/19 | 492119 | -112 17119 | 1492119 | 21 |
| -144171/19 | $1282 / 19$ | -14117/19 | -139 17/19 | -137 77/19 | -13517/19 | -133 $17 / 19$ | $-13177 / 19$ | - $129177 / 19$ | -12817/19 | $1262 / 19$ | $1242 / 19$ | 122 2/19 | 1202119 | $1182 / 19$ | $1162 / 19$ | 114 2/19 | 1302119 | $1472 / 19$ | 21 |
| $1642 / 19$ | 1462119 | 1482119 | $1502 / 19$ | 152 2/19 | $1542 / 19$ | $1562 / 19$ | $1582 / 19$ | 160 2/19 | -16217/19 | -163 17/19 | $-16517 / 19$ | -16717/19 | -16917/19 | -17117/19 | -17317/19 | -175 17/19 | -17717/19 | 162 2/19 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| -137 15/19 | -121 15/19 | -123 15/19 | -125 15/19 | -127 15/19 | -129 15/19 | -131 15/19 | -133 15/19 | -135 15/19 | 187 4/19 | 188 4/19 | $1904 / 19$ | $1924 / 19$ | $1944 / 19$ | $1964 / 19$ | $1984 / 19$ | $2004 / 19$ | $2024 / 19$ | $-13915 / 19$ | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 203 4/19 | -105 15/19 | $1664 / 19$ | $1644 / 19$ | 162 4/19 | 160 4/19 | 158 4/19 | $1564 / 19$ | 154 4/19 | 153 4/19 | -101 15/19 | -99 15/19 | -9715/19 | -9515/19 | -9315/19 | -9115/19 | -89 15/19 | -103 15/19 | -156 15/19 | 441 |
| $2014 / 19$ | -120 15/19 | -8815/19 | 128 4/19 | -59 15/19 | 29 4/19 | 107 4/19 | -5815/19 | $984 / 19$ | $-1415 / 19$ | 14 4/19 | $774 / 19$ | -43 15/19 | $684 / 19$ | 4/19 | 44 4/19 | $474 / 19$ | 167 4/19 | -154 15/19 | 441 |
| 199 4/19 | -118 15/19 | $304 / 19$ | $1194 / 19$ | -8715/19 | $1214 / 19$ | -6615/19 | 15 | 89 4/19 | $-5715 / 19$ | $914 / 19$ | $-2115 / 19$ | $454 / 19$ | 59 4/19 | $-4215 / 19$ | $614 / 19$ | -615/19 | 19 | -152 15/19 | 441 |
| $1974 / 19$ | -116 15/19 | 122 4/19 | -73 15/19 | 23 4/19 | 120 4/19 | -75 15/19 | $924 / 19$ | -28 15/19 | 8 4/19 | $904 / 19$ | $-4515 / 19$ | 62 4/19 | -13 15/19 | $384 / 19$ | $604 / 19$ | -30 15/19 | 163 4/19 | -150 15/19 | 441 |
| $1954 / 19$ | -114 15/19 | $1134 / 19$ | -7415/19 | $1344 / 19$ | -72 | 16 4/19 | 83 4/19 | -4415/19 | $1044 / 19$ | $-2715 / 19$ | 1 | 53 4/19 | -29 15/19 | 74 4/19 | $-1215 / 19$ | $314 / 19$ | $1614 / 19$ | -148 15/19 | 441 |
| $1934 / 19$ | -112 15/19 | -60 15/19 | $174 / 19$ | $1064 / 19$ | -81 15/19 | $1354 / 19$ | -1515/19 | 2 | $764 / 19$ | -51 15/19 | $1054 / 19$ | -15/19 | $324 / 19$ | $464 / 19$ | -36 15/19 | $754 / 19$ | 159 4/19 | -14615/19 | 441 |
| 191 4/19 | -110 15/19 | -86 15/19 | 127 4/19 | -61 15/19 | $274 / 19$ | 110 4/19 | -5615/19 | 97 4/19 | -16 15/19 | $124 / 19$ | $804 / 19$ | $-4115 / 19$ | $674 / 19$ | -115/19 | 42 4/19 | $504 / 19$ | 157 4/19 | -144 15/19 | 441 |
| 189 4/19 | -108 15/19 | 28 4/19 | $1174 / 19$ | $-8415 / 19$ | 123 4/19 | -6715/19 | 13 4/19 | $874 / 19$ | $-5415 / 19$ | 93 4/19 | $-2215 / 19$ | $434 / 19$ | 57 4/19 | $-3915 / 19$ | 63 4/19 | $-715 / 19$ | 155 4/19 | -142 15/19 | 441 |
| -138 15/19 | $1514 / 19$ | $1254 / 19$ | -71 15/19 | 22 4/19 | $1184 / 19$ | -77 15/19 | 95 4/19 | $-2615 / 19$ | $74 / 19$ | 88 4/19 | $-4715 / 19$ | $654 / 19$ | -11 15/19 | $374 / 19$ | 58 4/19 | -32 15/19 | -104 15/19 | 185 4/19 | 441 |
| -736 15/19 | 149 4/19 | $1124 / 19$ | -7615/19 | $1324 / 19$ | -69 15/19 | $184 / 19$ | 82 4/19 | -4615/19 | $1024 / 19$ | $-2415 / 19$ | 3 4/19 | $524 / 19$ | -31 15/19 | 72 4/19 | -9 15/19 | $334 / 19$ | -102 15/19 | 183 4/19 | 441 |
| -734 15/19 | $1474 / 19$ | -62 15/19 | $204 / 19$ | 108 4/19 | -82 15/19 | 133 4/19 | -17 15/19 | $54 / 19$ | $784 / 19$ | -52 15/19 | 103 4/19 | -2 15/19 | $354 / 19$ | $484 / 19$ | -3715/19 | 73 4/19 | -100 15/19 | $1814 / 19$ | 441 |
| -732 15/19 | $1454 / 19$ | -85 15/19 | 129 4/19 | -63 15/19 | $254 / 19$ | $1114 / 19$ | -55 15/19 | 99 4/19 | -18 15/19 | $104 / 19$ | $814 / 19$ | $-4015 / 19$ | $694 / 19$ | -315/19 | $404 / 19$ | $514 / 19$ | -98 15/19 | $1794 / 19$ | 441 |
| -130 15/19 | $1434 / 19$ | 26 4/19 | 115 4/19 | -83 15/19 | 124 4/19 | -65 15/19 | $114 / 19$ | $854 / 19$ | $-5315 / 19$ | $944 / 19$ | -20 15/19 | $414 / 19$ | $554 / 19$ | $-3815 / 19$ | 644/19 | $-515 / 19$ | -96 15/19 | 177 4/19 | 441 |
| -128 15/19 | $1414 / 19$ | 126 4/19 | -70 15/19 | 24 4/19 | $1164 / 19$ | -79 15/19 | $964 / 19$ | $-2515 / 19$ | 9 4/19 | $864 / 19$ | $-4915 / 19$ | $664 / 19$ | -10 15/19 | $394 / 19$ | $564 / 19$ | -34 15/19 | -9415/19 | $1754 / 19$ | 441 |
| -126 15/19 | $1394 / 19$ | $1144 / 19$ | -7815/19 | $1304 / 19$ | -68 15/19 | 19 4/19 | 84 4/19 | -4815/19 | 100 4/19 | $-2315 / 19$ | 4 4/19 | 54 4/19 | -33 15/19 | $704 / 19$ | -8 15/19 | 34 4/19 | -92 15/19 | 173 4/19 | 441 |
| -124 15/19 | $1374 / 19$ | -6415/19 | $214 / 19$ | 109 4/19 | -80 15/19 | $1314 / 19$ | -19 15/19 | $64 / 19$ | 79 4/19 | -50 15/19 | $1014 / 19$ | -4 $15 / 19$ | 36 4/19 | 49 4/19 | -35 15/19 | $714 / 19$ | -90 15/19 | $1714 / 19$ | 441 |
| -122 15/19 | $1504 / 19$ | -119 15/19 | -117 15/19 | -115 15/19 | -113 15/19 | -111 15/19 | -109 15/19 | -107 15/19 | -106 15/19 | 148 4/19 | $1464 / 19$ | $1444 / 19$ | 142 4/19 | $1404 / 19$ | $1384 / 19$ | $1364 / 19$ | $1524 / 19$ | 169 4/19 | 441 |
| $1864 / 19$ | 168 4/19 | 170 4/19 | 172 4/19 | $1744 / 19$ | $1764 / 19$ | 178 4/19 | 180 4/19 | 182 4/19 | -140 15/19 | $-14115 / 19$ | -143 15/19 | -145 15/19 | -147 15/19 | -14915/19 | -151 15/19 | -153 15/19 | -155 15/19 | 184 4/19 | 441 |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -5412/19 | -3812/19 | $-4012 / 19$ | $-4212 / 19$ | $-4412 / 19$ | $-4612 / 19$ | -4812/19 | -5012/19 | -52 12 | 270 7/19 | $2717 / 19$ | 273 7/19 | 275 7/19 | 277 7/19 | 279 7/19 | $2817 / 19$ | 283 7/19 | $2857 / 19$ | 5612/19 | 2021 |
| 286 | -22 12/19 | 249 | 247 | 24 | 243 | $2417 / 19$ | 239 | 237 | 236 7/19 | -1812/19 | -1612/19 | -1412/19 | -12 12/19 | -10 12/19 | -812/19 | -612/19 | 2012/19 | 219 | 2021 |
| 284 | -3712\| | -512/19 | $2117 / 19$ | 23 7/19 | 112 7/19 | $1907 / 19$ | $247 / 19$ | 1817/19 | $687 / 19$ | 97 7/19 | 160 7/19 | 397119 | $1517 / 19$ | $837 / 19$ | 127 7/19 | $1307 / 19$ | $2507 / 19$ |  | 2021 |
| $2827 / 19$ | -3512/19 | 113 7/19 | 202 7/19 | -412/19 | 204 7/19 | $167 / 19$ | 98 7/19 | 172 7/19 | $257 / 19$ | $1747 / 19$ | $617 / 19$ | 128 7/19 | $1427 / 19$ | $407 / 19$ | 1447/19 | 767/19 | 248 7/19 | -69 12/19 | 2021 |
| 280 | -3312/19 | 205 | 9 | 106 | 203 | $77 / 19$ | 175 | 54 | 917/19 | 173 7/19 | 37 7/19 | 145 7/19 | $697 / 19$ | $1217 / 19$ | $1437 / 19$ | 52 7/19 | 246 7/19 | 19 | 2021 |
|  | -31 12/19 | $1967 / 19$ | $87 / 19$ | 217 7/19 |  | $997 / 19$ | 166 7/19 | 387119 | 187 7/19 | 55 7/19 | 84 | 136 7/19 | 53 7/19 | 157 7/19 | 70 7/19 | 1147/19 | 244 7/19 | -65 12/19 | 2021 |
| $2767 / 19$ | -2912/19 | $227 / 19$ | 1007 | 1897 | 17/19 | 2187/19 | 67 7/19 | $857 / 19$ | $1597 / 19$ | $317 / 19$ | 188 7/19 | 82 7/19 | 115 7/19 | $1297 / 19$ | $467 / 19$ | $1587 / 19$ | $2427 / 19$ | -6312/19 | 2021 |
| 274 | -27 12/19 | -312/19 | 210 | 21 | 110 | $1937 / 19$ | 26 | 180 7/19 | $667 / 19$ | 957 | 163 7/19 | $417 / 19$ | $1507 / 19$ | $817 / 19$ | 125 7/19 | 133 7/19 | 240 7/19 | -61 12/19 | 2021 |
| 272 | -2512/19 | $1117 / 19$ | $2007 / 19$ | -112/19 | 206 7/19 | $157 / 19$ | 967119 | $1707 / 19$ | $287 / 19$ | $1767 / 19$ | $607 / 19$ | 126 7/19 | 140 7/19 | $437 / 19$ | 1467/19 | $757 / 19$ | $2387 / 19$ | -59 12/19 | 2021 |
| -55 | 234 7/19 | 208 | 11 | 1057 | 201 | $57 / 19$ | 1787 | $567 / 19$ | $907 / 19$ | $1717 / 19$ | $357 / 19$ | 148 7/19 | $717 / 19$ | 120 7/19 | 1417/19 | $507 / 19$ | 2112/19 | 19 | 2021 |
| -5312\| | 23271 | 1957 | 67 | 215 | $137 / 19$ | $1017 / 19$ | 1657 | $367 / 19$ | $1857 / 19$ | 58 | $867 / 19$ | $1357 / 19$ | $517 / 19$ | 155 7/19 | $737 / 19$ | 1167/19 | -1912/19 | 19 |  |
|  | 230 7/19 | $207 / 19$ | 1037 | 1917 | 719 | 2167119 | $657 / 19$ | $887 / 19$ | 161 7/19 | $307 / 19$ | $1867 / 19$ | 80 7/19 | 118 7/19 | $1317 / 19$ | $457 / 19$ | $1567 / 19$ | -1712/19 | 264 7/19 | 2021 |
| -49 | 228 | -2 12/19 | 2127 | $197 / 19$ | 108 | $1947 / 19$ | 27 7/19 | 182 7/19 | 647/19 | 937 | 164 7/19 | $427 / 19$ | 152 7/19 | $797 / 19$ | 123 7/19 | 1347119 | -1512/19 | 262 7/19 |  |
| -4712\| | 2267119 | 109 7/19 | 1987 | -12/19 | 207 7/19 | $177 / 19$ | $947 / 19$ | 168 7/19 | $297 / 19$ | 177 7/19 | $627 / 19$ | 124 7/19 | 138 7/19 | 44719 | 1477/19 | 77 7/19 | -1312/19 | 260 7/19 | 2021 |
| -45 | 2247119 | $2097 / 19$ | $127 / 19$ | 107 7/19 | $1997 / 19$ | 7/19 | $1797 / 19$ | 577119 | 92 7/19 | $1697 / 19$ | $337 / 19$ | 1497119 | 72 7/19 | 122 7/19 | $1397 / 19$ | $487 / 19$ | -1112/19 | 258 7/19 | 2021 |
| -4312/19 | 222 7/19 | 197 7/19 | $47 / 19$ | $2137 / 19$ | $147 / 19$ | $1027 / 19$ | 167 7/19 | $347 / 19$ | $1837 / 19$ | $597 / 19$ | 87 7/19 | 137 7/19 | $497 / 19$ | $1537 / 19$ | 74 7/19 | $1177 / 19$ | -912/19 | 2567119 | 21 |
| -4112/19 | 220 7/19 | $187 / 19$ | 104 7/19 | 192 7/19 | 27/19 | 214 7/19 | 63 7/19 | $897 / 19$ | 162 7/19 | $327 / 19$ | $1847 / 19$ | 78 7/19 | $1197 / 19$ | $1327 / 19$ | $477 / 19$ | 154 7/19 | -712/19 | 254 7/19 | 2021 |
| -3912/19 | 233 7/19 | -3612/19 | -3412/19 | -32 12/19 | -30 12/19 | -2812/19 | -2612/19 | $-2412 / 19$ | $-2312 / 19$ | $2317 / 19$ | 22971 | 227 7/19 | 225 7/19 | 223 7/19 | 2217119 | 2197119 | $2357 / 19$ | 252 7/19 | 2021 |
| $2697 / 19$ | 2517/19 | 253 7/19 | 255 7/19 | 257 7/19 | $2597 / 19$ | 2617/19 | 263 7/19 | 265 7/19 | $-5712 / 19$ | -58 12/19 | -60 12/19 | -62 12/19 | -6412/19 | -66 12/19 | -68 12/19 | -70 12/19 | -72 12/19 | 267 7/19 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 |

In this case, the blocks of order 5 are pandiagonal with equal magic sums. In each case the magic sums order 5 are given by

$$
\begin{aligned}
{\left[S_{5 \times 5}: S_{19 \times 19}:=21\right] } & :=\frac{105}{19} \\
{\left[S_{5 \times 5}: S_{19 \times 19}:=441\right] } & :=\frac{2205}{19} \\
{\left[S_{5 \times 5}: S_{19 \times 19}:=2021\right] } & :=\frac{10105}{19} .
\end{aligned}
$$

### 1.13 Block-Wise Magic Squares of Order 20

Below are block-wise magic squares of order 20 in two different ways giving magic sums $21,21^{2}$ and 2021. First, pandiagonal blocks of order 4 with equal magic sums. The second, pandiagonal blocks of order 5 with different magic sums.

### 1.13.1 First Type

|  | pan | 21 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -9.45 | 1.55 | -179.45 | 191.55 | -32.45 | 24.55 | -162.45 | 174.55 | -50.45 | 42.55 | -140.45 | 152.55 | -73.45 | 65.55 | -123.45 | 135.55 | -91.45 | 83.55 | -101.45 | 113.55 | 21 |
| 21 | -188.45 | 200.55 | -18.45 | 10.55 | -165.45 | 177.55 | -35.45 | 27.55 | -147.45 | 159.55 | -57.45 | 49.55 | -124.45 | 136.55 | -74.45 | 66.55 | -106.45 | 118.55 | -96.45 | 88.55 | 21 |
| 2 | 181.55 | -189.45 | 11.55 | 0.55 | 164.55 | -172.45 | 34.55 | -22.45 | 142.55 | -150.4 | 52.55 | -40.45 | 125.55 | -133.45 | 75.55 | -63.45 | 103.55 | -111.45 | 93.55 | -81.45 | 21 |
| 21 | 20.55 | -8.45 | 190.55 | -198.45 | 37.55 | -25.45 | 167.55 | -175.45 | 59.55 | -47.45 | 149.55 | -157.45 | 76.55 | -64.45 | 126.55 | -134.45 | 98.55 | -86.45 | 108.55 | -116.45 | 21 |
| 2 | -70.45 | 62.55 | -12 | 132.55 | -93 | 85.55 | -103. | 115 | -11. | 3.55 | -181 | 193.55 | -29.45 | 21.55 | -1 | 171.55 | 5 | 55 | -142.45 | 154.55 | 21 |
| 2 | -127.45 | 139.55 | -77.45 | 69.55 | -104.45 | 116.55 | -94.45 | 86.55 | -186.45 | 198.55 | -16.45 | 8.55 | -168.45 | 180.55 | -38.45 | 30.55 | -145.45 | 157.55 | -55.45 | 47.55 | 21 |
| 21 | 122.55 | -130.45 | 72.55 | -60.45 | 105.55 | -113.45 | 95.55 | -83.45 | 183.55 | -191.45 | 13.55 | -1.45 | 161.55 | -169.45 | 31.55 | -19.45 | 144.55 | -152.45 | 54.55 | -42.45 | 21 |
| 21 | 79. | -67 | 129.55 | -137.4 | 96.55 | -84.45 | 106.55 | -114.45 | 18.55 | -6.45 | 188.55 | -196.45 | 40.55 | -28.45 | 170.55 | -178.45 | 57.55 | -45.45 | 147.55 | -155.45 | 21 |
| 2 | -31.45 | 23.55 | -161.45 | 173.55 | -49.45 | 41.55 | -139.45 | 151.55 | -72.4 | 64.55 | -122.45 | 134.55 | -90.45 | 82.55 | -100.45 | 112.55 | -13.45 | 5.55 | -183.45 | 195.55 | 21 |
| 21 | -166.45 | 178.55 | -36.45 | 28.55 | -148.45 | 160.55 | -58.45 | 50.55 | -125.45 | 137.55 | -75.45 | 67.55 | -107.45 | 119.55 | -97.45 | 89.55 | -184.45 | 196.55 | -14.45 | 6.55 | 21 |
| 2 | 163.55 | -17 | 33.55 | -21.4 | 141.55 | -149.4 | 51.55 | -39.45 | 124.55 | -132.4 | 74.55 | -62 | 102.55 | -110.45 | 92.55 | -80.45 | 185.55 | -193.45 | 15.55 | -3.45 | 21 |
| 21 | 38.55 | -26.45 | 168.55 | -176.45 | 60.55 | -48.45 | 150.55 | -158.45 | 77.55 | -65.45 | 127.55 | -135.45 | 99.55 | -87.45 | 109.55 | -117.45 | 16.55 | -4.45 | 186.55 | -194.45 | 21 |
| 21 | -92.45 | 84.55 | -102.45 | 114.55 | -10.45 | 2.55 | -180.45 | 192.55 | -33.45 | 25.55 | -163.45 | 175.55 | -51.45 | 43.55 | -141.45 | 153.55 | -69.45 | 61.55 | -119.45 | 131.55 | 21 |
| 21 | -105.45 | 117.55 | -95.45 | 87.55 | -187.45 | 199.55 | -17.45 | 9.55 | -164.45 | 176.55 | -34.45 | 26.55 | -146.45 | 158.55 | -56.45 | 48.55 | -128.45 | 140.55 | -78.45 | 70.55 | 21 |
| 21 | 104.55 | -112.45 | 94.55 | -82.45 | 182.55 | -190.45 | 12.55 | -0.45 | 165.55 | -173.45 | 35.55 | -23.45 | 143.55 | -151.45 | 53.55 | -41.45 | 121.55 | -129.45 | 71.55 | -59.45 | 21 |
| 21 | 97.55 | -85.45 | 107.55 | -115.45 | 19.55 | -7.45 | 189.55 | -197.45 | 36.55 | -24.45 | 166.55 | -174.45 | 58.55 | -46.45 | 148.55 | -156.45 | 80.55 | -68.45 | 130.55 | -138.45 | 21 |
| 2 | -53.45 | 45.55 | -143.45 | 155.55 | -71.45 | 63.55 | -121.45 | 133.55 | -89.45 | 81.55 | -99.45 | 111.55 | -12.45 | 4.55 | -182.45 | 194.55 | -30.45 | 22.55 | -160.45 | 172.55 | 21 |
| 2 | -144.45 | 156.55 | -54.45 | 46.55 | -126.45 | 138.55 | -76.45 | 68.55 | -108.45 | 120.55 | -98.45 | 90.55 | -185.45 | 197.55 | -15.45 | 7.55 | -167.45 | 179.55 | -37.45 | 29.55 | 21 |
| 21 | 145.55 | -153.45 | 55.55 | -43.45 | 123.55 | -131.45 | 73.55 | -61.45 | 101.55 | -109.45 | 91.55 | -79.45 | 184.55 | -192.45 | 14.55 | -2.45 | 162.55 | -170.45 | 32.55 | -20.45 | 21 |
|  | 56.55 | -44.45 | 146.55 | -154.45 | 78.55 | -66.45 | 128.55 | -136.45 | 100.55 | -88.45 | 110.55 | -118.45 | 17.55 | -5.45 | 187.55 | -195.45 | 39.55 | -27.45 | 169.55 | -177.45 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 41 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | 11.55 | 22.55 | -158.45 | 212.55 | -11.45 | 45.55 | -141.45 | 195.55 | -29.45 | 63.55 | -119.45 | 173.55 | -52.45 | 86.55 | -102.45 | 156.55 | -70.45 | 104.55 | -80.45 | 134.55 | 441 |
|  | -167.4 | 221.55 | 2.55 | 31.55 | -144.4 | 198.55 | -14.45 | 48.55 | -126.45 | 180.55 | -36.4 | 70.55 | -103.45 | 157.55 | -53.45 | 87.55 | -85.45 | 139.55 | -75.45 | 109.55 | 441 |
|  | 202.5 | -168. | 32 | 21.55 | 185.5 | -15 | 55 | -1.45 | 16 | -1 | 73 | -19.45 | 146.55 | -1 | 96.55 | -42.45 | 5 | -90.45 | 5 | -60.45 | 441 |
|  | 41. | 12.55 | 211.55 | -177.4 | 58.55 | -4.45 | 188.55 | -154.45 | 80.55 | -26.45 | 170.55 | -136. | 97.55 | 5 | 147.55 | -113.45 | 119.55 | -65.45 | 129.55 | -95.45 | 1 |
|  | -49 | 83. | -99 | 153.55 | -72 | 106.55 | -82 | 136 | 9.55 | 24.55 | -160 | 21 | -8.45 | 55 | -138.45 | 192.55 | -31.45 | 65.55 | -121.45 | 175.55 | 441 |
|  | -10 | 160. | -5 | 90 | -8 | 13 | -7 | 10 | -16 | 21 | 4.55 | 29.55 | -147.45 | 20 | -17.45 | 51.55 | 5 | 178.55 | 5 | 68.55 | 441 |
|  | 143.55 | -109.4 | 93.55 | -39.45 | 126.55 | -92.45 | 116.55 | -62.4 | 204.55 | -170.45 | 34.55 | 19.5 | 182.55 | -148. | 52.55 | 1.5 | 165.55 | -131.45 | 75.55 | -21.45 | 441 |
| 441 | 100 | -4 | 150 | -11 | 117 | -63 | 12 | -93 | 39 | 14 | 209.55 | -1 | 61 | -7 | 191.55 | -1 | 78 | -24.45 | 168.55 | -134.45 | 441 |
|  | -10 | 44 | -1 | 19 | -2 | 62 | -1 | 17 | -5 | 85 | -1 | 15 | -69.45 | 10 | -79.45 | 13 | 7.55 | 26.55 | 5 | 216.55 | 441 |
|  | -145.45 | 199.55 | -15.45 | 49.55 | -127.4 | 181.55 | -37.45 | 71.55 | -104.45 | 158.55 | -54 | 88.55 | -86.45 | 140.55 | -76.45 | 110.55 | -163.45 | 217.55 | 6.55 | 27.55 | 441 |
| 441 | 18 | -15 | 54 | -0 | 16 | -12 | 72 | -18 | 14 | -1 | 95 | -4 | 123.55 | -89.45 | 113.55 | -59.45 | 206.55 | -172.45 | 36.55 | 17.55 | 441 |
|  | 59.5 | -5.4 | 189.5 | -155.4 | 81.5 | -27 | 171. | -137.4 | 98 | -4 | 148.55 | -11 | 120.55 | -6 | 130.55 | -96 | 37 | 5 | 207.55 | 5 | 1 |
|  | -71 | 105.55 | -81.45 | 135.55 | 10.55 | 23.55 | -159.4 | 213.55 | -12 | 46.55 | -142.45 | 196.55 | -30.45 | 64.55 | -120.45 | 174.55 | -48.45 | 82.55 | -98.45 | 152.55 | 441 |
| 441 | -8 | 138.55 | -74.4 | 108.55 | -166.4 | 220.5 | 3.5 | 30 | -143.4 | 197. | -1 | 47.55 | -125.45 | 179.55 | -35.45 | 69.55 | -107.45 | 161.55 | -57.45 | 91 | 441 |
| 441 | 125.5 | -91.45 | 115.55 | -61.45 | 203.55 | -169.45 | 33.55 | 20.55 | 186.55 | -152. | 56.55 | -2.45 | 164.55 | -130.45 | 74.55 | -20.45 | 142.55 | -108.45 | 92.55 | -38.45 | 441 |
| 441 | 118.55 | -64.45 | 128.55 | -94.45 | 40.55 | 13.55 | 210.55 | -176.45 | 57.55 | -3.45 | 187.55 | -153.4 | 79.55 | -25.45 | 169.55 | -135.45 | 101.55 | -47.45 | 151.55 | -117.45 | 441 |
| 441 | -32.45 | 66.55 | -122.45 | 176.55 | -50.45 | 84.55 | -100.45 | 154.55 | -68.45 | 102.55 | -78.45 | 132.55 | 8.55 | 25.55 | -161.45 | 215.55 | -9.45 | 43.55 | -139.45 | 193.55 | 441 |
|  | -123.45 | 177.55 | -33.45 | 67.55 | -105.4 | 159.55 | -55.45 | 89.55 | -87.45 | 141.55 | -77.45 | 111.55 | -164.45 | 218.55 | 5.55 | 28.55 | -146.45 | 200.55 | -16.45 | 50.55 | 441 |
| 44 | 166.55 | -132.45 | 76.55 | -22.45 | 144.55 | -110.45 | 94.55 | -40.45 | 122.55 | -88.45 | 112.55 | -58.45 | 205.55 | -171.45 | 35.55 | 18.55 | 183.55 | -149.45 | 53.55 | 0.55 | 441 |
|  | 77.55 | -23.45 | 167.55 | -133.45 | 99.55 | -45.45 | 149.55 | -115.45 | 121.55 | -67.45 | 131.55 | -97.45 | 38.55 | 15.55 | 208.55 | -174.45 | 60.55 | -6.45 | 190.55 | -156.45 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 202 | 90.55 | 101.55 | -79.45 | 291.55 | 67.55 | 124.5 | -62.45 | 274.55 | 49.55 | 142.55 | -40.45 | 252.55 | 26.55 | 165.55 | -23.45 | 235.55 | 8.55 | 183.55 | -1.4 | 213.55 | $\begin{aligned} & 2021 \\ & 2021 \end{aligned}$ |
| 202 | -88.45 | 300.5 | 81.55 | 110.55 | -65.45 | 277.5 | 64.5 | 127.5 | -47.45 | 259. | 42.55 | 149. | -24.45 | 236. | 25. | 166.55 | -6. | 218.55 | 3.55 | 188.55 |  |
| 2021 | 281.55 | -89.45 | 111.55 | 100.55 | 264.55 | -72.45 | 134.55 | 77.55 | 242.55 | -50.45 | 152.55 | 59.55 | 225.55 | -33.45 | 175.55 | 36.55 | 203.55 | -11.45 | 193.55 | 18.55 | 2021 |
| 202 | 120.5 | 91.55 | 290.55 | -98.4 | 137.5 | 74. | 26 | -75.4 | 159.5 | 52.55 | 24 | -57.45 | 17 | 35.55 | 226.55 | -34.45 | 198.55 | 13.55 | 208.55 | , 5 | 2021 |
| 20 | 29. | 162. | -20.4 | 232.5 | 6.55 | 185 | -3. | 21 | 88 | 103.5 | -8 | 29 | 70 | 12 | -5 | 271.55 | 47.55 | 144.55 | 5 | 254.55 | 2021 |
| 2021 | -27.45 | 239.55 | 22.55 | 169.55 | -4.45 | 216.5 | 5.55 | 186.55 | -86.45 | 298.55 | 83.55 | 108.55 | -68.45 | 280.55 | 61.55 | 130.55 | -45.45 | 257.55 | 44.55 | 147.55 | 2021 |
| 202 | 222 | -30.4 | 172 | 39 | 20 | -13 | 195 | 16. | 283 | -9 | 113 | 98 | 26 | -69. | 131. | 80.5 | 24 | -52.45 | 154.55 | 57.55 | 2021 |
| 202 | 179 | 32.5 | 229.5 | -37 | 196 | 15.5 | 206 | -1 | 118 | 93.55 | 288.5 | -96.45 | 14 | 71 | 270.55 | -78.45 | 15 | 54.55 | 247.55 | 5 | 2021 |
| 2021 | 68.55 | 123.5 | -61.45 | 273.55 | 50.55 | 141.55 | -39.45 | 251.55 | 27.55 | 164.55 | -22.45 | 234.5 | 9.55 | 182.5 | -0.45 | 212.55 | 86.55 | 105.55 | -83.45 | 295.55 | 2021 |
| 202 | -66.4 | 278.5 | 63.55 | 128.5 | -48.4 | 260.5 | 41.5 | 150.5 | -25.4 | 237.5 | 24.55 | 167.5 | -7.45 | 219.5 | 2.55 | 189.55 | -84. | 296.55 | 85.55 | 106.55 | 2021 |
| 202 | 263.5 | -71.4 | 133.5 | 78. | 24 | -49 | 151. | 60 | 22 | -32.45 | 17 | 37 | 202.5 | -10. | 192.55 | 19 | 285.55 | -93.45 | 115.55 | 96.55 | 2021 |
| 2021 | 138.55 | 73.55 | 268.55 | -76.4 | 160.55 | 51.55 | 250.55 | -58.4 | 177.5 | 34.55 | 227.5 | -35.45 | 199.5 | 12.55 | 209.55 | -17.45 | 116.55 | 95.55 | 286.55 | -94.45 | 2021 |
| 2021 | 7.55 | 184 | -2.45 | 214.55 | 89.55 | 102.5 | -80.4 | 292.5 | 66.55 | 125.5 | -63.45 | 275.55 | 48.55 | 143.55 | -41.45 | 253.55 | 30.55 | 161.55 | -19.45 | 231.55 | 2021 |
| 202 | -5.45 | 217.5 | 4.55 | 187.5 | -87.4 | 299.5 | 82.55 | 109.5 | -64.4 | 276.55 | 65.55 | 126.55 | -46.45 | 258.55 | 43.55 | 148.55 | -28.45 | 240.55 | 21.55 | 170.55 | 2021 |
| 2021 | 204.55 | -12.45 | 194.55 | 17.55 | 282.5 | -90.4 | 112.5 | 99.55 | 265.5 | -73.45 | 135.55 | 76.55 | 243.55 | -51.45 | 153.55 | 58.55 | 221.55 | -29.45 | 171.55 | 40.55 | 2021 |
| 2021 | 197.55 | 14.55 | 207.55 | -15.45 | 119.55 | 92.55 | 289.55 | -97 | 136.55 | 75.55 | 266.55 | -74.45 | 158.55 | 53.55 | 248.55 | -56.45 | 180.55 | 31.55 | 230.55 | -38.45 | 2021 |
| 2021 | 46.55 | 145.55 | -43.45 | 255.55 | 28.55 | 163.55 | -21.45 | 233.5 | 10.55 | 181.5 | 0.55 | 211.55 | 87.55 | 104.55 | -82.45 | 294.55 | 69.55 | 122.55 | -60.45 | 272.55 | 2021 |
| 2021 | -44.45 | 256.55 | 45.55 | 146.55 | -26.45 | 238.55 | 23.55 | 168.55 | -8.45 | 220.55 | 1.55 | 190.55 | -85.45 | 297.55 | 84.55 | 107.55 | -67.45 | 279.55 | 62.55 | 129.55 | 2021 |
| 2021 | 245.55 | -53.45 | 155.55 | 56.55 | 223.55 | -31.45 | 173.55 | 38.55 | 201.55 | -9.45 | 191.55 | 20.55 | 284.55 | -92.45 | 114.55 | 97.55 | 262.55 | -70.45 | 132.55 | 79.55 | 2021 |
|  | 156.55 | 55.55 | 246.55 | -54.45 | 178.55 | 33.55 | 228.55 | -36.45 | 200.55 | 11.55 | 210.55 | -18.45 | 117.55 | 94.55 | 287.55 | -95.45 | 139.55 | 72.55 | 269.55 | -77.45 | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the block of order 4 are pandiagonal magic squares with equal magic sums given by

$$
\begin{gathered}
{\left[S_{4 \times 4}: S_{20 \times 20}:=21\right]:=\frac{21}{5}=4.2} \\
{\left[S_{4 \times 4}: S_{20 \times 20}:=441\right]:=\frac{441}{5}=88.2} \\
{\left[S_{4 \times 4}: S_{20 \times 20}:=2021\right]:=\frac{2021}{5}=404.2}
\end{gathered}
$$

### 1.13.2 Second Type

|  | pan | 21 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -192.45 | -64.45 | -16.45 | 111.55 | 159.55 | -187.45 | -59.45 | -11.45 | 116.55 | 164.55 | -198.45 | -70.45 | -22.45 | 105.55 | 153.55 | -185.45 | -57.45 | -9.45 | 118.55 | 166.55 | 1 |
| 21 | 63.55 | 191.55 | -160.45 | -112.45 | 15.55 | 68.55 | 196.55 | -155.45 | -107.45 | 20.55 | 57.55 | 185.55 | -166.45 | -118.45 | 9.55 | 70.55 | 198.55 | -153.45 | -105.45 | 22.55 | 1 |
| 21 | -80.45 | -32.45 | 95.55 | 143.55 | -128.45 | -75.45 | -27.45 | 100.55 | 148.55 | -123.45 | -86.45 | -38.45 | 89.55 | 137.55 | -134.45 | -73.45 | -25.45 | 102.55 | 150.55 | -121.45 | 21 |
| 21 | 175.55 | -176.45 | -48.45 | -0.45 | . 55 | 180.55 | 71.45 | -43.45 | 4.55 | 52.55 | 169.55 | -182.45 | -54.45 | -6.45 | 41.55 | 182.55 | -169.45 | -41.45 | 6.55 | 4.55 | 11 |
| 21 | 31.55 | 79.55 | 127.55 | -144.45 | -96.45 | 36.55 | . 55 | 132.55 | -139.45 | -91.45 | 25.55 | 73.55 | 121.55 | -150.45 | -102.45 | 38.55 | 86.55 | 134.55 | -137.45 | -89.45 | 21 |
| 21 | -197.45 | -69.45 | -21.45 | 106.55 | 154.55 | -186.45 | -58.45 | -0.45 | 55 | 5.55 | -191.45 | -63.45 | -15.45 | 112.55 | 160.55 | -188.45 | -60.45 | -12.45 | 115.55 | 163.55 | 21 |
| 21 | 58.55 | 186.55 | -165.45 | -117.45 | 10.55 | 69.55 | 197.55 | -154.45 | -106.45 | 21.55 | 64.55 | 192.55 | -159.45 | -111.45 | 16.55 | 67.55 | 195.55 | -156.45 | -108.45 | 19.55 | 21 |
| 21 | -85.45 | -37.45 | 90.55 | 138.55 | -133.45 | -74.45 | -26.45 | 101.55 | 149.55 | -122.45 | -79.45 | -31.45 | 96.55 | 144.55 | -127.45 | -76.45 | -28.45 | 99.55 | 147.55 | -124.45 | 21 |
| 21 | 170.55 | -181.45 | -53.45 | -5.45 | 42.55 | 181.55 | -170.45 | -42.45 | 5.55 | 53.55 | 176.55 | -175.45 | -47.45 | 0.55 | 48.55 | 179.55 | -172.45 | -44.45 | 3.55 | 51.55 | 21 |
| 21 | 26.55 | 74.55 | 122.55 | -149.45 | -101.45 | 37.55 | 85.55 | 133.55 | -138.45 | -90.45 | 32.55 | 80.55 | 128.55 | -143.45 | -95.45 | 35.55 | 83.55 | 131.55 | -140.45 | -92.45 | 21 |
| 21 | -183.45 | -55.45 | -7.45 | 120.55 | 168.55 | -196.45 | -68.45 | -20.45 | 107.55 | 155.55 | -189.45 | -61.45 | -13.45 | 114.55 | 162.55 | -194.45 | -66.45 | -18.45 | 109.55 | 157.55 | 21 |
| 21 | 72.55 | 200.55 | -151.45 | -103.45 | 24.55 | 59.55 | 187.55 | -164.45 | -116.45 | 11.55 | 66.55 | 194.55 | -157.45 | -109.45 | 18.55 | 61.55 | 189.55 | -162.45 | -114.45 | 13.55 | 21 |
| 21 | -71.45 | -23.45 | 104.55 | 152.55 | -119.45 | -84.45 | -36.45 | 91.55 | 139.55 | -132.45 | -77.45 | -29.45 | 98.55 | 146.55 | -125.45 | -82.45 | -34.45 | 93.55 | 141.55 | -130.45 | 21 |
| 21 | 184.55 | -167.45 | -39.45 | 8.55 | 56.55 | 171.55 | -180.45 | -52.45 | -4.45 | 43.55 | 178.55 | -173.45 | -45.45 | 2.55 | 50.55 | 173.55 | -178.45 | -50.45 | -2.45 | 45.55 | 21 |
| 21 | 40.55 | 88.55 | 136.55 | -135.45 | -87.45 | 27.55 | 75.55 | 123.55 | -148.45 | -100.45 | 34.55 | 82.55 | 130.55 | -141.45 | -93.45 | 29.55 | 77.55 | 125.55 | -146.45 | -98.45 | 21 |
| 21 | -190.45 | -62.45 | -14.45 | 113.55 | 161.55 | -193.45 | -65.45 | -17.45 | 110.55 | 158.55 | -184.45 | -56.45 | -8.45 | 119.55 | 167.55 | -195.45 | -67.45 | -19.45 | 108.55 | 156.55 | 21 |
| 21 | 65.55 | 193.55 | -158.45 | -110.45 | 17.55 | 62.55 | 190.55 | -161.45 | -113.45 | 14.55 | 71.55 | 199.55 | -152.45 | -104.45 | 23.55 | 60.55 | 188.55 | -163.45 | -115.45 | 12.55 | 21 |
| 21 | -78.45 | -30.45 | 97.55 | 145.55 | -126.45 | -81.45 | -33.45 | 94.55 | 142.55 | -129.45 | -72.45 | -24.45 | 103.55 | 151.55 | -120.45 | -83.45 | -35.45 | 92.55 | 140.55 | -131.45 | 21 |
| 21 | 177.55 | -174.45 | -46.45 | 1.55 | 49.55 | 174.55 | -177.45 | -49.45 | -1.45 | 46.55 | 183.55 | -168.45 | -40.45 | 7.55 | 55.55 | 172.55 | -179.45 | -51.45 | -3.45 | 44.55 | 21 |
|  | 33.55 | 81.55 | 129.55 | -142.45 | -94.45 | 30.55 | 78.55 | 126.55 | -145.45 | -97.45 | 39.55 | 87.55 | 135.55 | -136.45 | -88.45 | 28.55 | 76.55 | 124.55 | -147.45 | -99.45 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan | 441 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 41 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 44 | -171.4 | -43.45 | 4.5 | 132.55 | 180.55 | -166.4 | -38.4 | 9.5 | 137.55 | 185.55 | -177.45 | -49.45 | -1.45 | 126.55 | 174.55 | -164.45 | -36.45 | 11.55 | 139.55 | 187.55 |  |
|  | 84.5 | 212.55 | -139.45 | -91.4 | 36.55 | 89.55 | 217.55 | -134.45 | -86. | 41 | 78.5 | 206.55 | -145.45 | -97. | 30 | 91.55 | 219.55 | -132.45 | -84.45 | 43.55 | 441 |
|  | -59 | -11. | 116.5 | 164.55 | -107.45 | -54.4 | -6. | 121.55 | 169.55 | -102.45 | -6 |  | 110.55 | 158.55 | -113.45 | -52.45 | 5 | 123.55 | 171.55 | -100.45 | 441 |
|  | 19 | -15 |  | 20 |  | 20 | -1 | -2 | 25.55 | 73.55 | 19 | -1 |  |  |  | 5 | 5 | -20.45 |  | 75.55 | 441 |
|  | 52 | 100.5 | 148.5 | -12 | -75 | 57 | 105.5 | 153.55 | -1 | -7 | 46.55 | 94 | 142.55 | -129.45 | -8 | 59.55 | 107.55 | 155.55 | -116.45 | 5 | 441 |
| 4 | -17 | -48.4 | -0.45 | 127 | 175.55 | -165.4 | -37 | 10.55 | 138.5 | 186.55 | -170.45 | -42 | 5.55 | 133.55 | 181.55 | -167.45 | -39.45 | 8.55 | 136.55 | 184.55 | 1 |
|  | 79 | 20 | -1 | -9 | 31 | 90 | 21 | -1 | -8 |  | 85 | 2 | -138.45 | -9 | 37.55 | 88.55 | 5 | 5 | 5 | 40.55 | 441 |
|  |  | -16 | 11 | 159 | -112 | -5 | -5 | 12 | 17 | -1 | -5 | -1 | 5 | 165.55 | -106.45 | -55.45 | -7.45 | . 55 | 168.55 | 5 | 441 |
| 4 | 19 | -16 | -3 | 15 | 63 | 202.55 | -1 | -2 |  | 74 | 19 | -154.45 | -2 | 21.55 | 69 | 200.55 | -151.45 | -23.45 | 24.55 | 5 | 1 |
| 441 | 47 | 95 | 14 | -1 | -8 | 58 | 10 | 15 | -1 | -6 | 53 | 10 | 149.55 | -1 | -7 | 56 | 104.55 | 152.55 | -119.45 | 5 | 441 |
|  | -16 | -3 | 13. | 14 | 189.55 | -175. | -4 | 0. | 12 | 176.55 | -168.45 | -4 | 7.55 | 135.55 | 183.55 | -173.45 | -4 | 5 | 130.55 | 55 | 1 |
| 4 | 93. | 22 | -13 | -8 | 45 | 80 | 208.55 | -143.45 | -9 | 32 | 87 | 215.55 | -136.45 | -88.45 | 39 | 82.55 | 210.55 | -141.45 | -93.45 | 34.55 | 1 |
|  |  | -2 | 12 | 17 | -9 | -6 | -1 | 11 | 16 | -1 | -5 | -8 |  | 167.55 | -104.45 | -61.45 | -13.45 | 114.55 | 162.55 | 5 | 1 |
|  | 205.55 | -146.4 | -18.45 | 29 | 77 | 192.55 | -159. | -31. | 16 | 64 | 199.55 | -152.45 | -24 | 23 | 71.55 | 194.55 | -157.45 | -29.45 | 18.55 | 55 | 441 |
| 4 | 61. | 109.55 | 157.55 | -11 | -66 | 48 | 96.5 | 144.55 | -127 | -79.45 | 55.55 | 103.55 | 151.55 | -120.45 | -72.45 | 50.55 | 98.55 | 146.55 | -125.45 | -77.45 | 441 |
| 4 | -169.45 | -41 | 6.5 | 134.55 | 182.55 | -172.45 | -44.4 | 3.55 | 131.55 | 179.55 | -163.45 | -35.45 | 12.55 | 140.55 | 188.55 | -174.45 | -46.45 | 1.55 | 129.55 | 177.55 | 441 |
|  | 86.55 | 214.55 | -137.45 | -89.4 | 38.55 | 83.55 | 211.55 | -140.45 | -92 | 35.55 | 92.55 | 220.55 | -131.45 | -83.45 | 44.5 | 81.55 | 209.55 | -142.45 | -94.45 | 33.55 | 441 |
| 4 | -57.45 | -9.45 | 118.55 | 166.55 | -105.45 | -60.45 | -12.45 | 115.55 | 163.55 | -108.45 | -51.45 | -3.45 | 124.55 | 172.55 | -99.45 | -62.45 | -14.45 | 113.55 | 161.55 | -110.45 | 441 |
| 44. | 198.55 | -153.45 | -25.45 | 22.55 | 70.55 | 195.55 | -156.45 | -28.45 | 19.55 | 67.55 | 204.55 | -147.45 | -19.45 | 28.55 | 76.55 | 193.55 | -158.45 | -30.45 | 17.55 | 65.55 | 441 |
|  | 54.55 | 102.55 | 150.55 | -121.45 | -73.45 | 51.55 | 99.55 | 147.55 | -124.45 | -76.45 | 60.55 | 108.55 | 156.55 | -115.45 | -67.45 | 49.55 | 97.55 | 145.55 | -126.45 | -78.45 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 20 | 2021 | 2021 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | -92.4 | 35.5 | 83.55 | 211.55 | 259.55 | -87. | 40.55 | 88.55 | 216.5 | 264.55 | -98.4 | 29.5 | 77 | 205.55 | 253.55 | -85 | 42 | 90.55 | 218.55 | 266.55 | 2021 |
| 2021 | 163.55 | 291.55 | -60.45 | -12.45 | 115.55 | 168.55 | 296.55 | -55.45 | -7.45 | 120.55 | 157.55 | 285.55 | -66.45 | -18.45 | 109.55 | 170.55 | 298.55 | -53.45 | -5.45 | 122.55 | 20 |
| 20 | 19.55 | 67. | 195.55 | 243.55 | -28.4 | 24.55 | 72.55 | 200.55 | 248.5 | -23 | 13 | 61.55 | 189.55 | 237.55 | -34. | 26.55 | 74.55 | 202.55 | 250.55 | -21.45 | 2021 |
| 20 | 275 | -7 | 51.55 | 99.55 | 147 | 28 | -71.45 | 56 | 10 | 15 | 26 | -8 | 45.55 | 93.55 | 141.55 | 28 | -69.45 | 5 | 5 | 5 | 2021 |
| 202 | 131.55 | 179.55 | 227.55 | -4 | 3.55 | 136.55 | 184.55 | 232.55 | -39.4 | 8.5 | 125.5 | 173.5 | 221.5 | -50.45 | -2.4 | 138.55 | 186.55 | 234.55 | -37.45 | 10.55 | 20 |
| 202 | -97 | 30. | 78.55 | 206.55 | 25 | -8 | 41.5 | 89 | 21 | 26 | -9 | 36 | 84 | 212.55 | 260.55 | -88.45 | 39.55 | 87.55 | 215.55 | 263.55 | 2021 |
| 20 | 158 | 286 | -65 | -17 | 110 | 169 | 29 | -5 | -6.45 | 121.55 | 16 | 292.55 | -5 | -11.45 | . 5 | 167 | 295.55 | -56.4 | 8.45 | 5 | 2021 |
| 202 | 14.55 | 62.55 | 190.55 | 238.55 | -33.4 | 25.55 | 73.55 | 201.55 | 249.5 | -22.45 | 20.55 | 68.55 | 196.55 | 244.55 | -27.45 | 23.55 | 71.55 | 199.55 | 247.55 | -24.45 | 20 |
| 202 | 270 | -81 | 46. | 94 | 142 | 28 | -70 | 57 | 105 | 153 | 27 | -75.45 | 52 | 100.5 | 148.55 | 279.55 | -72.45 | 55.55 | 103.55 | 151.55 | 2021 |
| 20 | 126 | 17 | 22 | -4 | -1. | 13 | 185 | 233 | -38 | 9. | 13 | 18 | 22 | -43.45 | 4.55 | 13 | 183.55 | 231.55 | -40.45 | 5 | 2021 |
| 2021 | -83 | 44.55 | 92.55 | 220.55 | 268.55 | -96 | 31.55 | 79.55 | 207.5 | 255.55 | -89 | 38.55 | 86.55 | 214.55 | 262.55 | -94.45 | 33.55 | 81.55 | 209.55 | 257.55 | 2021 |
| 202 | 172 | 300.5 | -5 | -3.45 | 12 | 159 | 287.5 | -64.45 | -16 | 11 | 166.5 | 294.55 | -5 | -9.45 | 118.55 | 161.55 | 289.55 | -62.45 | -14.45 | 113.55 | 2021 |
| 202 | 28.55 | 76. | 204 | 252.55 | -19 | 15. | 63. | 19 | 239 | -32.45 | 22 | 70.55 | 19 | 246.55 | -25. | 17.55 | 65.55 | 193.55 | 241.55 | -30.45 | 20 |
| 2021 | 284.5 | -67.4 | 60.55 | 108.55 | 156.5 | 271.55 | -80.4 | 47.55 | 95.55 | 143.55 | 278.55 | -73.45 | 54.55 | 102.55 | 150.55 | 273.55 | -78.45 | 49.55 | 97.55 | 145.55 | 2021 |
| 2021 | 140.55 | 188.55 | 236.55 | -35.45 | 12.55 | 127.55 | 175.5 | 223.55 | -48.4 | -0.45 | 134.5 | 182.55 | 230.55 | -41.45 | 6.55 | 129.55 | 177.55 | 225.55 | -46.45 | 1.55 | 202 |
| 2021 | -90.45 | 37.55 | 85.55 | 213.55 | 261.55 | -93.45 | 34.55 | 82.55 | 210.55 | 258.55 | -84.45 | 43.55 | 91.55 | 219.55 | 267.55 | -95.45 | 32.55 | 80.55 | 208.55 | 256.55 | 20 |
| 2021 | 165.55 | 293.55 | -58.4 | -10.45 | 117 | 162.55 | 290.5 | -61.45 | -13.4 | 114.55 | 171 | 299.55 | -52.4 | -4.45 | 123.55 | 160.55 | 288.55 | -63.45 | -15.45 | 112.55 | 2021 |
| 2021 | 21.55 | 69.55 | 197.55 | 245.55 | -26.4 | 18.55 | 66.55 | 194.55 | 242.55 | -29.45 | 27.55 | 75.55 | 203.55 | 251.55 | -20.45 | 16.55 | 64.55 | 192.55 | 240.55 | -31.45 | 202 |
| 2021 | 277.55 | -74.45 | 53.55 | 101.55 | 149.55 | 274.55 | -77.45 | 50.55 | 98.55 | 146.55 | 283.55 | -68.45 | 59.55 | 107.55 | 155.55 | 272.55 | -79.45 | 48.55 | 96.55 | 144.55 | 202 |
|  | 133.55 | 181.55 | 229.55 | -42.45 | 5.55 | 130.55 | 178.55 | 226.55 | -45.45 | 2.55 | 139.55 | 187.55 | 235.55 | -36.45 | 11.55 | 128.55 | 176.55 | 224.55 | -47.45 | 0.55 | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |  |

In this case, the blocks of order 5 are pandiagonal with different magic sums.

