Problem No.8

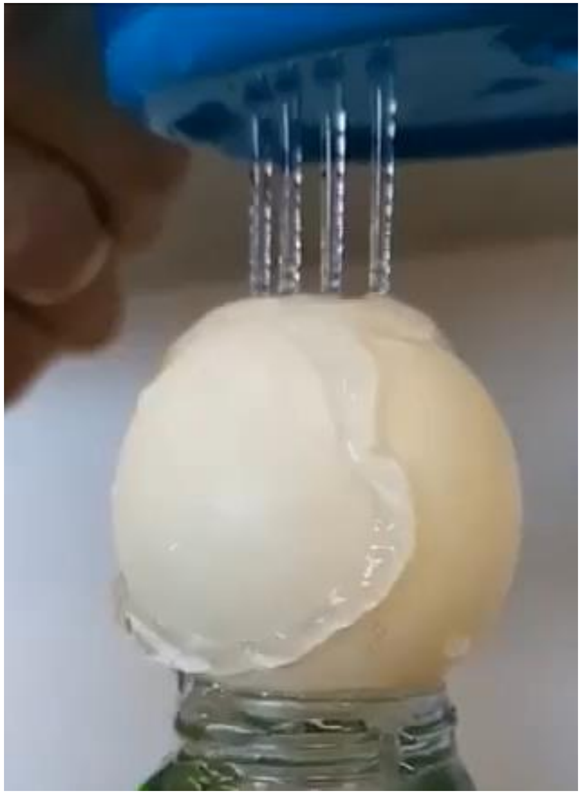
Rippled water columns

Reporter: Artem Sukhov

Team of Russia

International Physicists' Tournament 2020

When a vertical water jet hits a surface, ripples may appear. If **certain conditions** are met, the ripple **structure** is pronounced, **steady** and very reproducible. Describe the phenomenon. What **properties of the fluid and the flow** can be deduced **from the observations**?



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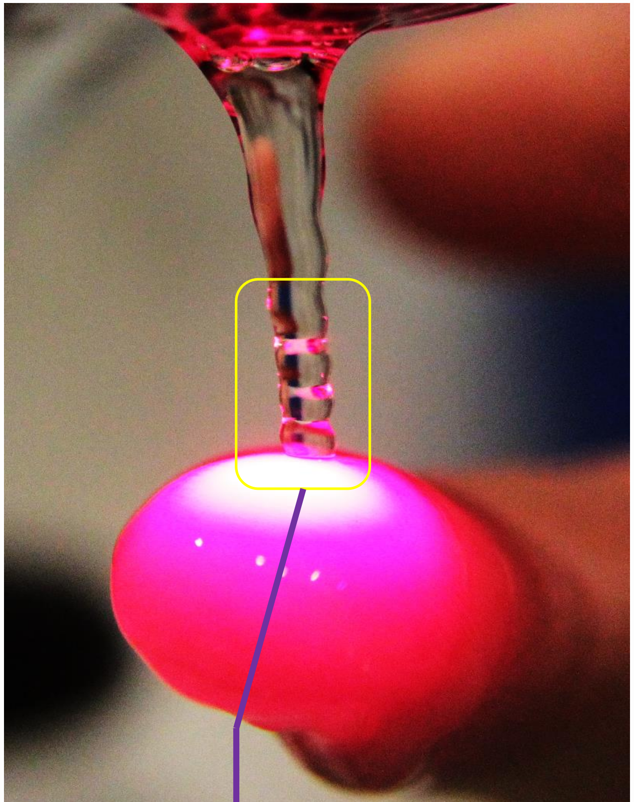
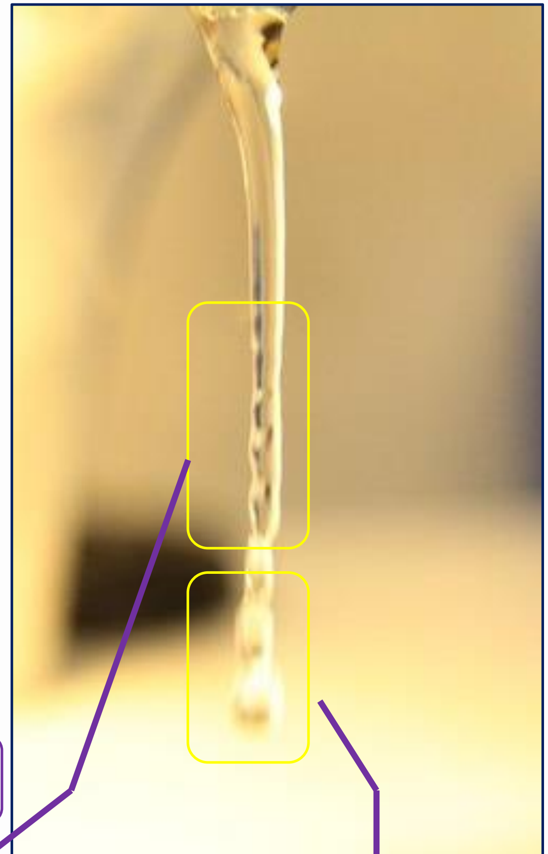
Identification of the investigation area



First observations:



Investigated ripple structure

Capillary waves?

Plateau–Rayleigh instability

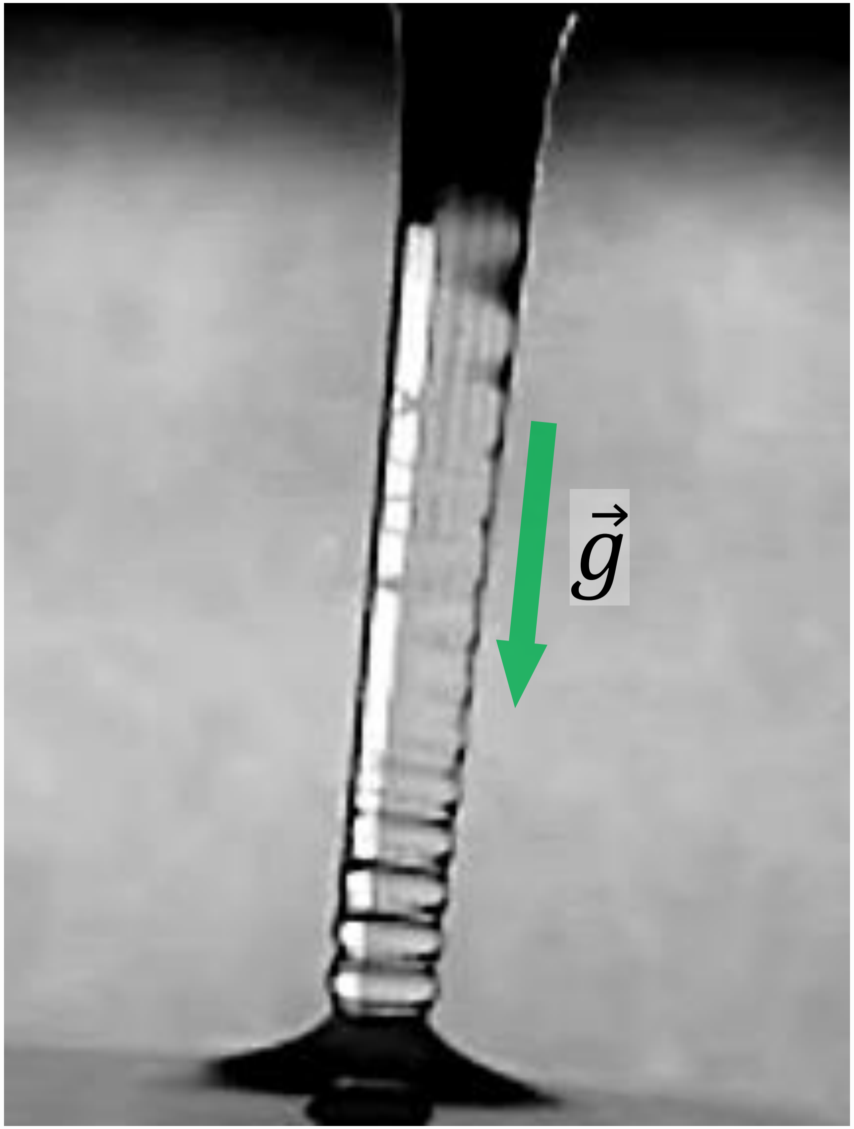
1. Ripples reproduce well with a **Reynolds number Re~100**
2. The shape of the ripples is **independent of the obstacles** that it encounters
3. **Ripples** also **form** at a height **where** it almost breaks up into **droplets**
4. Closer to the obstacle - more pronounced and has less distance between the peaks
5. In constant conditions it **looks stationary**

