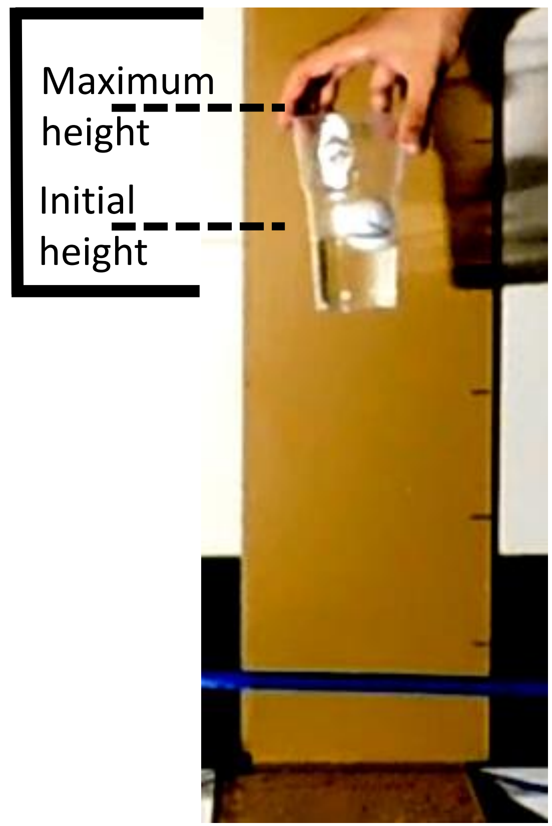
Problem No.1
Cumulative cannon

Reporter: Artem Sukhov

Team of Russia

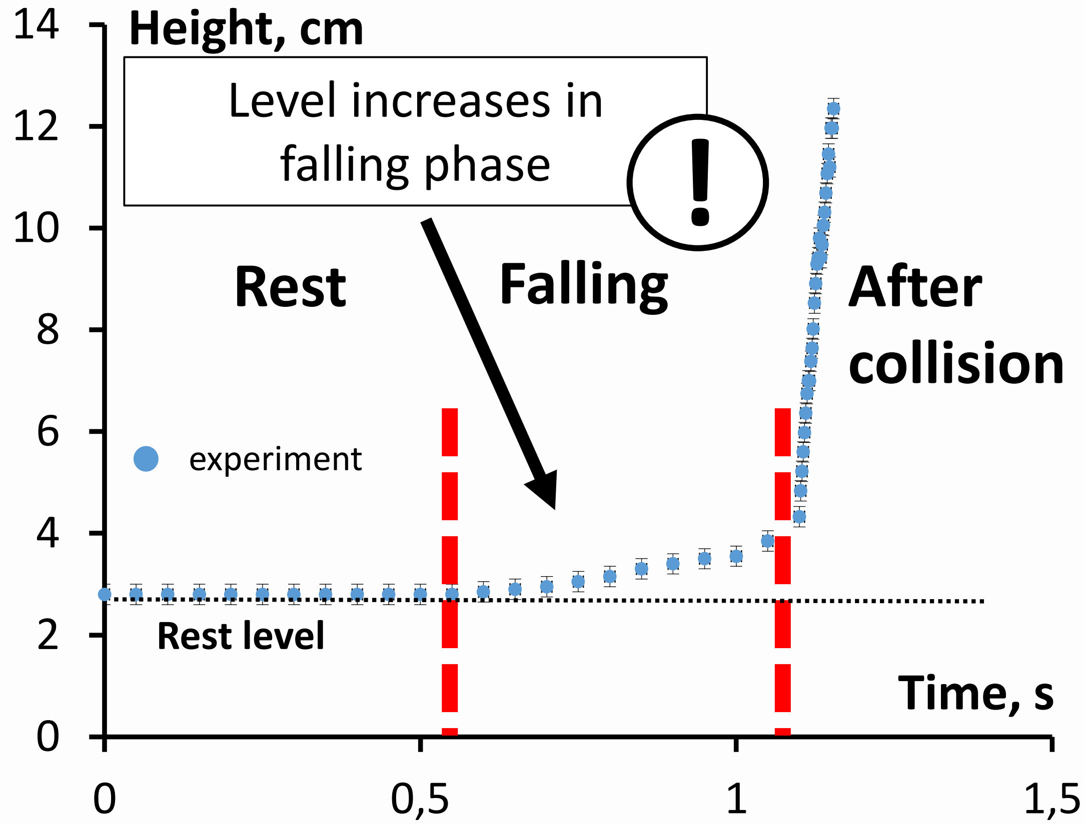
International Physicists' Tournament 2020

How high may a **ping-pong ball** jump using the setup on the video?
What is the **maximal fraction of the total kinetic** energy that can be transferred to the ball?



