Problem 9 - Optical Compass

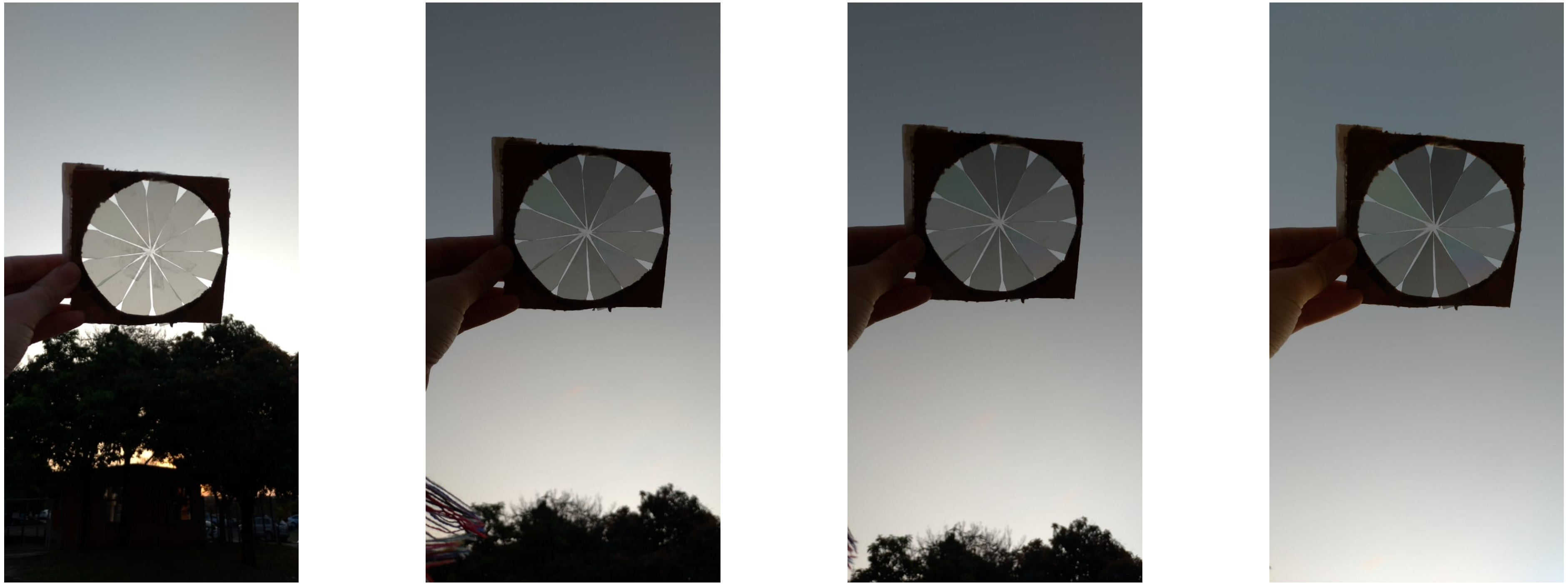
Maria Carolina Volpato
and Denise Christovam

Instituto de Física Gleb Wataghin
UNICAMP

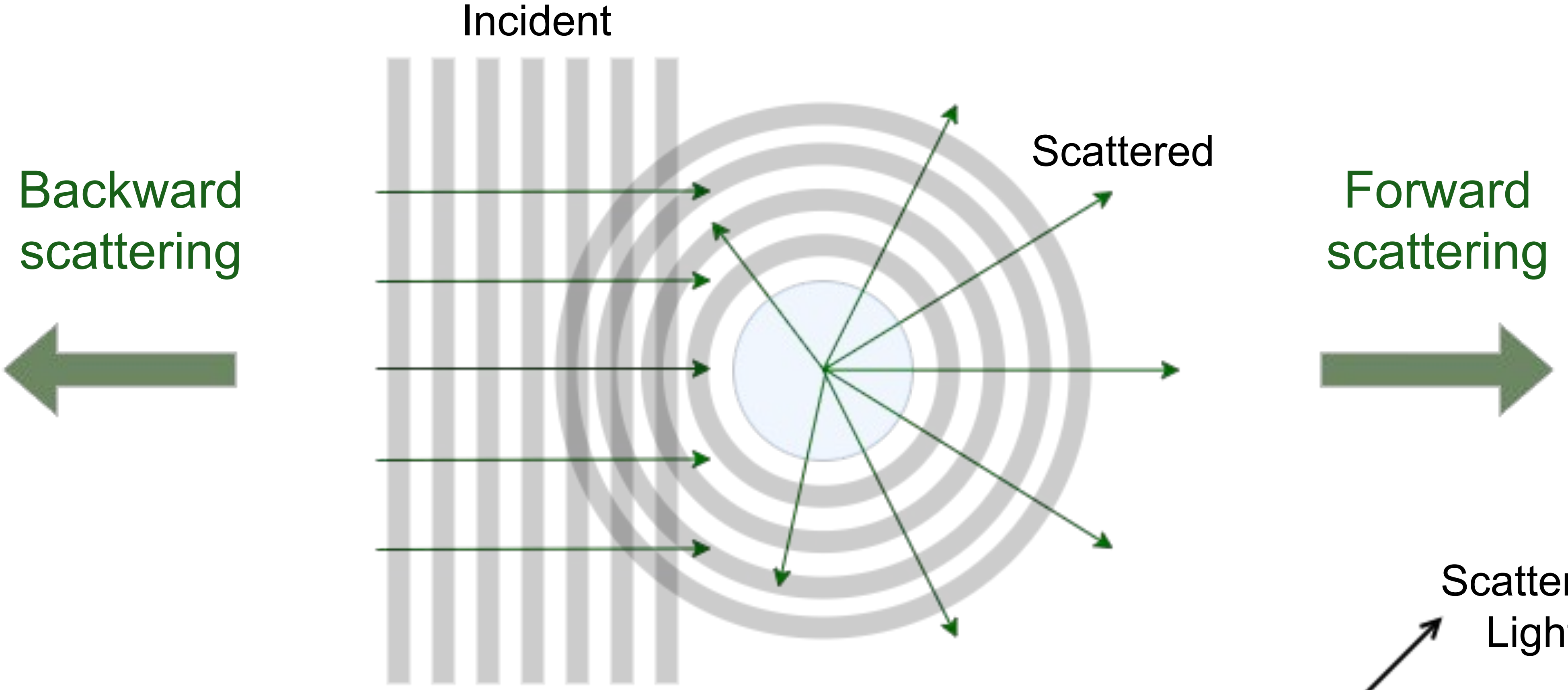
Problem Statement

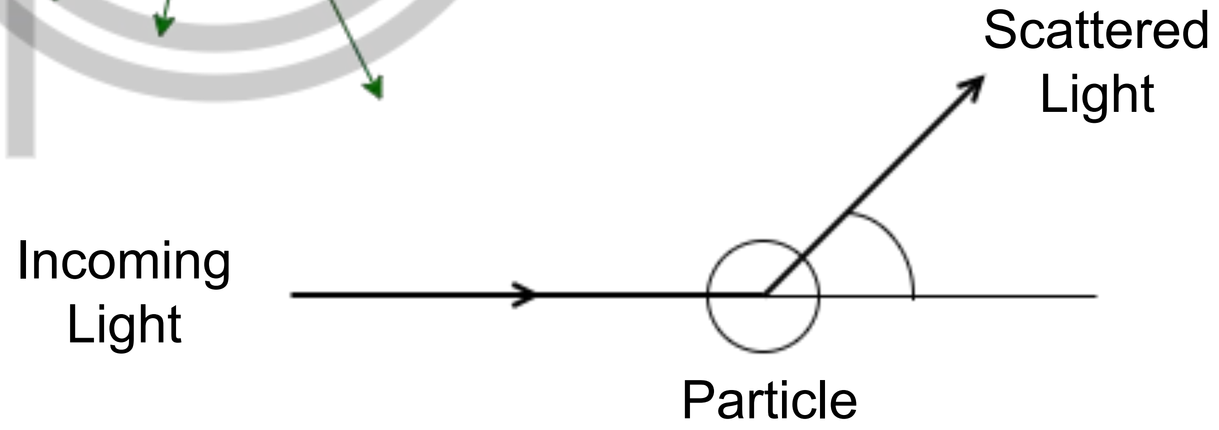
Bees locate themselves in space using their eyes' sensitivity to light polarization. Design an inexpensive optical compass using polarization effects to obtain the best accuracy. How would the presence of clouds in the sky change this accuracy?

Visualization of the phenomenon



Light Scattering





Light Scattering

$\lambda = 500nm$

Rayleigh Scattering

Mie Scattering

Conditions to Rayleigh scattering are satisfied!

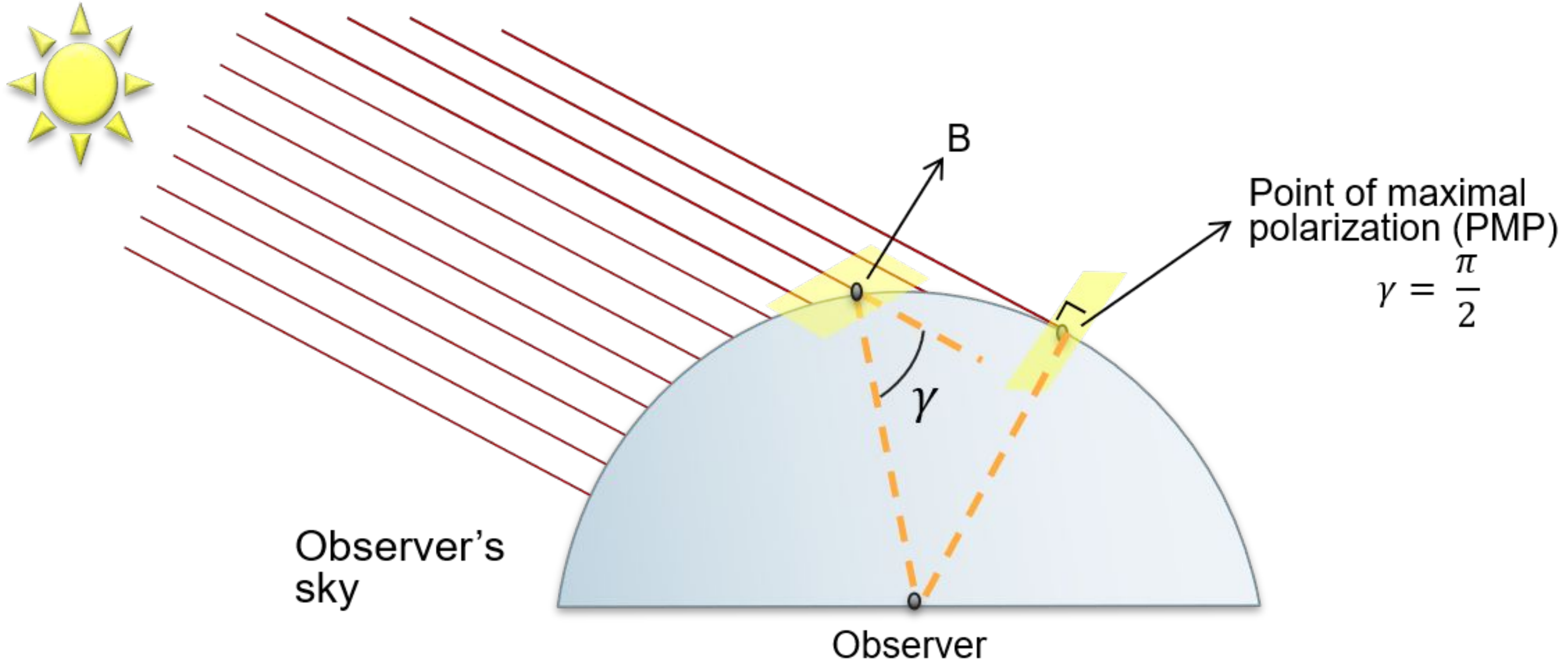
$$\text{Particle} < \frac{1}{10}\lambda$$

$$\text{Particle} < 50nm$$

$$\frac{1}{10}\lambda < \text{Particle} < \lambda$$

$$\text{Particle} \approx 50 - 500nm$$

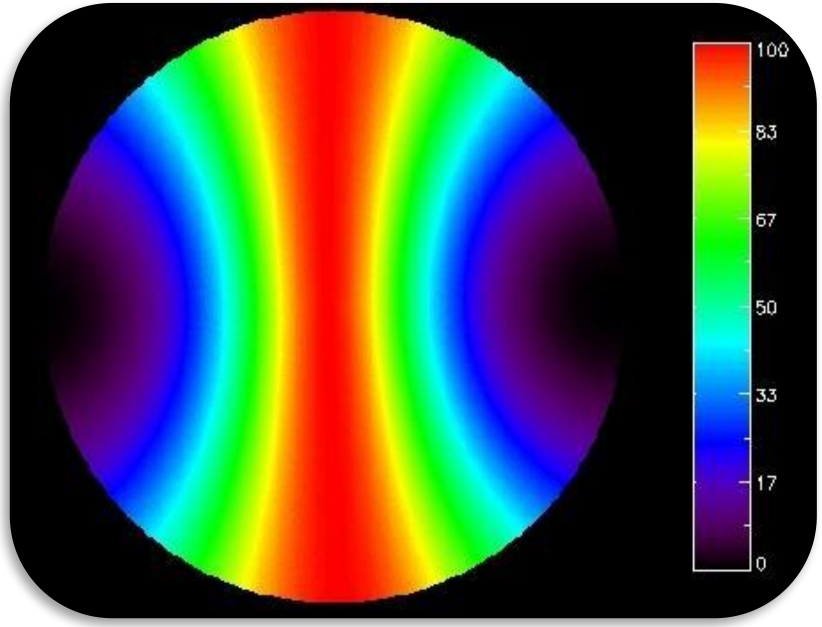
Linear Polarization



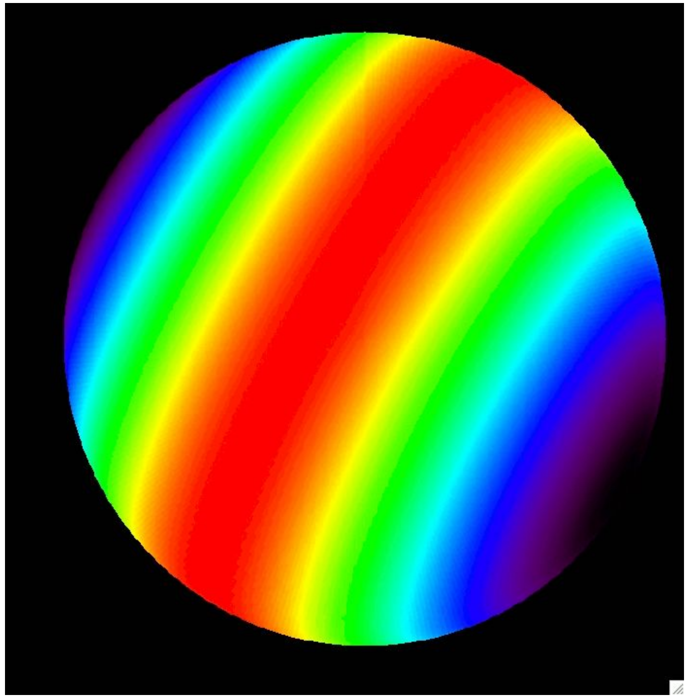
- ➔ For the observer at PMP polarization plane collapses into a line
- ➔ 90º from the source of light

Mapping the sky

- Rayleigh sky model describes how the maximally polarized light stripe varies with rotation



Degree of polarization at sunset or sunrise.