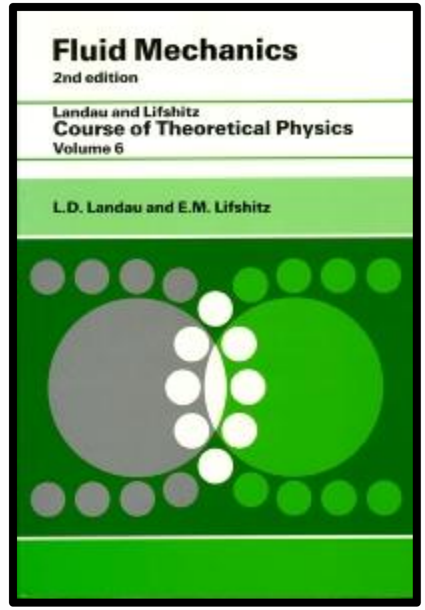
Experimental part

Qualitative explanation

Quantitative description

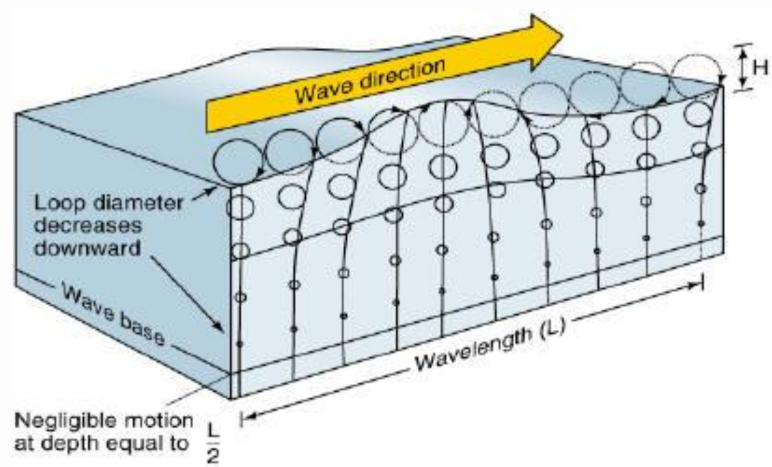
Stationary ripple pattern





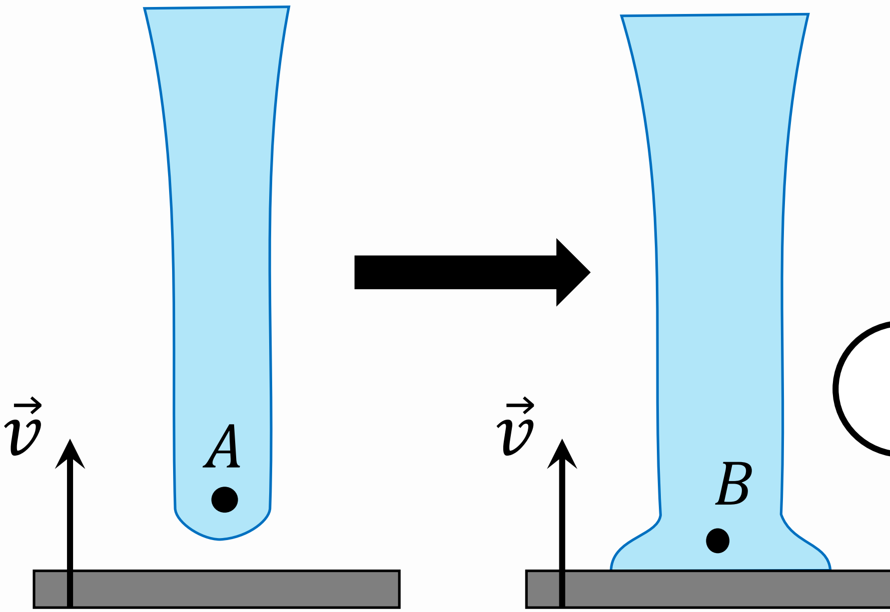
“Гидродинамика”, [Fluid mechanics], L.D. Landau and E.M. Lifshitz

“Гидродинамика”, [Fluid mechanics], G. Lamb

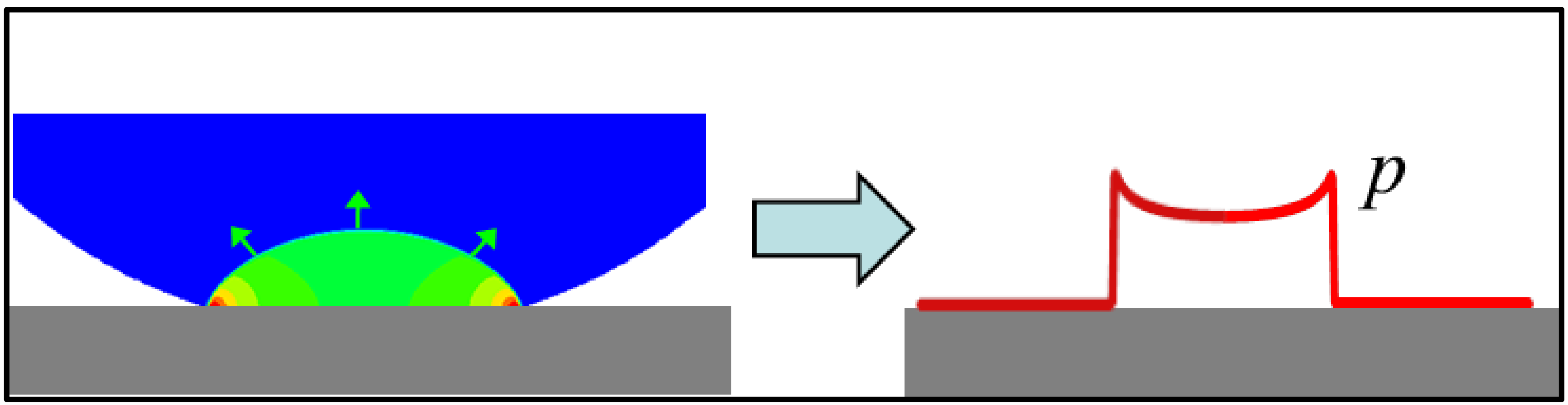


Experimental part | Qualitative explanation | Quantitative description

Disturbance in jet

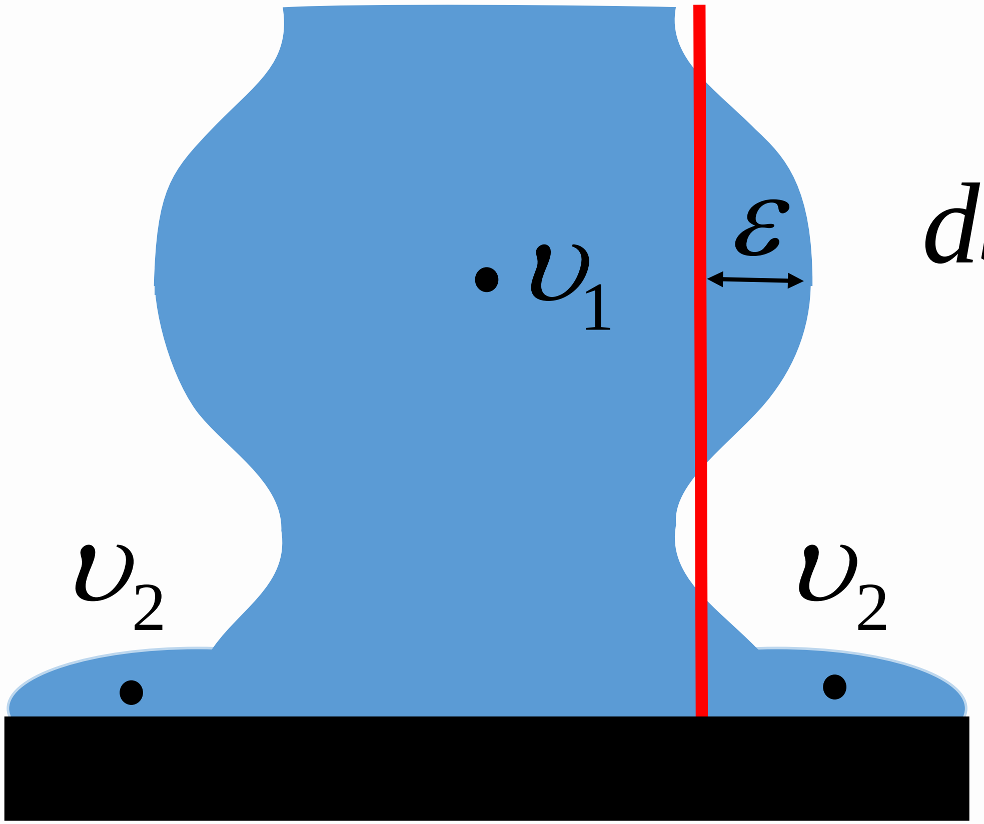


The initial disturbance in the jet arises due to the wave after impact



“Соударение струи с поверхностью, моделирование “ударной” волны”,
[“Shock” wave in the jet simulation],© IME - Subdivision of FIC KazanSC of RAS, 2020

Initial amplitude



$$\sigma dS = dE$$

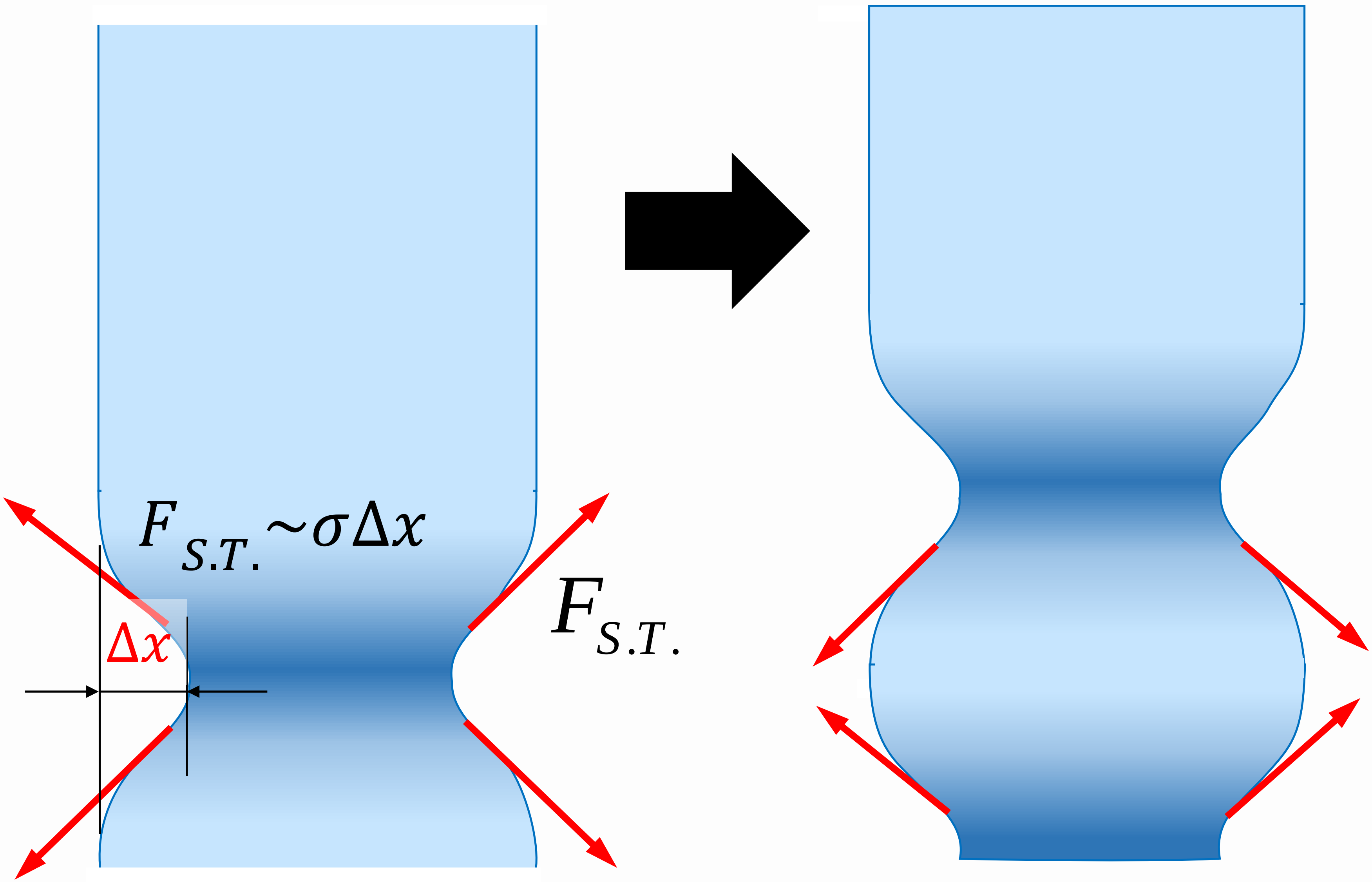
$$dS \sim d\varepsilon^2 \quad dE \sim \rho d\upsilon^2$$

$$\varepsilon \sim \upsilon\sqrt{\frac{\rho}{\sigma}}$$

! This dependence is confirmed experimentally well

Experimental part | Qualitative explanation | Quantitative description

Capillary waves



Experimental part

Qualitative explanation

Quantitative description

Why are the waves standing?

