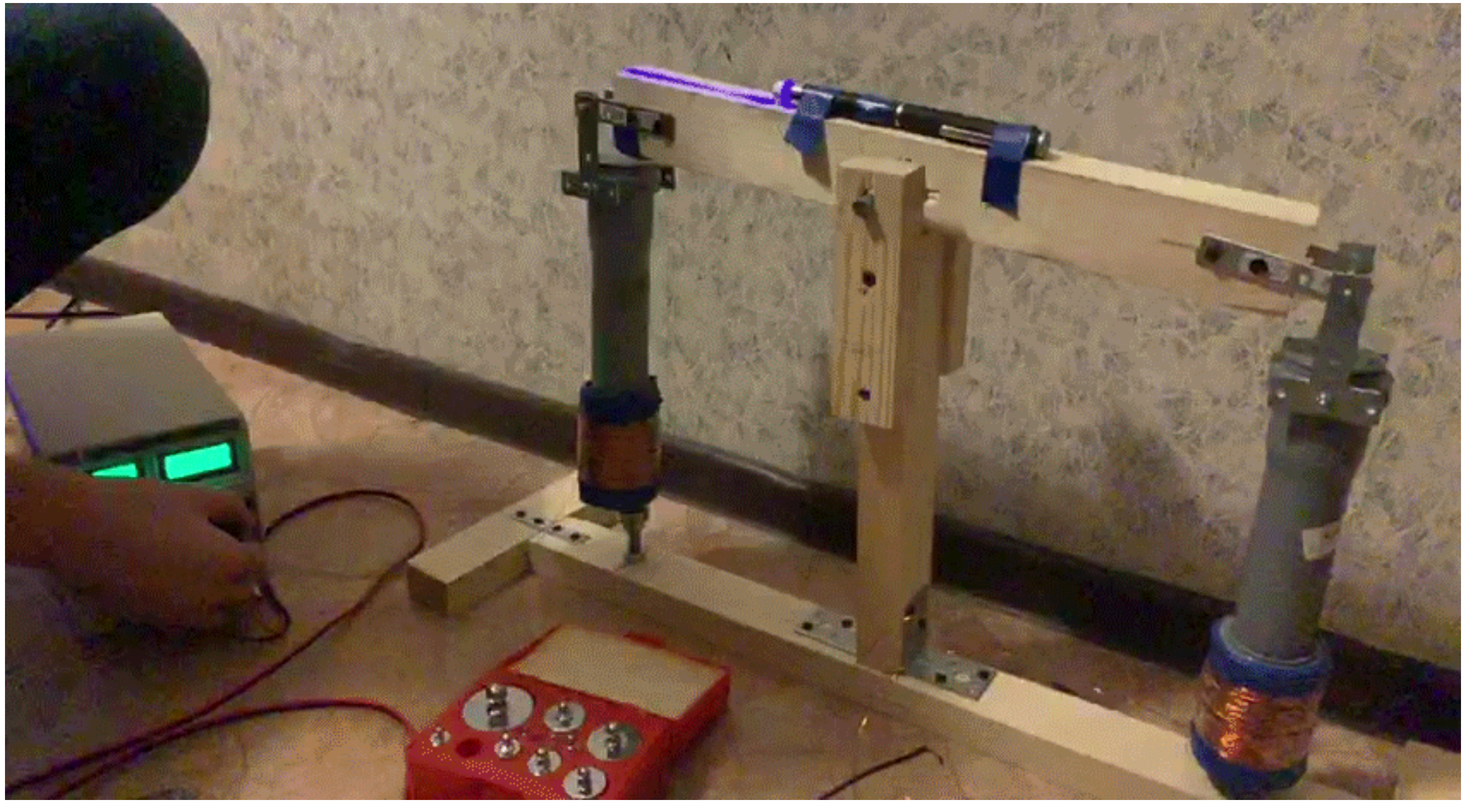
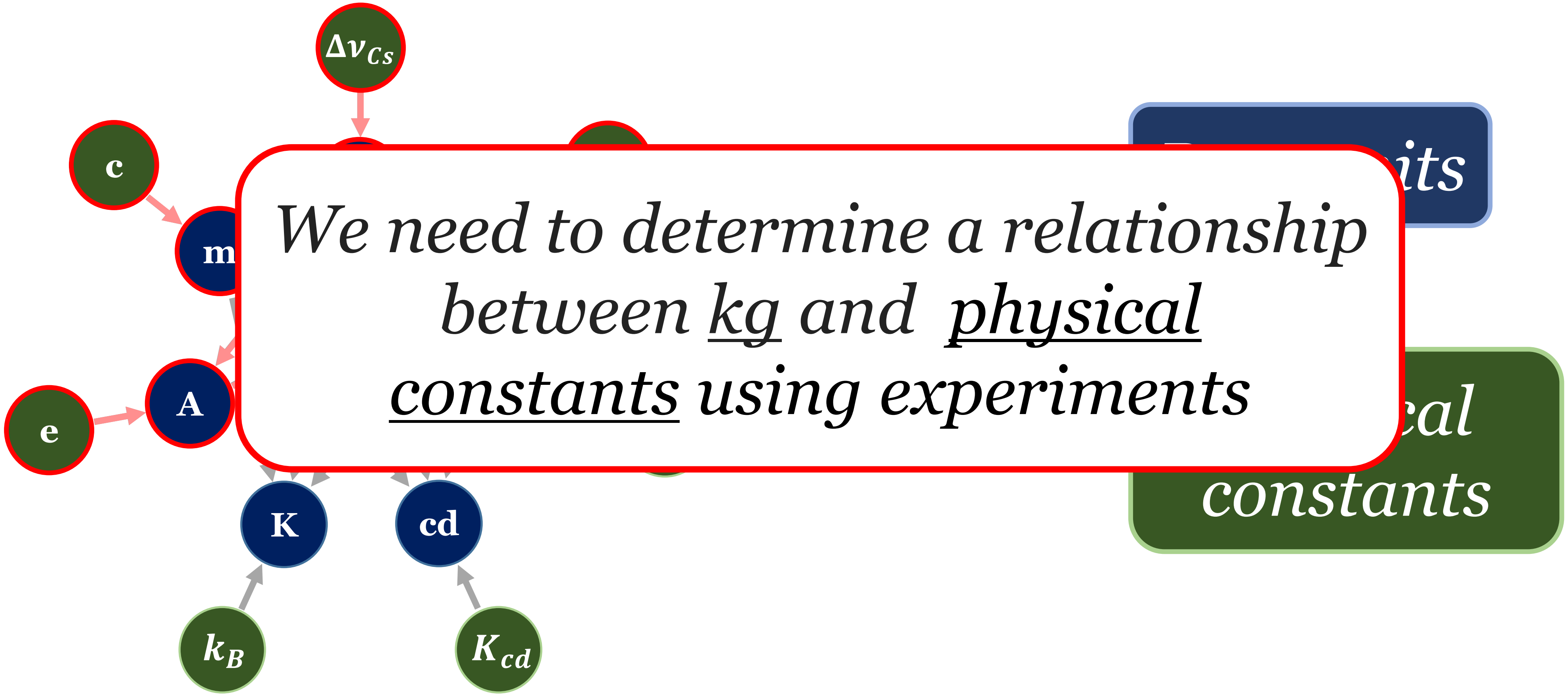
Quantum Gram
Problem No.17

Reporter:
Dmytro Spinov

As of 2019, the International System of **Units (SI)** defines the **kilogram** from the **Planck constant**, which is now defined exactly as 6.62607015×10^{-34} Js. Propose and make a **room-temperature experiment** to calibrate a **weight of one gram** with **maximal precision** using the new definition (you may freely measure the other primary units with your equipment considering them calibrated at the room temperature too).



Translation of the problem into the language of metrology

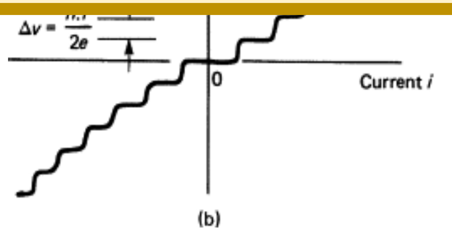


We need to determine a relationship between kg and physical constants using experiments