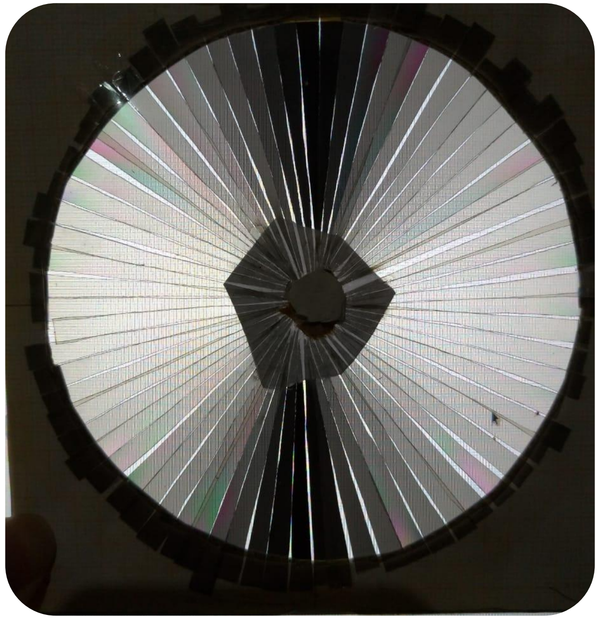
https://en.wikipedia.org/wiki/Rayleigh_sky_model

Materials and design

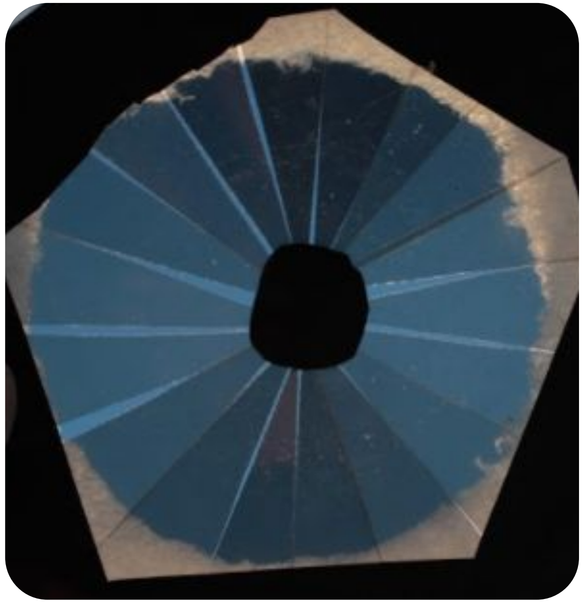
- ➔ Polarizing sheets
- ➔ Guillotine Paper Cutting Machine
- ➔ Adhesive tape
- ➔ Cardboard A4 (120 g/m²)

~ R$ 30.00 ($6.00)