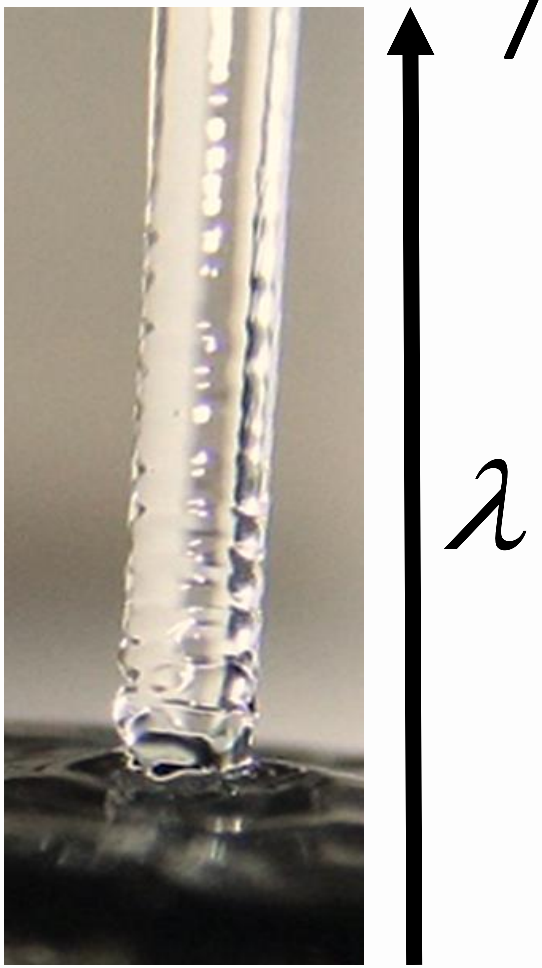
The jet at a point moves at a constant speed c.

⬇

Waves with wave number k when added at z give a nonzero amplitude if their phases at this point are equal

⬇

One disturbance was generated by t_0 earlier than the second, then the phase equality can be written as follows



$$k \cdot (z + ct_0) - \omega \cdot (t + t_0) = k \cdot z - \omega \cdot t \rightarrow \boxed{v_\varphi = \frac{\omega}{k} = c}$$

Wavelength from height

1) There is wave dispersion

$$\omega = \omega(k)$$

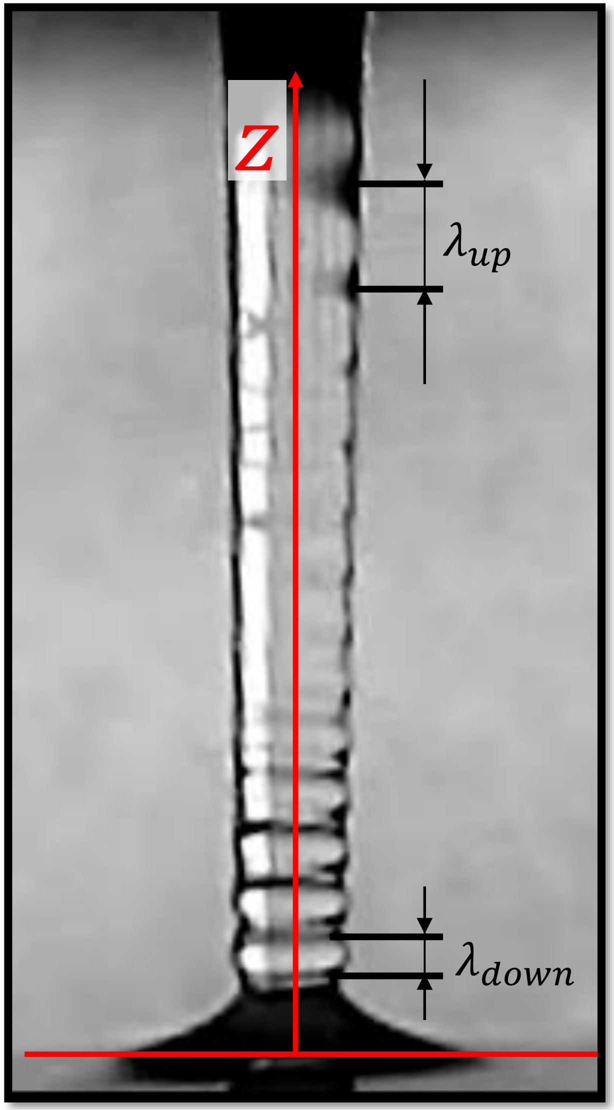
$$k = \frac{2\pi}{\lambda} \ll \sqrt{\frac{g\rho}{\sigma}} \qquad \omega = \frac{2\pi c}{\lambda}$$

2) The jet is accelerated by gravity

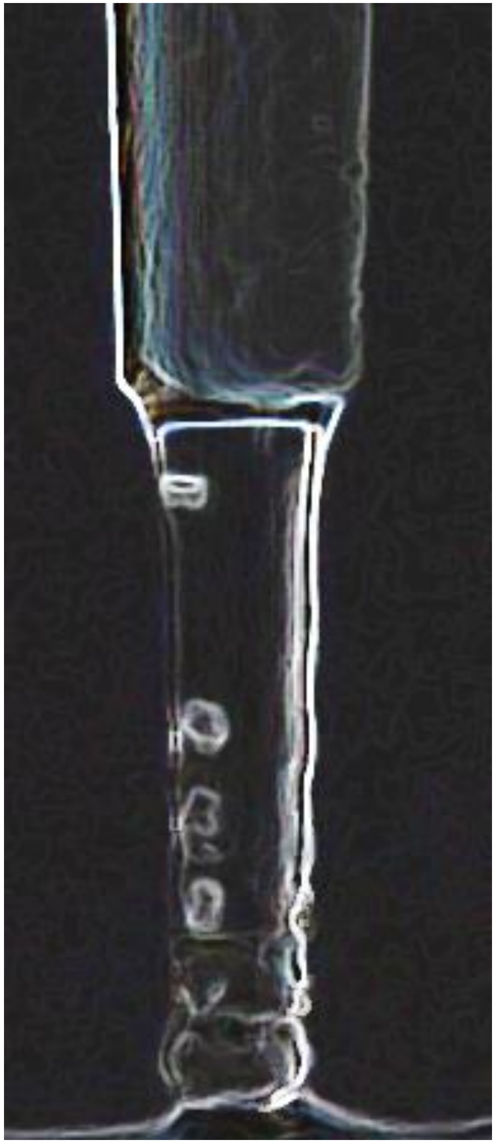
$$\upsilon(z) = \sqrt{\upsilon_o^{\,2} + 2g(h-z)}$$

3) From the equality of the phase velocity (c) and the jet velocity v (z), the wavelength can be found

$$\lambda = \lambda(z)$$



Plan of investigation



1. Derivation and verification of dispersion law

2. Investigation of ripples formation dynamics

3. Method for measuring initial jet parameters

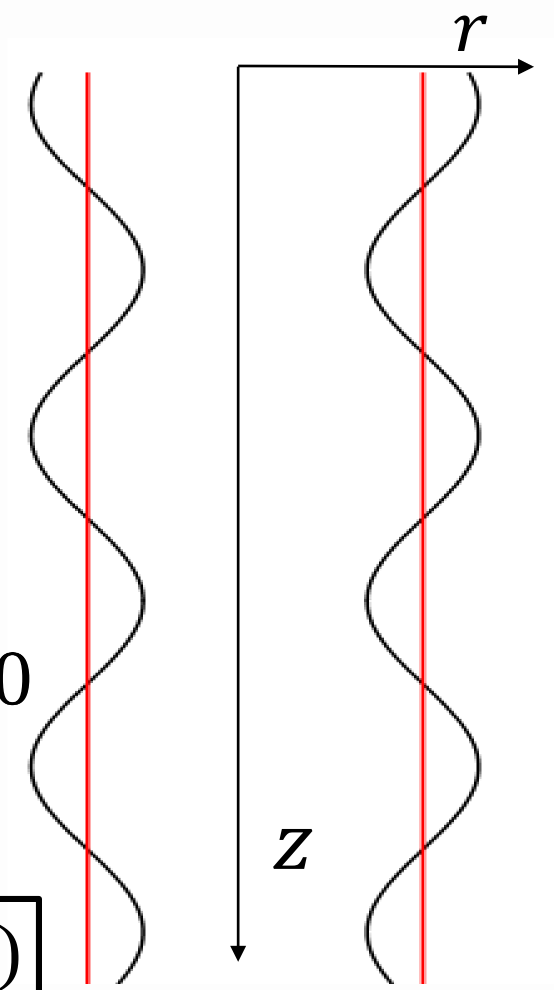
Dispersion law

(not a flat surface, to improve - gravity)

- Stationary waves on cylindrical fluid jets, K. M. Awati and T. Howes, American Journal of Physics 64, 808 (1996)

$$\begin{cases} \dfrac{\partial \vec{\upsilon}}{\partial t} + (\vec{\upsilon}, \nabla)\vec{\upsilon} = -\dfrac{\nabla p}{\rho} \\ \nabla \cdot \vec{\upsilon} = 0 \end{cases}$$

Incompressible Euler equations with constant and uniform density



We assume for potential flow:

$$\vec{\upsilon} = \nabla \times \phi$$

$$\begin{cases} \Delta \phi = 0 \\ \dfrac{\partial \phi}{\partial t} + \dfrac{p}{\rho} = 0 \end{cases}$$

Transition to cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$\boxed{\phi = \phi_0 I_0(kr)\sin(kz - \omega t)}$$

