

AUTOMORPHISM GROUPS OF CERTAIN ENRIQUES SURFACES: COMPUTATIONAL DATA

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1. INTRODUCTION

This note is the explanation of the computation data that are used to obtain the main results of the paper

[BS] S. Brandhorst, I. Shimada: Automorphism groups of certain Enriques surfaces.

The data is available from the author's webpage [2] as plain text files. The data consists of the following items.

`GramL10`, `oneBP`, `irecs`, `Enrs`, `E6EnrRDPs` .

They are written in 3 files:

```
Preliminaries.txt : GramL10, oneBP,
irecs.txt         : irecs,
Enrs.txt          : Enrs,
E6EnrRDPs.txt    : E6EnrRDPs.
```

The data are made by `GAP` (see [3]). In particular, the `Record` format of `GAP` is used everywhere. The results of Algorithms in Sections 6.3 and 6.4 of [BS] are too large to be put on a webpage, and hence we omit them.

2. THE FILE `Preliminaries.txt`

In the file `Preliminaries.txt`, we have the items `oneBP` and `oneBP`.

The item `GramL10` is the Gram matrix of the even unimodular hyperbolic lattice L_{10} of rank 10 with respect to the basis $\{e_1, \dots, e_{10}\}$ given in Figure 1.1 of the paper [BS]. Throughout this note, every computation data about the lattice L_{10} is expressed in terms the basis $\{e_1, \dots, e_{10}\}$.

The item `oneBP` is the number

$$\text{oneBP} := 2^{21} \cdot 3^5 \cdot 5^2 \cdot 7 \cdot 17 \cdot 31 = 46998591897600,$$

which is the unit of volume of chambers in \mathcal{P}_{10} .

3. THE FILE `irecs.txt`

The list `irecs` describes the primitive embeddings

$$\iota: L_{10}(2) \hookrightarrow L_{26}$$

classified in [1], except for the primitive embedding of type `infy`. Thus `irecs` consists of 16 records `irec`, and each has the following items. We use the notions

and notation in [1] and [BS]. Recall that R_ι denote the orthogonal complement of the image of ι in L_{26} .