### 1.14 Block-Wise Magic Squares of Order 21

Below are block-wise magic squares of order 21 in two different ways giving magic sums $21,21^{2}$ and 2021. First, semimagic squares blocks of order 3 with equal semi-magic sums. The second, pandiagonal blocks of order 7 with equal magic sums.

### 1.14.1 First Type

|  | pan | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |  | 21 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 26 | -109 | 86 | 54 | 155 | -206 | -33 | -131 | 167 | -8 | 199 | -188 | -52 | -153 | 208 | 35 | 133 | -165 | -15 | -87 | 105 | 21 |
| 21 | 80 | 23 | -100 | -202 | 67 | 138 | 163 | -43 | -117 | -179 | 1 | 181 | 204 | -65 | -136 | -161 | 45 | 119 | 102 | -21 | -78 | 21 |
| 21 | -103 | 89 | 17 | 151 | -219 | 71 | -127 | 177 | -47 | 190 | -197 | 10 | -149 | 221 | -69 | 129 | -175 | 49 | -84 | 111 | -24 | 21 |
| 27 | 53 | 120 | -170 | -28 | -77 | 108 | 27 | -108 | 84 | 68 | 143 | -208 | -51 | -118 | 172 | 9 | 184 | -190 | -71 | -137 | 211 | 21 |
| 21 | -174 | 40 | 137 | 112 | -18 | -91 | 81 | 21 | -99 | -214 | 65 | 152 | 176 | -38 | -135 | -194 | -1 | 198 | 220 | -62 | -155 | 21 |
| 21 | 124 | -157 | 36 | -81 | 98 | -14 | -105 | 90 | 18 | 149 | -205 | 59 | -122 | 159 | -34 | 188 | -180 | -5 | -146 | 202 | -53 | 21 |
| 21 | -9 | 197 | -185 | -54 | -152 | 209 | 34 | 136 | -167 | -10 | -90 | 103 | 14 | -98 | 87 | 69 | 144 | -210 | -37 | -130 | 170 | 21 |
| 21 | -181 | 4 | 180 | 205 | -64 | -138 | -158 | 43 | 118 | 99 | -23 | -73 | 91 | 24 | -112 | -213 | 63 | 153 | 164 | -40 | -121 | 21 |
| 21 | 193 | -198 | 8 | -148 | 219 | -68 | 127 | -176 | 52 | -86 | 116 | -27 | -102 | 77 | 28 | 147 | -204 | 60 | -124 | 173 | -46 | 21 |
| 21 | 56 | 154 | -207 | -36 | -129 | 168 | 5 | 185 | -187 | -72 | -139 | 214 | 51 | 121 | -169 | -29 | -74 | 106 | 32 | -111 | 82 | 21 |
| 21 | -203 | 66 | 140 | 165 | -42 | -120 | -193 | 2 | 194 | 218 | -59 | -156 | -173 | 41 | 135 | 115 | -20 | -92 | 78 | 19 | -94 | 21 |
| 21 | 150 | -217 | 70 | -126 | 174 | -45 | 191 | -184 | -4 | -143 | 201 | -55 | 125 | -159 | 37 | -83 | 97 | -11 | -107 | 95 | 15 | 21 |
| 21 | -12 | -89 | 104 | 13 | -95 | 85 | 74 | 141 | -212 | -49 | -119 | 171 | 6 | 186 | -189 | -58 | -151 | 212 | 33 | 134 | -164 | 21 |
| 21 | 100 | -22 | -75 | 94 | 22 | -113 | -216 | 61 | 158 | 175 | -39 | -133 | -192 | 0 | 195 | 206 | -61 | -142 | -160 | 46 | 117 | 21 |
| 21 | -85 | 114 | -26 | -104 | 76 | 31 | 145 | -199 | 57 | -123 | 161 | -35 | 189 | -183 | -3 | -145 | 215 | -67 | 130 | -177 | 50 | 21 |
| 21 | -57 | -150 | 210 | 47 | 122 | -166 | -30 | -76 | 109 | 30 | -110 | 83 | 55 | 157 | -209 | -31 | -132 | 166 | -7 | 196 | -186 | 21 |
| 21 | 207 | -63 | -141 | -172 | 44 | 131 | 113 | -17 | -93 | 79 | 20 | -96 | -200 | 64 | 139 | 162 | -44 | -115 | -182 | 3 | 182 | 21 |
| 21 | -147 | 216 | -66 | 128 | -163 | 38 | -80 | 96 | -13 | -106 | 93 | 16 | 148 | -218 | 73 | -128 | 179 | -48 | 192 | -196 | 7 | 21 |
| 21 | -50 | -116 | 169 | 11 | 183 | -191 | -70 | -140 | 213 | 48 | 123 | -168 | -16 | -88 | 107 | 12 | -97 | 88 | 72 | 142 | -211 | 21 |
| 21 | 178 | -41 | -134 | -195 | -2 | 200 | 217 | -60 | -154 | -171 | 42 | 132 | 101 | -19 | -79 | 92 | 25 | -114 | -215 | 62 | 156 | 21 |
|  | -125 | 160 | -32 | 187 | -178 | -6 | -144 | 203 | -56 | 126 | -162 | 39 | -82 | 110 | -25 | -101 | 75 | 29 | 146 | -201 | 58 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21. |


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | 46 | -89 | 106 | 74 | 175 | -186 | -13 | -111 | 187 | 12 | 219 | -168 | -32 | -133 | 228 | 55 | 153 | -145 | 5 | -67 | 125 | 441 |
| 441 | 100 | 43 | -80 | -182 | 87 | 158 | 183 | -23 | -97 | -159 | 21 | 201 | 224 | -45 | -116 | -141 | 65 | 139 | 122 | -1 | -58 | 441 |
| 441 | -83 | 109 | 37 | 171 | -199 | 91 | -107 | 197 | -27 | 210 | -177 | 30 | -129 | 241 | -49 | 149 | -155 | 69 | -64 | 131 | -4 | 441 |
| 4 | 73 | 140 | -150 | -8 | -57 | 128 | 47 | -88 | 104 | 88 | 163 | -188 | -31 | -98 | 192 | 29 | 204 | -170 | -51 | -117 | 231 | 441 |
| 4 | -154 | 60 | 157 | 132 | 2 | -71 | 101 | 41 | -79 | -194 | 85 | 172 | 196 | -18 | -115 | -174 | 19 | 218 | 240 | -42 | -135 | 441 |
| 4 | 144 | -137 | 56 | -61 | 118 | 6 | -85 | 110 | 38 | 169 | -185 | 79 | -102 | 179 | -14 | 208 | -160 | 15 | -126 | 222 | -33 | 441 |
| 4 | 11 | 217 | -165 | -34 | -132 | 229 | 54 | 156 | -147 | 10 | -70 | 123 | 34 | -78 | 107 | 89 | 164 | -190 | -17 | -110 | 190 | 441 |
|  | -161 | 24 | 200 | 225 | -44 | -118 | -138 | 63 | 138 | 119 | -3 | -53 | 111 | 44 | -92 | -193 | 83 | 173 | 184 | -20 | -101 | 441 |
| 4 | 213 | -178 | 28 | -128 | 239 | -48 | 147 | -156 | 72 | -66 | 136 | -7 | -82 | 97 | 48 | 167 | -184 | 80 | -104 | 193 | -26 | 441 |
| 4 | 76 | 174 | -187 | -16 | -109 | 188 | 25 | 205 | -167 | -52 | -119 | 234 | 71 | 141 | -149 | -9 | -54 | 126 | 52 | -91 | 102 | 441 |
| 441 | -183 | 86 | 160 | 185 | -22 | -100 | -173 | 22 | 214 | 238 | -39 | -136 | -153 | 61 | 155 | 135 | 0 | -72 | 98 | 39 | -74 | 441 |
| 4 | 170 | -197 | 90 | -106 | 194 | -25 | 211 | -164 | 16 | -123 | 221 | -35 | 145 | -139 | 57 | -63 | 117 | 9 | -87 | 115 | 35 | 441 |
| 4 | 8 | -69 | 124 | 33 | -75 | 105 | 94 | 161 | -192 | -29 | -99 | 191 | 26 | 206 | -169 | -38 | -131 | 232 | 53 | 154 | -144 | 441 |
| 4 | 120 | -2 | -55 | 114 | 42 | -93 | -196 | 81 | 178 | 195 | -19 | -113 | -172 | 20 | 215 | 226 | -41 | -122 | -140 | 66 | 137 | 441 |
| 4 | -65 | 134 | -6 | -84 | 96 | 51 | 165 | -179 | 77 | -103 | 181 | -15 | 209 | -163 | 17 | -125 | 235 | -47 | 150 | -157 | 70 | 441 |
| 441 | -37 | -130 | 230 | 67 | 142 | -146 | -10 | -56 | 129 | 50 | -90 | 103 | 75 | 177 | -189 | -11 | -112 | 186 | 13 | 216 | -166 | 441 |
| 441 | 227 | -43 | -121 | -152 | 64 | 151 | 133 | 3 | -73 | 99 | 40 | -76 | -180 | 84 | 159 | 182 | -24 | -95 | -162 | 23 | 202 | 441 |
| 441 | -127 | 236 | -46 | 148 | -143 | 58 | -60 | 116 | 7 | -86 | 113 | 36 | 168 | -198 | 93 | -108 | 199 | -28 | 212 | -176 | 27 | 441 |
| 441 | -30 | -96 | 189 | 31 | 203 | -171 | -50 | -120 | 233 | 68 | 143 | -148 | 4 | -68 | 127 | 32 | -77 | 108 | 92 | 162 | -191 | 441 |
| 441 | 198 | -21 | -114 | -175 | 18 | 220 | 237 | -40 | -134 | -151 | 62 | 152 | 121 | 1 | -59 | 112 | 45 | -94 | -195 | 82 | 176 | 441 |
|  | -105 | 180 | -12 | 207 | -158 | 14 | -124 | 223 | -36 | 146 | -142 | 59 | -62 | 130 | -5 | -81 | 95 | 49 | 166 | -181 | 78 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pa | 2021 | 021 | 2021 | 2021 | 2021 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | $1215 / 21$ | -1316/21 | $1815 / 21$ | 1449 /21 | $2505 / 21$ | -11016/21 | 62 5/21 | -3516/21 | 1262 5/21 | 87 5/21 | $2945 / 21$ | -9216/21 | $435 / 21$ | -57 16/21 | 303 5/21 | 130 5/21 | 228 5/21 | -69 16/21 | $805 / 21$ | $85 / 21$ | 5/21 | 2021 |
| 2021 | 175 5/21 | $1185 / 21$ | -4 | -106 | 162 | $2335 / 21$ | $2585 / 21$ | 52 5/21 | -2116/21 | -8316/21 | $965 / 21$ | 276 5/21 | $2995 / 21$ | $305 / 21$ | 21 | -6516/21 | $1405 / 21$ | 214 5/21 | $1975 / 21$ | 74 5/21 | 17 5/21 | 21 |
| 2021 | -71 | 184 | 112 | 246 |  | 166 5/21 | -3116/21 | 2725 | $485 / 21$ | 285 |  | 105 5/21 | -5316/21 | $3165 / 21$ | 26 5/21 | 224 5/21 | -79 16/21 | 5/21 | $115 / 21$ | $2065 / 21$ | $715 / 21$ | 2021 |
| 2021 | $1485 / 2$ | 215 5/21 | -74 | 67 5/21 | $185 / 21$ | $2035 / 21$ | 122 5/21 | -12 16/21 | 179 5/21 | 1635 | $2385 / 2$ | -112 16/21 | $445 / 21$ | -22 16/21 | 267 5/21 | $1045 / 21$ | 279 5/21 | -9416/21 | 24 5/21 | -4116/21 | 21 | 2021 |
| 2021 | -78 16/2 | $1355 / 21$ | 2325 | $2075 / 2$ | 77 5/21 | $45 / 21$ | 176 5/21 | 116 5/21 | -316/21 |  | 160 | $2475 / 21$ | $2715 / 21$ | 57 5/21 | -39 16/21 | -9816/21 | $945 / 21$ | $2935 / 21$ | \|315 5/21 | $335 / 21$ | 121 | 2021 |
| 2021 | $2195 / 2$ |  | $1315 / 21$ | 14 5/21 | $1935 / 21$ | 81 5/21 |  | 185 | $1135 / 21$ | 244 |  | $1545 / 21$ | -2616/21 | 2545 | $615 / 21$ | $2835 / 21$ | 21 | 90 5/21 | -50 16 | 297 5/21 | $425 / 21$ | 2021 |
| 2021 | $865 / 21$ | $2925 / 21$ |  | $415 / 21$ | -56 | 304 | 129 5/21 | 231 |  | 85 5/2 | 5 5/21 | $1985 / 21$ | $1095 / 21$ | $-216 / 21$ | 5/21 | $1645 / 21$ | 2395 |  | $585 / 21$ |  | 265 5/21 | 2021 |
| 2021 | -8516/2 | $995 / 21$ | 275 | $3005 / 21$ | $315 / 21$ | -4216 | -6216/21 | $1385 / 21$ | 213 5/21 | $1945 / 21$ | 72 5/21 | 22 5/21 | 186 5/21 | $1195 / 21$ | -1616/21 | -117 16/21 | 158 5/21 | $2485 / 21$ | 259 5/21 | 55 5/21 | $2516 / 21$ | 2021 |
| 20 | $2885 /$ |  | 103 | -5216\| | 314 5/21 | 27 5/21 | 222 5/21 | -8 | $1475 / 21$ | $95 / 21$ | $2115 / 21$ | $685 / 21$ | -616 | 172 | 1235 | 242 5/21 | -108 16/21 | 155 5/21 | -2816/21 | 268 5/21 | 49 5/21 | 2021 |
| 202 | 1515 | 249 | -11 | $595 / 21$ | -33 | $2635 / 21$ | 100 | 2805 | 21 | 23 5/21 | -4316/21 | 309 5/21 | $1465 / 21$ | $2165 / 2$ | 21 | 66 5/21 | $215 / 21$ | 5/21 | 127 5/21 | -1516/21 | $775 / 21$ | 2021 |
| 202 | -107 | $1615 / 21$ | 2355 | 260 5/21 | 53 5/21 | -24 | -9716/21 | 97 | 289 5/21 | 313 5/21 | $365 / 21$ | -60 16/21 | -7716 | 136 | $2305 / 21$ | 210 5/21 | 75 5/21 | 3 5/21 | 173 5/21 | 114 5/21 | $15 / 21$ | 2021 |
| 20 | $2455 /$ |  | $1655 /$ | -30 | 269 | $505 / 21$ | $2865 / 21$ | -88 | $915 / 21$ | -4 | 2965 | $405 / 21$ | 220 5/21 | -6316 | $1325 / 21$ | 12 5/21 | 192 5/21 | $845 / 21$ | -1116/21 | 190 5/21 | 105 | 2021 |
| 2021 | $835 / 2$ | 6 | 199 | 1085 | 5/21 | $1805 / 21$ | $1695 / 21$ | 2365 | -11616/21 | 465 | $-2316 / 21$ | 266 5/21 | 101 5/21 | 281 | -9316/21 | $375 / 21$ |  | 307 5/21 | 128 | $2295 / 21$ | 21 | 2021 |
| 2021 | 195 5/2 | 73 5/21 | 20 5/2 | 1895 | 117 5/21 | -1716/21 | -120 |  | $2535 / 21$ | 270 | $565 / 21$ | -37 16/21 | -9616/21 | 95 5/21 | 290 5/21 | $3015 / 21$ | $345 / 21$ | -4616/21 | -6416 | $1415 / 21$ | 212 5/21 | 2021 |
| 2021 | 105 | 209 | 69 5/2 | -816/21 | 171 5/21 | $1265 / 2$ | $2405 / 21$ |  | 152 5/21 | -27 | 256 5/21 | $605 / 21$ | $2845 / 21$ | -87 16/21 | 92 5/21 | -49 16/21 | $3105 / 21$ | 28 5/21 | 225 5/21 | -8116\| | $1455 / 21$ | 21 |
| 2021 | $385 / 21$ | -5416/21 | 305 5/2 | 142 5/2 | 217 5/21 | -70 | $655 / 21$ | 19 | 204 5/21 | $1255 / 21$ | -1416/21 | 178 5/21 | $1505 / 21$ | 2525 | -11316/2 | 64 5/21 | -3616 | $2615 / 21$ | $885 / 21$ | $2915 / 21$ | $9016 / 21$ | 221 |
| 2021 | 302 5/21 | $325 / 21$ | -4516/ | -7616/2 | $1395 / 21$ | $2265 / 21$ | $2085 / 21$ | 78 5/21 | 2 5/21 | $1745 / 21$ | 115 5/21 | -16/21 | -10416/ | 159 5/21 | $2345 / 21$ | 257 5/21 | $515 / 21$ | -1916/21 | -8616/21 | $985 / 21$ | 277 5/21 | 2021 |
| 2021 | -51 16/21 | $3115 / 21$ | 29 5/21 | $2235 / 21$ | -67 16/21 | 133 5/21 | 15 5/21 | $1915 / 21$ | 82 5/21 | -10 16/21 | $1885 / 21$ | 111 5/21 | $2435 / 21$ | -12216 | $1685 / 21$ | -32 16/21 | 274 5/21 | 47 5/21 | $2875 / 2$ | -100 16/21 | 102 5/21 | 2021 |
| 202 | $455 / 21$ | -2016/21 | $2645 / 2$ | 106 5/2 | $2785 / 21$ | -9516/21 | 25 5/21 |  | 308 5/21 | 14335121 | $2185 / 21$ | -72 16/21 | 79 5/21 | $75 / 21$ | 202 5/21 | 107 5/21 | -116/21 | $1835 / 21$ | 1675/21 | $2375 / 2$ | -115 16/21 | 2021 |
| 2021 | 273 5/21 | 54 5/21 | -3816/21 | -9916/21 | 93 5/21 | $2955 / 21$ | 312 5/21 | 35 5/21 | -58 16/21 | -7516/21 | 137 5/21 | 227 5/21 | $1965 / 21$ | 76 5/21 | 16 5/21 | 187 5/21 | 120 5/21 | -1816/21 | -11996/21 | 157 5/21 | $2515 / 21$ | 2021 |
|  | -29 16/2 | 255 5/21 | 63 5/21 | $2825 / 21$ | -82 16/21 | 89 5/21 | -4816/21 | $2985 / 21$ | 39 5/21 | $2215 / 21$ | -6616/21 | $1345 / 21$ | $135 / 21$ | 205 5/21 | 70 5/21 | -516/21 | 170 5/21 | $1245 / 21$ | 241 5/21 | -10516 | $1535 / 21$ | 2021 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 021 | 2021 | 202 | 2021 | 2021 | 2021 | 202 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the blocks of order 3 are semi-magic squares, with following equal semi-magic sums given by

$$
\begin{aligned}
& {\left[S m_{3 \times 3}: S_{21 \times 21}:=21\right] }:=3 \\
& {\left[{S m_{3 \times 3}} S_{21 \times 21}:=441\right]:=63 } \\
& {\left[S m_{3 \times 3}: S_{21 \times 21}:=2021\right]:=\frac{2021}{7} . }
\end{aligned}
$$

### 1.14.2 Second Type

|  | pan | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -219 | -109 | -65 | 23 | 45 | 155 | 177 | -217 | -110 | -66 | 22 | 46 | 154 | 178 | -218 | -111 | -64 | 21 | 47 | 153 | 179 | 21 |
| 21 | 150 | 176 | -201 | -114 | -67 | 19 | 44 | 151 | 175 | -200 | -112 | -68 | 18 | 43 | 152 | 174 | -199 | -113 | -69 | 20 | 42 | 21 |
| 21 | 17 | 40 | 149 | 171 | -202 | -96 | -72 | 16 | 39 | 148 | 172 | -203 | -95 | -70 | 15 | 41 | 147 | 173 | -204 | -94 | -71 | 21 |
| 21 | -97 | -54 | 12 | 38 | 145 | 170 | -207 | -98 | -53 | 14 | 37 | 144 | 169 | -206 | -99 | -52 | 13 | 36 | 146 | 168 | -205 | 21 |
| 21 | 166 | -208 | -102 | -55 | 30 | 33 | 143 | 165 | -209 | -101 | -56 | 31 | 35 | 142 | 167 | -210 | -100 | -57 | 32 | 34 | 141 | 21 |
| 21 | 51 | 138 | 164 | -212 | -103 | -60 | 29 | 52 | 140 | 163 | -213 | -104 | -59 | 28 | 53 | 139 | 162 | -211 | -105 | -58 | 27 | 21 |
| 21 | -61 | 24 | 50 | 156 | 159 | -214 | -107 | -62 | 25 | 49 | 157 | 161 | -215 | -108 | -63 | 26 | 48 | 158 | 160 | -216 | -106 | 21 |
| 21 | -177 | -130 | -86 | 2 | 66 | 134 | 198 | -175 | -131 | -87 | 1 | 67 | 133 | 199 | -176 | -132 | -85 | 0 | 68 | 132 | 200 | 21 |
| 21 | 129 | 197 | -159 | -135 | -88 | -2 | 65 | 130 | 196 | -158 | -133 | -89 | -3 | 64 | 131 | 195 | -157 | -134 | -90 | 1 | 63 | 21 |
| 21 | -4 | 61 | 128 | 192 | -160 | -117 | -93 | -5 | 60 | 127 | 193 | -161 | -116 | -91 | -6 | 62 | 126 | 194 | -162 | -115 | -92 | 21 |
| 21 | -118 | -75 | -9 | 59 | 124 | 191 | -165 | -119 | -74 | -7 | 58 | 123 | 190 | -164 | -120 | -73 | -8 | 57 | 125 | 189 | -163 | 21 |
| 21 | 187 | -166 | -123 | -76 | 9 | 54 | 122 | 186 | -167 | -122 | -77 | 10 | 56 | 121 | 188 | -168 | -121 | -78 | 11 | 55 | 120 | 21 |
| 21 | 72 | 117 | 185 | -170 | -124 | -81 | 8 | 73 | 119 | 184 | -171 | -125 | -80 | 7 | 74 | 118 | 183 | -169 | -126 | -79 | 6 | 21 |
| 21 | -82 | 3 | 71 | 135 | 180 | -172 | -128 | -83 | 4 | 70 | 136 | 182 | -173 | -129 | -84 | 5 | 69 | 137 | 181 | -174 | -127 | 21 |
| 21 | -198 | -151 | -44 | -19 | 87 | 113 | 219 | -196 | -152 | -45 | -20 | 88 | 112 | 220 | -197 | -153 | -43 | -21 | 89 | 111 | 221 | 21 |
| 21 | 108 | 218 | -180 | -156 | -46 | -23 | 86 | 109 | 217 | -179 | -154 | -47 | -24 | 85 | 110 | 216 | -178 | -155 | -48 | -22 | 84 | 21 |
| 21 | -25 | 82 | 107 | 213 | -181 | -138 | -51 | -26 | 81 | 106 | 214 | -182 | -137 | -49 | -27 | 83 | 105 | 215 | -183 | -136 | -50 | 21 |
| 21 | -139 | -33 | -30 | 80 | 103 | 212 | -186 | -140 | -32 | -28 | 79 | 102 | 211 | -185 | -141 | -31 | -29 | 78 | 104 | 210 | -184 | 21 |
| 21 | 208 | -187 | -144 | -34 | -12 | 75 | 101 | 207 | -188 | -143 | -35 | -11 | 77 | 100 | 209 | -189 | -142 | -36 | -10 | 76 | 99 | 21 |
| 21 | 93 | 96 | 206 | -191 | -145 | -39 | -13 | 94 | 98 | 205 | -192 | -146 | -38 | -14 | 95 | 97 | 204 | -190 | -147 | -37 | -15 | 21 |
|  | -40 | -18 | 92 | 114 | 201 | -193 | -149 | -41 | -17 | 91 | 115 | 203 | -194 | -150 | -42 | -16 | 90 | 116 | 202 | -195 | -148 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | -199 | -89 | -45 | 43 | 65 | 175 | 197 | -197 | -90 | -46 | 42 | 66 | 174 | 198 | -198 | -91 | -44 | 41 | 67 | 173 | 199 | 441 |
| 441 | 170 | 196 | -181 | -94 | -47 | 39 | 64 | 171 | 195 | -180 | -92 | -48 | 38 | 63 | 172 | 194 | -179 | -93 | -49 | 40 | 62 | 441 |
| 441 | 37 | 60 | 169 | 191 | -182 | -76 | -52 | 36 | 59 | 168 | 192 | -183 | -75 | -50 | 35 | 61 | 167 | 193 | -184 | -74 | -51 | 441 |
| 441 | -77 | -34 | 32 | 58 | 165 | 190 | -187 | -78 | -33 | 34 | 57 | 164 | 189 | -186 | -79 | -32 | 33 | 56 | 166 | 188 | -185 | 441 |
| 441 | 186 | -188 | -82 | -35 | 50 | 53 | 163 | 185 | -189 | -81 | -36 | 51 | 55 | 162 | 187 | -190 | -80 | -37 | 52 | 54 | 161 | 441 |
| 441 | 71 | 158 | 184 | -192 | -83 | -40 | 49 | 72 | 160 | 183 | -193 | -84 | -39 | 48 | 73 | 159 | 182 | -191 | -85 | -38 | 47 | 441 |
| 441 | -41 | 44 | 70 | 176 | 179 | -194 | -87 | -42 | 45 | 69 | 177 | 181 | -195 | -88 | -43 | 46 | 68 | 178 | 180 | -196 | -86 | 441 |
| 441 | -157 | -110 | -66 | 22 | 86 | 154 | 218 | -155 | -111 | -67 | 21 | 87 | 153 | 219 | -156 | -112 | -65 | 20 | 88 | 152 | 220 | 441 |
| 441 | 149 | 217 | -139 | -115 | -68 | 18 | 85 | 150 | 216 | -138 | -113 | -69 | 17 | 84 | 151 | 215 | -137 | -114 | -70 | 19 | 83 | 441 |
| 441 | 16 | 81 | 148 | 212 | -140 | -97 | -73 | 15 | 80 | 147 | 213 | -141 | -96 | -71 | 14 | 82 | 146 | 214 | -142 | -95 | -72 | 441 |
| 441 | -98 | -55 | 11 | 79 | 144 | 211 | -145 | -99 | -54 | 13 | 78 | 143 | 210 | -144 | -100 | -53 | 12 | 77 | 145 | 209 | -143 | 441 |
| 441 | 207 | -146 | -103 | -56 | 29 | 74 | 142 | 206 | -147 | -102 | -57 | 30 | 76 | 141 | 208 | -148 | -101 | -58 | 31 | 75 | 140 | 441 |
| 441 | 92 | 137 | 205 | -150 | -104 | -61 | 28 | 93 | 139 | 204 | -151 | -105 | -60 | 27 | 94 | 138 | 203 | -149 | -106 | -59 | 26 | 441 |
| 441 | -62 | 23 | 91 | 155 | 200 | -152 | -108 | -63 | 24 | 90 | 156 | 202 | -153 | -109 | -64 | 25 | 89 | 157 | 201 | -154 | -107 | 441 |
| 441 | -178 | -131 | -24 | 1 | 107 | 133 | 239 | -176 | -132 | -25 | 0 | 108 | 132 | 240 | -177 | -133 | -23 | -1 | 109 | 131 | 241 | 441 |
| 441 | 128 | 238 | -160 | -136 | -26 | -3 | 106 | 129 | 237 | -159 | -134 | -27 | -4 | 105 | 130 | 236 | -158 | -135 | -28 | -2 | 104 | 441 |
| 441 | -5 | 102 | 127 | 233 | -161 | -118 | -31 | -6 | 101 | 126 | 234 | -162 | -117 | -29 | -7 | 103 | 125 | 235 | -163 | -116 | -30 | 441 |
| 441 | -119 | -13 | -10 | 100 | 123 | 232 | -166 | -120 | -12 | -8 | 99 | 122 | 231 | -165 | -121 | -11 | -9 | 98 | 124 | 230 | -164 | 441 |
| 441 | 228 | -167 | -124 | -14 | 8 | 95 | 121 | 227 | -168 | -123 | -15 | 9 | 97 | 120 | 229 | -169 | -122 | -16 | 10 | 96 | 119 | 441 |
| 441 | 113 | 116 | 226 | -171 | -125 | -19 | 7 | 114 | 118 | 225 | -172 | -126 | -18 | 6 | 115 | 117 | 224 | -170 | -127 | -17 | 5 | 441 |
|  | -20 | 2 | 112 | 134 | 221 | -173 | -129 | -21 | 3 | 111 | 135 | 223 | -174 | -130 | -22 | 4 | 110 | 136 | 222 | -175 | -128 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 | 2021 | 2021 | 282 | 2021 | 2021 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 202 | -123 16/21 | -1316/21 | $305 / 21$ | 118 5/21 | $1405 / 21$ | 250 | 272 5/21 | -121 16/21 | $-1416 / 21$ | 29 5/21 | $1175 / 21$ | $1415 / 21$ | $2495 / 21$ | $2735 / 21$ | -122 16/21 | -1516/21 | $315 / 21$ | $1165 / 21$ | 142 5/21 | $2485 / 21$ | 274 5/21 | 2021 |
| 2021 | 245 5/2 | $2715 / 2$ | -105 16 | -1816/21 | $285 / 21$ | $1145 / 21$ | $1395 / 21$ | $2465 / 21$ | $2705 / 21$ | -10416/2 | -1616/21 | $275 / 21$ | $1135 / 21$ | $1385 / 21$ | $2475 / 21$ | 269 5/2 | -10316/21 | -1716/21 | 26 5/21 | 115 5/21 | $1375 / 21$ | 2021 |
| 2021 | $1125 / 21$ | $1355 / 2$ | $2445 / 2$ | $2665 / 21$ | -106 | -16/21 | $235 / 21$ | $1115 / 21$ | $1345 / 21$ | $2435 / 21$ | 2675 | -107 16/21 | 5/21 | 25 5/21 | $1105 / 21$ | 136 5/21 | 242 5/21 | $2685 / 21$ | 21 | $15 / 21$ | 24 5/21 | 2021 |
| 2021 | -116/21 | $415 / 21$ | $1075 / 21$ | $1335 / 21$ | $2405 / 21$ | $2655 / 21$ | -111 | -2 16/21 | $425 / 21$ | $1095 / 21$ | 132 5/21 | $2395 / 21$ | 264 5/2 | -110 16/21 | $-316 / 21$ | $435 / 21$ | 108 5/21 | 131 5/21 | $2415 / 21$ | 263 5/21 | /21 | 2021 |
| 2021 | $2615 / 2$ | -112 | -616/21 | $405 / 21$ | 125 5/21 | 128 | $2385 / 21$ | $2605 / 21$ | -113 16 | $-516 / 21$ | $395 / 21$ | $1265 / 21$ | $1305 / 21$ | 237 5/21 | 262 5/2 | -11416/21 | -416/21 | $385 / 21$ | 127 5/21 | 129 5/21 | $2365 / 2$ | 2021 |
| 2021 | $1465 / 2$ | $2335 / 2$ | $2595 / 21$ | -116 | -716/21 | $355 / 21$ | 124 5/21 | $1475 / 21$ | $2355 / 21$ | 258 5/21 | -11716 | -816/21 | $365 / 21$ | $1235 / 21$ | 148 5/21 | 2345 | 257 5/21 | -11516/21 | -916/21 | $375 / 21$ | $1225 / 2$ | 2021 |
| 2021 | $345 / 21$ | $1195 / 2$ | 145 5/21 | $2515 / 21$ | 254 5/21 | -11816/21 | -1116/21 | $335 / 21$ | $1205 / 21$ | $1445 / 21$ | 2525 | $2565 / 21$ | -119 16/21 | -12 16/21 | $325 / 21$ | $1215 / 21$ | 143 5/21 | $2535 / 21$ | 255 5/21 | -120 $16 / 2$ | -10 16/21 |  |
| 202 | -81 | -34 | 9 5/21 | $975 / 2$ | 161 | 229 | $2935 / 21$ | -7916/21 | -35 | $85 / 21$ | 965/21 | 162 5/2 | 228 | $2945 / 2$ | -80 16/2 | -3616/21 | 10 5/21 | 95 5/21 | 1635 | 275 | 295 |  |
| 2021 | 224 | 292 | -63 | -39 | $75 / 21$ | $935 / 21$ | $1605 / 21$ | 225 5/21 | $2915 / 21$ | -62 16/2 | -3716 | $65 / 21$ | $925 / 21$ | $1595 / 21$ | 226 | 290 | -61 16/21 | -381 | 5 5/21 | 94 5/21 | 585/21 |  |
| 2021 | $915 / 21$ | 156 | 2235 | 2875 | -64 | -21 | $25 / 21$ | $905 / 21$ | 155 5/2 | 2225 | 288 | -65 16/21 | 20161 | $45 / 21$ | $895 / 21$ | 157 5/21 | 2215 | 2895 | -6616/21 | -1916 | $35 / 21$ |  |
| 2021 | $-2216 / 2$ | $205 / 21$ | $865 / 21$ | $1545 / 2$ | 219 5/21 | 2865 |  | -23 | $215 / 21$ | $885 / 21$ | 1535 | 2185 | 285 5/21 | $16 / 21$ | -2416/21 | 22 5/21 | 87 5/21 | 152 5/21 | $2205 / 21$ | 2845 | -67 16/21 |  |
| 2021 | 282 5/2 |  | -27 | 19 5/21 | $1045 /$ | 1495 | 217 5/21 | $2815 / 21$ | -7116/21 | -2616/21 | 18 5/21 | $1055 / 21$ | 151 5/21 | $2165 / 21$ | 2835 | -72 16/21 | 2516 | $175 / 21$ | $1065 / 21$ | $150 \quad 5 / 21$ | 215 5/21 |  |
| 2021 | $1675 / 2$ | 212 512 | $2805 / 2$ | -7416 | -2816/21 | 14 5/21 | $1035 / 21$ | $1685 / 21$ | $2145 / 21$ | 279 5/21 | -75 16/21 | -2916/21 | 15 5/21 | $1025 / 21$ | $1695 / 21$ | 213 5/21 | $2785 / 21$ | -73 16/21 | -30 16/21 | 16 5/21 | $1015 / 21$ |  |
| 2021 | $135 / 21$ | $985 / 21$ | $1665 / 2$ | 2305 | 275 5/2 | -7616 | -32 16 | 12 5/21 | 99 5/21 | $1655 / 21$ | $2315 / 21$ | 277 5/21 | -77 16/21 | -33 16/21 | $115 / 21$ | 1005 | $1645 / 2$ | 2325 | $2765 / 21$ | -7816/21 | -3116/21 |  |
| 2021 | -102 16 | -5516/21 | $515 / 21$ | 76 5/21 | 182 | 208 | 314 5/21 | -100 16/21 | -5616/21 | $505 / 21$ | $755 / 21$ | $1835 / 2$ | $2075 / 21$ | 315 5/21 | -10116 | -57 16/21 | 52 5/21 | 74 5/21 | $1845 / 21$ | 206 5/2 | 316 5/2 |  |
| 2021 | $2035 / 2$ | 313 5/2 | -8416/ | -60 16/21 | $495 / 21$ | 72 5/21 | $1815 / 21$ | $2045 / 21$ | 312 5/21 | -83 16/2 | -5816/21 | $485 / 21$ | 71 5/21 | 180 5/21 | 205 5/2 | $3115 / 21$ | -8216/21 | -59 16/21 | 47 5/21 | 73 5/21 | 179 5/21 |  |
| 2021 | $70 \quad 5 / 2$ | 177 | 202 | $3085 /$ | -8516/2 | -42 16/21 | $445 / 21$ | $695 / 21$ | $1765 / 21$ | 2015 | $3095 / 21$ | -8616/21 | $-4116 / 21$ | $465 / 21$ | $685 / 21$ | 1785 | $2005 / 2$ | 310 5/21 | -8716/21 | -40 16/21 | $45 \quad 5 / 21$ |  |
| 202 | $-4316 / 21$ | 62 5/21 | $655 / 21$ | $1755 / 21$ | $1985 / 21$ | $3075 / 2$ | -9016/21 | -4416/21 | 63 5/21 | $675 / 21$ | 174 5/21 | $1975 / 21$ | $3065 / 21$ | -8916/21 | -4516/21 | $645 / 21$ | $665 / 21$ | 173 5/21 | $1995 / 21$ | 305 5/21 | -8816/21 | 2021 |
| 2021 | $3035 / 21$ | -9116/21 | -4816/21 | $615 / 21$ | $835 / 21$ | $1705 / 2$ | $1965 / 21$ | 302 5/21 | -9216/21 | -47 16/21 | $605 / 21$ | $845 / 21$ | 172 5/21 | $1955 / 21$ | 304 5/21 | -9316/21 | -4616/21 | 59 5/21 | $855 / 21$ | $1715 / 21$ | $1945 / 21$ | 2021 |
| 2021 | $1885 / 21$ | $1915 / 21$ | $3015 / 21$ | -9516/21 | -4916/21 | $565 / 21$ | $825 / 21$ | 189 5/21 | $1935 / 21$ | $3005 / 21$ | -9616/21 | -5016/21 | $575 / 21$ | $815 / 21$ | $1905 / 21$ | 192 5/21 | $2995 / 21$ | -9416/21 | -5116/21 | $585 / 21$ | $805 / 21$ | 21 |
|  | 55 5/21 | 71 5/21 | $1875 / 21$ | $2095 / 21$ | $2965 / 21$ | -9716/21 | -5316/21 | 54 5/21 | 78 5/21 | $1865 / 21$ | 210 5/21 | $2985 / 21$ | -9816/21 | -5416/21 | $535 / 21$ | 79 5/21 | 185 5/21 | $2115 / 21$ | $2975 / 21$ | -9916/21 | -52 16/21 | 202 |
|  | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 | 202 | 2021 | 2021 | 202 | 2021 | 202 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 | 2021 |

In this case, blocks of order 7 are pandiagonal with equal magic square sums. These sums are given by

$$
\begin{gathered}
{\left[S_{7 \times 7}: S_{21 \times 21}:=21\right]:=7} \\
{\left[S_{7 \times 7}: S_{21 \times 21}:=441\right]:=147} \\
{\left[S_{7 \times 7}: S_{21 \times 21}:=2021\right]:=\frac{2021}{3} .}
\end{gathered}
$$

### 1.15 Block-Bordered Magic Squares of Order 22

Below are block-bordered magic squares of order 22 in two different ways giving magic sums $21,21^{2}$ and 2021. The blockwise magic square considered is of order 20 with blocks of orders 4 and 5 .First, the pandiagonal blocks of order 4 with
equal magic sums. The second, pandiagonal magic squares of order 5 with different magic sums. The magic squares sums of order 20 are given by

$$
\begin{gathered}
{\left[S_{20 \times 20}: S_{22 \times 22}:=21\right]:=\frac{210}{11}} \\
{\left[S_{20 \times 20}: S_{22 \times 22}:=441\right]:=\frac{4410}{11}} \\
{\left[S_{20 \times 20}: S_{22 \times 22}:=2021\right]:=\frac{20210}{11} .}
\end{gathered}
$$

