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12th International Physicists' Tournament

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IPT

Problem no.11 – Flat fog

IPT 2020
Team Slovenia
Presenter: Luka Školč

University of Ljubljana
Faculty of Mathematics and Physics

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Problem Statement

After pouring liquid nitrogen into a mug, you will notice that the mug starts to cast a mist. The **mist’s border** is a clearly marked **thin plane** at a certain height from the mug. Investigate the phenomenon.

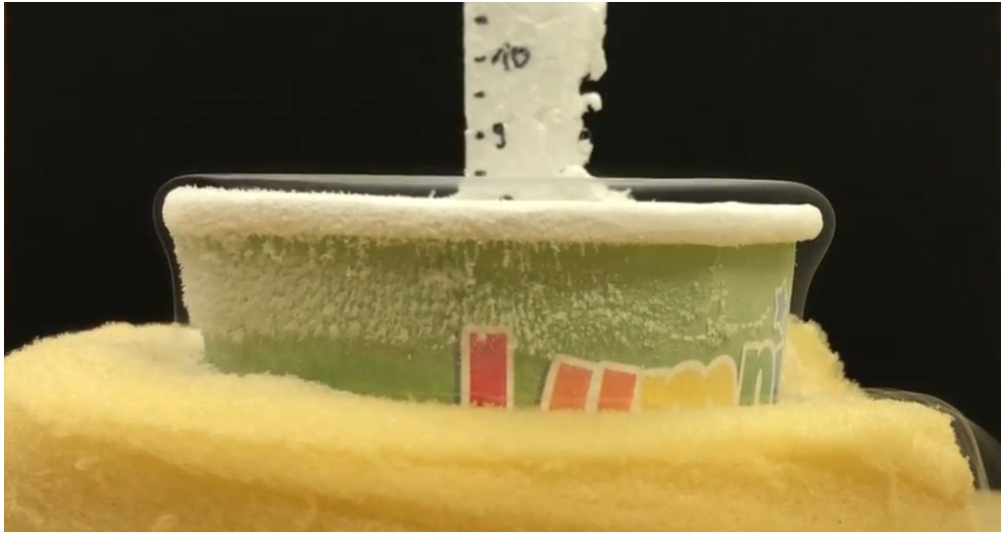
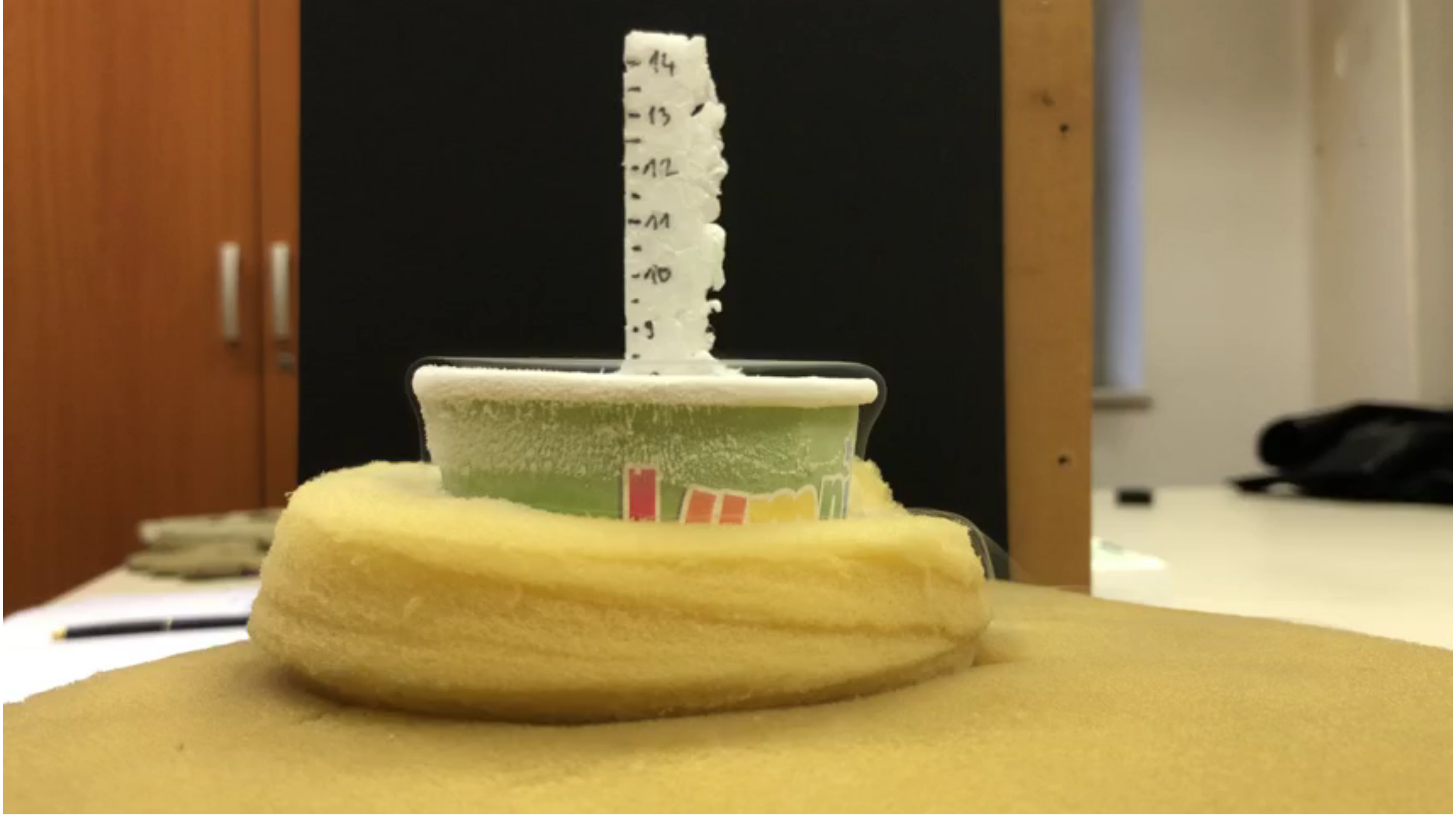