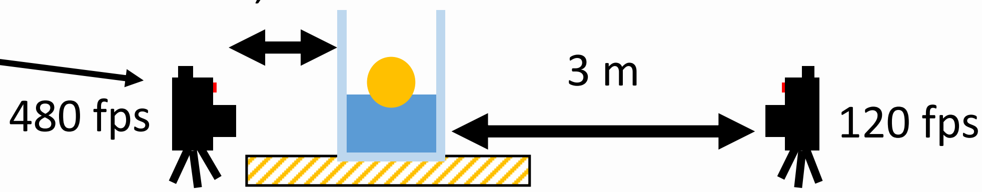
Level of water in cup

Video from nearby camera



$$V_{water} = 100\,ml$$

$$V_{cup} = 500\,ml$$

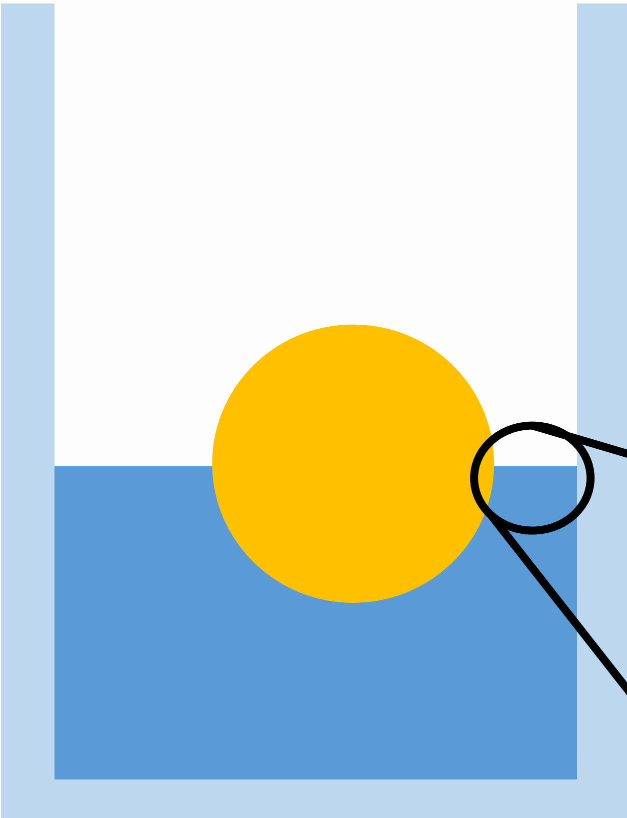


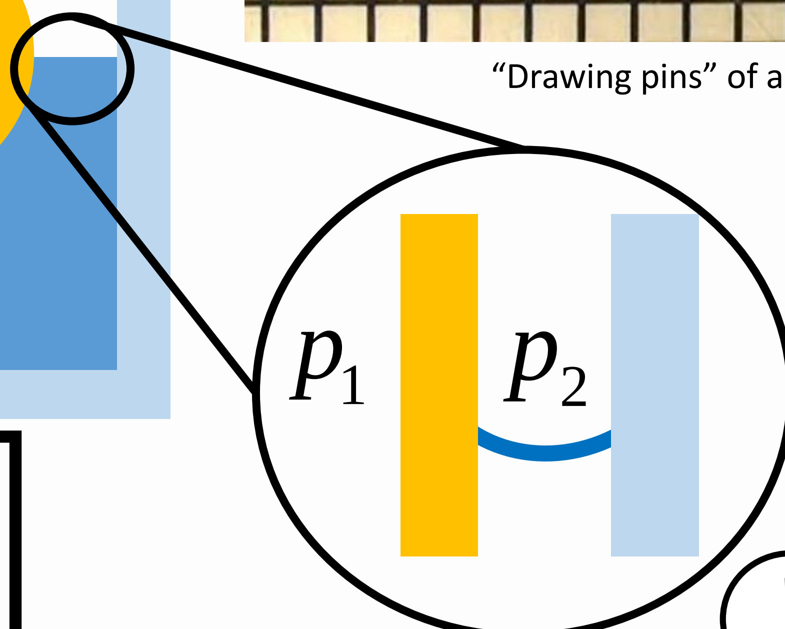
Experimental part

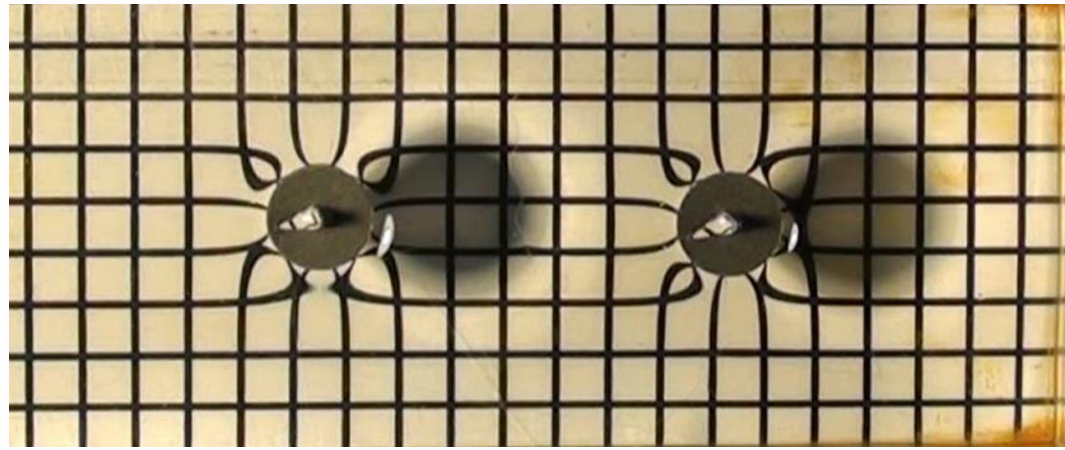
Qualitative explanation

Calculation part

Rest phase







“Drawing pins” of a team of Russia (2012), Alex Krotov

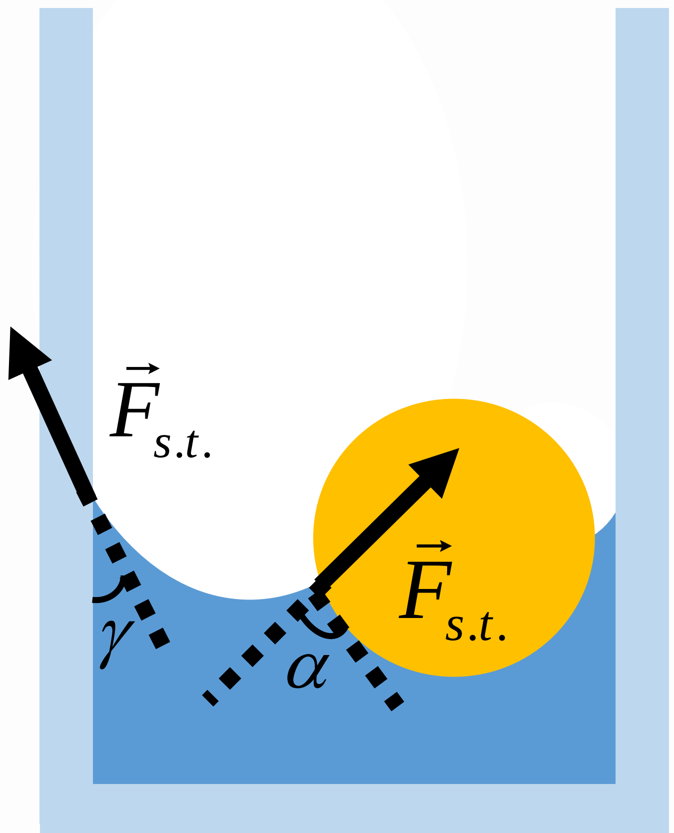
$$\Delta p \sim \frac{\sigma}{R}$$

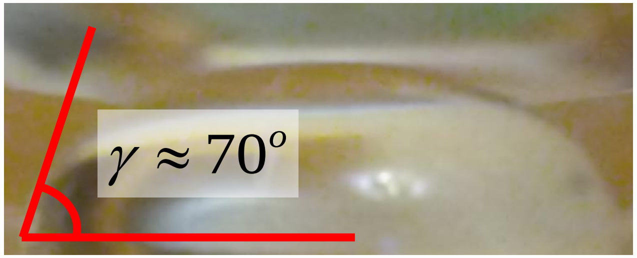
$$p_1 > p_2$$

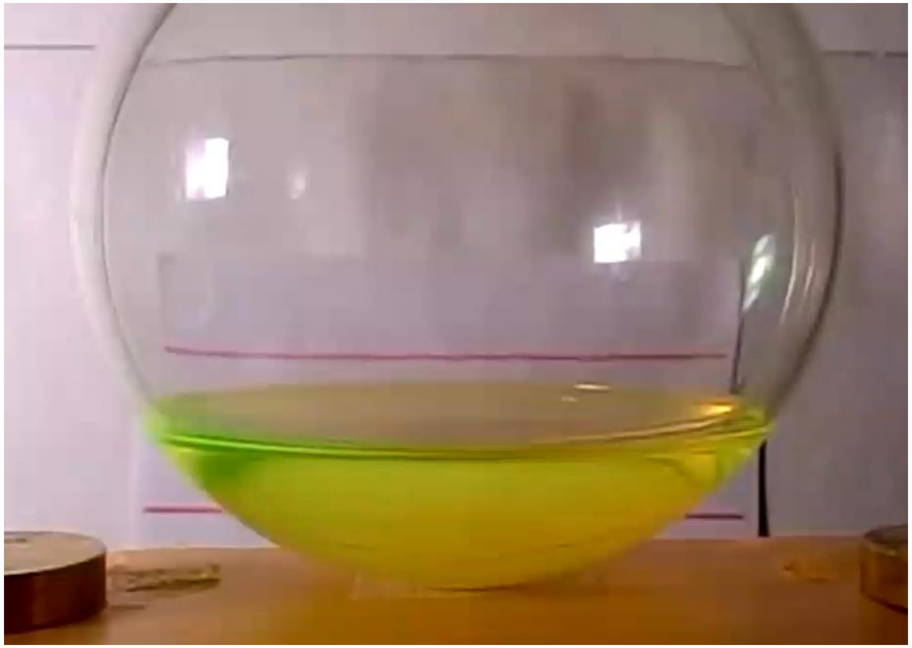
! Ball moves to wall

Experimental part | Qualitative explanation | Calculation part

Falling phase







Experiment by GetAClass

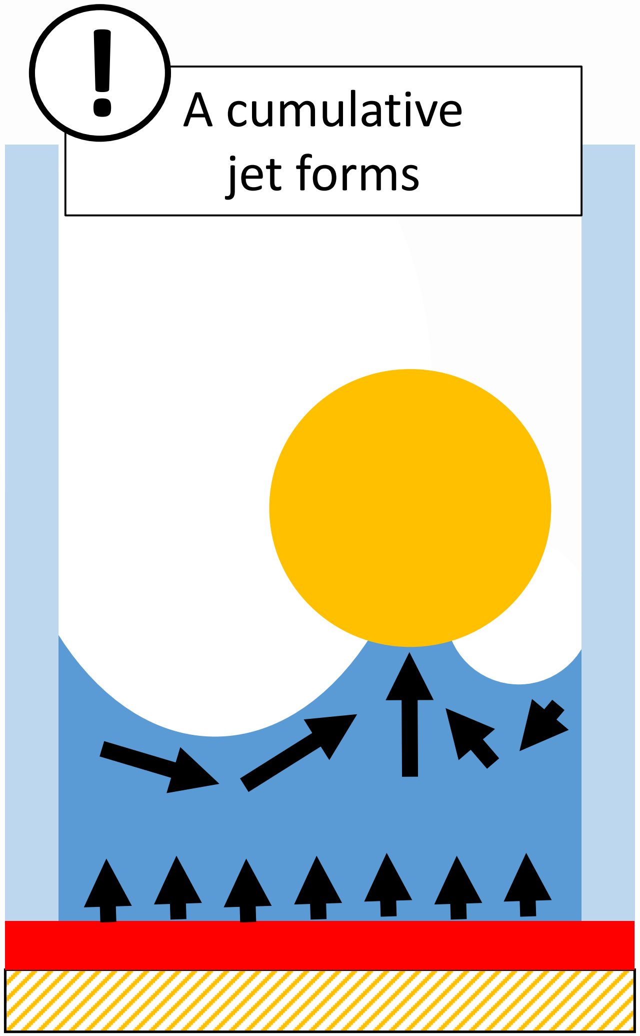
$$\Sigma\vec{F} = \vec{F}_{g.} + \vec{F}_{in.} + \vec{F}_{s.t.} + \vec{F}_{res.}$$

$$\vec{F}_{g.} + \vec{F}_{in.} \approx 0$$

→ Surface tends to hemisphere (Full wetting case)

Experimental part | Qualitative explanation | Calculation part

Collision phase



There is no effect in the cup with the walls lubricated with paraffin.

Effect still occurs in the cup with rigid walls.