### 1.15.1 First Type

| -220 6/11 | 2315 | -228 6/11 | 229 5/11 | -226 6/11 | 227 5/11 | -224 6/11 | 225 5/11 | -222 6/11 | 223 5/11 | $-2406 / 11$ | $2325 / 11$ | -218 6/11 | 219 5/11 | -216 6/11 | 217 5/11 | -214 6/11 | 215 5/11 | -212 6/11 | 213 5/11 | -210 6/11 | 221 5/11 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -208 6/11 | -9 6/11 | $15 / 11$ | -179 6/11 | 191 5/11 | -32 6/11 | 24 5/11 | -162 6/11 | 174 5/11 | -50 6/11 | 42 5/11 | -140 6/11 | 152 5/11 | -73 6/11 | 65 5/11 | -123 6/11 | 135 5/11 | -91 6/11 | 83 5/11 | -101 6/11 | 113 5/11 | 210 /11 | 21 |
| 209 5/11 | -188 6/11 | $2005 / 11$ | -18 6/11 | 10 5/11 | -165 6/ | 177 5/11 | -35 6/11 | 27 5/11 | -147 6/11 | 159 5/11 |  | 49 5/11 | -124 6/11 | 136 5/11 | -74 6/11 | 66 5/11 | -106 6/11 | 118 5/11 | -96 6/11 | 88 5/11 | -207 6/11 | 21 |
| -2 | 181 | -18 | 11 | 5/11 | 16 | -17 | 34 | -2 | 142 | -15 | 52 | -40 6/11 | 125 5/11 | -133 6/11 | 75 5/11 | 11 | 11 |  | 11 | 11 | 111 | 21 |
| 20 | 20 | -8 | 190 | -1981 | 37 | -25 6/11 | 167 | -17 | 59 | -476/11 | $1495 / 11$ | -1 | 76 | 1 | 11 | 11 | 98 5/11 | 11 | /11 | 11 | 11 | 21 |
| -204 | -70 | 625 | -120 6/1 | 132 5/11 | -93 | 85 5/11 | -103 6/11 | 115 | -11 | $35 / 11$ | -181 6/11 | $1935 / 11$ | -29 | $215 / 11$ | -159 6/11 | $1715 / 11$ | -52 6/11 | 44 5/11 | -142 6/11 | $1545 / 11$ | 11 | 21 |
| 20 | -12 | 139 | -7 | 69 | -10 | 116 | -9 | 86 | -18 | 198 5/11 | -1 | 8 5/11 | -168 6/11 | 180 5/11 | -38 6/11 | $305 / 11$ | 11 | 11 | 11 | 11 | 11 | 21 |
| -2 | 122 | -13 | 72 | -6 | 105 | -1 | 95 | -83 6/11 | 18 | -191 | 13 | -1 6/11 | 161 5/11 | -1 | $315 / 11$ | 11 | 11 | 11 | 11 | 11 | 11 | 21 |
| 203 | 79 | -6 | 129 | -13 | 96 | -84 | 106 | -11 | 18 | -6 | 188 5/11 | -19 | 40 | -28 6/11 | 170 5/11 | -178 6/11 | 57 5/11 | -45 6/11 | $1475 / 11$ | -155 6/11 | 11 | 21 |
| -2 | -3 | 23 | -16 | 173 | -4 | 41 | -139 6/11 | 15 | -7 | 64 | -122 6/11 | $1345 / 11$ | -90 6/11 | 82 5/11 | -100 6/11 | 112 5/11 | 11 | 5 5/11 | 11 | 195 5/11 | 11 | 21 |
| 20 | -16 | 178 | -3 | 28 | -1 | 160 | -5 | 50 | -12 | 13 | - | $675 / 11$ | -107 6/11 | 119 5/11 | -97 6/11 | 11 | 11 | 196 5/11 | 11 | 6 5/11 | 11 | 21 |
| 211 | 163 | -171 | 335 | -2 | 141 | -149 | 51 | -39 | 124 | -13 | 74 | -62 | 102 5/11 | -110 6/11 | 92 5/11 | -80 6/11 | 185 5/11 | -193 6/11 | 15 5/11 | 11 | 11 | 21 |
| 233 | 385 | -2 | 168 | -17 | 605 | -4 | 1505 | -158 | 77 | -6 | 127 5/11 | -13 | 99 5/11 | -87 6/11 | 109 5/11 | -117 6/11 | 16 5/11 | -4 6/11 | $1865 / 11$ | -194 6/11 | -231 6/11 | 21 |
| -2 | -92 | 84 | -102 | 114 | -10 | 2 | -180 6/11 | 192 | -3 | 25 | -163 6/11 | 175 5/11 | -51 6/11 | 43 5/11 | -141 6/11 | 153 5/11 | -69 6/11 | 61 5/11 | 11 | $1315 / 11$ | 11 | 21 |
| 235 5/1 | -105 6/1 | 117 5/11 | -95 | 875 | -187 6/ | 199 5/11 | -17 | 9 5/11 | -164 6/11 | $1765 / 11$ | -34 | 26 5/11 | -146 6/11 | 158 5/11 | -56 6/11 | 48 5/11 | -128 6/11 | $1405 / 11$ | -78 6/11 | 70 5/11 | 6/11 | 21 |
| -234 | 1045 | -112 | 945 | -82 | 1825 | -190 | 125 | - 6 | 165 | -173 6/11 | 35 5/11 | -23 6/11 | $1435 / 11$ | -151 6/11 | 53 5/11 | $-416 / 11$ | $1215 / 11$ | -129 6/11 | $715 / 11$ | -59 6/11 | $2365 / 11$ | 21 |
| 2375 | $975 /$ | -85 | $1075 / 11$ | -115 6 | 195 | -7 6 | 1895 | -197 6/11 | 365 | -24 6/11 | 166 5/11 | -174 6/11 | 58 5/11 | -46 6/11 | 148 5/11 | -156 6/11 | 80 5/11 | -68 6/11 | 130 5/11 | -138 6/11 | -235 6/11 | 21 |
| -236 6/1 | -53 6/1 | 45 5/11 | -143 6/11 | 155 5/11 | -71 6/11 | 63 5/11 | -121 | $1335 / 11$ | -89 6/11 | $815 / 11$ | -99 6/11 | $1115 / 11$ | -12 6/11 | $45 / 11$ | -182 6/11 | $1945 / 11$ | -30 6/11 | 22 5/11 | -160 6/11 | 172 5/11 | $2385 / 11$ | 21 |
| $2395 / 11$ | -144 6/11 | 156 5/11 | -54 6/11 | 46 5/11 | -126 6/11 | 138 5/11 | -76 6/11 | 68 5/11 | -108 6/11 | 120 5/11 | -98 6/11 | 90 5/11 | -185 6/11 | 197 5/11 | -15 6/11 | 7 5/11 | -167 6/11 | 179 5/11 | -37 6/11 | 29 5/11 | -2376/11 | 21 |
| -238 6/11 | 145 / | -153 6/11 | 55 5/11 | -43 6/ | 123 5/11 | -131 6/1 | 73 5/11 | -61 6/11 | $1015 / 11$ | -109 6/11 | 91 5/11 | -79 6/11 | $1845 / 11$ | -192 6/11 | 14 5/11 | -2 6/11 | 162 5/11 | -170 6/11 | 32 5/11 | -20 6/11 | 240 5/11 | 21 |
| $2415 / 11$ | 56 5/11 | -44 6/11 | 146 5/11 | -154 6/11 | 78 5/11 | -66 6/11 | $1285 / 11$ | -136 6/11 | $1005 / 11$ | -88 6/11 | 110 5/11 | -118 6/11 | $175 / 11$ | -5 6/11 | $1875 / 11$ | -195 6/11 | 39 5/11 | -27 6/11 | 169 5/11 | -177 6/11 | -239 6/11 | 21 |
| -219 6/11 | -229 6/11 | $2305 / 11$ | -227 6/11 | 228 5/11 | -225 6/11 | 226 5/11 | -223 6/11 | 224 5/11 | -221 6/11 | 242 5/11 | -230 6/11 | $2205 / 11$ | -217 6/11 | 218 5/11 | -215 6/11 | 216 5/11 | -213 6/11 | 214 5/11 | -211 6/11 | 212 5/11 | 222 5/11 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| -201 5/11 | 250 6/1 | -209 5/11 | $2486 / 11$ | -207 5/11 | 246 6/11 | -205 5/11 | 244 6/11 | -203 5/11 | 242 6/11 | -221 5/11 | 251 6/11 | -199 5/11 | 238 6/11 | -197 5/11 | $2366 / 11$ | -195 5/11 | $2346 / 11$ | -193 5/11 | 232 6/11 | -191 5/11 | $2406 / 11$ | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -189 5 | 9 6/11 | 20 6/11 | -160 5/11 | 210 6/11 | $-135 / 11$ | $436 / 11$ | -143 5/11 | 193 6/11 | -31 5/11 | 61 6/11 | -121 5/11 | 171 6/11 | -54 5/11 | $846 / 11$ | -104 5/11 | 154 6/11 | -72 5/11 | $1026 / 11$ | -82 5/11 | $1326 / 11$ | 229 6/11 |  |
| 228 6/1 | -169 | 219 | 6/11 | 2 | -146 5 | 196 6/11 | -16 | 46 | -128 5 | 178 6/11 | -3 | 68 6/11 | -105 5/11 | $1556 / 11$ | , | 85 6/11 | -87 5/11 | 137 6/11 | 77-11 | 107 6/11 | 188 5/11 |  |
| -187 5/ | 2006 | -170 5/11 | 306 | 19 | 183 6/ | -153 5/1 | 53 6/11 | -3 5/11 | 161 | -131 5/1 | , | -21 | 44 | -114 5/11 | 4 | -44 5/11 | 122 6/11 | -92 5/11 | 112 6/11 | -62 5/11 | 227 6/11 |  |
| 226 |  |  | 209 | -179 5/ |  | -6 5/1 | 186 | -156 5/1 |  | -28 5/1 | 168 6/11 | -138 5 |  |  |  |  | 117 6/11 |  |  | -97 5/11 | -186 5/11 |  |
| -185 5/11 | -51 5/ | 81 6/11 | -101 5/1 | 151 6/11 | -74 5/17 | $1046 / 11$ | -84 5/11 | $1346 / 11$ | 76 | 22611 | -162 5/11 | 212 6/11 | -10 5/1 | 40 6/11 | -140 5/11 | 100 | -33 5/11 | 63611 | -123 5/11 | 173 6/11 | 225 6/11 |  |
| 22 | -1085 | 158 | -58 | 80 | -85 | 135 | -75 | 1 | -167 | 21 | 2 | 27 6/11 | -149 5/11 | 199 6/11 | - | 49 | -126 5/11 | 176 6/11 | 11 | 11 | 11 |  |
| -183 5/11 | 1416 | -1115/1 | 91 6/11 | -415/11 | 1246 | -94 5/11 | 114 | -64 5/11 | 202 6/11 | -172 5/ | 32 6/11 |  | 180 6/11 | -150 5/11 | 50 6/11 | - 11 | 163 6/11 | -133 5/11 | 73 6/11 | -23 5/11 | 223 6/11 |  |
| 22 | 98 | -4 | 14 | -118 | 115 | -6 | 12 | -9 | 37 6/11 | 12 6/11 | 207 6/11 | -1 | 59 6/11 | -9 5/11 | 11 | - | 76 6/11 | 11 | 11 | 11 | 11 | 441 |
| -18 | -12 | 42 | -142 | 192 | -30 | 60 | -120 | 170 | -53 | 83 | -10 | 153 | -71 | 10 | -8 | 131 6/11 | 5 | 24 | -164 5/11 | 214 6/11 | 11 |  |
| 220 | -14 | 197 | -17 | 47 | -12 | 17 | -3 | 69 | -106 5/11 | 156 | -5 | 86 | -88 5/11 | $1386 / 11$ | -78 5/11 | 108 6/11 | -165 5/11 | 11 | 4 | 11 | 11 | 44 |
| 230 | 182 | -152 | 52 | -2 | 160 | -130 | 70 | -20 | 14 | -113 | 93 | -4 | 12 | -91 5/11 | 1 | -61 5/11 | 204 6/11 | -174 5/11 | 34 6/11 | 15 6/11 | 5/11 |  |
| 25 | 57 | -7 | 187 | -15' | 79 | -29 | 169 | -139 | 96 | -46 | 146 | -116 5/ | 118 6/11 | -68 5/11 | 128 6/11 | -985 | 356 | 14 | 205 6/11 | -175 5/11 | -212 5/11 |  |
| -21 | -73 | 103 | -8 | 133 | 8 | 21 | -16 | 21 | -1 | 44 | -14 | 194 | -32 | 62 6/11 | -122 5/11 | 172 6/11 | -50 5/11 | $806 / 11$ | -100 5/11 | 150 6/11 | 11 |  |
| 254 | -86 5/ | $1366 /$ | -76 5/11 | 106 6/11 | -168 5/ | 218 6/1 | 16 | 28 6/11 | -145 5/11 | 195 6/11 | -15 5/11 | $456 / 11$ | -127 5/11 | 177 6/11 | -37 5/11 | 67 6/11 | -109 5/11 | 159 6/11 | -59 5/11 | 89 6/11 | -214 5/11 |  |
| -215 5/ | 123 6/1 | -93 | 1136 | -63 | 201 6/11 | -171 | 316 | 186 | $1846 / 11$ | -154 5/ | 546 | -4 5/11 | 162 6/11 | -132 5/11 | 72 6/11 | -22 5/11 | $1406 / 11$ | -110 5/11 | 90 6/11 | -40 5/11 | 255 6/11 |  |
| 256 6/1 | 116 6/1 | -66 5 | 126 | -96 5/1 | 386 | 116 | 208 6/11 | -178 5/11 | 556 | -5 5/11 | 185 6/11 | -155 5/11 | 77 6/11 | -27 5/11 | 167 6/11 | -137 5/11 | 99 6/11 | -49 5/11 | $1496 / 11$ | -119 5/11 | -216 5/11 |  |
| -217 5/11 | -34 5/ | $646 / 11$ | -124 5/11 | 174 6/11 | -52 5/1 | 82 6/11 | -102 5/11 | 152 6/11 | -70 5/11 | 100 6/11 | -80 5/11 | 130 6/11 | 6 6/11 | 23 6/11 | -163 5/11 | 213 6/11 | -11 5/11 | $416 / 11$ | -141 5/11 | 191 6/11 | 257 6/11 |  |
| 258 6/11 | -125 5/11 | 175 6/11 | -35 5/11 | $656 / 11$ | -107 5/11 | 157 6/11 | -57 5/11 | 87 6/11 | -89 5/11 | 139 6/11 | -79 5/11 | 109 6/11 | -166 5/11 | 216 6/11 | $36 / 11$ | 26 6/11 | -148 5/11 | 198 6/11 | -18 5/11 | 48 6/11 | -218 5/11 |  |
| -219 5/11 | 164 6/11 | -134 5/11 | $746 / 11$ | -24 5/11 | 142 6/11 | -112 5/11 | 92 6/11 | -42 5/11 | 120 6/11 | -90 5/11 | 110 6/11 | -60 5/11 | 203 6/11 | -173 5/11 | 33 6/11 | 16 6/11 | 181 6/11 | -151 5/11 | 51 6/11 | -1 5/11 | 259 6/11 |  |
| 260 6/11 | 75 6/11 | -25 5/11 | 165 6/11 | -135 5/11 | $976 / 11$ | -47 5/11 | $1476 / 11$ | -117 5/11 | 119 6/11 | -69 5/11 | 129 6/11 | -99 5/11 | 36 6/11 | 13 6/11 | 206 6/11 | -176 5/11 | 58 6/11 | -8 5/11 | 188 6/11 | -158 5/11 | -220 5/11 | 441 |




| -129 7/11 | 322 4/11 | -137 | 3204 | -135 7/11 | $3184 / 111$ | -133 7/11 | $3164 / 11$ | $-1317 / 11$ | 314 4/11 | -149 7/11 | 323 4/11 | -127 7/11 | 310 4/11 | -125 7/11 | $3084 / 11$ | -123 7/11 | $3064 / 11$ | -1217/11 | $3044 / 11$ | -119 7/11 | 312 4/11 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -117 7/1 | $814 / 11$ | $924 / 11$ | -887 | $2824 / 1$ | $584 / 11$ | $1154 / 11$ | -71 7/11 | $2654 / 11$ | $404 / 1$ | $1334 / 11$ | -497/11 | $2434 / 11$ | $174 / 11$ | $1564 / 11$ | -32 7/11 | $2264 / 11$ | -7/11 | 174 4/11 | -10 7/11 | 204 4/11 | $3014 / 11$ |  |
| 300411 | -977/11 | 2914 | 724 | 101 | -74 7/11 | $2684 / 11$ | $554 / 11$ | $1184 / 11$ | -56 | $2504 / 11$ | 33 | $1404 / 11$ | -33 7/11 | 2274 | 16 | $1574 / 11$ | -15 7/11 | 209 4/11 | -5 7/11 | $1794 / 11$ |  |  |
| -115 7/1 | 272 4/11 | -987/1 | $1024 / 11$ | $914 / 11$ | $2554 / 11$ | -81 7/11 | $1254 / 11$ | 68 4/11 | 233 4/11 | -59 7 | $1434 / 11$ | $504 / 11$ | 216 4/11 | -42 7/11 | $1664 / 11$ | $274 / 11$ | $1944 / 11$ | -20 7/11 | $1844 / 11$ | 4 | $2994 / 11$ |  |
| 2984 | $1114 / 11$ | $824 / 11$ | $2814 / 1$ | -10 | 1284 | $654 / 11$ | $2584 / 11$ | -84 | 1504 | 43 | 240 | -66 7/11 | 167 4/11 | $264 / 11$ | $2174 / 11$ | -43 7/11 | $1894 / 11$ | $44 / 11$ | $1994 / 11$ | -25 7/11 |  |  |
| -113 | $204 / 11$ | 153 | -29 | 223 | -2 | 176 | -12 | 206 | 79 | 94 | -90 | 284 4/11 | 111 | 112 4/11 | 7/11 | 262 4/11 | 38 | $1354 / 11$ | -517/11 | $2454 / 11$ | 29 |  |
| $2264 / 11$ | -367/11 | 2304 | $134 / 1$ | 1604 | -13 7/11 | 2074 | -37/11 | 177 4/11 | -95 7 | 2894 | 74 | 99 | -77 7/11 | $2714 / 11$ | $524 / 11$ | $1214 / 11$ | -54 7/11 | $2484 / 11$ | $354 / 11$ | $1384 / 11$ | -112 7/11 |  |
| -111 | 213 4/11 | -39 7 | 163 | 30 | 196 | -22 7 | 186 | $74 / 11$ | 274 | -100 | 104 | 89 | $2524 / 11$ | -78 7/11 | 122 | $714 / 11$ | $2354 / 11$ | 11 | 11 | $484 / 11$ | 29 |  |
| 294 | 170 | 23 | 2204 | -46 | 187 | $64 / 11$ | 197 | -23 7 | 109 | 84 | 279 | -10 | 131 | 62 | 11 | -87 7/11 | $1484 / 11$ | $454 / 11$ | $2384 / 11$ | 111 | -110 7/11 |  |
|  | 594 | 114 | -70 7 | 264 |  | 132 | -48 |  | 18 | 155 | -31 | 22 | 4/11 | 173 4/11 |  | 203 4/11 | 77 | $964 / 11$ | -92 7/11 | $2864 / 11$ | $2934 / 11$ |  |
| 292 | -75 7/1 | 2694 | 544 | 1194 | -57 7/1 | 251 | $324 / 11$ | 141 | -34 7 | 228 | 15 | 158 4/11 | -16 7 | 2104 | -67/11 | 180 4/11 | -93 7/11 | $2874 / 11$ | $764 / 11$ | 11 | -108 7/11 |  |
| 302 | 2544 | -80 7 | 1244 | 69 4/11 | $2324 / 1$ | -58 7/11 | 142 4/11 | 51411 | 2154 | -41711 | $1654 / 11$ | $284 / 11$ | 193 4/11 | -19 7/11 | 183 4/11 | $104 / 11$ | 276 | -102 | $1064 / 11$ | 87 4/11 | -118 |  |
| 324 | 129 | 64 | 259 | -85 | 151 | 42 | 241 | -67 | 168 | 25 | 218 4/11 | -44 | 190 | $34 / 11$ | $2004 / 11$ | -26 7/11 | $1074 / 11$ | $864 / 11$ | 277 4/11 | 37 | 11 |  |
|  | -1 | 175 | -11 | 205 | 80 | 93 | -89 | 283 | 57 | 116 | -72 7/11 | 26 | 39 | $1344 / 11$ | -50 7/11 | $2444 / 11$ | $214 / 11$ | 152 | -287/11 | 222 4/11 | 325 |  |
| $3264 / 1$ | -14 7/11 | 208 | -4 | 178 | -96 | 290 | 73 | 100 | -73 | 267 | 56 | 11 | -55 | $2494 / 11$ | $344 / 11$ | $1394 / 11$ | -37 7/11 | 231 | 12 | 4/11 | -142 7/11 |  |
|  | $1954 /$ | -21 | 185 | 8 | 273 | -9 | 103 | $904 / 11$ | 256 | -82 | 126 | 67 4/11 | $2344 / 11$ | -60 7/11 | $1444 / 11$ | 49 4/11 | 11 | -38 | 162 | $4 / 11$ | 32 |  |
| 3284 | 1884 | 5 | 1984 | -24 | 1104 | 83 | $2804 / 11$ | -106 7/11 | $1274 / 11$ | 664 | $2574 / 11$ | -83 7/11 | $1494 / 11$ | $444 / 11$ | $2394 / 11$ | -65 7/11 | $1714 / 11$ | $224 / 11$ | $2214 / 11$ | -477/11 | -144 |  |
| -145 | $374 / 11$ | $1364 / 11$ | -52 | 246 | $194 / 11$ | 154 4/11 | -30 7/11 | 224 4/11 | $14 / 11$ | 172 4/11 | -87/11 | 202 4/11 | $784 / 11$ | 954/11 | -917/11 | $2854 / 11$ | $604 / 11$ | 113 4/11 | -69 7/11 | 263 4/11 | 3294 |  |
| 3304111 | -53 7/11 | $2474 / 11$ | $364 / 11$ | $1374 / 11$ | -35 7/11 | $2294 / 11$ | 14 4/11 | $1594 / 11$ | -17 7/11 | $2114 / 11$ | -7 7/11 | $1814 / 11$ | -94 7/11 | $2884 / 11$ | $754 / 11$ | $984 / 11$ | -76 7/11 | $2704 / 11$ | $534 / 11$ | $1204 / 11$ | -146 7/11 |  |
| -147 7/11 | $2364 / 11$ | -62 7/11 | $1464 / 11$ | $474 / 11$ | $2144 / 11$ | -40 7/11 | 164 4/11 | 29 4/11 | $1924 / 11$ | -187/11 | $1824 / 11$ | $114 / 11$ | $2754 / 11$ | -1017/11 | $1054 / 11$ | $884 / 11$ | $2534 / 11$ | -79 7/11 | $1234 / 11$ | $704 / 11$ | $3314 / 11$ |  |
| $3324 / 11$ | $1474 / 11$ | $464 / 11$ | $2374 / 11$ | -63 7/11 | 169 4/11 | 24 4/11 | 219 4/11 | -457/11 | $1914 / 11$ | 2 4/11 | $2014 / 11$ | -27 7/11 | 108 4/11 | $854 / 11$ | $2784 / 11$ | -104 7/11 | $1304 / 11$ | 63 4/11 | 260 4/11 | -86 7/11 | -148 7/11 |  |
| -128 7/1 | -138 | $13214 / 11$ | -136 | 319 4/11 | -134 7 | $3174 / 1$ | -132 7/11 | $3154 / 11$ | -130 7 | $3334 / 11$ | -139 7/11 | $3114 / 11$ | -126 7/11 | 1309 4/11 | -124 7/11 | $3074 / 11$ | -122 7/11 | $3054 / 11$ | -120 7/11 | $3034 / 11$ | $3134 / 11$ |  |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |  |

In this case, the blocks of order 4 are pandiagonal with equal magic sums. In each case the magic sums of order 5 are given by

$$
\begin{gathered}
{\left[S_{4 \times 4}: S_{22 \times 22}:=21\right]:=\frac{42}{11}} \\
{\left[S_{4 \times 4}: S_{22 \times 22}:=441\right]:=\frac{882}{11}} \\
{\left[S_{4 \times 4}: S_{22 \times 22}:=2021\right]:=\frac{4042}{11} .}
\end{gathered}
$$

### 1.15.2 Second Type

| -220 6/11 | 231 5/11 | -228 6 | 229 5/11 | -226 6/11 | 227 5/11 | -224 6 | 225 5/11 | -222 6 | 223 5/11 | -240 6/11 | 232 5/11 | $-2186 / 11$ | 219 5/11 | -216 6/11 | 2175/11 | -214 6/11 | 215 5/11 | -212 6/11 | 213 5/11 | -210 6/11 | $2215 / 11$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -2086/11 | -192 6/1 | -64 6/ | -16 6/11 | $1115 / 11$ | 159 5/11 | -187 6/11 | -59 6/11 | -11 6/11 | 116 5/11 | $1645 / 11$ | -198 6/11 | -70 6/11 | -22 6/11 | 105 5/11 | $1535 / 11$ | -185 6/11 | -57 6/11 | -9 6/11 | $1185 / 11$ | 166 5/11 | 5/11 | 1 |
| 209 | 635 | $1915 /$ | -160 6 | -112 | 15 5/11 | 68 | $1965 / 11$ | -155 | -107 | 20 5/11 | 57 5/11 | 185 5/11 | -166 6/11 | -118 | 9 5/11 | 70 5/11 | 1985/11 | -153 | -105 6/11 | 22 5/11 | 207 6/11 | 1 |
| -206 | -80 6 | -32 | 95 | 1/3 | -128 | -75 | -27 | 1005 | 1485 | -12 | -86 | -38 | 11 | 137 | 11 | -73 6/11 | -25 | 102 5/11 | 150 5/11 | -121 6/11 | 208 5/11 | 21 |
| 207 | 1755 | -17 | -48 |  | $475 / 11$ | $1805 / 11$ | -17 | -4 |  | 52 5/11 | 169 | -182 6/11 |  | -66/11 | 11 | 182 5/11 | -169 6/11 | -41 6/11 | 6 5/11 | 11 |  | 21 |
| -204 | 31 511 | 5/1 | 127 5/11 | -144 6 | -96 | 565 | 84 ¢1 | 132 5/11 | -139 6 | -91 6/11 | 25 5/11 | 73 5/11 | $1215 / 11$ | -150 | 11 | $385 / 11$ | 86 5/11 | $1345 / 11$ | -137 6/11 | -89 6/11 | 11 | 21 |
| 205 5/1 | -197 6/ | -69 6/1 | $-216 / 11$ | $1065 / 11$ | 154 5/11 | -186 6/11 | -58 6/11 | -10 6/11 | 117 5/11 | 165 5/11 | -191 6/11 | -63 6/11 | -15 6/11 | 112 5/11 | $1605 / 11$ | -188 6/11 | -60 6/11 | -12 6/11 | $1155 / 11$ | 163 5/11 | 11 | 21 |
| -202 | 585/11 | $1865 / 11$ | -165 6/ | -117 6 | 10 5/11 | 69 5/11 | $1975 / 11$ | -154 6 | -106 6/1 | 21 5/11 | 64 5/11 | 192 5/11 | -159 6/11 | -111 6/11 | 16 5/11 | 67 5/11 | 195 5/11 | -156 6/11 | -1086/11 | 19 5/11 | 204 5/11 | 21 |
| $2035 / 11$ | -85 6/1 | -37 6/1 | 905/11 | $1385 / 11$ | -133 6/11 | -74 6/11 | -26 6/11 | $1015 / 11$ | 1495/11 | -122 6/11 | -79 6/11 | -31 6/11 | $965 / 11$ | 144 5/11 | -127 6/11 | -76 6/11 | -286/11 | 99 5/11 | 1475/11 | -124 6/11 | -201 6/11 | 21 |
| -200 6/11 | 170 5/11 | -181 6/1 | -53 6/1 | -5 6/11 | 42 5/1 | $1815 / 11$ | -170 6/11 | -42 6/1 | 55 | 53 5/11 | $1765 / 11$ | -175 6/11 | -47 6/1 | 5/11 | $485 / 11$ | 179 5/11 | -172 6/11 | -44 6/11 | 3 5/11 | $515 / 11$ | 202 5/11 | 21 |
| $2015 / 11$ | $265 / 11$ | 745/11 | 122 5/11 | -149 6/11 | -1016/11 | 37 5/11 | 85/11 | $1335 / 11$ | -138 6/11 | -90 6/11 | 32 5/11 | 80 5/11 | 128 5/11 | -143 6/11 | -95 6/11 | 35 5/11 | 83 5/11 | $1315 / 11$ | -140 6/11 | -92 6/11 | -199 6/11 | 21 |
| $2115 / 11$ | -183 6/1 | -55 6/ | -7 6/1 | 120 /1 | $1685 / 11$ | -196 6/1 | -68 6/1 | -20 6/11 | $1075 / 11$ | $1555 / 11$ | -189 6/11 | -61 6/11 | -13 6/11 | $1145 / 11$ | 162 5/11 | -194 6/11 | -66 6/11 | -186/11 | 109 5/11 | $1575 / 11$ | 11 | 21 |
| $2335 / 1$ | 72 5/1 | 200 5/11 | -151 6/1 | -103 6 | 24 5/1 | $595 / 11$ | $1875 / 11$ | -164 6/1 | -116 6 | 11 | $665 / 11$ | $1945 / 11$ | -157 6 | -109 6 | 18 5/1 | 61 5/11 | 189 5/11 | -162 6/11 | -114 6/11 | 13 5/1 | -231 6/11 | 21 |
| -232 61 | -71 6/11 | -23 6/ | 104 5/1 | 152 5/11 | -119 | -84 6/1 | -36 6/1 | 111 | $1395 / 11$ | -132 6 | -77 6/1 | -296/1 | $985 / 11$ | $1465 /$ | -125 6/11 | -82 6/11 | -34 6/11 | 93 5/11 | 141 5/11 | -130 6/11 | 234 | 21 |
| $2355 / 11$ | 1845/11 | -167 6/1 | -39 6/1 | 85 | $565 / 11$ | 171 5/11 | -180 6 | -52 6/11 | $-46 / 11$ | $435 / 11$ | $1785 / 11$ | -173 6/11 | -456/1 | 25 | $505 / 11$ | 173 5/11 | -178 6/11 | -50 6/11 | -2 6/11 | 45 5/11 | 111 | 21 |
| -234 6/ | 40 5/1 | 88 5/11 | 136 5/11 | -135 6/1 | -87 617 | 27 5/11 | 75 5/11 | $1235 / 11$ | -148 6 | -1006/11 | 34 5/11 | 82 5/11 | 130 5/11 | -141 6/11 | -93 6/11 | 29 5/11 | 77 5/11 | 125 5/11 | -1466/11 | -98 6/11 | 23 | 21 |
| $2375 / 11$ | -190 6/1 | -62 6/1 | -14 6/1 | $1135 / 11$ | $1615 / 11$ | -193 6/11 | -65 6/11 | -17 6/11 | $1105 / 11$ | 1585/11 | -184 6/11 | -56 6/11 | -8 6/11 | $1195 / 11$ | 1675/11 | -195 6/11 | -67 6/11 | -196/11 | 108 5/11 | 156 5/11 | -235 6/11 | 1 |
| -236 6/11 | 65 5/11 | 193 5/11 | -158 6/ | -110 6/1 | 17 5/11 | 62 5/11 | $1905 / 11$ | -161 6/11 | -113 6/1 | 14 5/11 | 71 5/11 | $1995 / 11$ | -152 6/11 | -104 6/11 | 23 5/11 | 60 5/11 | 1885/11 | -163 6/11 | -115 6/11 | $125 / 11$ | $2385 / 11$ | 1 |
| 239 5/11 | -78 6/11 | -30 6/1 | $975 / 11$ | 145 5/11 | -126 6 | -81 6/11 | -33 6 | $945 / 11$ | 142 5/11 | -1296/11 | -72 6/11 | -24 6/11 | 103 5/11 | $1515 / 11$ | -120 6/11 | -83 6/11 | -35 6/11 | 92 5/11 | 140 5/11 | -131 6/11 | -237 6/11 | 21 |
| -2386/11 | $1775 / 11$ | -174 6/1 | -46 6/11 | $15 / 11$ | 49 5/11 | 174 5/11 | -177 6/11 | -496/11 | -1 6/11 | $465 / 11$ | $1835 / 11$ | -168 6/11 | -40 6/11 | 7 5/11 | $555 / 11$ | 172 5/11 | -179 6/11 | -51 6/11 | -36/11 | 445/11 | $2405 / 11$ | 21 |
| $2415 / 11$ | $335 / 11$ | $815 / 11$ | 129 5/11 | -142 6/11 | -94 6/11 | 30 5/11 | 78 5/11 | $1265 / 11$ | -145 6/11 | -97 6/11 | 39 5/11 | 87 5/11 | 135 5/11 | -136 6/11 | -88 6/11 | 28 5/11 | $765 / 11$ | $1245 / 11$ | -147 6/11 | -996 | -239 6/11 | 21 |
| -219 6/11 | -2296/1 | 230 5/11 | -227 6/11 | $2285 / 11$ | -225 6/11 | 226 5/11 | -223 6/11 | 224 5/11 | -221 6/11 | 242 5/11 | -230 6/11 | 220 5/11 | -217 6/11 | 218 5/11 | -215 6/11 | $2165 / 11$ | -213 6/11 | 2145/11 | -211 6/11 | 212 5/11 | 222 5/11 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| -201 | 2506 | -209 5 | 248 6/11 | -207 5/11 | 246 6/11 | -205 5 | 244 | -203 5 | 242 6/11 | -221 5/11 | 251 6/11 | -199 5/11 | 238 6/11 | -197 5/11 | $2366 / 11$ | -195 5/11 | $2346 / 11$ | -193 5/11 | 232 6/11 | -191 5/11 | $2406 / 11$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -189 5/11 | -173 5/11 | -45 5/11 | 2 6/11 | 130 6/1 | 178 6/11 | -168 5/11 | -40 5/11 | 7 6/11 | $1356 / 11$ | 183 6/11 | -179 5/11 | -51 5/11 | -3 5/11 | 124 6/11 | 172 6/11 | -166 5/11 | -38 5/11 | $96 / 11$ | $1376 / 11$ | 185 6/11 | $2296 / 11$ |  |
| 228 | 82 | 2106 | -141 | -93 5/11 | 34 | 87 | 215 | -136 5 | -88 5/11 | 39 | 76 | 204 6/11 | -147 5/11 | -99 5/11 | 28 |  | 217 6/11 | -134 5/11 | -86 5/11 |  | /11 |  |
|  | -61 | -13 |  | 162 | -109 5/ | -5 |  |  |  | -104 5 | -67 5 | -19 5/11 | 108611 | 1 | -115 5 | 54 | 6 |  | 169 6/11 | - |  |  |
| 226 | 194 | -15\% | -29 | 18 | 66 | 199 | -15 | -2 | 23 | 71 6/11 | 188 | -16 | -3 | 12 | 60 6/11 | 201 6/11 | -150 5/11 | -22 5/11 | 25 6/11 | 1 | -186 5/11 |  |
|  | 506 | 98 |  | -125 |  |  | 103 | 151 | -120 |  |  |  |  | -131 5/11 |  |  | $1056 / 11$ | 153 6/11 | -118 5/11 | -70 5/11 |  |  |
| 224 6/11 | -178 5/ | -50 5/1 | -2 5 | 125 | 1736 | -167 5/ | -39 5 | 8 | 136 | 1046 | -172 5/11 | -44 5/1 | 36 | 1 |  | -169 5/11 | - | 6 | T34 | 182 | -184 5/11 |  |
| -183 5/11 | 77 | 20 | -14 | -98 |  | 88 6/11 | 21 | -13 | -8 |  | 83 6/11 | 211 6/11 | -140 5/11 |  | 35 6/11 | 86 6/11 | 1 | -137 5/11 | 11 | 38 6/11 | 11 |  |
| 222 | -66 5 | -18 5/1 | 109 | 157 | -114 | -55 | -7 | 120 | 168 | -103 | -60 5/11 |  | 115 | 163 | -108 5/11 | -5 | -9 | 118 6/11 | 166 6/11 | -105 5/11 | -182 5/11 |  |
| -18 | 189 | -16 | -3 | 13 |  | 20 | -15 | -2 | $246 / 11$ | 72 6/11 | 195 6/11 | -1 | -2 | 19 6/11 | 67 6/11 | 198 6/11 | -153 5/11 | -25 5/11 | 22 6/11 | 70 6/11 | 11 |  |
| 220 | 45 | 93 | 14 | -130 5 | -82 | 56 | 10 | 15 | -119 | -7 | 51 6/11 | 99 6/11 | 141 | -124 5/11 | -76 | 54 6/11 | 11 | 150 6/11 | -121 5/11 | 11 | -180 5/11 |  |
| 230 | -164 | -36 |  | 13 |  | -17 |  |  |  | 174 | -170 5/11 |  | 5 |  | 18 | -175 5/11 | -47 | 6/11 | 128 6/11 | 176 6/11 | -190 5/11 |  |
| 252 6/11 | 916 | 219 6/ | -132 5/ | -84 5/11 | 43 | 78 | 206 | -145 5 | -97 5/1 | 30 | 85 | 213 | -138 5/11 | -90 5/1 | 37 6/1 | $806 / 11$ | 208 6/11 | -143 5/11 | -95 5/11 | 32 6/11 | -212 5/11 |  |
| -213 | -52 | -4 | 12 | 17 | -100 | -65 | -17 | 110 | 158 | -11 | -58 | -10 | 117 | 165 | -106 5/11 | -63 | -15 5/11 | 112 6/11 | 160 6/11 | -111 5/11 | 11 |  |
| 254 | 203 6/1 | -148 5 | -20 | 27 | 75 | 190 | -161 | -33 | 14 | 62 | 197 6/11 | -154 5/11 | -26 5/1 | 21 | 69 | 192 6/11 | -159 5/11 | -31 5/11 | 16 6/11 | 64 6/11 | -214 5/11 |  |
| -21 | 59 | 107 | 155 | -116 | -68 | 46 | 94 | 14 | -129 | -81 | 53 | 101 | 149 | -122 | -74 | 48 | 96 6/11 | $1446 / 11$ | -127 5/11 | -79 5/11 | 55 6/11 |  |
| 256 6/1 | -171 5 | -43 | 4 | 132 | 1806 | -174 | -46 5 | 1 | 129 | 177 6/11 | -165 5/11 | -37 5/11 | 10 | $1386 / 11$ | 186 6/11 | -176 5/11 | -48 5/11 | - 5/11 | 127 6/11 | 175 6/11 | -216 5/11 |  |
| -217 5/1 | 846 | 212 6/1 | -139 5/11 | -91 5/11 | 366 | 816 | 209 6/1 | -142 5 | -94 5 | 33 6/11 | 90 6/11 | 218 6/11 | -133 5/11 | -85 5/11 | 42 | 79 6/11 | 207 6/11 | -144 5/11 | -96 5/11 | 31 6/11 | 257 6/11 | 441 |
| 258 6/11 | -59 5/1 | -115 | 116 | 164 6/11 | -107 5/11 | -62 5/11 | -14 5 | 113 | 161 | -110 5/11 | -53 5/11 | -5 5 | 122 6/11 | 170 6/11 | -1015/11 | -64 5/11 | -16 5/11 | 111 6/11 | 159 6/11 | -112 5/11 | -218 5/11 |  |
| -219 5/11 | $1966 / 11$ | -155 5/ | -27 5/11 | 206 | 686 | 193 6/11 | -158 5/11 | -30 5/ | 176 | 65 6/11 | 202 6/11 | -149 5/11 | -21 5/11 | 26 6/11 | 7461 | 191 6/11 | -160 5/11 | -32 5/11 | 15 6/11 | 63 6/11 | $2596 / 11$ |  |
| 260 6/11 | 52 6/11 | $1006 / 1$ | $1486 / 11$ | -123 5/11 | -75 5/11 | 49 6/11 | 97 6/11 | 145 6/11 | -126 5/11 | -78 5/11 | 58 6/11 | 106 6/11 | $1546 / 11$ | -117 5/11 | -69 5/11 | 47 6/11 | $956 / 11$ | 143 6/11 | -128 5/11 | -80 5/11 | -220 5/11 | 441 |
| -200 5/11 | -210 5/11 | $2496 / 11$ | -208 5/11 | 247 6/11 | -206 5/11 | 245 6/11 | -204 5/11 | 243 6/11 | -202 5/11 | 261 6/11 | -211 5/11 | $2396 / 11$ | -198 5/11 | $2376 / 11$ | -196 5/11 | $2356 / 11$ | -194 5/11 | 233 6/11 | -192 5/11 | $2316 / 11$ | 241 6/11 |  |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -129 7/ | 3224 | -137 7/11 | 32041 | -135 7/ | 318 4/11 | -133 7/11 | 316 4/11 | -1317/11 | 314 4/11 | -149 7/11 | 323 4/11 | -127 7/11 | 310 4/11 | -125 7/11 | 308 4/11 | -123 7/11 | $3064 / 11$ | -121 7/11 | 304 4/11 | -119 7/11 | 312 4/11 | 2021 |
| -117 7/1 | -101 | 264 | 74 | 202 | 2504 | -96 7/ | 31 | 79 | 207 | 255 4/11 | -107 7/11 | 20 4/11 | 68 4/11 | $1964 / 11$ | 244 4/11 | -94 7/11 | 33 4/11 | $814 / 11$ | 209 4/11 | 57 4/11 | 301 4/11 | 2021 |
| 300 | 154 | 282 | -69 | -21 | 106 | 159 | 287 | -64 |  | 111 | $1484 / 11$ | $2764 / 11$ | -75 7/11 | -27 7/11 | $1004 / 11$ | 161 4/11 | 289 4/11 | -62 7/11 | -14 7/11 | 113 4/11 | 11 | 2021 |
| -11 | 10 | 58 | 186 | 234 | -37 | 15 | 63 | 19 | 239 4/11 | -32 | 4 | $524 / 11$ | 180 4/11 | 228 4/11 | -43 7/11 | 17 4/11 | $654 / 11$ | $1934 / 11$ | $2414 / 11$ | -30 7/11 | 11 | 2021 |
| 29 | 266 | -85 | 42 | 90 | 138 | 27 | -80 | 47 | 95 | 14 | 26 | -917/11 | 36 | 84 4/11 | 132 4/11 | 11 | 1 | 49 4/11 | 11 | 11 | 11 | 2021 |
| -113 | 122 | 170 | 218 | -53 | -5 | 127 | 175 | 223 | -48 | - |  | 164 4/11 | 21 | -59 7/11 |  | 129 4/11 | 177 4/11 | $2254 / 11$ | -46 7/11 | $14 / 11$ | 11 | 2021 |
| 29 | -10 | 21 | 69 | 197 | 24 | -9 | 32 | 80 | 20 | $2564 / 11$ | -10 | 27 | 75 | 203 4/11 | $2514 / 11$ | 11 | 30 4/11 | 78 4/11 | 11 | 254 4/11 | 11 | 2021 |
| -1 | 14 | 27 | -74 | -2 | 10 | 160 | 288 4/11 | -6 | -1 | 11 | 15 | 283 4/11 | -687/11 | -20 7/11 | 107 4/11 | 11 | 11 | 11 | 11 | 11 | 11 | 2021 |
| 294 | 5 | 53 | 181 | 229 | -4 | 16 | 64 | 192 | 240 | -3 |  | 59 | 187 4/11 | $2354 / 11$ | -36 | 14 | 62 4/11 | $1904 / 11$ | 238 4/11 | -33 7/11 | 11 | 2021 |
| -10 | 26 | -90 | 37 | 85 | 13 | 27 | -7 | 48 | 96 | 14 | 267 4/11 | -84 7/11 | 43 | 91 4/11 | $1394 / 11$ | 270 4/11 | -817/11 | 46 4/11 | 94 4/11 | 11 | 11 | 2021 |
| 29 | 117 | 165 | 213 | -58 | -10 | 128 | 17 | 224 4/11 | -4 | 4/11 | 12 | 17 | 219 4/11 | -52 7/11 | -4 7/11 | 11 | 11 | 11 | 11 | -17/11 | -108 7/11 | 2021 |
| 302 | -927/17 | 35 | 83 | 211 | 259 | -105 7/11 | 22 | 70 | 198 | 246 | -98 | 29 4/11 | 77 4/11 | $2054 / 11$ | 253 4/11 | -103 7/11 | 24 4/11 | 72 4/11 | 200 4/11 | 248 4/11 | -118 7/11 | 2021 |
| 324 | 163 | 291 | -60 | -12 | 115 | 150 | 27 | -7 | -2 | 102 | 157 4/11 | 285 4/11 | -66 7/11 | -187/11 | 109 4/11 | 152 4/11 | 280 4/11 | -71 7/11 | -23 7/11 | 104 4/11 | -140 7/11 | 2021 |
| -141 | 19 | 67 | 195 | 243 | -28 | 6 | 54 | 182 | $2304 / 11$ | -4 | 13 |  | 189 4/11 | $2374 / 11$ | -34 7/11 | 8 | 56 4/11 | 184 4/11 | 232 4/11 | -39 7/11 | 11 | 2021 |
| 326 4/1 | $2754 / 11$ | -76 7/ | 514 | 99 | 1474 | 262 4/11 | -8971 | 38 4/11 | 86 | $1344 / 11$ | 269 4/11 | -82 7/11 | 45 4/11 | 93 4/11 | 1414/11 | 264 4/11 | -87 7/11 | 40 4/11 | 88 4/11 | $1364 / 11$ | -142 7/11 | 2021 |
| -143 | 131 | 179 | 227 | -447 | 34 | 1184 | 1664 | 214 | -57 7/11 | -97 | 125 4/11 | 173 4/11 | 221 4/11 | -50 7/11 | -2 7/11 | $1204 / 11$ | 168 4/11 | 216 4/11 | -55 7/11 | -7 7/11 | 327 4/11 | 2021 |
| 328 4/1 | -99 | 284 | 764 | 2044 | 2524 | -102 7/11 | 25 | 73 | 201 4/11 | 249 4/11 | -93 7/11 | 34 | 82 4/11 | 210 4/11 | 258 4/11 | -104 7/11 | 23 4/11 | 71 4/11 | $1994 / 11$ | 247 4/11 | -144 7/11 | 2021 |
| -145 7/11 | 156 4/11 | 284 4/11 | -67 7/11 | -19 7/11 | 108 4/11 | 153 4/11 | 281 4/11 | -70 7/11 | -22 7/11 | $1054 / 11$ | 162 4/11 | 290 4/11 | -61 7/11 | -13 7/11 | 114 4/11 | $1514 / 11$ | 279 4/11 | -72 7/11 | -24 7/11 | 103 4/11 | 329 4/11 | 2021 |
| $3304 / 1$ | 12 4/11 | 60 4/11 | $1884 / 1$ | $2364 / 11$ | -35 7/11 | 9 4/11 | $574 / 11$ | 185 4/11 | 233 4/11 | -38 7/11 | 18 4/11 | 66 4/11 | 194 4/11 | 242 4/11 | -29 7/11 | 7 4/11 | $554 / 11$ | $1834 / 11$ | $2314 / 11$ | -407/11 | -146 7/11 | 2021 |
| -147 7/11 | 268 4/11 | -83 7/11 | 44 4/11 | 92 4/11 | 140 4/11 | 265 4/11 | -86 7/11 | 41 4/11 | 89 4/11 | $1374 / 11$ | 274 4/11 | -77 7/11 | $504 / 11$ | 98 4/11 | 146 4/11 | 263 4/11 | -88 7/11 | 39 4/11 | 87 4/11 | $1354 / 11$ | $3314 / 11$ | 2021 |
| 332 4/11 | 124 4/11 | 172 4/11 | 220 4/11 | -51 7/11 | -3 7/11 | 121 4/11 | 169 4/11 | 217 4/11 | -54 7/11 | -6 7/11 | $1304 / 11$ | 178 4/11 | 226 4/11 | -457/11 | 2 4/11 | 119 4/11 | 167 4/11 | 215 4/11 | -56 7/11 | -87/11 | -148 7/11 | 2021 |
| -128 7/11 | -138 7/11 | 321 4/11 | -136 7/11 | 319 4/11 | -134 7/11 | 317 4/11 | -132 7/11 | 315 4/11 | -130 7/11 | 333 4/11 | -139 7/11 | $3114 / 11$ | -126 7/11 | 309 4/11 | -124 7/11 | 307 4/11 | -122 7/11 | $3054 / 11$ | -120 7/11 | 303 4/11 | 313 4/11 | 2021 |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |

In this case, the block order 5 are pandiagonal with different magic sums.