Research plan

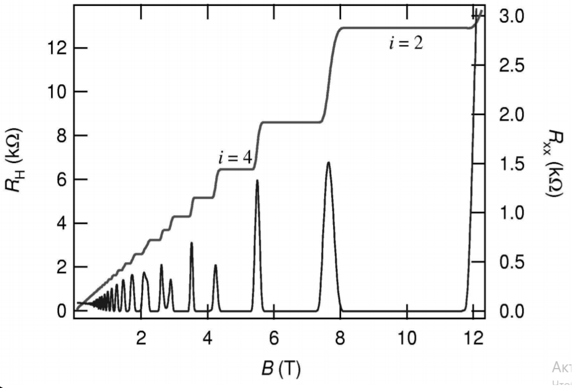
$$m = \frac{UI}{vg}$$

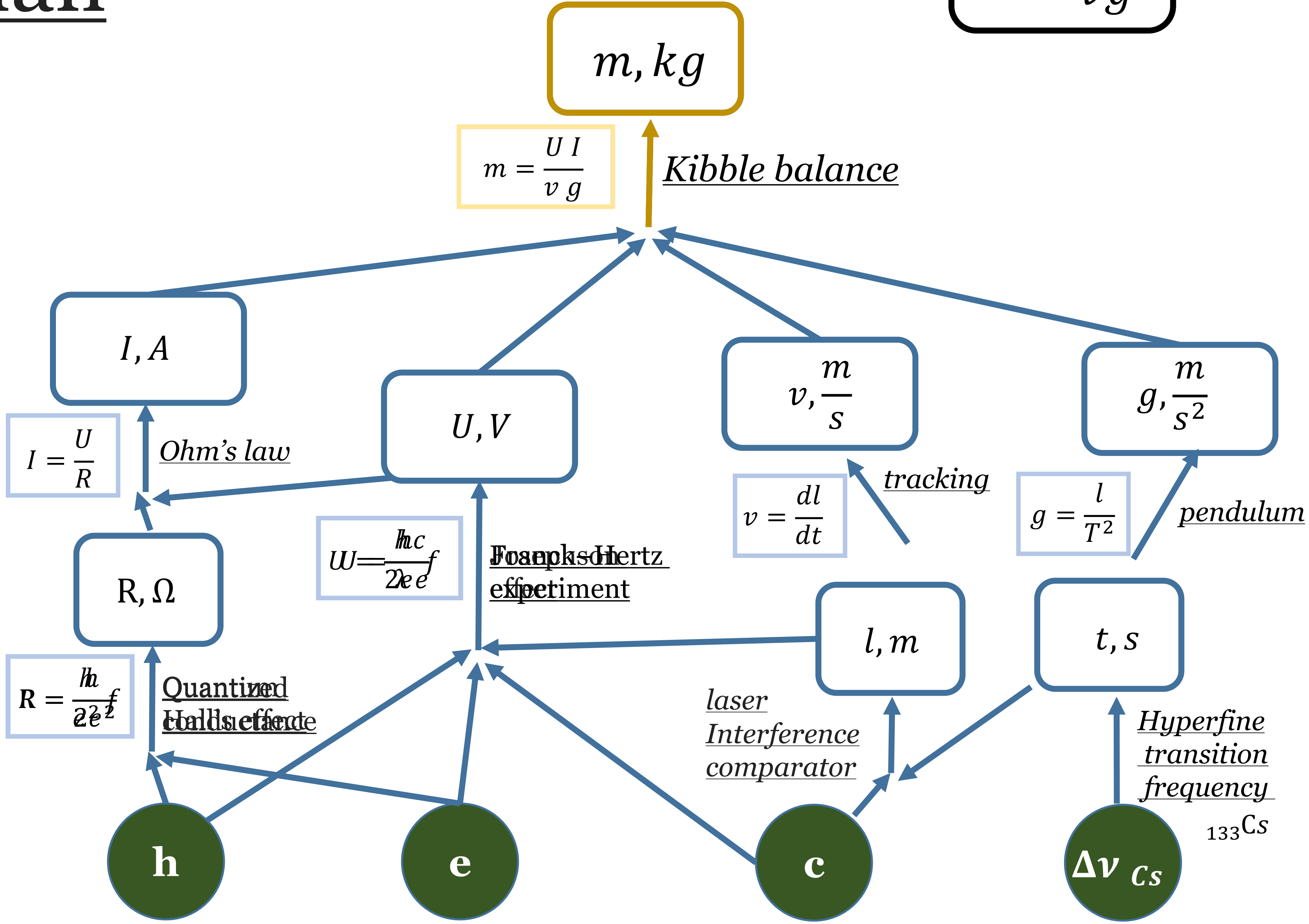
Kibble balance



These experiments need a temperature around 0 K

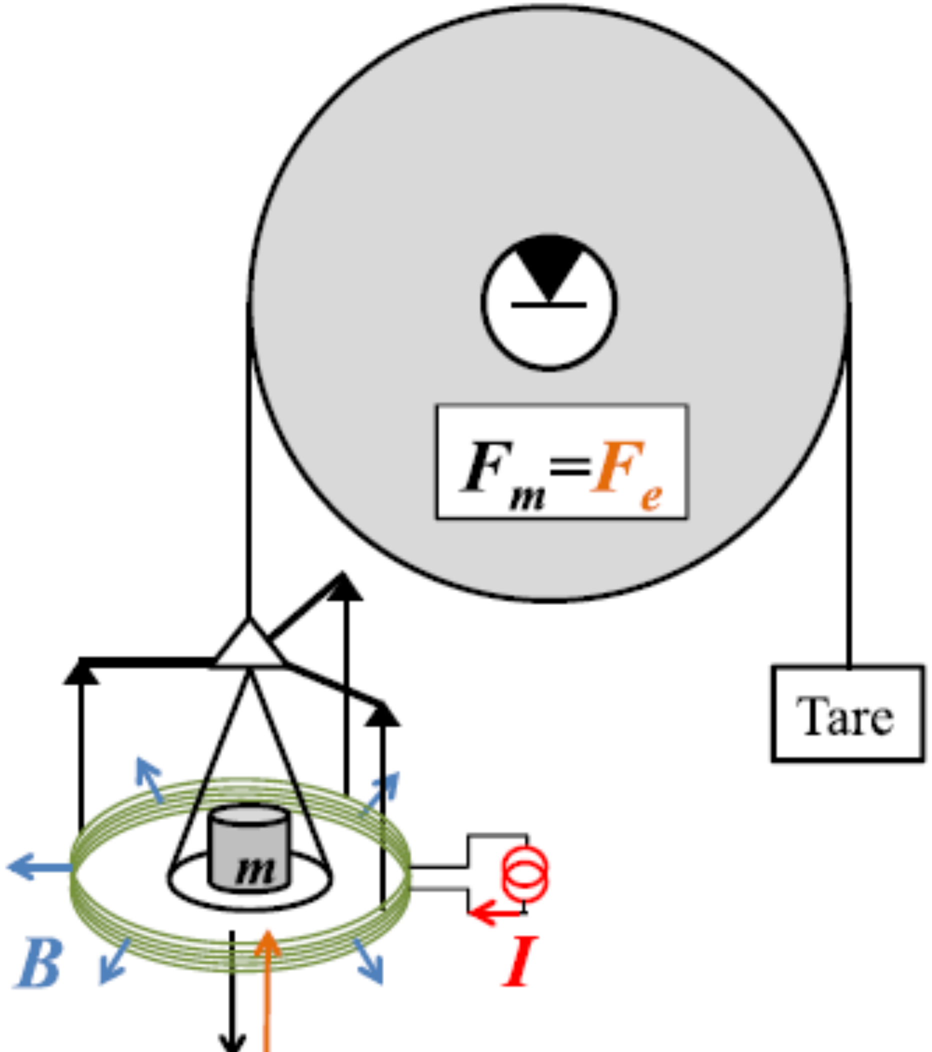
Quantum Hall's effect



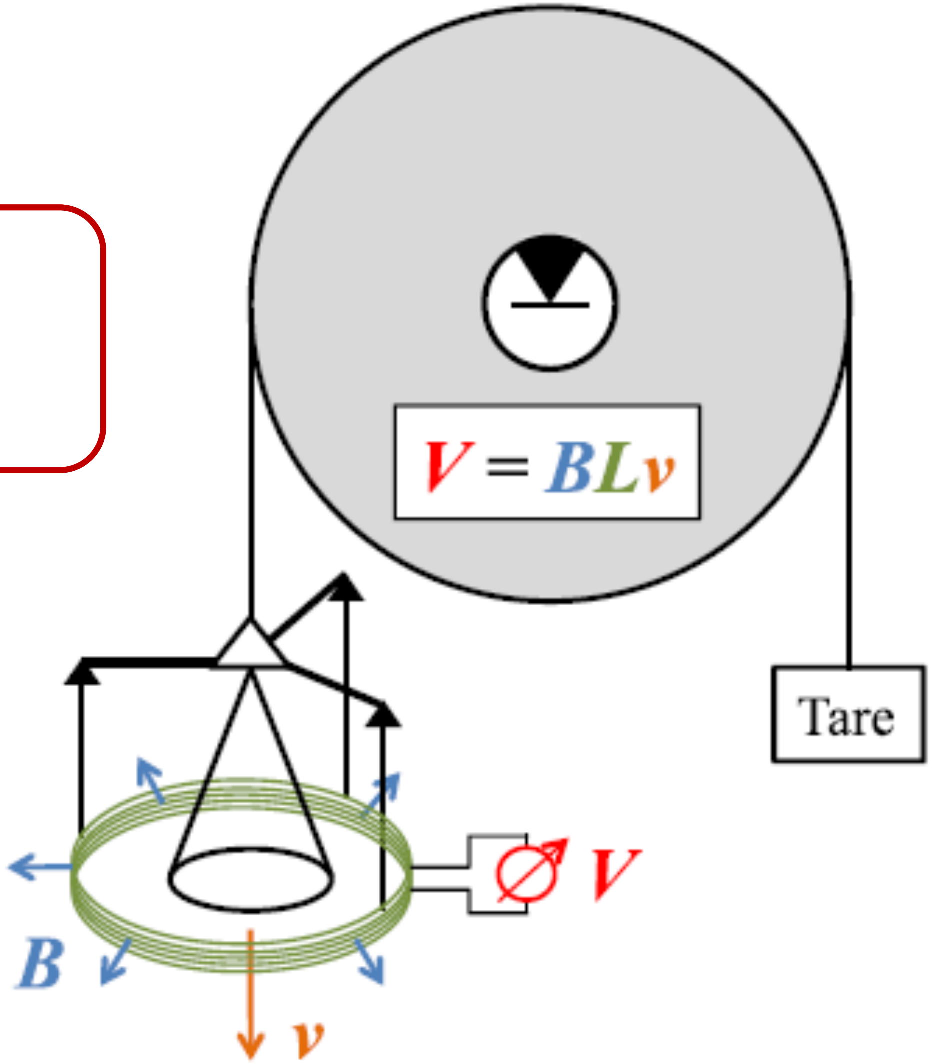


Kibble balance

$$m = \frac{UI}{vg}$$



$$m = \frac{VI}{gv}$$



$F = mg$ $F = BLI$

$V = BLv$

Research plan

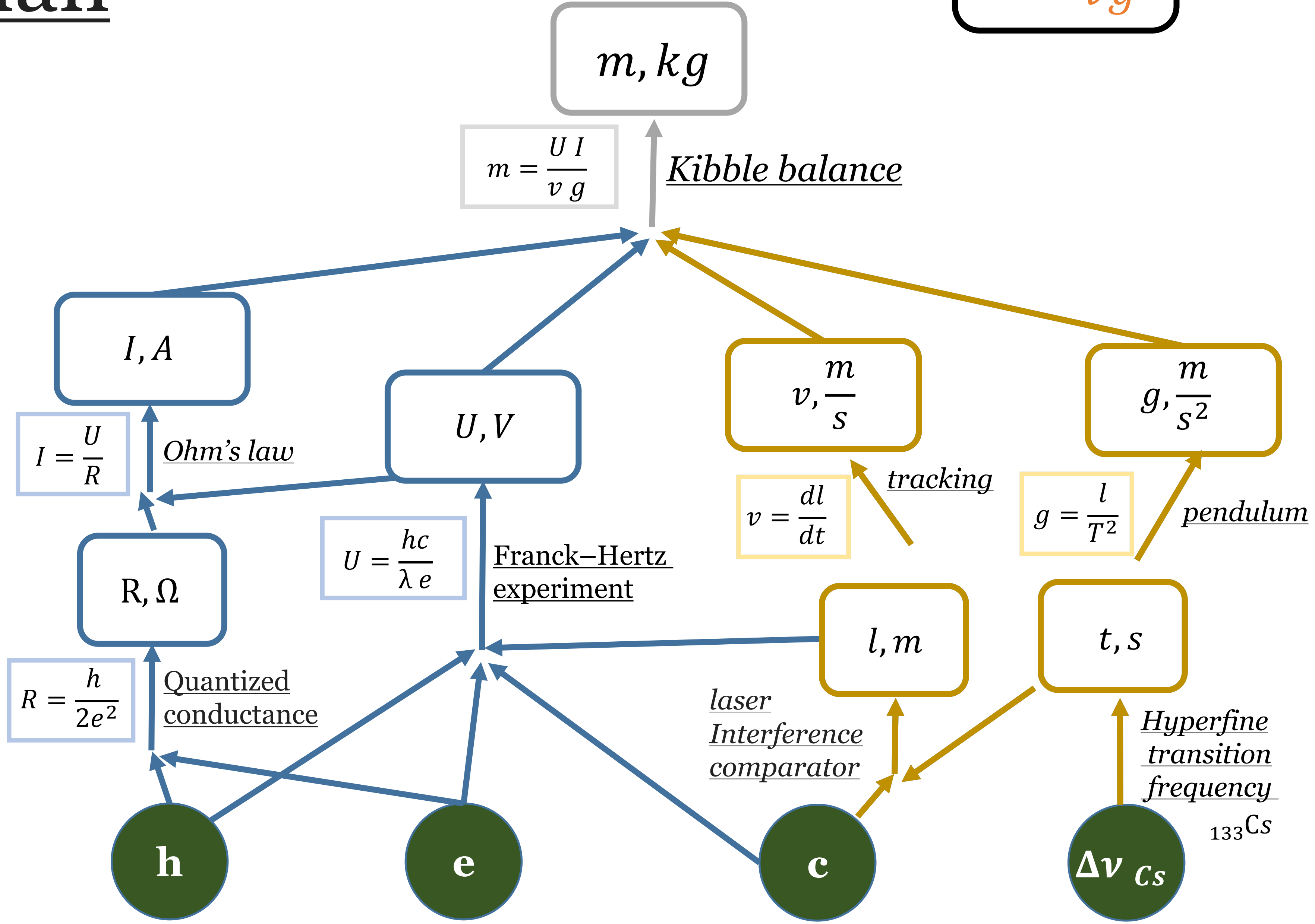
$$m = \frac{UI}{vg}$$

Kibble balance

Speed and gravitational Constant

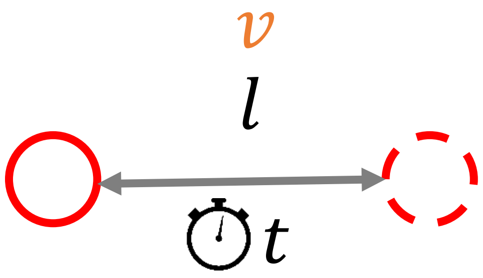
Franck–Hertz experiment

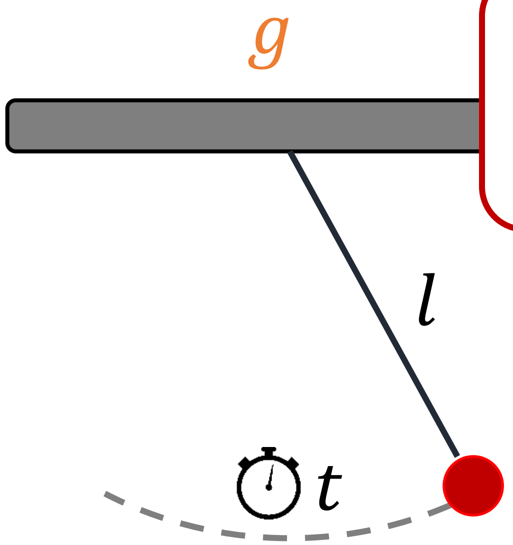
Quantized conductance



Speed and gravitational constants

$$m = \frac{UI}{vg}$$

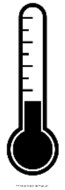




seconds

Hyperfine transition frequency $_{133}Cs$

Experiments are reproduced at room temperature



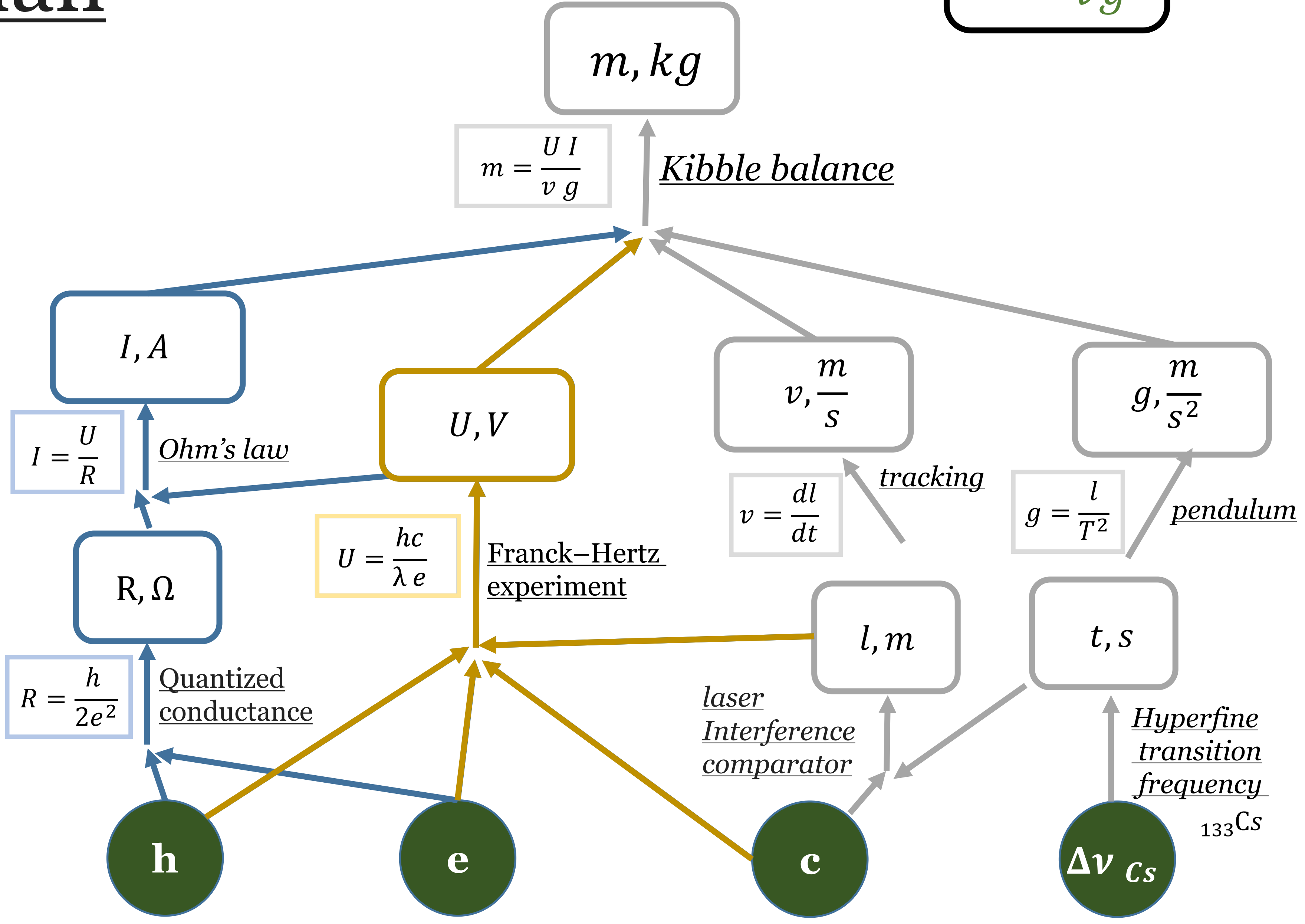
Laser Interference comparator

image source:Ekaterina Zhdanova

Research plan

$$m = \frac{UI}{vg}$$

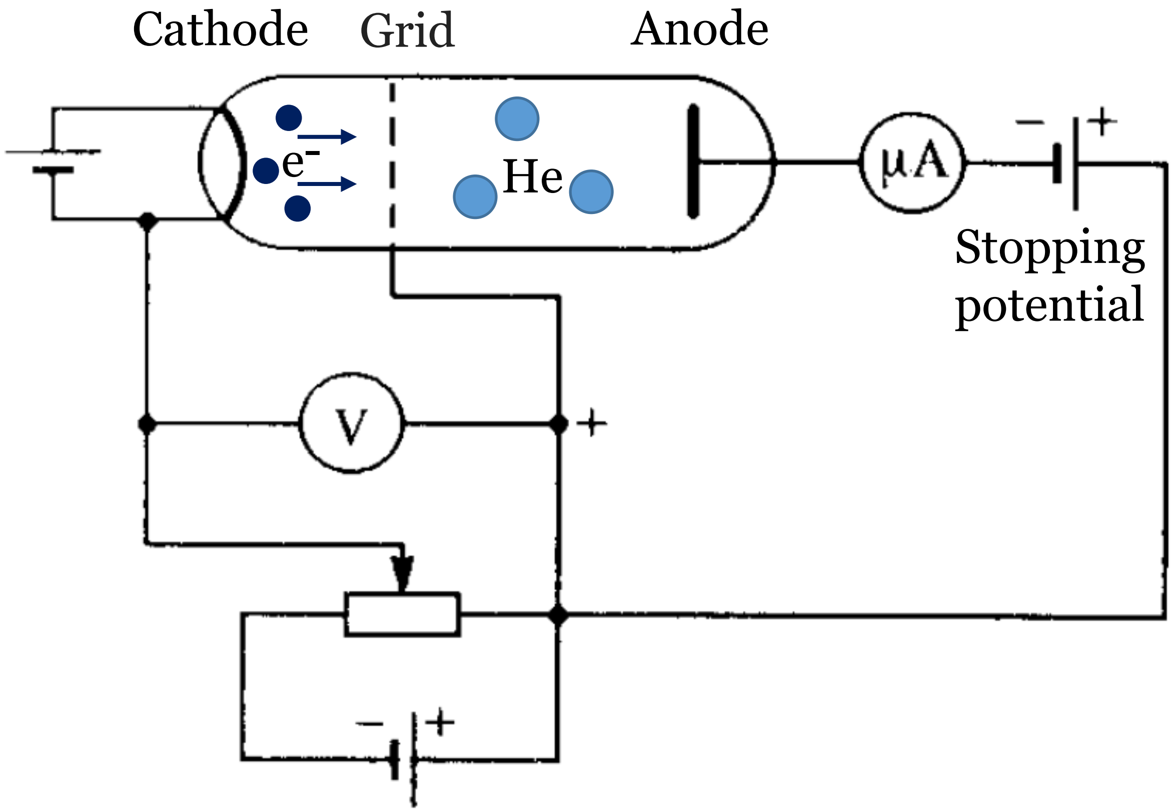
- *Kibble balance*
- *Speed and gravitational Constant*
- *Franck–Hertz experiment*
- *Quantized conductance*