I_i $-$ *modificated Bessel function of the first kind of the i order*

Dispersion law

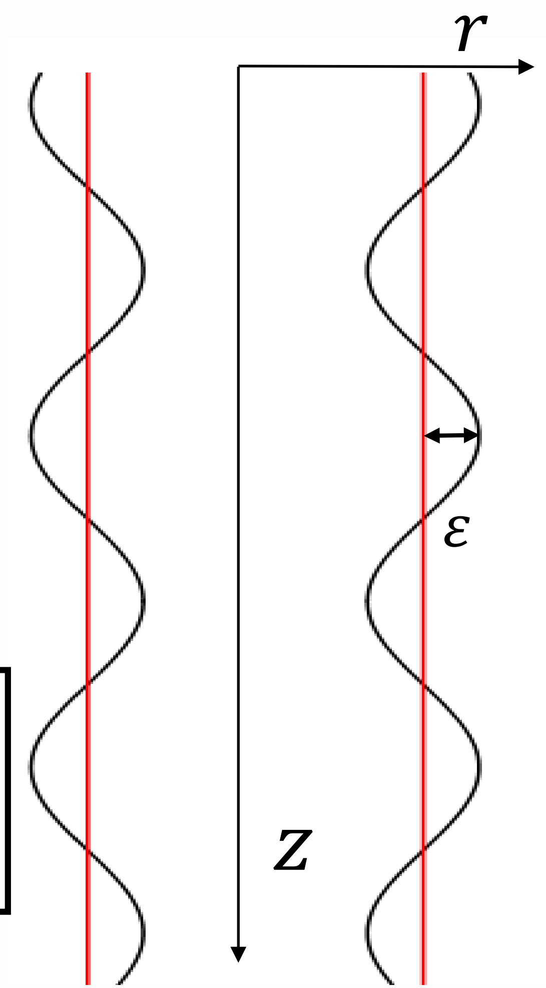
(not a flat surface, to improve - gravity)

- Stationary waves on cylindrical fluid jets, K. M. Awati and T. Howes, American Journal of Physics 64, 808 (1996)

$$\vec{\upsilon} = \nabla \times \phi \qquad \frac{d\varepsilon}{dt} = \upsilon_r \qquad p - p_0 = -\sigma\left(\frac{d^2\varepsilon}{dz} - \frac{1}{\varepsilon}\right)$$

$$\omega^2 = \frac{\sigma k\left(k^2 - \frac{1}{r^2}\right)}{\rho} \cdot \frac{I_1(kr)}{I_0(kr)}$$

$$\omega = \frac{2\pi c}{\lambda} \qquad \left(\frac{2\pi c}{\lambda}\right)^2 = \frac{2\pi\sigma}{\lambda\rho} \cdot \left[\left(\frac{2\pi}{\lambda}\right)^2 - \frac{1}{r^2}\right] \cdot \frac{I_1\left(\frac{2\pi}{\lambda} r\right)}{I_0\left(\frac{2\pi}{\lambda} r\right)}$$



Numerical solution fit

In literature:

Hancock, M. J., & Bush, J. W. M. (2002). Fluid pipes. *Journal of Fluid Mechanics*, 466, 285–304.

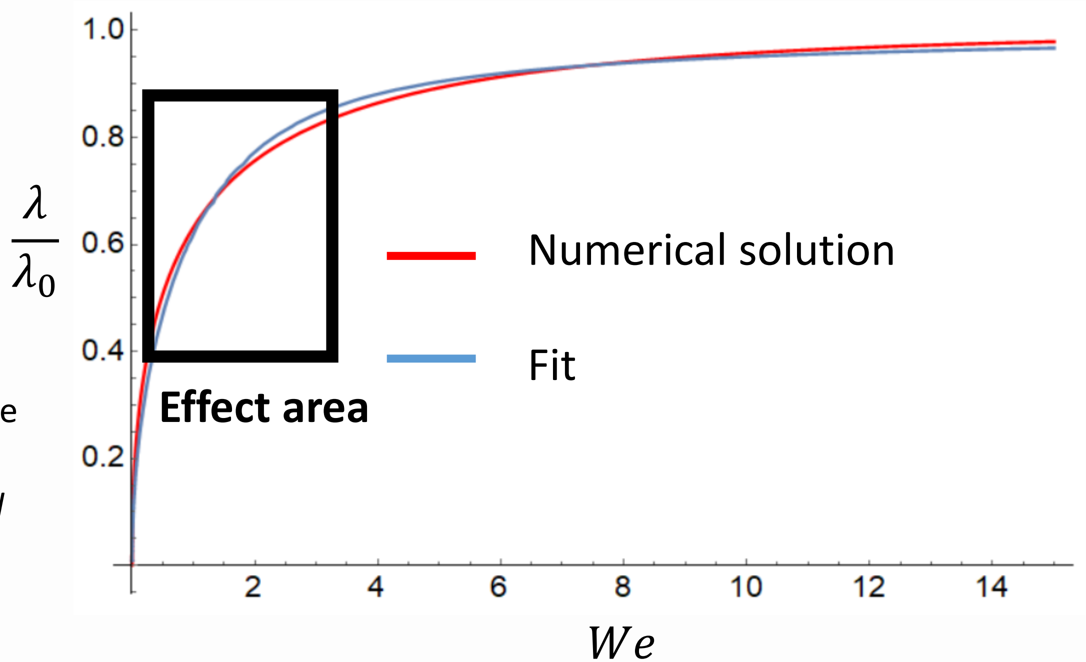
Wavelength on a flat surface:

$$\lambda_0 = \frac{2\pi\sigma}{\rho\upsilon^2}$$

$$We = \frac{\rho\upsilon^2 R_0}{\sigma}$$

Fit of the numerical solution:

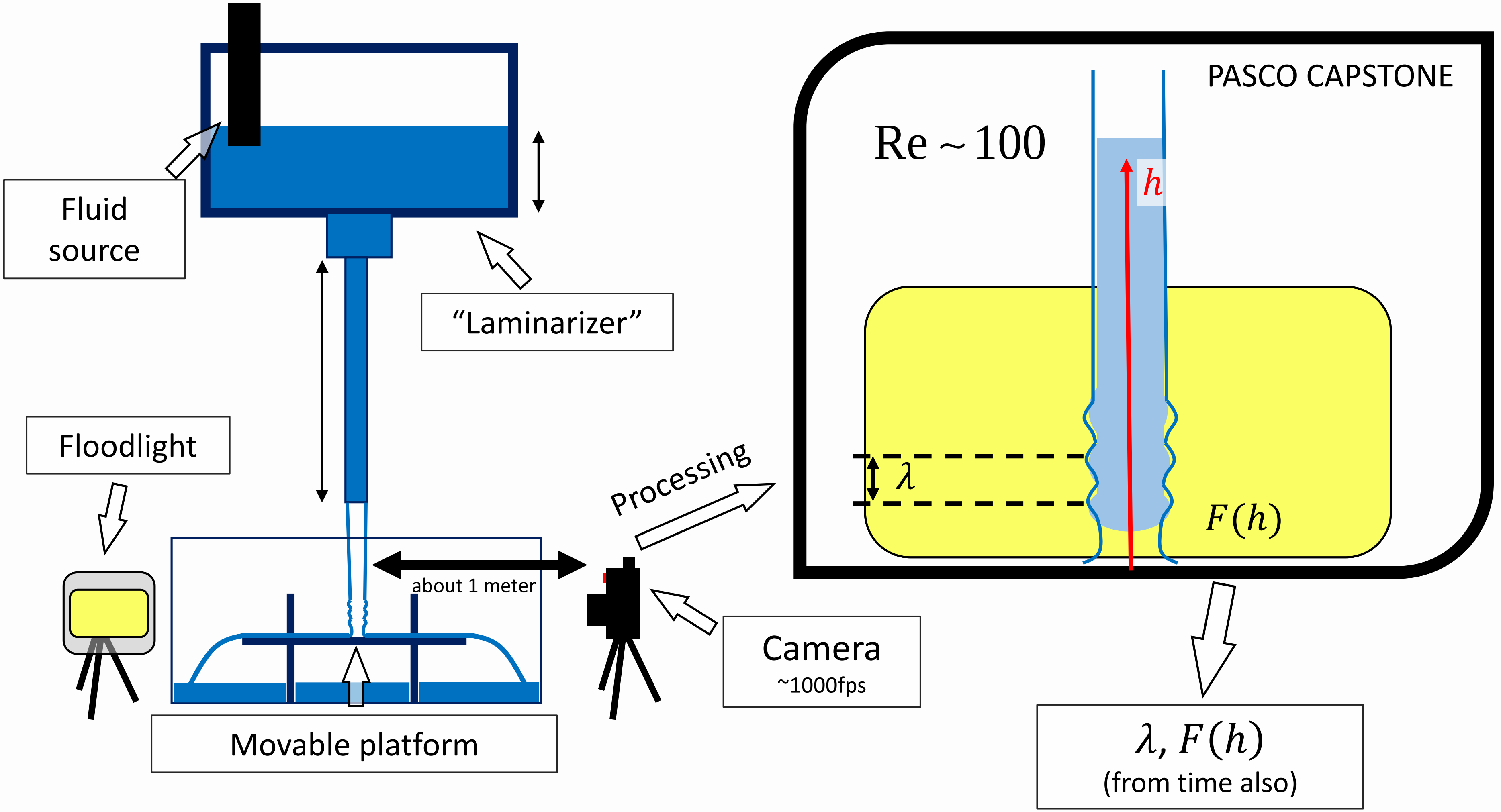
$$\frac{\lambda}{\lambda_0} = 1 - \exp(-\sqrt{We})$$



The correction taking into account the curvature corrected the plane model **by 2 times!**
Gravity is neglected everywhere for a beautiful solution. Amendment less than 10%.

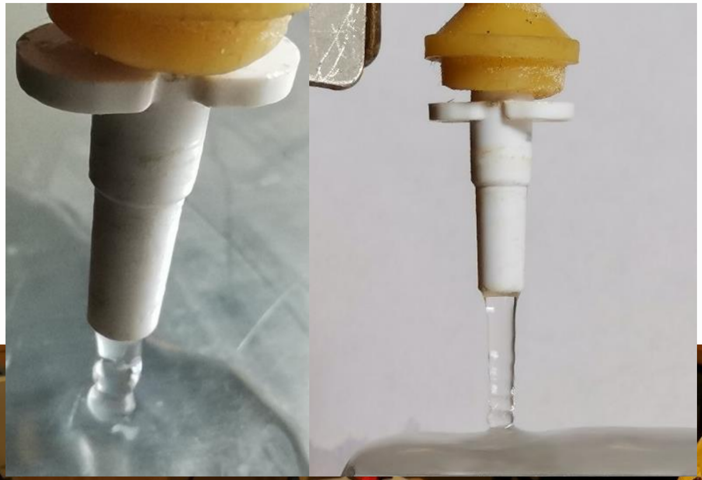
Experimental part | Qualitative explanation | Quantitative description