### 1.16 Block-Bordered Magic Squares of Order 23

Below are block-bordered magic squares of order 23 in two different ways giving magic sums $21,21^{2}$ and 2021. The blockwise magic square considered is of order 21 with blocks of orders 3 and 7 . The blocks of order 3 are semi-magic squares with equal semi-magic sums. The block of order 7 are pandiagonal magic squares with different magic sums. In this case, the magic sums of order 21 are given by

$$
\begin{gathered}
{\left[S_{21 \times 21}: S_{23 \times 23}:=21\right]:=\frac{441}{23}} \\
{\left[S_{21 \times 21}: S_{23 \times 23}:=441\right]:=\frac{9261}{23}} \\
{\left[S_{21 \times 21}: S_{23 \times 23}:=2021\right]:=\frac{42441}{23} .}
\end{gathered}
$$

### 1.16.1 First Type

| 243 21/23 | -221 2/23 | -223 2/23 | -225 2/23 | -227 2/23 | -229 2/23 | -231 2/23 | -233 2/23 | -235 2/23 | -237 2/23 | -239 2/23 | -241 2/23 | 24621/23 | 24821/23 | 250 21/23 | 252 21/23 | 254 21/23 | 256 21/23 | 258 21/23 | 260 21/23 | 262 21/23 | 264 21/23 | -240 2/23 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $22121 / 23$ | 25 21/23 | -109 2/23 | 85 21/23 | 53 21/23 | 154 21/23 | -206 2/23 | -33 2/23 | $-1312 / 23$ | 166 21/23 | -8 2/23 | 198 21/23 | -188 2/23 | -52 2/23 | -153 2/23 | 20721/23 | 3421/23 | 132 21/23 | -165 2/23 | -15 2/23 | -87 2/23 | 104 21/23 | -220 2/23 |  |
| 22 | 79 | 22 21/23 | -100 2/23 | -202 2/23 | $6621 / 23$ | 137 21/23 | 162 21/23 | -43 2/23 | -117 2/23 | -179 2/23 | 21 | 18 | $20321 / 23$ | -65 2/23 | -136 2/23 | -161 2/23 | 23 | 23 | 23 | -21 2/23 | 123 | 23 |  |
| 22 | -103 | 88 | 16 | 150 21/23 | -219 2/23 | 70 21/23 | -127 2/23 | 176 21/23 | -47 2/23 | 189 21/23 | -19 | 921 | -149 2/23 | $22021 / 23$ | 23 | 128 21/23 | 23 | 48 21/23 | 23 | 23 | -24 2/23 | 23 |  |
| 227 21/23 | 52 21/23 | 119 21/23 | -170 2/23 | -28 2/23 | -77 2/23 | 107 21/23 | $2621 / 23$ | -108 2/23 | 83 21/23 | 67 21/23 | 142 21/23 | -208 2/23 | -51 2/23 | -118 2/23 | 17121/23 | $821 / 23$ | 183 21/23 | -190 2/23 | -71 2/23 | -137 2/23 | 210 21/23 | -226 2/23 |  |
| 2292 | -174 | $3921 / 23$ | 136 21/23 | 111 21/23 | -18 2/23 | -91 2/23 | 8021/23 | $2021 / 23$ | -99 2/23 | -214 2/23 | 6421/23 | 151 21/23 | 175 21/23 | -38 2/23 | -135 2/23 | -194 2/23 | -1 2/23 | 23 | 219 21/23 | -62 2/23 | -155 2/23 | 23 |  |
| 23 | 123 | -157 | 35 | -8 | 97 | -14 2/23 | -10 | 89 | 17 | 14821/23 | -20 | 58 | 23 | 158 21/23 | $-342 / 23$ | 23 | -180 2/23 | -5 2/23 | 23 | 20121/23 | 23 | 23 |  |
| $23321 / 23$ | -9 2/23 | 196 21/23 | -185 2/23 | -54 2/23 | -152 2/23 | 208 21/23 | 33 21/23 | 135 21/23 | -167 2/23 | -10 2/23 | -90 2/23 | 102 21/23 | 13 21/23 | -98 2/23 | 8621/23 | 68 21/23 | 143 21/23 | -210 2/23 | -37 2/23 | -130 2/23 | 169 21/23 | -232 2/23 |  |
| 23 | -18 | 32 | 17 | 20421/23 | -64 2/23 | -138 2/23 | -15 | 42 | 11 | 98 | -23 2/23 | -7 | 90 | 23 | -112 2/23 | -213 2/23 | 3 | 23 | 163 21/23 | -40 2/23 | -121 2/23 | 23 |  |
| 23 | 192 | -198 2/2 | 721 | -148 2/23 | 218 21/23 | -68 2/23 | 126 21/23 | -176 | 51 | -86 2/23 | 115 21/23 | -27 2/23 | -102 2/23 | 76 | 27 21/23 | 14621/23 | -204 2/23 | 59 21/23 | 2/23 | 172 21/23 | 23 | -236 2/23 |  |
| $23921 / 2$ | 55 21/23 | 153 21/23 | -207 2/23 | -36 2/23 | -129 2/23 | 167 21/23 | 421/23 | 184 21/23 | -187 2/23 | -72 2/23 | -139 2/23 | 21321/23 | 50 21/23 | 120 21/23 | -169 2/23 | -29 2/23 | $-742 / 23$ | 105 21/23 | 3121/23 | -111 2/23 | $8121 / 23$ | -238 $2 / 23$ |  |
| -243 | -203 | 65 | 139 21/23 | 164 21/23 | -42 2/23 | -120 2/23 | -19 | 12 | 19321/2 | 21721/23 | -59 2/23 | -156 2/23 | -173 2/23 | $4021 / 23$ | 134 21/23 | 114 21/23 | $-202 / 23$ | -92 2/23 | 77 21/23 | 1821/23 | -94 2/23 | 23 |  |
| -244 2/23 | 149 21/23 | -217 2/23 | 69 21/23 | -126 2/23 | 173 21/23 | -45 2/23 | 190 21/23 | -184 2/23 | -4 2/23 | -143 2/23 | 200 21/23 | -55 2/23 | 124 21/23 | -159 2/23 | $3621 / 23$ | -83 2/23 | 9621/23 | -11 2/23 | -107 2/23 | 9421/23 | 1421/23 | 21/23 |  |
| -246 2/23 | -12 2/23 | -89 2/23 | 103 21/23 | 12 21/ | -95 2/23 | 8421/23 | 73 21/23 | 140 21/23 | -212 2/23 | -49 2/23 | -119 2/23 | 170 21/23 | 521/23 | 185 21/23 | -189 2/23 | -58 2/23 | -151 2/23 | 21121/23 | 32 21/23 | 133 21/23 | -164 2/23 | 247 21/23 |  |
| -248 2/2 | 99 21/23 | -22 2/23 | -75 2/23 | 93 21/ | $2121 /$ | -113 2/23 | -216 $2 /$ | $6021 / 23$ | $15721 / 23$ | 174 21/23 | -39 2/23 | -133 2/23 | -192 | - 2/23 | 19421/23 | 205 21/23 | -61 2/23 | -142 2/23 | -160 2/23 | 45 21/23 | 116 21/23 | 23 |  |
| -250 2/23 | -85 2/23 | 113 21/23 | $-262 / 23$ | -104 2/23 | 7521/23 | $3021 / 23$ | 144 21/23 | -199 2/23 | $5621 / 23$ | -123 2/23 | 160 21/23 | -35 2/23 | 188 21/23 | -183 2/23 | -3 2/23 | -145 2/23 | 214 21/23 | -67 2/23 | 129 21/23 | -177 2/23 | 49 21/23 | $25121 / 23$ |  |
| -252 2/23 | -57 2/23 | -150 2/23 | 209 21/23 | $4621 / 23$ | 121 21/23 | -166 2/23 | -30 2/23 | -76 2/23 | 108 21/23 | 29 21/23 | -110 2/23 | 82 21/23 | 5421/23 | 156 21/23 | -209 2/23 | -31 2/23 | -132 2/23 | 165 21/23 | -7 2/23 | 195 21/23 | -186 2/23 | 253 21/23 |  |
| -254 2/23 | 20621/23 | -63 2/23 | -141 2/23 | -172 2/23 | 4321/23 | 130 21/23 | 112 21/23 | -17 2/23 | -93 2/23 | 78 21/23 | 19 21/23 | -96 2/23 | -200 2/23 | $6321 / 23$ | 138 21/23 | 161 21/23 | -44 2/23 | -115 2/23 | -182 2/23 | 221/23 | 181 21/23 | 255 21/23 |  |
| -256 2/23 | -147 2/23 | 215 21/23 | -66 2/23 | 127 21/23 | -163 2/23 | 37 21/23 | -80 2/23 | 95 21/23 | -13 2/23 | -106 2/23 | 92 21/23 | 15 21/23 | 147 21/23 | -218 2/23 | 72 21/23 | -128 2/23 | 178 21/23 | -48 2/23 | 19121/23 | -196 2/23 | $621 / 23$ | 25721/23 |  |
| -258 2/23 | -50 2/23 | -116 2/23 | 168 21/23 | 10 21/23 | 182 21/23 | -191 2/23 | -70 2/23 | -140 2/23 | 212 21/23 | 47 21/23 | 122 21/23 | -168 2/23 | $-162 / 23$ | -88 2/23 | 106 21/23 | 1121/23 | -97 2/23 | 87 21/23 | 7121/23 | 14121/23 | -211 2/23 | 259 21/23 |  |
| -260 2/23 | 177 21/23 | -41 2/23 | -134 2/23 | -195 2/23 | -2 2/23 | 19991/23 | 216 21/23 | -60 2/23 | -154 2/23 | -171 2/23 | 4121/23 | 131 21/23 | 100 21/23 | -19 2/23 | -79 2/23 | 9121/23 | 2421/23 | -114 2/23 | -215 2/23 | 6121/23 | 155 21/23 | 261 21/23 |  |
| -262 2/23 | -125 2/23 | 159 21/23 | -32 2/23 | 186 21/23 | -178 2/23 | -6 2/23 | -144 2/23 | 202 21/23 | -56 2/23 | 125 21/23 | -162 2/23 | $3821 / 23$ | -82 2/23 | $10921 / 23$ | -25 2/23 | -101 2/23 | 7421/23 | 28 21/23 | 145 21/23 | -201 2/23 | 57 21/23 | 263 21/23 |  |
| 24121/23 | 222 21/23 | $22421 / 23$ | 226 21/23 | 228 21/23 | $23021 / 23$ | 232 21/23 | 234 21/23 | 236 21/23 | 238 21/23 | 240 21/23 | 242 21/23 | -245 2/23 | -247 2/23 | -249 2/23 | -251 2/23 | -253 2/23 | -255 2/23 | -257 2/23 | -259 2/23 | -261 2/23 | -263 2/23 | -242 2/23 |  |


| 262 4／23 | －202 19／23 | －204 19／23 | －206 19／23 | －20819／23 | －210 19／23 | －212 19／23 | $-21419 / 23$ | －216 19／23 | －218 19／23 | －220 19／23 | $-22219 / 23$ | $2654 / 23$ | 267 4／23 | 269 4／23 | $2714 / 23$ | 273 4／23 | $2754 / 23$ | 277 4／23 | 279 4／23 | $2814 / 23$ | $2834 / 23$ | －221 19／23 | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 240 4／23 | 444 | －90 19／23 | 104 4／23 | 72 4／23 | $1734 / 23$ | －18719／23 | $-1419 / 23$ | -112 19／23 | $1854 / 23$ | $104 / 23$ | $2174 / 23$ | －169 19／23 | －33 19／23 | －134 19／23 | $2264 / 23$ | 53 4／23 | $1514 / 23$ |  | $34 / 23$ | －68 19／23 | $1234 / 23$ | 23 |  |
|  |  |  |  | －18 |  | 156 |  | －2 |  | －16 |  |  |  | －4619／23 |  |  |  |  | $1204 / 23$ | 23 | 23 | 19／23 |  |
| $2444 / 23$ | －84 | 107 4／23 |  | 169 4／23 | －20 | 89 | －108 19／23 | 19 |  | $2084 / 23$ | －17 | 28 | －130 19／23 | 239 4／23 | 3 | 3 | 23 | 67 4／23 | 23 | 129 4／23 | 23 | 23 | 44 |
|  | 71 | 138 | －15 | －9 | －58 19／23 | $1264 / 23$ |  | －89 |  |  |  | －189 19／23 |  | －99 |  |  | 123 | 3 | －5 | －11 | $2294 / 23$ | 23 | 44 |
|  | －155 |  | 15 | 13 | 4／23 | －7 |  |  | －80 | －195 |  |  |  | －19 | －116 19／23 | －175 19／23 |  | $2164 / 23$ | 238 4／23 | $-4319 / 23$ | 23 | －20919／23 | 44 |
|  | 14 |  |  |  | 116 |  |  | 10 |  | 167 4／23 | －18 |  |  | 177 4／23 | －1519／23 | 3 | 3 | 13 4／23 | 23 | 3 | 23 | 3 |  |
|  | 94 | 215 | －166 | －35 19 | －133 19／23 | $2274 / 23$ |  |  |  | $84 / 23$ |  |  |  | －79 19／23 | $1054 / 23$ |  | 162 4／23 | －191 19／23 | －1819／23 | $-11119 / 23$ | $1884 / 23$ | $-21319 / 23$ | 位 |
| 25 | －16 |  |  | 22 | －4 | －11 | －13 |  |  |  |  | －54 19／23 | 10 |  | －93 19／23 | 3 |  | 1714／23 | 23 | $-2119 / 23$ | 23 | －215 19／23 | 4 |
| 25 | 21 | －17 | 26 | －12 | 23 | －4919／23 | 145 | －15＇ | 70 | －67 19／23 | 134 | －8 | －8 | $954 / 23$ | 46 4／23 | 165 4／23 | －185 19／23 | 78 4／23 | －105 19／23 | 23 | 23 | －217 19／23 |  |
|  |  |  | －18 |  | －11 | $1864 / 23$ |  | $2034 / 23$ | －168 19／23 | －5319／23 | －120 19／23 |  |  | 139 4／23 | －150 19／23 | －10 19／23 | －5519／23 | 23 | 23 | 23 | $1004 / 23$ | －21919／23 |  |
| －22 | －184 | 84 | 158 | 183 | －23 | －10119／23 | －17 | 20 | 21 | 23 | －40 | －137 19／23 | －154 19／23 | 59 | 153 4／23 | 13 | －19／23 | －7319／23 | 96 4／23 | 23 | 23 | 263 4／23 |  |
| －225 19／23 | 168 |  | 88 | －10 | 192 | $-2619 / 23$ | 209 4／23 |  | 14 | －124 19／23 | $2194 / 23$ | －36 | $1434 / 23$ |  | 23 | 3 | $1154 / 23$ | 7 4／23 | －8819／23 | 113 4／23 | 33 4／23 | 23 |  |
|  | 64 | －70 | 122 | 31 | －76 19／23 | $1034 / 23$ | 92 | 159 4／23 | －193 19／23 | －30 19／23 | －100 19／23 | 189 4／23 | 24 | 204 4／23 | －170 19／23 | －39 19／23 | －132 19／23 | $2304 / 23$ | 23 | 152 4／23 | －145 19／23 | $2664 / 23$ | 位 |
| －22 | 118 | －3 | －5 | 112 | 40 | －9419／23 | －19719／23 | 79 | 176 | 193 | －20 | －114 19／23 | －173 19／23 | 18 | 213 4／23 | 224 4／23 | －42 19／23 | －123 19／23 | －141 19／23 | 64 4／23 | $1354 / 23$ | 268 4／23 | 位 |
| －231 | －66 19／2 | 132 | －7 | －85 | 94 | 49 | 163 4／23 | －180 19 | $754 / 23$ | －104 | 179 4／23 | －16 | 207 4／23 | －164 19／23 | 15 | －126 19／23 | 233 4／23 | $-4819 / 23$ | $1484 / 23$ | －158 19／23 | 68 4／23 | $2704 / 23$ |  |
| －233 19／23 | －38 19／23 | －131 | 2284 |  | $1404 / 23$ | －14719／23 | －111 | －57 | $1274 / 23$ |  |  | $1014 / 23$ | 73 4／23 | $1754 / 23$ | －190 19／23 | $-1219 / 23$ | $-11319 / 23$ | $1844 / 23$ | $114 / 23$ | 214 4／23 | －167 19／23 | 272 4／23 | 4 |
| －235 19／23 | $2254 / 23$ | －44 | －122 19／23 | －153 19／23 | 62 | 149 4／23 | $1314 / 23$ | 1 | －74 | 97 | 38 4／23 | －77 19／23 | －181 19／23 | 82 4／23 | 157 4／23 | 180 4／23 | －25 19／23 | －9619／23 | －163 19／23 | 21 4／23 | $2004 / 23$ | 274 4／23 |  |
| －237 19／23 | －128 19／2 | 234 4／23 | －47 | 146 4／23 | －144 19／23 | $564 / 23$ | －61 19／23 | 114 | $54 / 23$ | －87 |  | 34 | 166 4／23 | －199 19／23 | $914 / 23$ | －109 19／23 | $1974 / 23$ | $-2919 / 23$ | $2104 / 23$ | －177 19／23 | $254 / 23$ | 276 4／23 |  |
| －239 19／23 | －31 19／23 | －97 | $1874 / 23$ | 29 4／23 | 201 4／23 | －172 19／23 | －51 19／23 | $-12119 / 23$ | $2314 / 23$ | $664 / 23$ | $1414 / 23$ | －14919／2 | $24 / 23$ | －69 19／23 | $1254 / 23$ | $304 / 23$ | －78 19／23 | $1064 / 23$ | $904 / 23$ | $1604 / 23$ | －192 19／23 | $2784 / 23$ |  |
| －24119／23 | $1964 / 23$ | $-2219 / 2$ | －115 19／23 | －176 19／23 | 164 | 218 4／23 | $2354 / 23$ | $-4119 / 23$ | －135 19／23 | －152 19／23 | $604 / 23$ | $1504 / 23$ | 119 4／23 | －19／23 | －60 19／23 | $1104 / 23$ | 43 4／23 | －95 19／23 | －196 19／23 | 80 4／23 | $1744 / 23$ | 280 4／23 | 4 |
| －243 19／23 | －106 19／23 | 178 4／23 | －13 19／23 | $2054 / 23$ | －159 19／23 | 12 4／23 | －125 19／23 | 221 4／23 | －3719／23 | 144 4／23 | －14319／23 | 57 4／23 | －63 19／23 | 128 4／23 | －619／23 | －82 19／23 | 93 4／23 | 474／23 | 164 4／23 | －182 19／23 | 76 4／23 | 282 4／23 | 44 |
| 260 4／23 | $2414 / 23$ | $2434 / 23$ | $2454 / 23$ | $2474 / 23$ | 249 4／23 | $2514 / 23$ | 253 4／23 | $2554 / 23$ | $2574 / 23$ | 259 4／23 | $2614 / 23$ | －226 19／23 | －228 19／23 | －230 19／23 | －232 19／23 | －234 19／23 | －236 19／23 | －238 19／23 | －240 19／23 | －242 19／23 | －244 19／23 | －223 19／23 | 441 |


|  | -134 | -136 3/23 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 20/23 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 30 | 112 20/23 | -22 3/23 | 172 | 140 20/23 | 24 |  | $5320 / 23$ | -44 3/23 | 123 | 7820/23 | 285 20/23 |  | 3420/23 | -66 3/23 | 23 | 3 | 2 |  | 7120/23 |  | 20/23 |  |  |
| 310 | 16620/23 | 10920/2 | -13 3/23 | -115 3/23 | 15 | 23 | 2 | 43 |  | -92 | 123 | 123 | 290 20/23 | 2/23 | 3/23 | 3/23 | 20/23 | 20/23 | 0/23 | 65 20/23 |  |  |  |
| 2/23 | $-163 / 23$ | 175 20/23 | 103 | 237 | -132 3/23 | 157 20/23 | -40 | 263 | 0/23 | 276 | 3/23 | 9620/23 | -62 3/23 | 307 20/23 | 2023 | 2150/23 | $3 / 23$ | 135 20/23 | 200/23 | 20/2 | 6200/23 | -37 3/23 |  |
| 2/23 | 139 20/2 | 206 | -83 3/23 | 58 | 920/23 | 19420/23 | 113 | 3/23 | 170 20/23 | $15420 / 2$ | 22920/2 | -121 3/23 | 3520/23 | 3/23 | 2580/23 | 9500/23 | 270 20/23 | -103 3/23 | 1520/23 | 50 3/23 | 2972 | -139 3/23 |  |
| 316 | -87 | 12600/2 | 22 | 198 20/23 | 68 20/23 | -4 3/23 | 167 20/23 | 107 20/23 | -12 | -12 | 15120/23 | 238 | 262 | 48 20/23 | 3 | -107 3/23 | 85 20/23 | 20/23 | 30620/23 | 2420/23 | -68 3/23 | -141 3/23 |  |
| 318 | 210 | -70 | 122 20/23 | 520/23 | 18420/23 | 72 | -18 | 17620/23 | 104 20/23 | 235 | -118 3/23 | 23 | -35 | 245 20/23 | 5220/23 | 274 20/23 | 23 | 23 | -59 3/23 | 288 20/23 | 23 | -143 3/23 |  |
| 32 | 77 | 283 | -98 | 32 | -65 3/23 | 29520/23 | 120 | 222 20/23 | -80 3/23 | 76 | -3 3/23 | 189 20/23 | 100 | 23 | 23 | 155 20/23 | $23020 / 23$ | 23 | 49 20/23 | $-433 / 23$ | 256 20/23 |  |  |
|  | -94 | 90 20/23 |  | 29120/23 |  |  |  |  |  |  |  | 1300/23 | 177 | 110 20/23 | -25 3/23 | -126 3/23 | 149 20/23 | 239 20/23 | 3 | 4620/23 | -34 3/23 |  |  |
|  | 279 | -111 | 94 |  |  |  |  |  |  |  | 202 | 5920/23 | -153/3 | 163 20/23 | 17420 | 23 | , | 3 | -J J1 | 25 | 40 20/23 |  |  |
| 326 | 142 | 240 | -12 | 50 |  | 254 20/23 | 91 | 27120/ |  |  | -52 3/2 |  |  | 207 | 3/23 | 5720/23 | 1220/23 | 120 | 118 | -24 3/23 | 220 |  |  |
|  | -116 | 152 | 226 | 251 | 44 | -33 3/23 | -106 3 | 88 | 280 | 304 | 27 | -69 3/23 | -86 3/23 | 127 20/23 | 221 20/23 | 2 | 6620 | -5 3/23 | 16420 | 105 20/23 | -73/23 |  |  |
|  | 236 | -130 | 156 | -39 | 260 | 41 | 271 |  | 8220/23 |  | 28720/23 | 3120/23 | 21120 | -72 3/3 | 123 20/23 | 300/23 | 183 20/23 | 75 20/23 | -20 3/ | 18120 | $20 / 2$ |  |  |
|  | 74 | -2 | 190 20/23 | 99 | -8 3/23 | 171 20/23 | 160 | 227 | -125 3/23 | 37 | -32 3/23 | 257 20/23 | 92 | 272 20/23 | -102 3/23 | 28 | -64 3/23 | 29820/23 | 119 20/23 | 220 20/23 | -77 3/23 |  |  |
|  | 186 | 64 | 11 | 180 20/23 | 108 | $-263 / 23$ |  |  | 24 | 261 | 47 20/23 | / | -10 | 86 | 281 20/23 | 2920/23 | 2520 | -55 3/23 | -73 | 3220 | 20 | 336 20/23 |  |
|  | 12 | 20020 | 23 |  | , | 2 | 231 | , | 1432023 | -36 | 247 20/23 | $5120 / 23$ | 27 | - | 832,2 | ) | Jo | 20/23 | 21620 | -90 3/23 | 3620/23 | ) |  |
| -165 3/23 | 2900/23 | 23 | 29620 | 133 | 208 | -79 3/23 | 5620/23 | 1020/23 | 193023 | 116 | -23 32 | 169 20/23 | 14 | 24 | -122 3/23 | 55 | -45 3/23 | 252 20/ | 790/23 | 282 20, | -993/23 | 340 |  |
| /23 | 293 20/23 | 23 20/23 | -54 3/23 | -85 3/23 | 13020 | 217 20/23 | 19920/23 | 69 20/2 | -63/23 | 165 20/23 | 106 20/23 | -9 3/23 | -113 3/23 | $15020 / 2$ | 225 20/23 | 24820 | 4220/23 | -28 3/23 | -95 3/23 | 8920/23 | 268 20/2 | 3422 |  |
| 3/23 | -60 3/23 | 302 20/23 | $2020 / 23$ | 21420 | -76 3/23 | 124 | 620/23 | 182 20/23 | 73 20/23 | -193/23 | 179 20/23 | 102 20/23 | $23420 / 23$ | -131 3/23 | 159 20/23 | $-413 / 23$ | 26520 | 38 20/23 | 27820 | $1093 /$ | 930/23 | 34420 |  |
| -171 3/23 | 3620/23 | $-293 / 23$ | 25520 | 97 | 269 | -104 3/23 | 1620 | -5 | 29900/23 | 134 | 209 | -81 3/23 | 70 20/23 | $-13 / 23$ | 19320/23 | 9820/23 | -10 3/23 | 17420/2 | 15820 | 22820 | -124 | 34620 |  |
| 3/23 | 26420/2 | $4520 / 23$ | -47 | -1083 | 8420/23 | 286 | 303 20/23 | 2620/23 | 3/23 | -84 3/23 | 128 20/23 | 21820/23 | 187 20/23 | 67 20/23 | 720/23 | 178 20/23 | $11120 / 23$ | -273/23 | -1283/ | 14820 | 242 20/ | 34820 |  |
| -175 3/23 | -38 3/23 | 24620/2 | 5420/2 | 27320 | -91 3/23 | 80 20/23 | -57 3/23 | 28920 | $3020 / 23$ | 21200/23 | -75 3/23 | 125 20/23 | 420/23 | 19600/23 | $6120 / 23$ | -14 3/23 | 16120/2 | $11520 / 2$ | 23220 | -1143/ | 14420/2 | 35020 |  |
| 328 20/2 | 30 | $31120 / 2$ | 313 20/23 | 315 | 31720 | 319 20/23 | 321 | 323 | 325 | 327 | 329 | 158 | -160 | 162 | 164 | -166 3/23 | -168 3/23 | 170 | 172 | 74 | 173 | -155 3/23 |  |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 202 | 2021 | 2021 | 2021 | 2021 |  |  |

In this case, the blocks of order 3 are semi-magic squares, with following equal semi-magic sums given by

$$
\begin{aligned}
& {\left[{\left.S m_{3 \times 3}: S_{23 \times 23}:=21\right]}:=\frac{63}{23}\right.} \\
& {\left[\text { Sm }_{3 \times 3}: S_{23 \times 23}:=441\right] }:=\frac{1323}{23} \\
& {\left[S_{3 \times 3}: S_{23 \times 23}:=2021\right] }:=\frac{6063}{23}
\end{aligned}
$$

### 1.16.2 Second Type

| $24321 / 23$ | -221 2/23 | -223 2/23 | -225 2/23 | -227 2/23 | -229 2/23 | -2312/23 | -233 2/23 | -235 2/23 | -237 2/23 | -239 2/23 | -241 2/23 | $24621 / 23$ | $24821 / 23$ | $25021 / 23$ | 252 21/23 | 25421/23 | 256 21/23 | $25821 / 23$ | $26021 / 23$ | 262 21/23 | 264 21/23 | 2/23 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 22121/23 | -219 2/23 | -109 2/23 | -65 2/23 | $2221 / 23$ | $4421 / 23$ | 15421/23 | 17621/23 | -217 2/23 | -110 2/23 | -66 2/23 | 2121/23 | 4521/23 | $15321 / 23$ | $17721 / 23$ | -218 2/23 | -1112/23 | -64 2/23 | $2021 / 23$ | $4621 / 23$ | 152 21/23 | 178 21/23 | 23 | 21 |
| $22321 / 23$ | 149 21/23 | 175 21/23 | -201 2/23 | -114 2/23 | -61 | $1821 / 23$ | $4321 / 23$ | 150 21/23 | 17421/23 | -200 2/23 | -112 2/23 | -68 | 123 | $4221 / 23$ | 15121/23 | 173 21/23 | -1999 2/23 | 113 2/23 | -69 2/23 | 1921/23 | 21/23 | 222 2/23 | 21 |
| 22 | 1621/23 | 3921/23 | 14821/23 | 170 | -202 2/23 | -96 | -72 2/23 | 123 | $3821 / 23$ | 14 | 1/23 | -203 | -95 | -70 2/23 | 23 | 1/23 | 21/23 | -172 21/23 | -204 2/23 | 2/23 | -712/23 | -224 2/23 | 21 |
| 22 | -97 | -5 | 1121/23 | 3721/23 | 14421/23 | 16921/23 | 23 | -98 | -53 | 1321/23 | 123 | 14321/23 | 168 | -206 2/23 | 223 | -52 2/23 | 1221/23 | 23 | 14521/23 | 21/23 | -205 2/23-20 | -226 2/23 | 21 |
| $22921 / 23$ | 165 | -208 | -102 2/23 | -55 |  |  |  | 164 | -20 | -101 | -56 | 30 | 34 | 23 | 166 21/23 | -210 | -100 2/23 | -57 2/23 | 23 | 21/23 | $14021 / 23$ | -228 2/23 | 21 |
|  | $5021 / 23$ | 13721/23 | 16321/23 | -21 | -103 2/23 |  |  |  | 139 21/23 | 162 21/23 | -213 2/23 | -104 2/23 | -59 | $2721 / 23$ | 52 21/23 | 23 | 23 | -211 2/23 | 23 | 23 | 2621/23 | $2302 / 23$ | 21 |
| 2332 |  | 2321/23 |  | 155 21/23 |  |  |  | -62 | 2421/23 | 4821/23 | 156 | 160 | -21 |  | -63 2/23 | 251/23 | 4, 212 | 23 | 23 | 216 2/23 | -106 2/23 | , | 21 |
| $23521 / 23$ | -177 2/23 | -130 2/23 | -86 2/23 |  |  | 133 21/23 |  | -175 2\| | -131 2 | -87 2/23 | 21/23 | 2/23 | 132 21/23 | 19821/23 | -176 2/23 | -132 2/23 | -85 2/23 | 2/23 | 123 | 13121/23 | 19921/23 | -234 2/23 | 21 |
| 237 21/23 | 128 21/23 | 19621/23 | -159 2/23 | -135 2/23 | -88 | 123 | 23 | 129 21/23 | 19521/23 | -158 2\| | -133 2/23 | -89 2/23 | -3 2/23 | 6321/23 | 130 21/23 | 19421/23 | -157 2/23 | 1342 | -90 2/23 | 2/23 | 6221/23 | -236 2/23 | 21 |
| $23921 / 23$ | -4 | 60 21/23 | $1 / 23$ | 19121/23 |  | -117 | -93 |  |  | $12621 / 23$ | $19221 / 23$ | 2/23 | -116 2/23 | -912 | -6 2/23 | 6121/23 | 12521/23 | .19321/23 | -162 2/23 | -115 2/23 | -92 2\|2 | 238 2/23 | 21 |
| -24 | -118 | -75 2/23 | -9 | $5821 / 23$ | 12321/23 | 190 21/23 | -165 2/23 | -119 | -74 | -7 2/23 | 5721/23 | 122 21/23 | 18921/23 | -164 2/23 | -120 2/23 | -73 2/23 | -8 2/23 | 5621/23 | 12421/23 | 18821/23 | 2/23 | 24421 | 1 |
| -244 2/23 | 18621/23 | -166 2/23 | -123 2/23 | -76 2/2 | 82 | 5321/23 | 12121/23 | 185 21/23 | -161 | -122 21 | -77 2\|2 | 92 | 5521/23 | $12021 / 23$ | 187 21/23 | 2/23 | -121 2/23 | -78 2/23 | $1021 / 23$ | 5421/23 | 119 21/23 | 24521 | 21 |
| -246 | 7121/23 | 11621/23 | 18421/23 | -170 | -124 | -81 | 721/23 | 7221/23 | 11821/23 | 18321/ | -17 | -125 | -80 2 | 62 | 7321/2 | 11721/23 | 182 21/23 | -169 2/23 | -126 2/23 | -79 2/23 | 521/23 | 24 | 21 |
| -248 2/23 | -82 2/23 | 22 | $7021 / 23$ | 13421/2 | 179 | -172 | -128 | -83 | 321/23 | $6921 / 23$ | 135 21/23 | 18121/23 | -173 2/23 | -129 2/23 | -84 2/23 | 421/23 | $6821 / 23$ | 136621/23 | 180 21/23 | -174 2/23-12 | -127 2/23 | 24921 | 21 |
| -250 2/23 | -198 2/23 | -151 2/23 | -44 2/23 | -19 2/2 | $8621 / 23$ | 11221/23 | 21821/23 | -196 | -152 2/23 | $-452 / 23$ | $-202 / 23$ | 8721/23 | 21/23 | 21921/23 | -197 2/23 | -153 2/23 | -43 2/23 | $-212 / 23$ | $8821 / 23$ | $11021 / 23$ | $22021 / 23$ | 25121 | 21 |
| -252 2/23 | $10721 / 23$ | 21721/23 | -180 2/23 | -156 2/23 | -462/23 | -23 2/2 | 8521/23 | $10821 / 23$ | 21621/23 | -179 2/23 | -154 2/23 | $-472 / 23$ | $-242 / 23$ | 8421/23 | 109 21/23 | 215 21/23 | -178 2/23 | 155 2/23 | -48 2/23 | -22 2/23 | $8321 / 23$ | 253 | 21 |
| -254 2/23 | -25 2/23 | 8121/23 | 10621/23 | 21221/23 | -181 2/23 | -138 2/23 | -51 2/23 | $-262 / 23$ | $8021 / 23$ | 105 21/23 | 21321/23 | -182 2/23 | -137 2/23 | -49 2/23 | -27 2/23 | 8221/23 | 10421/23 | 21421/23 | -183 2/23 | -136 2/23 | -50 2/23 | 25521 | 21 |
| -256 2/23 | -139 2/23 | -33 2/23 | -30 2/23 | 7921/23 | 10221/23 | 21121/23 | -186 2/23 | -140 2/23 | -32 2/23 | -28 2/23 | 7821/23 | 10121/23 | 210 21/23 | -185 2/23 | -141 2/23 | $-312 / 23$ | $-292 / 23$ | $7721 / 23$ | 10321/23 | $20921 / 23$ | -184 2/23 | 257 21/2 | 1 |
| -258 2/23 | $20721 / 23$ | -187 2/23 | -144 2/23 | -34 2/23 | -12 2/23 | 7421/23 | $10021 / 23$ | $20621 / 23$ | -188 2/23 | -143 2/23 | -35 2/23 | -112/23 | 7621/23 | 9921/23 | $20821 / 23$ | -189 2/23 | -142 2/23 | -36 2/23 | -10 2/23 | 7521/23 | 9821/23 | 259 21/ | 1 |
| -260 2/23 | 9221/23 | 9521/23 | 205 21/23 | -1912/23 | -145 2/23 | -39 2/23 | -13 2/23 | 9321/23 | 9721/23 | 20421/23 | -192 2/23 | -146 2/23 | -38 2/23 | -14 2/23 | 9421/23 | 9621/23 | 203 21/23 | -190 2/23 | -147 2/23 | -37 2/23 | -15 2/23 | 26121/2 | 1 |
| -262 2/23 | -40 2/23 | -18 2/23 | 9121/23 | 11321/23 | 200 21/23 | -193 2/23 | -149 2/23 | $-412 / 23$ | -172/23 | 9021/23 | 11421/23 | 202 21/23 | -194 2/23 | -150 2/23 | -42 2/23 | $-162 / 23$ | 8921/23 | 115 21/23 | 20121/23 | -195 2/23 | -148 2/23 | 26321/2 | 21 |
| $24121 / 23$ | 22221/23 | 22421/23 | $22621 / 23$ | $22821 / 23$ | $23021 / 23$ | 232 21/23 | $23421 / 23$ | $23621 / 23$ | $23821 / 23$ | 24021/23 | 24221/23 | -245 2/23-21 | -247 2/23-21 | -249 2/23 | -251 2/23 | -253 2/23 | -255 2/23 | -257 2/23 | -259 2/23 | -261 2/23- | -263 2/23- | -242 2/23 | 21 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


| 26 | -202 1 | -204 19/23 | -206 19/23 | -208 19/23 | -210 19/23 | -212 19/23 | -21419/23 | -216 19/23 | -218 19/23 | -220 19/23 | -222 19/23 | $2654 / 23$ | $2674 / 23$ | 269 4/23 | $2714 / 23$ | 273 4/23 | 275 4/23 | $2714 / 23$ | 279 4/23 | $2814 / 23$ | 283 4/23 | -22119/23 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 240 | -200 19/23 | -90 19/23 | -4619/2 | 41 | 63 | 173 4/23 | $1954 / 23$ | -198 19/23 | -91 19/2 | $-4719 / 23$ | 40 | 64 | 172 4/23 | $1964 / 23$ | -199 19/23 | -92 19/23 | -4519/23 | 39 | 65 | 23 | $1974 / 23$ | -201 19/23 |  |
|  | 168 | 194 | -182 | -95 | $-4819 / 23$ |  |  |  | 193 |  |  |  |  |  |  |  | -18 | -9 |  |  |  | 23 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | 18 |  |  |  |  |  |  | 187 4/23 |  |  |  |  |  | 164 4/23 | 3 | 23 |  |  |
|  | 184 | -18 |  | -36 |  |  |  | 183 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | -193 |  |  |  |  |  |  |  | -85 |  |  |  |  |  | 位 |  |  |  | - |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $1764 / 23$ | 178 4/23 | 23 | 23 | -213 19/23 |  |
|  | -158 |  |  |  |  |  |  |  |  |  |  |  |  | $2174 / 23$ | -157 19/23 |  |  |  |  | 23 |  | -215 19/23 |  |
|  |  |  | -1 |  |  |  |  | 14 |  | -13 |  |  |  |  |  |  | -138 19/23 |  |  |  |  | -217 19/23 |  |
| 258 4/23 |  |  |  | 21 |  | -9 |  |  |  | $1454 / 23$ | 21 | -142 19/23 | -9719/23 | -72 19/23 | 12 4/23 | 80 4/23 | 144 4/23 | 212 4/23 | 3 | -96 19/23 | -73 19/23 | -219 19/23 |  |
|  | -99 |  |  |  |  | 209 |  |  |  |  |  |  | $2084 / 23$ | -14519/23 |  |  |  |  | 23 | 123 | 23 | 3 |  |
| -2 | 205 | -14 | -10 | -57 |  |  | 14 | 20 | -14819/23 | -10 |  |  |  | 13 | 23 | -149 19/23 | -10 | -59 19/23 | 29 4/23 | 73 4/23 | 23 | 264 4/23 |  |
|  |  |  |  |  |  |  |  |  | 13 | 20 | -152 19/23 | -106 19/23 |  | $254 / 23$ | $924 / 23$ | 3 | $2014 / 23$ | -15 | -107 19/23 | 23 | 23 | 266 4/23 |  |
|  |  |  |  | 153 | 198 4/23 |  |  |  |  |  | 154 | $2004 / 23$ | -154 19/23 | -1 |  | 23 4/23 | 874/23 | 155 4/23 | $1994 / 23$ | -155 19/23 | -108 19/23 | 23 |  |
| -23 | -17 | -132 | -25 |  |  | 13 | 23 | -17 | -13 | -2 |  |  | $1304 / 23$ | 238 4/23 | -178 19/23 | -13 | -2 |  | 107 4/23 | 129 4/23 | 239 4/23 | 270 4/23 |  |
| -23 | 126 | 236 | -161 | -137 |  |  |  | 127 | 235 |  | -135 | -28 |  | 103 | 128 4/23 | $2344 / 23$ | -159 | -136 19 | -2919/23 | -319/23 | 123 | 23 |  |
|  | -6 | 100 | 125 | 23 | -162 19/23 | -119 19/23 |  |  |  |  | 232 4/23 | -163 19/23 | -118 19/23 | -30 |  | 10 | 123 4/23 | 233 4/23 | -164 19/23 | -117 19/23 | -31 19/23 | 274 4/23 |  |
| -23719/23 | -120 19/2 | -14 | -11 | 98 | 121 | $2304 / 23$ | -167 19/23 | -12 | -13 |  |  | $1204 / 23$ | 229 4/23 | -166 19/23 | -122 19/23 | -12 19/23 | -10 19 | $964 / 23$ | $1224 / 23$ | 228 4/23 | -165 19/23 | 276 4/23 |  |
| -2391 | $2264 / 23$ | -168 19/23 | -125 | -15 | 6 |  | 119 | $2254 / 23$ | -169 | -124 19/23 | -16 | 7 | 95 | $1184 / 23$ | 227 4/23 | -170 19/23 | -123 19/23 | -1719 | $84 / 23$ | 94 4/23 | 4/23 | $2784 / 23$ |  |
| -241 19/23 | 111 | 114 | 224 | -172 | -126 19/23 | -20 | 54/23 | 112 | 116 | 223 4/23 | -173 19/23 | -127 19/23 | -19 | 4 | $1134 / 23$ | $1154 / 23$ | 222 4/23 | -171 19/23 | -128 19/23 | $-1819 / 23$ | $34 / 23$ | 280 4/23 |  |
| -24319/23 | $-2119 / 23$ | 4/23 | $1104 / 23$ | $1324 / 2$ | 219 4/23 | -174 19/23 | -130 19/23 | $-2219 / 23$ | $14 / 23$ | 109 4/23 | $1334 / 23$ | $2214 / 23$ | -175 19/23 | $-13119 / 23$ | $-2319 / 23$ | $24 / 23$ | 108 4/23 | $1344 / 23$ | $2204 / 23$ | -176 19/23 | -129 19/23 | 282 4/23 |  |
| 260 4/23 | $2414 / 23$ | $2434 / 23$ | $2454 / 23$ | $2474 / 23$ | 249 4/23 | 251 4/23 | 253 4/23 | $2554 / 23$ | $2574 / 23$ | $2594 / 23$ | $2614 / 23$ | -226 19/23 | -228 19/23 | -230 19/23 | -232 19/23 | -234 19/23 | -236 19/23 | -238 19/23 | -240 19/23 | -242 19/23 | -244 19/23 | -223 19/23 |  |
| 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 44 | 441 | 441 |  |


| $33020 / 23$ | -134 3/23 | -136 3/23 | -138 3/23 | -1403/2 | -142 3/23 |  | -146 3/23 | $-1483 / 23$ | -150 3/23 | -152 3/23 | -154 3/23 | 333 20/23 | 335 20/23 | 337 20/23 | 339 20/23 | $34120 / 23$ | 343 20/23 | 345 20/23 | 347 20/23 | 349 20/23 | 351 20/23 | 153 3/23 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -13 | -22 | 21 | 109 | 13120/23 | $24120 / 23$ | 263 20/23 | -130 3/23 | -23 | 20-20/23 | 108 | 132 | 24 | 26 |  |  | 22 20/23 | 23 | $13320 / 23$ | 239 20/23 | /23 | -133 3/23 |  |
|  | 236 |  |  |  | 19 20/23 |  | $13020 / 23$ | 23 |  | -113 3/23 |  | 18 20/23 |  |  |  | 23 |  | -26 3/23 |  | 23 | 23 |  |  |
| 312 20/23 | 1032 |  | 235 |  |  |  |  | 102 20/23 |  | 234 |  |  |  |  |  |  | 23 | 259 20/23 |  |  |  |  |  |
|  | -10 |  |  | 12 | 23 | 25 | -120 |  |  |  |  | 23 | 25 |  |  |  |  |  | 3 |  | 23 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 23 | 23 |  |  |
|  | 137 | 224 | 250 | -12 | -16 |  |  | 138 |  | 249 |  |  |  |  |  | 225 20/23 | 248 20/23 | -124 3/23 | -18 3/23 | 23 | 23 | 143 3/23 |  |
|  |  |  | 136 |  |  |  | -2 |  |  |  |  |  |  |  |  |  |  |  |  | 23 | 23 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $8620 / 23$ | 3 | 3 | 286 20/23 | -147 3/23 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -70 3/23 | -473/23 |  | 23 | 23 | 23 |  |
| 32 | 82 | 147 | 21 | 27 | -73 | -30 |  |  | 146 20/23 | 21 | 27 | -7 | -2 |  |  | 14 | 212 20/23 | 280 20/23 | -75 3/23 | -28 3/23 |  | -151 3/23 |  |
| -156 3/23 | -3 |  |  |  |  |  | -7 |  |  |  |  |  |  | -77 3/23 | -33 3/23 |  |  | 3 | 3 | 275 20/23 | 2 | 331 20/23 |  |
| -157 3/23 |  | -7 | -3 |  |  |  |  |  | -80 |  |  | 96 | 14 |  |  | -0 | -34 3/23 | 8 20/23 | 9720/23 | 123 | $20620 / 23$ | 332 20/23 |  |
| -159 3/23 | 15 | 20 | 27 | -8 | -37 |  |  | 159 20/23 | 20 |  | -8 | -3 |  | 93 | 16 | 20 | 269 20/23 | -82 3/23 | -39 3/23 | $720 / 23$ | 23 | 23 |  |
|  | 4 | 89 | 15 | 22 | 266 | -85 | -4 |  | 90 | 15 | 222 | 268 | -86 | -4 |  | 91 | 155 20/23 | 223 20/23 | 267 20/23 | -87 3/23 | 23 | $33620 / 23$ |  |
|  | -111 |  |  |  |  |  | $305 \text { 20/23 }$ | -10 | -65 |  |  |  |  | 30 | -110 | -66 3/23 | 43 20/23 | 65 20/23 | 175 20/23 | 197 20/23 | 307 20/23 | 338 20/23 |  |
| -165 3/23 | 19 | 30 | -93 | -6 |  | 63 | 172 | 195 | 30 | -92 | -6 | 39 | 62 | 171 | 196 | 30 | -9 | -68 3/23 | 38 20/23 | 64 20/23 | 170 20/23 | 34 |  |
| -16 | 612 | 168 | 193 | 299 | -94 | -5 | 35 | 60 | 167 | 192 | 300 | -95 | -50 | 37 | 59 | 169 | 191 | 301 20/23 | -96 3/23 | 23 | 36 20/23 | 342 20/23 |  |
| -16 | -52 3 |  |  |  |  | 298 | -99 | -53 |  | 58 | 165 | 188 |  | -98 | -54 | 55 | 57 20/23 | 164 20/23 | 190 20/23 | $29620 / 23$ | -97 3/23 | 344 20/23 |  |
| -171 3/23 | $29420 / 2$ | -100 | -57 | 52 | 74 | 161 | 187 20/2 | 293 20/2 | -101 3/23 | -56 | 51 | 75 20/2 | 163 20/23 | 186 20/2 | 29520 | -102 3/23 | -5 | 50 | 76 | 162 20/23 | 185 20/23 | 34 |  |
| -173 3/23 | 179 20/23 | 182 20/ | 2922 | -104 3/23 | -58 3/2 | 47 | 73 20/ | 180 20/23 | 184 20/23 | 29120/23 | -105 3/2 | -59 3/23 | 48 20/23 | 72 20/23 | 181 20/23 | 183 20/2 | 290 20/23 | -103 3/23 | -60 3/23 | 49 20/23 | 7120/23 | 348 20/23 |  |
| -175 3/23 | 46 20/23 | 68 20/23 | 178 20/2 | $20020 / 2$ | 287 20/2 | -106 3/2 | -62 3/2 | 45 20/23 | $6920 / 23$ | 177 20/23 | $20120 / 23$ | 289 20/23 | -107 3/23 | -63 3/23 | 44 20/23 | 70 20/23 | 176 20/2 | 202 20/2 | 288 20/ | -108 3/23 | -61 3/23 | 350 20/23 |  |
| 0/2 | 309 20/23 | 311 20/23 | 313 20/2 | 315 20/2 | 317 20/2 | 319 20/23 | $32120 / 2$ | 323 20/23 | 325 20/23 | 327 20/23 | 329 20/23 | -158 3/23 | -160 3/23 | -162 3/23 | -164 3/23 | -166 3/23 | -168 3/23 | -170 3/23 | -172 3/23 | -174 3/23 | -176 3/23 | -155 3/23 |  |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 | 2021 | 2021 | 2021 | 202 | 202 | 202 | 202 | 202 | 202 | 2021 | 202 | 202 |  |

In this case, the blocks of order 7 are pandiagonal with equal magic square sums. These sums are given by

$$
\begin{gathered}
{\left[S_{7 \times 7}: S_{23 \times 23}:=21\right]:=\frac{147}{23}} \\
{\left[S_{7 \times 7}: S_{23 \times 23}:=441\right]:=\frac{3087}{23}} \\
{\left[S_{7 \times 7}: S_{23 \times 23}:=2021\right]:=\frac{141471}{23} .}
\end{gathered}
$$

### 1.17 Block-Wise Magic Squares of Order 24

Below are block-wise magic squares of order 24 in four different ways giving magic sums $21,21^{2}$ and 2021. First, the blocks of order 3, magic squares with different magic sums, resulting in pandiagonal magic squares and semi-bimagic squares of order 24. Second, the blocks of order 4, pandiagonal magic squares with equal magic sums, resulting in pandiagonal magic squares of order 24 . Third, blocks of order 6 magic squares with equal magic sums. Forth, blocks of order 8 bimagic or semi-bimagic squares of order 8 , resulting in semi-bimagic square of order 24 .