Plan of investigation

1 **Cumulative jet calculation**

Parameters of a fluid and a cup

2 **Energy transfer to a ball**

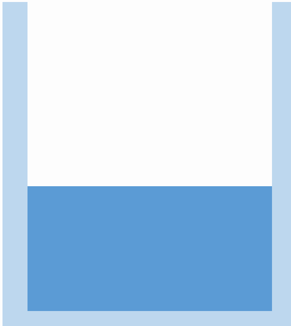
Parameters of the ball

3 **Height maximization**

And ***maximal fraction of the energy***

4 Some questions are covered in the hidden part of the investigation.

Collapse of crater



$$Ma_y = Mg - F_d.$$

Mass of braking water

Mass of cup fluid system

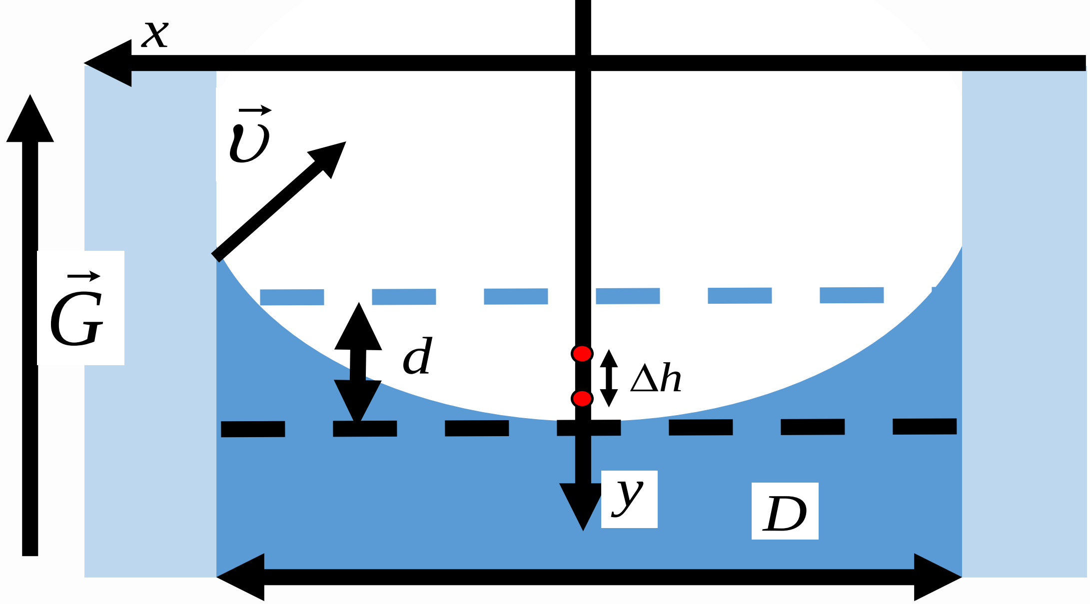
$$\upsilon_0 \approx \sqrt{2gh_0}$$

$$G \gg g$$

$$A = mG\Delta h$$

$$\frac{A}{V} = \rho G\Delta h = \frac{\rho \upsilon^2}{2}$$

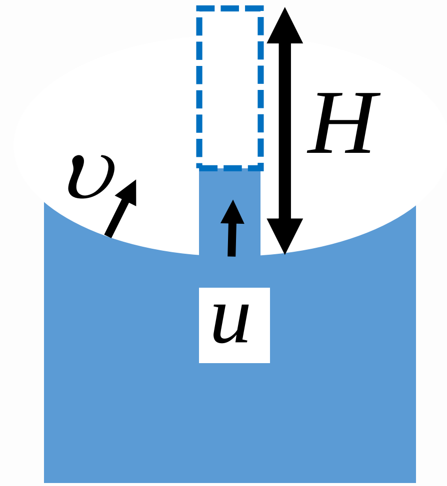
Fluid density



Calculation of cumulative jet height

$$\frac{A}{A_{d.}} \sim 10^3$$

! Air drag force can be neglected



$$H = \frac{u^2}{2g} = \frac{k\upsilon^2}{2g} = k\Delta h \frac{G}{g}$$

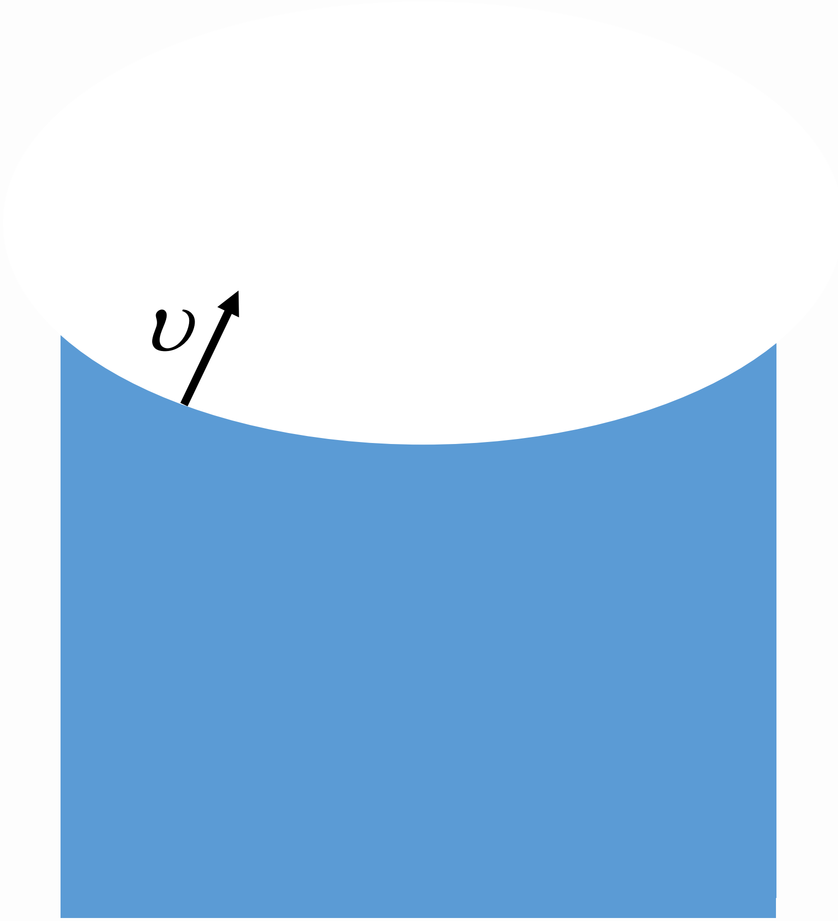
Cumulation coefficient

$$\frac{G}{g} = \frac{h_0}{\Delta h}$$

$$H = kh_0$$

! Maximum height is proportional to initial

Energy cumulation coefficient



Let's consider that water front already has velocity

$$\frac{u^2}{\upsilon^2} = ctg^2 \frac{\beta}{2}$$

Cone case

For part of sphere, there is an equivalent cone

$$V_{air} = V_{cone}$$

Hemisphere case

$$\frac{u^2}{\upsilon^2} = 18$$

Experimentally confirmed

$$\frac{u^2}{\upsilon^2} = k$$

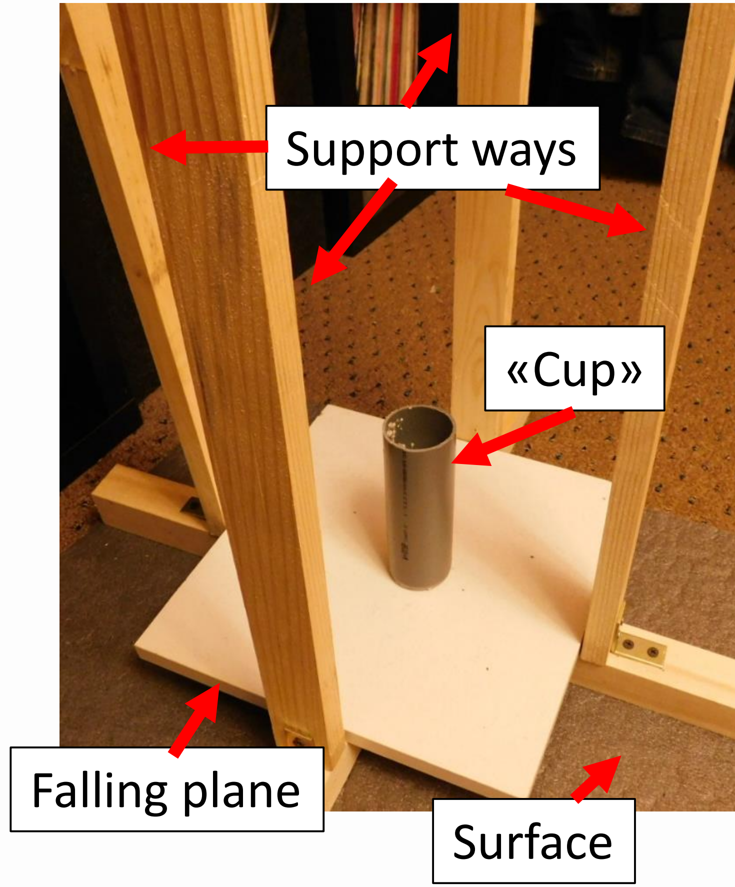
Experimental part

Qualitative explanation

Calculation part

- “Кумулятивный эффект в простых опытах” [Cumulative effect in simple experiments] , V. V. Mayer