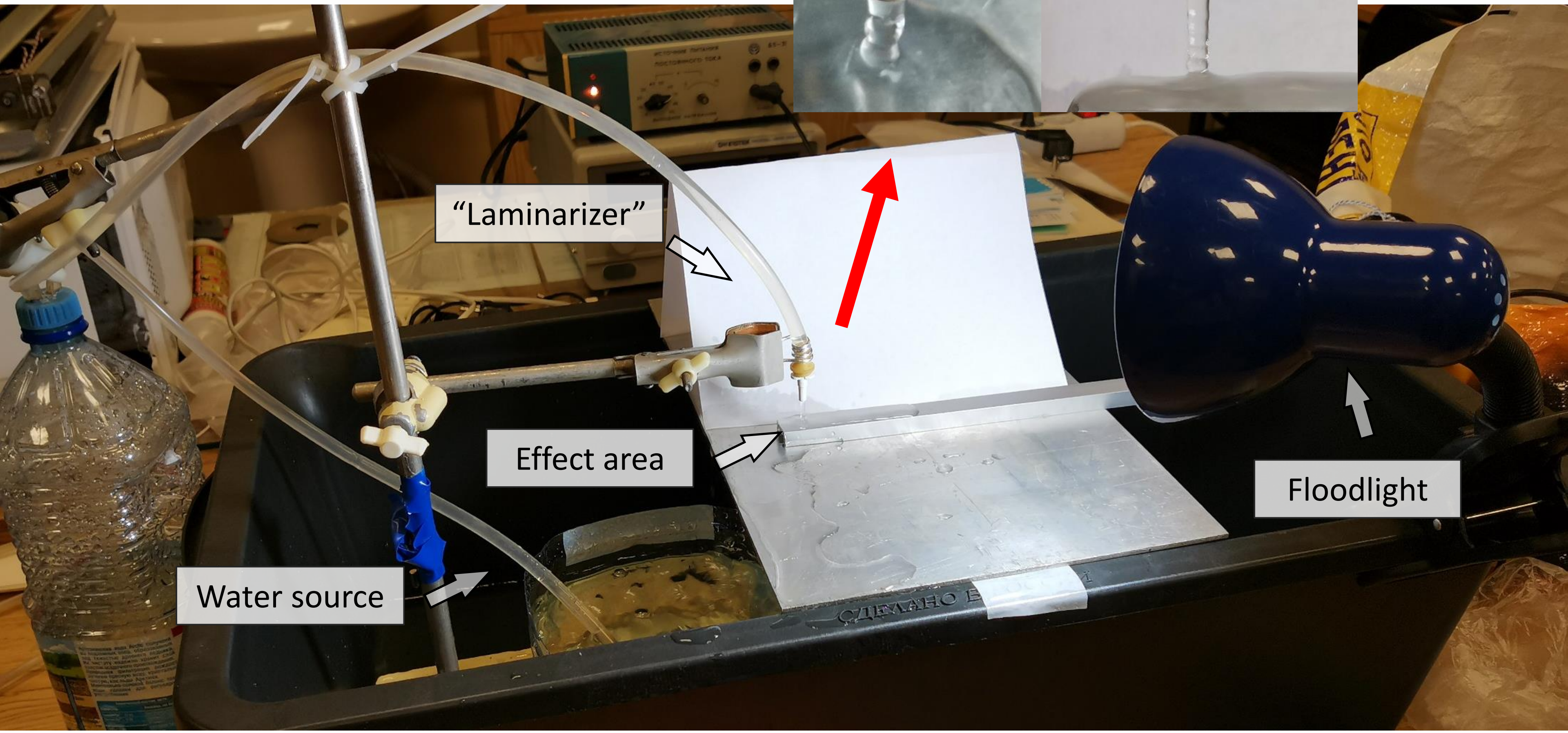
Experimental setup



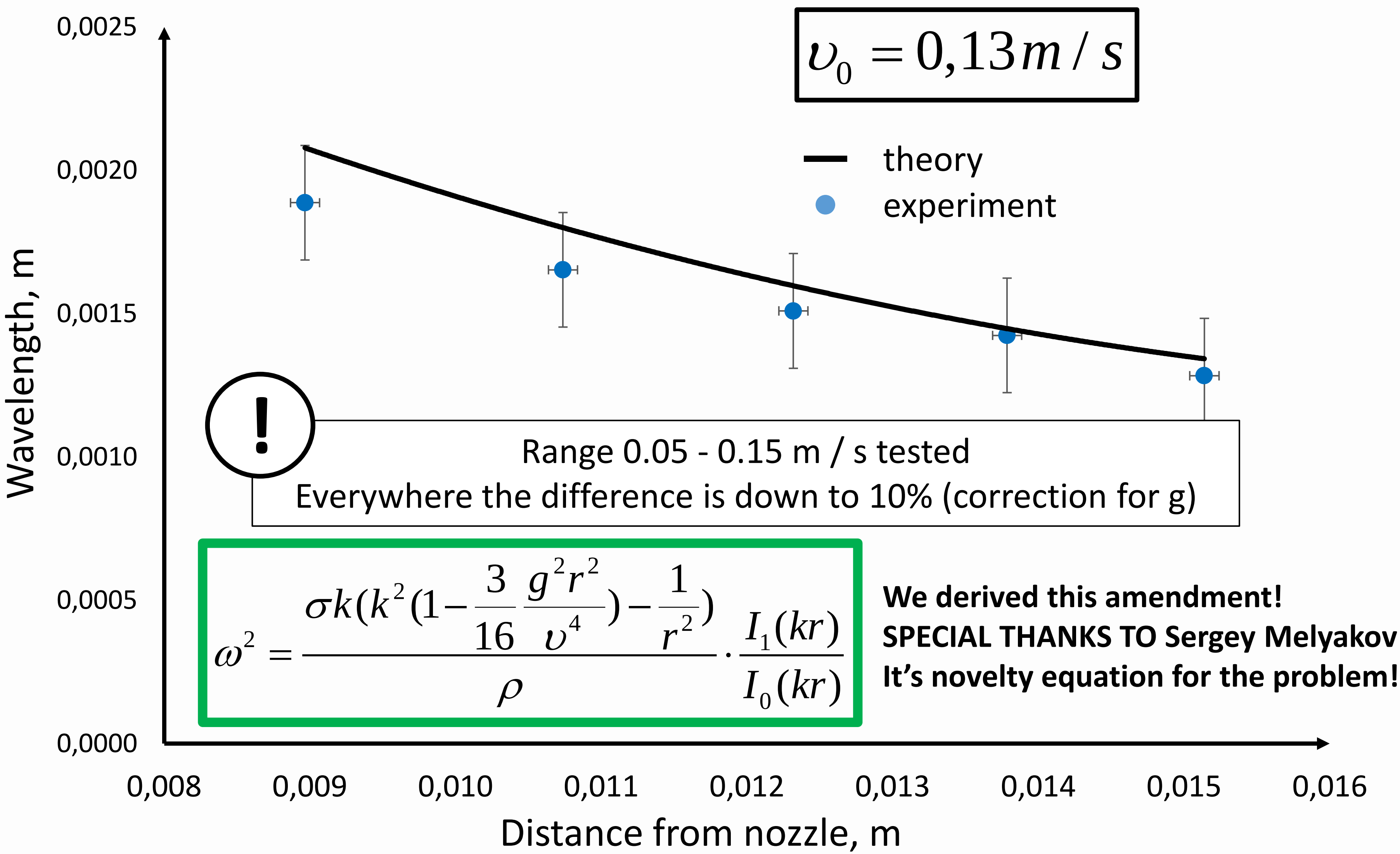
Experimental setup

Effect area





Verification of dispersion law



!

Range 0.05 - 0.15 m / s tested

Everywhere the difference is down to 10% (correction for g)

$$\omega^2 = \frac{\sigma k\left(k^2\left(1 - \frac{3}{16}\frac{g^2 r^2}{\upsilon^4}\right) - \frac{1}{r^2}\right)}{\rho} \cdot \frac{I_1(kr)}{I_0(kr)}$$

We derived this amendment!
SPECIAL THANKS TO Sergey Melyakov!
It's novelty equation for the problem!

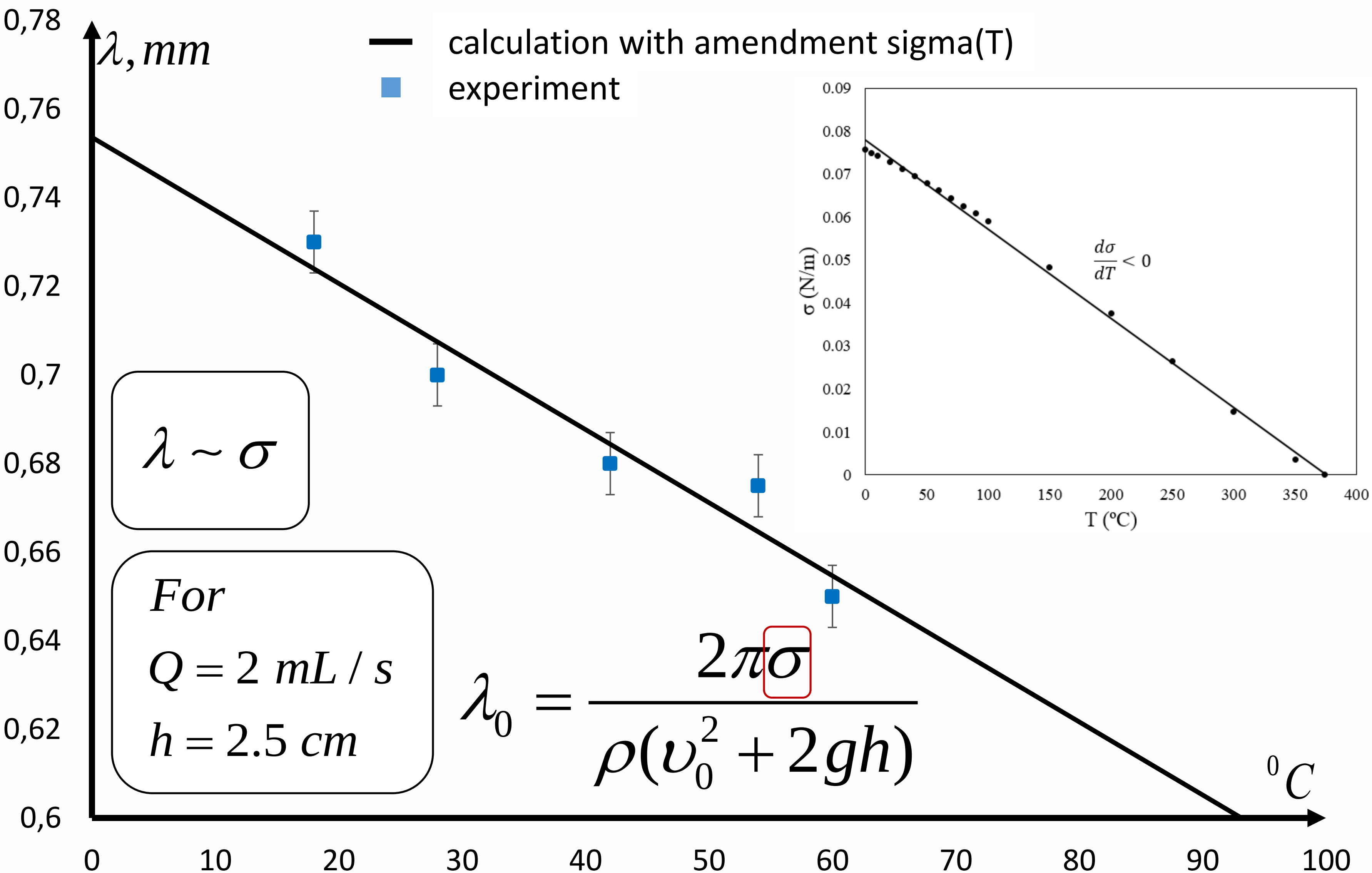
Surface tension effect

The surface tension increases the compression ratio of the jet

Warm water

Cold water

Wavelength at obstacle



Experimental part

Qualitative explanation

Quantitative description

Plan of investigation

1. Derivation and verification of dispersion law

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Amplitude of waves

$f(z)$

Wave packet propagation speed

$$v_g = \frac{\partial \omega}{\partial k}$$

Amplitude depending on the coordinate

$$f(z) = \int_{-\infty}^{0} b \cdot e^{-\alpha(k)(t+t_0)} \cdot \cos\big(k(x + ct_0) - \omega(t + t_0)\big)\, dt_0$$