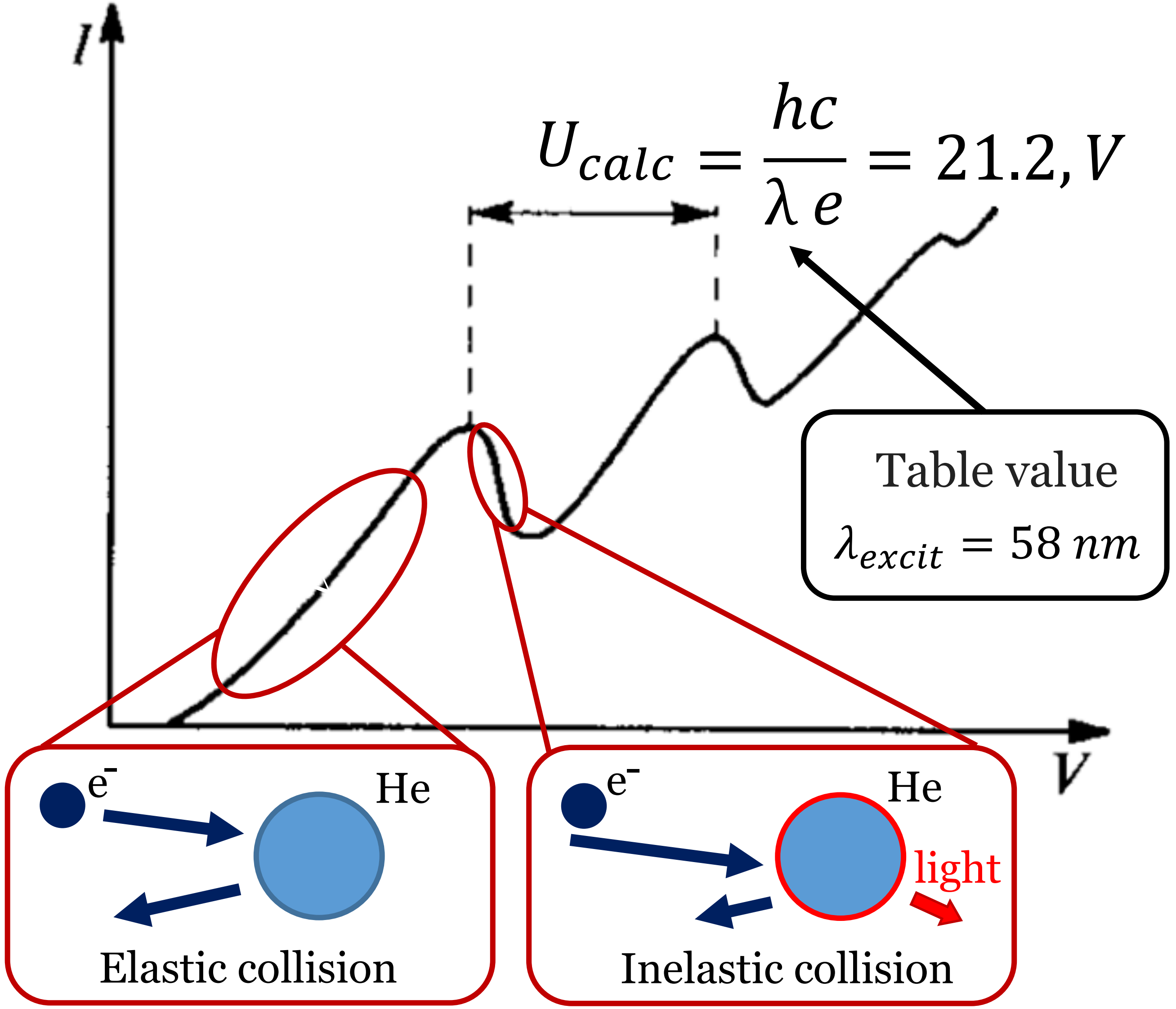


$$m = \frac{UI}{vg}$$

Franck–Hertz experiment

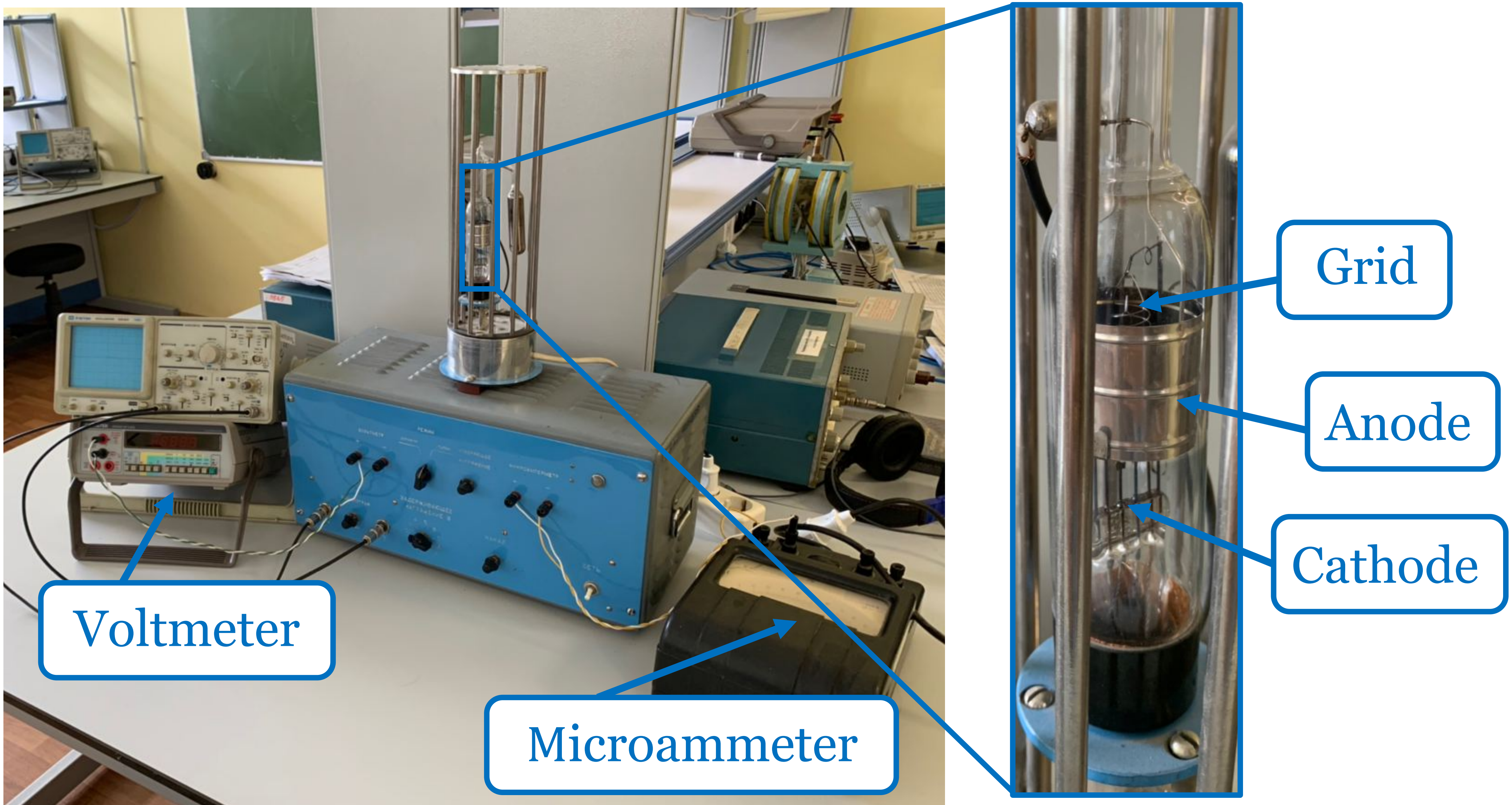
$$eU_{calc} = \Delta E = \frac{hc}{\lambda}$$