Experimental setup



$$a_{plane} \approx 9{,}8\,\frac{m}{s^2}$$

We carry out 5 measurements per 1 point of dependence

We put the camera at a distance of 2 meters to avoid the parallax effect

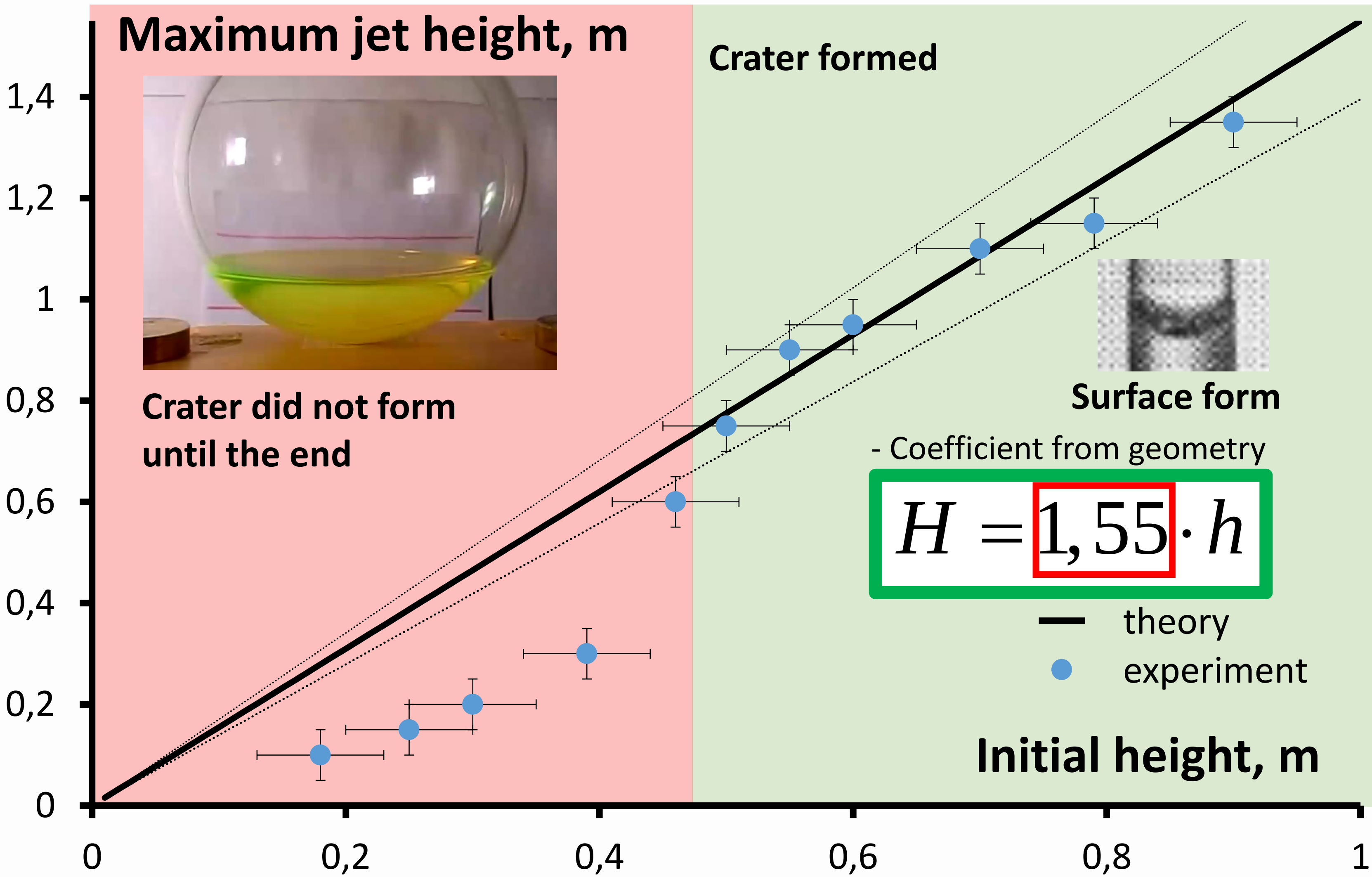
To save the angle of contact with the plane, we will improve the experimental setup.

The experiment was repeatable. (Standard deviation about 5%)

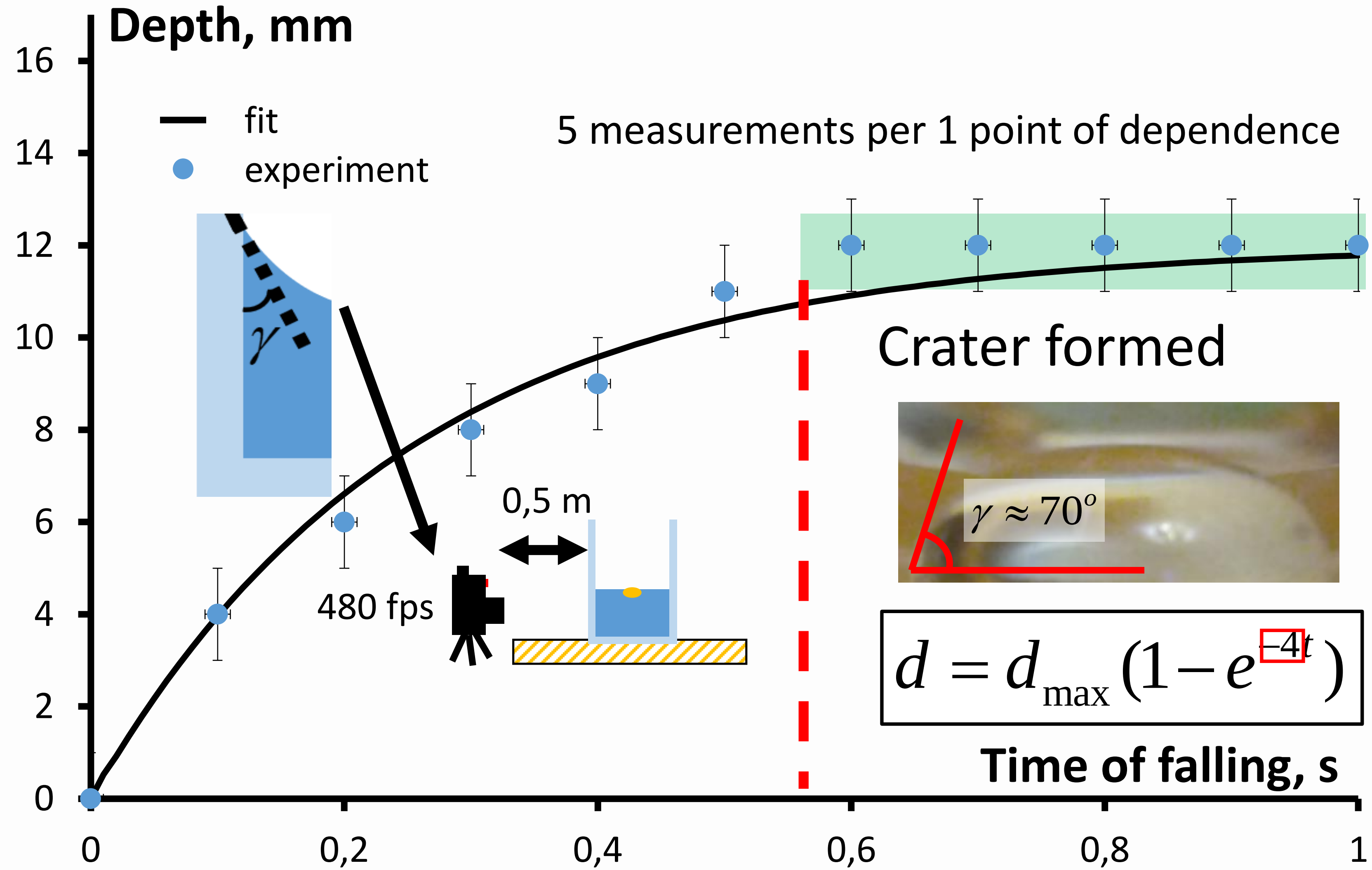
Maximum jet height vs. initial height

Analytical solution

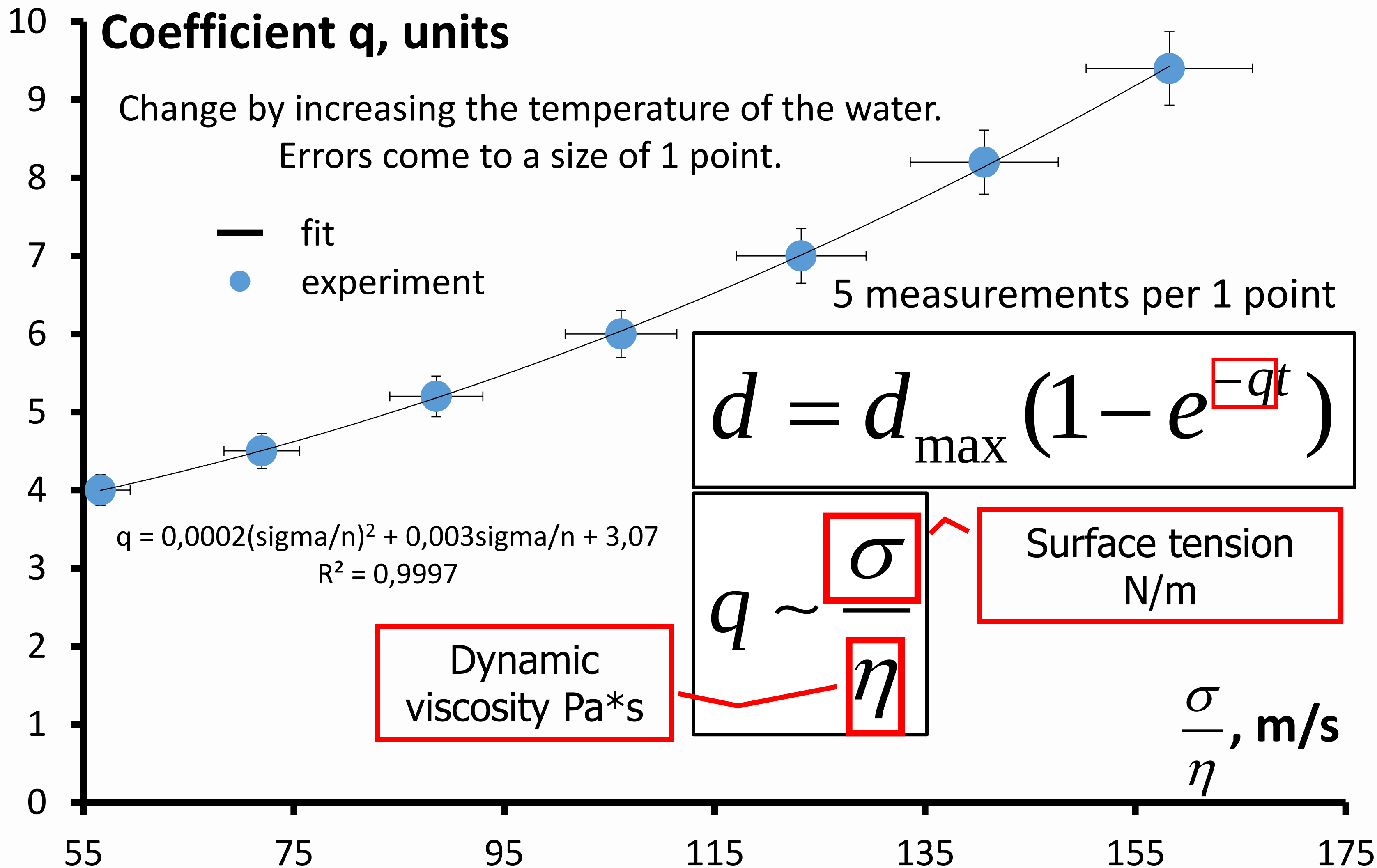
Water 20 ^oC, cup diameter 40 mm, 100 ml water