12 petals – 30º

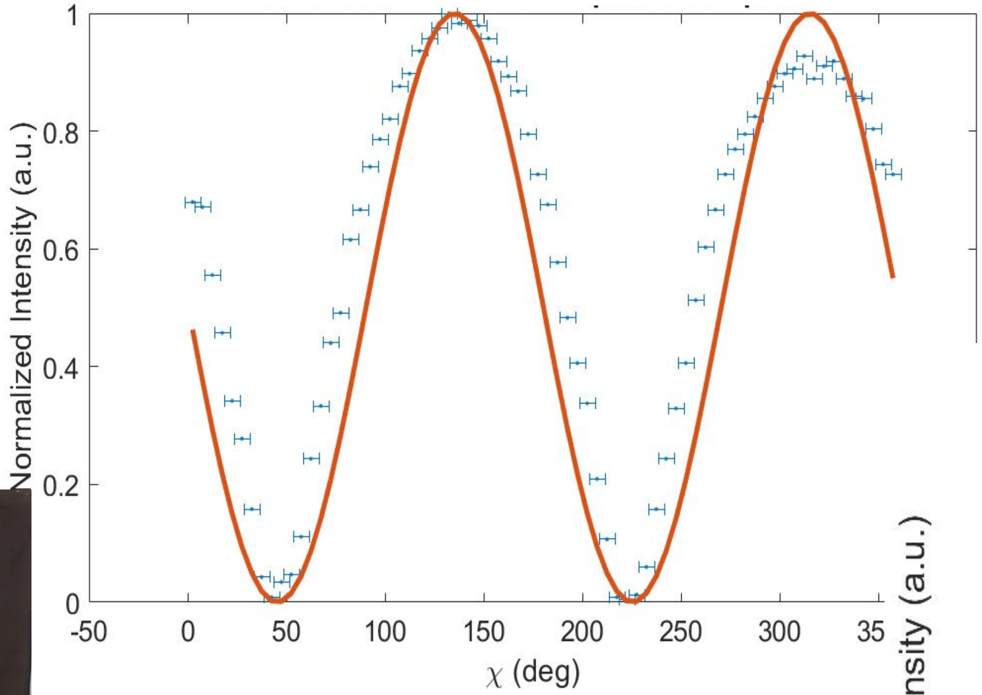


72 petals – 5º

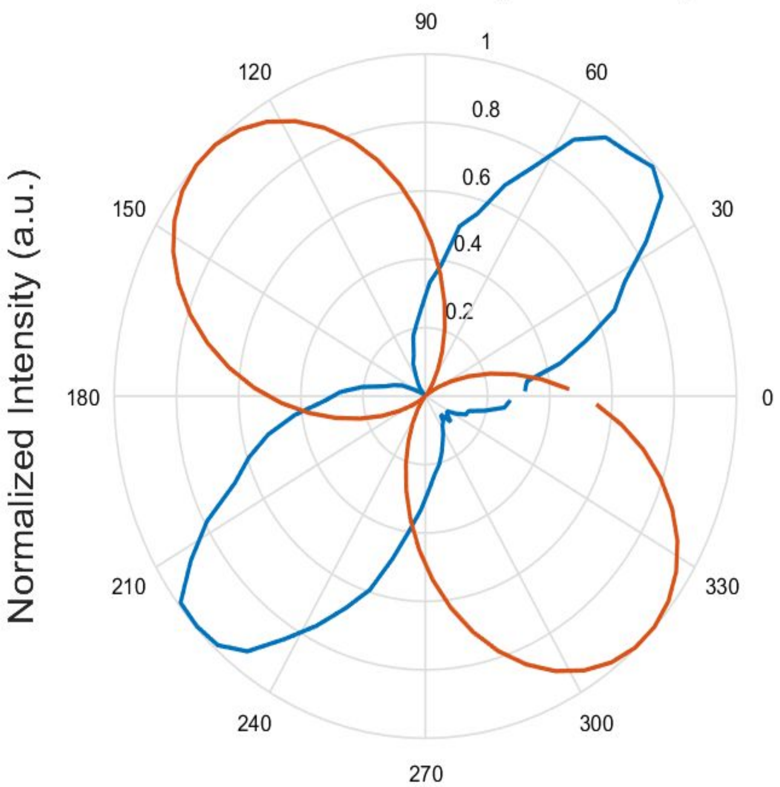


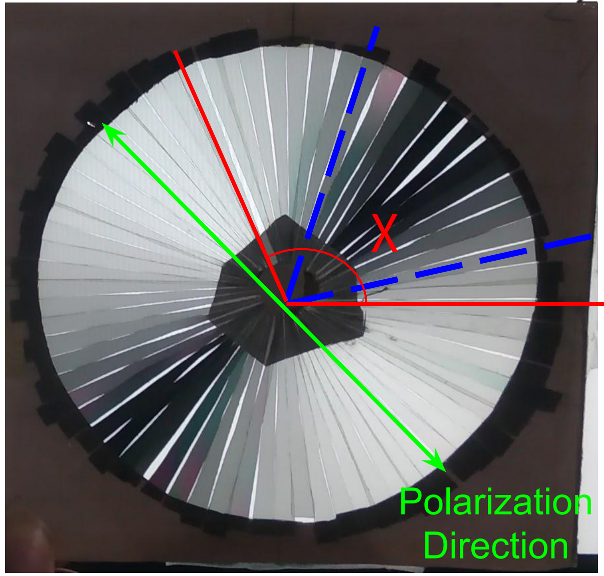
18 petals – 20º

Angular Resolution - Malus Law

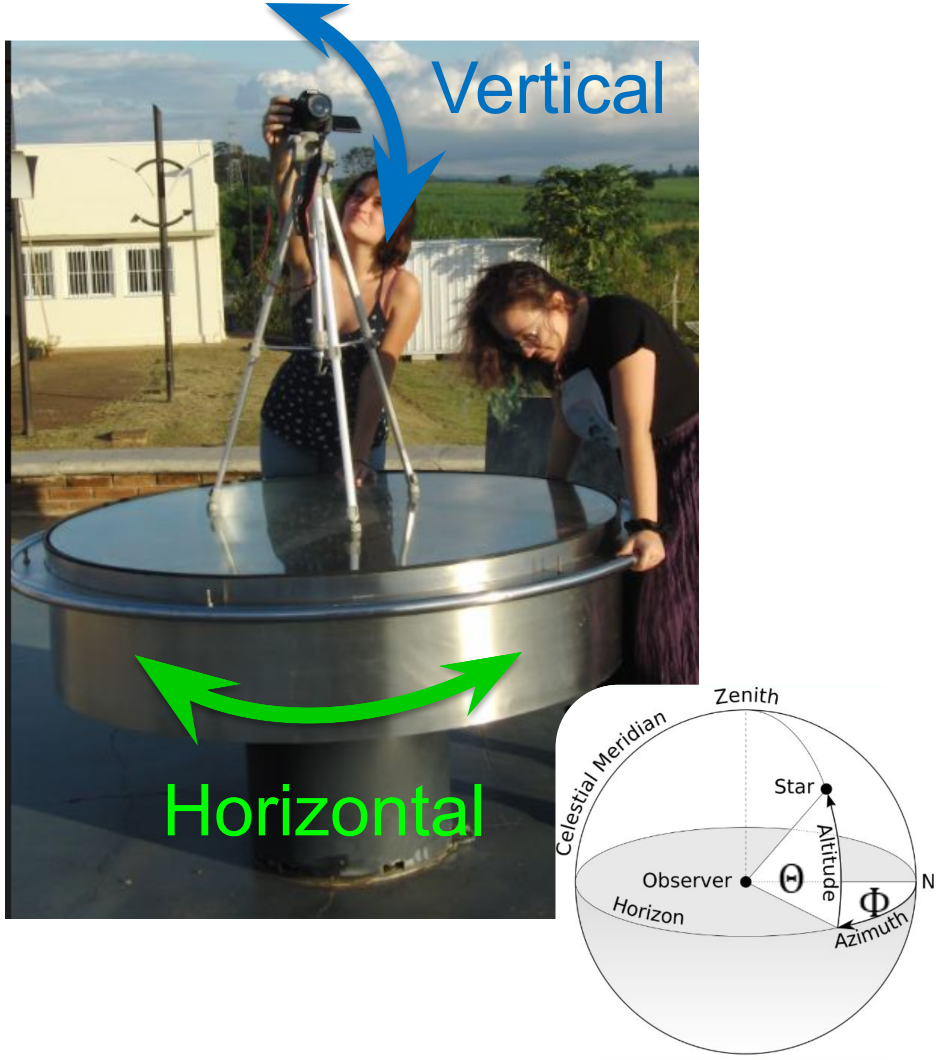


Angular Resolution
~ 20º

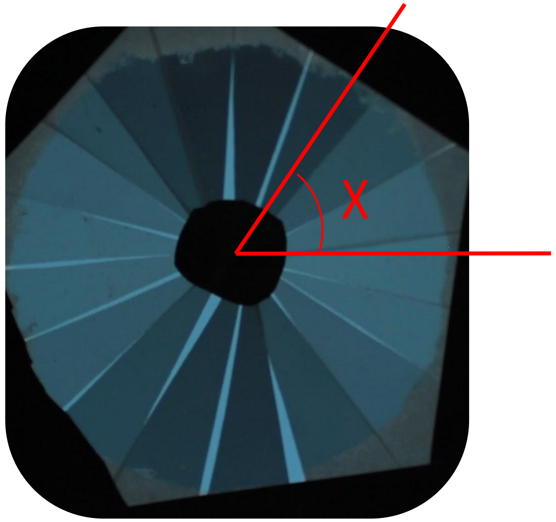


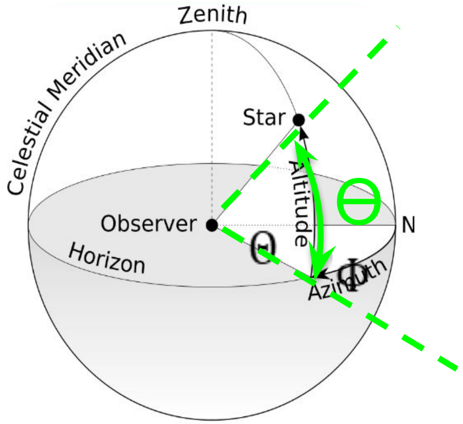


Setup- Vertical and Horizontal Mapping

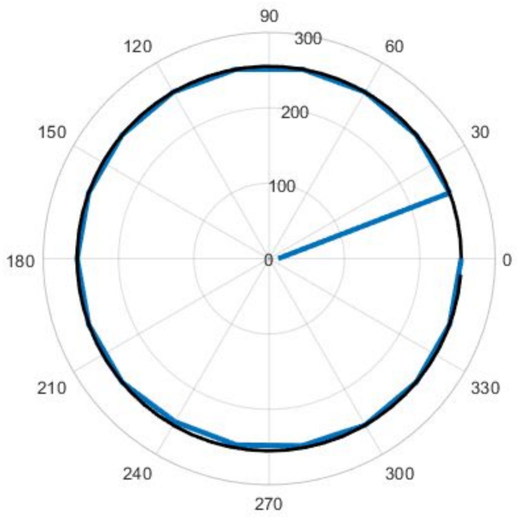


Vertical Angle

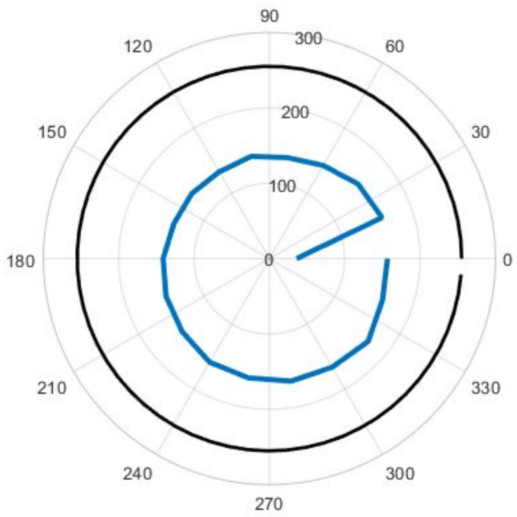




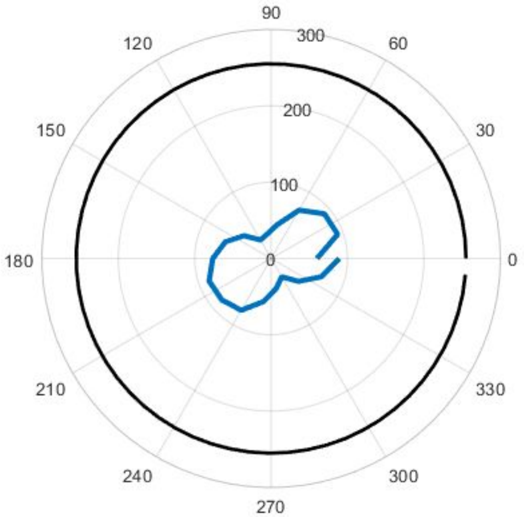
Φ= 335º



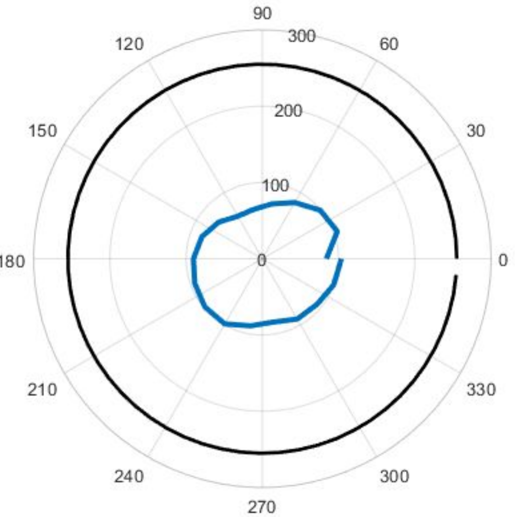
0º



40º

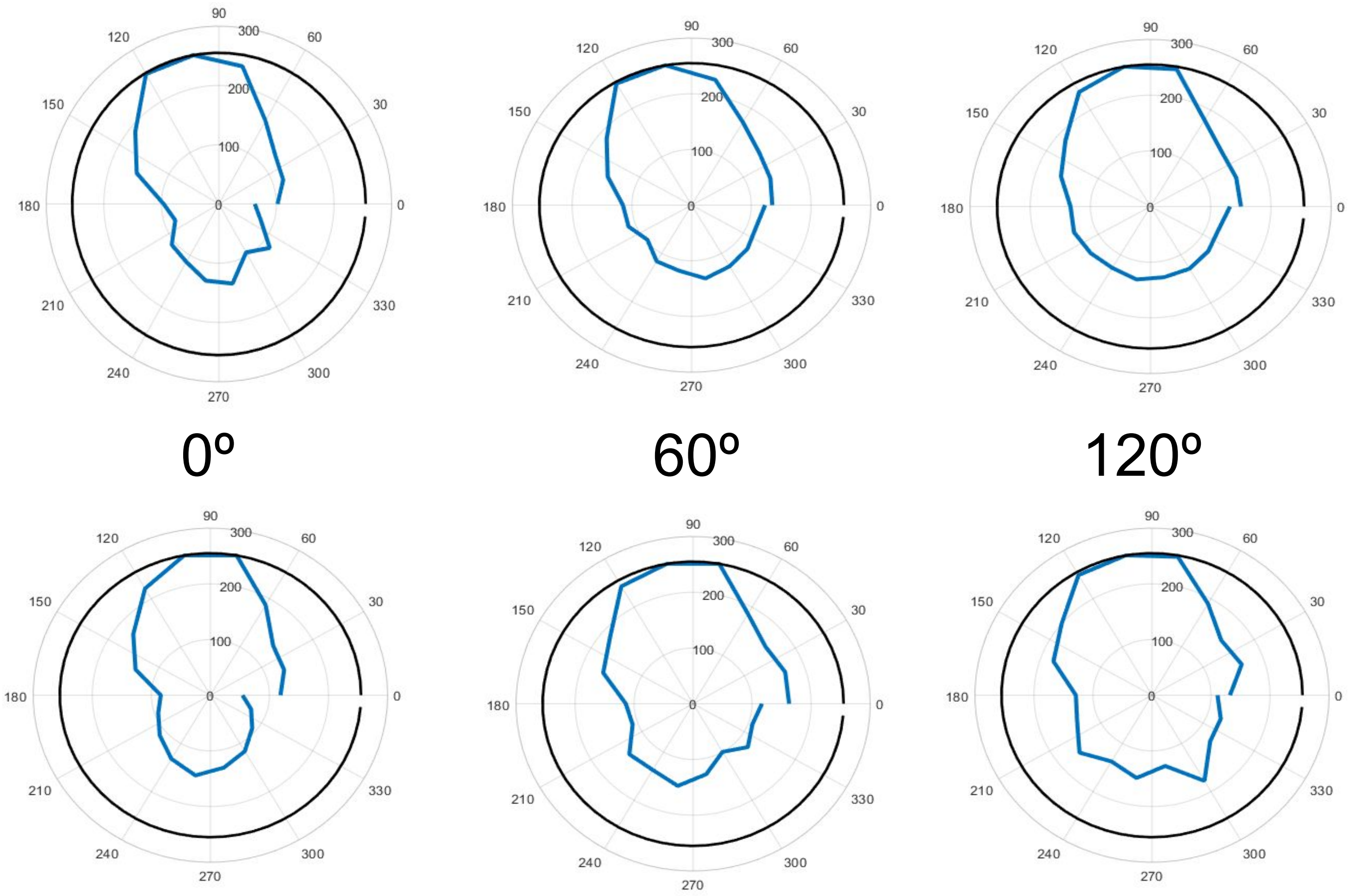


80º



120º

Horizontal Angle



0º

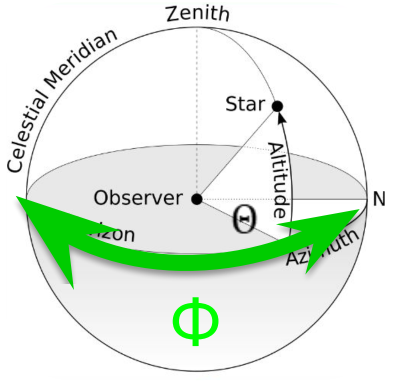
60º

120º

180º

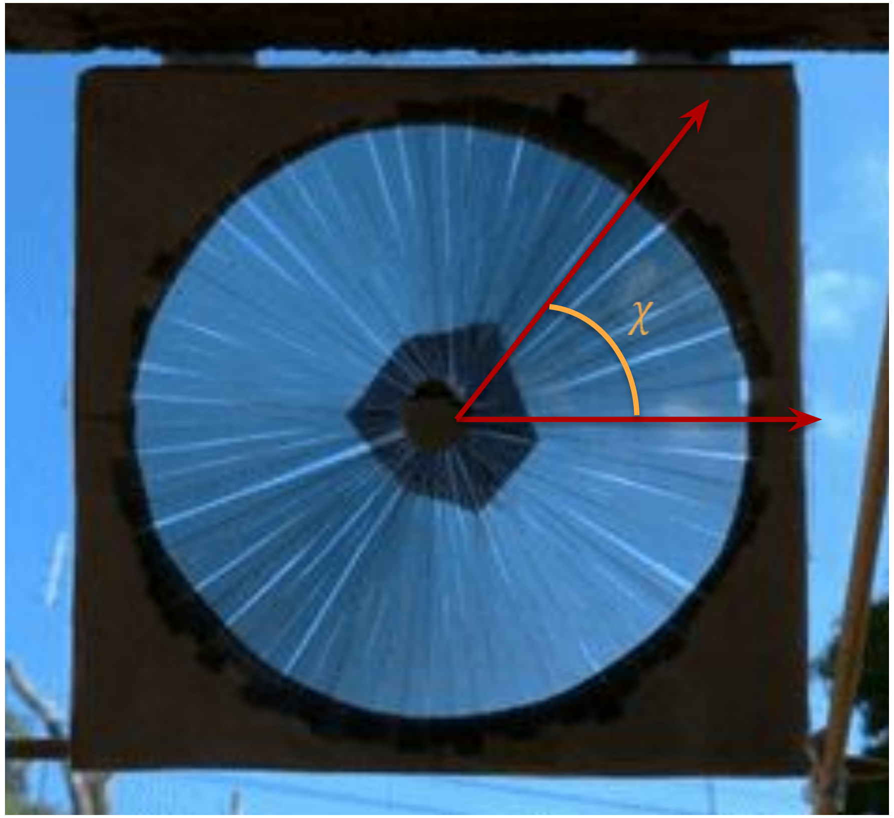
240º

300º

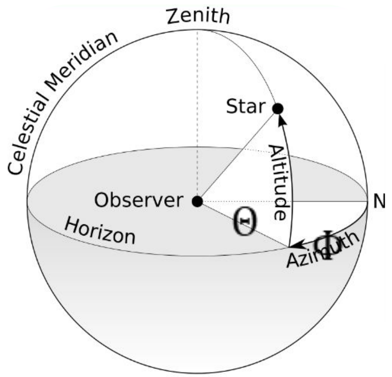


ϴ ~ 10º

Temporal evolution of the shadow

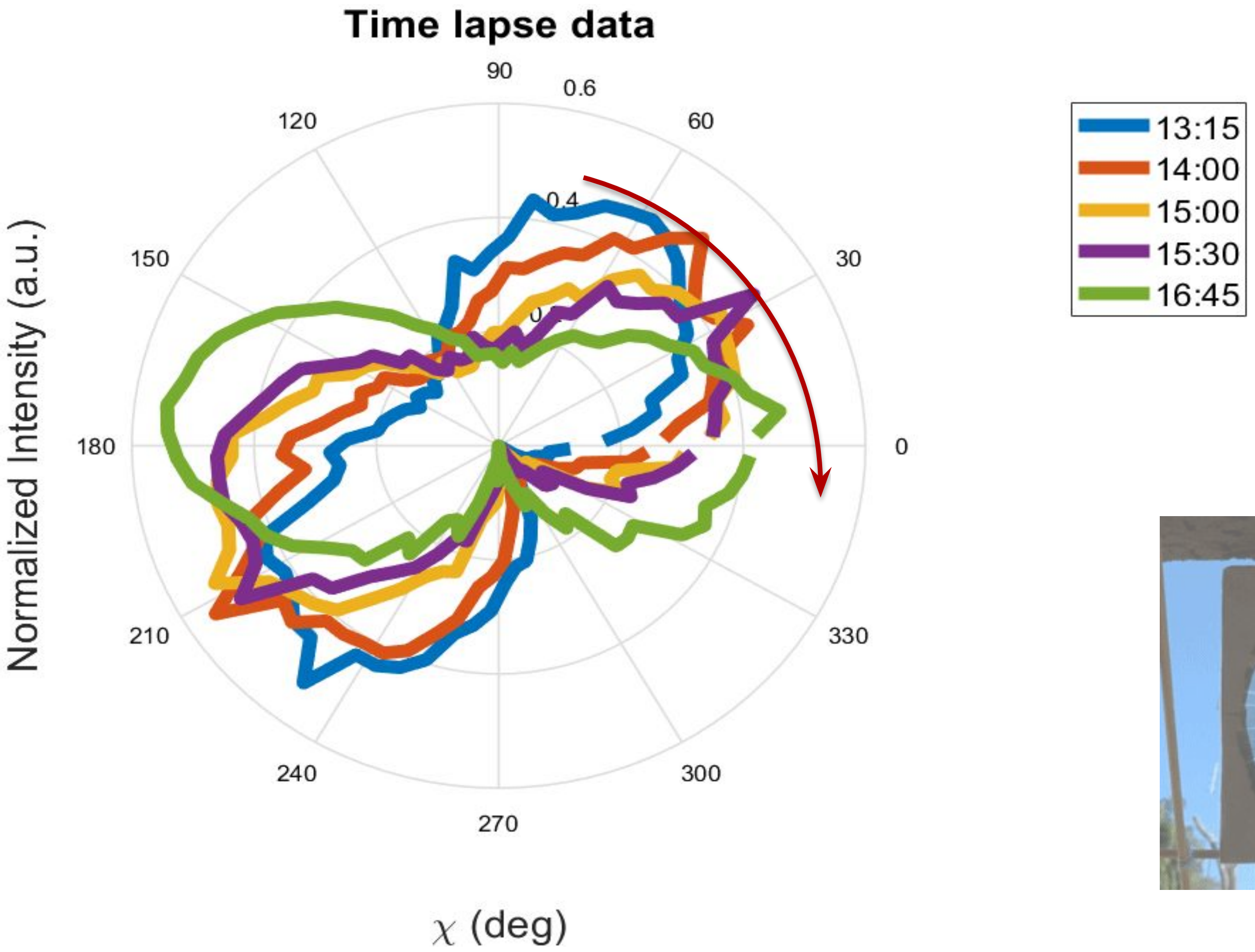


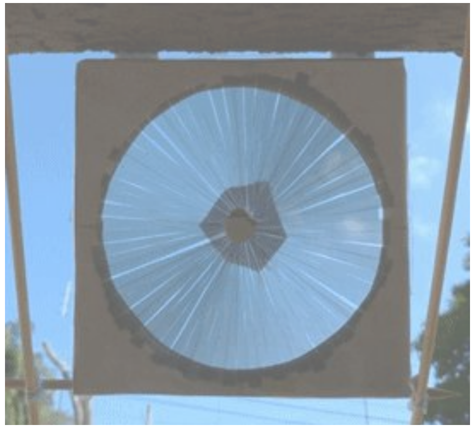
- ➔ Angle in relation to the ground ϴ= 25±1º
- ➔ Approximate geographical position:
  - ◆ Latitude: 23º 00’ 21” S
  - ◆ Longitude: 46º 50’ 20” W
- ➔ Window facing SSW (276º form north)
- ➔ Total time of acquisition 266 min (from 12 PM to sunset)
- ➔ Performed at 09/24/2019