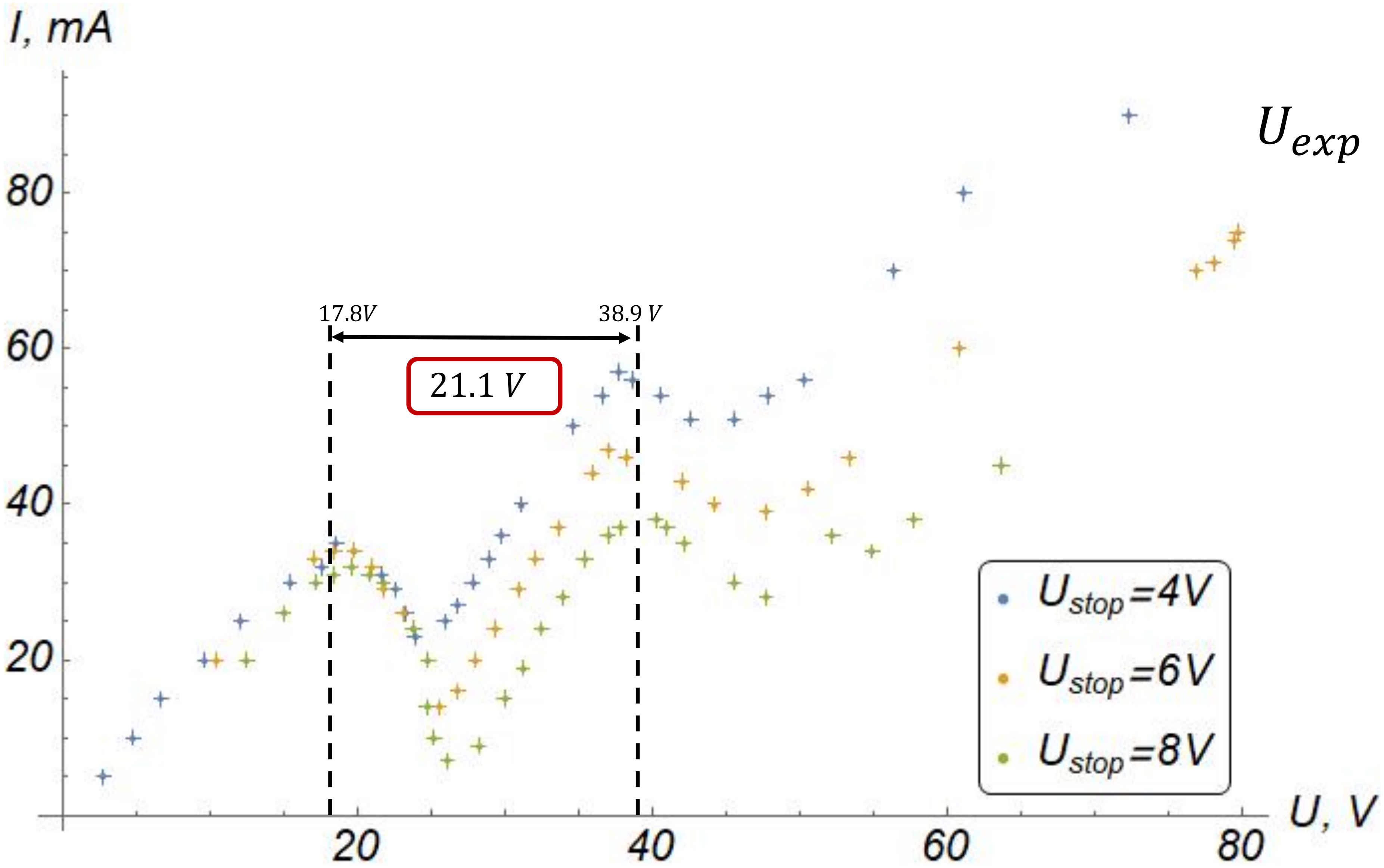
Franck–Hertz experiment setup

$$m = \frac{UI}{vg}$$



Anode current vs grid voltage

$$m = \frac{UI}{vg}$$



$$U_{exp} = (21.1 \pm 0.3), V$$

$$\delta_U = 0.014$$

$$U_{calc} = 21.2, V$$

Research plan

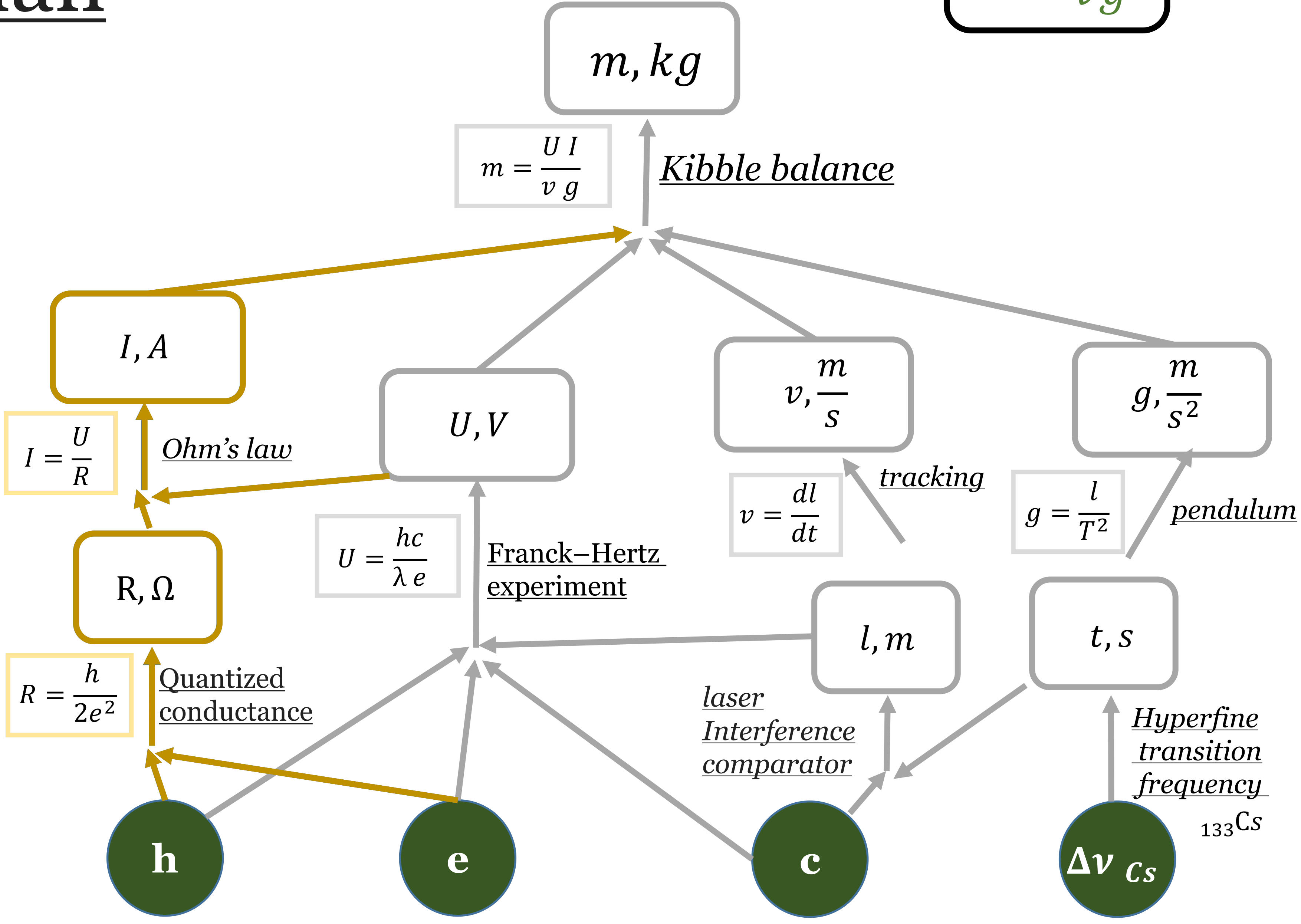
$$m = \frac{UI}{vg}$$

Kibble balance

Speed and gravitational Constant

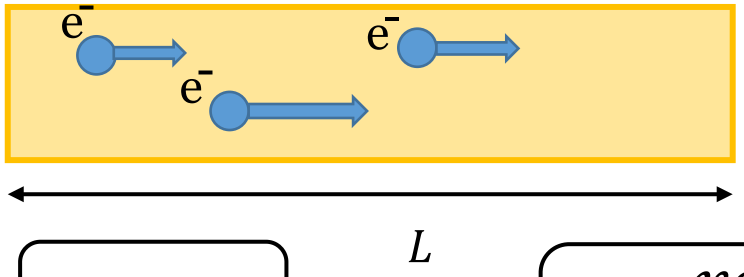
Franck–Hertz experiment

Quantized conductance



Quantized conductance

$$m = \frac{UI}{vg}$$



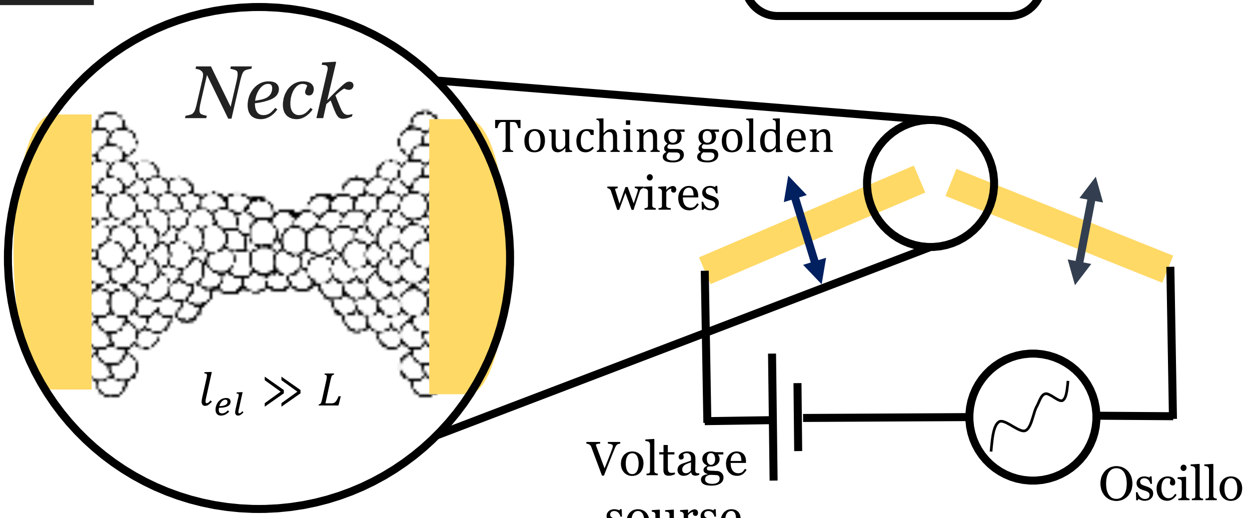
$$\Delta E = eV$$

$$I = \frac{veN}{L}$$

$$G = \frac{I}{V} = \frac{ve^2N}{L\Delta E}$$

$$N = \frac{2\,L\,\Delta E}{v\,h}$$

$$G = \frac{2e^2}{h}$$



kinetic energy

$$E = \frac{mv^2}{2}$$

$$\Delta E = m\,v\,\Delta v$$

in range Δv

$$N = \frac{2\,L\,m\,\Delta v}{h}$$

Electron spin degeneracy

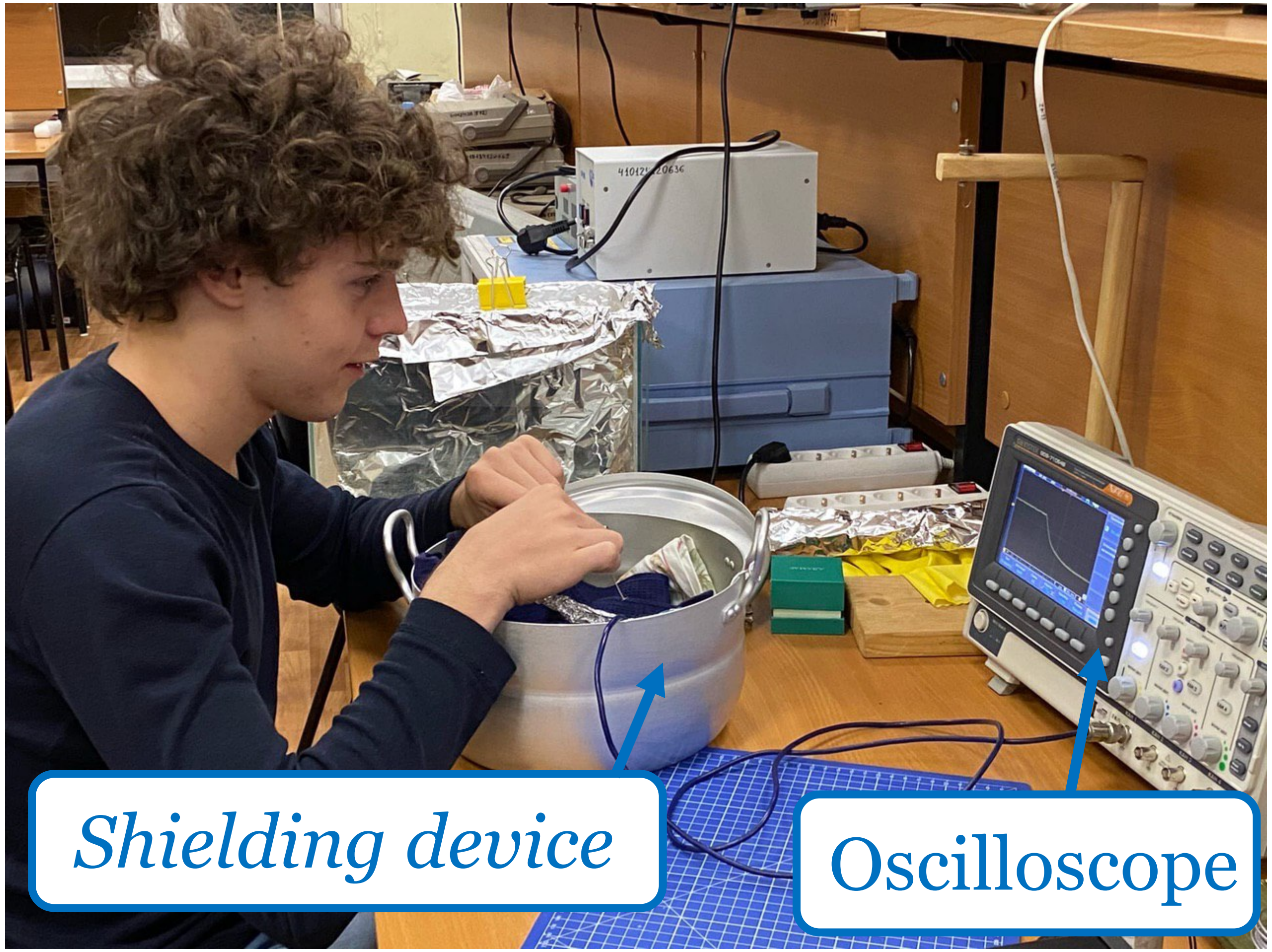
de Broglie wavelength

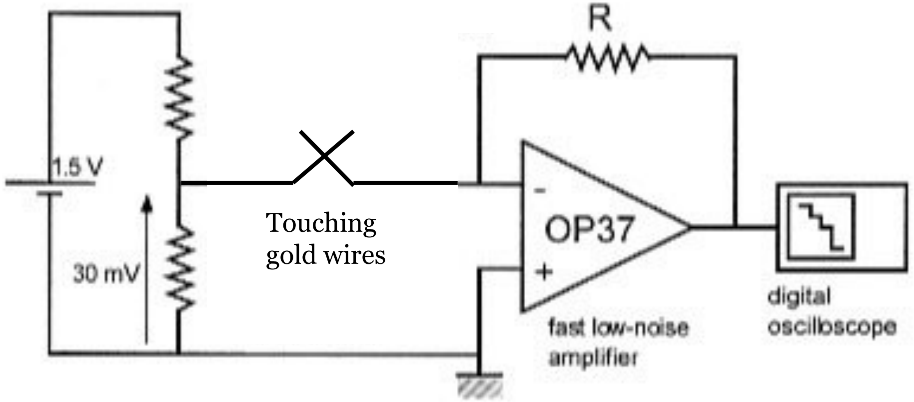