Figure 1: characteristic flat fog

Intuitive explanation

1. There is only nitrogen gas below the mist
2. Mist forms as ambient air in contact with cold nitrogen gas cools below its dew point
3. Droplets are levitated by the upward gas flow



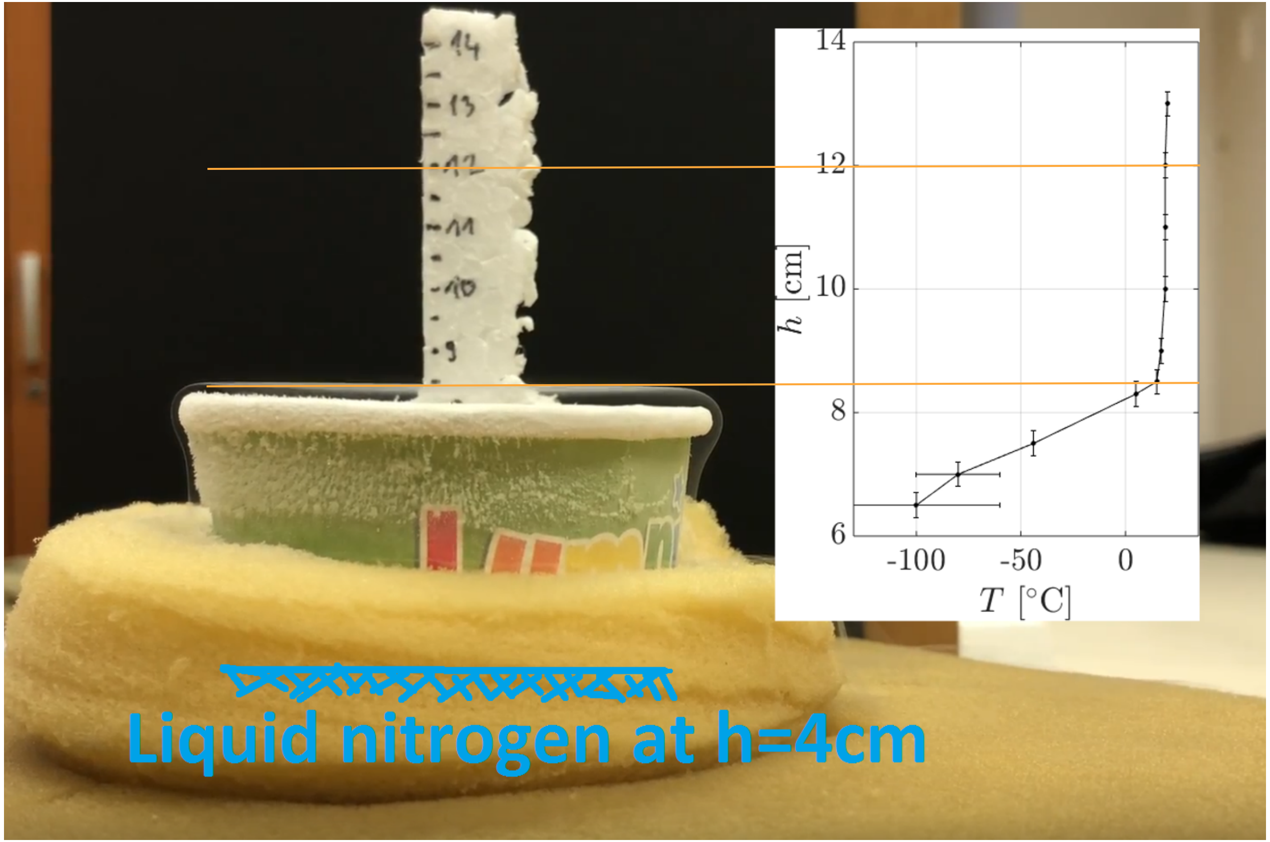