https://www.timeanddate.com/sun/@3460598?month=9&year=2019

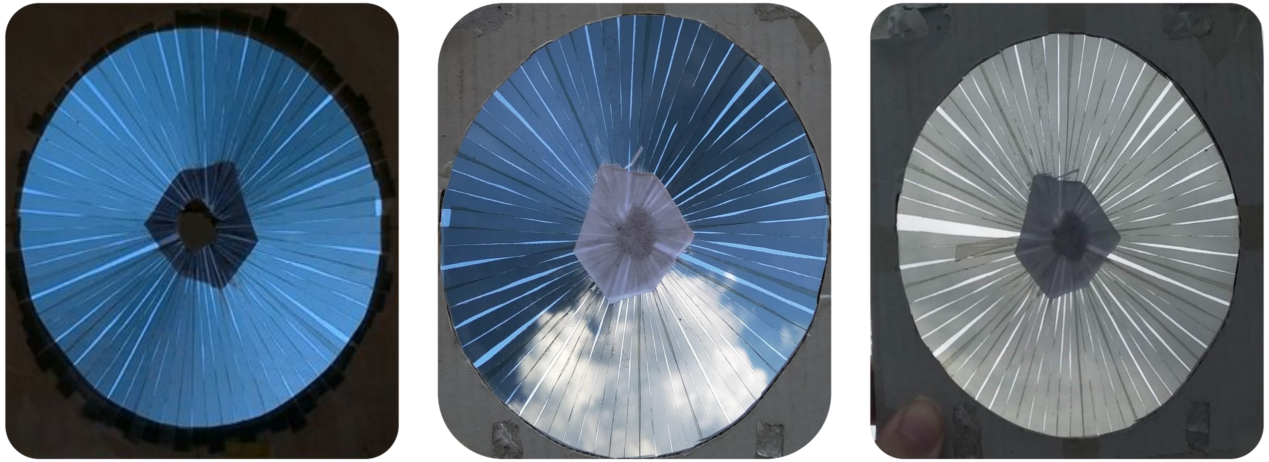
Temporal evolution of the shadow





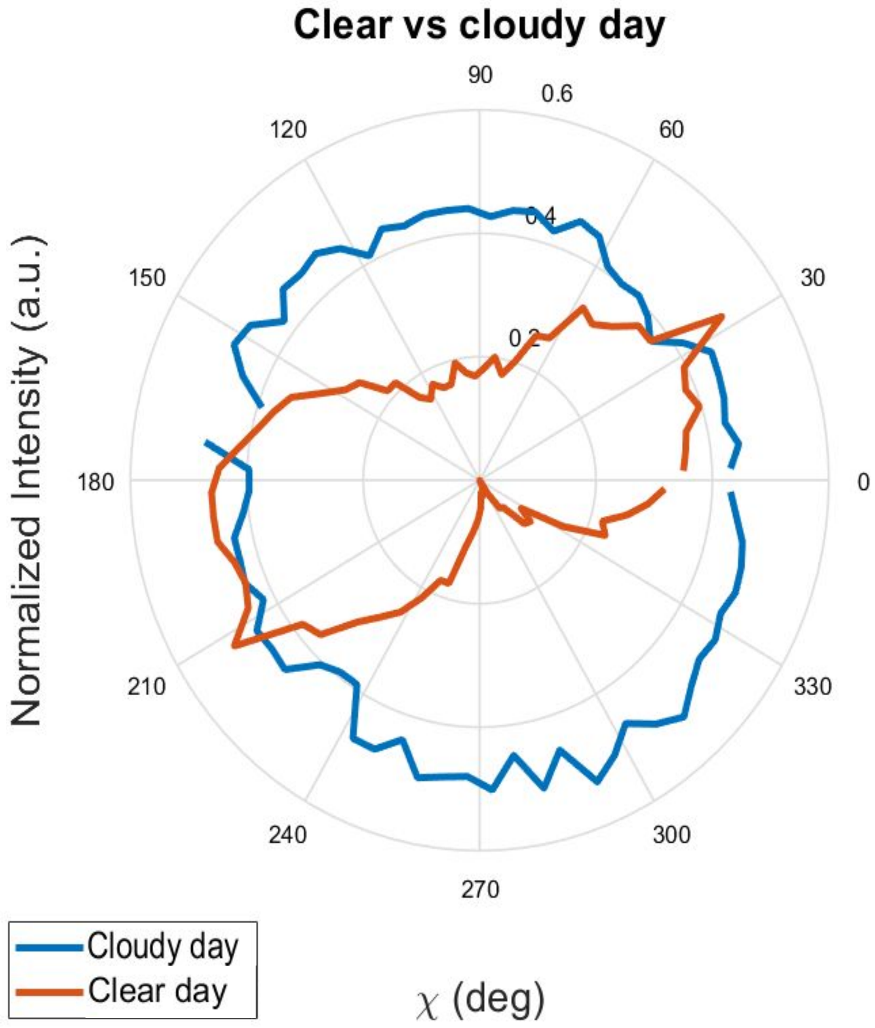
Presence of clouds

Clear sky | Partly Cloudy | Cloudy



Typical radius of the droplets ~ 10^2 nm ~ λ

Mie Scattering dominates



Conclusions

- ➔ It is possible to build a cheap device

  R$ 30.00 - $ 6.00

- ➔ It indicates N-S direction

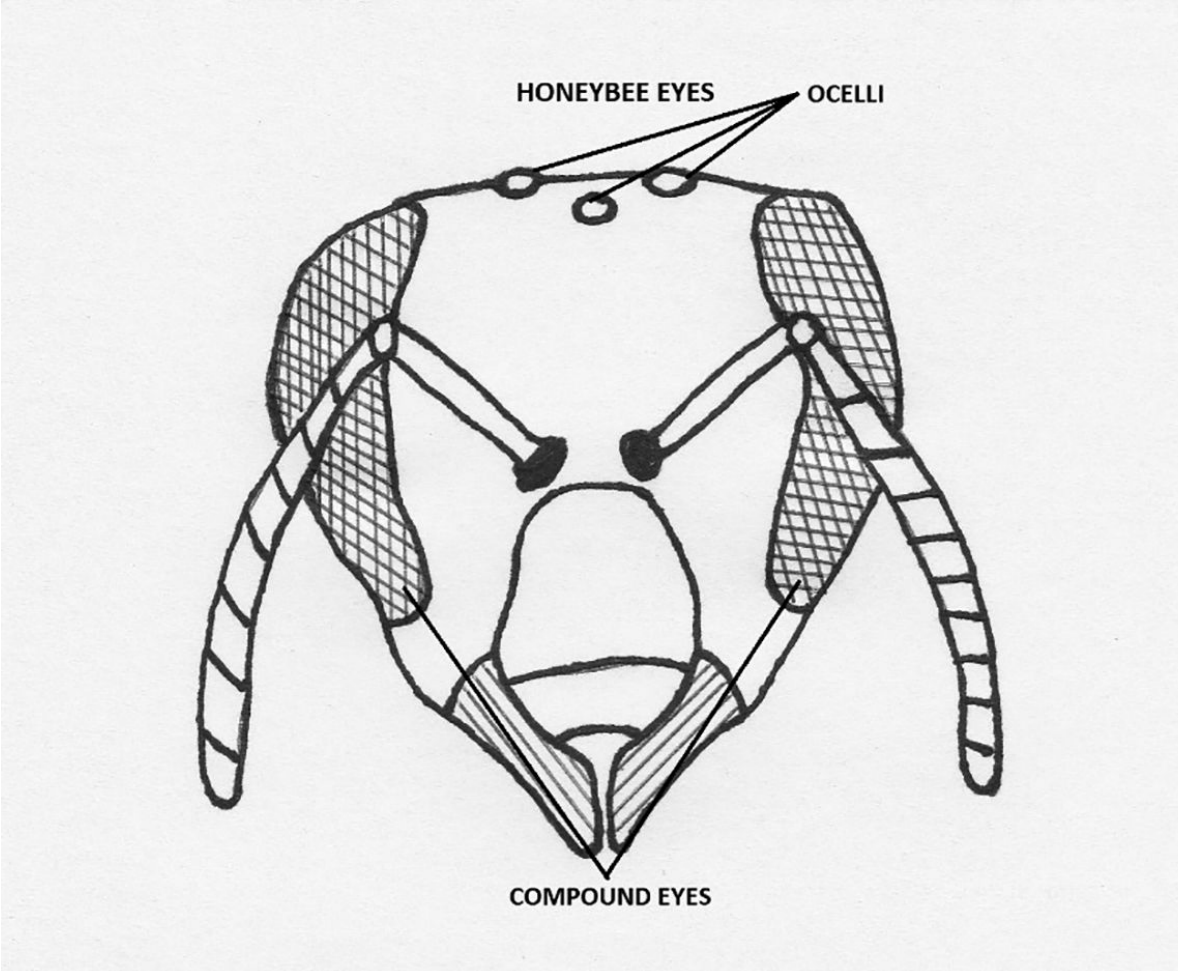
  Rayleigh Scattering

- ➔ Clouds reduces the accuracy

  Mie Scattering

Thank You!

How do bees locate themselves?



https://www.beeculture.com/bees-see-matters/

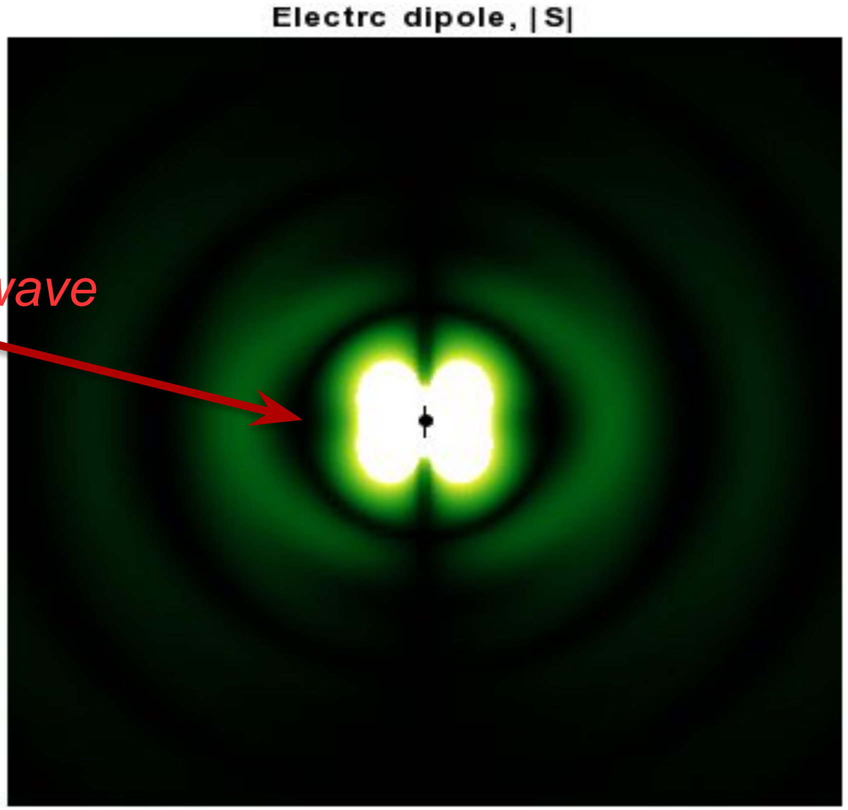
A model for radiation

The electric fields propagate radially

Time-averaged Poynting vector:

$$\langle \mathbf{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2(\theta)}{r^2} \hat{\mathbf{r}}$$

"p" wave



Total time-averaged power radiated:

$$P = \frac{\mu_0 \omega^4 p_0^2}{12\pi c}$$

David J. Griffiths, Introduction to Electrodynamics, Prentice Hall, 1999

Rayleigh x Mie

If α≪λ : Rayleigh

$$I_{Rayl} = I_0 \left(\frac{1 + cos^2\theta}{2R^2}\right)\left(\frac{2\pi}{\lambda}\right)^4 \left(\frac{n_{sph}{}^2 - 1}{n_{sph}{}^2 + 2}\right)^2 \left(\frac{a}{2}\right)^6$$

Angular dependence

Distance between particle and observer

Same dependence with frequency

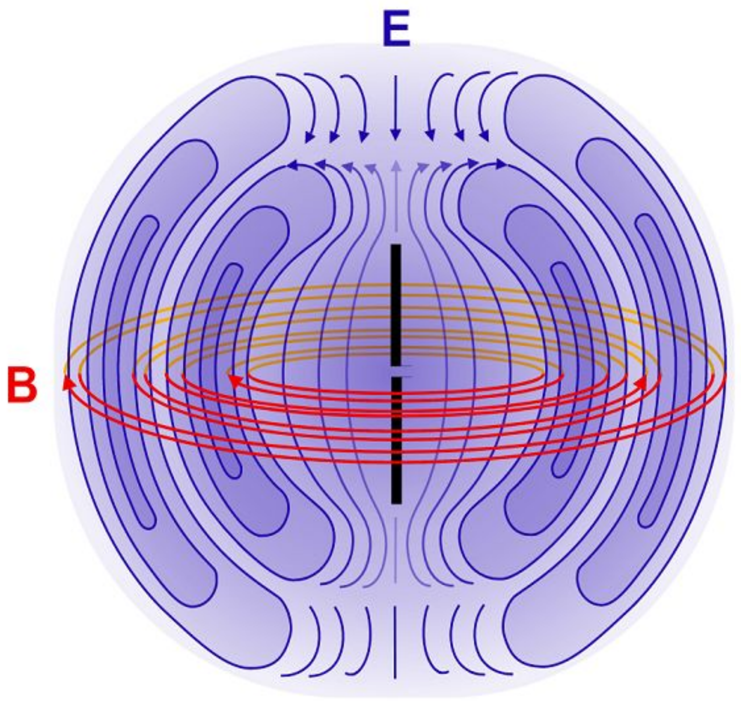
We recover dipole radiation!

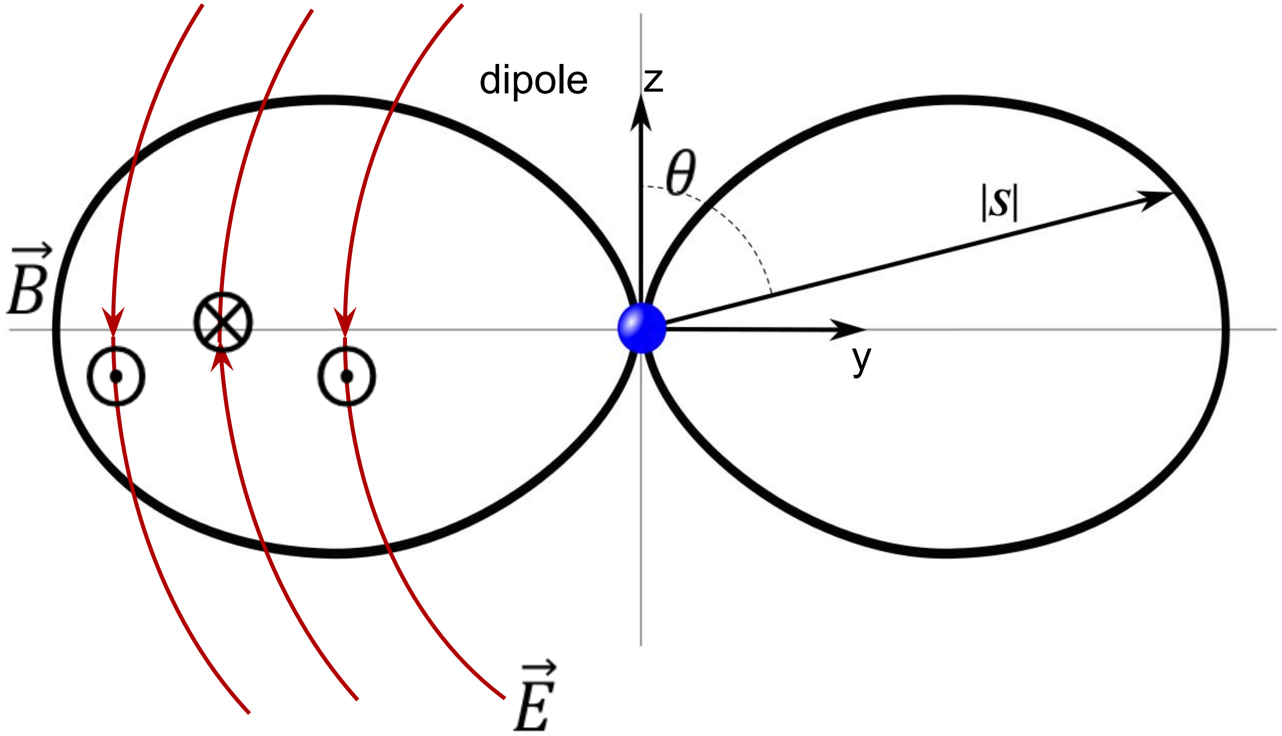
If α is close to λ: Mie

Distortion of the radiation in one direction

Rayleigh x Mie

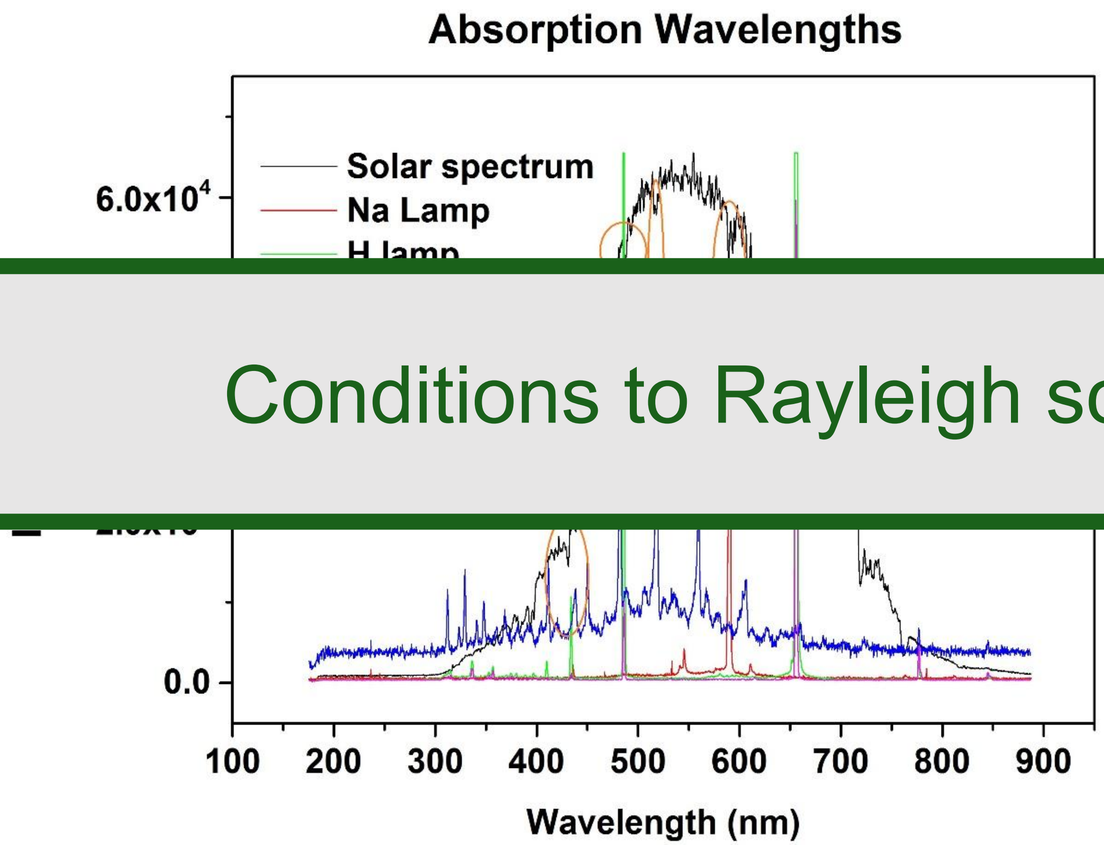
Recovering the toroidal profile of dipole radiation:





Light scatters at *xy* plane – polarization depends on our position in relation to the Sun

Atmospheric Conditions

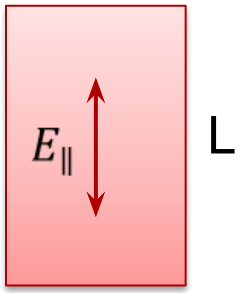


	%	Kinetic Diameter (nm)
N_2	78	0.364
H_2O	3	0.265
Ar	0.9	0.340

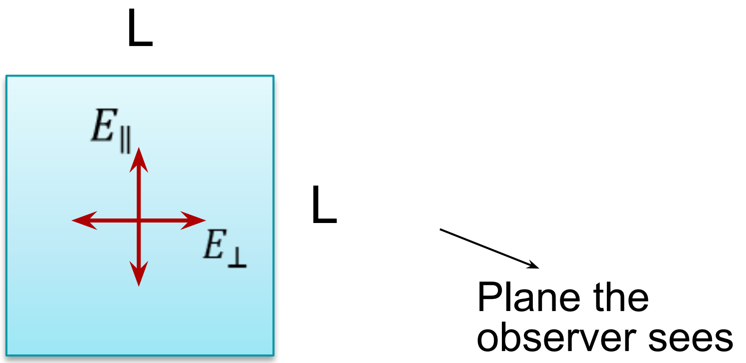
Conditions to Rayleigh scattering are satisfied!