$$\lambda = \frac{L}{n}$$

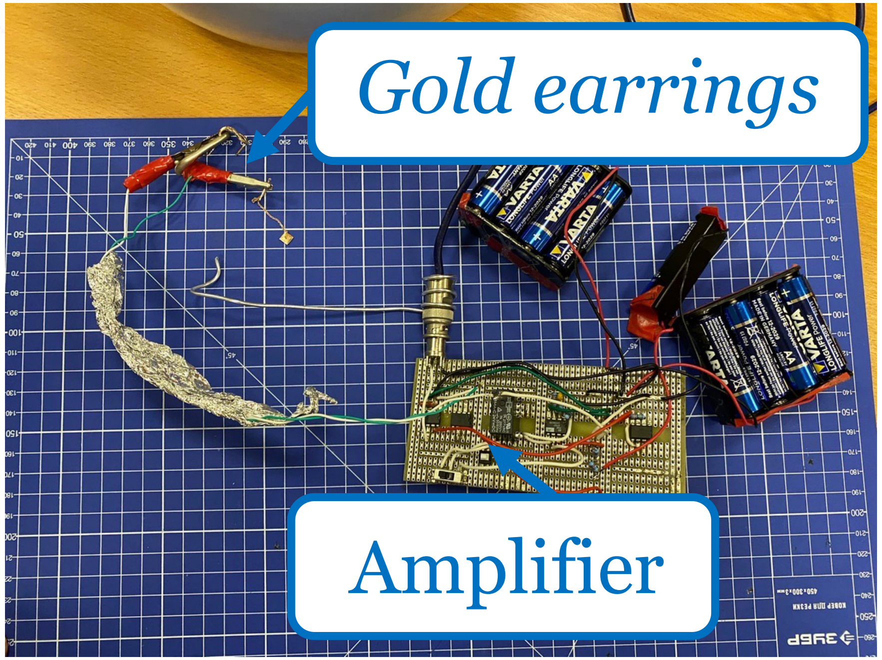
$$v_n = \frac{n\,h}{L\,m}$$

Quantized conductance setup

$$m = \frac{UI}{vg}$$

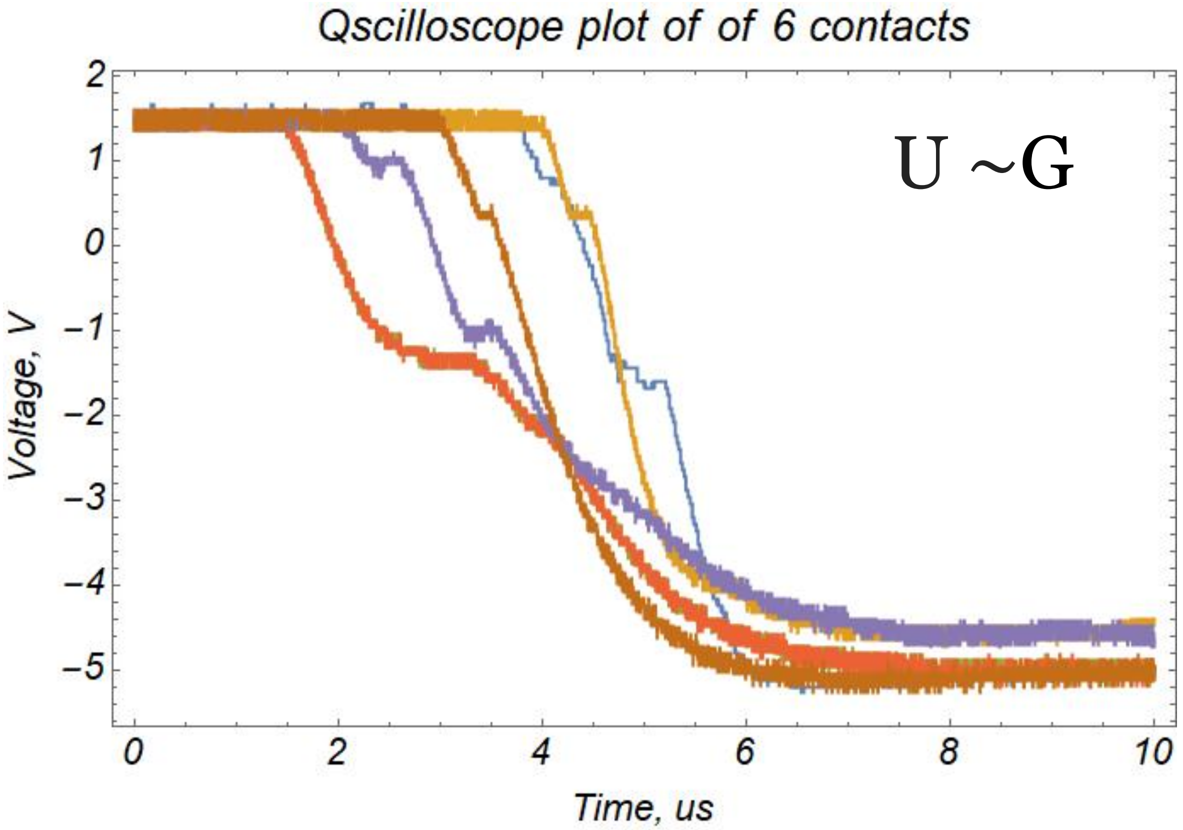


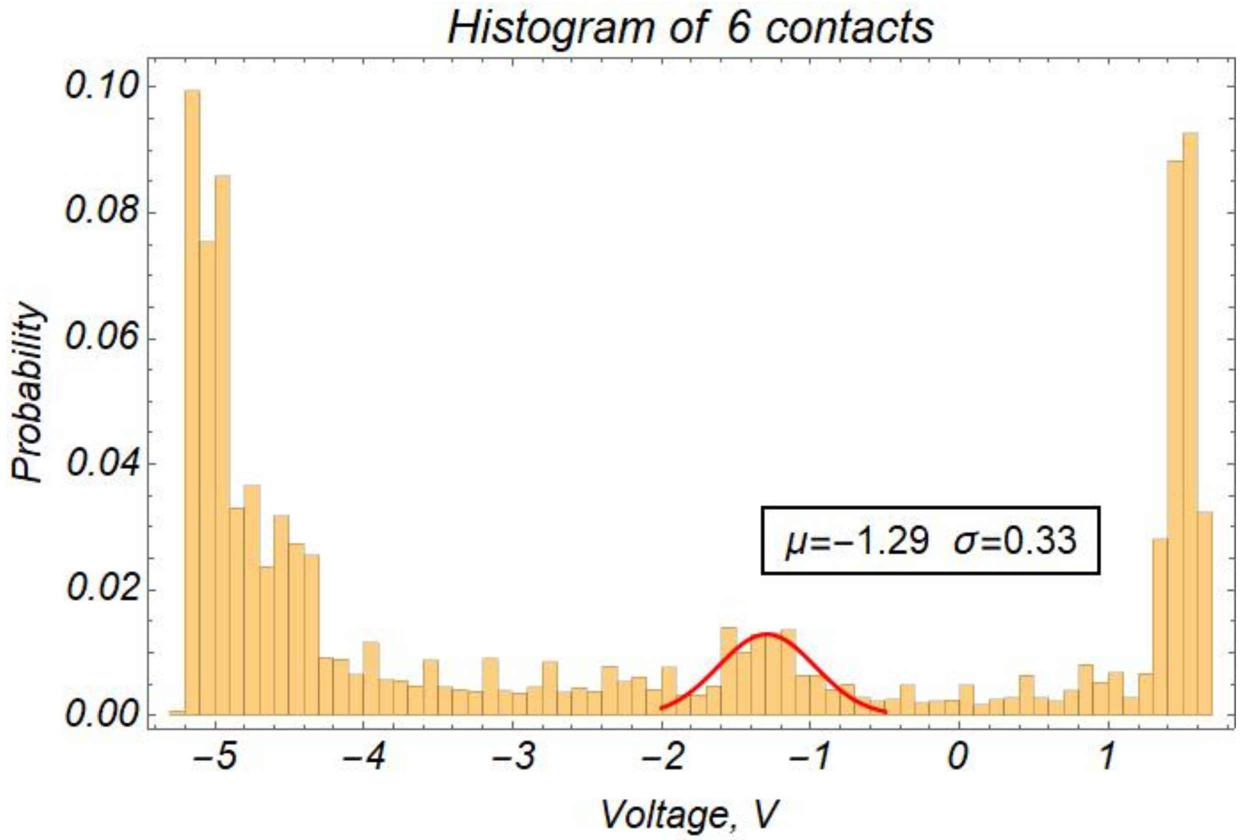




Oscillogram of current flow through contacts

$$m = \frac{UI}{vg}$$





$$\delta_U = 0.014$$

$$\delta_I = \sqrt{{\delta_R}^2 + {\delta_U}^2} = 0.25$$

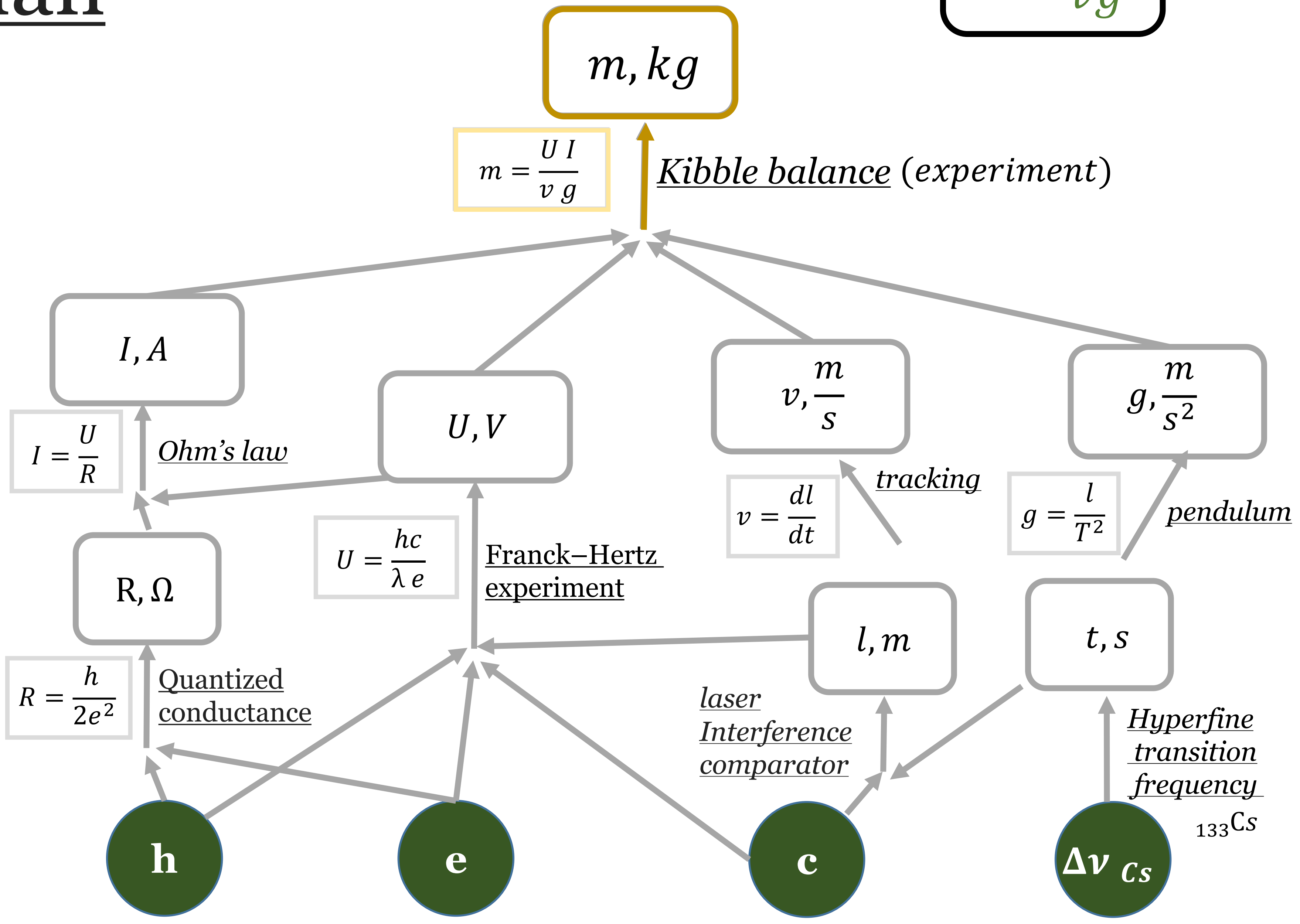
$$I = \frac{U}{R}$$

$$\delta_R = \frac{\sigma}{\mu} = 0.25$$

$m = \frac{UI}{vg}$

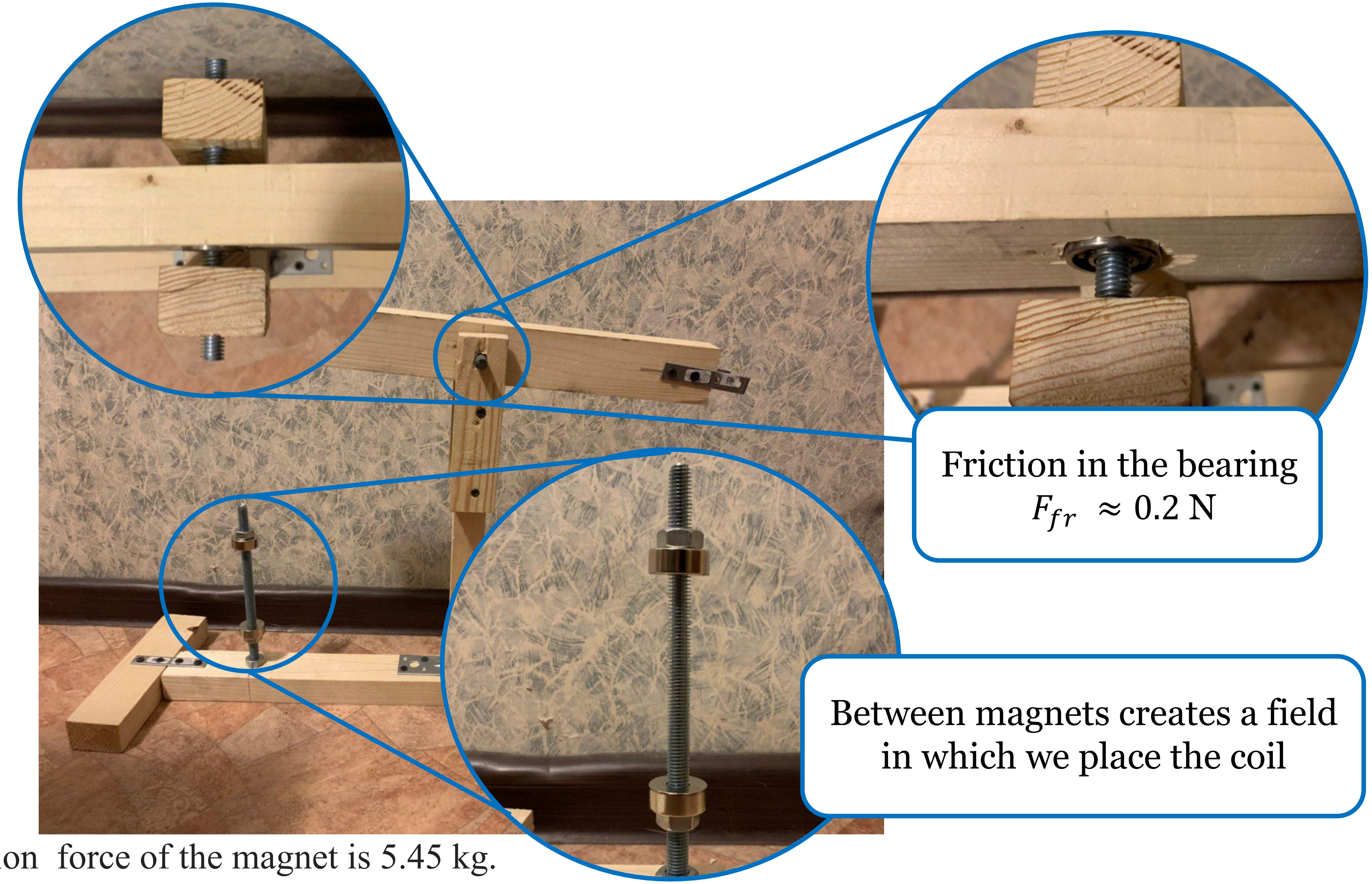
Research plan

- *Kibble balance (experiment)*
- *Speed and gravitational Constant*
- *Franck–Hertz experiment*
- *Quantized conductance*



Kibble balance experimental setup

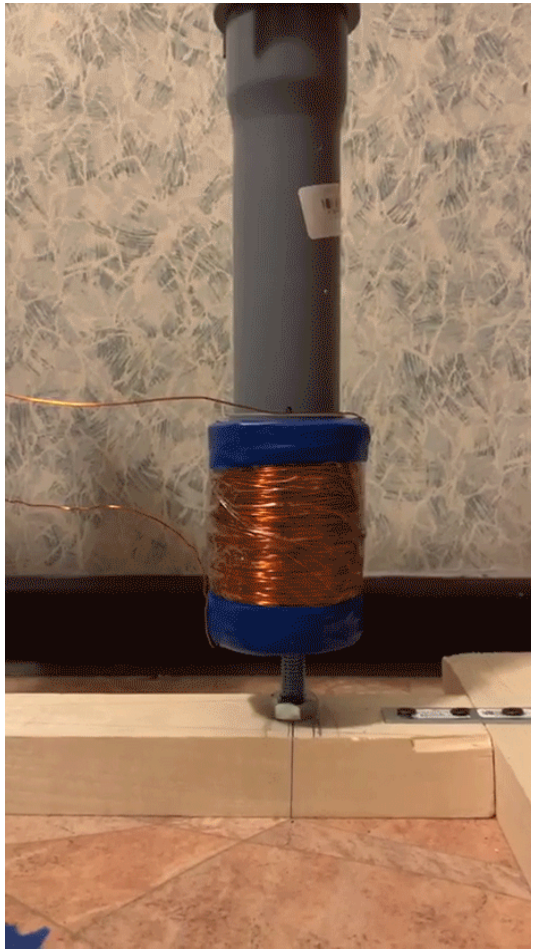
$$m = \frac{UI}{vg}$$

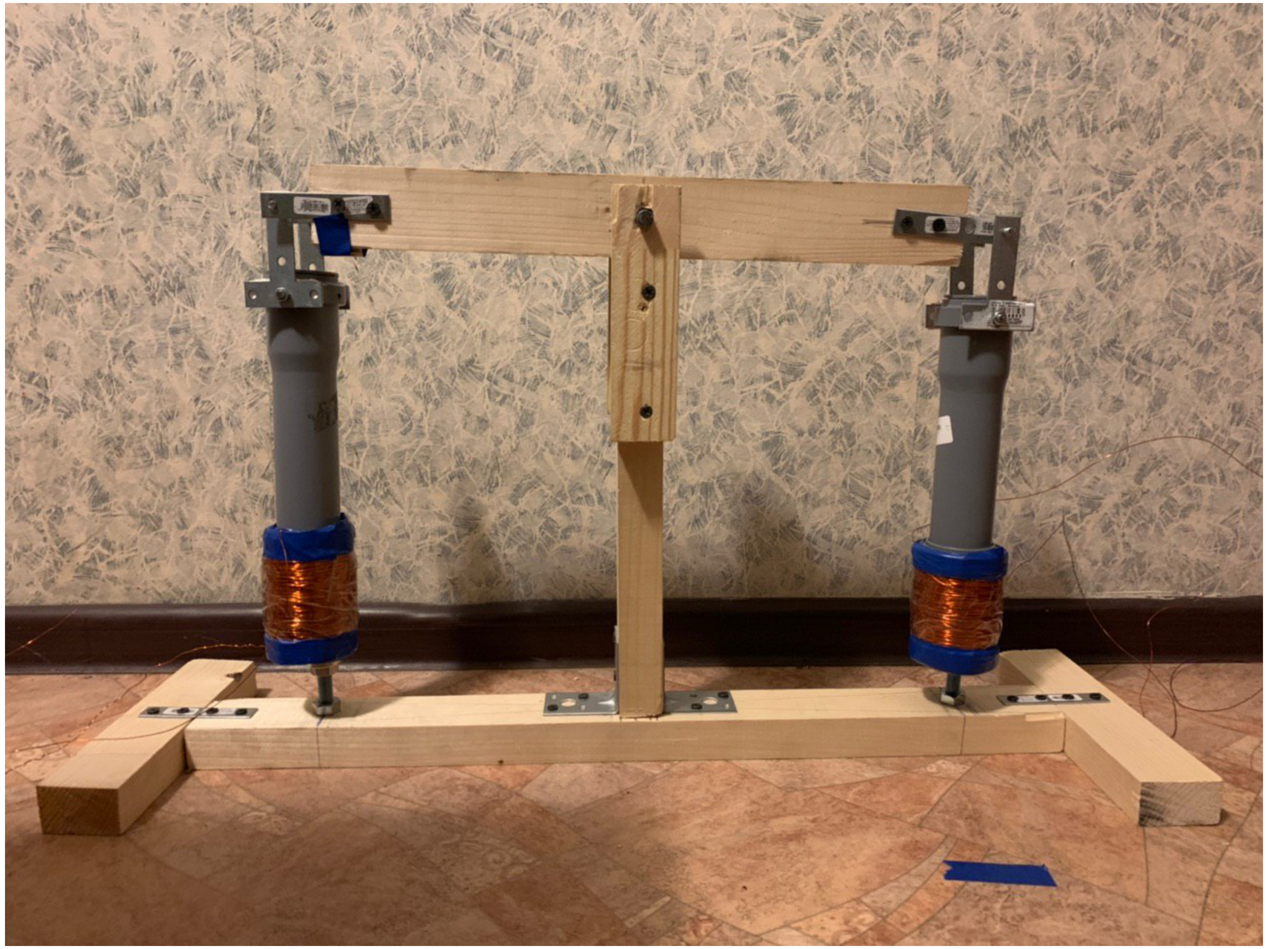


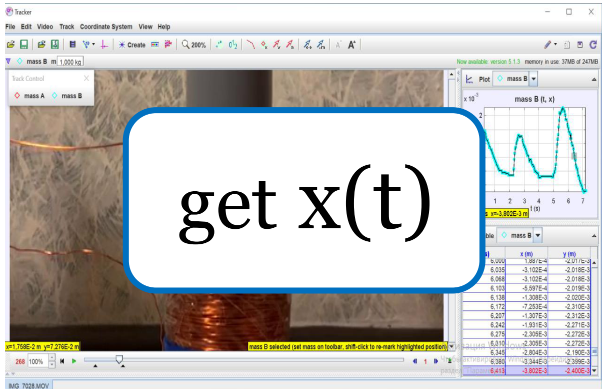
The attraction force of the magnet is 5.45 kg.