Depth of crater vs. time of falling

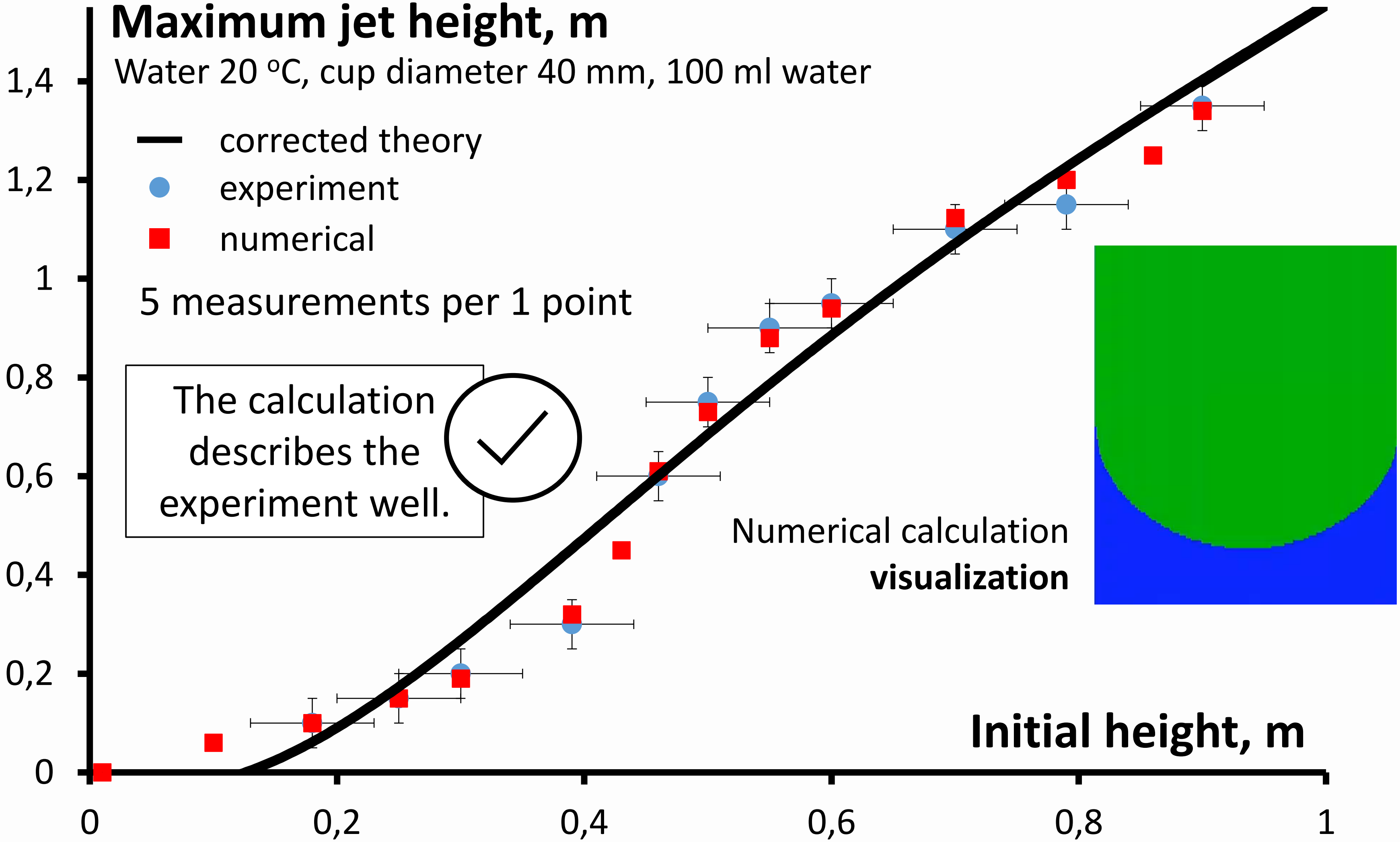


Coefficient of lift velocity



Maximum jet height vs. initial height

Added numerical calculation with dynamic of crater



Experimental part | Qualitative explanation | Calculation part

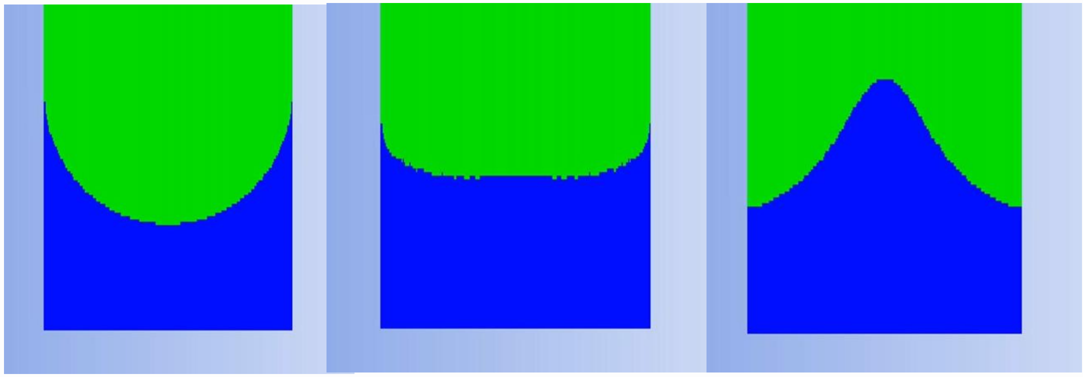
Numerical calculation **ANSYS AUTODYN**

Comparison

1. We can calculate lifting in zero gravity
2. We can calculate collision with large negative accelerations

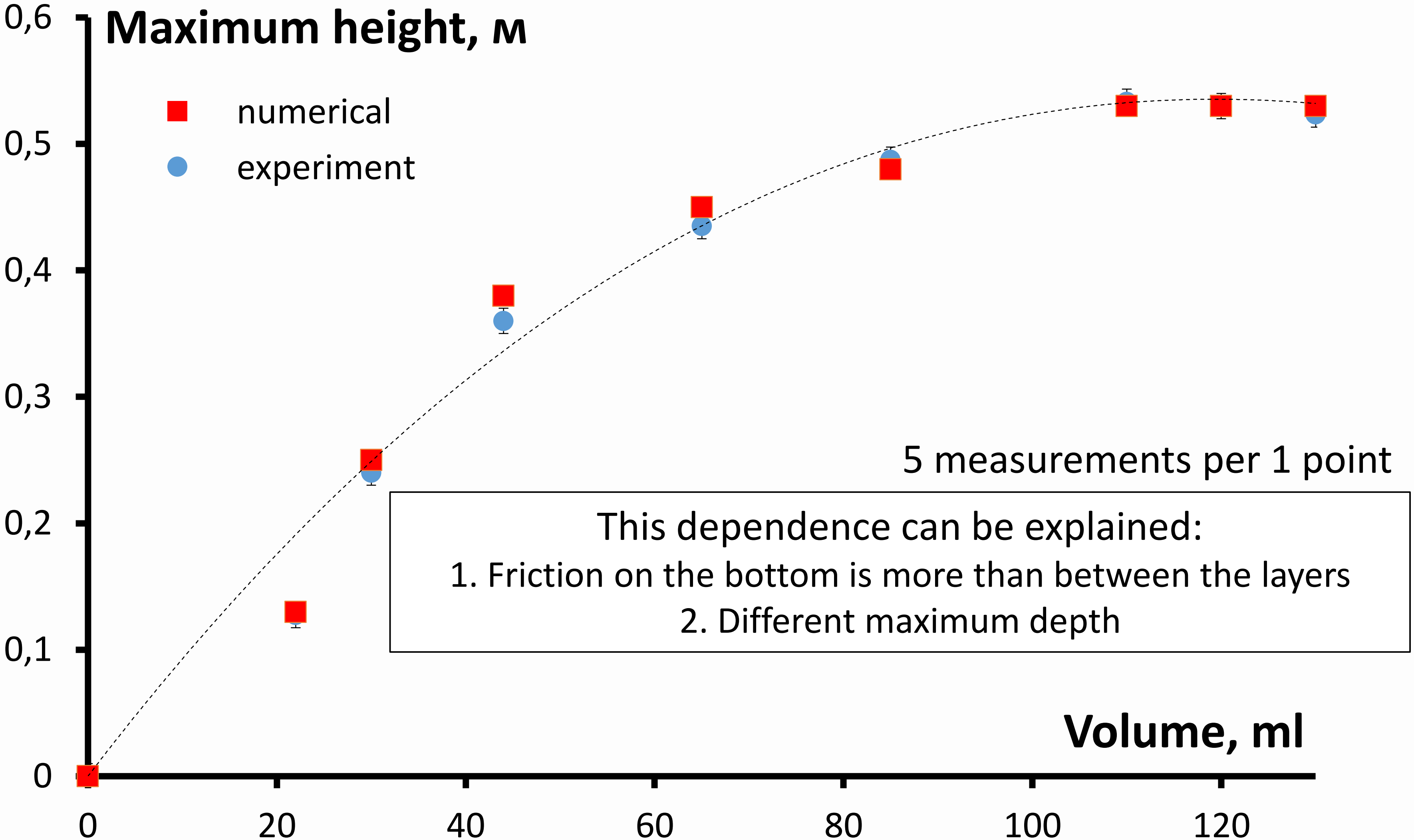
Numerically calculate the fluid motion by solving **the Navier-Stokes** differential equation **by FEM**

The calculation procedure is in the hidden part



Maximum jet height vs. fluid volume

Ping-pong ball, initial height – 35 cm



This dependence can be explained:

1. Friction on the bottom is more than between the layers
2. Different maximum depth

Plan of investigation

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Parameters of the ball

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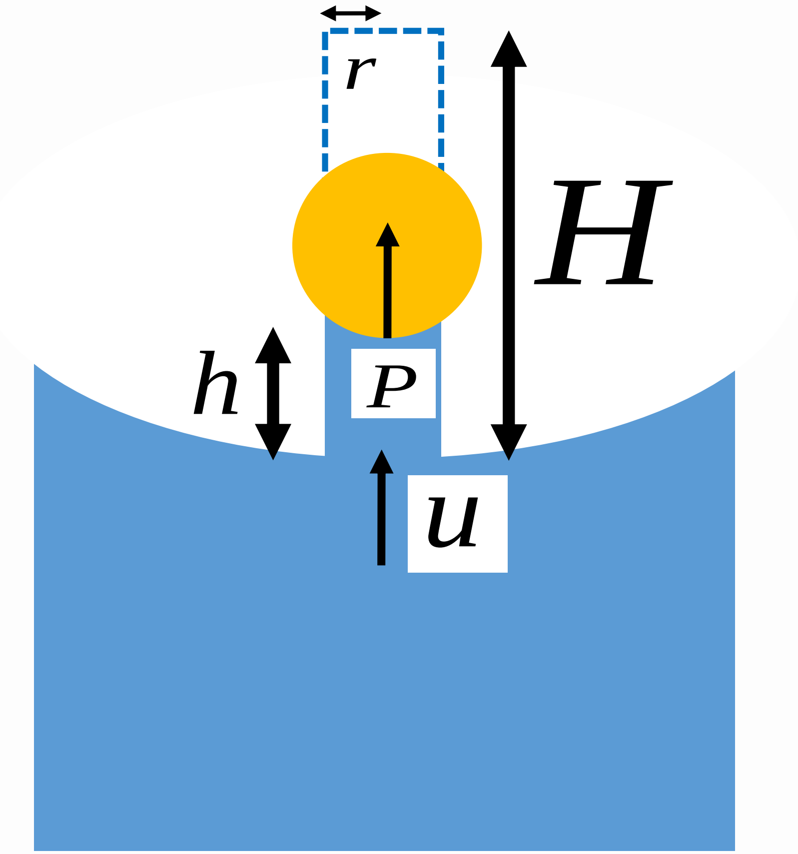
And ***maximal fraction of the energy***

Energy transfer to ball from jet



Law of energy conservation:

$$\frac{\rho u^2}{2} + \rho g h = \rho g H$$

Law of change of momentum:

$$\Delta p = F\Delta t = m\Delta u$$

Law of mass conservation:

$$\pi r_0^{\ 2} u_0 = \pi r^2 u$$

$$-F = P = \frac{m\Delta u}{\Delta t} = \rho \frac{u_0 \pi r_0^{\ 2}}{u}(u - \upsilon_{ball})^2$$

Ball maximum height calculation

$$P = \rho \pi r_0^2 \sqrt{\frac{H}{H-h}} \left(\sqrt{2g(H-h)} - \upsilon_{ball}\right)^2$$

$$a_{ball-i} = \frac{P_i}{m} - g$$

$$\upsilon_{ball-i+1} = \upsilon_{ball-i} + a_i \Delta t$$

$$\upsilon_0 = 0\, m/s \quad h_0 = 0\ \text{m}$$

$$h_{i+1} = h_i + \upsilon_{ball-i}\Delta t + \frac{a_i \Delta t^2}{2}$$

Jet radius

Height of jet

Fluid density

Maximum jet height without ball

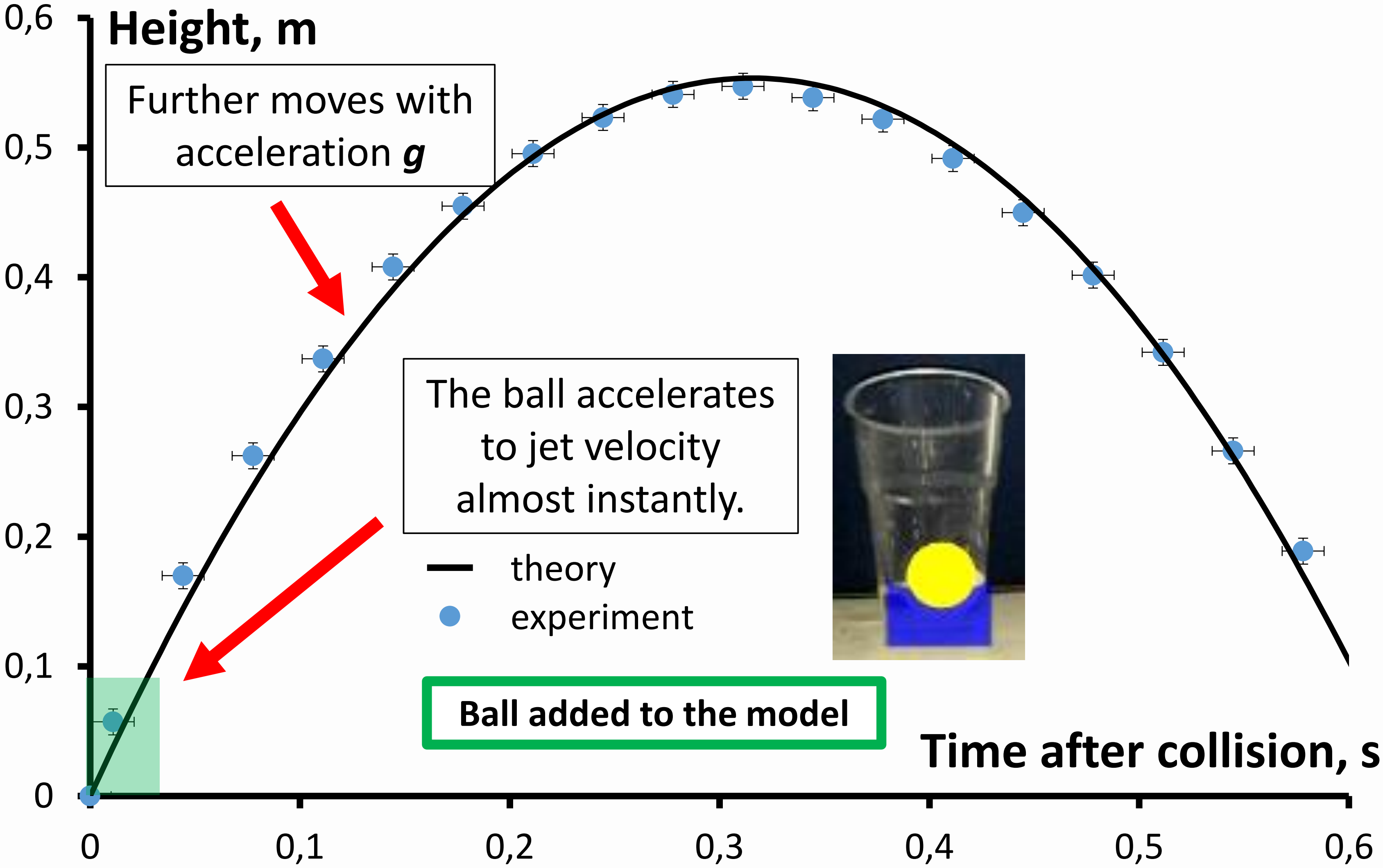
Velocity of the ball

The resulting differential equations are solved by the **Euler method with correction** and a dependent step.