- **irec.name** is the name of ι , which is one of the strings "12A", "12B", ..., "96C".
- **irec.GramL26** is the Gram matrix of L_{26} with respect to a certain fixed basis. We use this basis for other data in this record **irec**.
- **irec.embs** is the 10×26 integer matrix M such that $v \mapsto vM$ gives the embedding $\iota: L_{10}(2) \hookrightarrow L_{26}$, where L_{10} is equipped with the basis $\{e_1, \dots, e_{10}\}$.
- **irec.embr** is the 16×26 integer matrix M' such that $v \mapsto vM'$ gives the embedding of R_ι into L_{26} with respect to a certain fixed basis of R_ι .
- **irec.GramR** is the Gram matrix of R_ι with respect to the fixed basis.
- **irec.rootsR** is the list of roots of R_ι .
- **irec.rootstypeR** is the ADE-type of the system of roots of R_ι .
- **irec.m4RVects** is the list of (-4) -vectors of R_ι .
- **irec.weyl** is the Weyl vector $\mathbf{w} \in L_{26}$ such that $D_0 := \iota_{\mathcal{P}}^{-1}(C(\mathbf{w}))$ is an induced chamber in $\mathcal{P}(L_{10})$, where $C(\mathbf{w}) \subset \mathcal{P}(L_{26})$ is the Conway chamber corresponding to \mathbf{w} .
- **irec.weylprime** is another Weyl vector $\mathbf{w}' \in L_{26}$ such that $\langle \mathbf{w}, \mathbf{w}' \rangle_{26} = 1$ and that $a_{26} = 2\mathbf{w} + \mathbf{w}'$ is an interior point of $C(\mathbf{w})$.
- **irec.walls** is the list of roots of L_{10} defining the walls of the induced chamber D_0 .
- **irec.alphas** is the list of pairs $\alpha = (r, v)$, where r is a root of L_{10} defining a wall of D_0 and v is a (-4) -vector of R_ι such that $(r + v)/2 \in L_{26}$.
- **irec.volindex** is $1_{\text{BP}}/\text{vol}(D_0)$, where $1_{\text{BP}} = 46998591897600$.
- **irec.OL10D0** is the list of matrices of the elements of $O(L_{10}, D_0)$.
- **irec.ample** is a primitive vector of L_{10} belonging to the interior of D_0 that is fixed under the action of $O(L_{10}, D_0)$.
- **irec.codim2facewallpairs** is the list of pairs of two indexes $\{i, j\}$ such that $D_0 \cap (r_i)^\perp \cap (r_j)^\perp$ is a face of codimension 2 of D_0 , where r_i and r_j are the i th and the j th elements of **irec.walls**, respectively.
- **irec.isotropicrays** is the list of primitive isotropic rays $f \in L_{10}$ contained in \bar{D}_0 .
- **irec.wallrecs** is the list of records **wrec**, each of which contains the data about a wall w of D_0 . Let **wrec** be a record in **irec.wallrecs** corresponding to a wall $w = D_0 \cap (r)^\perp$ of D_0 . Then **wrec** has the following items. See the proof of Proposition 2.7 of [1] for notation.
 - **wrec.r** is the root r of L_{10} that defines the wall $w = D_0 \cap (r)^\perp$.
 - **wrec.rlifts** is the list of roots \tilde{r} of L_{26} such that $\langle \mathbf{w}, \tilde{r} \rangle_{26} = 1$ and $\iota_{\mathcal{P}}^{-1}((\tilde{r})^\perp) = (r)^\perp$.
 - **wrec.thegtilde** is the isometry $\tilde{g} \in O^{\mathcal{P}}(L_{26})$ such that \tilde{g} preserves the image of $\iota: L_{10}(2) \hookrightarrow L_{26}$ and that its restriction $\tilde{g}|_{L_{10}(2)}$ to $L_{10}(2)$ maps D_0 to the induced chamber adjacent to D_0 across the wall $w = D_0 \cap (r)^\perp$. The existence of this isometry proves Proposition 2.7 of [1].
 - **wrec.theg** is the isometry $g = \tilde{g}|_{L_{10}(2)}$ of L_{10} . We can check that this isometry is equal to the reflection with respect to the root **wrec.r**.
- **irec.facerecs** is a complete list of representatives of orbits of the action of $O(L_{10}, D_0)$ on the set of faces of D_0 . Here we include isotropic rays

contained in \overline{D}_0 to the set of faces of D_0 . The list `irec.facerecs` consists of records `frec`, each of which expresses a representative face f of an orbit of faces under the action of $O(L_{10}, D_0)$. The record `frec` has the following items:

- `frec.dim` is the dimension of f .
- `frec.walls` is the list of indexes i such that $f \subset (r_i)^\perp$, where $D_0 \cap (r_i)^\perp$ is the i th wall in the list `irec.walls` above.
- `frec.adetype` is the ADE-type of the Dynkin diagram formed by the roots r_i that define walls of D_0 containing f . This ADE-type is an ordinary ADE-type when f is not an isotropic ray, whereas it is an *affine* ADE-type when f is an isotropic ray.
- `frec.basis` is a basis of the minimal linear space $\langle f \rangle$ of $L_{10} \otimes \mathbb{R}$ containing f . When $\dim f = 1$ so that `frec.basis` consists of a single element v , we can determine whether f is an isotropic ray or not by calculating $\langle v, v \rangle$.
- `frec.orbitsize` is the size of the orbit of f under the action of $O(L_{10}, D_0)$.

4. GENERALITIES

Let $\pi: X \rightarrow Y$ be the universal covering of a complex Enriques surface Y , and ε the deck-transformation of π . The lattice S_X is equipped with a basis.