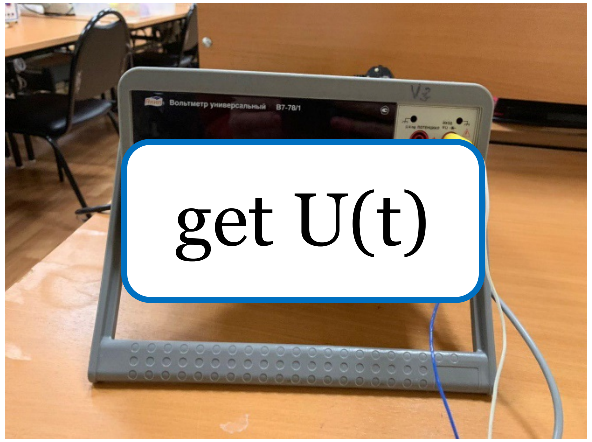
Kibble balance calibration

$$m = \frac{UI}{vg}$$





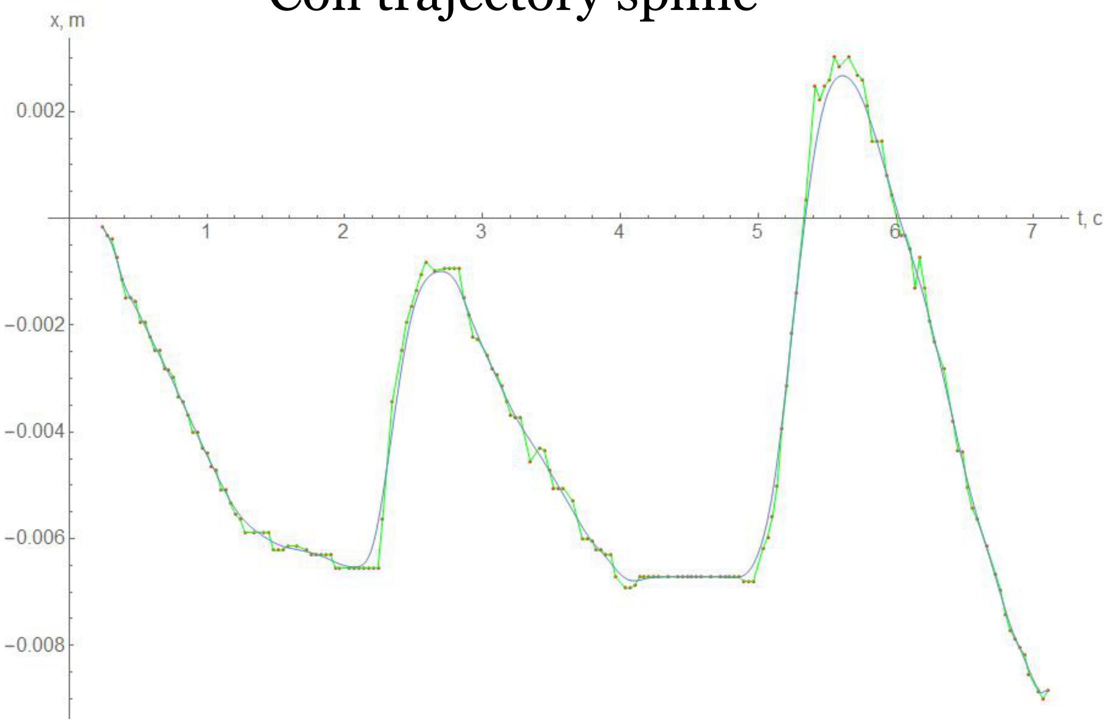




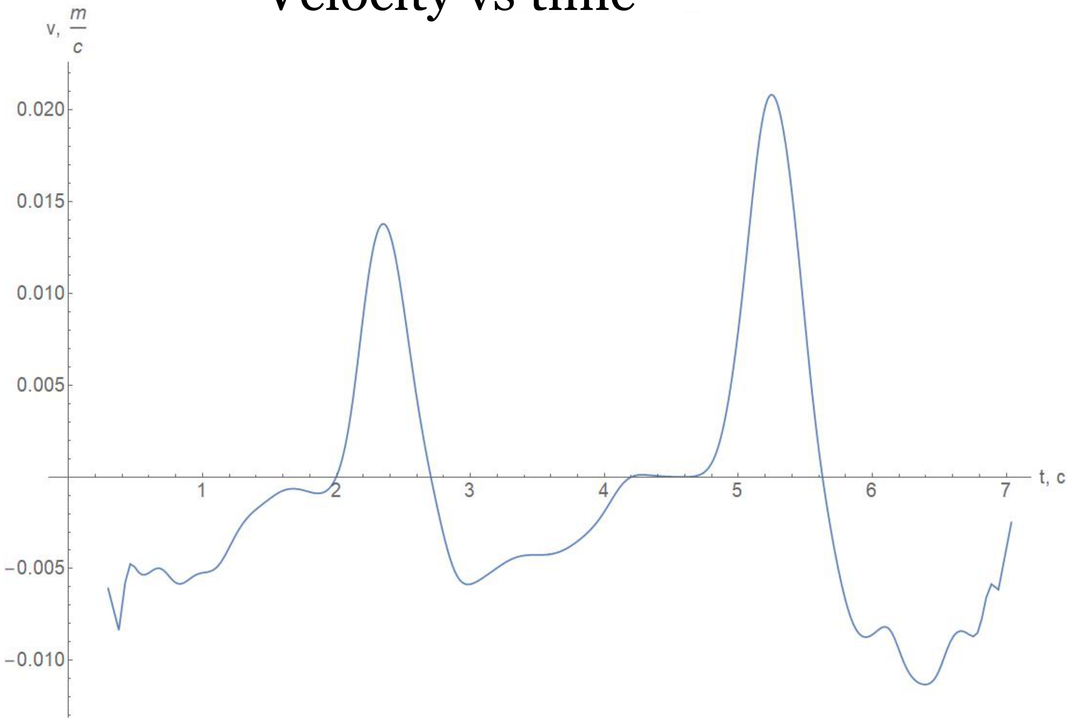
Coil velocity

$$m = \frac{UI}{vg}$$

Coil trajectory spline

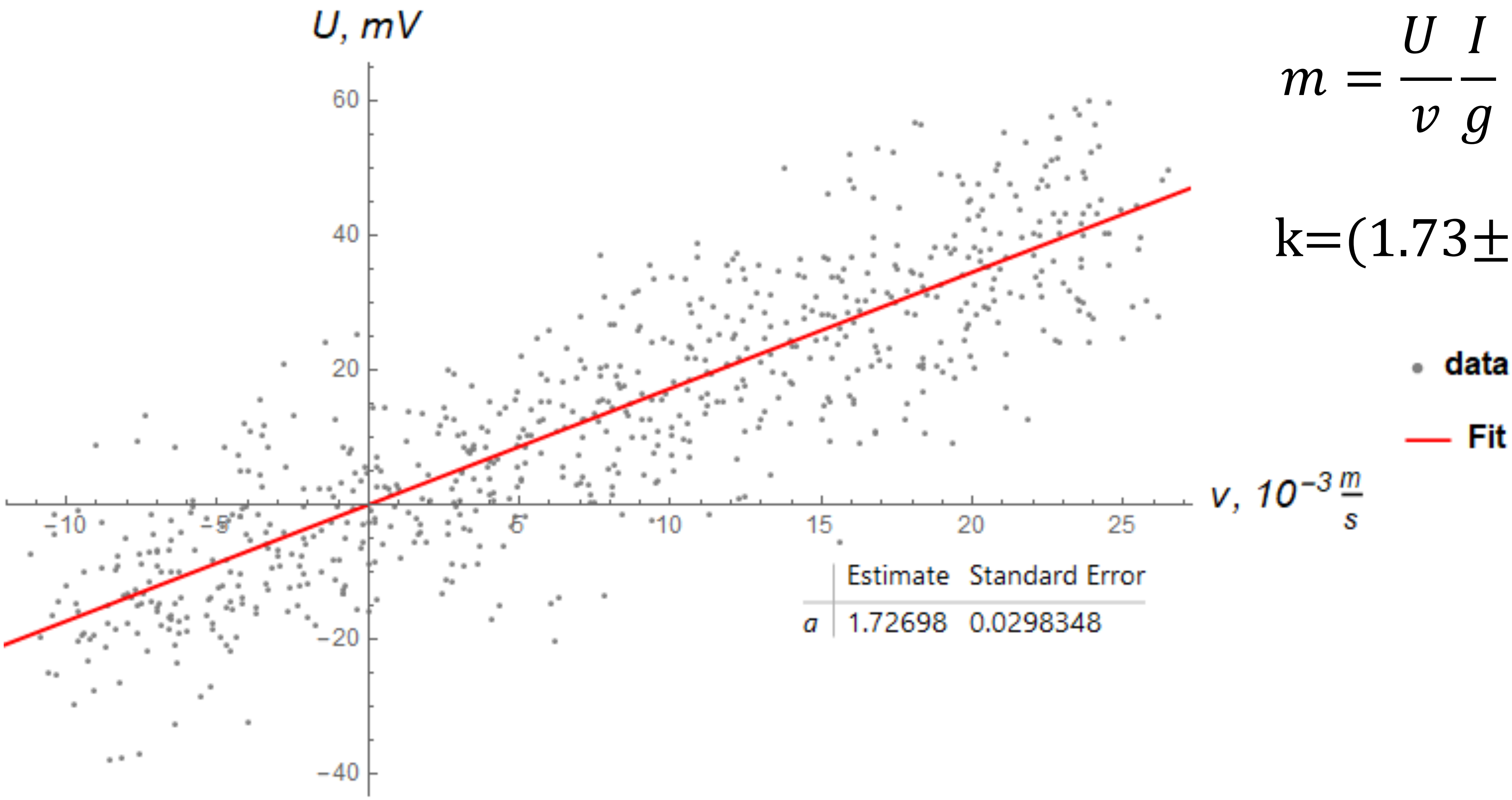


Velocity vs time



BL value from calibration

$$m = \frac{UI}{vg}$$



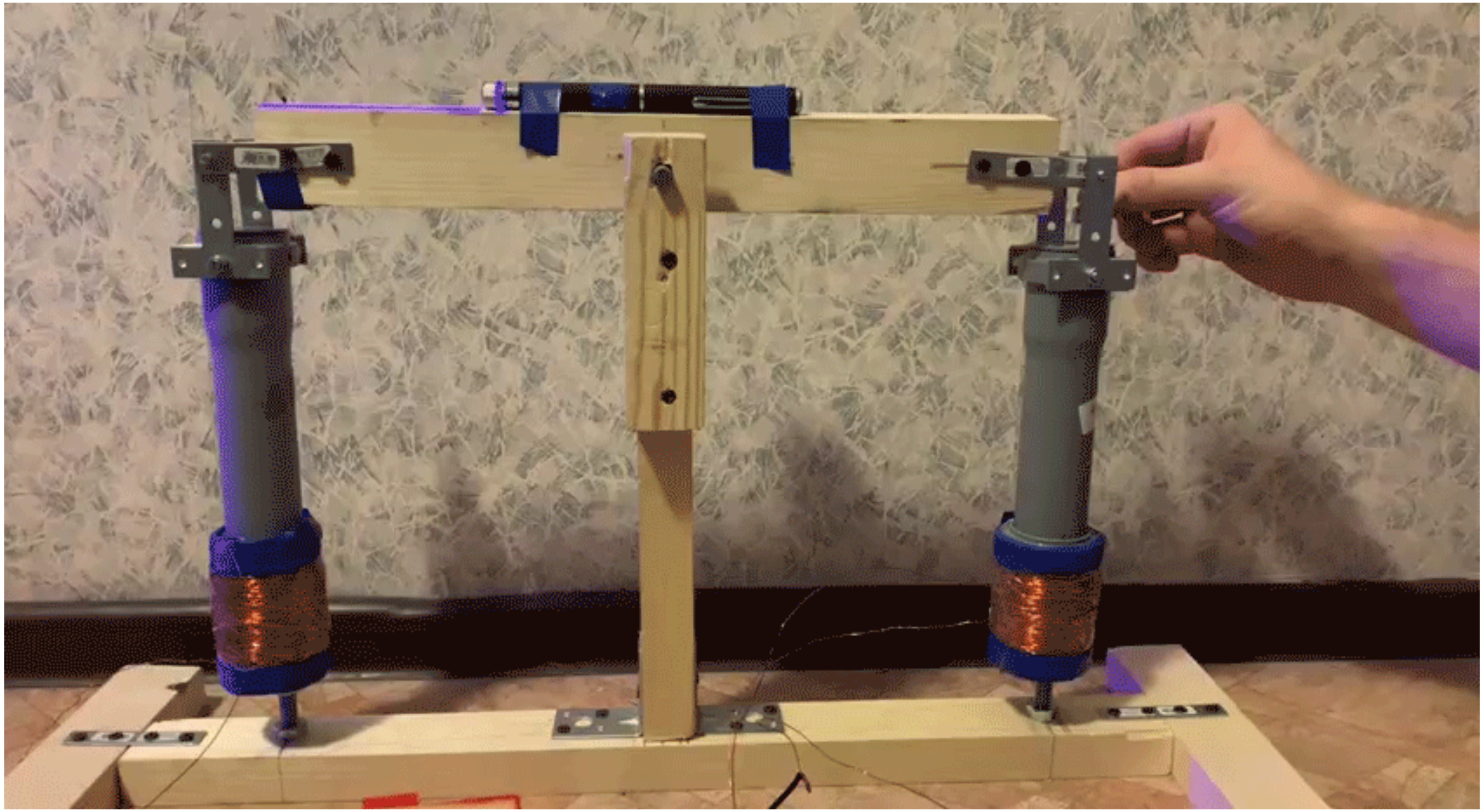
$$m = \frac{U}{v}\frac{I}{g} = k\frac{I}{g}$$