### 1.17.1 First Type

|  | pan | 27 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -167.625 | -193.625 | -144.625 | 97.375 | 75.375 | 122.375 | 36.375 | 58.375 | 11.375 | -250.625 | -224.625 | -273.625 | $-40.625$ | -62.625 | -15.625 | 258.375 | 232.375 | 281.375 | 187.375 | 213.375 | 164.375 | -113.625 | -91.625 | -138.625 | 21 |
| 2 | -145.625 | -168.625 | -191.625 | 123.375 | 98.375 | 73.375 | 10.375 | 35.375 | 60.375 | -272.625 | -249.625 | -226.625 | -14.625 | -39.625 | -64.625 | 280.375 | 257.375 | 234.375 | 165.375 | 188.375 | 211.375 | -139.625 | -114.625 | -89.625 | 21 |
| 2 | -192.625 | -143.625 | -169.625 | 74.375 | 121.375 | 99.375 | 59.375 | 12.375 | 34.375 | -225.625 | -274.625 | -248.625 | -63.625 | -16.625 | -38.625 | 233.375 | 282.375 | 256.375 | 212.375 | 163.375 | 189.375 | -90.625 | -137.625 | -115.625 | 21 |
|  | -41.625 | -67.625 | -18.625 | 259.375 | 237.375 | 284.375 | 186.375 | 208.375 | 161.375 | -112.625 | -86.625 | -135.625 | -178.625 | -200.625 | -153.625 | 108.375 | 82.375 | 5 | 75 | 75 | 75 | -239.625 | -217.625 | 25 | 21 |
|  | -19.625 | -42 | -65 | 285.375 | 260 | 235 | 160.375 | 185.375 | 210.375 | -134.625 | -111.625 | -88.625 | -152.625 | -177.625 | -202.625 | 130.375 | 107.375 | 84 | 5 | 5 | 5 | -265.625 | -240.625 | 25 | 21 |
| 2 | -6 | -17 | -4 | 236.375 | 283 | 26 | 20 | 162.375 | 18 | -87.625 | -136.625 | -110.625 | -201.625 | -154.625 | -17 | 83.375 | 132.375 | 106.375 | 5 | 1.375 | 5 | -216.625 | -263.625 | 5 | 21 |
|  | -262.62 | -23 | -28 | 48 | 70 | 23.375 | 10 | 87 | 13 | -179.625 | -205.625 | -156.625 | -10 | -79.625 | -126.625 | 175.375 | 201.375 | 5 | 5 | 220.375 | 269.375 | 5 | 25 | 25 | 21 |
| 21 | -284 | -26 | -238 | 22 | 47.375 | 72.375 | 13 | 11 | 85 | -157.625 | -180.625 | -203.625 | -127.625 | -10 |  | 153.375 | 176.375 | 199.375 | 268.375 | 5 | 222.375 | -2.625 | 5 | -52.625 | 21 |
| 21 | -237.6 | -286.62 | -260.6 | 71. | 24 | 46.375 | 86.375 | 13 | 111.375 | -204.625 | -155.625 | -18 | -7 | -125.625 | -103.625 | 200 | 15 | 177.375 | 221.375 | 270.375 | 244.375 | -51.625 | -4.625 | -26.625 | 21 |
| 21 | -100 | -74 | -12 | 174 | 196 | 14 | 24 | 225.375 | 27 | -2 | -5 | -6.625 | -251.625 | -229.625 | -2 | 37.375 | 63.375 | 14.375 | 120.375 | 94.375 | 143.375 | 25 | -212.625 | 25 | 21 |
| 21 | -122. | -99 | -76 | 148 | 173 | 19 | 27 | 248.375 | 22 | -7 | -3 | -5 | -277.625 | -252.625 | -2 | 15.375 | 38.375 | 61.375 | 142.375 | 119.375 | 96.375 | 5 | 25 | 5 | 21 |
| 21 | -75.62 | -124 | -98. | 197 | 150.3 | 172 | 224.37 | 271.375 | 249.37 | -5 | -5 | -3 | -228.625 | -275.625 | -253 | 62.375 | 13.375 | 39 | 5 | 144.375 | 118.375 | -213.625 | -166.625 | 55 | 21 |
| 21 | 42.3 | 64. | 17. | -256.625 | -230.62 | -279 | -185.62 | -211.625 | -162 | 115 | 93.3 | 140 | 169 | 195.375 | 146.375 | -95.625 | -73.625 | -120.625 | -34.625 | -5 | -9.625 | 5 | 226.375 | 275.375 | 21 |
| 21 | 16.37 | 41.3 | 66.375 | -278.625 | -255.62 | -232.625 | -163.625 | -186.625 | -209.62 | 141.375 | 116 | 91. | 147 | 170.375 | 193.375 | -12 | -96 | -71.62 | 25 | -33.625 | -58.625 | 274.375 | 251.375 | 5 | 21 |
| 21 | 65.37 | 18.375 | 40.375 | -231.625 | -280.625 | -254.625 | -210.625 | -161.625 | -187.625 | 92.375 | 139.375 | 117.375 | 194.375 | 145.375 | 171.375 | -72.625 | -119.625 | -97.625 | -57.625 | -10.625 | -32.625 | 227.375 | 276.375 | 250.375 | 21 |
| 21 | 180.375 | 202.375 | 155.375 | -106.625 | -80.625 | -129.625 | -23.625 | -49.625 | -0.625 | 241.375 | 219.375 | 266.375 | 43.375 | 69.375 | 20.375 | -257.625 | -235.625 | -282.625 | -184.625 | -206.625 | -159.625 | 114.375 | 88.375 | 137.375 | 21 |
| 21 | 154.375 | 179.375 | 204.375 | -128.625 | -105.625 | -82.625 | -1.625 | -24.625 | -47.625 | 267.375 | 242.375 | 217.375 | 21.375 | 44.375 | 67.375 | -283.625 | -258.625 | -233.625 | -158.625 | -183.625 | -208.625 | 136.375 | 113.375 | 90.375 | 21 |
| 21 | 203.375 | 156.375 | 178.375 | -81.625 | -130.625 | -104.625 | -48.625 | 0.375 | -25.625 | 218.375 | 265.375 | 243.375 | 68.375 | 19.375 | 45.375 | -234.625 | -281.625 | -259.625 | -207.625 | -160.625 | -182.625 | 89.375 | 138.375 | 112.375 | 21 |
| 21. | 103.375 | 81.375 | 128.375 | -173.625 | -199.625 | -150.625 | -244.625 | -218.625 | -267.625 | 30.375 | 52.375 | 5.375 | 264.375 | 238.375 | 287.375 | -46.625 | -68.625 | -21.625 | -107.625 | -85.625 | -132.625 | 181.375 | 207.375 | 158.375 | 21 |
| 27 | 129.375 | 104.375 | 79.375 | -151.625 | -174.625 | -197.625 | -266.625 | -243.625 | -220.62 | 4.375 | 29.375 | 54.375 | 286.375 | 263.375 | 240.375 | -20.625 | -45.625 | -70.625 | -133.625 | -108.625 | -83.625 | 159.375 | 182.375 | 205.375 | 21 |
| 27 | 80.375 | 127.375 | 105.375 | -198.625 | -149.625 | -175.625 | -219.625 | -268.625 | -242.625 | 53.375 | 6.375 | 28.375 | 239.375 | 288.375 | 262.375 | -69.625 | -22.625 | -44.625 | -84.625 | -131.625 | -109.625 | 206.375 | 157.375 | 183.375 | 21 |
| 21. | 253.375 | 231.375 | 278.375 | -35.625 | -61.625 | -12.625 | -118.625 | -92.625 | -141.625 | 192.375 | 214.375 | 167.375 | 102.375 | 76.375 | 125.375 | -172.625 | -194.625 | -147.625 | -245.625 | -223.625 | -270.625 | 31.375 | 57.375 | 8.375 | 21 |
| 27 | 279.375 | 254.375 | 229.375 | -13.625 | -36.625 | -59.625 | -140.625 | -117.625 | -94.625 | 166.375 | 191.375 | 216.375 | 124.375 | 101.375 | 78.375 | -146.625 | -171.625 | -196.625 | -271.625 | -246.625 | -221.625 | 9.375 | 32.375 | 55.375 | 21 |
|  | 230.375 | 277.375 | 255.375 | -60.625 | -11.625 | -37.625 | -93.625 | -142.625 | -116.625 | 215.375 | 168.375 | 190.375 | 77.375 | 126.375 | 100.375 | -195.625 | -148.625 | -170.625 | -222.625 | -269.625 | -247.625 | 56.375 | 7.375 | 33.375 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| 4 | -150.12 | -17 | -127 | 114 | 92.875 | 13 | 53.875 | 75.875 | 8.87 | -23 | -2010 | -2 | -23.125 | -45.125 | 75 | 5.875 | 9.875 | 8.8 | 4.875 | 230.875 | 81.875 | -96.125 | -7.125 | -121. |  |
| 4 | -128.12 | -151 | -17 | 140 | 115.875 | 75 |  | 52.87 | 75 | -25 | -232.125 |  |  | 125 |  | 297.875 | 875 | 875 | 5 | . 875 | 228.875 | 122.125 | 97.125 | 2.12 |  |
|  | -175.12 | -12 | -152 |  | 138.875 | 116.875 |  | 29.875 |  | -20 |  | 125 |  |  |  | 250.875 | 875 | . 875 | 229.875 | 180.875 | 206.875 | 73.125 | -120.125 | -98.125 |  |
| 4 |  | -5 |  | 276.875 | 25 | 301.875 | 203.875 | 225.875 | 178 | 125 |  | -118.125 |  | -183.12 | -136.125 | 5 | 99.875 | \% 87 | 42.8 | 875 | 875 | -222.125 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | -160.125 |  | 147.875 | 124.875 | 101.875 |  | 43.875 | 66.875 | -248.125 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | -137.125 |  | 100.875 | 149.875 | 123.875 |  |  | 44.875 | -199.125 | -246.125 |  |  |
|  |  |  |  |  |  | 40.875 | 12 | 104.875 | 151.875 | -162 | -1881 | -13 |  | -62.125 | -109.125 | 192.875 | 75 | 169.875 | 26 | 237.8 | 286 |  | -33.125 | 13.875 |  |
|  |  |  |  |  |  |  |  |  |  | -14 |  |  | -11 | -85.125 |  | 170.875 | 193.875 | 216.875 | 5 | 5 | 239.875 | 14.875 | 10.125 | -35.125 |  |
|  | -22 | -26 | -24 | 88 |  |  | 875 | 150.875 | 128.875 | -18 | -138.125 |  | 125 | -108.125 |  | 875 | 168.875 | 4.875 | 238.875 | . 875 | 261.875 | -34.125 | 12.875 | -9.125 |  |
|  | -83 | -57 | -10 |  | 213.875 | 166.875 | 26 | 24 | 28 | -12.125 |  | 75 | -23 | -212.125 |  | 54.875 | 80.875 | 1.875 | 137.875 | 875 | 60.875 | -173.125 | -195.125 | -148.125 |  |
|  | -10 | -82 | -59 | 165.875 | 190.875 | 21 | 29 | 26 | 240.875 | 9.875 |  | -36.125 | -26 | -23 | -20 | 32.875 | 55.875 | 78.875 | 159.875 | 136.875 | 113.875 | -147.125 | -172.125 | -197.125 |  |
|  |  |  | -8 |  |  |  | 241.875 |  |  | -37.125 |  |  |  |  |  |  | 30.875 |  |  | 1.875 | 135.875 | -196.125 | -149.125 |  |  |
|  |  |  |  |  |  |  |  |  |  | 132.875 |  |  |  | 212.875 |  |  | -56.125 | 5 |  |  |  | 269.875 | 243.875 |  |  |
|  |  |  |  |  |  |  |  |  |  | 158 |  |  | 16 | 187.875 |  |  | -79.125 | -54.125 |  |  | -41.125 | 291.875 | 268.875 |  |  |
|  |  |  | 57.875 |  |  |  | -193 |  |  | 109.875 | 156.875 | 134.875 | 211.875 | 162.875 | 188.875 | -55.125 | -102.125 | -80.125 | $-40.125$ | 6.875 | -15.125 | 244.875 | 293.875 | 267.875 |  |
|  |  | 21 | 172.875 |  |  |  |  |  |  |  | 236.875 | 28 |  |  |  | -240.125 | -218.125 | -265.125 | -1 | -189.125 | -142.125 | 131.875 | 105.875 | 154.875 |  |
|  |  |  | 221.875 |  |  |  |  |  |  | 28 | 259.875 | 23 | 38.875 |  |  | -2 | -241.125 | 5 | - |  | 1.125 | 153.875 | 3.8 | 107.875 |  |
|  | 22 | 17 | 195.875 |  |  |  |  |  |  | 23 |  |  | 85.875 | 36.875 | 62.875 | -21 | -264.125 | -2 | -19 | -143.125 | -165.12 | 106.8 | 55.8 | 129.875 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | 255.875 |  |  |  |  |  | -68.125 | -115.125 | 198.875 | 224.875 |  |  |
|  |  | 121.87 | 96.875 |  | -15 |  |  |  |  |  |  |  | 30 | 28 | 257.875 |  | -28.125 | .125 | -116.125 | -91.125 | 25 | 176.8 | 9.875 |  |  |
|  | 97.875 | 14 | 12 | -18 | -132 | -15 | -2021 | -251.125 | -22 | 70 | 23.875 | 45.875 | 256.875 | 305.875 | 279.875 | -52.125 | -5.125 | -27.125 | -67.125 | -114.125 | -92.125 | 223.8 | 174.875 | 200.875 |  |
| 44 | 270 | 248.875 | 295.87 | -18120 |  | 4.875 | -101.125 | -75.125 |  | 209.875 | 231.875 | 184.875 | 11 | 75 | 142.875 | -155.12 | -177.125 | -130.125 | -22 | -206.125 | -253.12 | 48.8 | 74.875 | 25.875 |  |
| 441 | 296.875 | 271.875 | 246.87 | 3.875 | -19.125 | -42.125 | -123.125 | -100.12 | -77.125 | 183.875 | 208.8 | 233.875 | 141.875 | 118.87 | 5.875 | -129.125 | -154.12 | -179.1 | -254. | -229.1 | -204.12 | 26.8 | 49.875 | 72.8 |  |
|  | 247.87 | 294.875 | 272.875 | -43.125 | 5.875 | -20.125 | -76.125 | -125.12 | -99.125 | 232.875 | 185.875 | 207.875 | 94.875 | 143.875 | 117.875 | -178.125 | -131.125 | -153.125 | -205.125 | -252.125 | -230.125 | 73.875 | 24.875 | 50.875 |  |
|  | 441 |  |  |  | 41 | 41 | 41 | 41 | 41 | 441 | 441 | 441 | 441 | 441 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 441 |  |


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|  | -84 7/24 | -110 7/24 | -617/24 | 180 17/24 | 15817/24 | 20517/24 | 11917/24 | 14117/24 | 9417/24 | -167 7 | -1417/2 | $1907 / 24$ | $4217 / 24$ | 20 | 71/24 | 34117/24 | 31517/24 | 36417/24 | 270 17/24 | 29617/24 | $24717 / 24$ | $307 / 24$ | $87 / 24$ | -55 7/24 |  |
|  |  |  |  | 206 |  | 124 | 93 | /24 | 124 |  | -166 7/24 |  | 68 |  | 1817/24 | 363 | 340 17/24 | 17/24 | 24817/24 | 17/24 | 29417/24 | 7/24 | -317/24 |  |  |
|  |  | -60 |  | 157 | 20417/2 | 182 | 14217/24 |  | 124 | -142 | -1917/24 |  |  |  | 4417/24 | 124 | 24 | $33917 / 24$ | 295 | 4 | 27217/24 | -7 7/24 | -547/24 | -32 7/24 |  |
|  |  |  |  | 342 | 320 | 367 | 269 | 29117/24 | 24417/24 |  |  | -52 7/24 | -95 7/24 | -117 7/24 | -70 7/24 | 19117/24 | 12 | 21417/24 | 10817/24 | 24 | 517/24 | -156 7/24 |  |  |  |
|  |  | 4017/24 |  | 368 | 343 | 318 | 243 | 268 | 293 | -51 7/24 | -287/2 | -57/24 | -69 7/24 | -947/24 | -119 7/24 | 21317/24 | 19017/24 | 16717/24 | 24 | 17/24 | 17/24 | -182 7\| | -157 7/24 |  |  |
|  |  |  | 3917/24 | 319 | 366 | 344 | 292 | 24517/24 | 267 | -47/24 | -53 7/24 | -27 | -118 | -717/24 | -93 7/24 | 24 | 21517/24 | 18917/24 | 133 17/24 | 8417/24 | 110 17/24 | 133 | -180 712 |  |  |
|  | -179 | -153 7/24 | 2027124 | 131 | 153 | 10617/24 | 192 | 170 $17 / 24$ | $21717 / 24$ | -96 | -122 7/24 | -73 7/24 | -18 | $317 / 24$ | 7/24 | 25 | 28417/2 | 23517/24 | $32917 \mid$ | $30317 / 24$ | 35217/24 | 17/24 | 3277/24 | 7917/24 |  |
|  |  |  |  | 105 | 130 17/24 | 15517/24 | 218 | 19317/24 | 168817/24 |  | -97 7/24 | -120 7/24 | -447/24 |  |  | 23617/24 | 25917/24 | 28217/24 | $35177 / 24$ | $32817 / 24$ | 17/24 | $8017 / 24$ | 124 | $3017 / 24$ |  |
|  |  |  |  | 154 | 10717/24 | 129 | 169 | 21617/24 | 194171/2 | -1217/24 | -72 7/24 |  |  | -42 7/24 | -20 7/24 | 283 | 17/2 | 26017/24 | 304 | 35317/24 | $32717 / 24$ | 3117/24 | 7817/24 | 17/24 |  |
|  |  |  |  | 257 | 279 | 232 | 330 | $30817 / 24$ | 355 |  |  |  |  |  |  | 12017/24 | 24 | 171/24 | 20317/24 | 1724 | 17/24 | -1077 |  |  |  |
|  |  |  |  | 231 | 256 | 28117/24 | 356 | $33117 / 24$ | 30 |  | 5217/24 | $2917 / 24$ | -19471 |  |  | 9817/24 | 12117/24 | 14417/24 | 22517/24 | 4 | 17/2 | -817/24 |  |  |  |
|  |  |  | -15 | 280 | 233 | 25 | 307 | 354 | 332 |  |  |  | -14 |  |  | 14517/24 | 17/24 | 12217/24 | 1781712 | 17/24 | 20117 | -130 7 | -83 |  |  |
|  |  |  | 100 | -173 |  |  |  | -1287/2 | 1/24 | 198 | 17 | 17/2 | 25217/24 | 278 | 22917/24 | -12 |  | -377/24 | $4817 / 24$ | 2617/24 | $7317 / 24$ | $33517 \mid$ | 30917 | $35817 / 24$ |  |
|  |  |  | 149 | -195 7 | -172 7/2 | -149 7/24 | -80 |  |  | 224 | 19917 | 17417/24 | 23017\|20 | 253 | 27617/24 | -38 | -13 | 1117/24 | 24 | $4917 / 24$ | 2417/2 | $35717 / 24$ | 33417 | 31117/24 |  |
|  |  |  | 12317/24 | -148 |  |  | -127 |  |  | 175 | 222 | 20 | $27717 / 24$ | 22811 | 254 | 1017/24 | -367\| | -14 | 2517/24 | 7271 | $5017 / 2$ | 310 17/24 | 35917 | 3317 |  |
|  | 263 |  | 238 |  |  | -467/24 |  |  |  |  | 302 | 34917/24 | 12617/24 | 152 | 10317/24 | -174 | -152 7/24 | -199 7/24 | -101 | -123 7/24 | -7671 | 2 | 171 | 22017/24 |  |
|  |  |  |  | -45 7/24 |  |  |  |  |  |  | 32517/24 | $30017 / 24$ | 10417/24 |  | 150 17/24 | -20 |  | -150 7/24 | -75 7/24 | -100 7/24 | -125 7/24 | 21917/24 | 19617/24 | 17317/24 |  |
|  |  | 239 |  |  |  |  |  |  |  |  | 34817/24 | 32617/24 | 15117/24 | 10217/24 | 128 17/24 | -15 | -198 7/24 | -176 7/24 | -124 7/24 | 4 | -997/24 | 17217/24 | 2211 | 19517/24 |  |
|  |  |  |  |  |  |  |  |  |  |  | 135 17/24 |  |  |  | 370 17/24 |  |  |  |  | -2 7/24 | -497/24 | 264 | 29 |  |  |
|  | 21217/2 |  | 162 | -68 | -91 |  |  |  | -137 7/24 | 87 | 112 | 137 | 36917/2 | 346 | 323 | $6217 / 24$ | 3717/24 | 1217/24 | -50 7/24 | -25 7 | - 7/24 | $24217 \mid$ | 26511 |  |  |
|  | 163 | 210 | 188 17/24 | -115 7/2 | -667/2 | -92 71/ | $-1367$ | 1857 | -159 | 136 | 89 | 17/24 | 322 17/24 | 371 | 34 | 1317/24 | $6017 / 24$ | 3817 | -17/24 | -487/24 | $-267 / 24$ | 28917 | 24017 | $26617 /$ |  |
|  | 336 | 314 |  |  |  |  |  |  | -58 7/24 | 27517 | $29717 / 24$ | 250 17/24 | 185517/24 | $15917 \mid$ | 20817/2 | -897/24 | -1117/24 | -647/24 | 1627 | -140 7/24 | -187712 | 11417/24 | $14017 /$ | 11712 |  |
| 2021 | 36217/24 | 337 | 31217/24 | $6917 / 24$ | 124 | 124 | -57 7/24 | -34 7/24 | 7/24 | 249 | 27417/24 | 29917/24 | $20717 / 24$ | 18417/20 | 17/24 | -63 7/24 | -887/24 | -113 7/24 | -1887 | 7 | -138 7/24 | 9217/24 | 11517/24 | 17 | 202 |
|  | 31317/24 | $36017 / 2$ | 338 17/2 | $2217 / 24$ | 7117/24 | 4517/24 | -10 7/24 | -59 7/24 | -33 7/24 | 29817/24 | 25117/24 | 27317/24 | $16017 / 24$ | 20917/24 | 18317/24 | -112 7/24 | -65 7/24 | -87 7/24 | -139 7 | 24 | -647/24 | 13917/24 | 9017/24 | 1617/24 | 2021 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

In this case, the block of order 3 are magic squares with different magic sums. Moreover, the magic squares of order 24 are semi-bimagic square with semi-bimagic sums given by

$$
\begin{aligned}
{\left[S m b_{24 \times 24}: S_{24 \times 24}:=21\right]:=663568.375 } \\
{\left[S m b_{24 \times 24}: S_{24 \times 24}:=441\right]:=671653.375 } \\
{\left[S m b_{24 \times 24}: S_{24 \times 24}:=2021\right]:=\frac{20009641}{24} }
\end{aligned}
$$

### 1.17.2 Second Type

|  | pan |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  | -70.625 | 144.3 | -286.62 | 216.375 | -69.625 | 143.375 | -285.625 | 215.375 | -68.625 | 142.375 | -284.625 | 214.375 | -67.625 | 141.375 | -283.625 | 213.375 | -66.625 | 140.375 | -282.625 | 212.375 | -65.625 | 39.375 | 281.625 | 211.375 |  |
|  | -215.625 | 145.375 | 0.375 | 73.375 | -216.625 | 146.375 | -0.625 | 74.375 | -217.625 | 14 |  | 75.375 | -21 | 148.375 | -2.625 | 75 | -21 | 149.375 | 3.625 | 375 | -220.625 | 75 | -4.625 | 78.375 |  |
|  | 288.375 | -21 | 72.375 | -142 | 287.375 | -213.625 | 71.375 | 25 | 286.375 | -2 | 70.375 | -140.625 | 28 | -211.625 | 375 | 5 | 28 | -210.625 | 375 | 3.625 | . 375 | -209.625 | 75 | -137.625 |  |
|  | 1.375 | -71.625 | 217.375 | -143.625 | 2.375 | -72.625 | 218.375 | -144.625 | 75 | -73.625 | 375 | -145.625 | 75 | 25 | 220.375 | -146.625 | 375 | 625 | . 375 | -147.625 | 375 | 6.625 | 222.3 | -148.625 |  |
|  | -64.625 | 375 | -28 | 210.375 | -63.625 | 375 | -27 | 209.375 | -62.625 | 136.375 | -278.625 | 208.375 | -61.625 | 375 | . 625 | 7375 | -6 | 375 | 6.625 | 206.37 | -59.62 | 33.375 | 275.625 | 205.37 |  |
|  | -221.62 | 151.375 | -5.625 |  | -22 | 15 |  | 75 | -22 | 153.375 |  |  | -224.625 | 15 | -8.625 | 375 | -225.625 | 375 | . 625 | 375 | -226.6 | 156.375 | -10.625 | 84.375 |  |
|  | 282.37 | -20 | 66 | -13 | 281 | -207.625 | 65.375 | -13 | 280.375 | -20 | 75 | -134.625 | 27 | -20 | 63.375 | 3.625 | 278 | -204.625 | 62.375 | -132.62 | 27.37 | 203.6 | 61.375 | -131.62 |  |
|  |  |  | 22 |  |  |  | 22 |  | 9.375 |  | 22 |  |  | -80.625 | 22 | -152.625 |  | -81.625 | 227.375 | -153.625 | 12.375 | 25 | 228.3 |  |  |
|  |  | 132.375 | -27 | 204.375 |  |  | -27 | 20 |  | 130.375 | -272.625 | 202 |  | 129.375 | -271.625 |  |  | 375 | 5 | 200.375 | 25 | 5 | 5 |  | 1 |
|  | -227 |  |  |  | -22 |  |  |  |  |  |  |  | -230.625 |  |  | 88.375 | -231.625 | 161.375 | -15.625 | 75 | -232.625 | 162.375 | -16.625 | 90.375 |  |
|  | 276.375 | -202 | 60.375 | -13 | 275.375 |  |  |  |  | -200.625 | 75 | -128.625 | 27 |  | 375 | -127.625 | 272.375 | -198.625 | 375 | -126.625 | 271.375 | -197.625 | 5.375 | -125.625 |  |
|  | 13.375 | -83.625 | 22 | -15 | 14.375 | -84.625 | 230.375 | -156 | 15.375 | -85.625 | 375 | -157.625 | 16.375 | -86.625 | 232.375 | -158.625 | 17.375 | 625 | 23.375 | -159.625 | 375 | 8.625 | 234.3 | -160.625 |  |
|  | -52 | 126.375 | -26 | 198.375 | -5 | 125.375 | -267 | 375 | -50.625 | 124.375 | -266.625 | 375 | -49.625 | 123.375 | -265.625 | 375 | -48.625 | 375 | 5 | . 37 | -47.625 | 1.375 | -263.625 | 193.375 |  |
|  | -233.62 | 163.375 |  |  | -23 | 164.375 | -18.625 |  | -235.625 | 165.375 | -19.625 | 93.375 | -23 | 166.375 | -20.62 | 94.375 | -237 | 167.375 | -21.625 | 375 | -238.625 | 168.375 | -22.6 | 96.375 |  |
|  | 270 | -196.62 |  |  |  | -195.625 |  |  | 26 |  |  |  |  | -193.625 | 51.375 | -121.625 | 26.375 | -192.625 | 50.3 | -120.625 | 265.375 | -191.625 | 49.375 | -119.625 |  |
|  |  |  | 235.37 |  |  |  |  |  |  |  | 237.375 | -163 | 2375 | 625 | 238.375 | -164.625 | 23.37 | -93.625 | 239.375 | -165.625 | 24.375 | -94.625 | 240.3 | -166.625 |  |
| 2 |  |  | -262 | 192.375 |  |  |  |  |  |  | -260.625 | 375 |  | 117.375 | -259.625 | 375 | -4 | 375 | -258.625 | 188.37 | -41.625 | 5.375 | 257.6 | 187.375 | 1 |
| 2, | -239 | 169 | -2 | 97.375 | -24 | 5 |  | 98.375 | -2 | 171.375 | -25.625 | 99.375 | -242 | 17 | -26.625 | 375 | -24 | 173.375 | -27.625 | 375 | -244.62 | 174.375 | -28.6 | 102.375 |  |
| 2 | 264.375 | -190 | 48.375 | -118.625 | 263.375 | -189.625 | 75 | -117.625 | 26 | -188.625 | 375 | -116.625 | 261.375 | -187.625 | 375 | -115.625 | 260.375 | -186.625 | . 37 | -114.625 | 99.37 | -185.6 | 3.37 | -113.625 | 1 |
| 2 | 25.375 | -95.625 | 24 | -167 | 26.375 | -96.625 | 242.375 | -168 | 27.375 | -97.625 | 243.375 | -169.625 | 28.375 | 625 | 244.375 | -170.625 | 375 | . 625 | 5.375 | 1.62 | 30.375 | -100.625 | 246.3 | -172.62 |  |
| 2 | -40, | 375 | -256.62 | 186.375 | -39.625 | 375 | -255.625 | . 375 | -38 | 112.375 | -254.625 | 375 | -37 | 375 | -253.625 | 3 375 | -36.625 | . 375 | -252.625 | 182.37 | -35.625 | 09.375 | -251.625 | 81.37 | 21 |
| 2 | -245.62 | 175.375 | -29.62 | 103.375 | -246.62 | 176.375 | -30.625 | 375 | -247 | 177.375 | -31.625 | . 375 | -248.625 | 178.375 | -32.625 | . 375 | -249.625 | 179.375 | -33.625 | 107.37 | -250.6 | 180.375 | -34.625 | 108.3 | 21 |
| 21 | 258.375 | -184.62 | 42.375 | -112.62 | 257.375 | -183.62 | 375 | . 62 | 256.375 | -182.6 | 40.375 | -110.62 | 255.37 | -181.62 | 39.375 | -109.625 | 254.375 | -180.62 | 38.375 | -108.625 | 253.375 | -179.625 | 37.375 | -107. |  |
|  | 31.375 | -101.625 | 247.375 | -173.62 | 32.375 | -102.625 | 248.375 | -174.62 | 33.375 | -103.625 | 249.375 | -175.625 | 34.375 | -104.625 | 250.375 | -176.62 | 35.375 | -105.625 | 251.375 | -177.62 | 36.375 | -106.625 | 252.375 | -178.625 | 2 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 21 |  |  |  |  |  |  |  |  |  |


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | -53 | 161.875 | -269.12 | 233 | -52 | 160.875 | -26 | 23 | -51 | 159.875 | -267.12 | 23 | -50.125 | 158.875 | -266.125 | 230 | -49.125 | . 87 | 265.125 |  | 8.125 | 6.875 | 4.12 | 228.875 |  |
|  | -1981 | 162.875 | 17.875 | 90.875 | -199.125 | 163.875 | 16.875 | 91.875 | -20 | 164 | 15.875 | 92.875 | -201.125 | 75 | 14.875 | 75 | -202.125 | 166.875 | 3.875 | 75 | 5 | 167.875 | 12.875 | 95.875 |  |
|  | 30 | -19 | 89.875 | -125.125 | 30 | -196.125 | 88.875 | -124.125 | 30 | -195.125 | 87.875 | -123.125 | 30 | -194.125 | 86.875 | -122.125 | 301.875 | -193.125 | 85.875 | -121.125 | 875 | -192.125 | . 87 | 120.125 |  |
|  | 18.875 | -54.125 | 234.87 |  | 19.875 | 25 | 75 |  | 20.875 | 25 | 236.875 | -128.125 |  | 125 | 237.875 | -129.125 | 75 | . 125 | 238.875 | -130.125 | . 875 | 9.125 | .87 | -131.125 |  |
|  |  | 155 | -263.12 | 227.875 |  | 154.875 | -262.125 | 875 |  | 875 | -261.125 | 225.875 |  | 875 |  | 22 | -43.125 | 151.875 | -259.125 | 22 | 2.125 | 50.875 | -258.125 | 222.875 |  |
|  |  |  |  |  | -20 |  |  |  |  |  |  |  | -207.125 | 171.875 |  |  | -208.125 | 172.875 |  | 100.875 | 9.12 | 173.875 | 6.875 |  |  |
|  |  |  | 83.875 |  | 298 |  | 82.875 | -118.125 | 29 |  |  |  | 29 | -188.125 | 80.875 |  | 295.875 |  | 79.875 | -115.125 | 294.875 | -186.125 | 78.87 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 29.875 | -65.125 | 245.875 |  |  |
|  |  |  |  |  |  |  |  |  |  |  | -255.125 |  |  | 146.875 | -254.125 |  |  | 145.875 | -253.125 | 217.875 | -36.125 | 144.875 | -252.125 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  | -2 | 177.875 | 2.875 |  | -214.125 | 178.875 | 1.875 | 106.875 | 5 | 179.875 | 0.875 |  |  |
|  | 29 |  |  |  |  |  | 76 |  |  | -181 | 75.875 |  | 29 | -182.125 | 74.875 |  | 28 | 5 | 875 | -109.125 | 288.875 | 5 | 2.875 |  |  |
|  |  |  | 246.87 | -138.125 |  |  | 247.875 |  |  | 25 | 248.875 |  | 33.875 | -69.125 | 249.875 |  | 34.875 | -70.125 | 250.875 | -142.125 | 35.87 | -71.125 | 1.8 |  |  |
|  |  |  |  |  |  |  |  |  |  |  | -249.125 | 213.875 | -32 | 140.875 | -248.125 | 21 | -31.125 | 139.875 | -247.125 | 211.875 | -30.125 | 138.875 | -246.125 | 210.875 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | 183.875 | -3.125 |  |  | 184.875 | -4.125 | 12.875 | 5 | 185.875 | -5.125 |  |  |
|  |  |  |  |  |  |  | 70 |  | 28 |  |  |  | 28 | -176.125 |  |  | 28 | -175.125 | 67.875 | -103.125 | 282.875 | -174.125 | 6.8 |  |  |
|  |  |  | 25 |  |  |  | 253 |  |  |  | 25 |  | 39 | 125 | 25 |  | 40. | 125 | 256.875 |  | 4.87 | -77.125 | 257.8 |  |  |
|  |  |  |  |  |  |  |  | 208.875 |  | 135.875 | -243.125 | $20$ |  | 134.875 | -242.125 | 206.875 | -25.125 | 133.875 | -241.125 | 205.875 | -24.125 | 132.875 | -240.125 | 204.875 |  |
|  |  | 186.875 |  |  | -2 |  |  | 115.875 |  | 188.875 |  | 116.875 | -225.125 | 189.875 | -9.125 | 117.875 |  | 190.875 | -10.125 | 118.875 |  | \% 81 | 11.125 | 119.875 |  |
|  | 28 | -17 | 65.875 | -101.125 | 28 | -172 | 64.875 |  | 27 | -1 | 63.875 | -99.125 | 27 | -170.125 | 62.875 | 5 | 27 | -169.125 | 61.875 | , 25 | 6.875 | 8.1 | 0.8 | -96.125 |  |
|  | 42.875 | -7 | 258.87 | -150.125 | 43 | -79.125 | 259.87 | -151.125 | 44.875 | -80.125 | 26 | -152 | 45.875 | -81.125 | 261.875 | -15 | 46.875 | 2.12 | 262.8 | -154.125 | 47.875 | 83.12 | 263.8 | -155. |  |
|  |  |  | -239.125 | 203 | -22 |  | -23 | 20 | -21.125 | 129.875 | -2 | 20 | -2 | 128.875 | -236.125 | 20 | -19.125 | 127.875 | -235.125 | 199.875 | -18.12 | 126.875 | -234. | 198.875 |  |
|  | -22 | 192 | -12.125 | 120.875 | -22 |  | -13.125 |  | -23 | 194.875 | -14.125 |  | -23 | 195 | -15.125 | 123.8 | -232 | 196.87 | -16.125 | 124.87 | -233.125 | 7.8 | -17.12 | 125.875 |  |
| 44 | 275.87 | -16 | 875 | -95.125 | 27 | -16 | 875 |  | 273.8 | -16 | 57.875 | -93.125 | 272.87 | -16 | 56.875 | -92.125 | 271.875 | -163.12 | 55.875 | , 25 | 270.875 | -162.1 | 54.87 | -90.125 |  |
|  | 48.875 | -84.125 | 264.87 | -156.12 | 49.875 | -85.125 | 265.875 | -157.125 | 50.875 | -86.125 | 266.875 | -158.12 | 51.875 | -87.125 | 267.875 | -159.125 | 52.875 | -88.125 | 268.87 | -160.12 | 53.875 | -89.125 | 269.875 | -161.12 | 441 |
|  | 441 |  |  |  |  |  |  | 4 | 4 |  | 4 | 4 |  | 4 | 4 | 4 |  |  | 44 |  | 44 | 44 | 44 |  |  |


|  | pan | 2021 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | 1217/24 | 22717/24 | -203 7\| | 29917/24 | $1317 / 24$ | 22617/24 | -2027/24 | 29817/24 | 1417/24 | 22517/24 | -2017/24 | $29717 / 24$ | 1517/24 | 22417/24 | $2007 / 24$ | 29617/24 | 1617/24 | 22317/24 | -1997/24 | 29517/24 | 1717/24 | 22217/24 | 1988724 | 29417/24 |  |
|  | -132 | 228 | 8317/24 | 15617/24 | -133 7/24 | 22 | 8217/24 | 15717/24 | -134 | 23 | 24 | 124 | -135 | 23117/24 | 124 | 124 | 136 | 23217/24 | 917/24 | 17/24 | -137 7/24 | $23317 / 24$ | $817 / 24$ | 16171/24 |  |
|  | 371 | -1317/24 | 15517/24 |  | 370 | -130 7/24 | 15417/24 | -587/24 | 36917/24 | -129 7/24 | 15317/24 | 124 | 36817/24 | -128 7/24 | 152 | 24 | 36717/24 | -127 7/24 | 15117/24 | 7/24 | 36617\| | -126 7/24 | 15017/24 | 54 7/24 |  |
|  | 8417/24 |  | 300 |  |  | 124 | 30117/24 |  | 8617/24 |  | 30217/24 | -62 7/24 |  | /24 | 303 | -63 7/24 | $8817 / 24$ | /24 | 17/24 | -64 7/24 | 8917/24 | 17/24 | 30517/24 | -65 7/24 |  |
|  |  | 22117/24 |  | 29317/24 | 1917/24 | 22017/2 | -19 | 29217/24 | 20 | 219 | -195 7/24 | 124 | 2117/24 | 218 | -1947/24 | $29017 / 24$ | 2217 | 17/2 | 4 | 17/24 | $2317 / 24$ | 21617/24 |  |  |  |
|  |  | 234 |  | 162217/24 | -139712 | 23517\|20 | /24 | 124 | -14 | 236 | 7517/24 | 7/24 | -1417/24 | $23717 / 24$ | 17/24 | 16517/24 | -142 7/24 | 238 17/24 | 7317/24 | 17/24 | 1437 | $23917 / 24$ | 7217/24 | 16717/24 |  |
|  | 365 |  |  |  | 364 |  |  | -52 | $36317 / 24$ | -123 | 14717/24 | 7/24 | 362 | -122 7/24 | 7/24 |  | 361 | -1217/24 | 24 | 124 | $36017 / 24$ | . 120 7/24 | 14417/24 | -487/24 |  |
|  |  |  | 306 |  | 91 | 417/24 | 307 |  | 9217/24 | 24 | 308 | -687/24 |  | /24 | 30917/24 | -69 7/24 | 9417/24 | /24 | 17/24 | -70 7/24 | 9517/2 | 124 | 31117/24 | -717/24 |  |
|  |  | 215 | -19 | 287 | 251 | 21417/24 | -190 7 | 286 | 261 | 21317/24 | -18 | 285 17/24 | 27 | 21217/2 | -188 | 284 | $2817 / 24$ | 17/24 | 2 | 28317/24 | 2917/24 | 210 17/24 | -186 7/24 | $28217 / 24$ |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | /24 | 17/24 | -148 7/24 | 24417/24 | $6717 / 24$ | 17217/24 | -149 7/24 | 24517/24 | $6617 / 24$ | 17317 |  |
|  |  |  |  |  |  | -118 7/24 |  |  |  |  |  |  | 35617/24 |  | 140 17/24 | -44 7/24 | $35517 / 24$ | -115 7/24 | 4 | -43 7/24 | 35417/24 | 4 | 13817/24 | -427/24 |  |
|  |  |  |  |  |  |  |  |  |  |  | 31417/24 |  |  |  | 315 | -75 7/24 | 100 17/24 | $-47 / 24$ | 4 | -767/24 | 10117/24 | -57/24 | 31717 |  |  |
|  |  | $20917 / 24$ |  |  | 3117/24 | 20 |  | $28017 / 24$ | 32 | $20717 / 24$ | -183 7/24 | 27917/24 | $3317 / 24$ | 20617/24 | -182 7/24 | $27817 / 24$ | 3417/24 | 20517/24 | -1817/24 | 27717/24 | $3517 / 2$ | 20417/24 |  | $27617 / 24$ |  |
|  | -15 | 246 | 6517/24 |  | -15 | 24 | 6417/24 | 17517/24 | -15 | 248 | $6317 / 24$ |  | -153 | 24917/24 | 62 | 17717/24 | -154 7/24 | 25 | 17/24 | 178 17/24 | -155 7 | 25117/24 | 6017 | 17917/24 |  |
|  | 353 | -113 7/24 |  |  | 352 | -112 | 136 | -407/24 | 351 | -1117/24 | 13517/24 |  |  | -110 7/24 | 134 | -387/24 | 349 | -1097/24 | 133 17/24 | -37 7/24 | 34817 | -1087/24 | 1321 | -36 7/24 |  |
|  |  | -67/24 | 318 | -78 | 103 | -7 7/24 | 319 | -79 7/24 | 10417/24 | -87/24 | 320 | -80 7/24 | 105 | -97/24 | 32117\|22 | -817/24 | 10617 | -10 7/24 | $32217 / 24$ | -82 71 | $10717 / 24$ | -117/24 | 32317 | -83 |  |
|  |  | 203 | -179 | 275 | $3717 / 24$ | 20217/2 | -178 | 27417/24 | 38 | 20 | -177 7\| | 27 | 39 | 20 | -176 7/24 | 27 | 40 | 19917/24 | -175 7/24 | 17/2 | 4117/24 | 198 17/24 | -174 7/24 | 270 17/24 |  |
|  |  |  |  |  |  |  |  |  | -158 7/24 |  |  |  |  | 25517/24 | /24 | 18317/24 | -160 7/24 | 25617/24 | 5517/24 | 18417/24 | -161 7/24 | 2517 | 517 | 18517/24 |  |
|  |  |  | 131 |  | 34 |  |  | $-347 / 24$ | 34517/24 |  | 12917/24 | -33 7/24 |  |  | 128 | -32 7/24 | 343 | -103 7/24 |  | 7/24 | 34217 | -102 7/24 | 1617 | -30 7/24 |  |
|  | 10817/24 | -12 | 324 | -847/24 | 109 | -13 7/24 | 32 | -85 7/24 | 110 | -147/24 | 32617/24 | -86 7/24 | 124 | 124 | 327 17/24 | -01724 | 120 | -167/24 | $32817 / 24$ | -88 12 | $11317 / 24$ | -177/24 | 32917 | -8 |  |
|  | 4217/24 | 19717/24 | -173 7/24 | $26971 / 24$ | $4317 / 24$ | 19617/24 | -172 7/24 | 26817/24 | 44 | 19517/24 | -171 7/24 | 26717/24 | 4517/24 |  | -170 7/24 | $26617 / 24$ | 4617/24 | 19317/24 | 4 | 24 | $4717 / 24$ | 19217/24 | -168 7/24 | 26417/24 |  |
|  |  | 25817/2 | 53 |  | -163 | 259 | 52 | 18717/24 | -16 | 260 | 5117/24 |  | -165 | 261 | $5017 / 2$ | 18917/24 | -166 7 | 26217 | $4917 / 24$ | 19017/24 | -167 7 | 26317/24 | 4817/24 | 19117/24 |  |
| 2021 | 34171/24 | -1017/2 | 12517/2 | -29 | $34017 / 2$ | -100 7/2 | 124 | -28 | $33917 \mid 2$ | -99 7/24 | 12317/2 | -27 7/2 | 338 | -987\| | 122171/2 | -267 | 33717 | -977/24 | 12117/24 | -25 7/24 | 33617/24 | -967/24 | 120 17/2 | -24 |  |
|  | 11417/2 | -187/24 | $33017 / 2$ | -90 712 | 11517/2 | -197/24 | $33117 \mid$ | -91712 | 11617/2 | -20 7/24 | 3321712 | -92712 | $11717 / 24$ | -21712 | 333 17/24 | -937 | 11817/24 | -22 7/24 | 33417/24 | -94 7/24 | 11917/24 | -23 7/24 | 33517/24 | . 957 | 2021 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

In this case, the blocks of order 4 are pandiagonal with equal magic sums given by

$$
\begin{gathered}
{\left[S_{4 \times 4}: S_{24 \times 24}:=21\right]:=3.5} \\
{\left[S_{4 \times 4}: S_{24 \times 24}:=441\right]:=73.5} \\
{\left[S_{4 \times 4}: S_{24 \times 24}:=2021\right]:=\frac{2021}{6} .}
\end{gathered}
$$

### 1.17.3 Third Type

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -286.625 | 257.375 | 256.375 | 225.375 | -255.625 | -191.625 | -285.625 | 258.375 | 255.375 | 226.375 | -256.625 | -192.625 | -284.625 | 259.375 | 254.375 | 227.375 | -257.625 | -193.625 | -283.625 | 260.375 | 253.375 | 228.375 | -258.625 | 4.625 |  |
| 192.375 | -159.625 | 160.375 | -158.625 | -126.625 | 97.375 | 191.375 | -160.625 | 159.375 | -157.625 | -125.625 | 98.375 | 190.375 | -161.625 | 158.375 | -156.625 | -124.625 | 99.375 | 189.375 | -162.625 | 157.375 | -155.625 | -123.625 | 100.375 | 1 |
| 96.375 | 65.375 | -62.625 | -31.625 | 32.375 | -94.625 | 95.375 | 66.375 | -61.625 | -32.625 | 31.375 | -93.625 | 94.375 | 67.375 | -60.625 | -33.625 | 30.375 | -92.625 | 93.375 | 68.375 | -59.625 | -34.625 | 29.375 | -91.625 | 1 |
| 0.375 | -63.625 | 33.375 | 64.375 | -30.625 | 1.375 | -0.625 | -64.625 | 34.375 | 63.375 | -29.625 | 2.375 | -1.625 | -65.625 | 35.375 | 62.375 | -28.625 | 3.375 | -2.625 | -66.625 | 36.375 | 61.375 | -27.625 | 4.375 | 1 |
| -190.625 | 128.375 | -127.625 | 129.375 | 161.375 | -95.625 | -189.625 | 127.375 | -128.625 | 130.375 | 162.375 | -96.625 | -188.625 | 126.375 | -129.625 | 131.375 | 163.375 | -97.625 | -187.625 | 125.375 | -130.625 | 132.375 | 164.375 | -98.625 | 21 |
| 193.375 | -222.625 | -254.625 | -223.625 | 224.375 | 288.375 | 194.375 | -221.625 | -253.625 | -224.625 | 223.375 | 287.375 | 195.375 | -220.625 | -252.625 | -225.625 | 222.375 | 286.375 | 196.375 | -219.625 | -251.625 | -226.625 | 221.375 | 285.375 | 21 |
| -282.625 | 261.375 | 252.375 | 229.375 | -259.625 | -195.625 | -281.625 | 262.375 | 251.375 | 230.375 | -260.625 | -196.625 | -280.625 | 263.375 | 250.375 | 231.375 | -261.625 | -197.625 | -279.625 | 264.375 | 249.375 | 232.375 | -262.625 | -198.625 | 21 |
| 188.375 | -163.625 | 156.375 | -154.625 | -122.625 | 101.375 | 187.375 | -164.625 | 155.375 | -153.625 | -121.625 | 102.375 | 186.375 | -165.625 | 154.375 | -152.625 | -120.625 | 103.375 | 185.375 | -166.625 | 153.375 | -151.625 | -119.625 | 104.375 | 21 |
| 92.375 | 69.375 | -58.625 | -35.625 | 28.375 | -90.625 | 91.375 | 70.375 | -57.625 | -36.625 | 27.375 | -89.625 | 90.375 | 71.375 | -56.625 | -37.625 | 26.375 | -88.625 | 89.375 | 72.375 | -55.625 | -38.625 | 25.375 | -87.625 | 21 |
| -3.625 | -67.625 | 37.375 | 60.375 | -26.625 | 5.375 | -4.625 | -68.625 | 38.375 | 59.375 | -25.625 | 6.375 | -5.625 | -69.625 | 39.375 | 58.375 | -24.625 | 7.375 | -6.625 | -70.625 | 40.375 | 57.375 | -23.625 | 8.375 | 21 |
| -186.625 | 124.375 | -131.625 | 133.375 | 165.375 | -99.625 | -185.625 | 123.375 | -132.625 | 134.375 | 166.375 | -100.625 | -184.625 | 122.375 | -133.625 | 135.375 | 167.375 | -101.625 | -183.625 | 121.375 | -134.625 | 136.375 | 168.375 | -102.625 | 21 |
| 197.375 | -218.625 | -250.625 | -227.625 | 220.375 | 284.375 | 198.375 | -217.625 | -249.625 | -228.625 | 219.375 | 283.375 | 199.375 | -216.625 | -248.625 | -229.625 | 218.375 | 282.375 | 200.375 | -215.625 | -247.625 | -230.625 | 217.375 | 281.375 | 21 |
| -278.625 | 265.375 | 248.375 | 233.375 | -263.625 | -199.625 | -277.625 | 266.375 | 247.375 | 234.375 | -264.625 | -200.625 | -276.625 | 267.375 | 246.375 | 235.375 | -265.625 | -201.625 | -275.625 | 268.375 | 245.375 | 236.375 | -266.625 | -202.625 | 21 |
| 184.375 | -167.625 | 152.375 | -150.625 | -118.625 | 105.375 | 183.375 | -168.625 | 151.375 | -149.625 | -117.625 | 106.375 | 182.375 | -169.625 | 150.375 | -148.625 | -116.625 | 107.375 | 181.375 | -170.625 | 149.375 | -147.625 | -115.625 | 108.375 | 21 |
| 88.375 | 73.375 | -54.625 | -39.625 | 24.375 | -86.625 | 87.375 | 74.375 | -53.625 | -40.625 | 23.375 | -85.625 | 86.375 | 75.375 | -52.625 | -41.625 | 22.375 | -84.625 | 85.375 | 76.375 | -51.625 | -42.625 | 21.375 | -83.62 | 21 |
| -7.625 | -71.625 | 41.375 | 56.375 | -22.625 | 9.375 | -8.625 | -72.625 | 42.375 | 55.375 | -21.625 | 10.375 | -9.625 | -73.625 | 43.375 | 54.375 | -20.625 | 11.375 | -10.625 | -74.625 | 44.375 | 53.375 | -19.625 | 12.375 | 21 |
| -182.625 | 120.375 | -135.625 | 137.375 | 169.375 | -103.625 | -181.625 | 119.375 | -136.625 | 138.375 | 170.375 | -104.625 | -180.625 | 118.375 | -137.625 | 139.375 | 171.375 | -105.625 | -179.625 | 117.375 | -138.625 | 140.375 | 172.375 | -106.6 | 21 |
| 201.375 | -214.625 | -24.625 | -231.625 | 216.375 | 280.375 | 202.375 | -213.625 | -245.625 | -232.625 | 215.375 | 279.375 | 203.375 | -212.625 | -244.625 | -233.625 | 214.375 | 278.375 | 204.375 | -211.625 | -243.625 | -234.625 | 213.375 | 277.37 | 21 |
| -274.625 | 269.375 | 244.375 | 237.375 | -267.625 | -203.625 | -273.625 | 270.375 | 243.375 | 238.375 | -268.625 | -204.625 | -272.625 | 271.375 | 242.375 | 239.375 | -269.625 | -205.625 | -271.625 | 272.375 | 241.375 | 240.375 | -270.625 | -206.625 | 21 |
| 180.375 | -171.625 | 148.375 | -146.625 | -114.625 | 109.375 | 179.375 | -172.625 | 147.375 | -145.625 | -113.625 | 110.375 | 178.375 | -173.625 | 146.375 | -144.625 | -112.625 | 111.375 | 177.375 | -174.625 | 145.375 | -143.625 | -111.625 | 112.375 | 21 |
| 84.375 | 77.375 | -50.625 | -43.625 | 20.375 | -82.625 | 83.375 | 78.375 | -49.625 | -44.625 | 19.375 | -81.625 | 82.375 | 79.375 | -48.625 | -45.625 | 18.375 | -80.625 | 81.375 | 80.375 | -47.625 | -46.625 | 17.375 | -79.625 | 21 |
| -11.625 | -75.625 | 45.375 | 52.375 | -18.625 | 13.375 | -12.625 | -76.625 | 46.375 | 51.375 | -17.625 | 14.375 | -13.625 | -77.625 | 47.375 | 50.375 | -16.625 | 15.375 | -14.625 | -78.625 | 48.375 | 49.375 | -15.625 | 16.375 | 21 |
| -178.625 | 116.375 | -139.625 | 141.375 | 173.375 | -107.625 | -177.625 | 115.375 | -140.625 | 142.375 | 174.375 | -108.625 | -176.625 | 114.375 | -141.625 | 143.375 | 175.375 | -109.625 | -175.625 | 113.375 | -142.625 | 144.375 | 176.375 | -110.625 | 21 |
| 205.375 | -210.625 | -242.625 | -235.625 | 212.375 | 276.375 | 206.375 | -209.625 | -241.625 | -236.625 | 211.375 | 275.375 | 207.375 | -208.625 | -240.625 | -237.625 | 210.375 | 274.375 | 208.375 | -207.625 | -239.625 | -238.625 | 209.375 | 273.375 | 21 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 21 |  |  |  |  |  |