Video 1: Well-insulated container

Hypothesis: Only nitrogen below mist

Video 2: match dying out due to lack of oxygen

Temperature profile in gas

Hypothesis: Fog formation



Liquid nitrogen at h=4cm

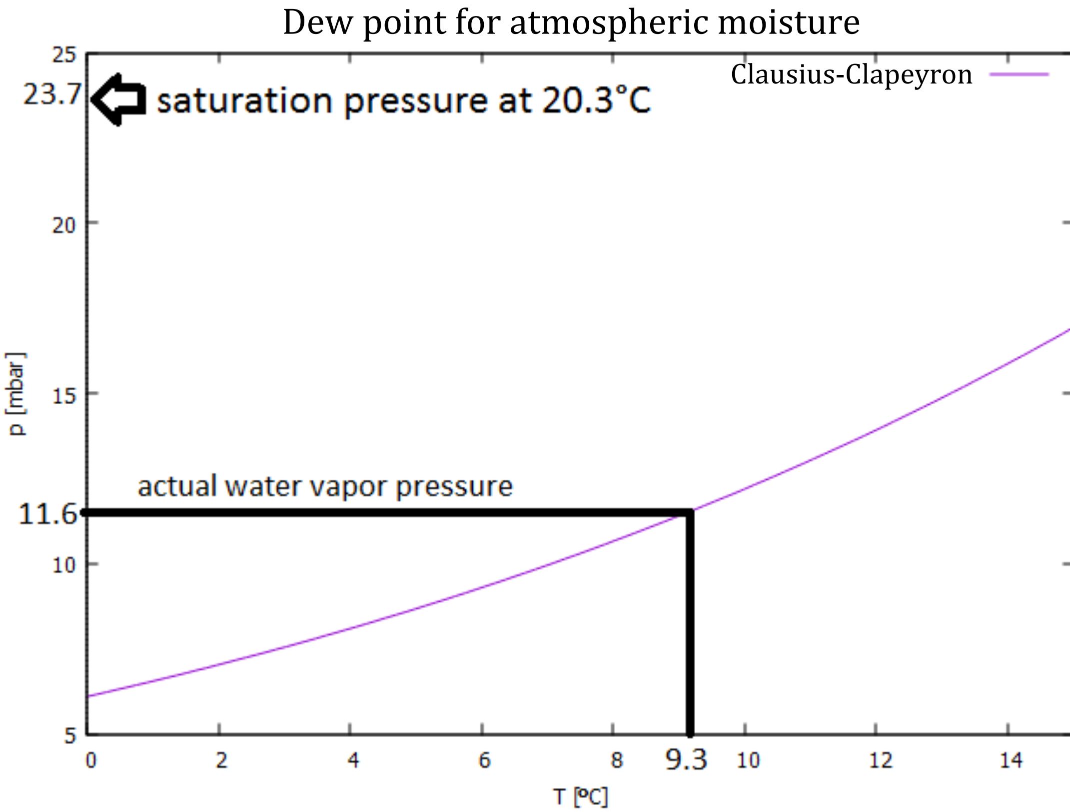
Measured with a thermocouple thermometer