Experimental part | Qualitative explanation | Calculation part

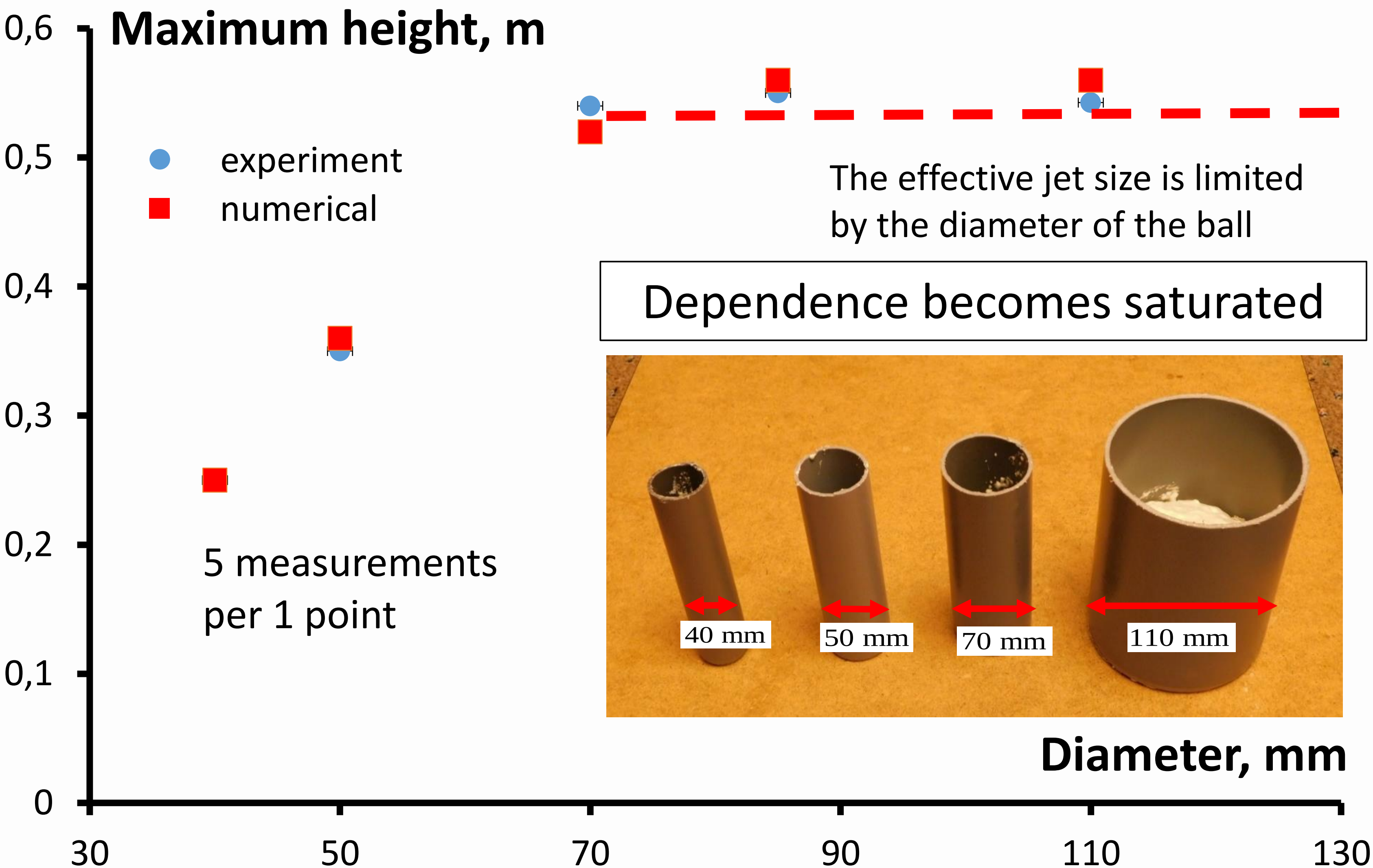
Ball height vs. time after collision

Numerical calculation, plastic cup, ping-pong ball, initial height - 35 cm



Maximum height of ball vs. diameter of cup

Ping-pong ball, initial height – 35 cm, 150 ml water



Plan of investigation

1 Cumulative jet calculation

Parameters of a fluid and a cup

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3 Height maximization

*And **maximal fraction of the energy***

Boundaries of model applicability

The limit of jet speeds is subsonic speed.

$$\upsilon \le 331\frac{m}{s}$$

The height limit is the breaking point of the cup.

$$h_0 \le 2{,}1\ \text{m}$$

The acceleration time limit by the size of the cup.

$$\tau_{acceleration} \ll \frac{d_{character}}{\upsilon_{sound}}$$

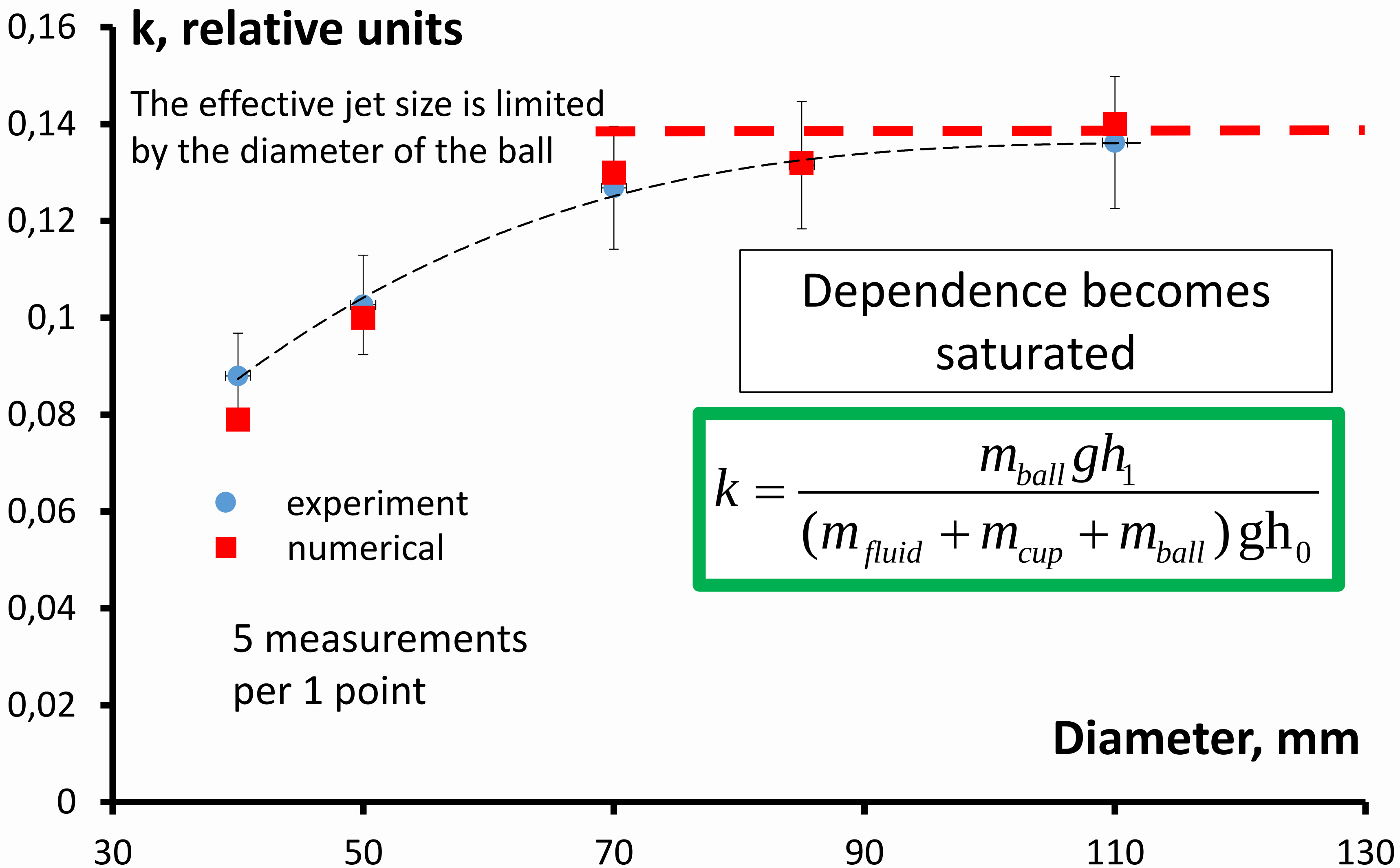
The limit on the maximum height is air resistance.