The time in which the wave packet reaches the point

$$t(z) = \int_{z_0}^{z} \frac{dz}{v_g - c}$$

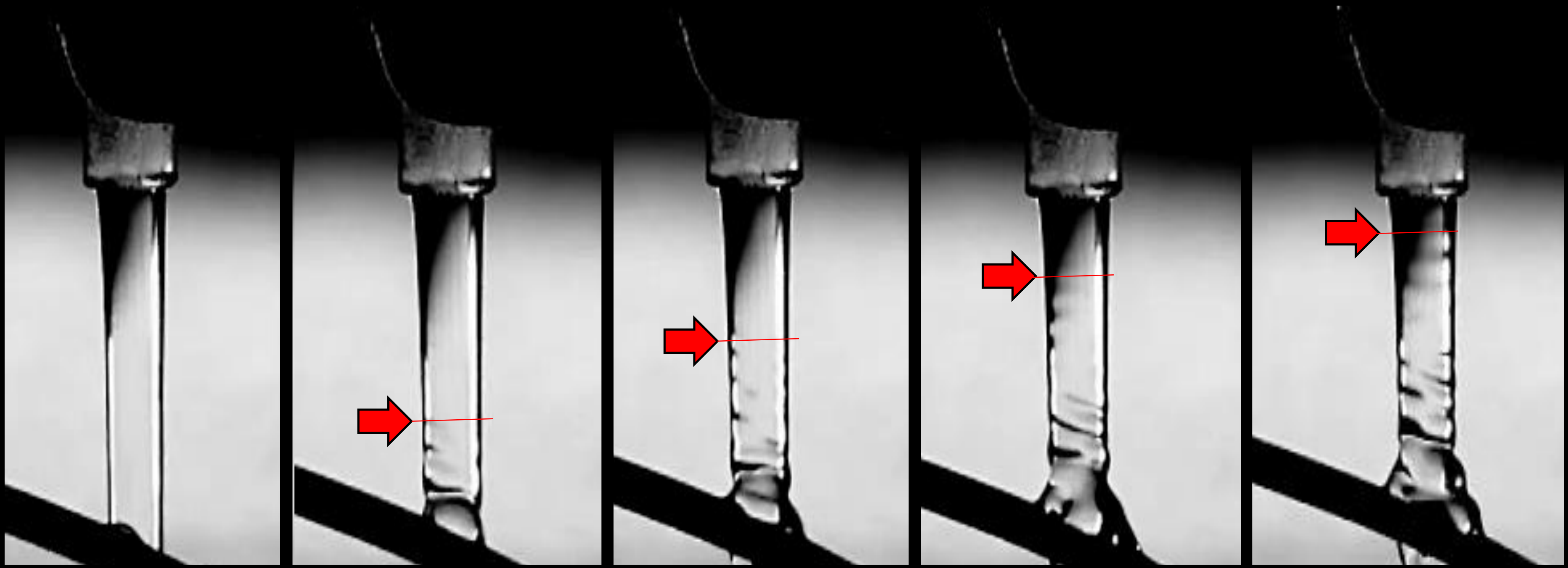
$$\boxed{f(z) = a_0 \cdot e^{-\alpha(k)\cdot t}}$$

a_0 - parameter determined from the width of the spectrum and the initial amplitude

Measurement method

Errors about 0.5 mm (shadow area)

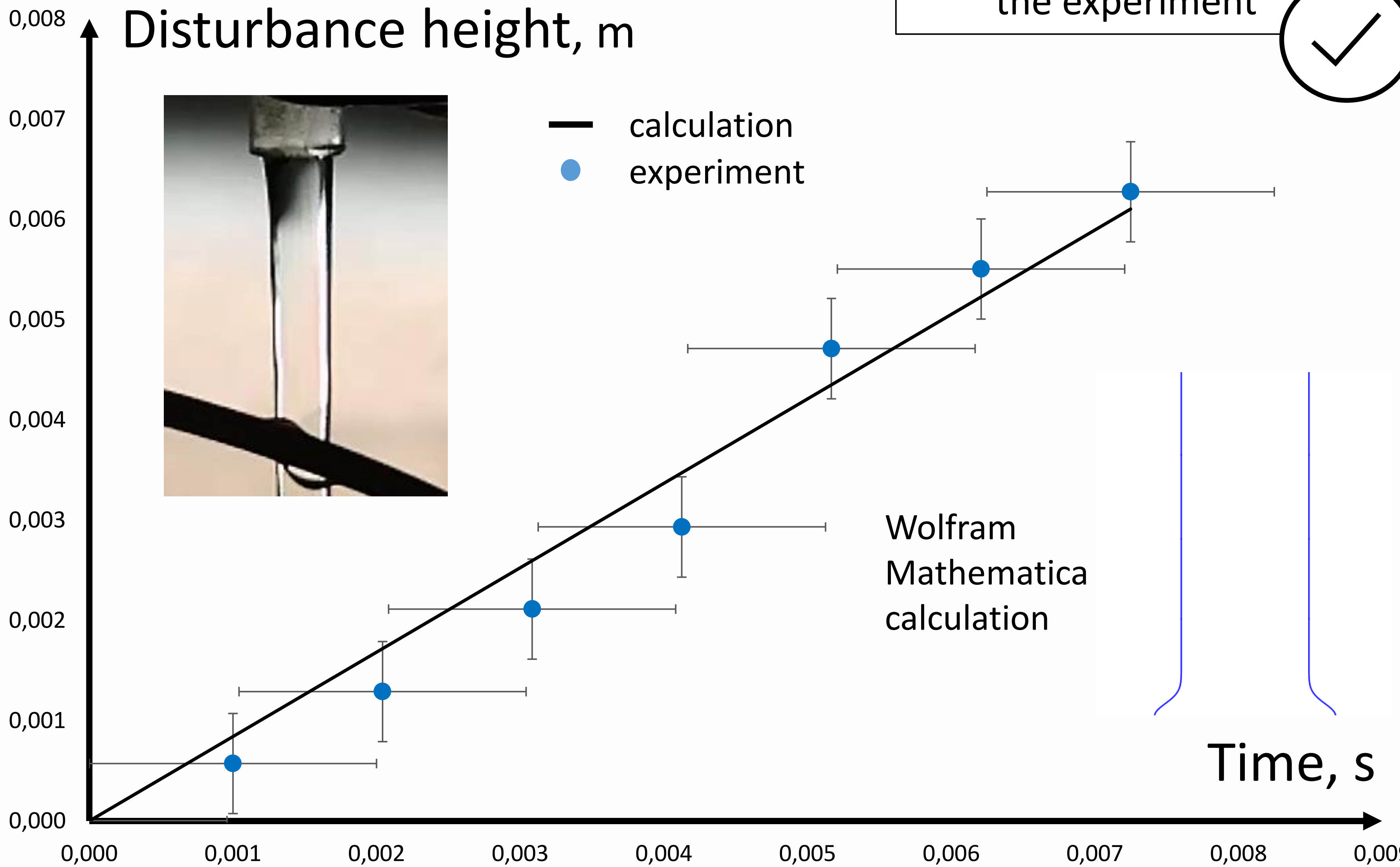
Ripple formation dynamics



Video, 1000 fps

Wave packet propagation

The calculation describes the experiment



Experimental part

Qualitative explanation

Quantitative description

Coefficient determination

The Navier-Stokes equation for two-dimensional flow at low Reynolds numbers

$$\begin{cases} \frac{\partial v_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \Delta v_x \\ \frac{\partial v_y}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \Delta v_y \\ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \end{cases}$$

Dispersion ratio taking into account viscosity:

$$\omega = -2i\nu k^2 + \sqrt{\omega(k)}$$

The imaginary part of the frequency - viscosity

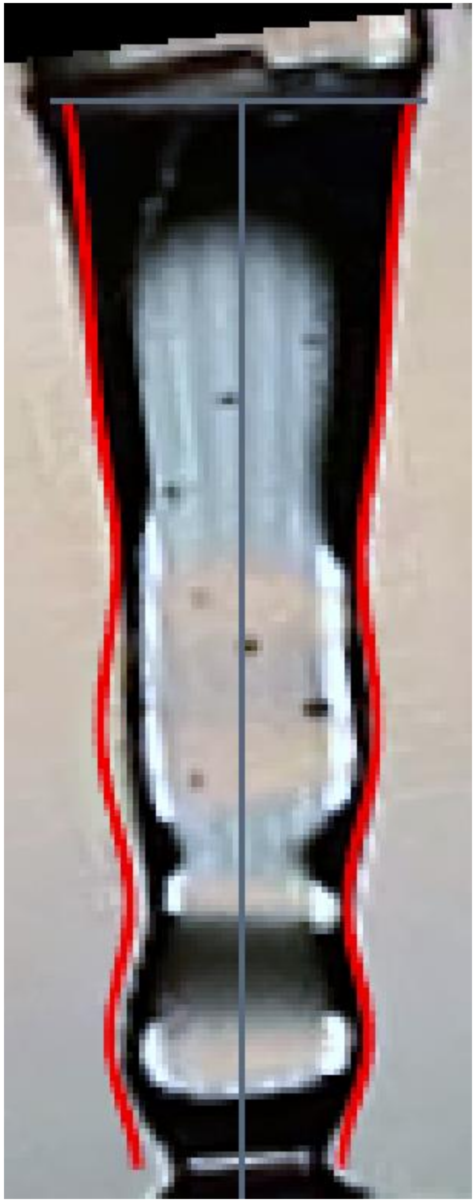
ν – kinematic viscosity
σ – tension
ρ – fluid density