| mgc |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -203 7/24 | $34017 / 24$ | $33917 / 24$ | $30817 / 24$ | -172 7/24 | -108 7/24 | -202 7/24 | 34177/24 | $338171 / 24$ | $30917 / 24$ | -173 7/24 | -109 7/24 | -201 7/24 | 34217/24 | 337 17/24 | 310 17/24 | -174 7/24 | -110 7/24 | -200 7/24 | 34317/24 | 33617/24 | 31117/24 | 175 7/24 | $1117 / 24$ |  |
| 27517/24 | -76 7/24 | 24317/24 | -75 7/24 | -43 7/24 | 180017/24 | 27417/24 | -77 7/24 | 24217/24 | -74 7/24 | -42 7/24 | 1817/24 | 27317/24 | -78 7/24 | 24117/24 | -73 7/24 | -417/24 | 18217/24 | 272 17/24 | -79 7/24 | 24017/24 | -72 7/24 | -407/24 | 17/24 |  |
| 17917/24 | 148 17/24 | $2017 / 24$ | 517/24 | 11517/24 | -117/24 | 178 17/24 | 149717/24 | 17/24 | $5017 / 24$ | 11417/24 | -107/24 | 17171/24 | 150 17/24 | $2217 / 24$ | 4917/24 | 11317/24 | 724 | 17617/24 | 15117/24 | $2317 / 24$ | 4817124 | 1217/24 | 24 |  |
| $8317 / 24$ | 1917/24 | 11617/24 | 14717/24 | 52 | 84 | 8217/24 | 124 | 124 | 14617/24 | 5317/24 | 8517/24 | 8117/24 | 1717/24 | /24 | 4 | $1 / 24$ | 124 | 24 | $1 / 24$ | 7/24 | 124 | 7/24 | 8717/24 |  |
| -107 7/24 | 21117/2 | -44 7/24 | 21217/24 | 24417/24 | -12 7/24 | -106 | 21017/24 | -45 7/24 | 21317/24 | 24517/24 | -13 | -105 | 20 | -46 | 21417/24 | 24617/24 | 24 | -104 7/24 | 24 | 124 | 2517/24 | 124 | 24 |  |
| 27617/24 | -139 | -171 | -140 7/2 | $30717 / 24$ | 37117/24 | 27717/24 | 138 | -170 | -1417/24 | 30617/24 | 370 17/24 | 27817/24 | -137 7/24 | -169 | -142 7/24 | 30517/24 | 36917/24 | $27917 / 24$ | 24 | 24 | 7/24 | 30417/24 | 24 |  |
| -19997/24 | $34417 / 24$ | $33517 / 24$ | 31217/24 | -176 7/24 | -112 71 | -198 | 34517/24 | 33417/24 | 31317/24 | -1771/24 | -113 7/24 | -197 7/24 | 34617/24 | $33317 / 24$ | 31417/24 | -178 7/24 | -114 7/24 | $1967 / 24$ | 34717/24 | 33217/24 | 31517/24 | 797 | -115 7/24 |  |
| 27117/24 | -80 7/24 | $23917 / 24$ | -717/24 | -397/24 | 18417/24 | $27017 / 24$ | -81 7/24 | $23817 / 24$ | -70 7/24 | -38 7/24 | 18517/24 | 26917/24 | -82 7/24 | $23717 / 24$ | -69 7/24 | -37 7/24 | 18617/24 | 26817/24 | -83 7/24 | 23617/24 | -687/24 | $367 / 24$ | 124 | 2021 |
| 17517/24 | 15217/24 | $2417 / 24$ | $4717 / 24$ | 11117/24 | -77/24 | 17417/24 | 15317/24 | 2517/24 | 4617/24 | 110 17/24 | -67/24 | $17317 / 24$ | 15417/24 | 2617/24 | 4517/24 | 10917/24 | -5 7/24 | 17217/24 | 15517/24 | $2717 / 24$ | 4417/24 | 10817/24 | -47/24 |  |
| 7917/24 | 1517/24 | 120 17/24 | 14317/24 | 5617/24 | $8817 / 2$ | 7817/24 | 1417/24 | 12117/24 | 14217/24 | $5717 / 24$ | 8917/24 | $7717 /$ | 1317/24 | 12217/24 | 14117/24 | $5817 / 24$ | 9017/24 | 7617/24 | 1217/24 | 12317/24 | 140017/24 | 5917/24 | 9117/24 |  |
| -103 7/24 | $20717 / 24$ | -487/24 | 21617/24 | 24817/24 | -167/24 | -102 7/24 | 20617/24 | -497/24 | $21717 / 24$ | 24917/24 | -177/24 | -1017/24 | 20517/24 | -50 7/24 | 21817/24 | 250 17/24 | -187/24 | 10071 | 20417/24 | -51 7/24 | 21917/24 | 25117/24 | 124 |  |
| 28017/24 | -135 | -167 | -144 | $30317 / 24$ | 3671712 | 28117/24 | 134 | -166 712 | -145 7/24 | 30217/24 | 36617/24 | 28217/24 | -133 7/2 | -165 7/2 | -1467/24 | 30117/24 | 36517/24 | 28317/2 | -132 7/24 | -164 7/24 | 7/24 | 300 17/2 | 36417/24 |  |
| -195 7/24 | $34817 / 24$ | 33117/24 | 31617/24 | -180 7/24 | -116 | -194 7/24 | 34917/24 | 330 17/24 | 31717/24 | -1817/24 | -117/24 | -193 7/24 | 350 17/24 | $32917 / 24$ | 31817/24 | -182 7/24 | -1187/24 | -192 7/24 | 35117/24 | 32817/24 | 31917/24 | 183 7/24 |  |  |
| 26717/24 | -847/24 | $23517 / 24$ | -67/24 | -35 7/24 | 18887/24 | 26617/24 | -85 7/24 | 23417/24 | -66 7/24 | -34 7/24 | 18997/24 | 26517/24 | -86 7/24 | 23317/24 | -65 7/24 | -33 7/24 | 190 17/24 | 26417/24 | -87 7/24 | 23217/24 | -64 7/24 | -32 7/24 | 19117/24 |  |
| 17117/24 | 15617/24 | $2817 / 24$ | $4317 / 24$ | 10717/24 | -37/24 | 170017/24 | 15717/24 | 2917/24 | 4217/24 | 10617/24 | -27/24 | 16917/24 | 15817/24 | $3017 / 24$ | 4117/24 | 10517/24 | -17/24 | 16817/24 | 15917/24 | 3117/24 | 4017/24 | 10417/24 | 7/24 |  |
| 7517/24 | $1117 / 24$ | 12417/24 | 13917/24 | $6017 / 24$ | 9217/2 | 7417/24 | 17/24 | 12517/24 | 138 17/24 | $6117 / 24$ | 9317/24 | 7317/24 | 917/24 | 12617/24 | 13717/24 | 6271/24 | 9417/24 | 7217/24 | $817 / 24$ | 12717/24 | 13617/24 | $6317 / 24$ | 9517/2 |  |
| -997/24 | $20317 / 24$ | -52 7/24 | $22017 / 24$ | 25217/24 | -20 7/24 | -987/24 | 20217/24 | -53 7/24 | 22117/24 | 25317/24 | -217/24 | -977/24 | 20117/24 | -54 7/24 | 22217/24 | 25417/24 | -22 7/24 | -967/24 | 20017/24 | -55 7/24 | 22317/24 | 25517/24 | 237 |  |
| 28417/24 | -1317/24 | -163 7/24 | -148 7/24 | 29917/24 | $36317 / 24$ | $28517 / 24$ | -130 7/24 | -162 7/24 | -149 7/24 | $29817 / 24$ | $36217 / 24$ | 28617/24 | -129 7/24 | -161 7/24 | -150 7/24 | $29717 / 24$ | 36117/24 | 28717/24 | -128 7/24 | -160 7/24 | -1517/24 | 29617/24 | 360171 |  |
| -1917/24 | $35217 / 24$ | $32717 / 24$ | $32017 / 24$ | -1847/24 | -120 712 | -190 7/24 | $35317 / 24$ | 32617/24 | 32117/24 | -185 7/24 | -121 7/24 | -18 | 35417/24 | $32517 / 24$ | 32217/24 | -186 7/24 | -122 7/24 | -188 7/2 | 35517/24 | 32417/24 | 32317/24 | 4 | -123 7/24 |  |
| 26317/24 | -887/24 | 23117/24 | -63 7/24 | -317/24 | 19217/24 | 262 | -897/24 | $23017 / 24$ | -62 7/24 | -30 7/24 | 19317/24 | 26117/24 | -90 7/24 | $22917 / 24$ | -617/24 | -29 7/24 | 19417/24 | $26017 / 24$ | -917/24 | 22817/24 | -60 7/24 | -287/24 | 124 |  |
| 16717/24 | 160 17/24 | 3217/24 | 3917/24 | 10317/2 | 17/24 | 16617/24 | 16117/24 | 3317/24 | 3817/24 | 10217/24 | 117/24 | 16517/24 | 16217/24 | 3417/24 | 3717/24 | 10117/2 | 217/24 | 16417/24 | 16317/24 | 3517/24 | 3617/24 | $10017 / 24$ | 317 |  |
| 7117/24 | $717 / 24$ | 128 17/24 | 13517/24 | 6417/24 | 9617/24 | 7017/24 | 17/24 | 12917/24 | 13417/24 | $6517 / 24$ | 9717/24 | 6917/24 | 517/24 | 130 17/24 | 13317/24 | 6617/24 | 9817/24 | $6817 / 24$ | 417/24 | 13117/24 | 132 17/24 | $6717 / 24$ | 9917/24 |  |
| -957/24 | 19917/24 | -56 7/24 | 22417/24 | 25617/24 | -24 7/24 | -94 7/24 | 19817/24 | -57 7/24 | 22517/24 | $25717 / 24$ | -25 7/24 | -93 7/24 | 19717/24 | -587/24 | 22617/24 | 25817/24 | -26 7/24 | -927/24 | 19617/24 | -597/24 | $22717 / 24$ | $25917 / 24$ | -27 7/24 |  |
| $28817 / 24$ | -127 7/24 | -159 7/24 | -152 7/24 | 29517/24 | $35917 / 24$ | 28917/24 | -126 7/24 | -158 7/24 | -153 7/24 | 29417/24 | $35817 / 24$ | 29017/24 | -125 7/24 | -157 7/24 | -154 7/24 | 29317/24 | $35717 / 24$ | 29117/24 | -124 7/24 | -156 7/24 | -155 7/24 | 292171/2 | 5617/24 | 2021 |
| 2021 | 021 | 2021 | 202 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 202 | 202 | 2021 | 2021 | 202 | 202 | 2021 | 2021 | 2021 |  |

In this case, the blocks of order 6 are with equal magic sums given by

$$
\begin{aligned}
{\left[S_{6 \times 6}: S_{24 \times 24}:=21\right] } & :=5.25 \\
{\left[S_{6 \times 6}: S_{24 \times 24}:=441\right] } & :=110.25 \\
{\left[S_{6 \times 6}: S_{24 \times 24}:=2021\right] } & :=505.25 .
\end{aligned}
$$

### 1.17.4 Forth Type

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -144.625 | 122.375 | 11.375 | -273.625 | -15.625 | 281.375 | 164.375 | -138.625 | -193.625 | 75.375 | 58.375 | -224.625 | -62.625 | 232.375 | 21.375 | -97.625 | -167.625 | 97.375 | 36.375 | -250.625 | -40.625 | 258.375 | 187.375 | -113.625 |  |
| -18.625 | 284.375 | 161.375 | -135.625 | -153.625 | 131.375 | 2.375 | -264.625 | -67.625 | 237.375 | 208.375 | -86.625 | -200.625 | 82.375 | 51.375 | -217.625 | -41.625 | 259.375 | 186.375 | -112.625 | -178.625 | 108.375 | 25.375 | 239.625 |  |
| -285.625 | 23.375 | 134.375 | -156.625 | -126.625 | 152.375 | 269.375 | -3.625 | -236.625 | 70.375 | 87.375 | -205.625 | -79.625 | 201.375 | 220.375 | -50.625 | -262.625 | 48.375 | 09.375 | -179.625 | -101.625 | 175.375 | 246.375 | -28.625 |  |
| -123.625 | 149.375 | 272.375 | -6.625 | -276.625 | 14.375 | 14.375 | -165.625 | -74.625 | 196.375 | 225.375 | -55.625 | -229.625 | 63.375 | 94.375 | -212.625 | -100.625 | 174.375 | 247.375 | -29.625 | -251.625 | 37.375 | 120.375 | 0.6 |  |
| 17.375 | -279.62 | -162.625 | 140.375 | 146.375 | -120.625 | -9.625 | 275.375 | 64.375 | -230.625 | -211.625 | 93.375 | 195.375 | -73.625 | -56.625 | 226.375 | 42.375 | -256.625 | -185.625 | 115.375 | 169.375 | -95.625 | -34.625 | 252.3 |  |
| 155.375 | -129.625 | -0.625 | 266.375 | 20.375 | -282.625 | -159.625 | 137.375 | 202.375 | -80.625 | -49.625 | 219.375 | 69.375 | -235.625 | -206.625 | 88.375 | 180.375 | -106.625 | -23.625 | 24.375 | 43.375 | -257.62 | -184.625 | 114.37 |  |
| 128.375 | -150.625 | -267.625 | 5.375 | 287.375 | -21.625 | -132.625 | 158.375 | 81.375 | -199.625 | -218.625 | 52.375 | 238.375 | -68.625 | -85.625 | 207.375 | 103.375 | -173.625 | -244.625 | 30.375 | 264.375 | -46.625 | -107.625 | 81.37 |  |
| 278.375 | -12.625 | -141.625 | 167.375 | 125.375 | -147.625 | -270.625 | 8.375 | 231.375 | -61.625 | -92.625 | 214.375 | 76.375 | -194.625 | -223.625 | 57.375 | 253.375 | -35.625 | -118.625 | 192.375 | 102.375 | -172.625 | -245.625 | 31.375 |  |
| -191.625 | 73.375 | 60.375 | -226.625 | -64.625 | 234.375 | 211.375 | -89.625 | -168.625 | 98.375 | 35.375 | -249.625 | -39.625 | 257.375 | 188.375 | -114.625 | -145.625 | 123.375 | 10.375 | -272.625 | -14.625 | 280.375 | 165.375 | 139.625 |  |
| -65.625 | 235.375 | 210.375 | -88.625 | -202.625 | 84.375 | 49.375 | -215.625 | -42.625 | 260.375 | 185.375 | -111.625 | -177.625 | . 375 | 26.375 | -240.625 | -19.625 | 285.375 | 160.375 | -134.625 | -152.625 | 130.375 | 3.375 | -265.625 |  |
| -238.625 | 72.375 | 85.375 | -203.625 | -77.625 | 199.375 | 222.375 | -52.625 | -261.625 | 47.375 | 110.375 | -180.625 | -102.625 | 176.375 | 245.375 | -27.625 | -284.625 | 22.375 | 135.375 | -157.625 | -127.625 | 153.375 | 268.375 | -2.625 |  |
| -76.625 | 198.375 | 223.375 | -53.625 | -227.625 | 75 | 96.375 | -214.625 | -99.625 | 173.375 | 248.375 | -30.625 | -252.625 | 38.375 | 119.375 | -189.625 | -122.625 | 148.375 | 273.375 | -7.625 | -277.625 | 15.375 | 142.375 | -164.63 |  |
| 66.375 | -232.625 | -209.625 | 91.375 | 193.375 | -71.625 | -58.625 | 228.375 | 41.375 | -255.625 | -186.625 | 116.375 | 170.375 | -96.625 | -33.625 | 251.375 | 16.375 | -278.625 | -163.625 | 141.375 | 147.375 | -121.625 | -8.625 | 274.3 |  |
| 204.375 | -82.625 | -47.625 | 217.375 | 67.375 | -233.625 | -208.625 | 90.375 | 179.375 | -105.625 | -24.625 | 242.375 | 44.375 | -258.625 | -183.625 | 113.375 | 154.375 | -128.625 | -1.625 | 267.37 | 21.375 | -283.625 | -158.625 | 36.3 |  |
| 79.375 | -197.625 | -220.625 | 54.375 | 240.375 | -70.625 | -83.625 | 205.375 | 104.375 | -174.625 | -243.625 | 29.375 | 263.375 | -45.625 | -108.625 | 182.375 | 129.375 | -151.625 | -266.625 | 4.375 | 286.375 | -20.625 | -133.625 | 159.37 |  |
| 229.375 | -59.625 | -94.625 | 216.375 | 78.375 | -196.625 | -221.625 | 55.375 | 254.375 | -36.625 | -117.625 | 191.375 | 101.375 | -171.625 | -246.625 | 32.375 | 279.375 | -13.625 | -140.625 | 166.375 | 124.375 | -146.625 | -271.625 | 9.375 |  |
| -169.625 | 99.375 | 34.375 | -248.625 | -38.625 | 256.375 | 189.375 | -115.625 | -143.625 | 121.375 | 12.375 | -274.625 | -16.625 | 282.375 | 163.375 | -137.625 | -192.625 | 74.375 | 59.375 | -225.625 | -63.625 | 233.375 | 212.375 | -90.62 |  |
| -43.625 | 261.375 | 184.375 | -110.625 | -176.625 | 106.375 | 27.375 | -241.625 | -17.625 | 283.375 | 162.375 | -136.625 | -154.625 | 132.375 | 1.375 | -263.625 | -66.625 | 236.375 | 209.375 | -87.625 | -201.625 | 83.375 | 50.375 | -216.6 |  |
| -260.625 | 46.375 | 1.375 | -181.625 | -103.625 | 177.375 | 244.375 | -26.625 | -286.625 | 24.375 | 133.375 | -155.625 | -125.625 | 151.375 | 270.375 | -4.625 | -237.625 | 71.375 | 86.375 | -204.625 | -78.625 | 200.375 | 221.375 | -51.625 |  |
| -98.625 | 172.375 | 249.375 | -31.625 | -253.625 | 39.375 | 118.375 | -188.625 | -124.625 | 150.375 | 271.375 | -5.625 | -275.625 | 13.375 | 144.375 | -166.625 | -75.625 | 197.375 | 224.375 | -54.625 | -228.625 | 62.375 | 95.375 | -213.625 |  |
| 40.375 | -254.625 | -187.625 | 117.375 | 171.375 | -97.625 | -32.625 | 250.375 | 18.375 | -280.625 | -161.625 | 139.375 | 145.375 | -119.625 | -10.625 | 276.375 | 65.375 | -231.625 | -210.625 | 92.375 | 194.375 | -72.625 | -57.625 | 227.3 |  |
| 178.375 | -104.625 | -25.625 | 243.375 | 45.375 | -259.625 | -182.625 | 112.375 | 156.375 | -130.625 | 0.375 | 265.375 | 19.375 | -281.625 | -160.625 | 138.375 | 203.375 | -81.625 | -48.625 | 218.375 | 68.375 | -234.625 | -207.625 | 89.375 |  |
| 105.375 | -175.625 | -242.625 | 28.375 | 262.375 | -44.625 | -109.625 | 183.375 | 127.375 | -149.625 | -268.625 | 6.375 | 288.375 | -22.625 | -131.625 | 157.375 | 80.375 | -198.625 | -219.625 | 5.375 | 239.375 | -69.625 | -84.625 | 206.3 |  |
| 255.375 | -37.625 | -116.625 | 190.375 | 100.375 | -170.625 | -247.625 | 33.375 | 277.375 | -11.625 | -142.625 | 168.375 | 126.375 | -148.625 | -269.625 | 7.375 | 230.375 | -60.625 | -93.625 | 215.375 | 77.375 | -195.625 | -222.625 | 56.375 |  |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |  |

## mgc

| 127.125 | 139.875 |  |  | 1.875 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 301.875 | 178.875 |  | -136.125 |  | 19.875 |  |  |  |  |  |  |  |  |  |  | 276.875 |  |  |  |  |  |  |  |
|  | 40.875 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 166.875 | 289 | 10.875 |  | 31.875 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 54.875 |  |  |  |
|  | -262.12 | -145.12 | 15 | 163.875 | -103.125 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -112.125 | 16.875 | 28.875 | 37.875 |  |  |  |  |  |  |  |  |  |  |  |  |  | -6125 | 258.875 |  |  |  |  |  |
|  | -133.12 | 250.12 | 22.875 | 304.87 | -4.125 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 875 |  | 184.87 | 142.87 | -130.12 |  |  | 248.8 | -44.125 |  |  | 93.875 |  |  |  | 270.875 |  |  |  | 119.875 |  |  |  |  |
|  | 90.87 | 77.875 |  |  |  | 228.875 |  |  | 115.875 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 252.875 | 227.87 | -71.125 |  | 101.875 | 66.875 |  | -25.125 |  |  |  |  |  |  |  | -2.125 | 302.875 |  |  |  | 147.875 |  |  |  |
|  | 89.875 | 102.87 |  |  | 216.87 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 108.875 |  |  |  |  | 58.875 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -65.125 |  |  | 84.875 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -180.125 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -42.125 |  | 233.875 | 95.875 |  |  |  | 271.875 | -19.125 |  |  | 118.875 |  |  | 49.875 | 296.8 |  |  | 183.875 | 141.875 | -129.125 |  |  |  |
|  | 116.875 | 51.87 |  |  |  | 20.875 |  |  |  |  |  | 0.875 |  | 180.875 |  |  |  |  |  |  |  | 229.875 |  |  |
|  | 278.875 | 201.875 | -93.125 |  |  |  |  | -0.125 |  |  |  |  |  |  |  |  | 253.875 |  |  |  |  |  |  |  |
|  | 63.875 | 128.875 | -164.125 |  |  | 261.875 |  |  | 41.87 | 150.875 | -138.125 |  |  | 287.875 |  |  |  |  |  |  | .875 | 238.8 |  |  |
|  | 189.875 | 266.875 | -14.125 | -236.125 | 56.875 | 135.875 |  | -107.125 |  | 288.875 | 11.875 |  | 30.875 | 161.875 | -149.125 | 58.125 | 214.87 | 24.875 | 37.12 |  | 79.875 | 12.8 |  |  |
|  | 237.12 | -170.12 | 134.875 | 188.875 | -80.12 | -15.125 |  | 35.875 |  |  | 156.875 |  |  | 6.875 |  | 82.8 |  | 193.12 | 109.875 | 211.875 | -55.125 | -40. |  |  |
|  | -87.125 | -8.125 | 260.875 | 62.875 |  | -165.125 | 129.875 | 173.875 | -113.125 | 17.875 | 282.875 | 36.875 | 264.12 | -143.125 | 155.875 | 220.8 | -64.12 | -31.125 | 235.8 | 85.875 | 217.125 | -190.125 |  |  |
| 122.875 | -158.125 | -225.125 | 45.875 | 279.875 | -27.125 | -92.125 | 200.875 | 14 | -132.125 | -251.125 | 23.87 | 305.875 | -5.12 | -114.125 | 174.875 | 97.8 | -181.125 | 202.125 | 70.875 | 256 | 52 | -67.125 |  |  |
| 272.87 | -20.12 | -99.12 | 207.8 | 117.8 | -153.12 | -230.12 | 50.8 | 294.8 | 5.875 | -125 | 185.8 | 143.8 | -131.12 | -252 | 24.875 | 247.875 | -43.125 | -76.125 | 232.8 | 44.875 | -178.125 | -205.125 | 73.875 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -61 7/24 | 20517/24 | 9417/24 | -190 7/24 | $6717 / 24$ | 36417/24 | 24177/24 | -55 7/24 | -11007/24 | 15817/24 | 14117/24 | -1417/24 | 2017/24 | 31517/24 | 29617/24 | -87/24 | -84 7/24 | 180 17/24 | 11917/24 | -167 7/24 | 4217/24 | 34171/24 | 27017/24 | $307 / 24$ |  |
| $6417 / 24$ | $36717 / 24$ | $24417 / 24$ | -52 7/24 | -70 7/24 | 21417/24 | 8517/24 | -1817/24 | 1517/24 | 320 17/24 | 29117/24 | -3 7/24 | -1177/24 | 16517/24 | 13417/24 | -1347/24 | 417/24 | $342171 / 24$ | 26917/24 | -29 7/24 | -957/24 | 19117/24 | 10817/24 | 567/24 | 021 |
| -202 | 10617/2 | 21717/24 | -73 7/24 | -43 7/24 | $23517 / 24$ | 352 17/24 | 7917/24 | -153 7/24 | 15317/24 | $17017 / 24$ | -122 7/24 | 317/24 | 28417/24 | $30317 / 24$ | 3217/24 | -1797/24 | 13117/24 | 19217/24 | -967/24 | -187/24 | 25817/24 | $32917 / 24$ | 24 |  |
| -40 7/24 | 232 17/24 | $35517 / 24$ | 7617/24 | $-1937 / 24$ | 9717/24 | 22617/24 | -82 7/24 | 817/24 | 27917/24 | 30817/24 | $2717 / 24$ | -146 7/24 | 14617/24 | 17717/24 | -129 7/24 | -177/24 | $25717 / 24$ | 330 17/24 | 5317/24 | -1687/24 | 120 17/24 | 20317/24 | 124 |  |
| 100 17/24 | -1967/2 | -79 7/24 | 22317/24 | 22917/24 | -37 7/24 | 7317/24 | $35817 / 24$ | 14717/24 | -1471/24 | -128 7/24 | 17617/24 | 278 17/24 | 24 | 2617/24 | $30917 / 24$ | 12517/24 | -173 7/24 | -1027/24 | 198 17/24 | 25217/24 | -127/24 | 4817/24 | 124 |  |
| $23817 / 24$ | -467/24 | $82177 / 24$ | 34917/24 | 10317/24 | -199 7/24 | -76 7/24 | 220 17/24 | 285 17/24 | 217/24 | $3317 / 24$ | $30217 / 24$ | 15217/24 | -152 7/24 | -123 7/24 | 17117/24 | 26317/24 | -23 7/24 | 5917/24 | 32417/24 | 12617/24 | -1747/24 | -101712 | 197717/24 | 2021 |
| 21117/24 | -67 7/24 | -184 7/24 | $8817 / 24$ | 370 17/24 | 617/24 | -497/24 | 24117/24 | 16417/24 | -116 7/24 | -135 7/24 | 13517/24 | 32117/24 | 1417/24 | -2 7/24 | 29017/24 | 18617/24 | -90 7/24 | -1617/24 | 11317/24 | 34717/24 | 3617/24 | -247/24 | 26417 | 2021 |
| $36177 / 24$ | 7017/24 | -58 7/24 | $25017 / 24$ | 20817/24 | -64 7/24 | -1877/24 | 124 | 31417/24 | 2117/24 | /24 | $29717 / 24$ | 15917/24 | -1117/24 | -140 7/24 | 140017/24 | 33617/24 | $4717 / 24$ | -35 7/24 | 27517/24 | 18517/24 | -897/24 | -162 7 | 17/2 | 021 |
| -108 7/24 | 15617/24 | 14317/24 | -143 7/24 | 1817/24 | 31717/24 | 29417/24 | -67/24 | -85 7/24 | 18117/24 | $11817 / 24$ | -166 7/24 | 4317/24 | $34017 / 24$ | 27117/24 | 7/24 | -62 7/24 | 20617/24 | 9317/24 | -189 7/24 | $6817 / 24$ | $36317 / 24$ | 24817/24 | -56 7/24 |  |
| 1717/24 | 31817/24 | 29317/24 | $-57 / 24$ | -119 7/24 | 16717/24 | 13217/24 | -132 7/24 | $4017 / 24$ | 34317/24 | 26817/24 | -287/24 | -94 7/24 | 19017/24 | 10917/24 | -157 7/24 | $6317 / 24$ | 36817/24 | 24317/24 | -51 7/24 | -69 7/24 | 21317/24 | $8617 / 24$ | $7 / 2$ | 2021 |
| -155 | 15517/24 | 168 17/24 | -120 7/24 | 517/24 | 28217/24 | 30517/24 | $3017 / 24$ | -178 7/24 | 130 17/24 | 19317/24 | -977/24 | -197/24 | $25977 / 24$ | 32817/24 | $5517 / 24$ | -2017/24 | 10517/24 | 21817/24 | -74 7/24 | -44 7/24 | 23617/24 | 35177/24 | $8017 / 24$ | 2021 |
| 617/24 | 28117/24 | 30617/24 | 2917/24 | -144 7/24 | 14417/24 | 17917/24 | -1317/24 | -167/24 | 25617/24 | $33117 / 24$ | 5217/24 | -169 7/24 | 12117/24 | 20217/24 | -1067/24 | -397/24 | 23117/24 | 35617/24 | 7517/24 | -194 7/24 | 9817/24 | 22517/24 | -817/24 |  |
| 14917/24 | -1497/24 | -126 7/24 | 17417/24 | 27617/24 | 1171/24 | 2417/24 | 31117/24 | 12417/24 | -172 7/24 | -103 7/24 | 19917/24 | 25317/24 | -137/24 | 4917/24 | 33417/24 | 9917/24 | -195 7/24 | -80 7/24 | 22417/24 | $23017 / 24$ | -387/24 | 7417/24 | $35717 / 24$ |  |
| $28717 / 2$ | 17/24 | 3517/24 | $30017 / 24$ | 15017/24 | -150 7/2 | -125 7/24 | 17317/24 | 26217/24 | -22 7/24 | $5817 / 24$ | 32517/24 | 127 17/24 | -175 7\| | -100 7/24 | 19617/24 | $23717 / 24$ | -457/24 | 8117/24 | $35017 / 24$ | 10417/24 | -200 7/24 | -75 7/24 | 21917 |  |
| 16217/24 | -114 7/24 | -137 7/24 | 137 17/24 | 32317/24 | 1217/24 | 7/24 | 28817/24 | 18717/24 | -917/24 | -160 7/24 | 11217/24 | 34617/24 | 3717/24 | -257/24 | 26517/24 | 21217/24 | -687/24 | -183 7/24 | 8717/24 | 36917/24 | 6217/24 | -50 7/24 | 242171 | 2021 |
| 31217/24 | $2317 / 24$ | -117/24 | 29917/24 | 16177/24 | -113 7/24 | -138 7/24 | 138 17/24 | 337 17/24 | 4617/24 | -34 7/24 | 27417/24 | 18417/24 | -887/24 | -163 7/24 | 11517/24 | 362 17/24 | 6917/24 | -57 7/24 | 24917/24 | 20717/24 | -63 7/24 | -1887/24 | 9217/24 |  |
| -86 7/24 | 18217/24 | $11717 / 24$ | -165 7/24 | 4417/24 | 339 | 27217/24 | -32 7/24 | -60 7/24 | 20417/24 | 9517/24 | -19171/20 | 124 | $36517 / 24$ | 24617/24 | -54 7/24 | -1097/24 | 157 $71 / 24$ | 14217/24 | -142 7/24 | 1917/24 | 31617/24 | 29517/24 | 24 |  |
| 39 | 34417/2 | 267 | -27 | -93 | $18917 / 24$ | 110 17/24 | -15 | 65 | 24 | 24517/24 | -5 | -71 | 21517/24 | /24 | -188 | 1617/24 | 17/24 | 24 | -47/24 | -118 7/24 | 124 | 317 | -133 7/24 |  |
| -1777/24 | 12917/24 | 19417/24 | -98 7/24 | -20 7/24 | 260 17/24 | $32717 / 24$ | 5617/24 | -203 | 10717/24 | 21617/24 | -72 | -42 7/24 | 23417/24 | $35317 / 24$ | $7817 / 24$ | -154 7/24 | 15417/24 | 16917/24 | -1217/24 | 417/24 | 28317/24 | 30417/24 | 3117/24 |  |
| -15 7/24 | 25517/24 | 332 17/24 | 517/24 | -170 7/24 | 12217/24 | 20117/24 | -1057/24 | -417/24 | 23317/24 | 35417/24 | $7717 / 24$ | -192 7/24 | 9617/24 | $22717 / 24$ | -837/24 | 717/24 | $28017 / 24$ | $30717 / 24$ | $2817 / 24$ | -145 7/24 | 14517/24 | 178 17/24 | -130 7 | 2021 |
| 12317/24 | -1717/24 | -104 7/24 | $20017 / 24$ | 25417/24 | -147/24 | 5017/24 | 33317/24 | 10117/24 | -1977/24 | -787/24 | 22217/24 | 22817/24 | -367/24 | 7277/24 | $35977 / 24$ | 148817/24 | -1487/24 | -127 7/24 | 17517/24 | 27717/24 | $1017 / 24$ | 2517/24 | 310 17/24 | 2021 |
| 26117/24 | $-217 / 24$ | 5717/24 | 32617/24 | 12817/24 | -176 7/24 | -997/24 | 19517/24 | 23917/24 | -477/24 | $8317 / 24$ | $34817 / 24$ | 10217/24 | -1987/24 | -77/24 | 22117/24 | 28617/24 | 117/24 | 3417/24 | 30117/24 | 15117/24 | -1517/24 | -1247/24 | 17217/24 | 2021 |
| 18817/24 | -92 7/24 | -159 7/24 | 11117/24 | 34517/24 | $3817 / 24$ | $-267 / 24$ | 26617/24 | 210 17/24 | -66 7/24 | -185 7/24 | $8917 / 24$ | 37117/24 | $6017 / 24$ | -487/24 | 24017/24 | 16317/24 | -115 7/24 | -136 7/24 | 13617/24 | $32217 / 24$ | 1317/24 | -17/24 | 28917/24 | 2021 |
| $33817 / 24$ | 4517/24 | -33 7/24 | 27317/24 | 18317/24 | -87 7/24 | -164 7/24 | 11617/24 | $36017 / 24$ | 7117/24 | -59 7/24 | 25117/24 | 20917/24 | -65 7/24 | -186 7/24 | 9017/24 | 31317/24 | 2217/24 | -107/24 | 29817/24 | 160 17/24 | -112 7/24 | -139 7/24 | 13917/24 | 2021 |
| 221 | 2021 | 2021 | 2021 | 2021 | 2021 | 021 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 |

In this case, the blocks of order 8 are magic squares with equal magic sums given by

$$
\begin{gathered}
{\left[S_{8 \times 8}: S_{24 \times 24}:=21\right]:=7} \\
{\left[S_{8 \times 8}: S_{24 \times 24}:=441\right]:=63} \\
{\left[S_{8 \times 8}: S_{24 \times 24}:=2021\right]:=\frac{2021}{3} .}
\end{gathered}
$$

Moreover, the block of order 8 are either bimagic or semi-bimagic squares. The magic squares of order 24 are semibimagic squares with semi-bimagic sums given by

$$
\begin{aligned}
{\left[\text { Smb }_{24 \times 24}: S_{24 \times 24}:=21\right] } & :=663568.375 \\
{\left[\text { Smb }_{24 \times 24}: S_{24 \times 24}:=441\right] } & :=671653.375 \\
{\left[\text { Smb }_{24 \times 24}: S_{24 \times 24}:=2021\right] } & :=\frac{20009641}{24}
\end{aligned}
$$

These sums are same as given in First Type.

### 1.18 Block-Wise Bimagic Square of Order 25

Below is block-wise magic square of order 25 giving magic sums $21,21^{2}$ and 2021. The blocks of order 5 are pandiagonal magic squares of equal sums resulting in a bimagic square of order 25.

|  | pan | 21 | 21 | 2 |  |  |  | 21 |  |  | 2 |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | -311.16 | -143.16 | -5.16 | 162.84 | 300.84 | 130.84 | 268.84 | -188.16 | -175.16 | -32.16 | -77.16 | -39.16 | 98.84 | 241.84 | -220.16 | 234.84 | -247.16 | -109.16 | 53.84 | 71.84 | 26.84 | 164.84 | 207.84 | -279.16 | -116.16 | 21 |
| 21 | 144.84 | 312.84 | -299.16 | -161.16 | 6.84 | -163.16 | -25.16 | 117.84 | 280.84 | -206.16 | 248.84 | -233.16 | -70.16 | -52.16 | 110.84 | 40.84 | 78.84 | 221.84 | -240.16 | -97.16 | -267.16 | -129.16 | 33.84 | 176.84 | 189.84 | 21 |
| 21 | -149.16 | -11.16 | 156.84 | 294.84 | -287.16 | 267.84 | -194.16 | -181.16 | -13.16 | 124.84 | -45.16 | 97.84 | 260.84 | -226.16 | -83.16 | -253.16 | -90.16 | 52.84 | 65.84 | 228.84 | 183.84 | 201.84 | -285.16 | -117.16 | 20.84 | 21 |
| 21 | 306.84 | -305.16 | -137.16 | 0.84 | 138.84 | -31.16 | 136.84 | 274.84 | -207.16 | -169.16 | -214.16 | -76.16 | -58.16 | 104.84 | 247.84 | 77.84 | 215.84 | -246.16 | -103.16 | 59.84 | -135.16 | 32.84 | 170.84 | 208.84 | -273.16 | 21 |
| 21 | 12.84 | 150.84 | 288.84 | -293.16 | -155.16 | -200.16 | -182.16 | -19.16 | 118.84 | 286.84 | 91.84 | 254.84 | -227.16 | -64.16 | -51.16 | -96.16 | 46.84 | 84.84 | 227.84 | -259.16 | 195.84 | -266.16 | -123.16 | 4 | 182.84 | 21 |
| 21 | 222.84 | -239. | -10 | 41. | 79 | 34.84 | 177.84 | 190.84 | -271.16 | -128.16 | -298.16 | -1 | 7.8 | 145.84 | 308.84 | 113.84 | 281.84 | -205.16 | -162.16 | -24.16 | 16 | -56.16 | 111.84 | 249.84 | -232.16 | 21 |
| 2 | 48.8 | 66 | 229.84 | -2 | -8 | -2 | -121 | 21 | 184.84 | 202.84 | 157.84 | 295.84 | -291.16 | -1 | -1 | -1 | -12.16 | 125.84 | 263.84 | -193.16 | 261.84 | -225.16 | 16 | 6 | 93.84 | 21 |
| 21 | -245.16 | -102.16 | 60.84 | 73.84 | 216.84 | 171.84 | 209.84 | -272.16 | -134.16 | 28.84 | -141.16 | 1.84 | 139.84 | 307.84 | -304.16 | 275.84 | -211.16 | -168.16 | -30.16 | 137.84 | 16 | 105.84 | 243.84 | -213.16 | 6 | 21 |
| 2 | 85.84 | 223.84 | -258.16 | -95.16 | 47.84 | -122.16 | 15.84 | 178.84 | 196.84 | -265.16 | 289.84 | -292.16 | -154.16 | 8.84 | 151.84 | -18.16 | 119.84 | 287.84 | -199.16 | -186.16 | -231.16 | -63.16 | -50.16 | 92.84 | 255.84 | 21 |
| 21 | -108.16 | 54.84 | 72.84 | 235.84 | -251.16 | 203.84 | -278.16 | -115.16 | 27.84 | 165.84 | -4.16 | 158.84 | 301.84 | -310.16 | -142.16 | -187.16 | -174.16 | -36.16 | 131.84 | 269.84 | 99.84 | 242.84 | -219.16 | -81.16 | -38.16 | 21 |
| 21 | 126.84 | 264.84 | -192.16 | -179.16 | -16.16 | -86.16 | -43.16 | 94.84 | 262.84 | -224.16 | 230.84 | -256.16 | -88.16 | 49.84 | 67.84 | 22.84 | 185.84 | 198.84 | -283.16 | -120.16 | -290.16 | -147.16 | -9.16 | 153.84 | 296.84 | 21 |
| 21 | -167.16 | -29.16 | 133.84 | 276.84 | -210.16 | 244.84 | -212.16 | -74.16 | -61.16 | 106.84 | 61.84 | 74.84 | 217.84 | -244.16 | -106.16 | -276.16 | -133.16 | 29.84 | 172.84 | 210.84 | 140.84 | 303.84 | -303.16 | -140.16 | 2.84 | 21 |
| 21 | 283.84 | -198.16 | -185.16 | -17 | 120.84 | -49.16 | 88.84 | 256.84 | -230.16 | -62.16 | -257.16 | -94.16 | 43.84 | 86.84 | 224.84 | 179.84 | 197.84 | -264.16 | -126.16 | 16.84 | -153.16 | 9.84 | 152.84 | 290.84 | -296.16 | 21 |
| 21 | -35.16 | 132.84 | 270.84 | -191.16 | -173.16 | -218.16 | -80.16 | -3 | 100.84 | 238.84 | 68.84 | 236.84 | -250.16 | -107.16 | 55.84 | -114.16 | 23.84 | 166.84 | 204.84 | -277.16 | 302.84 | -309.16 | -146.16 | -3.16 | 159.84 | 21 |
| 21 | -204.16 | -166.16 | -23.16 | 114.84 | 282.84 | 112.84 | 250.84 | -236.16 | -68.16 | -55.16 | -100.16 | 42.84 | 80.84 | 218.84 | -238.16 | 191.84 | -270.16 | -127.16 | 35.84 | 173.84 | 3.84 | 146.84 | 309.84 | -297.16 | -159.16 | 21 |
| 21 | 30.84 | 168.84 | 211.84 | -275.16 | -132.16 | -302.16 | -139.16 | -1.16 | 141.84 | 304.84 | 134.84 | 277.84 | -209.16 | -171.16 | -28.16 | -73.16 | -60.16 | 107.84 | 245.84 | -216.16 | 213.84 | -243.16 | -105.16 | 62.84 | 75.84 | 21 |
| 21 | -263.16 | -125.16 | 17.84 | 180.84 | 193.84 | 148.84 | 291.84 | -295.16 | -152.16 | 10.84 | -184.16 | -21.16 | 121.84 | 284.84 | -197.16 | 257.84 | -229.16 | -66.16 | -48.16 | 89.84 | 44.84 | 87.84 | 225.84 | -261.16 | -93.16 | 21 |
| 21 | 167.84 | 205.84 | -281.16 | -113.16 | 24.84 | -145.16 | -2.16 | 160.84 | 298.84 | -308.16 | 271.84 | -190.16 | -172.16 | -34.16 | 128.84 | -41.16 | 101.84 | 239.84 | -217.16 | -79.16 | -249.16 | -111.16 | 56.84 | 69.84 | 237.84 | 21 |
| 21 | -131.16 | 36.84 | 174.84 | 192.84 | -269.16 | 310.84 | -301.16 | -158.16 | 4.84 | 147.84 | -22.16 | 115.84 | 278.84 | -203.16 | -165.16 | -235.16 | -67.16 | -54.16 | 108.84 | 251.84 | 81.84 | 219.84 | -237.16 | -99.16 | 38.84 | 21 |
| 21 | 199.84 | -282.16 | -119.16 | 18.84 | 186.84 | -8.16 | 154.84 | 297.84 | -289.16 | -151.16 | -196.16 | -178.16 | -15.16 | 127.84 | 265.84 | 95.84 | 258.84 | -223.16 | -85.16 | -42.16 | -87.16 | 50.84 | 63.84 | 231.84 | -255.16 | 21 |
| 21 | -65.16 | -47.16 | 90.84 | 253.84 | -228.16 | 226.84 | -260.16 | -92.16 | 45.84 | 83.84 | 13.84 | 181.84 | 194.84 | -262.16 | -124.16 | -294.16 | -156.16 | 11.84 | 149.84 | 292.84 | 122.84 | 285.84 | -201.16 | -183.16 | -20.16 | 21 |
| 21 | 240.84 | -221.16 | -78.16 | -40.16 | 102.84 | 57.84 | 70.84 | 233.84 | -248.16 | -110.16 | -280.16 | -112.16 | 25.84 | 163.84 | 206.84 | 161.84 | 299.84 | -307.16 | -144.16 | -6.16 | -176.16 | -33.16 | 129.84 | 272.84 | -189.16 | 21 |
| 21 | -53.16 | 109.84 | 252.84 | -234.16 | -71.16 | -241.16 | -98.16 | 39.84 | 82.84 | 220.84 | 175.84 | 188.84 | -268.16 | -130.16 | 37.84 | -157.16 | 5.84 | 143.84 | 311.84 | -300.16 | 279.84 | -202.16 | -164.16 | -26.16 | 116.84 | 21 |
| 21 | -222.16 | -84.16 | -46.16 | 96.84 | 259.84 | 64.84 | 232.84 | -254.16 | -91.16 | 51.84 | -118.16 | 19.84 | 187.84 | 200.84 | -286.16 | 293.84 | -288.16 | -150.16 | -7.16 | 155.84 | -14.16 | 123.84 | 266.84 | -195.16 | -177.16 | 21 |
|  | 103.84 | 246.84 | -215.16 | -72.16 | -59.16 | -104.16 | 58.84 | 76.84 | 214.84 | -242.16 | 212.84 | -274.16 | -136.16 | 31.84 | 169.84 | -0.16 | 142.84 | 305.84 | -306.16 | -138.16 | -208.16 | -170.16 | -27.16 | 135.84 | 273.84 | 21 |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |


|  | pan | 441 | 4 | 441 | 441 | 441 | 441 | 44 | 441 | 441 | 44 | 441 |  | 441 | 44 |  | 441 |  |  |  |  |  |  |  |  | 441 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 441 | -294.36 | -126.36 | 11.64 | 179.64 | 317.64 | 147.64 | 285.64 | -171.36 | -158.36 | -15.36 | -60.36 | -22.36 | 115.64 | 258.64 | -203.36 | 251.64 | -230.36 | -92.36 | 70.64 | 88.64 | 43.64 | 181.64 | 224.64 | -262.36 | -99.36 | 441 |
| 441 | 161.64 | 329.64 | -282.36 | -144.36 | 23.64 | -146.36 | -8.36 | 134.64 | 297.64 | -189.36 | 265.64 | -216.36 | -53.36 | -35.36 | 127.64 | 57.64 | 95.64 | 238.64 | -223.36 | -80.36 | -250.36 | -112.36 | 50.64 | 193.64 | 206.64 | 441 |
| 44 | -132.36 | 5.64 | 173.64 | 311.64 | -270.36 | 284.64 | -177.36 | -164.36 | 3.64 | 141.64 | -28.36 | 114.64 | 277.64 | -209.36 | -66.36 | -236.36 | -73.36 | 69.64 | 82.64 | 245.64 | 200.64 | 218.64 | -268.36 | -100.36 | 37.64 | 441 |
| 441 | 323.64 | -288.36 | -120.36 | 17.64 | 155.64 | -14.36 | 153.64 | 291.64 | -190.36 | -152.36 | -197.36 | -59.36 | -41.36 | 121.64 | 264.64 | 94.64 | 232.64 | -229.36 | -86.36 | 76.64 | -118.36 | 49.64 | 187.64 | 225.64 | -256.36 | 441 |
| 441 | 29.64 | 167.64 | 305.64 | -276.36 | -138.36 | -183.36 | -165.36 | -2.36 | 135.64 | 303.64 | 108.64 | 271.64 | -210.36 | -47.36 | -34.36 | -79.36 | 63.64 | 101.64 | 244.64 | -242.36 | 212.64 | -249.36 | -106.36 | 31.64 | 199.64 | 441 |
| 441 | 239.64 | -222.36 | -84.36 | 58.64 | 96.64 | 51.64 | 194.64 | 207.64 | -254.36 | -111.36 | -281.36 | -143.36 | 24.64 | 162.64 | 325.64 | 130.64 | 298.64 | -188.36 | -145.36 | -7.36 | -52.36 | -39.36 | 128.64 | 266.64 | -215.36 | 441 |
| 441 | 65.64 | 83.64 | 246.64 | -235.36 | -72.36 | -267.36 | -104.36 | 38.64 | 201.64 | 219.64 | 174.64 | 312.64 | -274.36 | -131.36 | 6.64 | -163.36 | 4.64 | 142.64 | 280.64 | -176.36 | 278.64 | -208.36 | -65.36 | -27.36 | 110.64 | 441 |
| 441 | -228.36 | -85.36 | 77 | 90.64 | 233.64 | 188.64 | 226.64 | -255.36 | -117.36 | 45.64 | -124.36 | 18 | 156.64 | 324.64 | -287.36 | 292.64 | -194.36 | -151.36 | -13.36 | 154.64 | -40.36 | 122.64 | 260.64 | -196.36 | -58.36 | 41 |
| 441 | 102 | 240 | -2 | -7 | 64 | -105.36 | 32 | 19 | 213.64 | -248.36 | 306.64 | -275.36 | -1 | 25 | 16 | -1 | 136.64 | 304.64 | -182.36 | -169.36 | -214.36 | -46.36 | -33.36 | 64 | 4 | 41 |
| 441 | -91 | 71. | 89 | 252.64 | -234.3 | 220.64 | -261.36 | -98 | 44 | 18 | 12 | 17 | 318.64 | -293.36 | -125.36 | -170.36 | -157.36 | -19.36 | 14 | 286.64 | 116.64 | 259.64 | -202.36 | -64.36 | -21.36 | 41 |
| 441 | 143 | 281.6 | -17 | -162 | 0. | -6 | -26.36 | 111 | 279.64 | -20 | 247.64 | -2 | -7 | 66 | 8 | 39.64 | 202.64 | 215.64 | -266.36 | -103.36 | -273.36 | -130.36 | 7.64 | 64 | 4 | 41 |
| 441 | -150.36 | -12.36 | 150.64 | 293.64 | -193.36 | 26 | -195.36 | -57.36 | -4 | 12 | 78 | 91. | 234.64 | -227.36 | -89.36 | -259.36 | -1 | 46.64 | 189.64 | 227.64 | 157.64 | 320.64 | -286.36 | -123.36 | 19.64 | 41 |
| 441 | 300.64 | -181.36 | -168.36 | -0.36 | 137. | -32.36 | 105.64 | 273.64 | -213.36 | -45.36 | -240.36 | -7 | 60.64 | 103.6 | 24 | 196.64 | 214.64 | -247.36 | -109.36 | 33.64 | -136.36 | 26.64 | 169.64 | 307.64 | -279.36 | 41 |
| 441 | -18.36 | 149.64 | 287.64 | -174.36 | -156.36 | -201.36 | -63.36 | -20.36 | 117.64 | 255.64 | 85.64 | 253.64 | -233.36 | -90.3 | 72.64 | -97.36 | 40.64 | 183.64 | 221.64 | -260.36 | 319.64 | -292.36 | -129.36 | 13.64 | 176.64 | 41 |
| 441 | -187.36 | -149.36 | -6.36 | 131.64 | 299.64 | 129.64 | 267.64 | -219.36 | -51.36 | -38.36 | -83.36 | 59.64 | 97.64 | 235.64 | -221.36 | 208.64 | -253.36 | -110.36 | 52.64 | 190.64 | 20.64 | 163.64 | 326.64 | -280.36 | -142.36 | 441 |
| 441 | 47.64 | 185.64 | 228.64 | -258.36 | -115.36 | -285.36 | -122.36 | 15.64 | 158.64 | 321.64 | 151.64 | 294.64 | -192.36 | -154.36 | -11.36 | -56.36 | -43.36 | 124.64 | 262.64 | -199.36 | 230.64 | -226.36 | -88.36 | 79.64 | 92.64 | 441 |
| 441 | -246.36 | -108.36 | 34.64 | 197.64 | 210.64 | 165.64 | 308.64 | -278.36 | -135.36 | 27.64 | -167.36 | -4.36 | 138.64 | 301.64 | -180.36 | 274.64 | -212.36 | -49.36 | -31.36 | 106.64 | 61.64 | 104.64 | 242.64 | -244.36 | -76.36 | 441 |
| 441 | 184.64 | 222.64 | -264.36 | -96.36 | 41.64 | -128.36 | 14.64 | 177.64 | 315.64 | -291.36 | 288.64 | -173.36 | -155.36 | -17.36 | 145.64 | -24.36 | 118.64 | 256.64 | -200.36 | -62.36 | -232.36 | -94.36 | 73.64 | 86.64 | 254.64 | 441 |
| 441 | -114.36 | 53.64 | 191.64 | 209.64 | -252.36 | 327.64 | -284.36 | -141.36 | 21.64 | 164.64 | -5.36 | 132.64 | 295.64 | -186.36 | -148.36 | -218.36 | -50.36 | -37.36 | 125.64 | 268.64 | 98.64 | 236.64 | -220.36 | -82.36 | 55.64 | 441 |
| 441 | 216.64 | -265.36 | -102.36 | 35.64 | 203.64 | 8.64 | 171.64 | 314.64 | -272.36 | -134.36 | -179.36 | -161.36 | 1.64 | 144.64 | 282.64 | 112.64 | 275.64 | -206.36 | -68.36 | -25.36 | -70.36 | 67.64 | 80.64 | 248.64 | -238.36 | 441 |
| 441 | -48.36 | -30.36 | 107.64 | 270.64 | -211.36 | 243.64 | -243.36 | -75.36 | 62.64 | 100.64 | 30.64 | 198.64 | 211.64 | -245.36 | -107.36 | -277.36 | -139.36 | 28.64 | 166.64 | 309.64 | 139.64 | 302.64 | -184.36 | -166.36 | -3.36 | 441 |
| 441 | 257.64 | -204.36 | -61.36 | -23.36 | 119.64 | 74.64 | 87.64 | 250.64 | -231.36 | -93.36 | -263.36 | -95.36 | 42.64 | 180.64 | 223.64 | 178.64 | 316.64 | -290.36 | -127.36 | 10.64 | -159.36 | -16.36 | 146.64 | 289.64 | -172.36 | 441 |
| 441 | -36.36 | 126.64 | 269.64 | -217.36 | -54.36 | -224.36 | -81.36 | 56.64 | 99.64 | 237.64 | 192.64 | 205.64 | -251.36 | -113.36 | 54.64 | -140.36 | 22.64 | 160.64 | 328.64 | -283.36 | 296.64 | -185.36 | -147.36 | -9.36 | 133.64 | 441 |
| 441 | -205.36 | -67.36 | -29.36 | 113.64 | 276.64 | 81.64 | 249.64 | -237.36 | -74.36 | 68.64 | -101.36 | 36.64 | 204.64 | 217.64 | -269.36 | 310.64 | -271.36 | -133.36 | 9.64 | 172.64 | 2.64 | 140.64 | 283.64 | -178.36 | -160.36 | 441 |
|  | 120.64 | 263.64 | -198.36 | -55.36 | -42.36 | -87.36 | 75.64 | 93.64 | 231.64 | -225.36 | 229.64 | -257.36 | -119.36 | 48.64 | 186.64 | 16.64 | 159.64 | 322.64 | -289.36 | -121.36 | -191.36 | -153.36 | -10.36 | 152.64 | 290.64 | 441 |
|  | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 |


|  | pan | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 202 | 2021 | 202 | 202 | 202 | 202 | 202 | 202 | 2021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2021 | -231.16 | -63.16 | 74.84 | 242.84 | 380.84 | 210.84 | 348.84 | -108.16 | -95.16 | 47.84 | 2.84 | 40.84 | 178.84 | 321.84 | -140.16 | 314.84 | -167.16 | -29.16 | 133.84 | 151.84 | 106.84 | 244.84 | 287.84 | -199.16 | -36. | 2021 |
| 2021 | 224.84 | 392.84 | -219.16 | -81.16 | 86.84 | -83.16 | 84 | 197.84 | 360.84 | -126.16 | 328.84 | -153.1 | 9.84 | 27.84 | 190.84 | 120.84 | 158.84 | 301.84 | -160.16 | -17.16 | -187.16 | -49.16 | 113.84 | 256.84 | 269.84 | 2021 |
| 2021 | -69.16 | 68.84 | 236.84 | 374.84 | -207.16 | 347.84 | -114.16 | -101.16 | 6.84 | 204.84 | 34.84 | 177.84 | 340.84 | -14618 | -3.16 | -173.16 | -10.16 | 132.84 | 145.84 | 308.84 | 263.84 | 281.84 | -205.16 | -37.16 | 0.8 | 2021 |
|  | 386.84 | -2 | -57.16 | 80.84 | 218.84 | 48.84 | 84 | 354.84 | -127 | 16 | -134 | 3.84 | 84 | 184.84 | 327.8 | 157.84 | 295.84 | -166.16 | -23.16 | 139.84 | -55.16 | 112.84 | 250.84 | 288.84 | -193.16 | 2021 |
|  | 92.84 | 230.84 | 368.84 | -213.16 | . 16 | -12 | -102.16 | 3 | 198.84 | 84 | 84 | 334.84 | -147.16 | 15.84 | 28.84 | -16.16 | 126.84 | 84 | 307.84 | 9.16 | 275.84 | -186.16 | -43.16 | 4.84 | 262.84 | 202 |
| 202 | 302.84 | -159.16 | -21.16 | 121.84 | 159.84 | 84 | 84 | 270.84 | . 16 | . 16 | . 16 | -80.16 | 87.84 | 225.84 | . 84 | 193.84 | . 84 | 5.16 | -82.16 | 84 | 10.84 | 3.8 | 91.84 | 329.8 | -152.16 | 2021 |
|  | 128.84 | 146.84 | 309.84 | -172.16 | 16 | -204.16 | 16 | . 84 | 264.84 | 282.84 | 237.84 | 375.84 | -2 | 6.16 | 84 | -10 | 67.84 | 205.84 | 343.84 | -113.16 | 341.84 | -145. | -2.16 | 35.84 | 3.8 | 2021 |
| 202 | -165 | -22.16 | 140.84 | 153.84 | 296.84 | 251.84 | 289.84 | -192.16 | -54.16 | 108.84 | -61.16 | 84 | 219.84 | 387.84 | -224.16 | 355.84 | -131.16 | -88.16 | 9.84 | 217.84 | 22.84 | 185.84 | 323.84 | -133.16 | 4.84 | 2021 |
| 2021 | 165.84 | 303.84 | -178.16 | -15.16 | 127 | -42.16 | 34 | 258.84 | 276.84 | -185.16 | 369.84 | -212.16 | -7 | 8.84 | 23 | 61.84 | 199.84 | 367.84 | -119.16 | -106.16 | -151.16 | 16.8 | 29.84 | 172.84 | 335.8 | 2021 |
|  | -28.16 | 134.84 | 152.84 | 315.84 | -171.16 | 283.84 | -198.16 | -35.16 | 107.84 | 245.84 | 75.84 | 238.84 | 381.84 | -230.16 | -62.16 | -107.16 | -94.16 | 43.84 | 211.84 | 349.84 | 179.84 | 322.84 | -139. | -1.16 | 41.8 | 2021 |
|  | 206.84 | 344.84 | -112.16 | -99.16 | 84 | -6.16 | , 34 | 174.84 | 342.84 | -144.16 | 310.84 | -176 | -8.16 | 129.84 | 147.84 | 102.84 | 265.84 | 278.84 | -203.16 | -40.16 | -210.16 | -67 | 0.84 | 233.84 | 76. | 2021 |
|  | -87.16 | 50.84 | 213.84 | 356.84 | -130.16 | 324.84 | -132.16 | 5.84 | 18.84 | 186.84 | 141.84 | 154.84 | 297.8 | -16 | -26.16 | -19 | -53.16 | 109.84 | 252.84 | 290.84 | 220.84 | 383.84 | -223.16 | -60.16 | 2.8 | 2021 |
| 202 | 363.84 | -118.16 | -105.16 |  | 200.8 | 30 | . 84 | 336.84 | -15 | 17.84 | -17 | -14.16 | 123.84 | . 84 | 4.84 | 259.84 | . 84 | -184.16 | -46.16 | 84 | -73.16 | 4 | 232.84 | 0.8 | -216.16 | 2021 |
| 2021 | 44 | 212.84 | 350.84 |  | -93.16 | -13 | -0.16 |  | 180.84 | 318.84 | 148.84 | . 84 | -170.16 | 16 | 5.84 | -3 | 103.84 | 6.84 | 284.84 | 97.16 | 382.84 | -229.16 | $-66.16$ | 6.84 | 239.84 | 2021 |
| 2021 | -12 | -8 | 56.84 | 84 | 362.84 | 192.84 | 330.84 | -156 | 11.84 | 24.84 | -20.16 | 284 | 160.84 | 298.84 | -158.16 | 271.84 | -190.1 | 47.1 | 115.84 | 253.84 | 83.84 | 226.84 | 389.84 | -217.16 | 79.1 | 2021 |
| 2021 | 110 | 248 | 291.84 |  | -5 | -22 |  | 78.84 | 22 | 384.84 | 214.84 | 357.84 | -129.16 | -91.16 |  | 6.84 |  | . 84 | 325.84 | 36.16 | 293.84 | -163.16 | -25.16 | 142.84 | 155.8 | 2021 |
| 2021 | -183 | -45.16 | 97.84 | 260.84 | 273.84 | 228.84 | 371.84 | -215.16 | -72 | 84 |  | 84 | 201.84 | 364.84 | -117.16 | 337.84 | -149.1 | 13.8 | 31.84 | 169.84 | 124.84 | 167.84 | 305.84 | -181.16 | -13.1 | 2021 |
| 202 | 247.84 | 285.84 | -201.16 | -33.16 | 104.84 | -65.16 | 77.84 | 240.84 | 378.84 | -228.16 | 351.84 | -110.16 | -92.16 | 45.84 | 208.84 | 38.84 | 181.84 | 319.84 | -137.1 | 0.84 | 6 | -31.16 | 136.84 | 149.84 | 317.8 | 2021 |
| 202 | -51.16 | 6. 84 | 254.8 | 272.84 | -18 | 390.84 | -221.16 | -78.16 | 84.84 | 227.84 | 57.84 | 195.84 | 358.84 | -123.16 | -85.16 | -155. | 12.84 | 5.84 | 188.84 | 331.84 | 161.84 | 299.8 | -157.16 | -19.16 | 118.8 | 2021 |
| 2021 | 279.84 | -202.16 | -39.16 | . 84 | 266.84 | 71.84 | 234.84 | 377.84 | -209.1 | -71.16 | 6.16 | -98.1 | 64.84 | 207.84 | 345.84 | 175.84 | 338.84 | -143. | -5.16 | 37.8 | -7.16 | 130.84 | 143.8 | 311.8 | -175. | 2021 |
| 2021 | 14.84 | . 84 | 170.84 | 333.84 | -148.16 | 306.84 | -180.16 | -12.16 | 125.84 | 3.84 | 93.84 | . 84 | 274.84 | -182.16 | -44.16 | -214.16 | -76.16 | 1.8 | 229.8 | 372.84 | 202.84 | 365.84 | -121.16 | -103. | 9.8 | 2021 |
| 2021 | 320.84 | -1 | 1.84 | 39.84 | 182.84 | 137.84 | 150.84 | 313.84 | -168.16 | -30.16 | -2 | -32.16 | 105.84 | 243.84 | 286.84 | 241.84 | 379.84 | -227.16 | -64.16 | . 84 | 16 | 46.84 | 209.84 | 352.8 | -109. | 2021 |
| 2021 | 26.84 | 189.84 | 332.84 | -15 | 84 | -161 | . 16 | 84 | 2.84 | 300.84 | 255.84 | 268.84 | -188.16 | -50.16 | . 84 | -77. | 85.84 | 223.84 | 391.8 | -220.16 | 359.84 | -122.16 | -84. | 53.84 | 196.84 | 2021 |
| 2021 | -142.16 | -4.16 | 33.84 | 176.84 | 339.84 | 144.84 | 312.84 | -174.16 | -11.16 | 131.84 | -38.16 | 99.84 | 267.84 | 280.84 | -206.16 | 373.84 | -208.16 | -70.16 | 72.84 | 235.84 | 65.84 | 203.84 | 346.84 | -115.16 | -97.1 | 2021 |
|  | 183.84 | 326.84 | -135.16 | 7.84 | 20.84 | -24.16 | 138.84 | 156.84 | 294.84 | -162.16 | 292.84 | -194.16 | -56.16 | 111.84 | 249.84 | 79.84 | 222.8 | 385.8 | -226. | -58.16 | -128.16 | -90.16 | 52.8 | 215.8 | 353.8 | 20 |
|  | 2021 | 202 | 2021 | 202 | 2021 | 2021 | 2021 | 21 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 202 | 2021 | 202 | 202 | 20 |

In this case, the magic squares are bimagic with bimagic sums given by

$$
\begin{aligned}
& {\left[S b_{25 \times 25}: S_{25 \times 25}:=21\right]:=813817.64 } \\
& {\left[S b_{25 \times 25}: S_{25 \times 25}:=441\right]:=821579.24 } \\
& {\left[S b_{25 \times 25}: S_{25 \times 25}:=2021\right] }:=977177.64
\end{aligned}
$$

### 1.19 Block-Bordered Magic Squares of Order 26

Below are block-bordered magic squares of order 26 in four different ways giving magic sums 21, 21 ${ }^{2}$ and 2021. The blockwise magic square considered is of order 24 with blocks of orders $3,4,6$ and 8 . First, the blocks of order 3, magic squares with different magic sums resulting in pandiagonal magic squares and semi-bimagic squares of order 24 . Second, the blocks of order 4, pandiagonal magic squares with equal magic sums resulting in pandiagonal magic squares of order 24. Third, the blocks of order 6 magic squares with equal magic sums. Forth, blocks of order 8 bimagic or semi-bimagic squares of order 8 , resulting in semi-bimagic square of order 24 . The magic squares sums of order 24 are given by

$$
\begin{gathered}
{\left[S_{24 \times 24}: S_{26 \times 26}:=21\right]:=\frac{252}{13}} \\
{\left[S_{24 \times 24}: S_{26 \times 26}:=441\right]:=\frac{5292}{13}} \\
{\left[S_{24 \times 24}: S_{26 \times 26}:=2021\right]:=\frac{24252}{13} .}
\end{gathered}
$$

### 1.19.1 First Type

| -312 9/13 | $3254 / 13$ | -322 9/13 | 323 4/13 | -320 9/13 | 321 4/13 | -318 9/13 | $3194 / 13$ | -316 9/13 | $3174 / 13$ | -3149/13 | $3154 / 13$ | -336 9/13 | $3264 / 13$ | -310 9/13 | $3114 / 13$ | 3089/13 | 309 4/13 | -3069/13 | 307 4/13 | -304 9/13 | $3054 / 13$ | -302 9/13 | 303 4/13 | 300 9/13 | 313 4/13 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -2989/13 | -167 9/13 | -19391 | -144 | 97 | 75 | 122 | 36 | $584 / 13$ | $114 / 13$ | -250 9/13 | -224 9/13 | -273 9/13 | -409/13 | -62 | -15 9/13 | $2584 / 13$ | $2324 / 13$ | $2814 / 13$ |  | 213 4/13 | 164 //13 | -113 9/13 | -919/13 | /13 | 4/13 |  |
|  | -145 | -168 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 | 13 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 20 |  |  |  |  |  | -20 |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  | -110 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -262 |  |  |  |  |  |  |  |  | -179 | -205 9/13 | -156 |  |  |  |  |  |  |  | $2204 / 13$ | 26 | -289/13 | -50 9/13 |  |  |  |
|  | -28 |  |  |  |  |  |  |  |  | -15 | -180 |  |  |  |  |  |  |  |  |  |  |  | - | -52 9/13 |  |  |
|  | -23 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | - | -26 9/13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $1434 / 13$ | 13 | 13 | -165 9/13 | -2899/13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $384 / 13$ |  |  | $1194 / 13$ | $964 / 13$ | 3 | 13 | -214 9/13 | $2904 / 13$ |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 118 4/13 | 13 | 13 | -1889/13 | 28 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $2524 / 13$ | 13 | 13 | -2999/13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -33 9/13 | -589/13 | 274 4/13 | 13 | 13 | -325 9/13 |  |
|  |  |  |  |  |  |  | -210 |  |  |  |  |  |  |  |  |  | -119 | -979/13 | -5 | 13 | -329/13 | $2274 / 13$ | 13 | 13 | 13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -2829/13 | -18 | -206 9/13 | -15 | 13 | 113 | 13 | 13 |  |
|  |  |  |  | -128 9/13 | -10 |  |  | -24 |  |  |  |  |  |  |  | -28 | -258 9/13 | -233 9/13 | -158 9/13 | -1839/13 | -2089/13 | $1364 / 13$ | 13 | 413 | 13 |  |
|  | 203 | 156 |  |  | -13 | -104 |  | 4/13 |  |  |  | 243 |  |  |  | -23 | -2819/13 | -259 9/13 | -207 9/13 | -160 9/13 | -182 9/13 | 89 4/13 | $1384 / 13$ | 112 4/13 | 9/13 |  |
|  | 103 |  |  | -173 9/13 | -191 | -150 | -244 9/13 | -218 | -26 |  |  |  |  | 238 | 287 4/13 | -4 | -68 | -219/13 | -10 | -85 9/13 | -132 9/13 | $1814 / 13$ | $2074 / 13$ | $1584 / 13$ | 13 |  |
|  | 129 |  | 79 | -151 9/13 | -17 | -197 | -266 9/13 | -243 | -220 |  |  |  | 286 | 263 | $2404 / 13$ | -20 | -45 | -70 9/13 | -133 9 | -1089/13 | -83 9/13 | 159 4/13 | 182 4/13 | $2054 / 13$ | -331 9/13 |  |
| -33 | 80 | 127 | 105 | -198 9/13 | -149 | -175 9/ | -219 9/13 | -268 | -2429/13 | 53 | 6 | 28 | $2394 / 13$ | 288 4/13 | 262 4/13 | -69 | -22 9/13 | -449/13 | -849/13 | -131 9/13 | -109 9/13 | $2064 / 13$ | $1574 / 13$ | 183 4/13 | 3344 |  |
| 335 | 253 | 231 | 278 | -35 | -61 9/13 | -12 | -118 9/13 | -92 9/13 | -141 9/13 | 192 4/13 | $2144 / 13$ | 167 | 102 4/13 | $764 / 13$ | $1254 / 13$ | -172 9/13 | -194 9/13 | -147 9/13 | -245 9/13 | -223 9/13 | -270 9/13 | $314 / 13$ | 574/13 | $84 / 13$ | -333 9/13 |  |
| -334 9/13 | 2794 | 254 4/13 | 2294 | -13 | -36 9/13 | -59 9/13 | -140 9/13 | -117 9/13 | -94 9 | $1664 / 13$ | $1914 / 13$ | 216 | 124 4/13 | 101 | 78 4/13 | -146 9/13 | -171 9/13 | -196 9/13 | -271 9/13 | -246 9/13 | $-2219 / 13$ | $94 / 13$ | 32 4/13 | $554 / 13$ | $3364 / 13$ |  |
| 337 4/13 | $2304 / 13$ | 277 4/13 | $2554 / 13$ | -60 9/13 | -119/13 | -379/13 | -93 9/13 | -142 9/13 | -116 9/13 | 215 4/13 | 168 4/13 | $1904 / 13$ | $774 / 13$ | $1264 / 13$ | 100 4/13 | -195 9/13 | -1489/13 | -170 9/13 | -222 9/13 | -269 9/13 | -2479/13 | 56 4/13 | $74 / 13$ | 33 4/13 | -335 9/13 | 2 |
| -3119/13 | -323 9/13 | 324 4/13 | -321 9/13 | 322 4/13 | -319 9/13 | $3204 / 13$ | -317 9/13 | 318 4/13 | -315 9/13 | $3164 / 13$ | -313 9/13 | $3384 / 13$ | -324 9/13 | 312 4/13 | -309 9/13 | $3104 / 13$ | -307 9/13 | $3084 / 13$ | -305 9/13 | $3064 / 13$ | -303 9/13 | 304 4/13 | -301 9/13 | $3024 / 13$ | 314 4/13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


| -29 | $3416 / 13$ | -30671 | 339 6/13 | -304 7/13 | 337 6/13 | -302 7/13 | 335 6/13 | -300 7/13 | 333 6/13 | -298 7/13 | 331 6/13 | -320 7/13 | 342 6/13 | -294 7/13 | 327 6/13 | -292 7/13 | 325 6/13 | -290 7/13 | 323 6/13 | -2887/13 | 321 6/13 | -286 7/13 | $3196 / 13$ | -284 7/13 | 329 6/13 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -282 7/13 | -1517/ | -177 7 | -1287 | 1136 | 91 6/13 | 138 | 52 | 14 | 27 | -234 7/13 | -208 7/13 | -257 7/13 | -247/ | -4 | 6, | 274 6/13 | 248 6/13 | 297 6/13 | 203 6/13 | 2296113 | 180 6/13 | -97 7/13 | -75 7/13 | -122 7/13 | 316 6/13 |  |
|  | -129 | -152 |  | 139 |  |  |  |  |  | -25 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  | -258 7/13 |  |  |  |  |  |  |  | 228 6/13 |  |  |  |  |  |  |  |
|  |  |  |  | 275 | 253 | $3006 / 13$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -223 7/13 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 178 | $2006 / 13$ |  |  |  |  | -138 |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -246 | -22 |  |  |  |  |  | 103 6/13 |  | -16 | -189 7/13 | -14 |  |  |  | 191 6/13 |  |  |  | $2366 / 13$ |  |  | 13 |  |  |  |
|  | -268 |  |  |  |  |  |  | 12 |  |  | -164 7/13 |  |  |  |  | 169 6/13 |  |  | 284 6/13 |  | 238 6/13 |  |  | 13 |  |  |
|  |  | -27 |  |  |  |  | 10 | 14 |  | -18 | -13 | -16 | -6 |  |  | $2166 / 13$ | 167 6/13 |  | 237 6/13 | 13 | 13 | -35 7/13 |  | 13 | 308 6/13 |  |
|  |  | -58 |  | 19 |  |  |  |  | 288 6/13 |  |  |  |  |  |  |  |  |  |  |  |  | -174 7/13 | 13 | 13 |  |  |
|  | -10 | -83 | -60 |  |  |  | 28 | 26 |  |  |  |  |  | -23 |  |  |  |  |  | $1356 / 13$ |  | -148 7/13 | 13 | 113 |  |  |
|  |  |  |  |  |  | 18 | 24 | 28 |  |  |  |  |  | -25 |  |  |  |  |  | 160 6/13 | 13 | -197 7/13 | 13 | 13 |  |  |
|  |  |  |  |  |  |  |  |  | $3-14$ |  |  |  |  |  |  |  |  |  | -187/13 |  |  | 268 6/13 | 13 | 13 |  |  |
|  |  |  |  |  | -239 | -21 | -14 | -170 | -193 7/13 | 157 |  |  |  |  | 20 | -105 7/13 |  | -55 7/13- | 7 6/13 | -17 7/13 |  | 13 | 13 | 13 | -309 7/13 |  |
|  |  |  |  |  |  |  | -194 |  | -171 | 108 |  |  |  |  |  | -5 |  |  |  | $56 / 13$ | -16 7/13 | $2436 / 13$ | 13 | $2666 / 13$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -266 7/13 | -168 7/13 | -190 7/13 |  | $1306 / 13$ | 13 | 153 6/13 |  |  |
|  | 170 |  | 220 |  |  |  |  |  |  |  |  |  |  |  |  | -26 |  | -217 7/13 | -142 7/13 | -167 7/13 | -192 7/13 | 152 6/13 | 129 6/13 | 106 6/13 |  |  |
|  |  |  |  |  | -114 |  |  |  |  |  |  |  |  |  |  |  | -265 | -243 7/13 | -191 7/13 | -144 7/13 | -166 7/13 | 105 6/13 | $1546 / 13$ | 128 6/13 | -313 7/13 |  |
|  | 119 |  |  |  | -183 | -13 | -228 | -202 | -25 |  |  |  |  |  | $3036 / 13$ |  |  |  |  |  | -116 7/13 | 197 6/13 | 223 6/13 | 113 |  |  |
|  | 145 | 120 | 95 | -13 | -158 |  | -250 | -22 | -20 | 20 |  |  |  | 27 | 25 |  | -2 | -54 7/13 | -11 |  | -6 | 175 6/13 | 198 6/13 | 221 6/13 | -315 7/13 |  |
|  | 96 |  |  | -182 | -133 | -15 | -20 | -25 | -22 | 69 |  |  | 25 | 30 | 13 | -5 | -67/13 | -28 7/13 | -6 | -115 | -9 | 222 6/13 | 173 6/13 | 13 | 350 6/13 |  |
|  | 269 | 24 | 294 | -19 | -4 | 3 | -102 7/13 | -76 | -125 7/13 | 20 | 23 | 18 | 118 | 92 |  | -15 | -178 | -1317/13 | -22 | -2077/13 | -254 7 | $476 / 13$ | 73 6/13 | 113 | -317 7/13 |  |
| -31 | 295 | 27 | 245 | 2 | -20 | -43 | -124 7/13 | -10 | -78 | 182 | 20 | 23 | 140 | 117 | 94 6/13 | -13 | -155 7 | -180 7/13 | -255 7/13 | -230 7 | -2057 | 25 6/13 | $486 / 13$ | 71 6/13 | 352 6/13 |  |
| 353 | 246 | 293 | 271 | -4 | 4 | -21 | -77 7/13 | -12 | -100 7/13 | 231 | 184 | $2066 / 13$ |  | 142 | $1166 /$ | -17 | -132 7 | -154 7/13 | -206 7/ | -253 7/13 | -2317 | 72 6/13 | $236 / 13$ | 49 | -318 |  |
| -295 7/13 | -307 | 3406 | -305 7/13 | 338 6/13 | -3037/ | $3366 / 13$ | -3017/13 | 334 6/13 | -299 7/13 | 332 6/13 | -2977/13 | 354 6/13 | -3087/13 | 328 6/13 | -293 7/13 | 326 6/13 | -2917/13 | 324 6/13 | -289 7/13 | 322 6/13 | -2877/13 | 320 6/13 | -285 7/13 | 318 6/13 | 330 6/13 |  |
| 44 | 441 | 441 | 44 | 441 | 441 | 44 | 441 | 441 | 44 | 441 | 441 | 44 | 441 | 441 | 441 | 441 | 441 | 44 | 441 | 441 | 441 | 441 | 441 | 44 | 44 |  |


|  | $4023 / 13$ | -245 | 400 3/13 | -243 10 | 398 3/13 | -241 10/13 | 396 3/13 | -239 10/13 | 3943/13 | -23710/13 | 392 3/13 | -25910/13 | 403 3/13 | -233 10/13 | 388 3/13 | -23110/13 | 386 3/13 | -22910/13 | 384 3/13 | -22710/13 | 382 3/13 | -225 10/13 | 380 3/13 | /13 | 113 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -90 | -116 | -67 | 174 | 152 | 199 |  | 135 3/13 | 883/13 | -173 10/13 | -147 10/13 |  |  |  |  | 13 | 309 3/13 | 13 | 13 | 13 |  |  | 13 |  |  |  |
|  |  |  |  |  |  |  |  |  | 137 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 253 3/13 |  |  |  | 299 3/13 |  |  | 24 3/13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $2283 / 13$ |  |  | 3 |  |  |  | 50 3/13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $2203 / 13$ |  | -135 10/13 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 3 | 13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 | 113 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 | /13 |  |  |
|  |  |  |  |  | -20 |  | -13 |  |  |  |  |  |  |  | 248 3/13 |  |  |  |  |  |  |  | 13 | 13 | 405 3/13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -158 10/13 | -2 | -107 10/13 | -129 10/13 | -82 10/13 |  | 3 | 13 | 13 |  |
|  | 231 |  | 28 | -5 | -28 |  |  |  |  |  |  |  | 98 3/13 |  |  | 3 | -18 | -15 | -8110/13 | -10 | -131 10/13 | 13 | 13 | 13 | 407 3/13 |  |
| 408 3/13 | 280 |  |  |  |  |  |  |  |  |  |  | 320 3/13 |  |  |  | 3 | -20 | -182 10/13 | -130 10/13 | -83 10/13 | -105 10/13 | 13 | /13 | 3/13 | 13 |  |
|  |  |  |  |  | -12 |  |  |  | -19 |  |  |  |  |  |  |  |  |  |  | -810/13 | 13 | 258 3/13 | 3/13 | $2353 / 13$ | 13 |  |
|  |  |  |  |  |  | -12 | -189 10/13 | -166 | -14 |  |  |  |  |  |  |  |  |  | -56 10/13 |  | -6 | $2363 / 13$ | 259 3/13 | 282 3/13 | 13 |  |
|  | 157 |  |  | -12 |  | -98 | -142 10/13 | -191 10/13 | -165 | 130 |  |  |  |  |  |  |  | 32 3/13 |  | -54 10/13 | -32 10/13 | 283 3/13 | $2343 / 13$ | $2603 / 13$ | 13 |  |
|  | 330 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -70 10 | -168 10/13 | -146 10 | -193 10 | 108 3/13 | 3/13 | 85 3/13 | 10/13 |  |
| -257 10/13 | 356 | 331 |  |  |  |  | -63 | -40 | -17 |  | 268 3/13 |  |  |  | 155 | -6910/13 | -94 | -119 10/13 | -194 10/13 | -169 10/13 | -144 10 | 86 3/13 | 109 3/13 | 132 3/13 | 413 3/13 |  |
| $4143 / 13$ | 307 | 354 | 332 | 16 | 65 | 39 | -16 | -65 10/13 | -39 | 292 | 24 | 26 | 154 | 203 | 171 | -118 10/13 | -7110/ | -9310/13 | -145 10/13 | -192 10/13 | -170 10/13 | 133 3/13 | 84 3/13 | $1103 / 13$ | 258 10/13 |  |
| -23410/1 | -246 | $4013 / 13$ | -24 | 399 | -242 10 | 397 3/13 | -240 10/13 | 395 3/13 | -238 | 393 3/13 | -23610/ | 415 3/13 | -24710/13 | 389 3/13 | -232 10/13 | 387 3/13 | -230 10/13 | 385 3/13 | -22810/13 | 383 3/13 | -226 10/13 | 381 3/13 | -224 10/13 | 379 3/13 | 391 3/13 |  |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |  |

In this case, the blocks of order 3 are magic squares with different magic sums.

### 1.19.2 Second Type

| -31 | 325 | -322 | 323 4/13 | -320 | $3214 / 13$ | -318 9/13 | $3194 / 13$ | -316 9/13 | $3174 / 13$ | -314 9/13 | 315 4/13 | -336 9/13 | $3264 / 13$ | -310 9/13 | $3114 / 13$ | -308 9/13 | 309 4/13 | -306 9/13 | 307 4/13 | -304 9/13 | $3054 / 13$ | 302 9/13 | /13 | 13 | 313 4/13 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -28 | -70 | 14 | -286 | 216 | -69 | 143 | -28 | 215 | -68 9/13 | 142 | -284 9/13 | 214 | -6 | $1414 / 13$ | -283 9/13 | 213 | -66 9/13 | 13 | -2829/13 | 212 4/13 | -65 9/13 | 139 4/13 | 13 | 13 | $3004 / 13$ |  |
|  | -215 | 145 | 4/13 | 73 4/13 |  |  | - 9/13 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 288 |  | 72 |  | 287 |  | $714 / 13$ |  | $2864 / 13$ |  | 70 4/13 | -140 9/13 | 28 |  |  |  | $2844 / 13$ |  | 68 4/13 |  | 13 |  |  |  |  |  |
|  |  |  | 217 4/13 |  |  |  | 218 |  |  |  |  | -14 |  |  | 22 |  |  |  |  | -147 9/13 |  |  | 13 |  |  |  |
|  | -64 |  | -280 |  |  | 137 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 282 |  |  |  |  |  | 65 |  | 280 4/13 |  |  |  |  |  |  | -13 | $2784 / 13$ |  | $624 / 13$ |  |  | -2 |  | 13 |  |  |
|  |  |  | 223 4/13 |  |  | -78 9/13 | 22 | -15 |  | -7 | $2254 / 13$ |  |  | -8 | $2264 / 13$ |  |  |  | 227 4/13 |  |  | -82 9/13 | 13 | 13 |  |  |
|  | -5 |  |  | 20 | -5 |  |  |  | -5 | 13 |  |  |  |  |  |  |  | 13 | 13 | 13 |  | $1274 / 13$ |  | 13 |  |  |
|  | -22 |  |  |  | -22 |  |  |  |  | 15 |  |  |  |  |  |  |  |  |  |  | -232 9/13 | 162 4/13 |  |  |  |  |
|  |  |  |  |  |  |  |  | -12 |  | -2 |  | -128 9/13 |  |  |  |  |  |  | $564 / 13$ | -126 9/13 | $2714 / 13$ | -197 9/13 | $554 / 13$ | 13 | $2904 / 13$ |  |
|  |  |  |  |  |  |  | 2304 |  |  |  |  | -157 |  |  | 2324 | -150 |  |  | 23 | -159 | 10413 | -88 | $2344 / 13$ | -160 | 287 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -264 9/13 | 194 4/13 | -47 9/13 | $1214 / 13$ | 13 | 13 | -2999/13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -238 9/13 | 13 |  |  | 13 |  |
|  |  |  |  |  |  |  |  | -12 | $2684 / 13$ |  |  | -122 9/13 |  | -19 |  |  | $2664 / 13$ | -1929/13 | $504 / 13$ | -120 9/13 | 265 4/13 | -19 |  | 13 |  |  |
|  |  |  |  |  |  |  | 236 | -16 |  |  | 231 | -16 |  |  | $2384 / 13$ | -16 |  | 13 | $2394 / 13$ | 3 | 24 4/13 | -949/13 | $2404 / 13$ | 9/13 | 13 |  |
| -32 | -4 | 12 | -26 | 192 |  |  | -2 | $1914 / 13$ |  | 118 | -260 9/13 | 190 | -4 | 117 | -25 | 189 |  | 1116 /13 | -258 9/13 | $1884 / 13$ | -419/13 | $1154 / 13$ | -257 9/13 | 4/13 | 330 4/13 |  |
|  | -23 | 169 | -2 | 97 | -24 | 170 | -2 | 98 | -24 | 171 | -2 | 99 | -2 | 172 | -2 | 100 | -243 9/13 | 173 4/13 | -2 | 13 | -2 | 174 4/13 | -289/13 | $1024 / 13$ | 13 |  |
| -33 | 26 | -19 | 48 | -11 | 263 | -18 | 47 | -111 | 262 4/13 | -18 | 46 | -116 9/13 | 26 | -18 | 45 | -11 | $2604 / 13$ | -186 9/13 | 44 4/13 | -114 9/13 | 259 4/13 | -185 9 | $434 / 13$ | 9/13 | $3324 / 13$ |  |
| 33 | 25 | -95 | 24 | -16 | 26 | -96 | 24 | -16 | 27 | -9 | 2434113 | -169 9/13 | $284 / 13$ | -989/13 | 244 4/13 | -17 | 29 4/13 | /13 | $2454 / 13$ | -171 9/13 | $304 / 13$ | -100 9/13 | $2464 / 13$ | -172 9/13 | 9/13 |  |
| -332 9/13 | -40 | 114 | -256 | 186 | -39 | $1134 / 13$ | -25 | 185 | -389/13 | 112 4/13 | -254 9/13 | 184 4/13 | -37 | 111 | -253 9/13 | 183 4/13 | -36 9/13 | $1104 / 13$ | -252 9 | 182 4/13 | -35 9/13 | 109 4/13 | -251 9/13 | $1814 / 13$ | $3344 / 13$ |  |
| 335 4/13 | -24591 | 175 | -29 9/13 | 103 | -24691 | 176 4/13 | -30 | 104 | -247 9/13 | 177 4/13 | -319/13 | $1054 / 13$ | -2489/13 | 178 4/13 | -32 9/13 | 106 4/13 | -249 9/13 | 179 4/13 | -33 9/13 | 107 4/13 | -250 9/13 | 180 4/13 | -34 9/13 | 108 4/13 | -333 9/13 |  |
| -334 9/13 | 258 4/13 | -184 | 424 | -112 | 257 4/13 | -183 9/1 | $414 / 13$ | -111 | $2564 / 13$ | -182 9/ | $404 / 13$ | -110 9/13 | 255 | -181 9/13 | $394 / 13$ | -109 9/13 | 254 4/13 | -180 9/13 | $384 / 13$ | -108 9/13 | 253 4/13 | -179 9/13 | $374 / 13$ | -107 9/13 | $3364 / 13$ |  |
| 337 4/13 | $314 / 13$ | -101 9/13 | $2474 / 13$ | -173 9/13 | $324 / 13$ | -1029/13 | $2484 / 13$ | -174 9/13 | 33 4/13 | -103 9/13 | 249 4/13 | -175 9/13 | $344 / 13$ | -104 9/13 | 250 4/13 | -176 9/13 | $354 / 13$ | -105 9/13 | $2514 / 13$ | -177 9/13 | $364 / 13$ | -106 9/13 | 252 4/13 | -178 9/13 | -335 9/13 |  |
| -3119/13 | -323 9/1 | 324 4/13 | -321 9/13 | 322 4/13 | -319 91 | $3204 / 13$ | -317 9/13 | 318 4/13 | -315 9/13 | $3164 / 13$ | -313 9/13 | $3384 / 13$ | -324 9/13 | 312 4/13 | -309 9/13 | 310 4/13 | -307 9/13 | $3084 / 13$ | -305 9/13 | $3064 / 13$ | -303 9/13 | 304 4/13 | -3019/13 | $3024 / 13$ | 314 4/13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


|  | $3416 / 13$ | -306 | 33961 | -3047/13 | $3376 / 13$ | -302 | 335 6/13 | -300 | 3336 | 298 7/13 | 33161 | 320 7/13 | 342 6/13 | 294 | 327 6/13 | -292 | 325 6/13 | $2907 / 13$ | 323 6/13 |  | 321 6/13 | 286 | 319 6/13 | 2847113 | 329613 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -54 7/13 | 160 | -270 7/13 | 2326113 | -537/1 | 159 | -269 | 231 | -52 | 158 | -268 | 2306113 | -51 | 157 | -267 7/13 | 229 | -50 | 156 6/13 |  | 228 6/13 | -497/13 | $1556 / 13$ | -265 7/13 | 6/13 |  |  |
| 315 6/13 | -199 | 161 |  |  | 201 | 162 6/13 |  |  | -20 | 163 6/13 |  |  | -202 | 164 6/13 |  |  | 203 | $1656 / 13$ | 6/13 | $936 / 13$ | -204 7/13 | 166 6/13 |  | 4 6/13 | $2817 / 13$ |  |
|  | 304 |  |  |  | 303 |  |  |  | 302 |  |  | -124 7/13 | 30 |  |  | -123 7/13 | 3006113 | 3 | 846113 | 122 7/13 | 2996113 | -193 7/13 | 83 6/13 | -121 7/13 | 314 6/13 |  |
|  | 17 6/13 | -55 7/13 | 233613 |  | 18 6/13 | -56 7/13 | 234613 |  |  | -57 7/13 | 235611 | -129 7/13 | /13 | -587/13 | 236613 | $1307 / 13$ | 216113 | -59 7/13 | 2376 | -1317/13 | 613 | 7/13 | 238 | $1327 / 13$ |  |  |
|  |  |  |  |  |  | $1536 / 13$ | -263 7/13 | $2256 / 13$ |  | 152 6/13 | -262 7/13 | 224 6/13 |  | $1516 / 13$ |  | 22 |  | 150 6/13 | -260 7/13 | 222 6/13 |  | 1496113 | -259 7/13 | 2216113 |  |  |
|  | -205 |  |  |  |  |  |  |  | -20 | 1696113 | $86 / 13$ |  |  | 170 |  |  |  | 1771 6/13 |  | 6/13 |  | 172 6/13 | $56 / 13$ | 1006113 |  |  |
|  | 298 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $2936 / 13$ |  |  |  |  |  |
|  | 23 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $2436 / 13$ |  |  | -66 7/13 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -38 7/13 |  | S | 216 6/13 | -37 7/13 |  | 200 | 13 |  |  |
|  |  |  |  |  |  |  |  |  | -213 |  |  |  |  |  |  |  |  | 177613 |  |  | -216 7/13 | 178 6/13 | -7/13 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 289 |  |  |  | 288 6/13 | B | 72 6/13 |  | 287 6/13 | -1817/13 | 71 6/13 |  |  |  |
|  |  |  |  |  | 30613 |  |  |  |  |  |  |  |  | -70 7/13 | 248 | -142 7/13 | 33 | -71 7/13 | 249 | -143 7/13 | 34 6/13 | -72 7/13 | 250 | -144 7/13 |  |  |
|  | -36 |  |  |  | -35 7/13 |  |  |  |  | 140 |  |  | -33 7/13 | 139 |  | 211 6/13 | -32 7/13 | $1388 / 13$ |  | 210 6/13 | -317 | 137 6/13 |  | 2096113 |  |  |
|  | -217 | 179 |  |  |  | 180 |  |  | -219 |  |  |  | -22 |  |  | 1106713 | -22 | $1836 / 13$ | -5 7/13 |  | 222 | 184 6/13 | -6 7/13 | $1126 / 13$ |  |  |
|  | 286 |  |  |  | 285 |  |  |  | 284 |  |  |  | 283 |  | 67 6/13 |  | 282 | 3 | 66 6/13 | -104 7/13 | $2816 / 13$ | -175 7/13 | 65 6/13 | -103 7/13 |  |  |
|  | 35 |  | 251 |  |  |  |  |  |  |  |  | -147 7/13 | 38 6/13 |  |  |  |  |  | 255 | -149 7/13 | 406113 | -78 7/13 | 256 |  |  |  |
|  | -30 |  |  |  |  |  |  |  |  |  |  |  | -27 | $1336 / 13$ |  |  | -26 7 | 1326 | -242 7/13 |  | -25 7/13 | $1316 / 13$ | 241 | 203 6/13 |  |  |
|  | -223 |  |  |  |  |  |  |  |  |  |  | 115613 |  | 188 | -10 7/13 | $1166 / 13$ | -227 | 18961 | -117/13 | $1176 / 13$ | -228 | 190 | -12 7 | 1186713 |  |  |
|  | 280 |  | 64 |  | 279 | -17 | $636 / 13$ |  | 278 | -172 7/13 | 62 6/13 | 3 | 271 | -1711/13 | $616 / 13$ | -99 7/13 | 276 | - | 606 | -987/13 | 2756 | -169 7/13 | 596 | 971 | $3486 / 13$ |  |
|  | $416 / 13$ | -79 7/13 | 257 6/13 |  | 42613 | -80 7/13 | 258 6/13 | -152 7/13 | 436113 | -81 7/13 | 259 6/13 | -153 7/13 | 446113 | -82 7/13 | 260 6/13 | -154 7/13 | 45 | -83 7/13 | 26161 | -15 | 46 6/13 | -84 7/13 | 262 | 156 | -315 7/13 |  |
|  | -24 | 130 | -240 7/13 | 202 | -23 | $1296 / 13$ | -239 7/13 |  | -22 7/13 | 128 6/13 | -238 7/13 | 20 | -21 | 127 | -237 7/13 | 6/13 | -20 71 | 126 6/13 | -236 7/13 | 19861 | -1971 | 6 | 235 | 1976 | $3506 /$ |  |
| 351 6/13 | -22971 | 19 | -137/13 | 1196113 | 230 | 192 | -147/13 | 6 | -2317 | 193613 | -15 | $1216 / 13$ | -232 | 194 | -16 7/13 | 6/13 | -233 7 | 19561 | -177/13 | 123 | 234 | 1966 | -187/13 | 146 | -317 7/13 |  |
| -3187/13 | 274 | -168 | 58 | -967/13 | 273 | -16 | 576 | -957/13 | 272 | -166 7/13 | $566 / 13$ | -94 7 | 271 6/13 | -165 | 556 | -93 7/ | 27061 | -164 7 | 546 | -92 7 | 2696 | -63 7 | $536 / 13$ | -917/13 | 352 6/13 |  |
| 353 6/13 | 47 | -85 | 263 6/13 | -157 7/13 | 48 | -86 | 264 | -158 7/13 | 496113 | -87 7/13 | 265 | -159 7/13 | 50 | -88 | 266 | -160 7/13 | 516 | -89 7 | 2676 | -16 | 52 | -90 7/13 | 268 | -162 7/13 | -319 7/13 |  |
| -2957/ | -307 7/1 | 340 6/13 | -30571 | 338 6/13 | -303 7/13 | 33661 | -3017/13 | $3346 / 13$ | -2997 | 33261 | -297 7 | 354 6/13 - | -3087 | 328 6/13 | -29371 | 326 6/13 | -2917/13 | 32461 | $2897 / 13$ | 322 6/13 | 287 | 320 6/13 | -285 7 | 318 6/13 | 330 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