4.1. **ADE-types.** An ADE-type is a list of strings "A1", "A2", ..., "E8".

4.2. **Elements of $\text{aut}(Y)$.** An element g of $\text{aut}(Y)$ is expressed by a record `grec` that has items `grec.gX` and `grec.gY`. Let $\tilde{g} \in \text{aut}(X, \varepsilon)$ be an element of $\text{aut}(X)$ commuting with ε such that $\tilde{g}|_{S_Y} = g$ and that \tilde{g} acts on the discriminant group S_X^\vee/S_X trivially. The item `grec.gX` is the matrix representing the action of \tilde{g} on S_X , and the item `grec.gY` is the matrix representing the action of g on $S_Y \cong L_{10}$. Let `emb` be the matrix such that $v \mapsto v \cdot \text{emb}$ is the embedding $\pi^*: S_Y(2) \cong L_{10}(2) \hookrightarrow S_X$. Then we have `emb · gX = gY · emb`.

4.3. **Smooth rational curves on Y .** A smooth rational curve C on Y is expressed by a record `rrec` that has items `rrec.ratY` and `rrec.lifts`. The item `rrec.ratY` is the class $[C] \in S_Y$ of the curve C , and the item `rrec.lifts` is the pair of classes $[\tilde{C}_1], [\tilde{C}_2] \in S_X$ of the two irreducible components of $\pi^{-1}(C) = \tilde{C}_1 + \tilde{C}_2$. Let `emb` be as above. Then we have `ratY · emb = lifts[1] + lifts[2]`.

4.4. **The lists V_0 and \mathcal{H} .** The outputs V_0 and \mathcal{H} of Procedure 4.1 of the paper [BS] are described by the following data. In our applications, a vertex of the graph (V, E) is an $L_{26}/S_Y(2)$ -chamber. For each Enriques surface Y , we fix an $L_{26}/S_Y(2)$ -chamber D_0 contained in Nef_Y , and hence each $L_{26}/S_Y(2)$ -chamber D is expressed as D_0^i by some isometry $\tau \in O^{\mathcal{P}}(L_{10})$. (The isometry τ is unique up to left multiplications of elements of $O(L_{10}, D_0)$.) The group G acting on (V, E) is a subgroup of $\text{aut}(Y)$, and hence each element of G is expressed by a record `grec` in the way explained in Section 4.2.

The ordered list V_0 of vertices is expressed by an ordered list `V0` of records `chamrec`. Each `chamrec` has the following items. Let D be the $L_{26}/S_Y(2)$ -chamber expressed by `chamrec`.

- `chamrec.pos` is the position of the `chamrec` in `V0`. **Caution:** The position starts from 1 so that `chamrec` is `V0[chamrec.pos]`, whereas, in the explanation of Procedure 4.1 in the paper [BS], the position starts from 0.
- `chamrec.from` and `chamrec.adjpos` indicate from which chamber the chamber D is obtained. Namely, the chamber D is the chamber adjacent to the chamber D_{prev} expressed by a record `prevchamrec` in `V0` whose position `prevchamrec.pos` is equal to `chamrec.from`, and the wall across which D is adjacent to D_{prev} is the `chamrec.adjpos`-th wall in the list `prevchamrec.adjrecs` of adjacent chambers of D_{prev} (see below). If the chamber D is the initial chamber of the list V_0 (that is, `chamrec.pos` is 1), both of `chamrec.from` and `chamrec.adjpos` are "none".
- `chamrec.taug` is a matrix $\tau \in O^{\mathcal{P}}(L_{10})$ such that $D = D_0^{\tau}$.
- `chamrec.autcham` is the list of elements of the stabilizer subgroup $T_G(D, D)$ of D in the group G . Since G is a subgroup of $\text{aut}(Y)$, each element of `chamrec.autcham` is expressed by a record `grec`.
- `chamrec.adjrecs` is the list of adjacent chambers D' of D . Each element of `chamrec.adjrecs` is a record `adjrec` with the following items.
 - `adjrec.wall` is the root r' of S_Y defining the wall of D across which D' is adjacent to D . When we apply our algorithm to the calculation of the group $\text{aut}(Y)$ (Section 6.1 of [BS]) or the stabilizer subgroup of an elliptic fibration $\phi: Y \rightarrow \mathbb{P}^1$ (Section 6.4 of [BS]), the chamber D' is the image $D^{s_{r'}}$ of D by the reflection $s_{r'}$ with respect to the root r' . When we apply our algorithm to the calculation of the stabilizer subgroup of a smooth rational curve C on Y (Section 6.3 of [BS]), the wall is in fact the face $D \cap (r)^\perp \cap (r')^\perp$ of D with codimension 2, where $r = [C]$, and the chamber D' is