Fog temperature and dew point

Hypothesis: Fog formation

- Mist forms when relative humidity η reaches 100 %
- Experiment:
  - Room temperature $T_R = 20.3 \pm 0.1\ °\mathrm{C}$
  - Relative humidity $\eta = 49 \pm 1\ \%$
  - Mist appears when border temperature reaches $\mathrm{T} = 9 \pm 0.3\ °\mathrm{C}$
- Clausius-Clapeyron predicts dew point at $T = 9.3\ °\mathrm{C}$

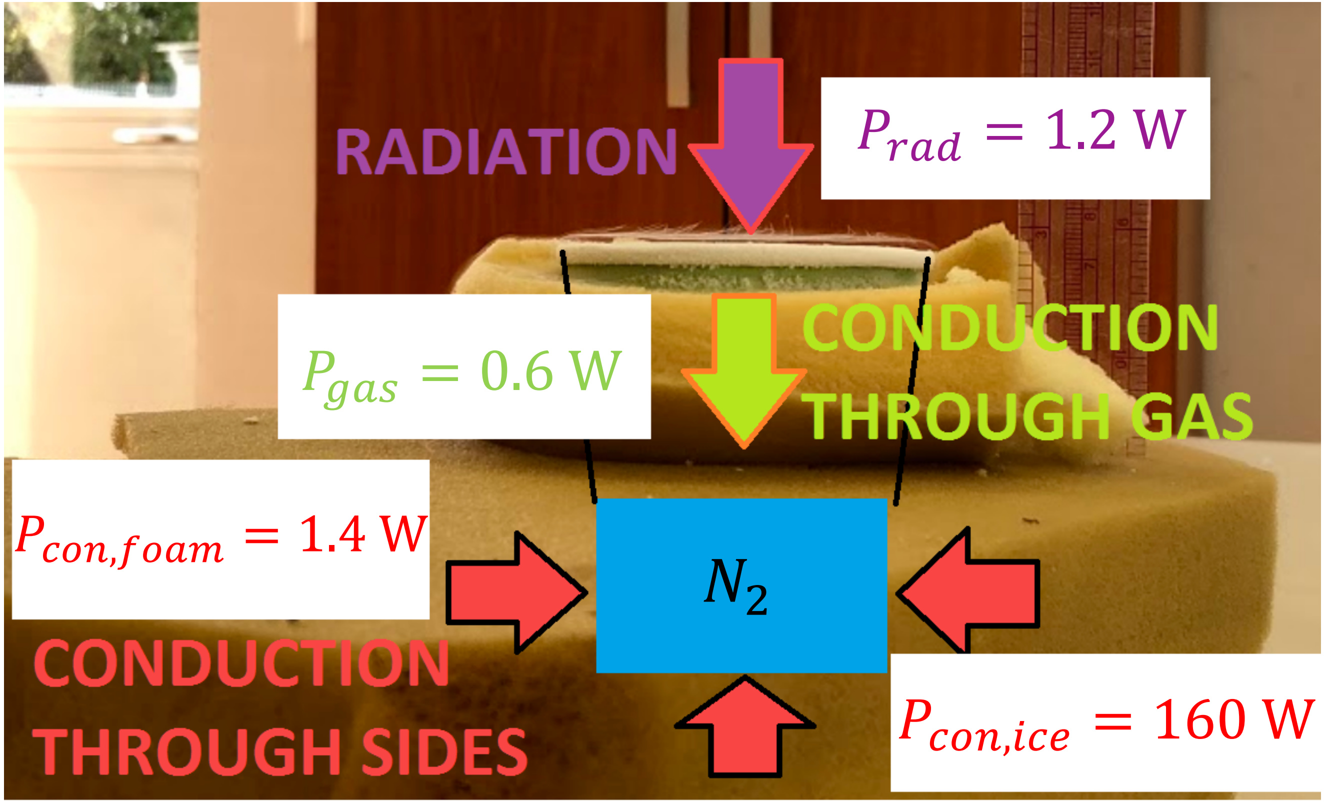
$$p_S(T) = p_0 e^{\frac{Mq_i}{R}\left(\frac{1}{T_0}-\frac{1}{T}\right)}$$



Liquid nitrogen evaporation

Mist height

Experiment: $\Delta V = 71 \pm 4\ \mathrm{cm}^3$ $\Delta t = 15\ \mathrm{min} \pm 5\ \mathrm{s}$ $\mathrm{P} = \frac{\Delta V\ \rho_l\ q_i}{\Delta t} = 13 \pm 1\ \mathrm{W}$