Linear polarization

At PMP, the polarization Plane:

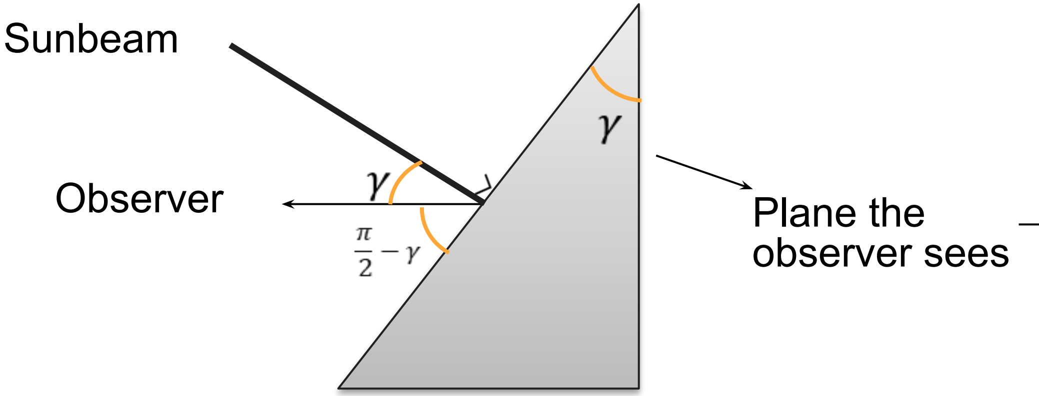


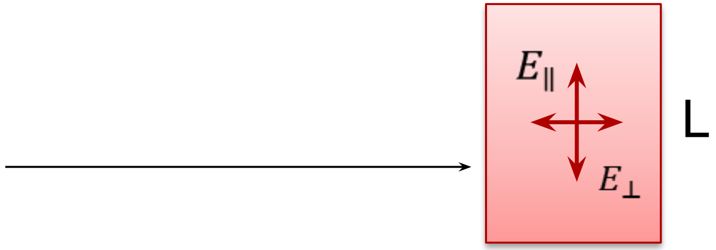
Observer's view



Plane of polarization

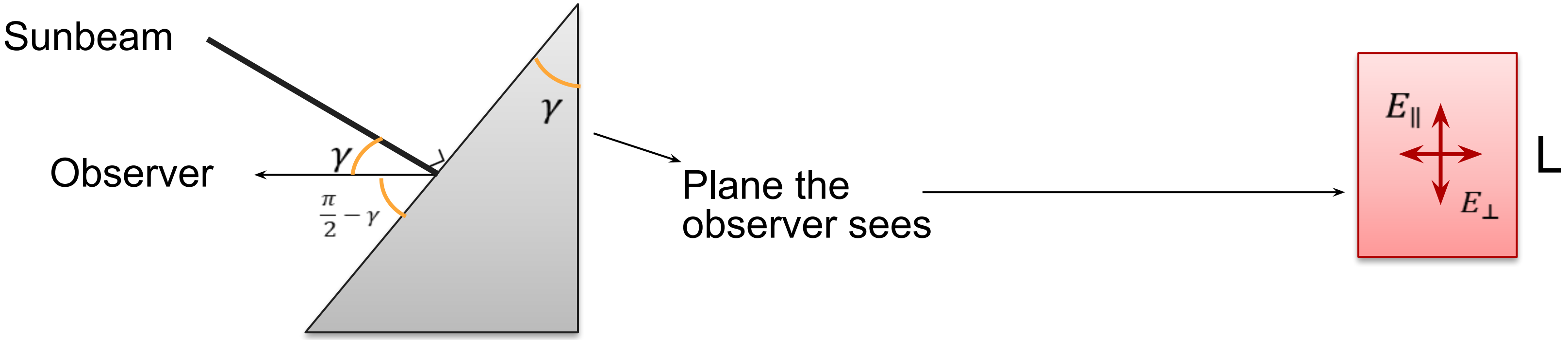
At B, the observer sees $E_{\perp}$:





Degree of polarization

At B, the observer sees $E_\perp$:



As sunlight is unpolarized at first

$$I_\parallel(\gamma) = I_0 \qquad I_\perp(\gamma) = I_0 cos^2\gamma$$

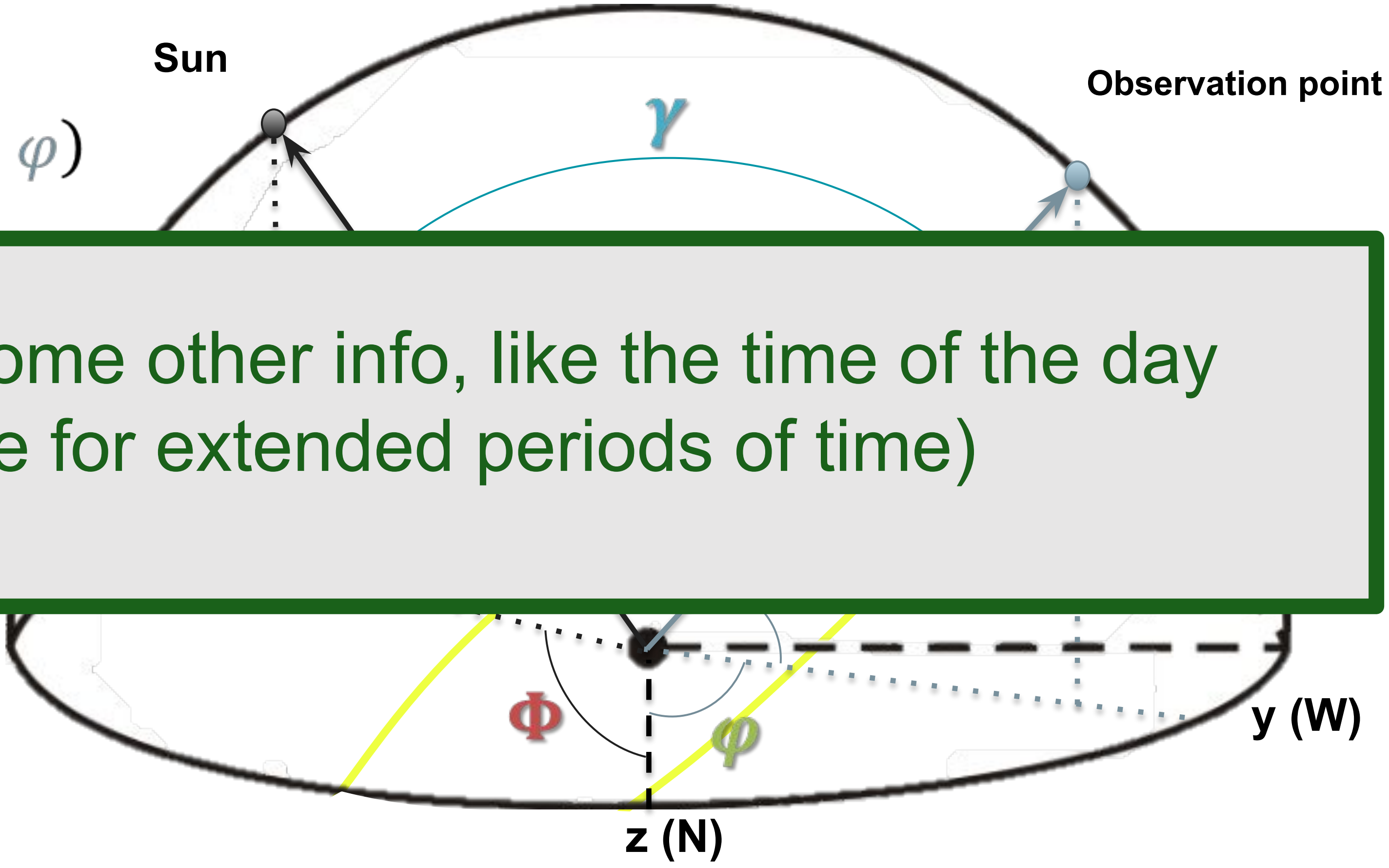
We can define the degree of polarization P(γ):

$$P(\gamma) = \frac{I_{max}(\gamma) - I_{min}(\gamma)}{I_{max}(\gamma) + I_{min}(\gamma)} \Rightarrow P = P_{max}\frac{sin^2\gamma}{1 + cos^2\gamma}$$

Navigation

Scattering Angle

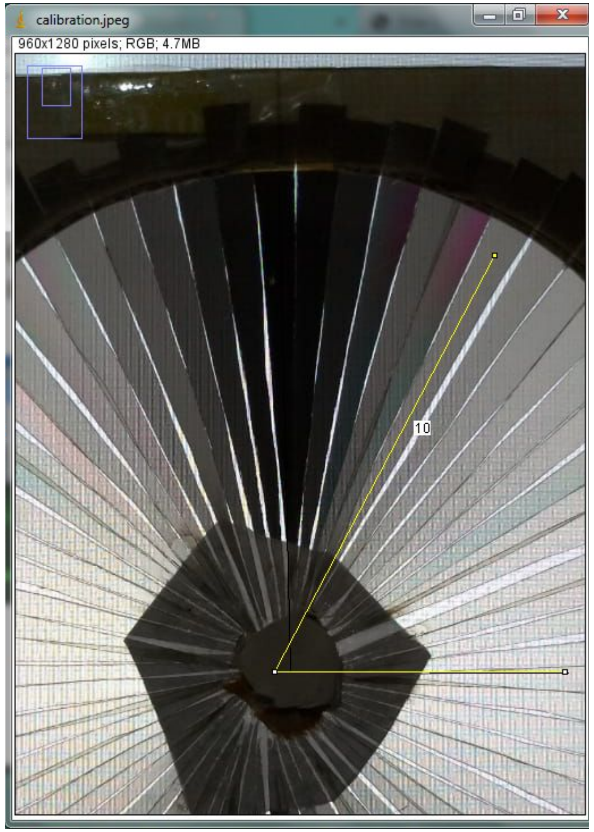
$$\cos\gamma = \cos\Theta\cos\theta\cos(\Phi - \varphi)$$
$$+ \sin\Theta\sin\theta$$

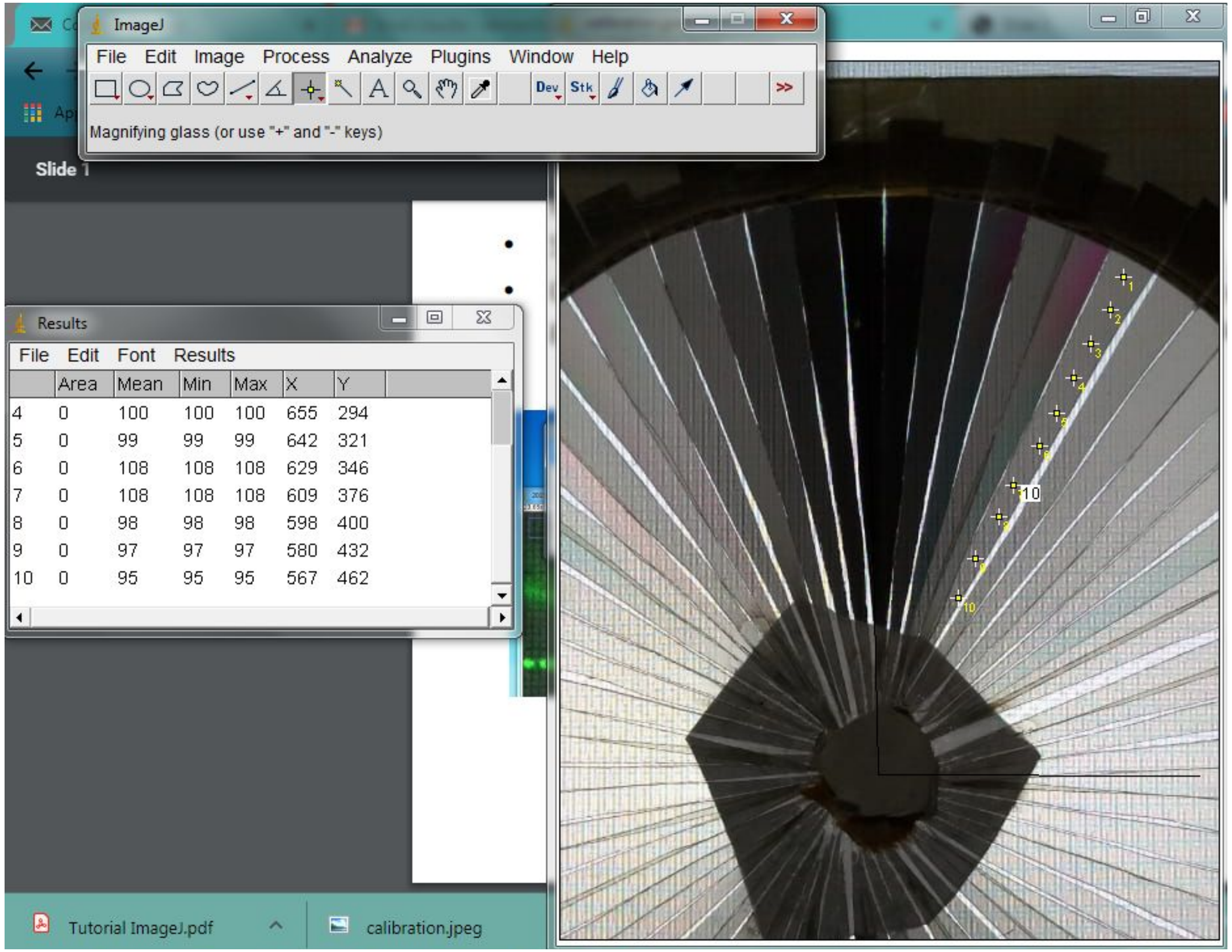


We always need some other info, like the time of the day (or measure for extended periods of time)