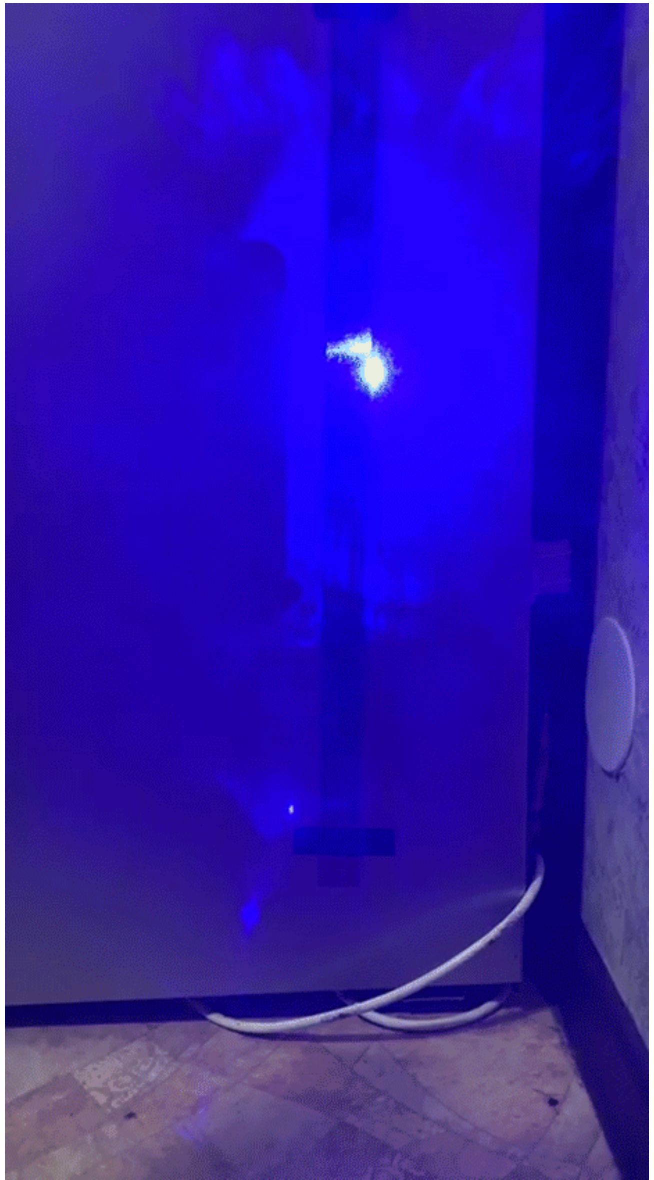
k=(1.73±0.05), $\frac{Vs^2}{m}$

δ_k=0.03

$$m = \frac{UI}{vg}$$

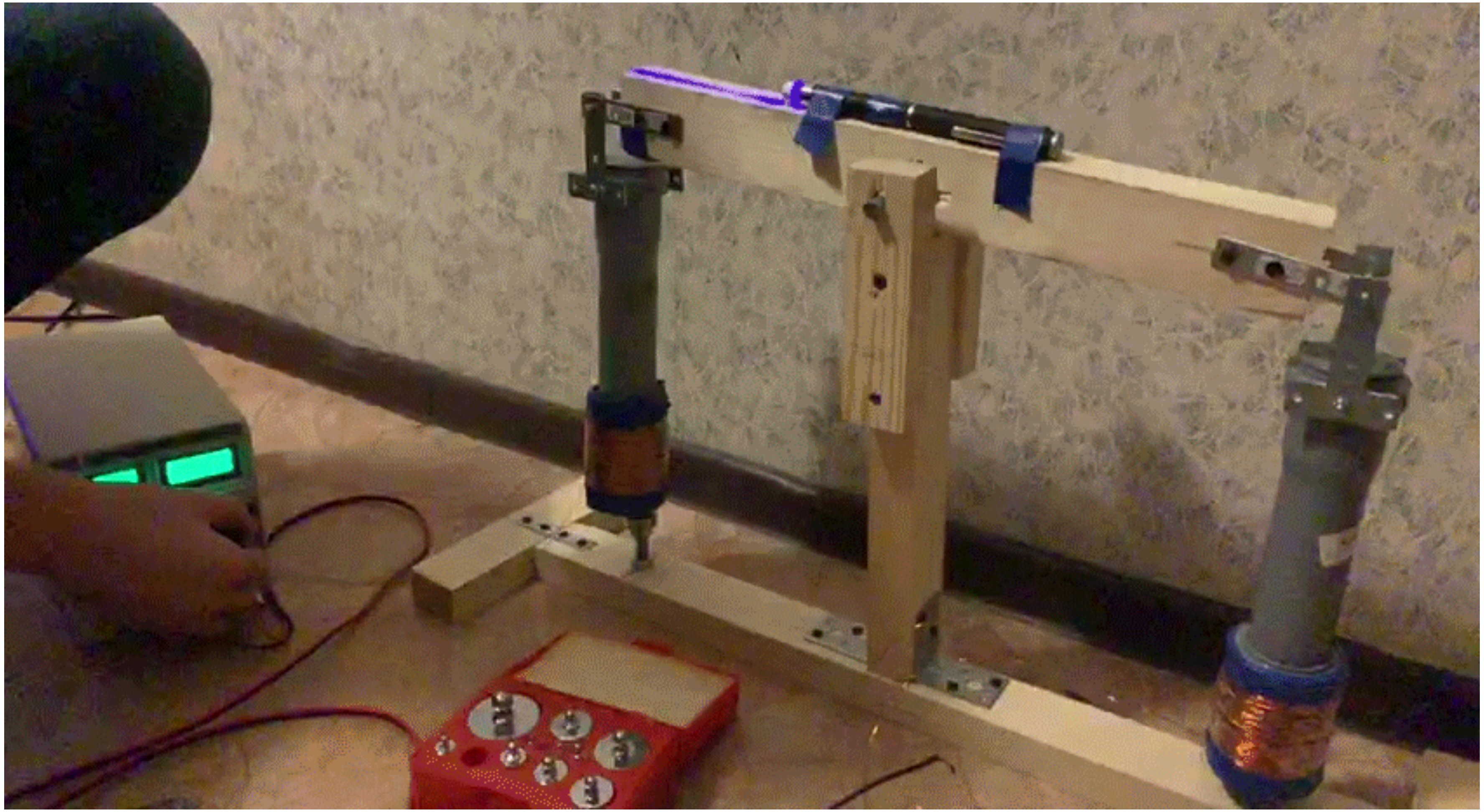
We calibrate the weights





Achieve equilibrium using a current source

$$m = \frac{UI}{vg}$$

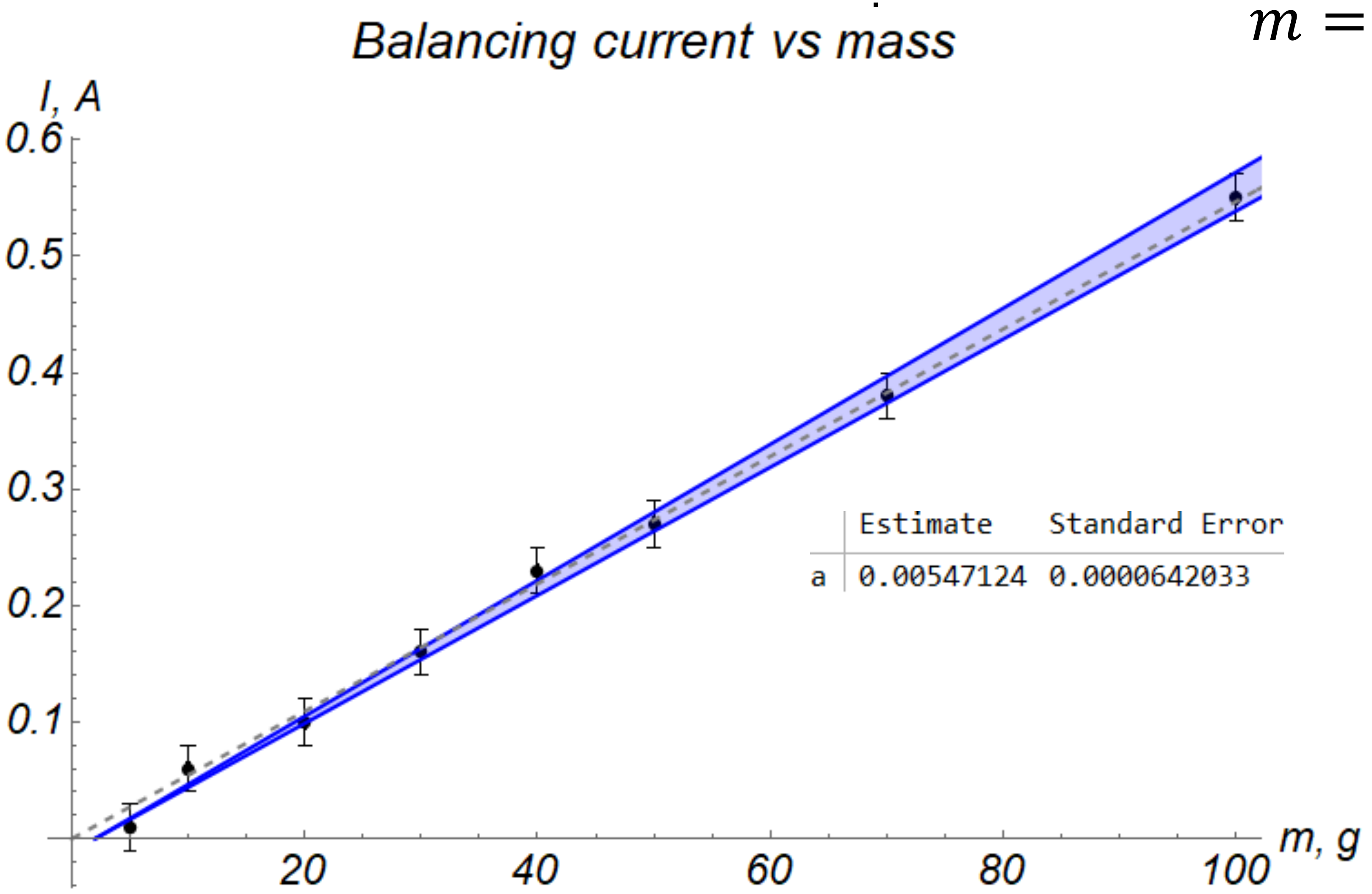


$$\boldsymbol{m} = \frac{UI}{vg}$$

Comparison of calibration with experiment data

$$m = \frac{V}{gv} I - \frac{F_{fr}}{g} = k^* I - \frac{F_{fr}}{g}$$

$$k^* = (0.176 \pm 0.005), \frac{kg}{A}$$



	Estimate	Standard Error
a	0.00547124	0.0000642033

Mass with the best accuracy we have achieved

$$m_{100} = (99 \pm 25), \mathrm{g}$$

$$\delta_m = 0.25$$

Conclusions

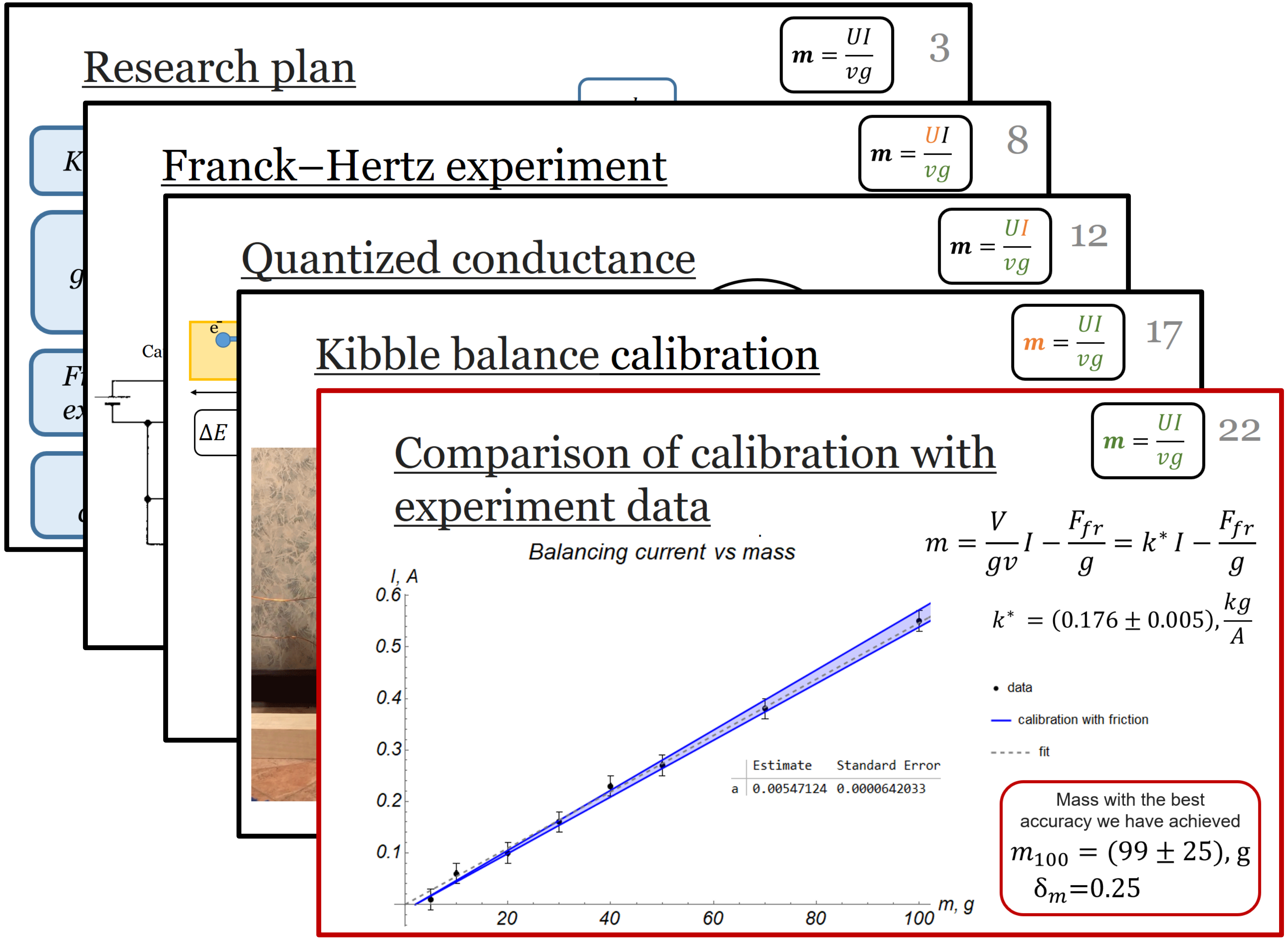
We proposed room-temperature experiments to determine the kilogram using Planck constant

We made the Franck-Hertz experiment to determine the voltage

We made the Quantized conductance experiment to determine resistance

We made and calibrated Kibble balance

We measured the mass.
The best precision we got is 25 %.
$m_{100} = (99 \pm 25), g$



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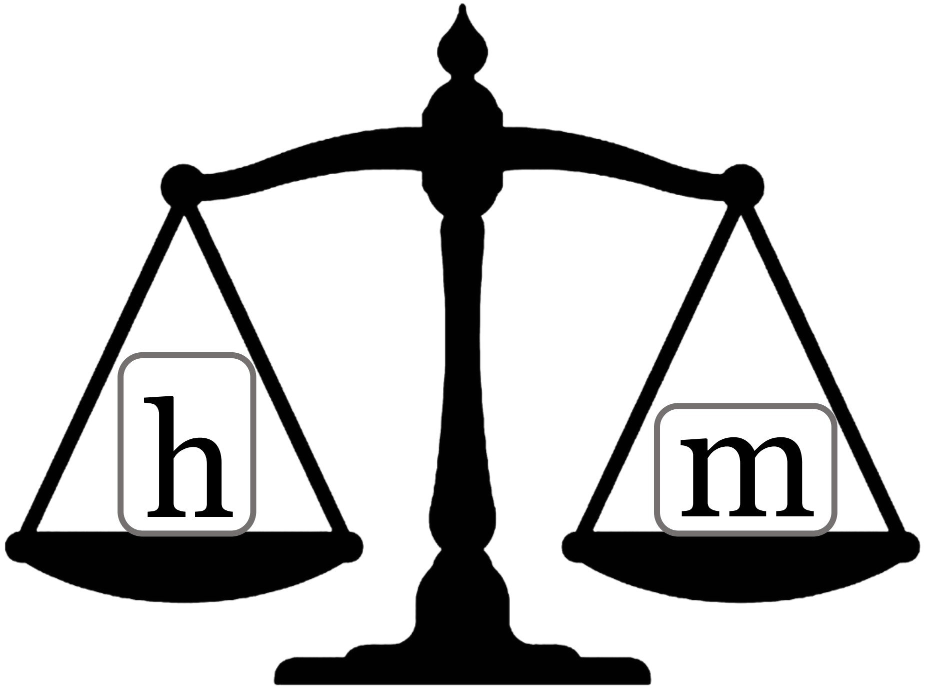
Fizmatbook, 2012. — 464 p. — ISBN 978-5-89155-206-7.

Thank you for your attention !



If you have any questions, I'll be happy to answer them

Compare definitions

NIST definition:

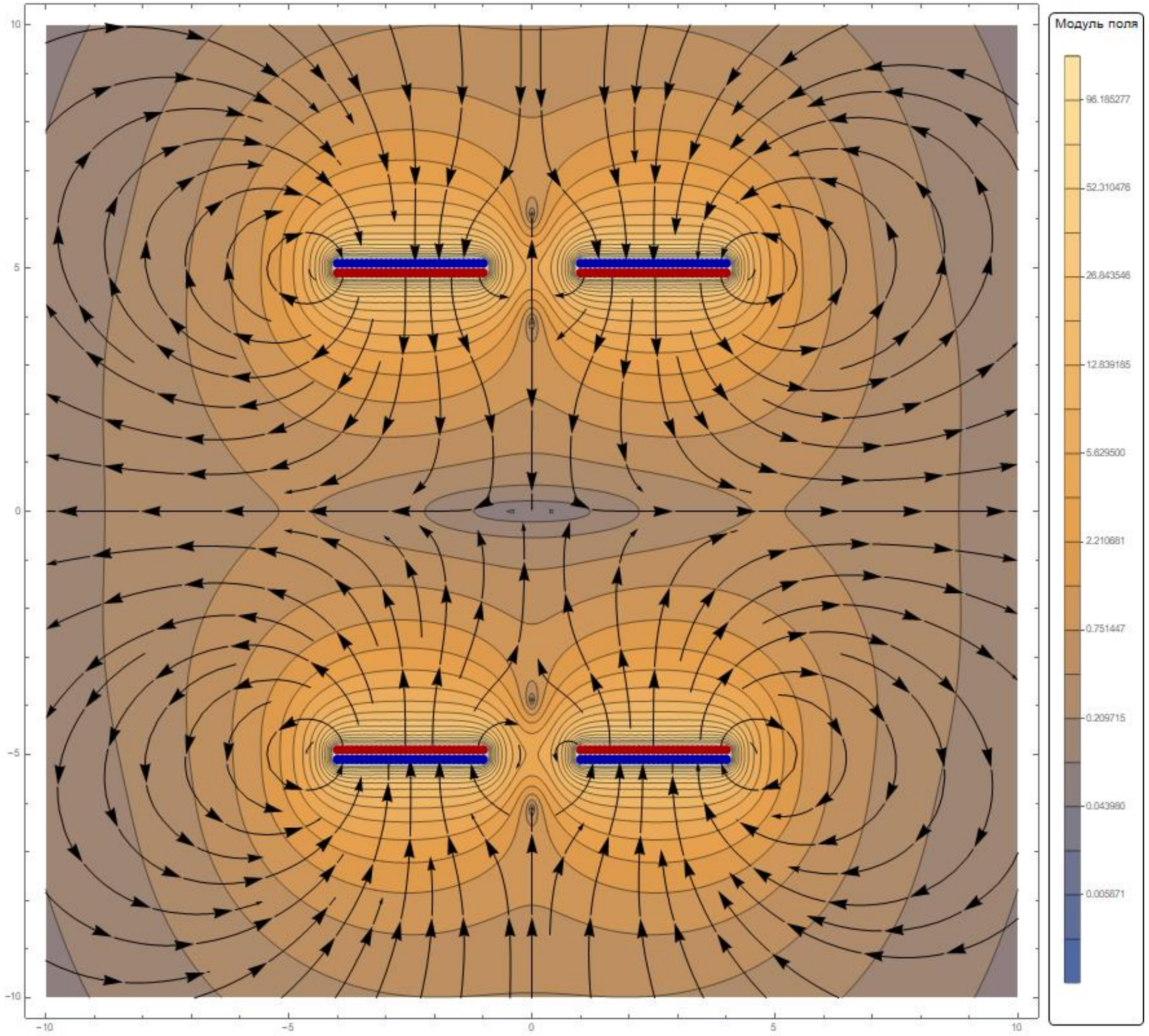
$$m = \frac{IV}{vg} = \frac{1}{vg}\frac{VV_R}{R} = \frac{1}{vg}Cf_1f_2\frac{h}{2e}\frac{h}{2e}\frac{e^2}{h} = \boxed{\frac{Cf_1f_2}{4}\frac{h}{gv}}$$