RADIATION

$P_{rad} = 1.2\ \mathrm{W}$

CONDUCTION THROUGH GAS

$P_{gas} = 0.6\ \mathrm{W}$

$P_{con,foam} = 1.4\ \mathrm{W}$

N_2

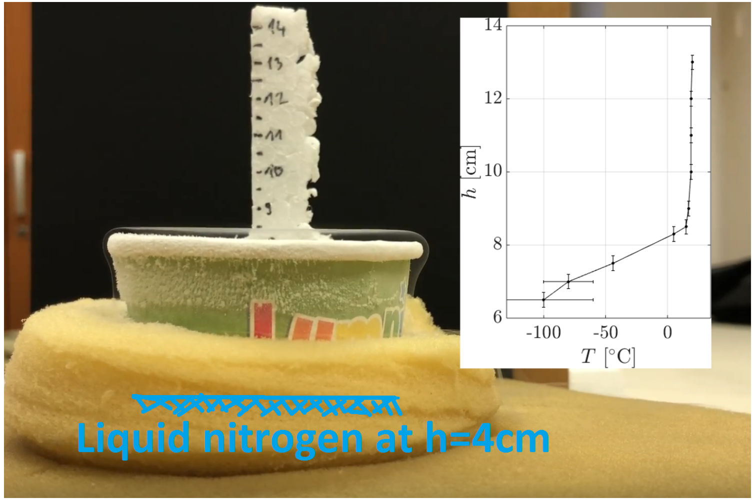
CONDUCTION THROUGH SIDES

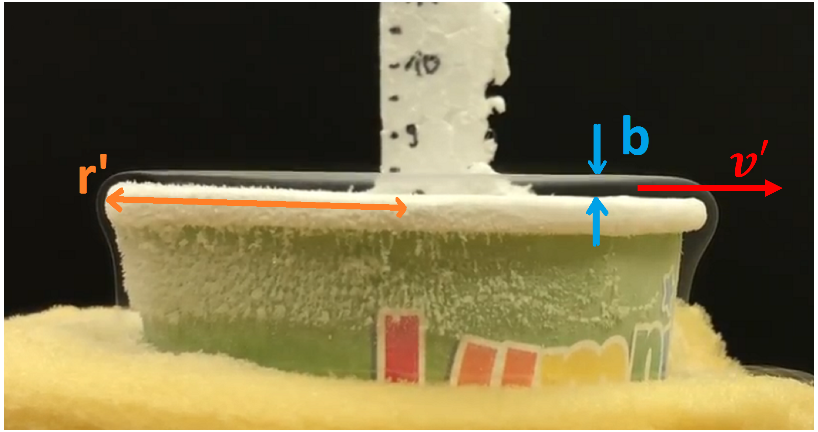
$P_{con,ice} = 160\ \mathrm{W}$

Nitrogen gas flow in a vertical tube

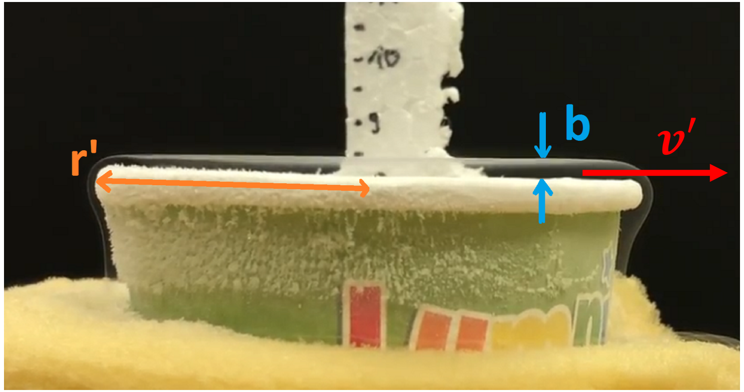
- Conservation of mass: $\Phi_m = \frac{P}{q_i} = S'v'\rho'$ $\qquad S' = 2\pi r'b$
- Experiment: nitrogen both warms up and speeds up on its way up