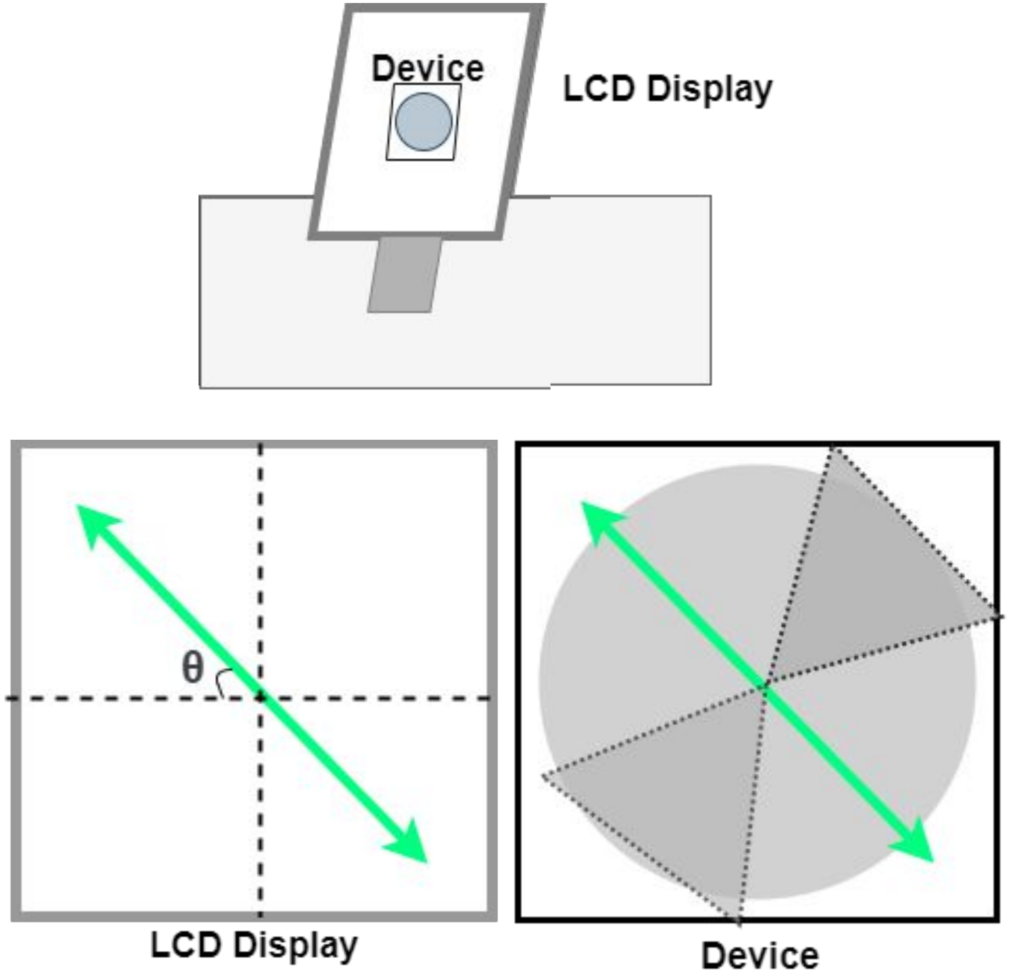
Analyse the data

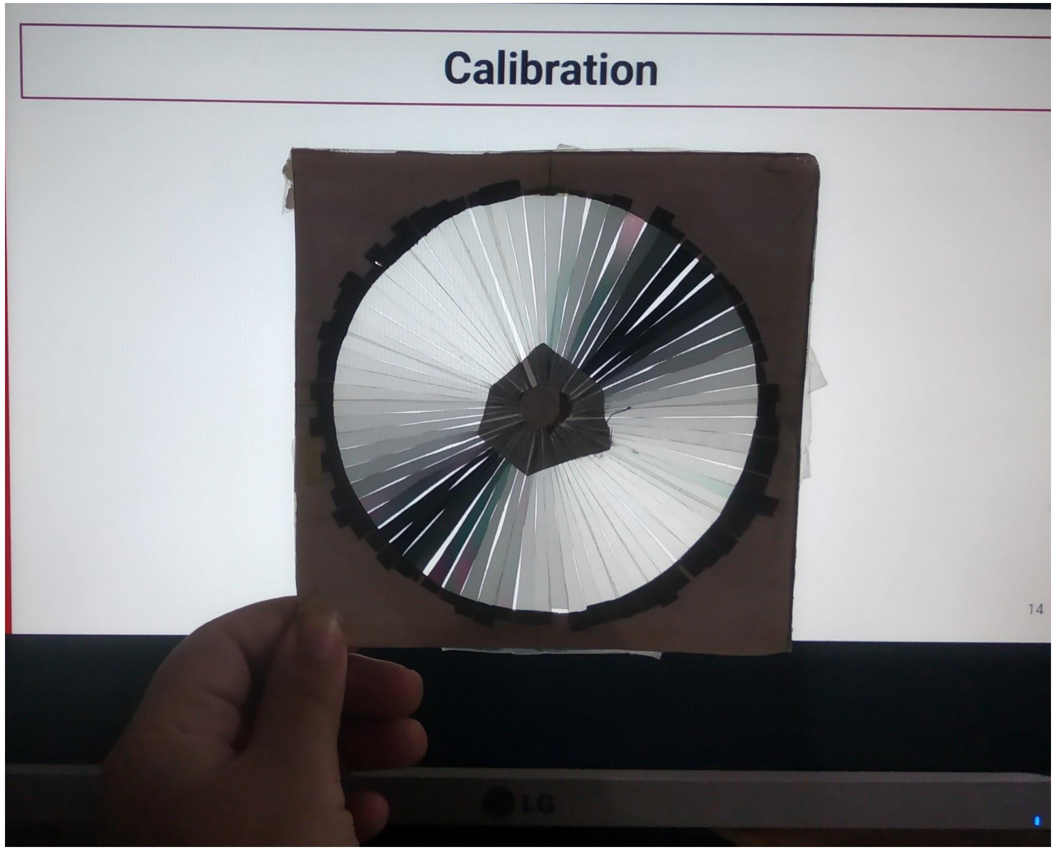
→ ImageJ



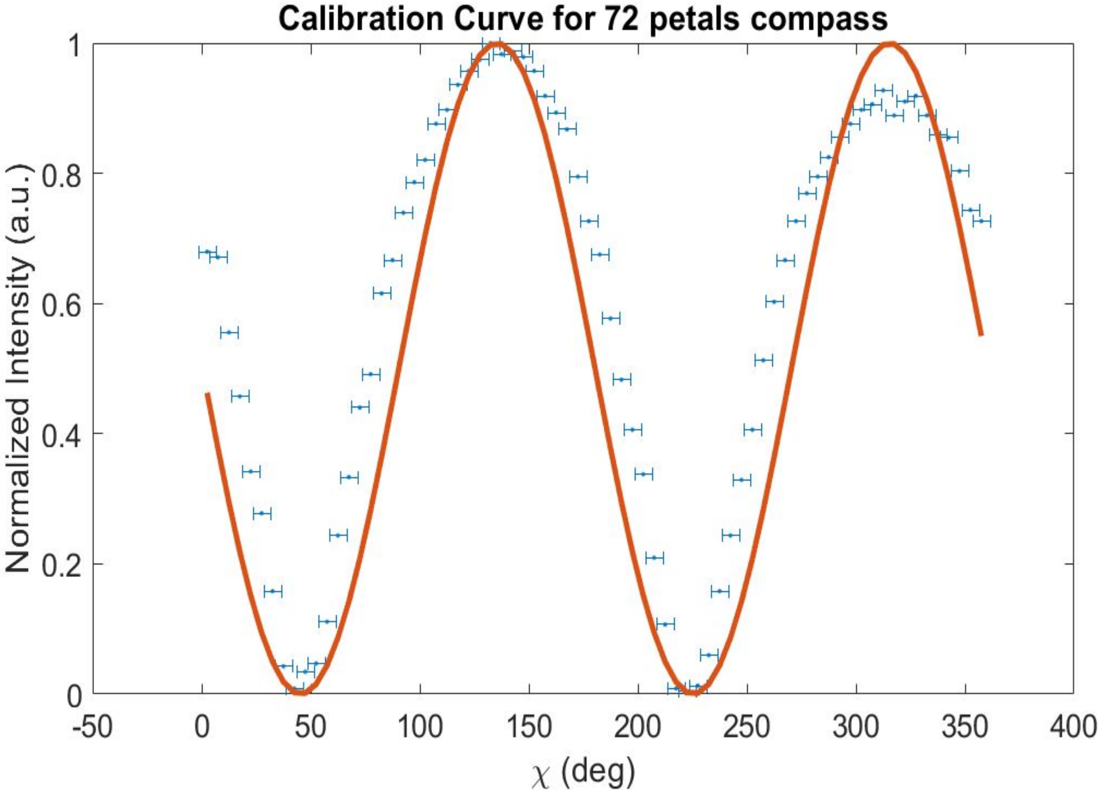


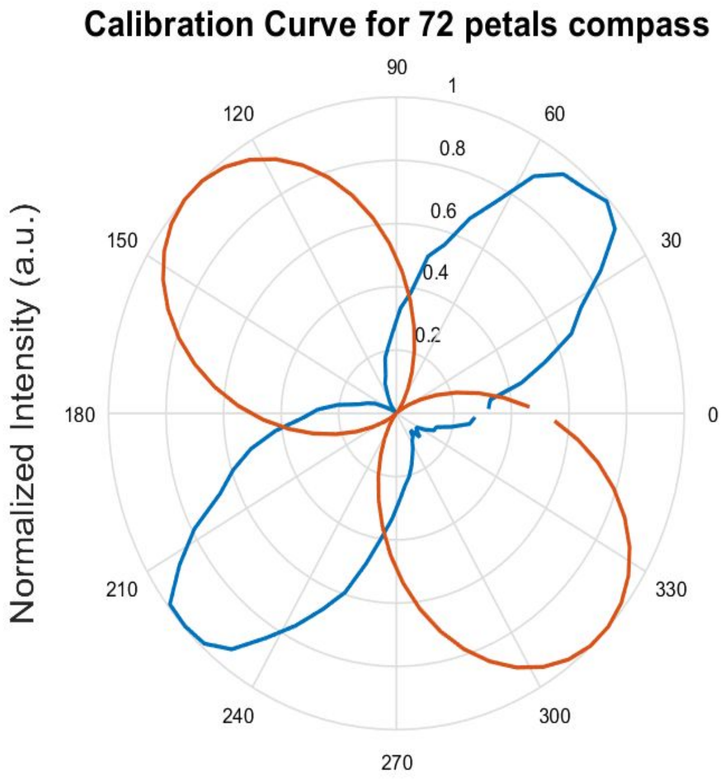
Defining angular resolution





Angular resolution - Malus Law





Deviation from Malus Law due to small misalignments/anisotropies

Intrinsic width of shadow: ~20º in 180º ("efficiency" of compass: ~89%)

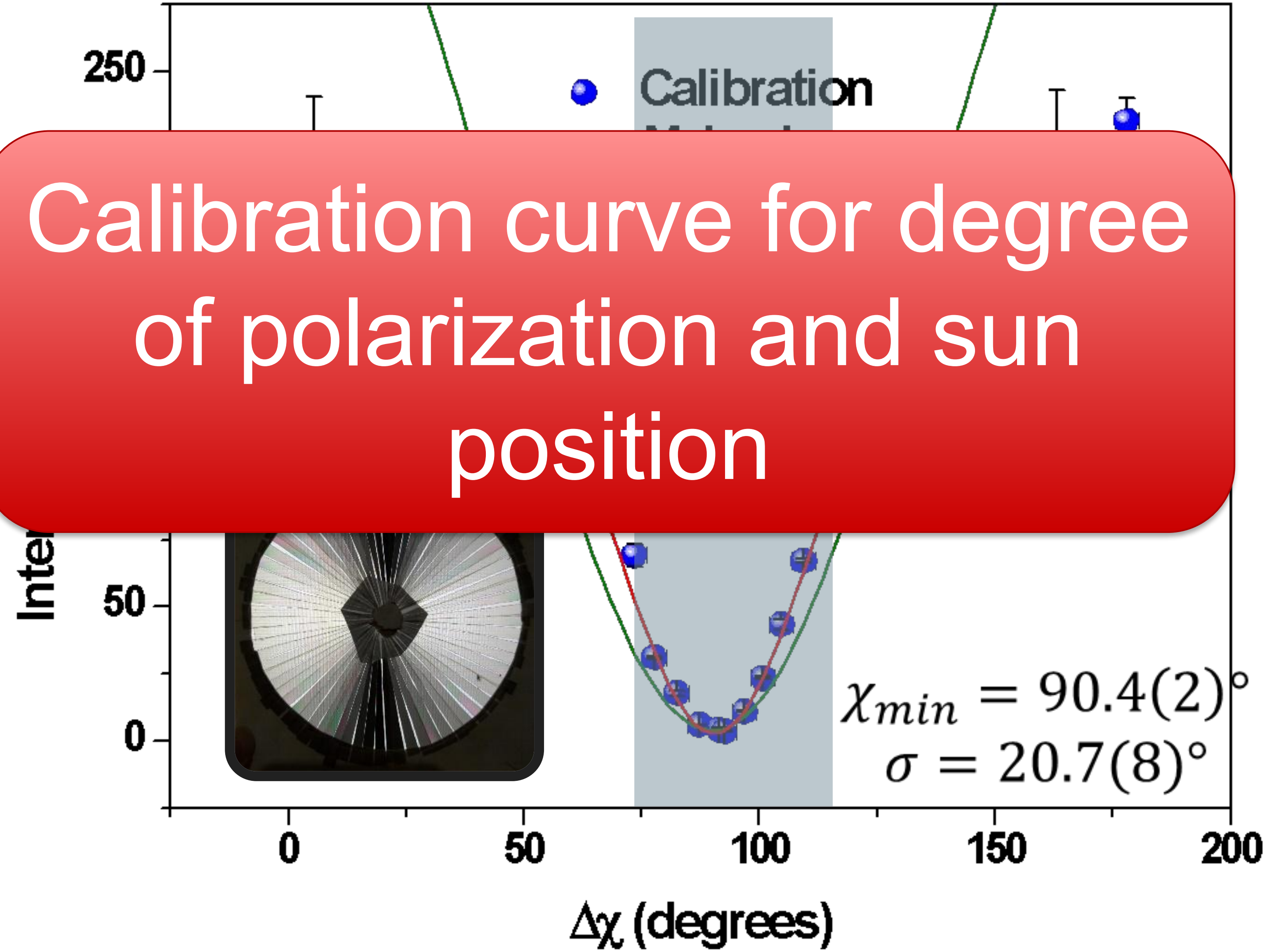
Limited resolution

Temporal evolution of the shadow

- ➔ Angle in relation to the ground ϴ= 25±1º
- ➔ Approximate geographical position:
    - Latitude: 23º 00’ 21” S
    - Longitude: 46º 50’ 20” W
- ➔ Window facing SSW (276º form north)
- ➔ Total time of acquisition 266 min (from 12 PM to sunset)
- ➔ Performed at 09/24/2019

Setup

Gaussian – Calibration



Calibration curve for degree of polarization and sun position

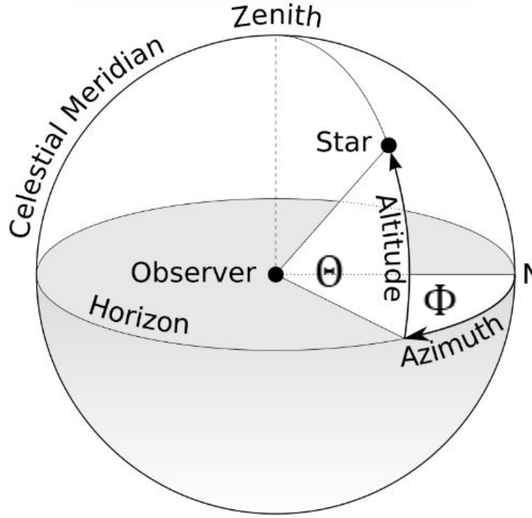
Deviation from Malus Law due to small misalignments/ anisotropies