| -235 10/13 | 402 3/13 | -24510/13 | 400 3/13 | -24310/13 | 398 3/13 | -24110/13 | 396 3/13 | -239 10/13 | 394 3/13 | -23710/13 | 392 3/13 | -25910/13 | 403 3/13 | -233 10/13 | 388 3/13 | -23110/13 | 386 3/13 | -22910/13 | 384 3/13 | -22710/13 | 382 3/13 | -22510/13 | 380 3/13 | -223 10/13 | $3903 / 13$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -221 | $63 / 13$ | $2213 / 13$ | -20910/ | 293 3/13 | $73 / 13$ | $2203 / 13$ | -20810/13 | 292 3/13 | $83 / 13$ | 219 3/13 | -20710/13 | $2913 / 13$ | $93 / 13$ | 2183/13 | -20610/13 | $2903 / 13$ | 10 3/13 | 2173/13 | -205 10/13 | 289 3/13 | $113 / 13$ | 216 3/13 | -20410/13 | 288 3/13 | 377 3/13 |  |
| 376 3/13 | -138 10/13 | 222 3/13 | 13 | 150 3/13 | -139 10/13 | 223 3/13 | $763 / 13$ | 151 3/13 | -140 10/13 | $2243 / 13$ | 75 3/13 | 152 3/13 | -141 10/13 | 225 3/13 | 74 3/13 | 153 3/13 | -142 10/13 | $2263 / 13$ | 73 3/13 | 154 3/13 | -14310/13 | $2273 / 13$ | 72 3/13 | 155 3/13 | /13 |  |
|  | 365 3/13 | -13 | 1493 | -65 10/13 | 364 3/13 | -136 10/ | 148 3/13 | -6410/13 | 363 3/13 | -135 10/13 | 147 3/13 | -63 10/13 | 362 3/13 | -134 10/13 | 146 3/13 | -62 10/13 | 13 | -133 10/13 | 145 3/13 | /13 | 360 3/13 | -132 10/13 | 3/13 | 13 | 113 |  |
|  | 78 | 5 3/13 | 29 | -6 | 79 | $43 / 13$ | $2953 / 13$ | -6 | 80 | 3 3/13 | $2963 / 13$ | -68 | 813/13 | $23 / 13$ | 297 3/13 |  | 82 3/13 |  | 13 | 13 | 83 3/13 | 3/13 | 13 | /13 | 13 |  |
|  | 12 | 215 | -2 | 28 | 13 | 21 | -202 | 28 | 14 | 213 3/13 | -20 | 28 | 15 3/13 | 13 | -200 10/13 | 28 |  | $2113 / 13$ | 13 | 13 |  | /13 | 13 | 13 | 13 |  |
| 37 | -144 | 228 | 71 | 156 | -14 | 229 | 70 | 15 | -14 | 230 | 69 | 158 | -147 10/13 | $2313 / 13$ | 68 3/13 | 13 | 13 | 23 | 67 3/13 | 13 | -14910/13 | 13 | 13 | 13 | 13 |  |
| -215 10/13 | 359 | -13 | 143 | -5 | 358 | -13 | 142 | -58 | 357 | -12 | 14 | -5 | 356 3/13 | -128 10/13 | 14 | -5610/13 | 13 | -127 10/13 | 13 | 13 | 13 | 13 | 13 | 13 |  |  |
| 37 | 84 | -10/13 | 30 | -72 10/13 | 85 | -110/13 | 301 3/13 | -7 | 86 3/13 | -210/13 | 302 3/13 | -7410/13 | 87 3/13 | -310/13 | 303 3/13 | -75 10/13 | 88 3/13 | -4 10/13 | 3043/13 | -7610/13 | 89 3/13 | -510/13 | 13 | 13 | 13 |  |
|  | 18 3/13 | 20 | -19 | 28 | 19 3/13 | 20 | -19 | 28 | $203 / 13$ | 20 | -195 10/13 | 27 |  | $2063 / 13$ | -194 10/13 | $2783 / 13$ | 22 3/13 | 205 3/13 | 13 | 277 3/13 | 23 3/13 | 13 | -192 10/13 | 13 | 369 3/13 |  |
| 368 3/13 | -150 10/13 | $2343 / 13$ | 65 | 162 3/3 | -151 10/1 | 2353 | 64 /13 | 163 313 | -152 10 | $2363 / 13$ | 63 | 164 313 | - $5310 / 3$ | 237 3/3 | 62 313 | 165 /3 | -154 10/13 | 238 3/3 | 6 Ј | 166 3/13 | -155 | 23) 315 | 60 3/3 | 167 3/3 | -2120 |  |
| -21110/13 | 353 | -12 | 137 |  | 35 | -12 | 136 | -5 | 35 | -12 | 135 | -5110/13 | 350 3/13 | -12 | 13 | -50 10/13 | 349 3/13 | -121 10/13 | 133 3/13 | -4910/13 | 13 | -120 10/13 | 13 | 3 | 367 3/13 |  |
|  | 90 |  | 306 | -78 |  |  | 307 3/13 | -79 |  | -8 | 308 3/13 | -80 | 93 | -910/13 | 30 | -8 | 943/13 | 13 | 310 3/13 | -82 10/13 | 13 | 13 | 13 | 13 | 13 |  |
| 37 | 24 | 203 3/13 | -191 | 275 | 25 | 2023 | -190 10/13 | 274 3/13 | $263 / 13$ | $2013 / 13$ | -189 10/13 | 273 3/ | 27 3/13 | $2003 / 13$ | -18 | 272 3/13 | $283 / 13$ | $1993 / 13$ | -187 10/13 | 271 3/13 | 13 | 198 3/13 | -186 10/13 | $2703 / 13$ | 13 |  |
|  | -156 | 24 | 59 | 168 3/13 | -15 | 2413 | 58 | 169 3/13 | -15 | 242 | 57 | 170 3/13 | -159 10/13 | 243 3/13 | 56 3/13 | 171 3/13 | -160 10/13 | 244 3/13 | 55 3/13 | $1723 / 13$ | -161 10/13 | 245 3/13 | 54 3/13 | $1773 / 13$ | -24810/13 |  |
| -2 | 347 | -11 | 131 |  | 346 3/13 | -118 | 130 | $-4610 / 13$ | 345 | -117 10/13 | 129 3/13 | -4510/13 | 344 3/13 | -116 10/13 | $1283 / 13$ | -4410/13 | 343 3/13 | -115 10/13 | 127 3/13 | $-4310 / 13$ | 342 3/13 | -114 10/13 | 126 3/13 | -42 10/13 | 405 3/13 |  |
| 406 3/13 | 96 | -12 10/13 | 312 | -8410/13 | 97 | -13 | 313 | -8 | 98 | -14 | 314 3/13 | -86 | 99 | -1510/13 | 315 3/13 | -8710/13 | $1003 / 13$ | -1610/13 | 316 3/13 | -8810/13 | $1013 / 13$ | -1710/13 | 317 3/13 | -8910/13 | 13 |  |
| -2 | 30 | 19 | -18 | 269 | 31 | 196 | -18 | 268 3/13 | 32 | 195 | -183 10/13 | 267 3/13 | 33 | $1943 / 13$ | -18 | 266 3/13 | $343 / 13$ | 193 3/13 | -181 10/13 | 265 3/13 | 35 3/13 | 192 3/13 | -180 10/13 | 264 3/13 | $4073 / 13$ |  |
| 408 | -162 | 24 | 53 | 174 | -16 | 247 | 52 | 175 3/13 | -16 | 248 | 51 | $1763 / 13$ | -165 10/13 | 249 3/13 | 50 | 177 3/13 | -166 10/13 | $2503 / 13$ | $493 / 13$ | 178 3/13 | -167 10/13 | 251 3/13 | 48 3/13 | 179 3/13 | -252 10/13 |  |
| -253 10/13 | 34 | -113 10/13 | 125 | -41 | 340 3/13 | -112 | $1243 / 13$ | $-4010 / 13$ | 339 3/13 | -111 10/13 | 123 3/13 | -39 10/13 | 338 3/13 | -110 10/13 | 122 3/13 | -38 10/13 | 337 3/13 | -109 10/13 | $1213 / 13$ | -3710/13 | 336 3/13 | -108 10/13 | $1203 / 13$ | -36 10/13 | 409 3/13 |  |
| 410 | 102 | -1810/13 | 318 3/13 | -90 10/13 | 103 3/13 | -1910/ | 319 3/13 | -9110/13 | $1043 / 13$ | -2010/13 | $3203 / 13$ | -92 10/13 | 105 3/13 | -21 10/13 | $3213 / 13$ | -9310/13 | 106 3/13 | -22 10/13 | 322 3/13 | -94 10/13 | 107 3/13 | -2310/13 | 323 3/13 | -9510/13 | -254 10/13 |  |
| -25510/13 | $363 / 13$ | 191 3/13 | -179 10/13 | 263 3/13 | 37 3/13 | $1903 / 13$ | -178 10/13 | 262 3/13 | $383 / 13$ | 189 3/13 | -177 10/13 | $2613 / 13$ | $393 / 13$ | 188 3/13 | -176 10/13 | $2603 / 13$ | $403 / 13$ | 187 3/13 | -175 10/13 | 259 3/13 | $413 / 13$ | 186 3/13 | -174 10/13 | 258 3/13 | $4113 / 13$ |  |
| 412 3/13 | -168 10/13 | 252 3/13 | 47 3/13 | $1803 / 13$ | -169 10/13 | 253 3/13 | $463 / 13$ | 181 3/13 | -170 10/13 | 254 3/13 | 45 3/13 | 182 3/13 | -171 10/13 | 255 3/13 | $443 / 13$ | 183 3/13 | -172 10/13 | $2563 / 13$ | 43 3/13 | 184 3/13 | -173 10/13 | 257 3/13 | $423 / 13$ | 185 3/13 | -256 10/13 |  |
| -25710/13 | 335 3/13 | -107 10/13 | $1193 / 13$ | -3510/13 | $3343 / 13$ | -106 10/13 | 118 3/13 | -3410/13 | 333 3/13 | -105 10/13 | $1173 / 13$ | -3310/13 | 332 3/13 | -104 10/13 | $1163 / 13$ | -32 10/13 | $3313 / 13$ | -103 10/13 | 115 3/13 | -31 10/13 | $3303 / 13$ | -102 10/13 | $1143 / 13$ | -30 10/13 | 413 3/13 |  |
| $4143 / 13$ | 108 3/13 | -2410/13 | 324 3/13 | -9610/13 | 109 3/13 | -25 10/13 | 325 3/13 | -9710/13 | $1103 / 13$ | $-2610 / 13$ | $3263 / 13$ | -9810/13 | $1113 / 13$ | -27 10/13 | 327 3/13 | -9910/13 | 112 3/13 | -2810/13 | $3283 / 13$ | -100 10/13 | 113 3/13 | -29 10/13 | $3293 / 13$ | -101 10/13 | -258 10/13 |  |
| -234 10/13 | -24610/13 | 401 3/13 | -244 10/13 | 399 3/13 | -242 10/13 | 397 3/13 | -240 10/13 | 395 3/13 | -23810/13 | 393 3/13 | -23610/13 | 415 3/13 | $-24710 / 13$ | 389 3/13 | -232 10/13 | 387 3/13 | -230 10/13 | 385 3/13 | -22810/13 | 383 3/13 | -22610/13 | 381 3/13 | -224 10/13 | 379 3/13 | $3913 / 13$ |  |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 |  |

In this case, the blocks of order 4 are pandiagonal magic squares with equal magic sums. These sums are given by

$$
\begin{gathered}
{\left[S_{4 \times 4}: S_{26 \times 26}:=21\right]:=\frac{42}{13}} \\
{\left[S_{4 \times 4}: S_{26 \times 26}:=441\right]:=\frac{882}{13}} \\
{\left[S_{4 \times 4}: S_{26 \times 26}:=2021\right]:=\frac{4042}{13} .}
\end{gathered}
$$

### 1.19.3 Third Type

|  | 325 | -322 | 323 4/13 | -320 9/13 | 321 4/13 | -318 9/13 | 319 4/13 | -316 9/13 | 317 4/13 | -314 9/13 | 315 4/13 | -336 9/13 | $3264 / 13$ | -310 9/13 | $3114 / 13$ | -3089/13 | 309 4/13 | -306 9/13 | 307 4/13 | -304 9/13 | $3054 / 13$ | -302 9/13 | 303 4/13 | -300 9/13 | 313 4/13 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -286 | 257 | 256 | 225 | -255 | -191 | -285 | 258 | 25 | 226 4/13 | -256 9/13 | -192 9/13 | -28 | 25 | 254 4/13 | 22 | -257 9/13 | -193 9/13 | 13 | 26 | 13 | 13 | 13 | 13 | $3004 / 13$ |  |
|  |  |  | 160 |  |  | 97 4/13 |  |  | 159 | -15 |  |  | $1904 / 13$ |  |  |  |  |  |  |  |  |  | 3 |  |  |  |
|  | $964 / 13$ |  |  |  |  |  |  | $664 / 13$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -190 |  |  |  |  |  |  |  | -128 9/13 |  |  |  |  |  |  |  |  |  |  |  | 13 |  |  |  |  |  |
|  |  |  |  |  | 224 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -282 |  |  | 229 | -25 |  |  | 262 |  | 230 | -260 9/13 |  | -28 |  |  |  |  |  |  |  |  | 13 | -262 9/13 | 13 |  |  |
|  | 188 |  | 156 |  |  |  | $1874 / 13$ | -16 |  |  |  |  |  |  |  | -15 | -120 9/13 |  |  |  |  | 13 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $1364 / 13$ | $1684 / 13$ | 13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | -2 |  |  | 218 4/13 | 282 4/13 | 13 |  | -247 9/13 | -2309/13 | 217 4/13 | $2814 / 13$ | -2879/13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 235 | -265 9/13 | -201 |  | 2684 | $2454 / 13$ | $2364 / 13$ | -260 | -202 9/13 | -269 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | -169 |  | -148 |  |  |  | - | 149 4/3 | -147 9/13 | -115 9/13 | $1084 / 13$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -42 9/13 |  | 13 | 328 4/13 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -749/13 |  |  | -199/13 |  |  |  |
|  |  | 12 |  |  |  | -10 |  | 119 | -13 | 138 | 170 |  | -18 |  | -13 | 139 |  | -105 9/13 | -179 9/13 | $1174 / 13$ | -138 9/13 | $1404 / 13$ | 172 4/13 | -106 9/13 | 33 |  |
|  | 20 | -21 | -24 | -23 | 216 | 280 | 20 | -21 | -24 | -23 | 215 | 279 4/13 | 203 | -212 9/13 | -24 | -23 | 214 /13 | 278 4/13 | 204 4/13 | -2119/13 | -243 9/13 | -234 9/13 | 213 4/13 | 277 4/13 | -329 9/13 |  |
| -33 | -27 | 269 | 24 | 23 | -26 | -203 9/13 | -27 | 270 | 243 | $2384 / 13$ | -268 | -204 9/13 | -27 | 271 | 242413 | 23 | -269 9/13 | -205 9/13 | -271 9/13 | $2724 / 13$ | $2414 / 13$ | $2404 / 13$ | -270 9/13 | -206 9/13 | 332 4/13 |  |
|  | 180 | -17 | 148 | -14 | -11 | 109 | 179 4/13 | -172 | 14 | -14 | -113 | $1104 / 13$ | 178 | -173 | 146 4/13 | -14 | -112 9/13 | $1114 / 13$ | $1774 / 13$ | -174 9/13 | $1454 / 13$ | -143 9/13 | -1119/13 | $1124 / 13$ | -331 9/13 |  |
|  | 84 | 71 | -50 | -4 | 20 | -82 |  | 78 4/13 | -49 | -4 | 19 | -81 |  | 79 | -489/13 | -4 | 18 4/13 | -80 9/13 | $814 / 13$ | $804 / 13$ | -47 9/13 | -46 9/13 | 17 4/13 | -79 9/13 | 334 4/13 |  |
| 335 | -11 | -75 | 45 | 52 | -18 | 13 |  | -76 | 46 | 51 | -17 | 14 | -13 | -77 9/13 | $474 / 13$ | $504 / 13$ | -16 9/13 | $154 / 13$ | -149/13 | -78 9/13 | $484 / 13$ | 49 4/13 | -159/13 | $164 / 13$ | -333 9/ |  |
| -334 9/13 | -178 9/13 | 1164 | -139 | 141 | 173 | -107 9/ | -177 9/13 | 115 4/13 | -14091 | $1424 / 13$ | 174 | -1089/13 | -176 9/13 | $1144 / 13$ | -141 9/13 | $1434 / 13$ | 175 4/13 | -109 9/13 | -175 9/13 | $1134 / 13$ | -1429/13 | $1444 / 13$ | $1764 / 13$ | -110 9/13 | 3364 |  |
| 337 4/13 | $2054 / 13$ | -210 9/1 | -24291 | -235 9/13 | 212 4/13 | 276 4/13 | $2064 / 13$ | -209 9/ | -2419/13 | -236 9/ | $2114 / 13$ | 275 4/13 | 207 4/13 | -2089/13 | -24091 | -2379/13 | $2104 / 13$ | 274 4/13 | 208 4/13 | -207 9/1 | -239 9/1 | -2389/13 | 209 4/13 | $2734 / 13$ | -335 9/13 |  |
| -3119/13 | -323 9/13 | 324 4/13 | -321 9/1 | 322 4/13 | -319 9/1 | $3204 / 13$ | -317 9/13 | 318 4/13 | -315 9/13 | $3164 / 13$ | -313 9/13 | $3384 / 13$ | -324 9/13 | 312 4/13 | -309 9/13 | 310 4/13 | -307 9/13 | $3084 / 13$ | -305 9/13 | $3064 / 13$ | -303 9/13 | 304 4/13 | -3019/13 | $3024 / 13$ | 314 4/13 |  |
|  | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |  |


|  | $3416 / 13$ | -3067 | 339 6/13 | 304 | 337 6/13 | 3027 | 335 6/13 | 300 | 3336113 | 2987113 | $3316 / 13$ | -320 7/13 | 3426113 | 2947113 | 327613 |  | 325 6/13 | -290 7/13 | 323 6/13 | 2887113 | 321613 | 2867113 | 319 6/13 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -270 7/1 | 2736113 | 272 6/13 | 24161 | -239 7 | -175 7/13 | -2697 | 274 | $2716 / 13$ | 242 6/13 | $2407 / 13$ | -176 | 268 | 275 6/13 | 270 6/13 | 243 6/13 | 2417 | -177 7/13 | 267 | $2766 / 13$ | $2696 / 13$ | $2446 / 13$ | 242 |  | $3166 / 13$ |  |
|  | 208 |  |  |  |  |  | 207 |  | 1756 | -1417/13 |  | 114 | 206 |  | 1746113 | -140 7/13 |  | 13 | 205 | -146 7/13 | 173613 | -139 7/13 |  | $1176 / 13$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $1096 / 13$ | 84 6/13 | $-437 / 13$ | -187/13 | $456 / 13$ | -757/13 |  |  |
|  | $166 / 13$ | -47 |  | $806 / 13$ | -14 7/13 |  | 15 6/13 | -487/13 | 50 6/13 |  |  |  |  | -497/13 | 51 6/13 | 78 6/13 |  | 19613 | $136 / 13$ | -50 7/13 | 52 6/13 | 6/13 | -117/13 | 206113 |  |  |
|  | -174 | 144 | -111713 | $1456 / 13$ | 177 6/13 |  | -173 7/13 | 1436113 | -112 7/13 | 146 6/13 | 178 6/13 | -80 7/13 | -172 7/13 | $1426 / 13$ | -113 7/13 | 147613 | 1796113 | -817/13 | -171 7/13 | 113 | -114 7/13 | 1486113 | 180 6/13 | 82 7/13 | 3126113 |  |
|  | 209613 |  |  |  |  | 304 6/13 | 210 |  |  |  | 23 | 303 | 211 | 204 | -236 7/13 |  | $2386 / 13$ | 30261 | 212 | -203 7/13 | -235 7/13 | -210 7/13 | 237 | 301 |  |  |
|  |  |  | 268 | $2456 / 13$ |  |  |  |  | 267 6/13 | 24 |  |  |  |  | 266 6/13 | 247 6/13 |  |  |  | 280 6/13 | 265 6/13 | $2486 / 13$ | -2467/13 |  |  |  |
|  | 204 |  |  | -138 |  |  | 203 |  | 171 6/13 |  |  | 6/3 | 202 |  | 170613 | -136 7/13 |  | 613 | $2016 / 13$ | -150 7/13 | 169613 | -135 7 | -103 7/13 | 120 6/13 |  |  |
|  | 108 |  |  |  |  |  | 107 |  |  |  |  |  | 106 6/13 |  |  |  | 42 6/13 |  | 105 | 88 6/13 |  |  | 613 |  |  |  |
|  | 12 |  | 53611 |  |  |  |  |  | 54 6/13 |  |  |  |  |  |  |  |  |  |  | -54 7/13 | 56 |  |  |  |  |  |
|  | 170 | 140 | -115 | 149 6/13 |  |  |  | 1396113 | -116 7/13 | $1506 / 13$ |  |  |  |  |  | $1516 / 13$ | $1836 / 13$ |  |  | $6 /$ | -118 | 152 6/13 | 46 | -86 7/13 |  |  |
|  | 21361 |  |  |  |  |  | 214 |  |  |  |  | $2996 / 13$ | 215 |  |  |  | 23 | 298 6/13 | 216 | -19971 | -231 7/13 | -214 7/13 | 2336 | 2976113 |  |  |
|  | 262 | 281 | 264 |  |  |  | -261 |  | 263 | 2506 |  |  | 260 | 2836 | 262 | 25 |  | -185 7/13 | -259 | 2846 | 261613 | 252 6/13 | 250 |  |  |  |
|  | 2006 |  |  |  |  |  |  |  | 167 |  |  | 122 6/13 | 198 | -153 7/13 | 166 | -13 | -100 7/13 | 123 6/13 | 197 | -154 7/13 | 165 | -1317/13 | -99 | 124 6/13 |  |  |
|  | 104 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -68 7/13 |  | 92 6/13 |  | -26 7/13 | 37 6/13 | 7/13 |  |  |
|  | $86 / 13$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -58 7/13 | 606113 | 696113 | -37/13 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $-897 / 13$ |  | $1336 / 13$ | -122 7/13 | 156 6/13 | 188 6/13 | -90 7/13 |  |  |
|  | 217 |  |  |  |  |  | 218 |  |  |  | 2316113 | 29 |  |  |  |  | 2306 | 2946113 | 220 | -195 7/13 | -227 7/13 |  | 2296113 | 2936113 |  |  |
|  | -258 | 285 | 260 | 253 6/13 |  |  |  |  | 259 6/13 |  | -252 7/13 |  |  | 287 | 258 6/13 | 255 | -253 7/13 | -189 7/13 |  | 288 6/13 | 2576 | 256 | 254 |  |  |  |
|  | $1966 /$ | -155 | 164 |  |  |  | 195 |  | 163 |  |  |  |  | -157 7/13 | 1626 | -12 | - | 127 6/13 | 1936 | -158 7/13 | 1616 | -127 7 | -95 | 128 |  |  |
|  | 1006 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $346 / 13$ |  |  | 966 | -317 | -30 7/13 | 336173 | 63 | 350 |  |
|  | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  | 636113 |  | - 7/13 | $316 / 13$ | $16 / 13$ | -62 7/13 | 64 | $656 / 13$ | 6/13 | 32 6/13 |  |  |
|  | -162 | 132 6/13 | 1237 | 157 6/13 | 1896113 | 13 | -161 7/13 |  | -124 7/13 | 158 6/13 | 1906113 | 7/13 | -160 | 130 | -125 7/13 | 1596113 | 1916113 | -93 | -159 | 1296 | -126 | 160 | 192 | -94 | 352 6/13 |  |
| 6/13 | 221613 | -194 | -226 | -219 | 2286 | 6/13 | 222 6/13 | -193 7/13 | -22 | -220 7/13 | 227613 | $2916 / 13$ | $2236 / 13$ | -1927 | -224 | -221 7/13 | $2266 / 13$ | 2906113 | 2246113 | -1917 | -223 | -222 7 | 22561 | 2896 | -319 |  |
| 2951/13 | -307 7/ | 34061 | -3057/13 | 33861 | -303 | 336 6/13 | -3017 | 3346 | -2997 | 332 6/13 | -29771 | 354 6/13 | -308 | 328 6/13 | -2937 | 326 6/13 | -2917 | 324 6/13 | -289 7 | 322 6/13 | 287 | 320 6/13 | -285 | 318 6/13 | 3306 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


|  | 402 3/13 | -24510/13 | $4003 / 13$ | -24310/13 | 398 3/13 | -24110/13 | $3963 / 13$ | -239 10/13 | 3943/13 | -23710/13 | 392 3/13 | -25910/13 | 403 3/13 | -233 10/13 | 388 3/13 | -231 10/13 | 386 3/13 | -22910/13 | 384 3/13 | -22710/13 | 382 3/13 | -22510/13 | 380 3/13 | -22310/13 | 390 3/13 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -209 10/13 | 334 3/13 | 3333 | 302 3/13 | -178 10/13 | -114 | -208 10/13 | 335 3/13 | 332 3/13 | 303 3/13 | -179 10/13 | -115 10/13 | -20710/13 | 336 3/13 | $3313 / 13$ | $3043 / 13$ | -180 10/13 | -116 10/13 | -206 10/13 | 337 3/13 | 330 3/13 | 3/13 | -181 10/13 | 10/13 | 113 |  |
|  | 269 |  | 237 |  |  |  | 268 3/13 |  | 236 |  |  | 175 3/13 |  |  | 235 |  |  |  |  | 13 |  |  |  | 13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 205 |  | 20 |  |  |  | 20 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 270 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 |  |  |  |
|  | -20 |  |  | 306 3/13 | -18 |  |  | 33 | 32 |  |  |  |  |  |  | 3083/13 |  |  |  |  | 326 3/13 | 13 | 13 | 13 |  |  |
|  |  |  |  |  |  |  |  |  | 232 3/13 |  |  |  |  |  |  |  |  |  |  |  | $2303 / 13$ |  |  | 181 3/13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 53 3/13 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1983/13 | -57 10/13 | 213 3/13 | 245 3/13 | -25 10/13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  | 360 3/13 |  |  |  |  |  | 359 3/13 | 277 3/13 | -138 10/13 | -170 10/13 | -153 10/13 | $2943 / 13$ | 358 3/13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 313 3/13 | -189 10/13 |  |  |  |
|  |  |  |  |  |  |  |  |  | 228 |  |  |  |  |  | 22 |  |  | 18 | 25 |  | $2263 / 13$ | -70 10/13 |  | 13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 3 | 13 | 13 | 13 |  |  |
|  |  |  |  |  |  |  |  | 19 |  |  | 247 3/13 | -2 |  | 195 |  |  | 248 3/13 | -2810/13 | -102 10/13 | $1943 / 13$ | 13 | 13 | 249 3/13 | -2910/13 | 407 3/13 |  |
|  | 27 | -137 10/13 | -16 | -15 | 29 | 35 | 27 | -13 | -16 | -15 | 29 | 35 | 280 3/13 | -135 10/13 | -16 | -156 10/13 | 29 | 355 3/13 | 13 | 10/13 | -166 10 | -157 10/13 | $2903 / 13$ | 3/13 | -252 10/13 |  |
|  | -197 | 34 | 32 | 314 3/13 | -19 | -12 | -19 | 34 | 32 | 31 | -19110/13 | -12 | -195 10/13 | 348 3/13 | 31 | 13 | -192 10/13 | -12 | -194 10/13 | 349 3/13 | 318 3/13 | 317 3/13 | -193 10/13 | -129 10/13 | 409 3/13 |  |
| 41 | 257 |  | 225 | -6 | -37 | 186 | 25 | -95 | 22 | -68 | -3 | 187 3/13 | 25 | -96 | 223 3/13 | -6710/13 | -3510/13 | 188 3/13 | $2543 / 13$ | 10/13 | 222 3/13 | -6610/13 | 10/13 | 189 3/13 | -254 10/13 |  |
| -25 | 161 | 154 | 26 | 33 | 91 |  |  | 155 | 27 | 32 |  |  | 15 | 156 | 28 | 3 | 13 | -310/13 | 13 | 157 3/13 | 29 3/13 | $303 / 13$ | /13 | -210/13 | $4113 / 13$ |  |
|  | 65 | 1 |  | 129 | 58 | 90 |  | 3/13 | 123 |  | 59 | 91 | 63 | - 1 | $1243 / 13$ | 127 3/13 | $603 / 13$ | 92 | 62 3/13 | -1 | 125 3/13 | $1263 / 13$ | $613 / 13$ | 93 3/13 | 13 |  |
| -257 10/13 | -101 | 193 3/13 | -62 | 218 3/13 | 250 3/13 | -30 | -100 10/13 | 192 3/13 | -63 10 | 2193/13 | $2513 / 13$ | -3110/13 | -9910/13 | 191 3/13 | -64 10/13 | $2203 / 13$ | 252 3/13 | -32 10/13 | -9810/13 | $1903 / 13$ | -65 10/13 | $2213 / 13$ | 253 3/13 | -33 10/13 | 413 3/13 |  |
| 414 3/1 | 282 3/13 | -133 10/ | -165 10/ | -158 10/13 | 289 3/13 | 353 3/13 | 283 3/13 | -132 10/13 | -164 10/ | -159 10/13 | 288 3/13 | 352 3/13 | $2843 / 13$ | -131 10/13 | -163 10 | -160 10/13 | 287 3/13 | $3513 / 13$ | 285 3/13 | -130 10/13 | -162 10/ | -161 10/13 | 286 3/13 | 350 3/13 | -258 10/13 |  |
| -23410/ | -24610/13 | 401 3/13 | -24410/ | 399 3/13 | -242 10/13 | 397 3/13 | -24010/13 | 395 3/13 | -23810/ | 393 3/13 | -236 10/13 | 415 3/13 | $-24710 / 13$ | 389 3/13 | -232 10/13 | 387 3/13 | -230 10/13 | 385 3/13 | -228 10/13 | 383 3/13 | -22610/13 | 381 3/13 | -224 10/13 | 379 3/13 | 391 3/13 |  |
| 2021 | 2021 | 021 | 021 | 2021 | 202 | 021 | 2021 | 202 | 202 | 202 | 2021 | 2021 | 202 | 2021 | 202 | 202 | 2021 | 202 | 2021 | 202 | 202 | 202 | 202 | 202 | 2021 |  |

In this case, the blocks of order 6 are magic squares with equal magic sums. These sums are given by

$$
\begin{gathered}
{\left[S_{6 \times 6}: S_{26 \times 26}:=21\right]:=\frac{63}{13}} \\
{\left[S_{6 \times 6}: S_{26 \times 26}:=441\right]:=\frac{1323}{13}} \\
{\left[S_{6 \times 6}: S_{26 \times 26}:=2021\right]:=\frac{6063}{13} .}
\end{gathered}
$$

### 1.19.4 Forth Type

| -31 | 325 | -322 9/13 | 323 4/13 | -320 9/13 | $3214 / 13$ | -3189 | 319 4/13 | -316 9/13 | 317 4/13 | -314 9/13 | 315 4/13 | -336 9/13 | 326 4/13 | -310 9/13 | $3114 / 13$ | -3089/13 | 309 4/13 | -306 9/13 | $3074 / 13$ | -304 9/13 | $3054 / 13$ | -302 9/13 | 303 4/13 | 300 9/13 | 13 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -28 | -144 | 122 | $114 / 13$ | -273 | -15 | 28 | 16 | -13 | -19 | $754 / 13$ | $584 / 13$ | -22 | -62 9/13 | $2324 / 13$ | 213 4/13 | -91 9/13 | -167 9/13 | $974 / 13$ | 36 4/13 | -250 9/13 | 13 | 13 | 13 | 13 | $3004 / 13$ |  |
|  | -18 |  | $1614 / 13$ | -135 |  |  | $24 / 13$ |  |  | 237 | 208 |  |  |  |  |  |  | 259 4/13 |  |  | 13 |  | $254 / 13$ |  |  |  |
|  |  |  |  |  |  |  | 26 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | -691 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $1204 / 13$ | -190 9/13 |  |  |
|  |  |  |  | $1404 / 13$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 155 |  | - |  | 20 | -282 | -159 | 137 | 202 |  |  |  |  |  |  |  |  |  |  | 13 |  | -257 9/13 |  |  |  |  |
|  | 128 | -150 | -267 | 5 |  | -21 |  | 158 |  | -199 | -218 |  |  |  |  |  | 103 4/13 | -17 | -244 9/13 |  | $2644 / 13$ | 13 |  |  |  |  |
|  | 278 |  |  | 167 |  |  |  | 8 |  |  |  |  |  |  |  |  | $2534 / 13$ |  | -1189/13 |  |  | -172 9/13 |  |  |  |  |
|  | -191 |  |  | -22 |  | 234 |  | -89 | -16 | 98 |  |  |  |  |  |  |  |  |  | -2 |  | 280 4/13 |  | 13 |  |  |
|  | -65 |  | 210 | -88 | -202 |  |  | -21 |  | 26 |  |  |  |  |  |  |  |  | 13 | -134 9/13 | 13 | 13 |  | 13 | 13 |  |
|  | -238 |  | 85 | -203 | -71 | 19 | 222 | -52 | -261 |  |  | -180 | -102 | $1764 / 13$ | $2454 / 13$ |  | -28 | $224 / 13$ | $1354 / 13$ |  | -127 9/13 | 13 | 268 | -29/13 | $2904 / 13$ |  |
|  | -76 | 19 | 22 |  |  |  |  |  |  | 17 | $2484 / 13$ |  |  |  |  |  | -122 9/13 | 14 | 273 4/13 |  | 3 | $154 / 13$ | $1424 / 13$ | 13 | 13 |  |
|  |  |  | -20 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -278 9/13 | -16 | 14 | $1474 / 13$ | -121 9/13 | -89/13 | 13 | 13 |  |
|  | 204 |  |  |  |  | -23 |  |  |  |  | -24 |  |  | -25 |  |  |  | -128 9/13 |  | $2674 / 13$ | $214 / 13$ | -28 | 13 | 13 | 13 |  |
|  |  |  |  |  |  |  |  |  |  |  | $-243$ |  |  |  |  |  | 129 4/13 | -15 |  |  | 13 | -20 9/13 | -133 9/13 | 13 | $3284 / 13$ |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -13 | -140 9/13 | 1664 |  | -146 9/13 | -2719 | 4/13 | 9/13 |  |
|  |  |  |  |  |  |  |  | -115 |  |  |  | -274 |  |  | 163 4/13 | -137 |  |  |  | -225 9 |  | $2334 / 13$ | 212 4/13 | -90 9/13 | $3304 / 13$ |  |
|  |  |  |  |  |  |  |  | $-24$ |  | $283$ | $162$ | -136 | $-154$ |  |  | -263 9/13 |  | $2364 / 13$ | $2094 / 13$ | -87 | -2019/13 | $834 / 13$ | 504/13 | -216 9/13 | -329 9/13 |  |
|  | -260 |  |  |  |  |  |  |  |  |  | 133 | -15 | $-12$ |  |  |  | -2 | $714 / 13$ |  | -20 | -789/13 | $2004 / 13$ | $2214 / 13$ | 13 | 332 4/13 |  |
|  |  |  | 24 | -3 |  |  |  | -18 | -12 | 150 | $2714 / 13$ |  | $-27$ |  | $1444 / 13$ | -16 |  |  | 13 | -5 | -2289/13 | 62 4/13 | 13 | -213 9/13 | -331 9/13 |  |
| -332 | 40 | -25 | -18 | $1174 / 13$ |  |  |  | 25 |  | -28 | -16 | 13 | $1454 / 13$ | -1199/13 | -10 | $2764 / 13$ | $654 / 13$ | -2 | -21 | $924 / 13$ | $1944 / 13$ | 13 | -57 | 13 | $3344 / 13$ |  |
| 33 | 178 | -10 | -2 | 24 | 45 | -259 9/13 | -182 | 112 | $1564 / 13$ | -130 | 4/13 | $2654 / 13$ | 19 | -2819/13 | -160 9/13 | $1384 / 13$ | $2034 / 13$ | -81 9/13 | -489/13 | $2184 / 13$ | $684 / 13$ | -234 9 | -2079/13 | $894 / 13$ | -333 9 |  |
| -334 9/13 | 1054 | -175 9/13 | -242 | 284 | 262 | -4491 | -109 9/13 | 183 4/13 | 127 4/13 | -149 9/13 | -2689/13 | $64 / 13$ | 288 4/13 | -22 9/13 | -131 9/13 | $1574 / 13$ | $804 / 13$ | -198 9/13 | -219 9/13 | 53 4/13 | $2394 / 13$ | -69 9/13 | -84 9/13 | $2064 / 13$ | 3364 |  |
| $3374 / 13$ | 2554 | -37 9/13 | -116 9/1 | $1904 / 13$ | $1004 / 13$ | -170 9/13 | -2479/13 | 33 4/13 | $2774 / 13$ | -119/13 | -1429/13 | 168 4/13 | $1264 / 13$ | -1489/13 | -269 9/13 | $74 / 13$ | $2304 / 13$ | -60 9/13 | -93 9/13 | 215 4/13 | $774 / 13$ | -195 9/13 | -222 9/13 | $564 / 13$ | -335 9/ |  |
| -311 9/13 | -323 9/ | 324 4/13 | -321 9/13 | 322 4/13 | -319 9/13 | $3204 / 13$ | -317 9/13 | $3184 / 13$ | -315 9/13 | $3164 / 13$ | -313 9/13 | $3384 / 13$ | -324 9/13 | 312 4/13 | -309 9/13 | 310 4/13 | -307 9/13 | $3084 / 13$ | -305 9/13 | $3064 / 13$ | 303 9/13 | 304 4/13 | -301 9/13 | $3024 / 13$ | $3144 / 13$ |  |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |  |


|  | 341 | -306 | 339 | -304 | 337 | -30 | 335 | -300 7/13 | 333 6/13 | -298 7/13 | 331 6/13 | -320 7/13 | 342 6/13 | -294 7/13 | 327 6/13 | -292 7/13 | 325 6/13 | -290 7/13 | 323 6/13 | -2887/13 | 321 6/13 | -286 7/13 | 319 6/13 | $2847 / 13$ | 329 6/13 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -282 | -128 | 138 | 27 | -25 | 6/13 | 297 6/13 | 180 | -122 7/13 | -17 | $916 / 13$ | 74 6/13 | -2087/13 | -467/13 | 248 6/13 | 229 6/13 | -75 7/13 |  |  |  | 3 | 3 | 13 | 13 | 713 | 316 6/13 |  |
|  |  | 30 |  |  | -137 |  | 18 6/13 |  |  | 25 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 158 6/13 | -148 7/13 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -262 7/13 | -147 7/13 |  |  | -105 7/13 | 7 6/13 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 | -1 | 152 6/13 |  |  |
|  |  |  |  |  |  |  |  |  |  | -15 |  |  |  |  |  |  |  | -13 | -250 7/13 |  |  | -4 | 13 | 13 |  |  |
|  | 24 |  | -7 |  |  | -18 |  |  |  |  |  | 207 |  | -15 | -23 |  |  | 2 6/13 | 13 | $1826 / 13$ | $1406 / 13$ | 7/13 | -255 7/13 | 13 | 13 |  |
|  |  |  |  |  | -22 |  | 20 |  |  |  |  | -2 |  | $2986 / 13$ | 17 | -121 7/13 | -176 | 90 | 75 6/13 | -20 | 13 | 13 | 13 | 13 | $3466 / 13$ |  |
|  | -27 | 27 | 20 |  | -16 |  | 43 6/13 | -22 |  |  | 178 | -12 | -138 | 148 |  | 13 | -5 | 252 6/13 | 225 6/13 | 13 | -185 7/13 | 99 6/13 | 13 | -200 7/13 | -313 7/13 |  |
|  | -24 | 62 | 12 | -16 | -87 | 19 | 260 | -10 | -27 | 40 | 1496113 | -13 | -109 7/13 | 167 6/13 | 28 | 11 6/13 | -221 7/13 | 87 6/13 | 13 | -188 7/13 | -62 7/13 | /13 | 237 6/13 | -35 7/13 | 13 |  |
|  | -82 | 188 | 265 | -15 | -2377/13 | 55 | 134 | -172 7/13 | -108 | 166 | 28 | 10 | -259 7/13 | 29 | $1606 / 13$ | -150 7/13 | -59 7/13 | 213 6/13 | 240 6/13 | -387/13 | -212 7/13 | 78 6/13 | 13 | /13 | 13 |  |
|  | 56 | -238 | -171 | 133 | 187 6/13 | -8 |  | $2666 / 13$ | 34 | -26 | -14 | 155 | 161 6/13 | -10 | 5 |  | $816 / 13$ | -215 7 | -1947/13 | 108 6/13 | 210 6/13 | -56 7/13 | -417/13 | 243 6/13 | 6/13 |  |
| 351 | 194 | -88 | -9 | 25 | 61 | -24 | -166 | 128 6/13 | 172 | -114 | 16 | 281 | 35 | -265 7/13 | -144 7 | 154 6/13 | 219 6/13 | -6 | -32 7/13 | $2346 / 13$ | 84 6/13 | -218 7/13 | -1917/13 | 105 6/13 | 13 |  |
| -318 | 121 6/1 | -159 7/13 | -22671 | 44 | 278 6/13 | -28 | -93 |  | 143 | -133 7/ | -252 7 | 22 6/13 | 304 6/13 | -67/13 | -115 | 173 6/13 | 96 6/13 | -1827 | -203 7/13 | $696 / 13$ | 255 6/13 | -53 7/13 | -687/13 | 222 6/13 | 352 6/13 |  |
| 353 6/13 | 2716 | -2171 | -100 7 | 206 | 116 | -154 | -231 | 49 | 293 | $46 / 13$ | -126 7/ | 184 | 142 6/13 | -132 7 | -253 7 | 23 6/13 | 246 6/13 | -447/13 | -77 7/13 | 231 6/13 | 93 6/13 | -179 7/13 | -2067 | 72 6/13 | -319 7/13 |  |
| -295 7/13 | -3077/13 | 340 6/13 | -305 7/13 | 338 6/13 | -303 7/13 | $3366 / 13$ | -3017/13 | 334 6/13 | -29971 | 3326 | -2977/13 | $3546 / 13$ | -3087/13 | 328 6/13 | -293 7/13 | $3266 / 13$ | -2917/13 | 324 6/13 | -289 7/13 | 322 6/13 | -2877/13 | 320 6/13 | -285 7/13 | 318 6/13 | 330 6/13 |  |
|  | 441 | 4 | 4 | 441 | 41 | 44 | 441 | 44 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 441 | 44 | 441 | 441 | 44 | 44 | 44 |  |  |


|  | 402 3/13 | -245 | $4003 / 13$ | -24310/ | 398 3/13 | -24110/13 | 396 3/13 | -239 10/13 | 394 3/13 | -23710/13 | 392 3/13 | -259 10/13 | 403 3/13 | -23310/13 | 388 3/13 | -23110/13 | 386 3/13 | -22910/13 | 384 3/13 | -22710/13 | 382 3/13 | 22510/13 | 380 3/13 | 22310/13 | 3/13 |  |
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|  | -67 | 199 | 88 | -19 | $613 / 13$ | 358 | 2413/13 | -61 10/13 | -116 10/13 | 152 | 135 3/13 |  |  |  |  |  |  |  |  | 13 | $363 / 13$ | 13 | 13 | 13 |  |  |
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|  |  |  |  |  |  |  |  |  |  |  | -166 10/13 |  | 340 3/13 |  |  |  |  |  | -189 10/13 |  |  | 56 3/13 | 13 | 13 | 405 3/13 |  |
|  |  |  |  |  |  |  |  |  |  | $403 / 13$ |  | 26 |  |  |  |  |  | 63 3/13 |  | 13 | $2013 / 13$ |  | -194 10/13 | 13 | -250 10/13 |  |
|  | -92 |  |  |  | 38 |  |  |  | -6 | 198 |  |  | 60 |  | 24 | -60 10/13 |  | 151 3/13 | 136 3/13 | -14810/13 | 13 3/13 | 13 | 13 | 13 | 407 3/13 |  |
|  | 33 |  | 26 |  |  | 18 |  | -16 | 59 3/13 | 36 | 23 | -5 | -71 | 20 |  | -18610/13 | $103 / 13$ | 313 3/13 | $2863 / 13$ | 13 | -124 10/13 | 13 | 13 | 13 | 13 |  |
|  | -183 | 123 |  | -10 | -26 |  |  |  |  | 101 | 210 | -78 | -48 | $2283 / 13$ |  | 72 | -160 10/13 | 148 |  | -127 10/13 | -110/13 | 13 | 3/13 | 13 | 13 |  |
|  |  |  |  |  |  | 116 |  |  |  | 22 | $3483 / 13$ |  |  | 90 |  | -89 |  | 274 3/13 |  | $223 / 13$ | -151 10/13 | 139 3/13 | /13 | -136 10/13 | -254 10/13 |  |
|  | 117 | -171 | -110 |  |  |  |  | 327 |  | -20 |  |  | 222 |  |  |  |  | -15 | -133 10/13 | 169 3/13 | 2713 | 13 | 19 3/13 | 304 3/13 | 13 |  |
|  | 255 |  | 51 | 320 | 122 | -182 | -105 | 189 | 233 | -53 | 77 |  | 96 | -204 10/13 | -831 |  | $2803 / 13$ | -4 | $283 / 13$ | 295 3/13 | 145 3/13 | -157 10/13 | -130 10/13 | 166 3/13 | -256 10/13 |  |
|  | 182 | -98 | -16 | 105 | 339 | 32 |  | 260 3/13 |  | -72 | -19 | 83 | 36 |  | -5 | $2343 / 13$ | 157 3/13 | -121 10/13 | -142 10/13 | 130 3/13 | 316 3/13 | 7 3/13 | -710/13 | 283 3/13 | 413 3/13 |  |
| 414 3/13 | 332 | 39 | -3 | 267 | 177 | -93 | -170 | 110 | 35 | 65 | -65 10/13 | 24 | 203 | -7110/13 | -192 | 84 3/13 | 307 3/13 | $163 / 13$ | -1610/13 | $2923 / 13$ | 154 3/13 | -118 10/13 | -145 10/ | 133 3/13 | -258 10/13 |  |
| 10/1 | -24 | $4013 / 13$ | -24410/13 | 399 3/13 | -242 10/13 | 397 3/13 | -240 10 | 395 3/13 | -23810 | 393 3/13 | -236 10/13 | $4153 / 13$ | -24710/13 | 389 3/13 | -232 10/13 | 387 3/13 | -230 10/13 | 385 3/13 | -22810/13 | 383 3/13 | -226 10/13 | 381 3/13 | -22410/13 | 379 3/13 | 391 3/13 |  |
| 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 2021 | 202 | 2021 | 202 | 2021 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 2021 |  |

In this case, the blocks of order 8 are magic squares with equal magic sums. These sums are given by

$$
\begin{gathered}
{\left[S_{8 \times 8}: S_{26 \times 26}:=21\right]:=\frac{63}{26}} \\
{\left[S_{8 \times 8}: S_{26 \times 26}:=441\right]:=\frac{1323}{26}} \\
{\left[S_{8 \times 8}: S_{26 \times 26}:=2021\right]:=\frac{6063}{26} .}
\end{gathered}
$$

## 2 Author's Contributions to Magic Squares

The item-wise author's work on magic squares is as follows:

1. Digital Numbers Magic Squares - [9, 10, 11, 12, 13, 14, 41];
2. Block-Wise Construction of Bimagic Squares - [15];
3. Connections with Genetic Tables and Shannon's entropy - [16];
4. Selfie and Palindromic-type Magic Squares - [17, 32, 41];
5. Intervally Distributed and Block-Wise Magic Squares - [18, 19, 20, 32];
6. Multi-digits and Number Patterns Magic Squares - [21, 31];
7. Perfect Square Sum Magic Squares with Uniformity, Minimum Sum and Pythagorean Triples - [22, 23];
8. Block-Wise Constructions of Magic and Bimagic Squares - [24, 25, 26, 27, 30, 33, 35];
9. Magic Crosses: Repeated and Non Repeated Entries - [28];
10. Representations of Letters and Numbers With Equal Sums Magic Squares of Orders 4 and 6 - [29].
11. Bordered Magic Squares - [36, 37, 38, 39, 40].
12. Block-Bordered Magic Squares - [49, [50, [51, 55].
13. 2-Digits Magic and Bimagic Squares - [42, 43, 44, 45, 46, 47, 48, 52, [53, [54].

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