NOT AN ADIABATIC PROCESS





Estimating the mist height

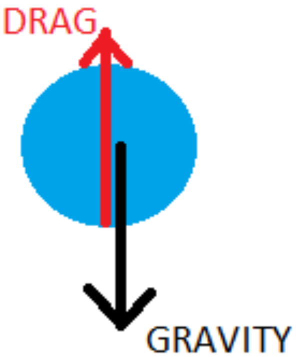


- Gas can only warm up to room temperature
- $P_{gas} \approx 13\ \mathrm{W}$ to heat gas for $200\ \mathrm{K}$
- Vertical throw: $\frac{v'^2}{2} = gb$
- $S' = 2\pi r' b$
- $b = \left(\frac{P}{2\sqrt{2}\pi r' \sqrt{g}\rho' q_i}\right)^{\frac{2}{3}}$

EXPERIMENT	ROOM TEMPERATURE ESTIMATE
$\boldsymbol{b = 1.1 \pm 0.2\ \mathrm{mm}}$ $r' = 3.5 \pm 0.1\ \mathrm{cm}$ $S' = 2.4 \pm 0.4\ \mathrm{cm}^2$	$\boldsymbol{b \leq 1.5\ mm}$ $v' \leq 17\ \frac{\mathrm{cm}}{\mathrm{s}}$

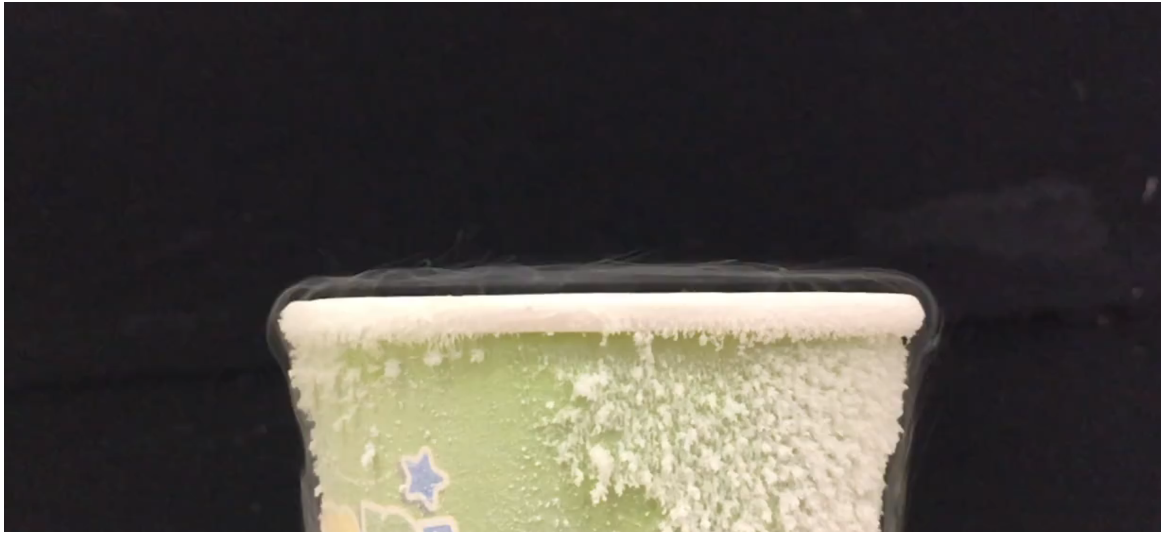
Droplets in the mist

- Droplets are levitated by the upward gas flow – as in **clouds**
- Radius by Stokes law: $\frac{4}{3}\pi\rho_{H_2O}R^3g = 6\pi R\eta v \xrightarrow[v = 10\ \frac{\mathrm{cm}}{\mathrm{s}}]{} R = 27\ \mu\mathrm{m}$
- Typical radius in clouds is $10\ \mu\mathrm{m}$ [8]