$$F_{gravity} = F_{drag}$$

$$H_{\exp} = 3{,}1\ \text{m} \approx \text{H}_{theor}$$

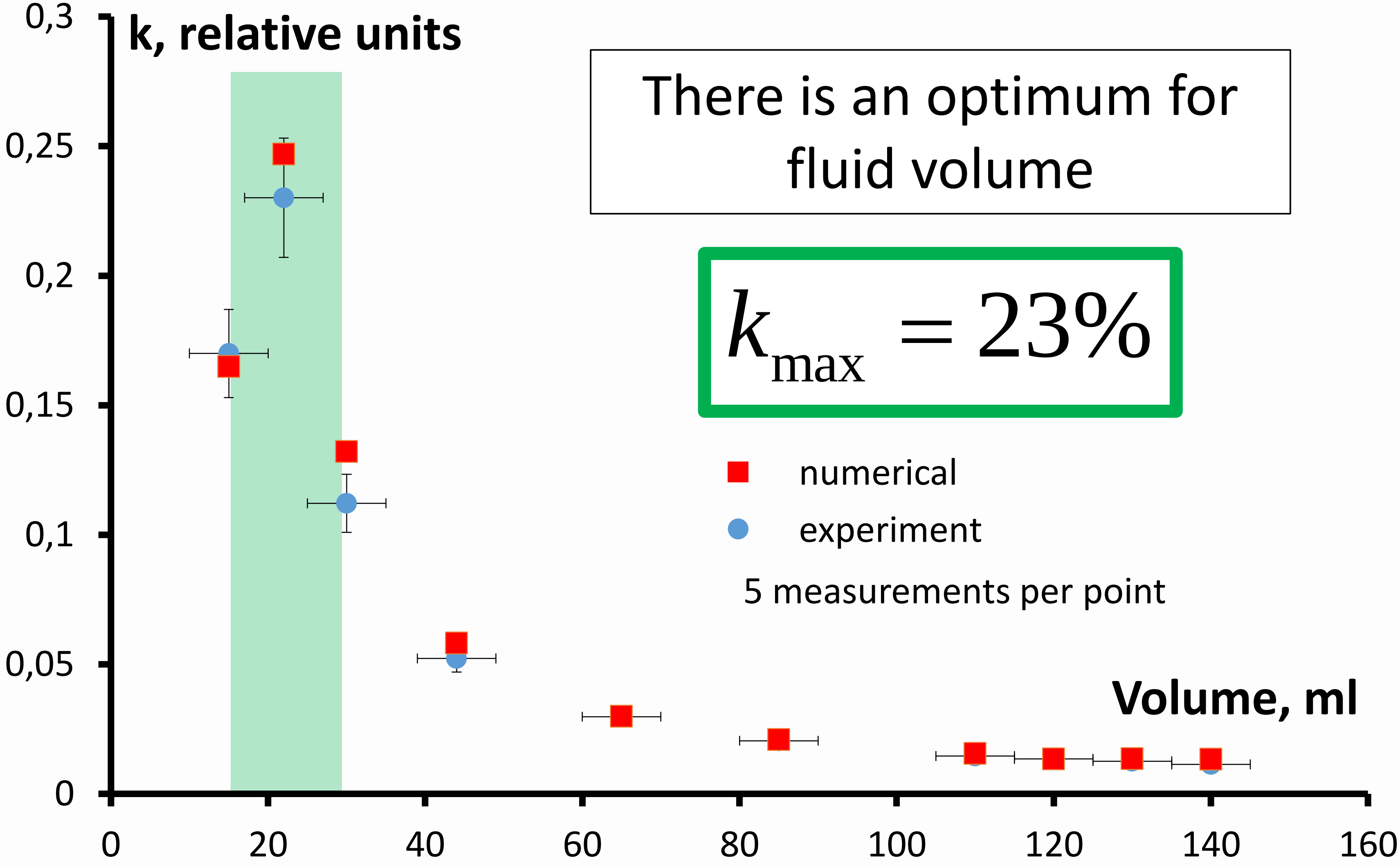
Fraction of energy vs. diameter of cup

Ping pong ball, initial height - 35 cm, 150 ml of water



Fraction of energy vs. fluid volume

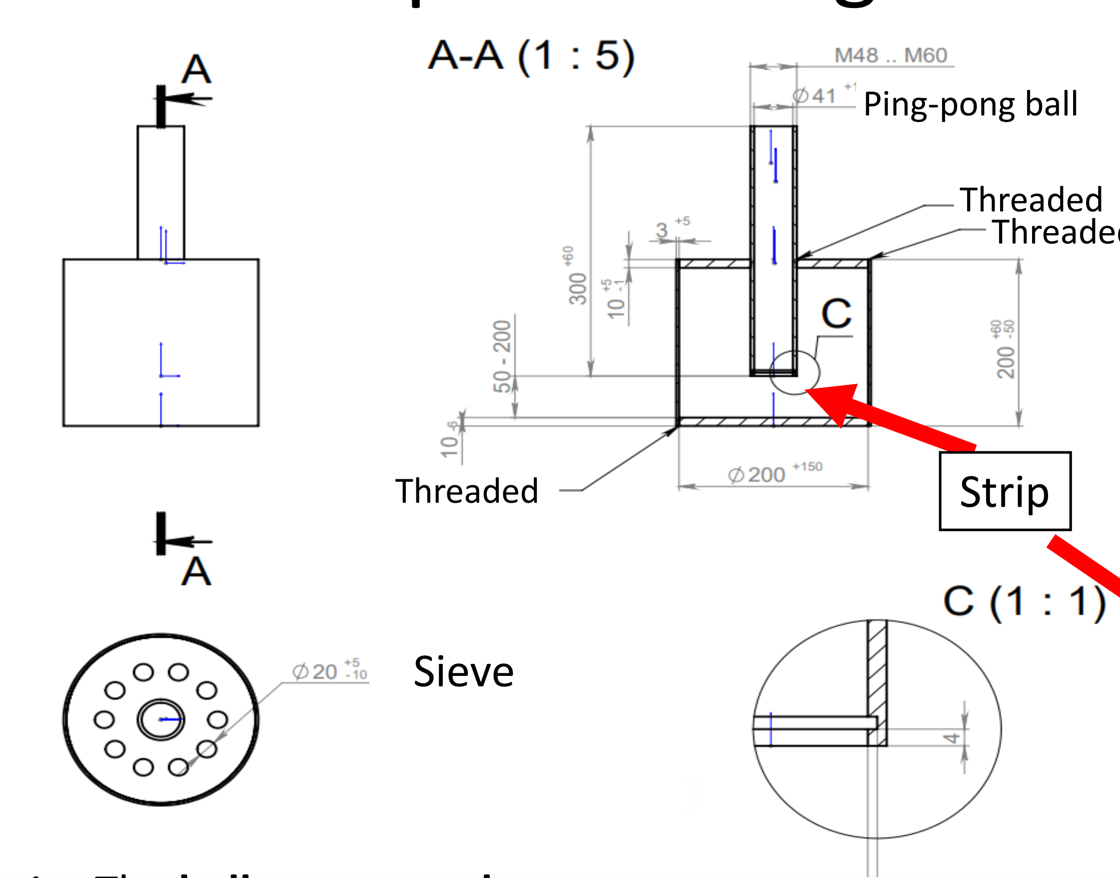
Ping pong ball, initial height - 55 cm



There is an optimum for fluid volume

$$k_{\max} = 23\%$$

Ideas for optimal design



1. The **ball moves** up **the center**
2. The whole **jet concentrates** to hit the ball and works like a piston
3. Sealing pipe strip slightly holds the ball (**empiric**)
4. Materials **are well wetted by water.**

Device can accelerate heavy objects well ☺

CUMULATIVE CANNON!

Toroidal bottom

Parabolic bottom

Flat bottom

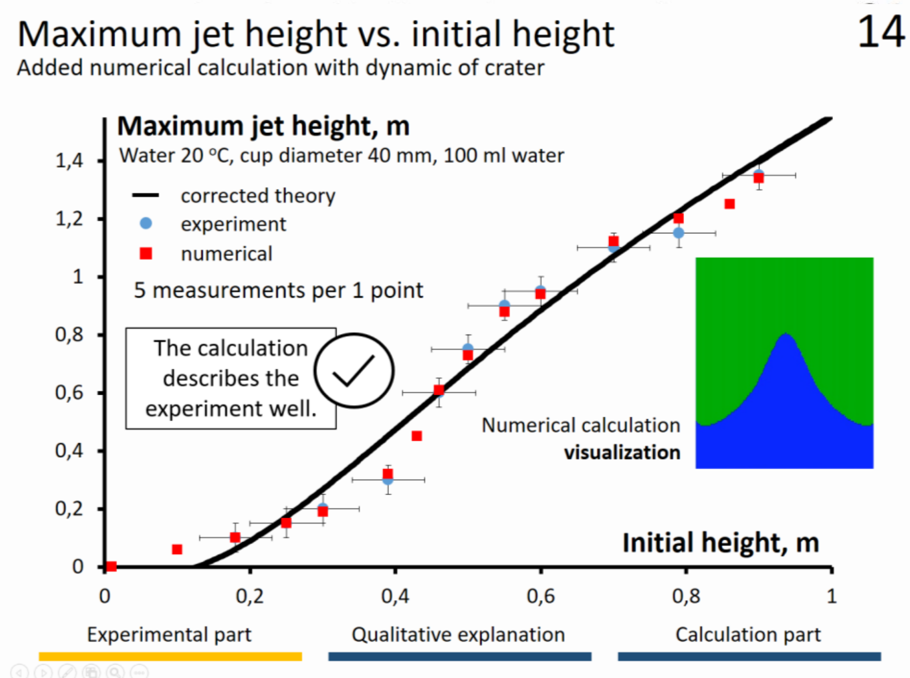
$$k \sim 1\%$$

$$H \sim 10m$$

Conclusions

Found the essence of the problem. ***The formed crater*** place ***the key role***. And **its collapse** under the influence of **large accelerations**.

An experimental setup has been designed to **improve the repeatability** of the phenomenon.



The **maximum height** of the jet from the **important parameters** of the liquid and the cup **is investigated**. An **rather accurate calculation is made**.

The **dynamics** of the formation **of a crater is investigated** depending on the properties of the liquid. **Empirical amendment**.

Conclusions

The **mechanism of energy transfer is investigated**. The energy transfer process is almost instantaneous. **The theory describes the experiment well.**

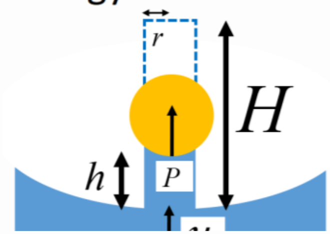
The maximum height is defined under conditions of limited parameters. The coefficient of energy transfer from the diameter and fullness of the cup is investigated.

In the end, the real

"Cumulative Cannon"

is developed.

Energy transfer to ball from jet 18

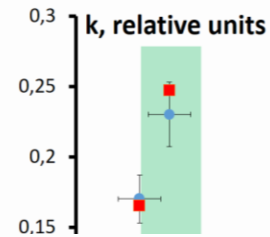


Law of energy conservation: $\frac{\rho u^2}{2} + \rho g h = \rho g H$

Law of change of momentum: $\Delta p = F \Delta t = m \Delta u$

Fraction of energy vs. fluid volume 25

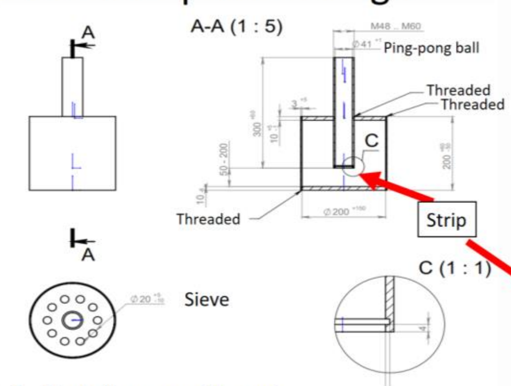
Ping pong ball, initial height - 55 cm



There is an optimum for fluid volume

$k_{max} = 23\%$

Ideas for optimal design 26



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Device can accelerate heavy objects well ☺

Experimental part | Qualitative explanation | Calculation part

Final thought

Bibliography

"Кумулятивный эффект в простых опытах",
[Cumulative effect in simple experiments], V. V. Mayer

"Аналитическая гидродинамика",
[Analytical fluid dynamics], A. G. Petrov

Solution of IYPT problem "Drawing pins",
Alex Krotov, Team of Russia (2012)

"Молекулярная физика",
[Molecular physics], G. Y. Myakishev, A. Z. Sinyakov

Further research:

1. Strut angle investigation
2. *__Focusing effect__* investigation

Problem №1
Cumulative cannon

Reporter: Artem Sukhov

Team of Russia

It was also investigated:

1. Strut angle of cone and another cups
   (change cumulation coefficient)
2. Mass of the ball
3. Radius of jet from radius of cup
4. Vortex in cup
5. Focusing effect
   (interesting phenomenon)
6. Surface
   (elasticity of collision)
7. Deformations effect
8. Decay into droplets
   (theory limitation)

COMPARISON

Final thought

Thank you! Questions?