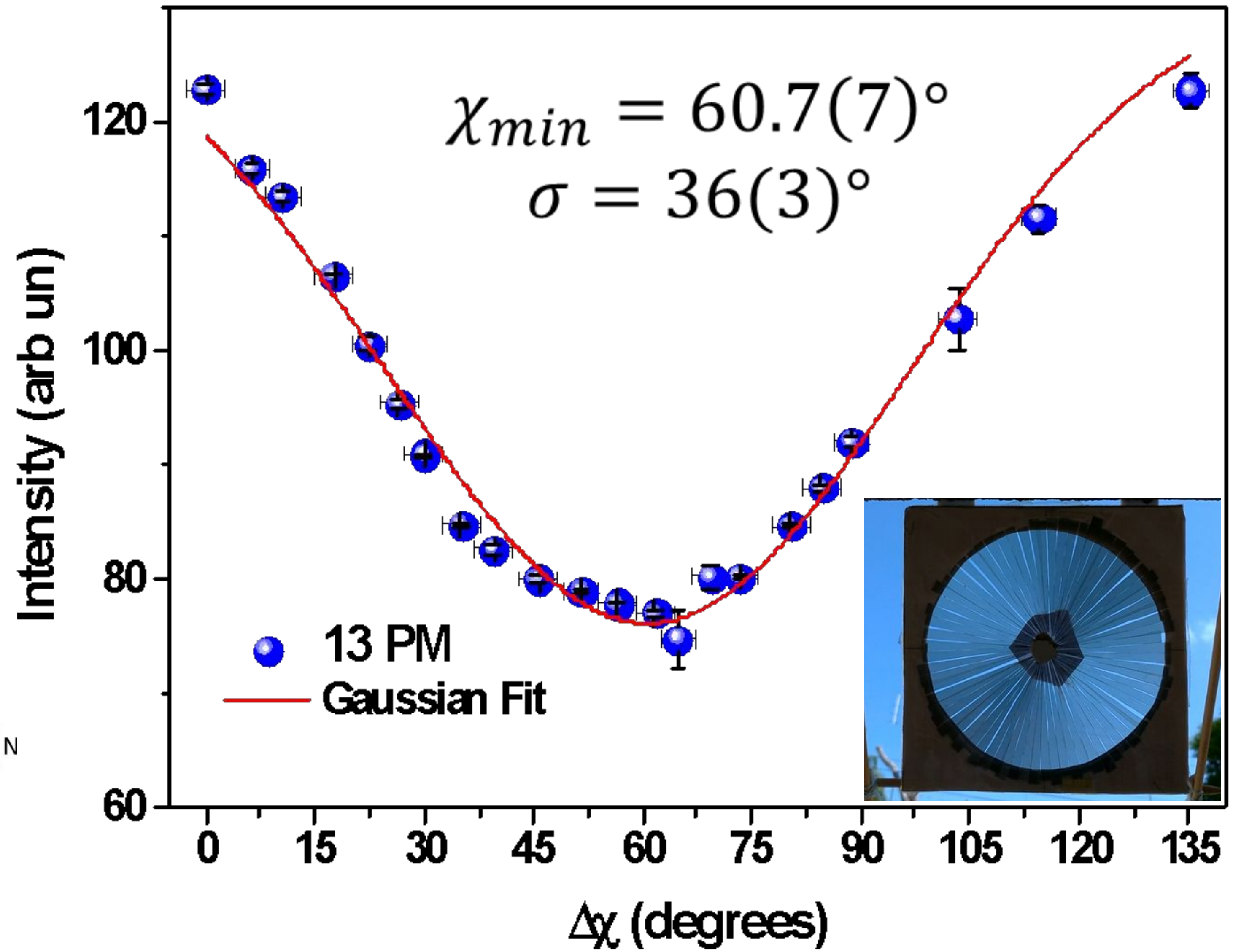
Intrinsic width of shadow: ~20º in 180º (“efficiency” of compass: ~89%)

Limited resolution

Timelapse – T_0 = 1 PM

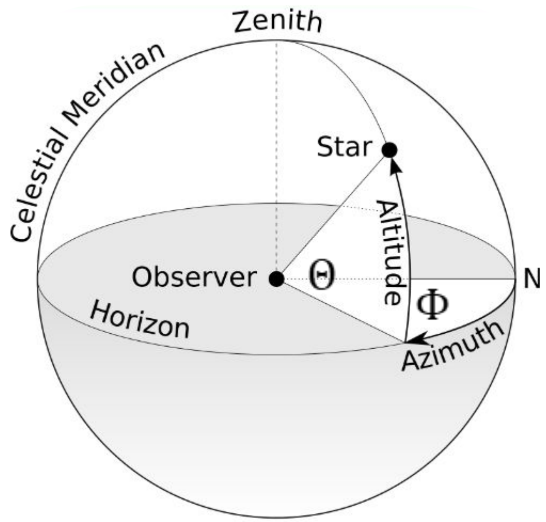
- Θ: 64°
- Φ: 321°
- Predicted angle of PMP: 154° or 26° to the ground
- $\frac{P(82^\circ)}{P_{max}} = 0.96$

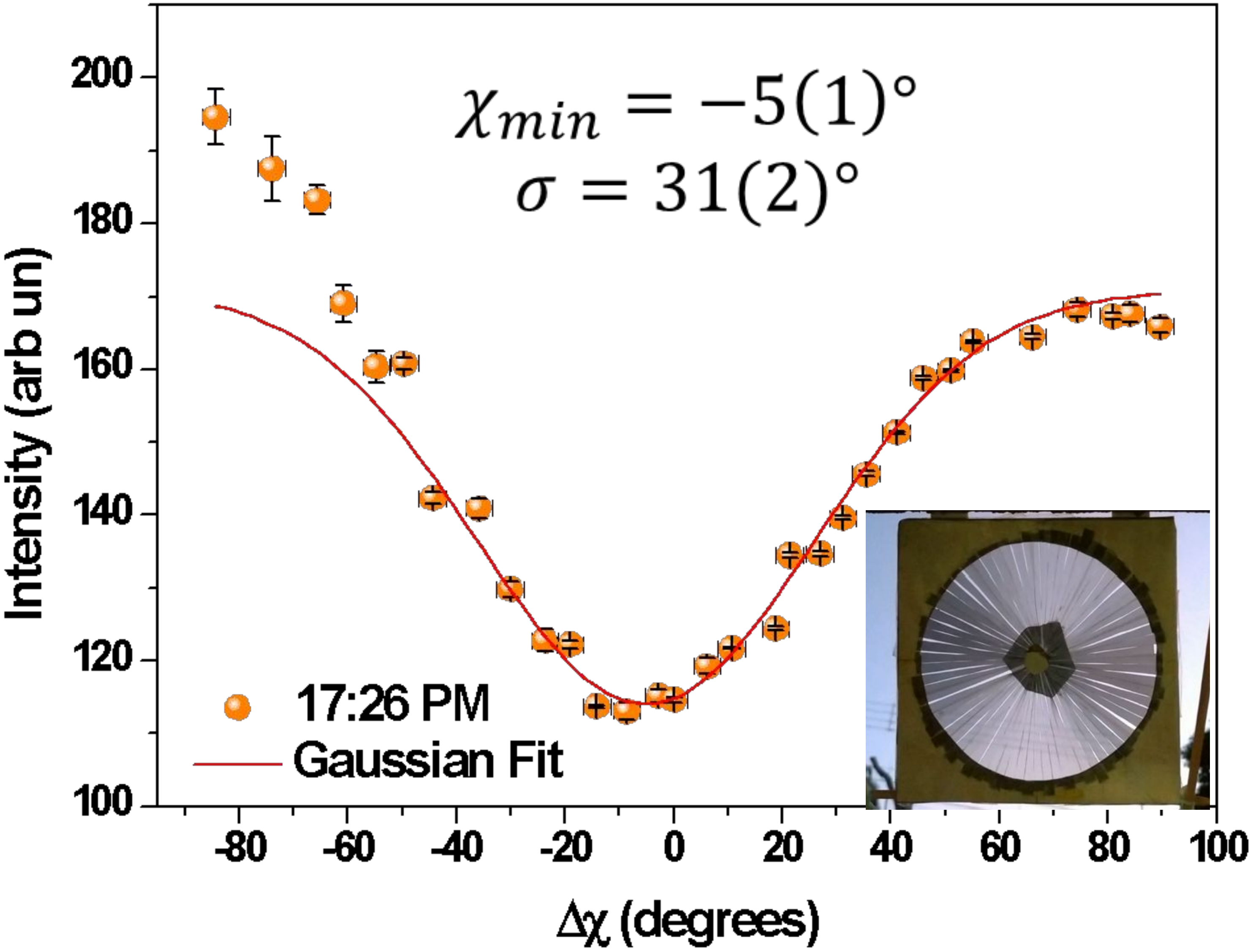




Timelapse – T_0 = 5:26 PM

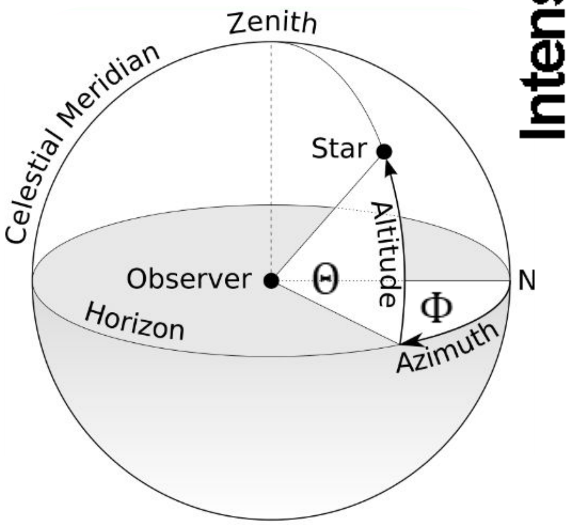
- Θ: 8°
- Φ: 273°
- Predicted angle of PMP: 98° or 82° to the ground
- $\frac{P(29^o)}{P_{max}} = 0.13$