Attenuation coefficient

$$\alpha(k) = 2\nu k^2$$

“Гидродинамика”, [Fluid dynamics], G. Lamb

Plan of investigation



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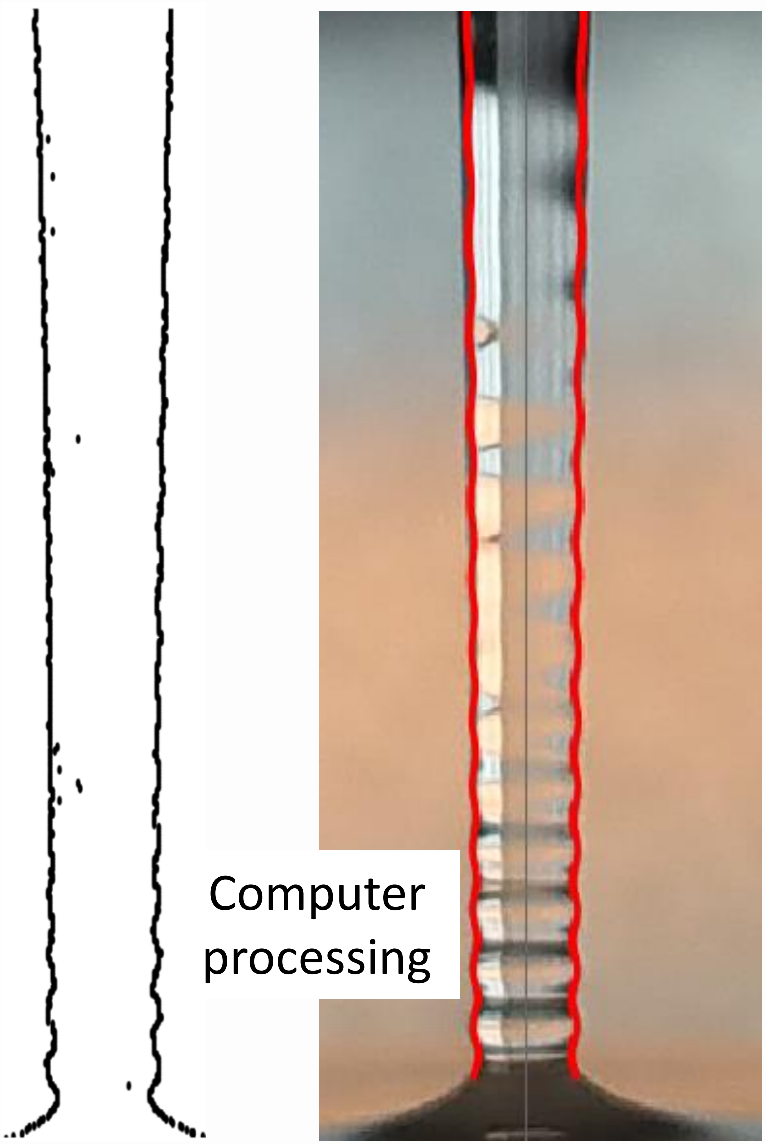
Measuring method



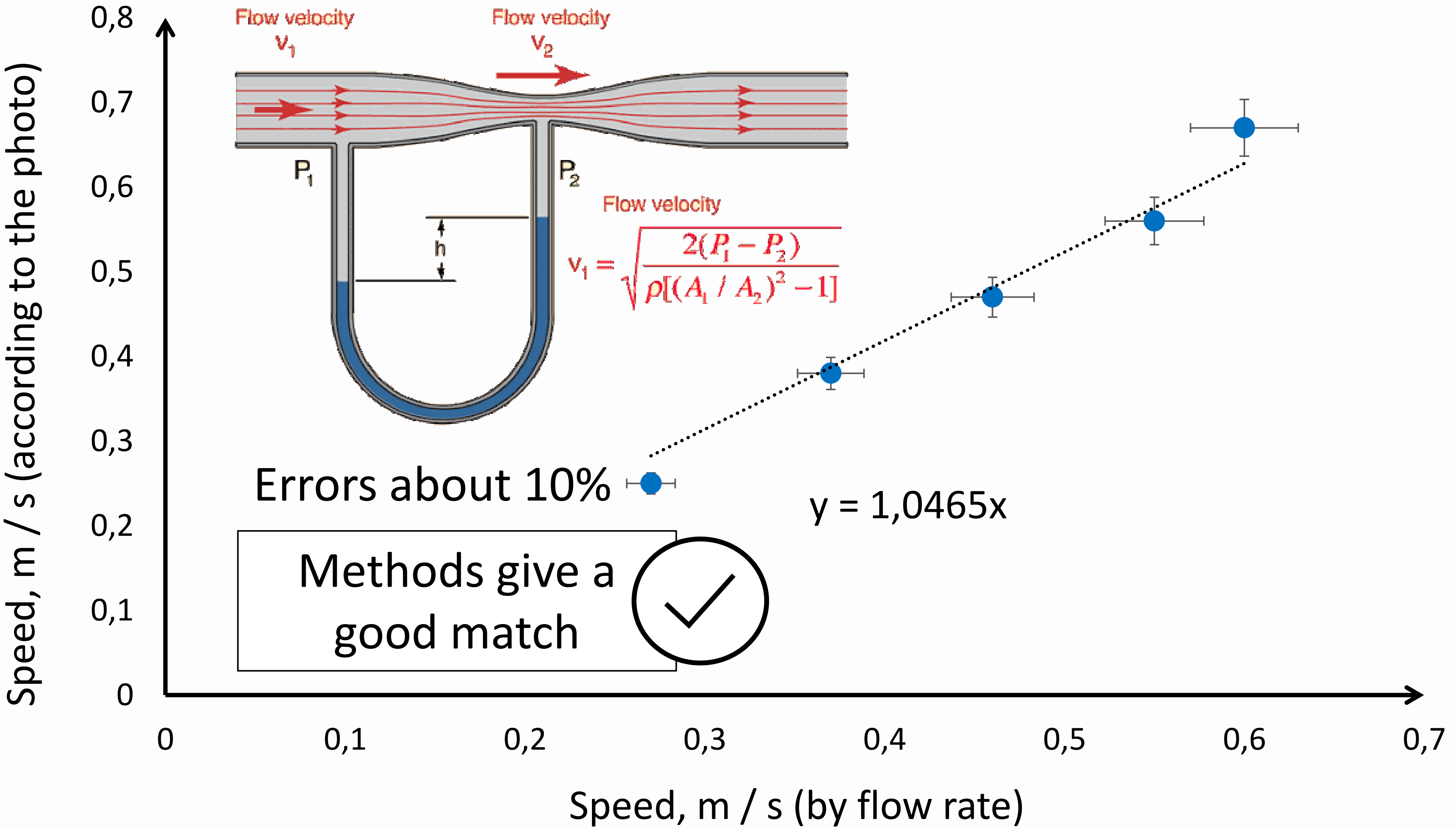
1. Track the jet boundary
2. Fit the boundary by the assumed dependence
3. Determine the attenuation coefficient
4. Spread out into a spectrum
5. Determine the flow rate and surface tension coefficient