$$\begin{cases} D^{s_{r'}} & \text{if } \langle r, r' \rangle = 0, \\ D^{s_r s_{r'}} & \text{if } \langle r, r' \rangle = 1, \end{cases}$$

where s_r is the reflection with respect to the root $r = [C]$. When we apply the algorithm to the graph (V_Γ, E_Γ) and the group G_Γ associated with an RDP-configuration Γ (Section 7.1 of [BS]), the root r' is a defining root of a wall of D such that

$$w_f := \mathcal{P}_{\langle \Gamma \rangle^\perp} \cap D \cap (r')^\perp$$

is the wall of the $L_{26}/\langle \Gamma \rangle^\perp(2)$ -chamber $f := \mathcal{P}_{\langle \Gamma \rangle^\perp} \cap D$. Note that f is a face of D with dimension $10 - |\Gamma| = \dim \mathcal{P}_{\langle \Gamma \rangle^\perp}$ (that is, f contains a non-empty open subset of $\mathcal{P}_{\langle \Gamma \rangle^\perp}$), whereas w_f is a face of D with dimension $9 - |\Gamma|$ contained in f . The adjacent chamber is the unique $L_{26}/S_Y(2)$ -chamber D' such that $D' \neq D$, that D' is contained in Nef_Y , that D' contains w_f , and that $\mathcal{P}_{\langle \Gamma \rangle^\perp} \cap D'$ contains a non-empty open subset of $\mathcal{P}_{\langle \Gamma \rangle^\perp}$.

- `adjrec.israt` is `true` if the root r' is the class of a smooth rational curve on Y (that is, the chamber D' is outside of Nef_Y), whereas `adjrec.israt` is `false` if r' is not the class of a smooth rational curve on Y and hence the chamber D' is contained in Nef_Y .

When `adjrec.israt` is `true` and r' is the class of a smooth rational curve C' on Y , the record `adjrec` has the following item.

- `adjrec.split` is the pair of roots $[\tilde{C}_1], [\tilde{C}_2]$ of S_X that are the classes of the two irreducible components of $\pi^{-1}(C') = \tilde{C}_1 + \tilde{C}_2$.

When `adjrec.israt` is **false** and the adjacent chamber D' is contained in Nef_Y , the record `adjrec` has the following item.

- `adjrec.isnew` is **true** or **false**. If `adjrec.isnew` is **true**, then the chamber D' is G -equivalent to none of chambers that had been added to V_0 when we processed D' , and hence D' was appended to V_0 . If `adjrec.isnew` is **false**, the chamber D' is G -equivalent to a chamber that had been already added to V_0 .

When `adjrec.israt` is **false** and `adjrec.isnew` is **true**, the record `adjrec` has the following item.

- `adjrec.positionInV0` indicates the position of V_0 at which D' is added, that is, the chamber D' is described by the record

$$\text{chamrec}' := V0[\text{adjrec.positionInV0}].$$

When `adjrec.israt` is **false** and `adjrec.isnew` is **false**, the record `adjrec` has the following item. Let D'' be the unique chamber in V_0 to which D' is G -equivalent.