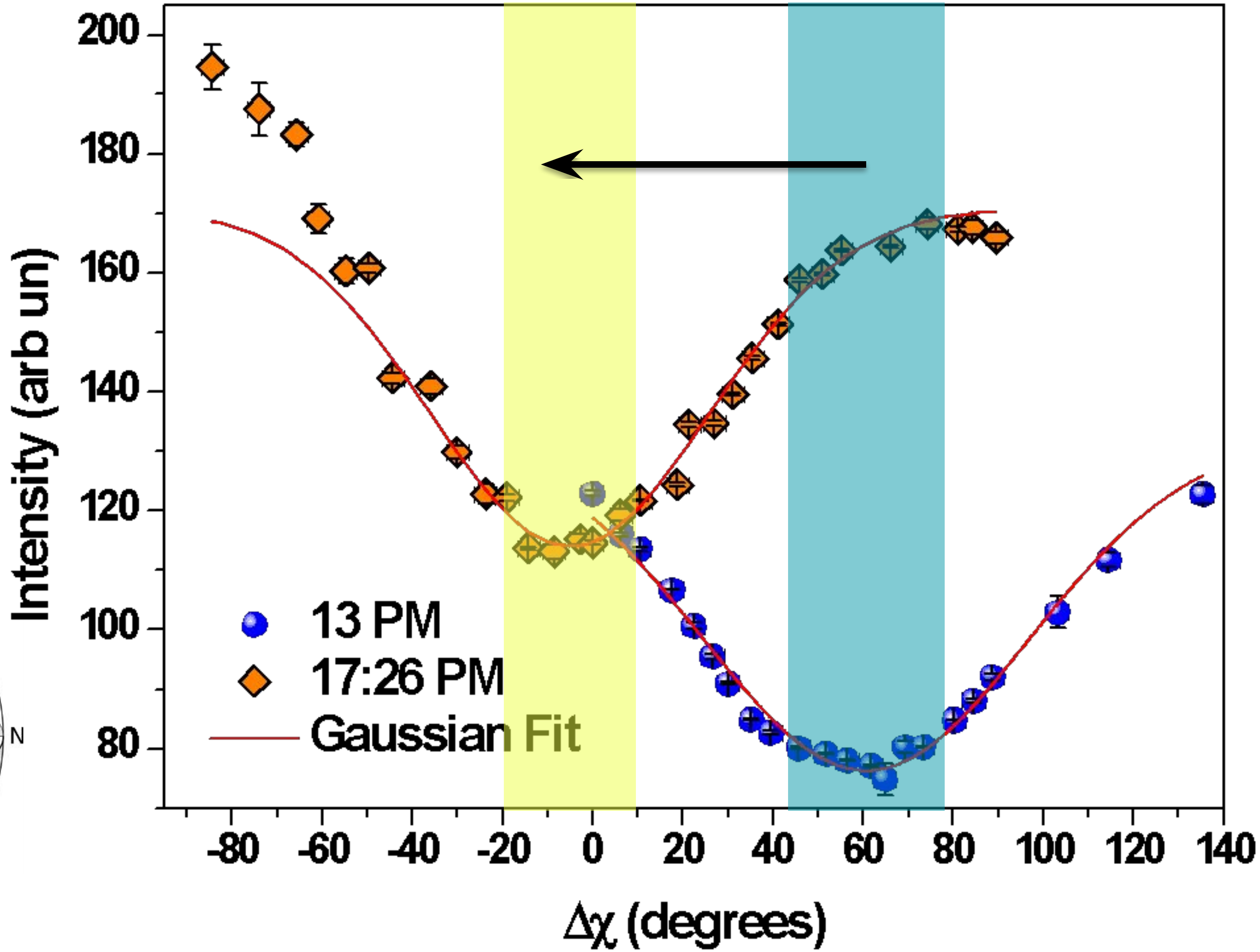




Temporal evolution of the shadow

- Shadow sharpens
- Strip rotates indicating solar position





Accuracy

$$SNR = \frac{peak\ value}{background\ value}$$

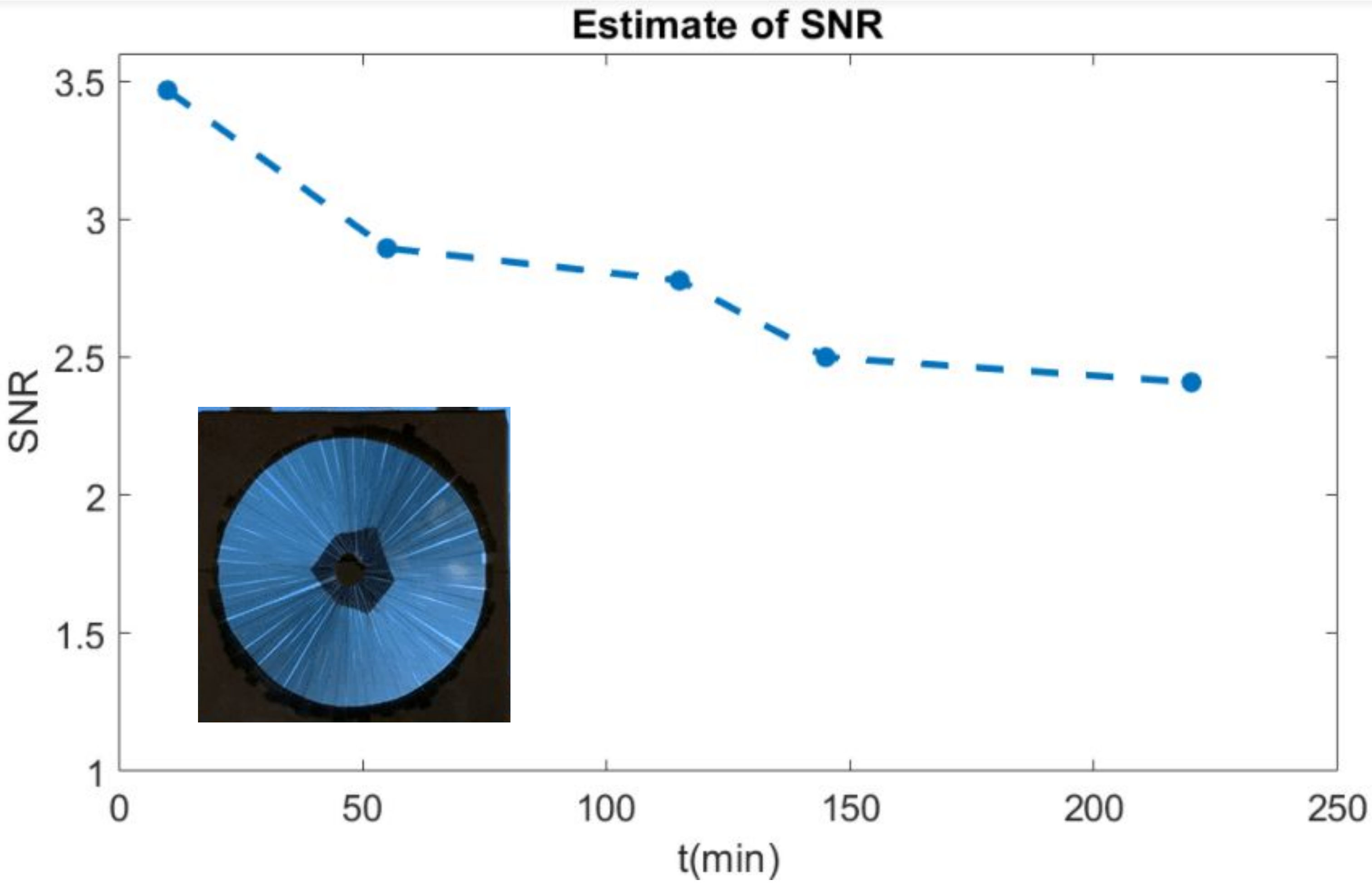
T_1 = 13:15h

T_2 = 14:00h

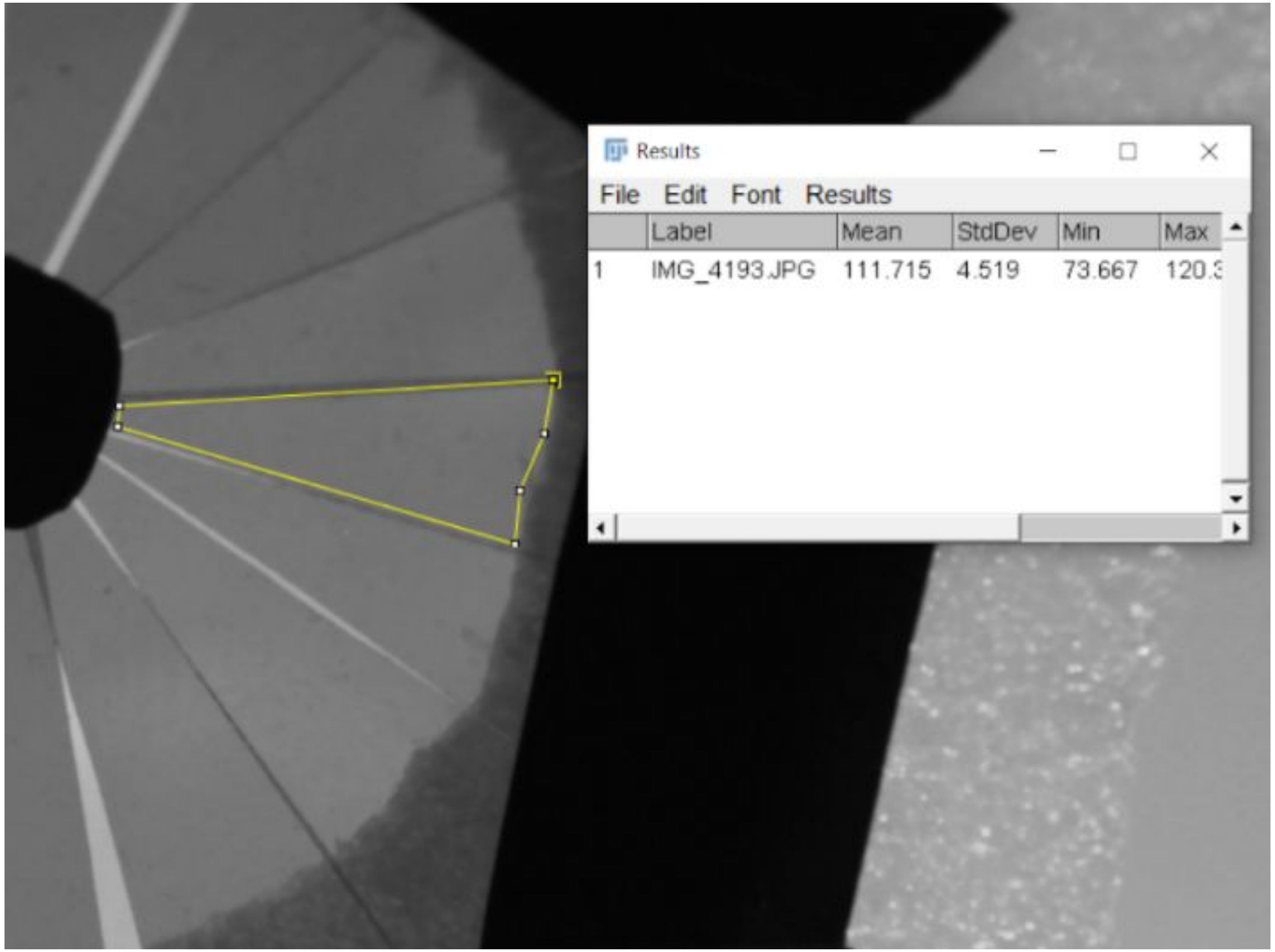
T_6 = 15:00h

T_8 = 15:30h

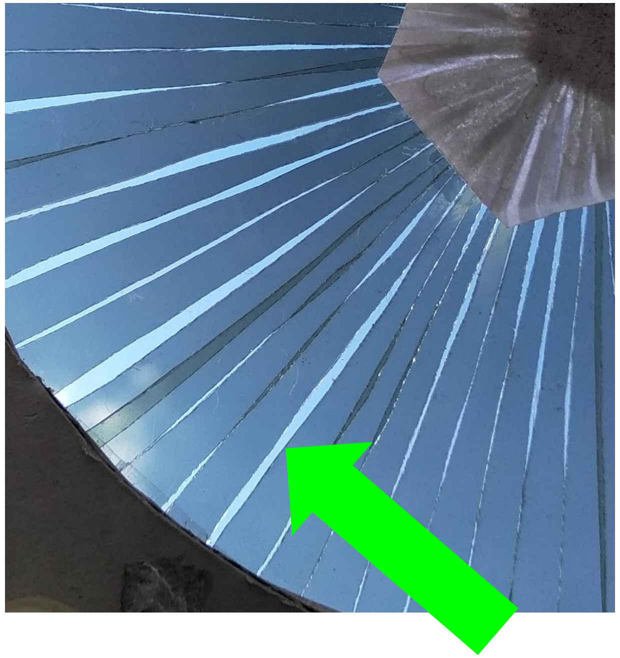
T_9 = 16:45h



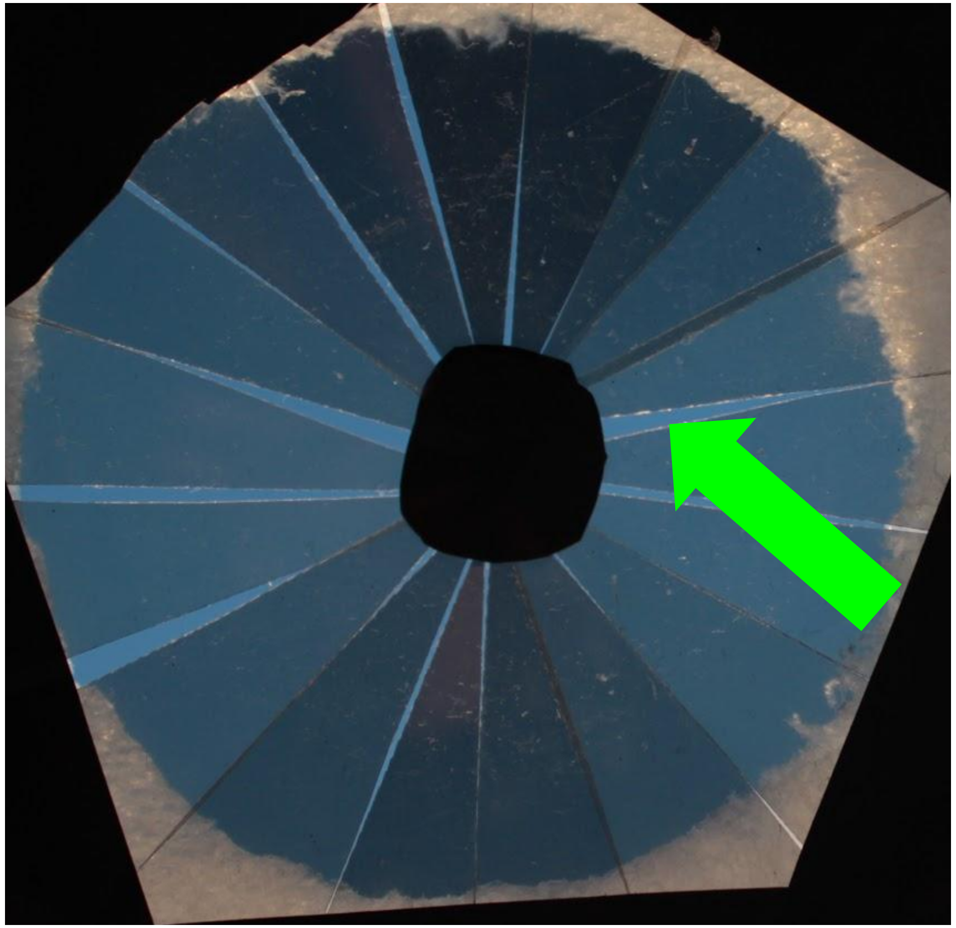
Error Analysis - Intensity



Error Analysis - Position

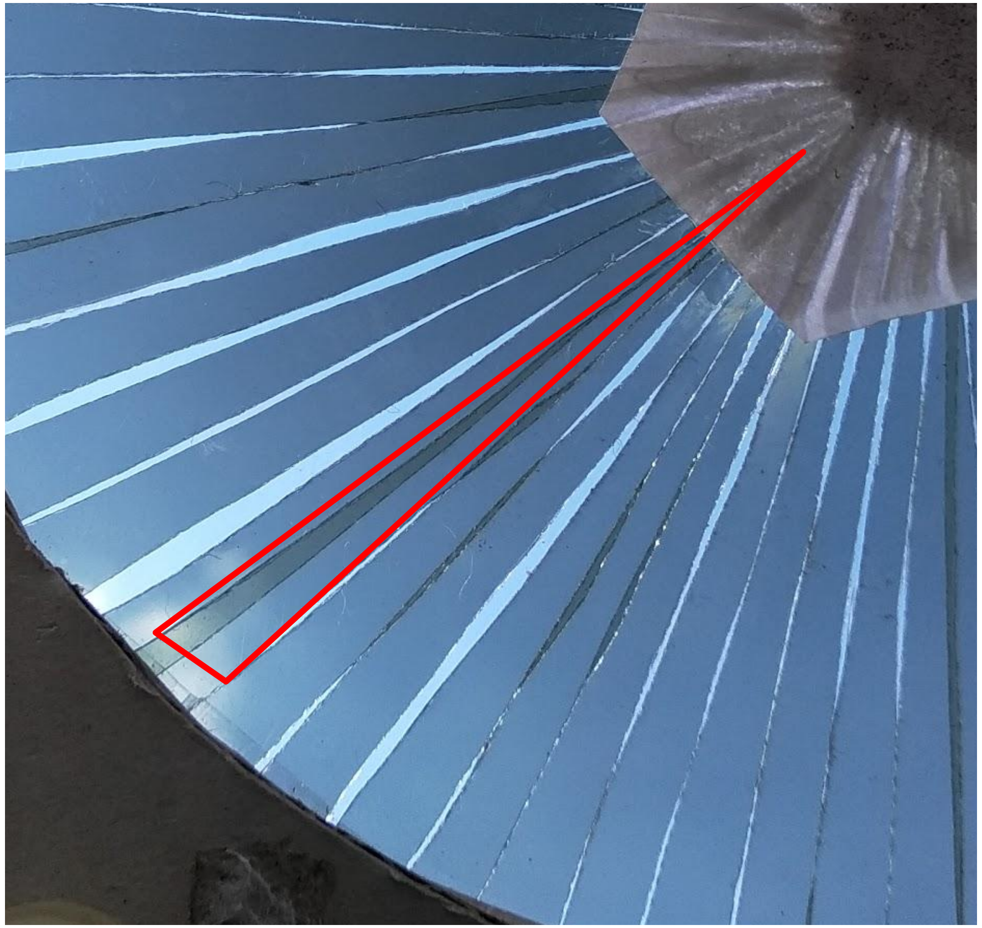


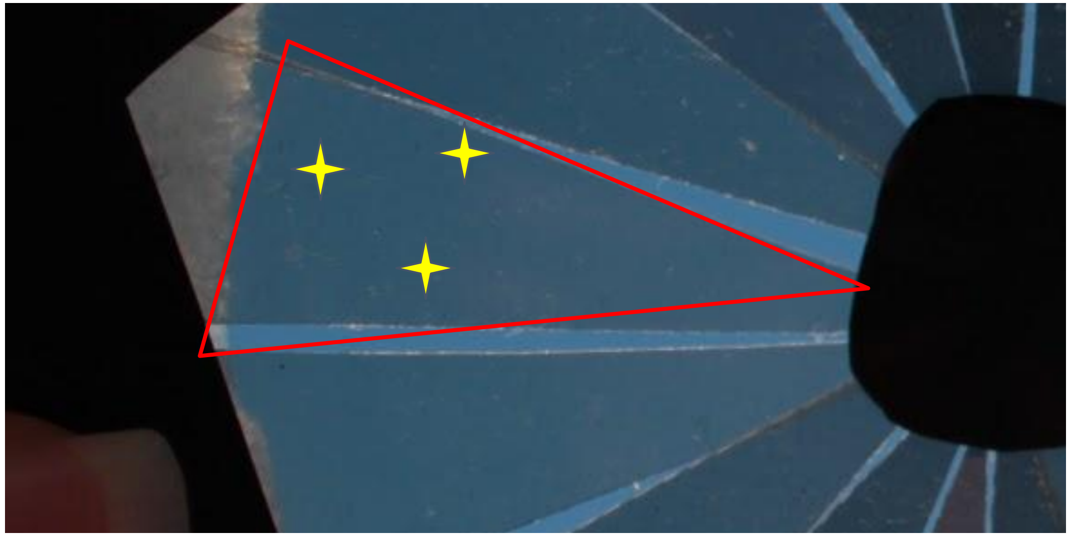
土5º



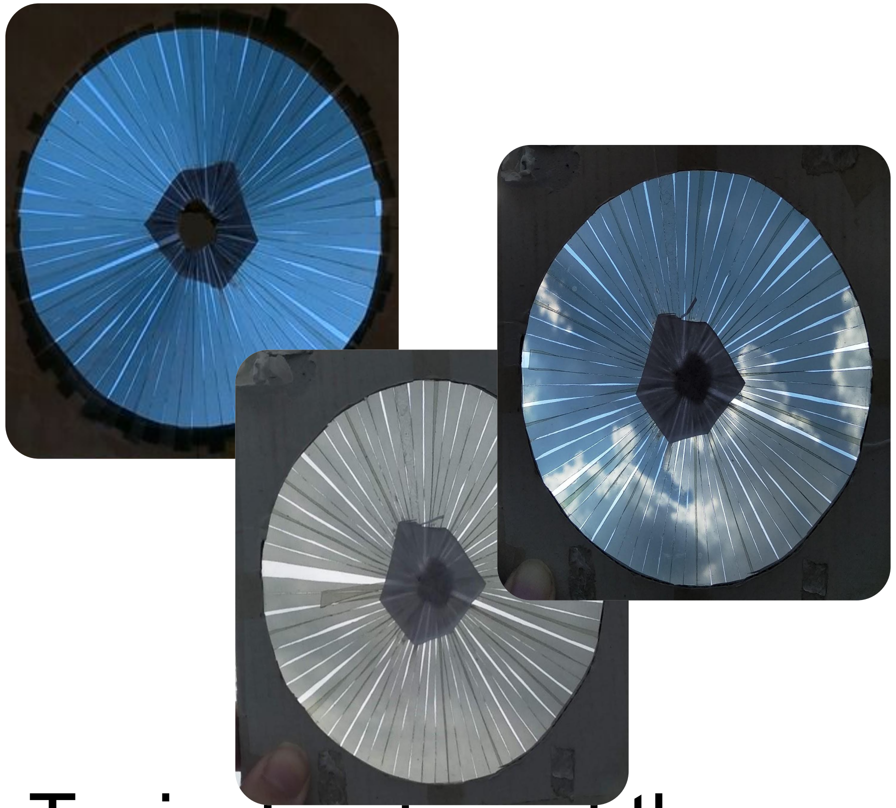
土8º

Error Analysis - Position





Atmospheric conditions - Cloudy weather



Typical radius of the droplets ~ 10^2 nm ~ λ

Conditions to Rayleigh scattering are not satisfied

⇩

Mie Scattering dominates

⇩

No pattern formed

References

Wehner, R.D., 1976. Polarized-light navigation by insects. *Scientific American*, *235*(1), pp.106-115.

Rossel, S., 1993. Navigation by bees using polarized skylight. Comparative Biochemistry and Physiology Part A: Physiology, 104(4), pp.695-708.