$$\nu = 1 \cdot 10^{-6} \frac{m^2}{s}$$

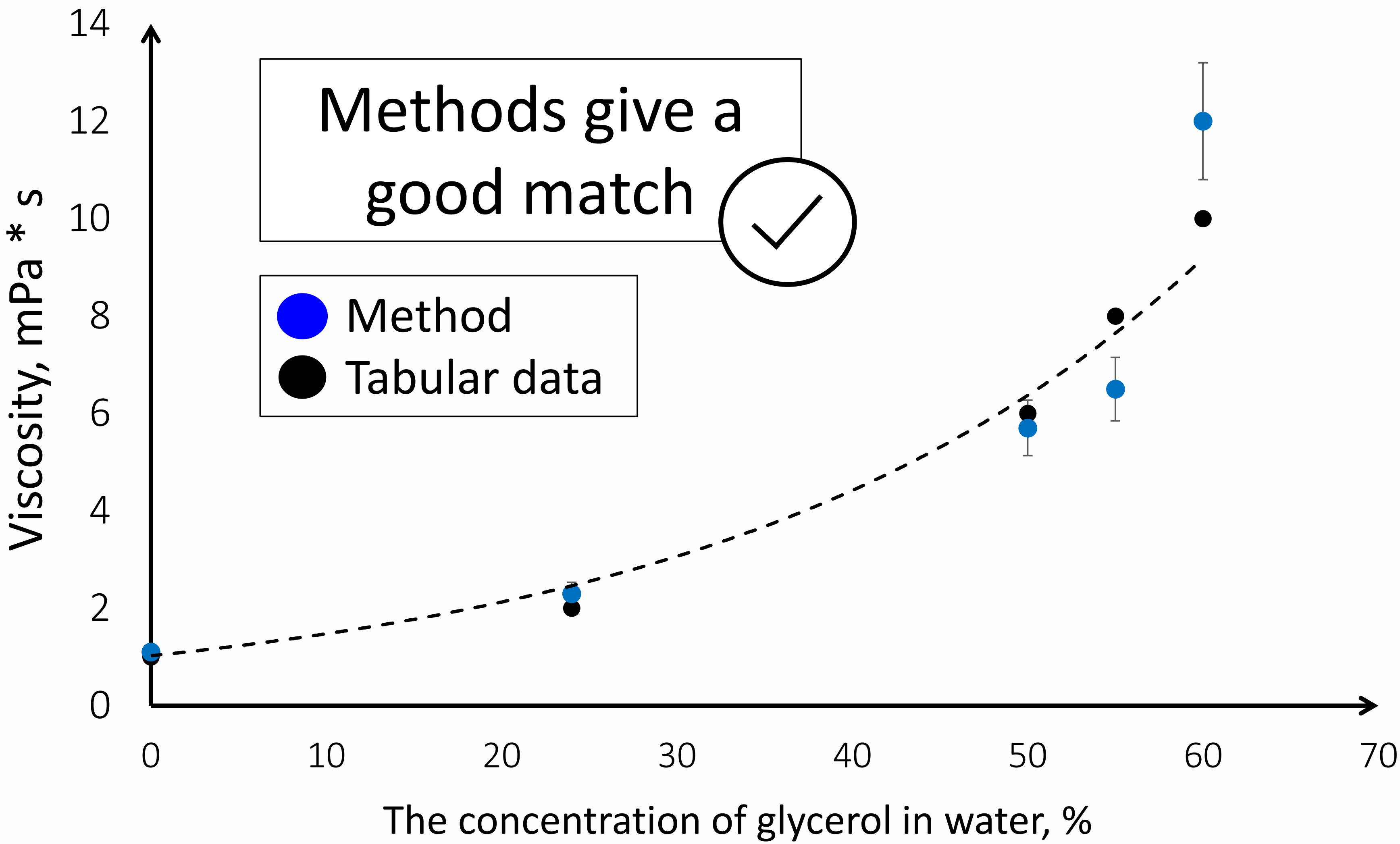
$$\sigma = 0{,}053 \ \frac{N}{m}$$

$$v_0 = 0{,}13 \frac{m}{s}$$

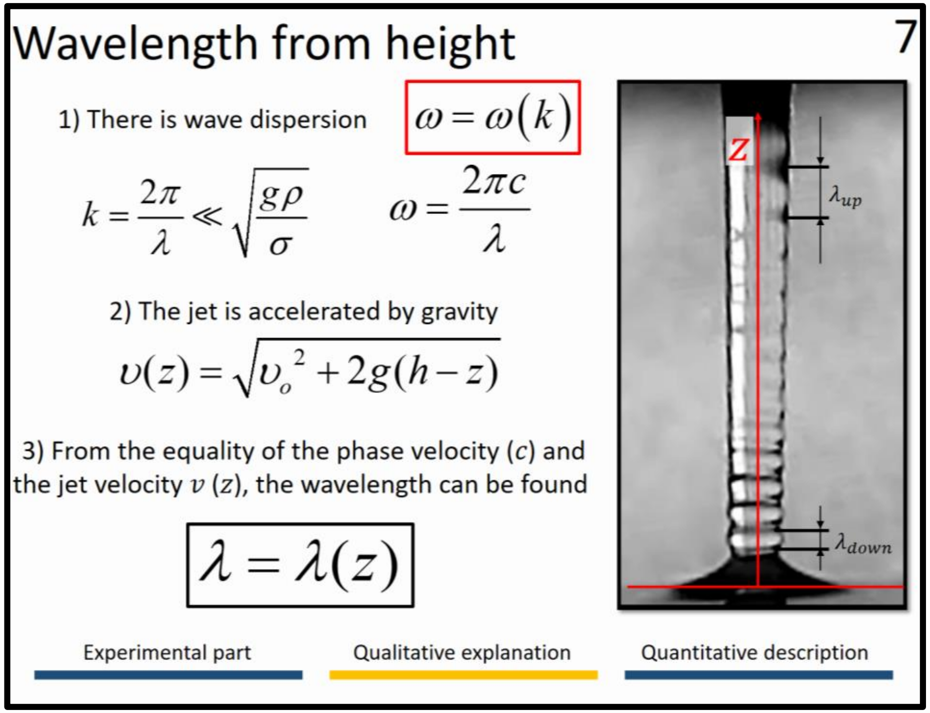
Flow rate measuring

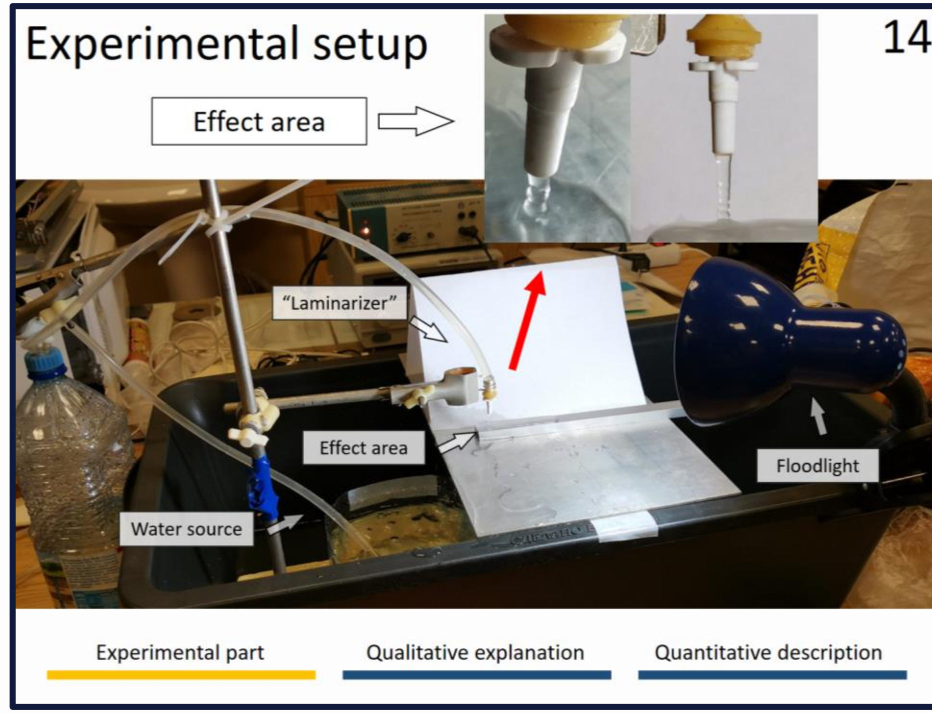


Kinematic viscosity measuring



Conclusions

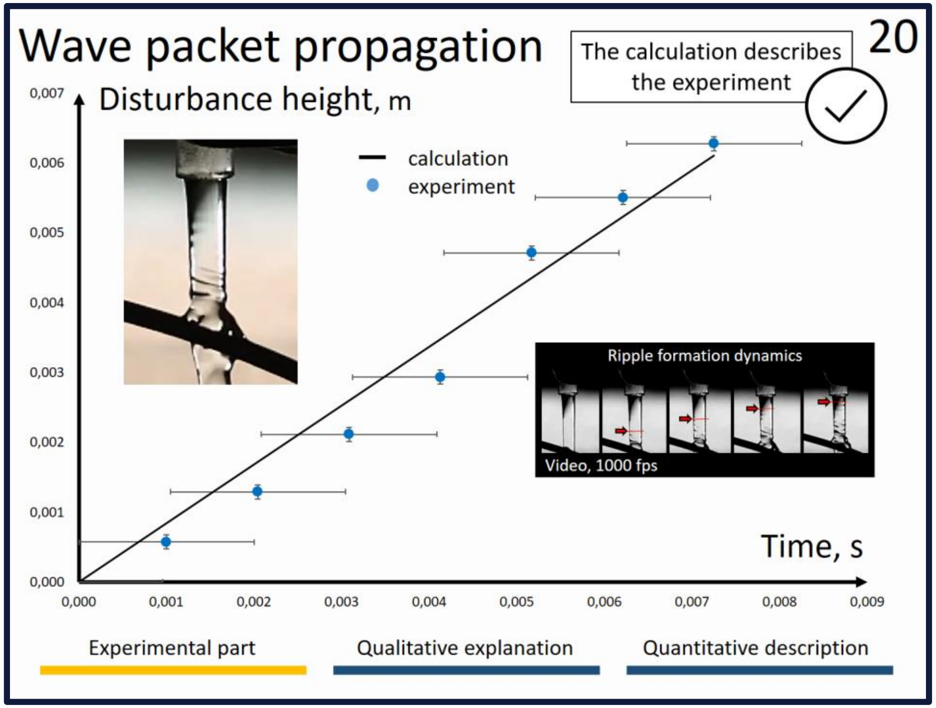


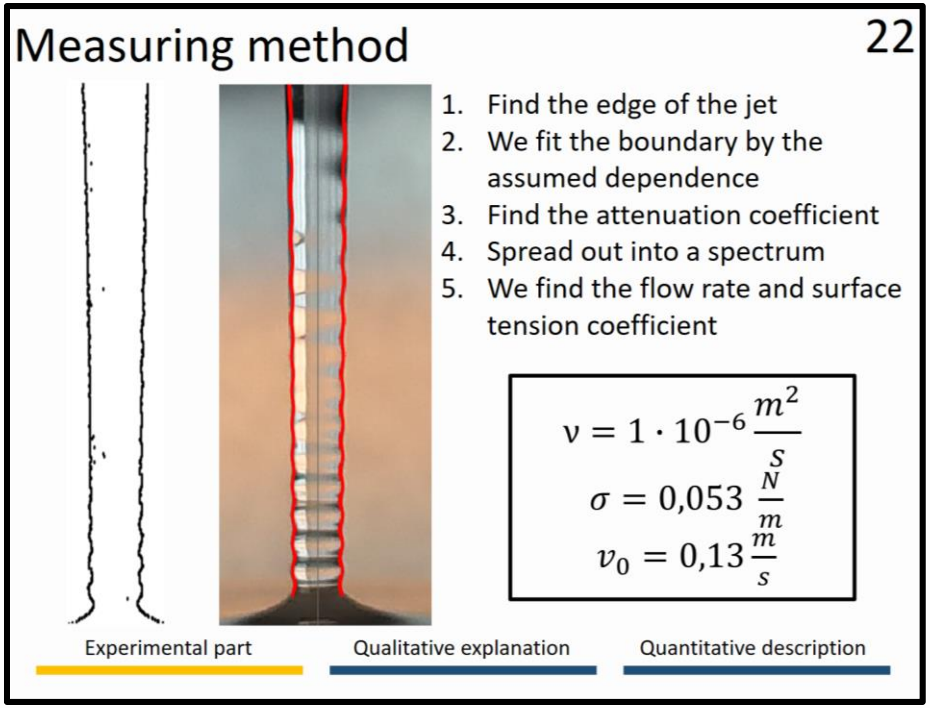


The nature of the formation of ripples - **Capillary waves**.

Stationarity is ensured by **wave dispersion**.
At each point of the jet, the velocity of the jet and the phase velocity of the wave are equal. **The ratio is** adjusted for surface curvature and **verified**.

Conclusions





The **ripples formation dynamics** is considered.

The wave **attenuation** from time is **described**.
Calculation model is confirmed by experiment.

The **method for measuring** viscosity, surface tension and flow rate **was determined** and verified.

Bibliography

- “Гидродинамика”, [Fluid dynamics], L.D. Landau and E.M. Lifshitz
- “Гидродинамика”, [Fluid dynamics], G. Lamb
- Fluid pipes, M. J. Hancock and J. W. M. Bush, J. Fluid Mech. (2002)
- Wave patterns on a water column, D. Sklavenites, American Journal of Physics 65, 225 (1997)
- Stationary waves on cylindrical fluid jets, K. M. Awati and T. Howes, American Journal of Physics 64, 808 (1996)
- Adachi, K. 1987 Laminar jets of a plane liquid sheet falling vertically in the atmosphere, J. Non-Newtonian Fluid Mech. 24.

Special thanks to Voronezh team “Rubicon”

Further research:

1. Fluid “pipes”
2. Initial amplitude – full description

Problem No.8

Rippled water columns

Reporter: Artem Sukhov

Team of Russia

It was also investigated:

1. Decay of jet into droplets
   (height, ripple)
2. Jet narrowing
   (radius from height, attenuation effect)
3. Fluid and flow parameters
   (dependencies for the wavelength at the foot)
4. Fluid “pipes”
   (areas where ripples do not form)
5. Waves on the plane

Final thought

Thank you! Questions?