My definition:

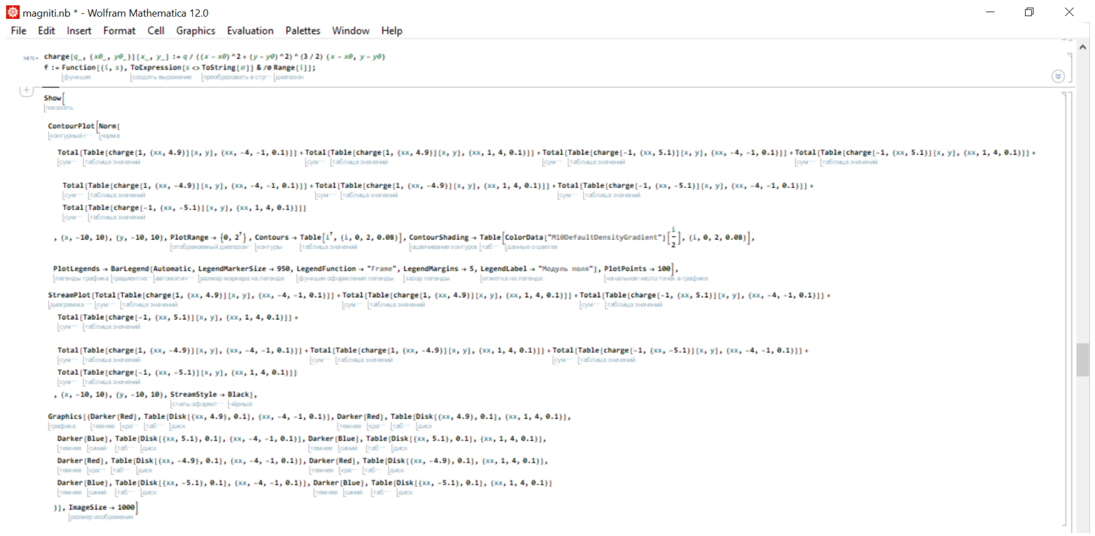
$$m = \frac{IV}{vg} = \frac{1}{vg}k_{U1}\frac{hc}{\lambda e}k_{U2}\frac{hc}{\lambda e}\frac{1}{k_R}\frac{2e^2}{h} = \boxed{\frac{2}{vg}\frac{k_{U1}k_{U2}}{k_R}\frac{hc^2}{\lambda^2}}$$

k Coefficient of the ratio of the measured value to the standard

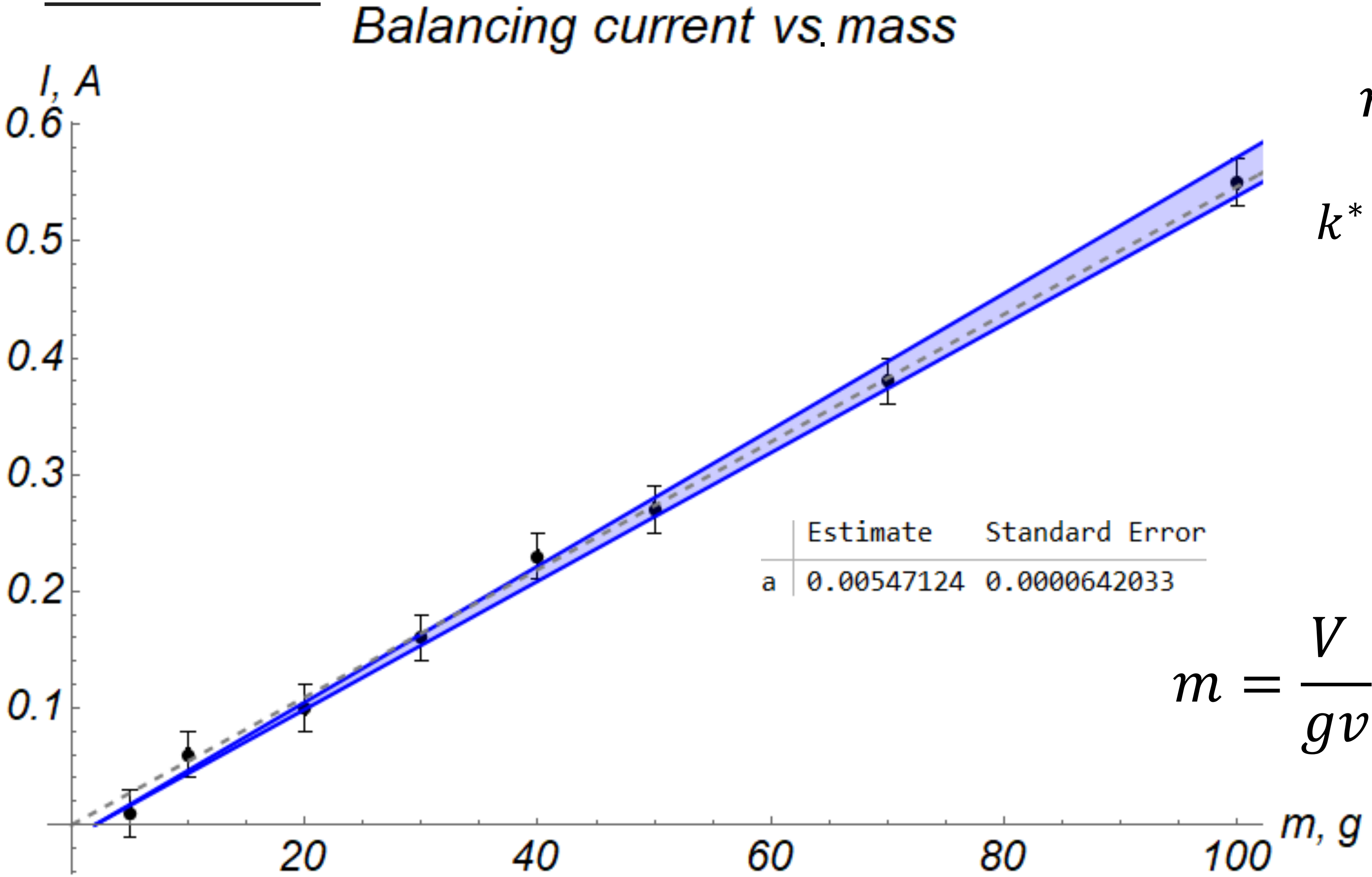
Field between magnets



Magnetic field simulation
Calculation in Wolfram mathematica
with a magnetic dipole model
(Approximate values □□are taken)



Comparison of calibration with and without friction



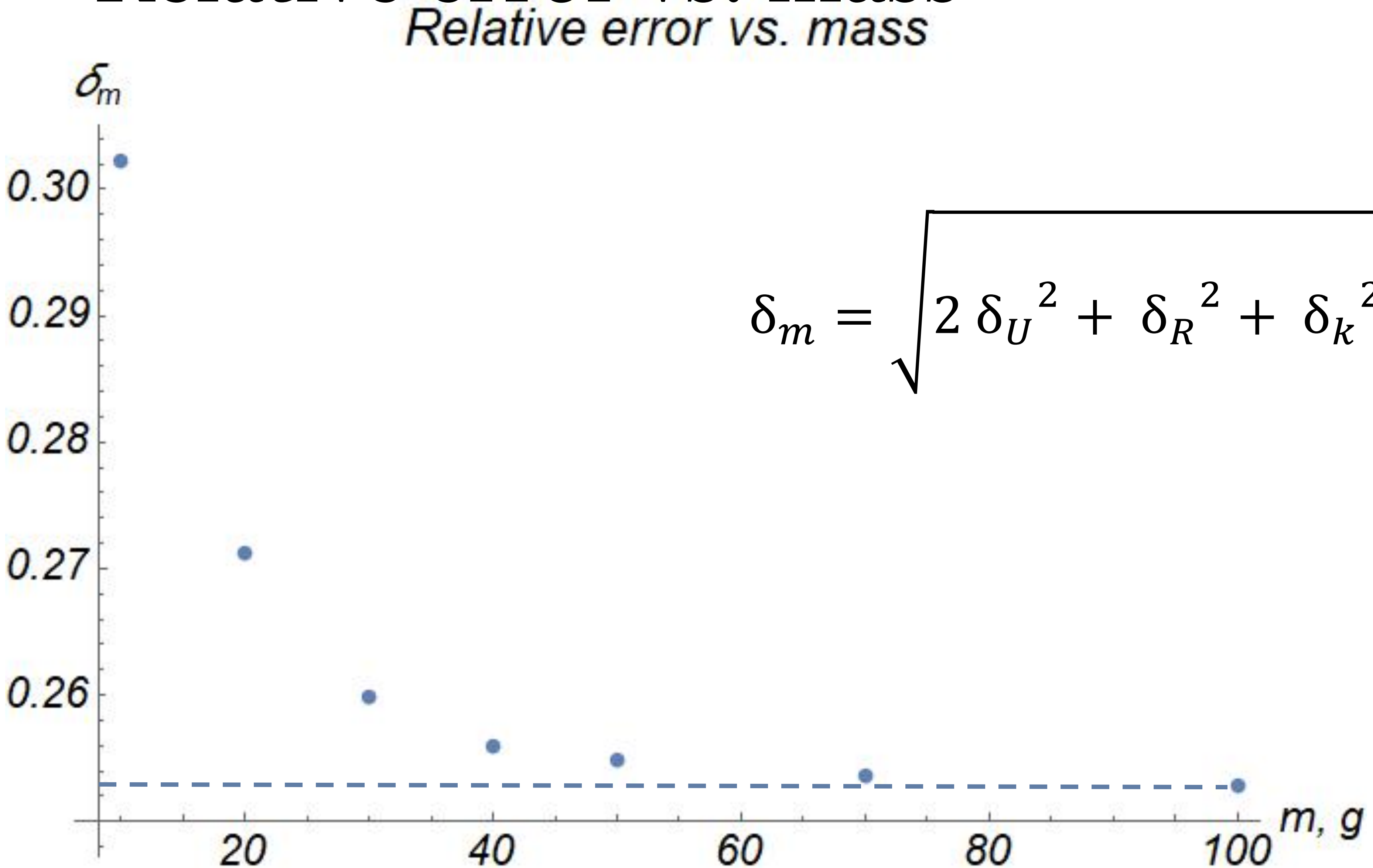
$$m = \frac{V}{gv} I = k^* I$$

$$k^* = (0.176 \pm 0.005), \frac{kg}{A}$$

$$m = \frac{V}{gv} I - \frac{F_{fr}}{g} = k^* I - \frac{F_{fr}}{g}$$

$$F_{fr} \approx 0.2 \text{ N}$$

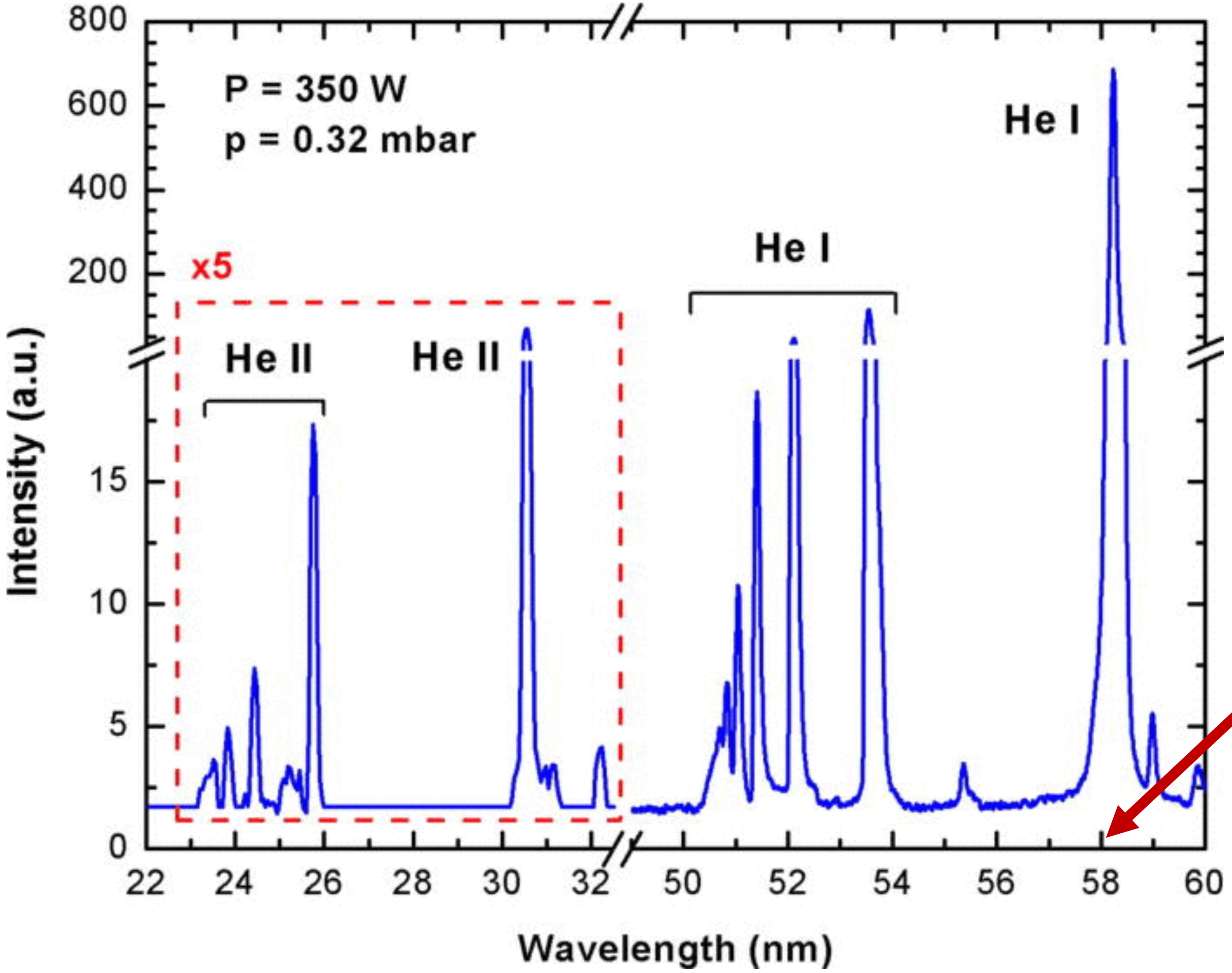
Relative error vs. mass



$$\delta_m = \sqrt{2\,\delta_U{}^2 + \delta_R{}^2 + \delta_k{}^2 + \frac{\Delta I}{I}^2} = 0.25$$

Oscillogram of current flow through contacts

Helium excitation wavelength



$\lambda = 58\ \text{nm}$