Parabolic vapour tracks above the mist

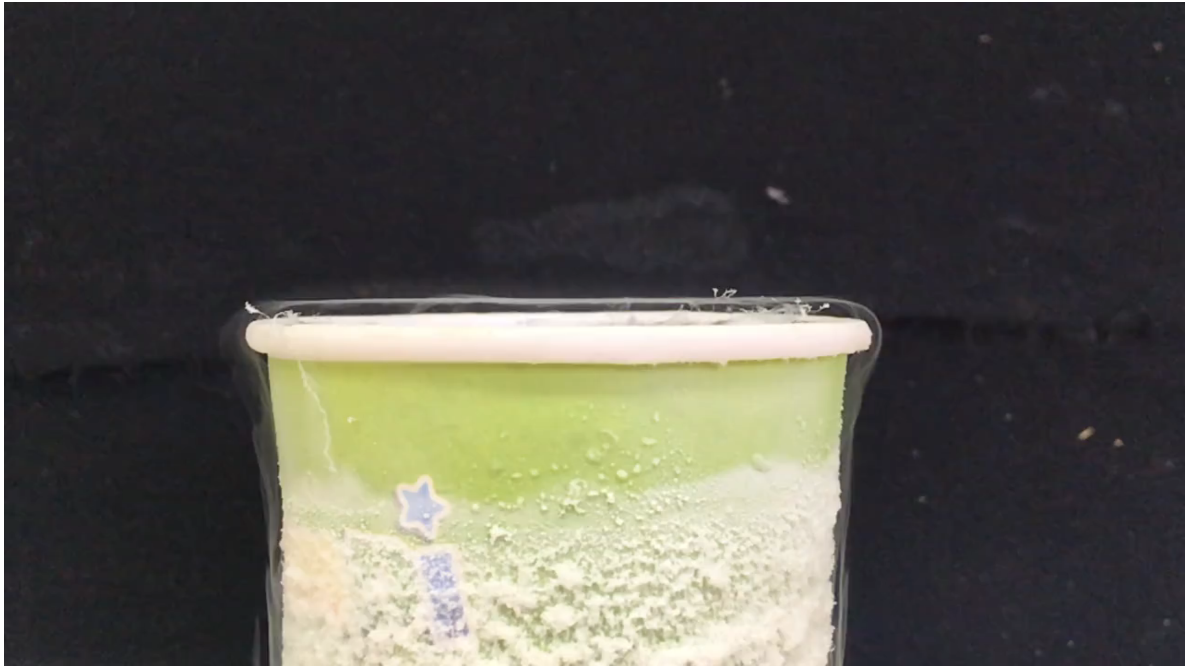
0.2x playback speed



Highlighted tracks

Blocking the droplets with cotton wool

0.2x playback speed



Conclusions

1. There is only nitrogen gas below the mist **CONFIRMED**
2. Mist forms as the air that comes in contact with cold nitrogen gas cools down below its dew point **CONFIRMED**
3. Droplets are levitated by the upward gas flow **PARTLY CONFIRMED**

- Next step: convective gas flow simulations

References

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Nitrogen data

$\rho_{liquid} = 0.804\ \frac{\mathrm{g}}{\mathrm{cm}^3}$ [3]

$q_i = 199\ \frac{\mathrm{kJ}}{\mathrm{kg}}$ [4]

$T_{boiling} = -195.8\ \mathrm{°C}$ [3]

$\rho_{gas,T=20.3\ \mathrm{°C}} = 1.16\ \frac{\mathrm{g}}{\mathrm{dm}^3}$ [4]

$\rho_{gas,T=-195.8\ \mathrm{°C}} = 4.4\ \frac{\mathrm{g}}{\mathrm{dm}^3}$

$c_p = 1040\ \frac{\mathrm{kJ}}{\mathrm{kgK}}$ specific heat capacity of Nitrogen gas at constant pressure

Derivations of formulae

- Clausius Clapeyron equation $p_S(T) = p_0 e^{\frac{Mq_i}{R}(\frac{1}{T_0} - \frac{1}{T})}$ $p_0 = 6.1\ \mathrm{mbar}$ $T_0 = 0\ °\mathrm{C}$
- Mass flow $\Phi_m = \frac{P}{q_i} = 0.063\ \frac{\mathrm{g}}{\mathrm{s}}$ is conserved
- Heat currents estimates:
  - Heat conduction through sides of the container
    - Taking conductivity of foam $\lambda_f = 0.03\ \frac{\mathrm{W}}{mK}$ [5] $P_{con,\,foam} = 1.4\ \mathrm{W}$
    - Foam partly freezes $\lambda_{ice,T=-100\ °\mathrm{C}} = 3.5\ \frac{\mathrm{W}}{mK}$ [2] $P_{con,\,ice} = 160\ \mathrm{W}$
  - Radiation $P_{rad} = r^2\pi\sigma(T_R^4 - T_B^4) = 1.2\ \mathrm{W}$ assuming both room and liquid nitrogen radiate as blackbodies
  - Conduction through nitrogen gas $P_{con,gas} = 0.6\ \mathrm{W}$