- `adjrec.isomto` indicates the position of D'' in V_0 .
- `adjrec.isomby` is the record `grec` of an element $g \in G$ such that $D'^g = D''$.

The subset \mathcal{H} of the group G is expressed by a set `HHH` of records `grec`. **Caution.** The identity is not contained in `HHH`.

5. THE FILE `Enrs.txt`

In the file `Enrs.txt`, we have a list `Enrs` of 182 records `Enr`. Each of these records corresponds to a $(\tau, \bar{\tau})$ -generic Enriques surface Y listed in Table 1.1 of [BS], except for the cases Nos. 88 and 146 A record `Enr` in this list has the following contents.

- `Enr.no` is the number of the corresponding row in Table 1.1 of [BS].
- `Enr.typeR` is the ADE-type $\tau(R)$.
- `Enr.typeRbar` is the ADE-type $\tau(\bar{R})$.
- `Enr.typeRtilde` is the ADE-type $\tau(\tilde{R})$.
- `Enr.exists` is **true** or **false**, and shows whether a $(\tau, \bar{\tau})$ -generic Enriques surface exists or not.
- `Enr.cde` is the triple $[c_{(\tau, \bar{\tau})}, d_{(\tau, \bar{\tau})}, e_{(\tau, \bar{\tau})}]$.
- `Enr.ker` is the list of elements of the kernel of the natural homomorphism $\text{aut}(X, \varepsilon) \rightarrow \text{aut}(Y)$. Each element is a matrix in $O^{\mathcal{P}}(S_X)$.
- `Enr.irecname` is the name if `irec` that is used in the computation.
- `Enr.SXrec` is a record describing the embeddings $S_Y(2) \hookrightarrow S_X \hookrightarrow L_{26}$ of lattices. See the subsection below for the details.
- `Enr.Autrec` is a record that describes the result of the computation in Section 6.1 of the paper [BS]. See the subsection below for the details.
- `Enr.Rats` is a record that describes the action of $\text{aut}(Y)$ on the list $\mathcal{R}_{\text{temp}}$ of smooth rational curves C on Y such that $[C]$ defines a wall of an $L_{26}/S_Y(2)$ -chamber D belonging to `Enr.Autrec.V0`. See the subsection below for the details.

- **Enr.Ells** is a record that describes the action of $\text{aut}(Y)$ on the list $\mathcal{E}_{\text{temp}}$ of elliptic fibrations $\phi: Y \rightarrow \mathbb{P}^1$ such the ray $\mathbb{R}_{\geq 0}[F_\phi]$, where F_ϕ is a general fiber of ϕ , is contained in the closure \overline{D} of an $L_{26}/S_Y(2)$ -chamber D belonging to **Enr.Autrec.V0**. See the subsection below for the details. **Caution.** This item **Enr.Ells** is *not* provided if **Enr.exists** is **false**; that is, we calculate **Enr.Ells** only when the data **Enr.SXrec** is geometrically realizable.

5.1. The record Enr.SXrec. The record **Enr.SXrec** is the record describing the lattice S_X and the primitive embeddings

$$\iota: S_Y(2) \cong L_{10}(2) \hookrightarrow S_X \hookrightarrow L_{26}$$

of lattices. The record **Enr.SXrec** has the following items, many of which are just the copies of items of the record **irec** describing the primitive embedding $\iota: S_Y(2) \hookrightarrow L_{26}$. In the paper [BS], the orthogonal complement of $S_Y(2)$ in S_X is denoted by S_{X-} . In this note, we use Q to denote the lattice S_{X-} . The lattices L_{26} , S_X and Q are equipped with certain bases. The lattice $S_Y \cong L_{10}$ is equipped with the basis $\{e_1, \dots, e_{10}\}$.