- Generalised Bernoulli equation: $h + gz + \frac{1}{2}\overline{v^2} = c_P T + gz + \frac{1}{2}\overline{v^2} = const.$ [6]
- Estimating fog height
  - Power to warm up gas for 200 K $P_{gas} = \Phi_m \Delta T c_p = 13$ W

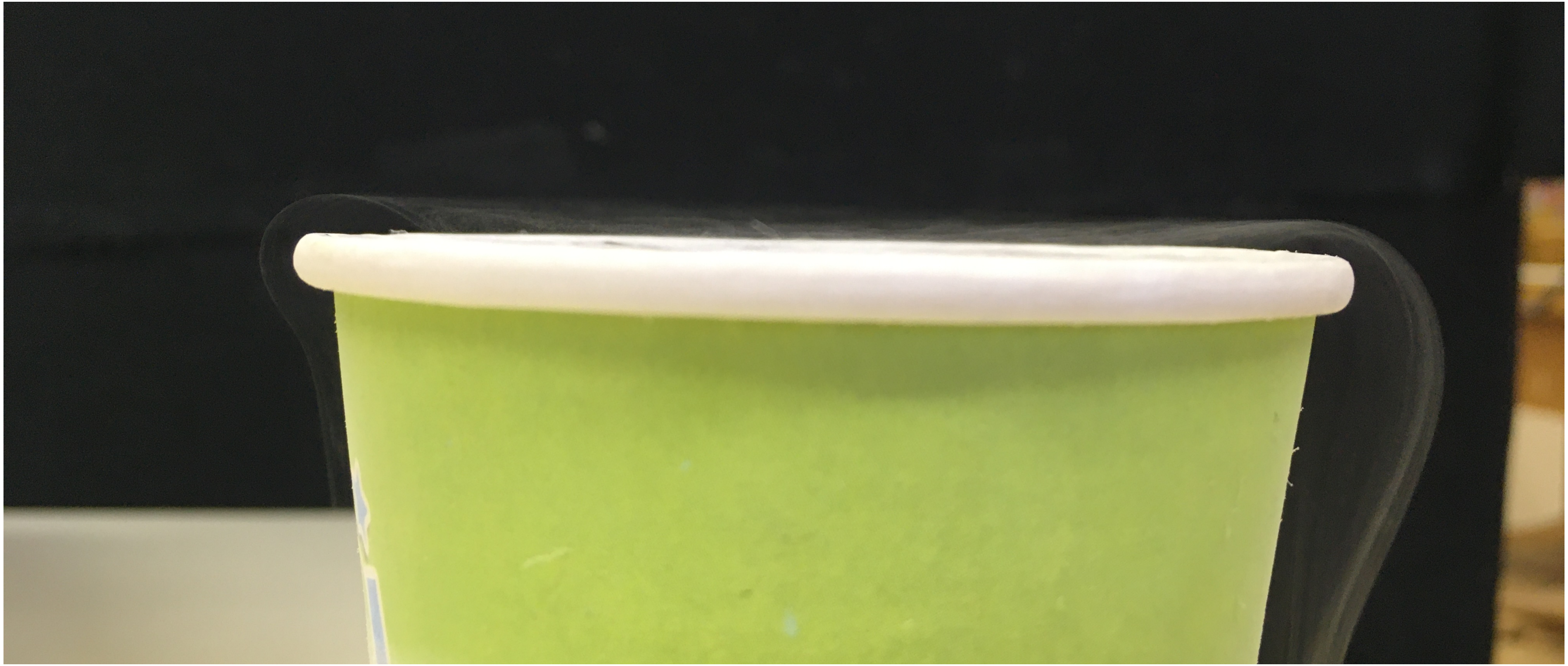
$$\frac{P}{q_i} = 2\pi r' b\sqrt{2gb}\rho' = 2\sqrt{2}\pi r'\sqrt{g}\rho' b^{\frac{3}{2}} \qquad b = \left(\frac{P}{2\sqrt{2}\pi r'\sqrt{g}\rho' q_i}\right)^{\frac{2}{3}} \qquad r' = 3.5 \pm 0.1 \text{ cm}$$

  - Estimating droplet size

$$\eta_{N_2, T=0°C} = 1.66\text{x}10^{-5} \text{ Pas } [7] \qquad Re = O(10^{-3}) \qquad mg = \frac{4}{3}\pi\rho_{H_2O}R^3 g = 6\pi R\eta v$$

Video 3: Container with no insulation

Uneven surface of the mist



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