- **SXrec.GramL26** is the Gram matrix of L_{26} .
- **SXrec.embSXL26** is the matrix M such that $v \mapsto vM$ is the embedding $S_X \hookrightarrow L_{26}$.
- **SXrec.embSYSX** is the matrix M' such that $v \mapsto vM'$ is the embedding $\pi^*: S_Y(2) \cong L_{10}(2) \hookrightarrow S_X$.
- **SXrec.embQSX** is the matrix M'' such that $v \mapsto vM''$ is the embedding $Q \hookrightarrow S_X$.
- **SXrec.GramSX** is the Gram matrix of S_X .
- **SXrec.configrats** is a list of roots $[\tilde{C}_1], \dots, [\tilde{C}_m]$ in S_X , where $\tilde{C}_1, \dots, \tilde{C}_m$ are distinct smooth rational curves on X such that the divisor $\sum \tilde{C}_i$ is an ADE-configuration of type R and that $\sum \tilde{C}_i$ is mapped isomorphically to a divisor on Y by $\pi: X \rightarrow Y$. The classes in this list together with the image S_{X+} of $\pi^*: S_Y(2) \hookrightarrow S_X$ generate S_X .
- **SXrec.walls** is the list of defining roots of the walls of the fixed $L_{26}/S_Y(2)$ -chamber D_0 .
- **SXrec.ampleY** is a primitive vector of $S_Y \cong L_{10}$ in the interior of the $L_{26}/S_Y(2)$ -chamber D_0 , which is an ample class of S_Y . This class is chosen in such a way that it is invariant under the action of the group $O(L_{10}, D_0)$.
- **SXrec.m4vsInQ** is the list of (-4) -vectors in Q .
- **SXrec.codim2faces** is the list of non-ordered pairs $\{i, j\}$ such that the intersection of the i th wall and the j th wall in **SXrec.walls** is a face of codimension 2 of D_0 .
- **SXrec.isotropicrays** is the list of primitive isotropic rays in the closure $\overline{D_0}$ of D_0 .
- **SXrec.volumeindex** is $1_{\text{BP}}/\text{vol}(D_0)$.
- **SXrec.enrinvol** is the matrix presentation of the Enriques involution $\varepsilon \in O^{\mathcal{P}}(S_X)$.

5.2. The record Enr.Autrec. The record **Enr.Autrec** is the record describing the results of the algorithm in Section 6.1 of the paper [BS] for the calculation of the

action of $\text{aut}(Y)$ on Nef_Y . This record `Enr.Autrec` is comprised of `Enr.Autrec.V0` and `Enr.Autrec.HHH` as are explained in Section 4.4.

5.3. **The record `Enr.Rats`.** The record `Rats := Enr.Rats` has the following items.

- `Rats.representatives` is the list of records `rrec` of smooth rational curves that are chosen as representatives of the orbit decomposition of $\mathcal{R}_{\text{temp}}$ by the action of $\text{aut}(Y)$.
- `Rats.Ratstemp` is the list of records describing elements of $\mathcal{R}_{\text{temp}}$. For $C \in \mathcal{R}_{\text{temp}}$, the corresponding record `temprrrec` has the following items:
 - `temprrrec.rat` is the record `rrec1` describing C .
 - `temprrrec.rep` is the record `rrec2` describing the representative C' of the orbit containing C . Hence `rrec2` is a member of the list `Rats.representatives`.
 - `temprrrec.by` is the record `grec` describing an automorphism $g \in \text{aut}(Y)$ that maps C to C' .

5.4. **The record `Enr.Ells`.** This item `Enr.Ells` is provided only when `Enr.exists` is `true`. The record `Ells := Enr.Ells` has the following items.

- `Ells.representatives` is the list of records `ellfib` of elliptic fibrations $\rho: Y \rightarrow \mathbb{P}^1$ chosen as representatives of the orbit decomposition of $\mathcal{E}_{\text{temp}}$ by the action of $\text{aut}(Y)$. Each `ellfib` has the following item:
 - `ellfib.ell` is the primitive isotropic ray $f_\rho := [F_\rho]/2$, where F_ρ is a general fiber of $\rho: Y \rightarrow \mathbb{P}^1$.
 - `ellfib.reduciblefibers` describes the reducible fibers of the elliptic fibration $\rho: Y \rightarrow \mathbb{P}^1$. This record is a list of the records `fiberrec` corresponding to reducible fibers. Each `fiberrec` has the following items.
 - * `fiberrec.adetype` is the ADE-type of the reducible fiber, which is one of the strings "A1", "A2", ..., "E8". This string expresses the *affine* ADE-type of the reducible fiber.
 - * `fiberrec.multiplicity` is the multiplicity m , which is 1 if the fiber has a reduced component, whereas $m = 2$ if the multiplicities of all components are even.
 - * `fiberrec.irredcomps` is the list of records `rrec` that describe irreducible components of the fiber.
- `Ells.Ellstemp` is the list of records describing elements of $\mathcal{E}_{\text{temp}}$. For $\phi \in \mathcal{E}_{\text{temp}}$, the corresponding record `tempphirec` has the following items:
 - `tempphirec.ell` is the primitive isotropic ray $f_\phi := [F_\phi]/2$, where F_ϕ is a general fiber of ϕ .
 - `tempphirec.rep` is the primitive isotropic ray f_ρ of the representative $\rho: Y \rightarrow \mathbb{P}^1$ of the orbit pf ϕ under the action of $\text{aut}(Y)$.
 - `tempphirec.by` is the record `grec` describing an automorphism $g \in \text{aut}(Y)$ that maps f_ϕ to f_ρ .

6. THE FILE `E6EnrRDPS.txt`

In the file `E6EnrRDPS.txt`, we have a list `E6EnrRDPS` of 750 records about the RDP-configurations on an (E_6, E_6) -generic Enriques surface Y . See Section 7.1 of the paper [BS]. Each record `RDPrec` in `E6EnrRDPS` describes an RDP-configuration

Γ obtained as $\Gamma(f)$ for some face f of the fundamental domain D_0 . Each `RDPre` has the following items.

- `RDPre.position` is the position of `RDPre` in the list `E6EnrRDPs`.
- `RDPre.rats` is the list of smooth rational curves in the RDP-configuration Γ . Each member of `RDPre.rats` is a record `rrec` explained in Section 4.3.
- `RDPre.adetype` is the ADE-type of the RDP-configuration Γ .
- `RDPre.isrepresentative` is `true` or `false`. This item being `true` means that Γ is chosen as a representative of the $\text{aut}(Y)$ -orbit of RDP-configurations containing Γ . In this case, the record `RDPre` has the following two additional items.
 - `RDPre.V0` and `RDPre.HHH` are the result of Procedure 4.1 of the paper [BS] applied to the graph (V_Γ, E_Γ) and the group G_Γ .

If `RDPre.isrepresentative` is `false`, the record `RDPre` has the following two additional items.

- `RDPre.isomto` is a pair $[i, j]$, which indicates the following. The i th RDP-configuration Γ' in `E6EnrRDPs` is the representative of the $\text{aut}(Y)$ -orbit containing Γ . Let `RDPre' = RDPrecs[i]` be the record describing the representative Γ' . The initial chamber D_0 of the graph (V_Γ, E_Γ) is mapped to the j th chamber D' in the list `RDPre'.V0` by an element g of $\text{aut}(Y)$ such that $\Gamma^g = \Gamma'$, which is described by `RDPre.isomby` below.
- `RDPre.isomby` is a record `grec` explained in Section 4.2 that describes the automorphism $g \in \text{aut}(Y)$ that maps D_0 to D' and Γ to Γ' .

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