5. Whirlpool in a bottle

Russia, Voronezh

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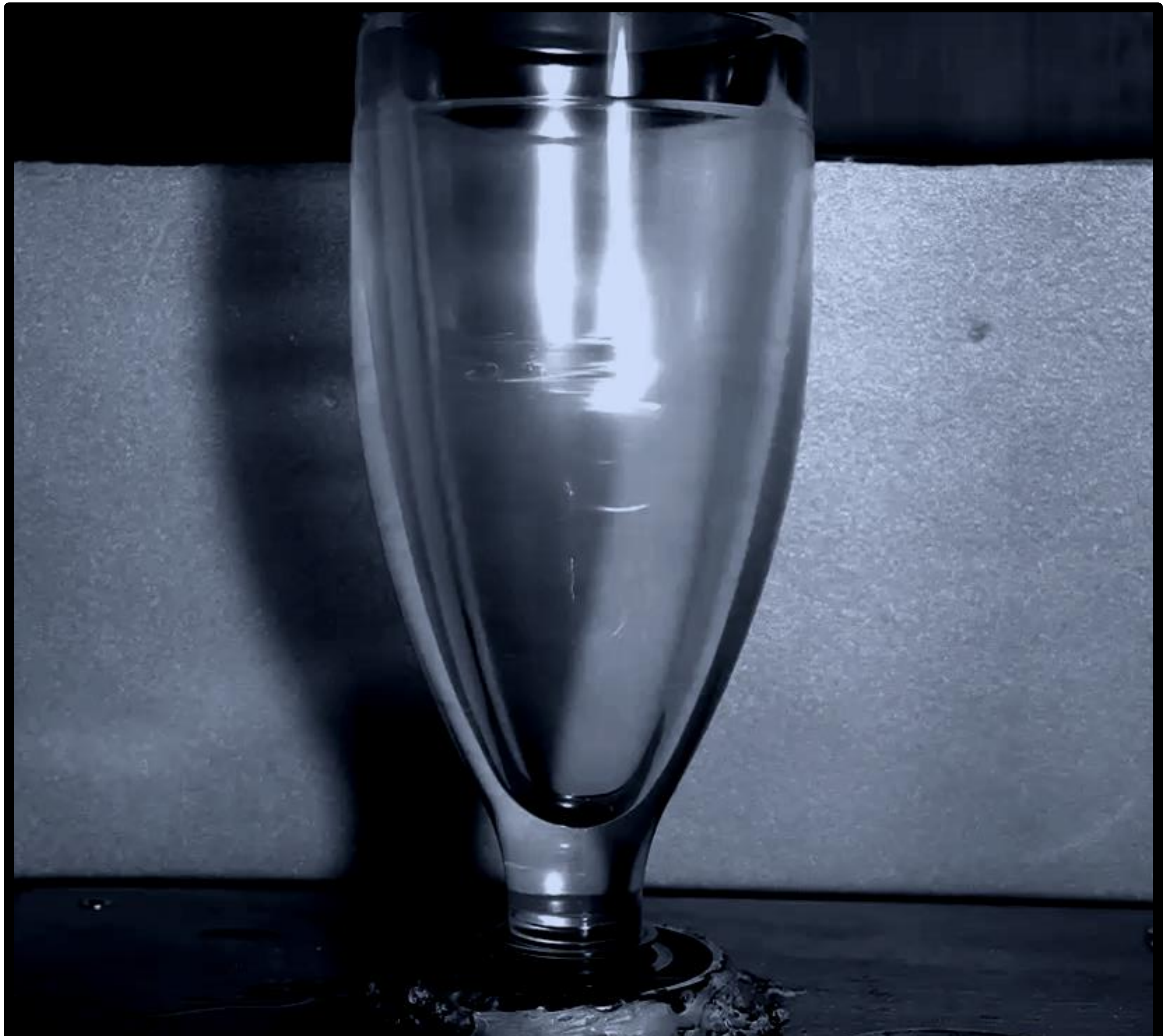
(Text of each slide is under the slide in notes)

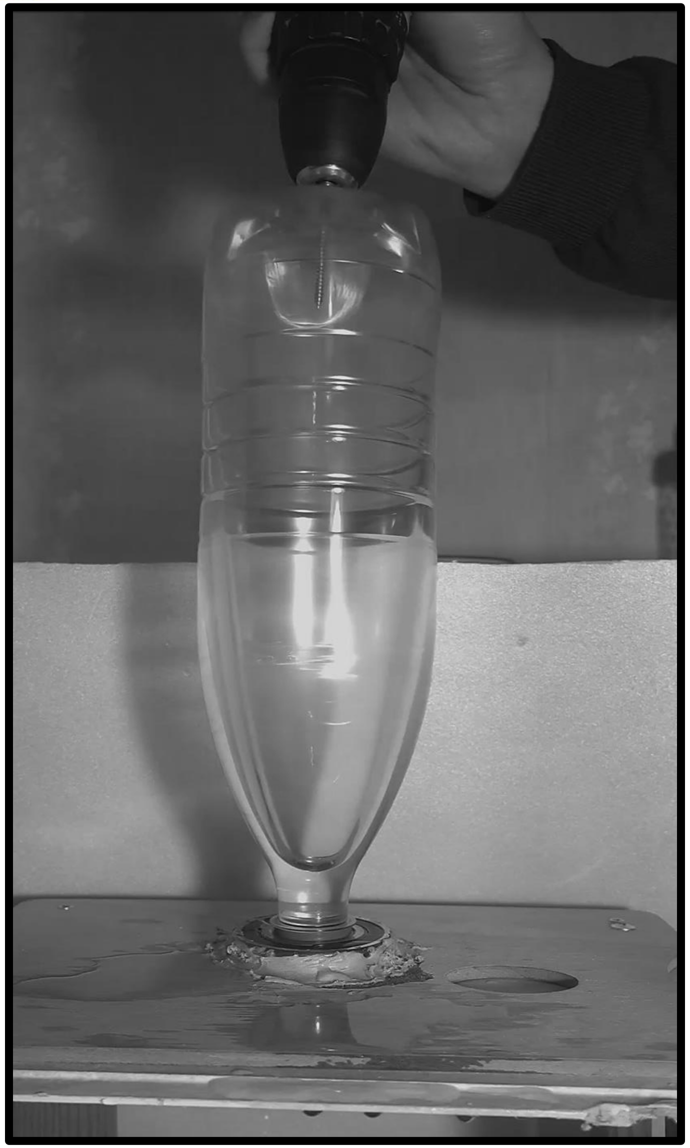
The task

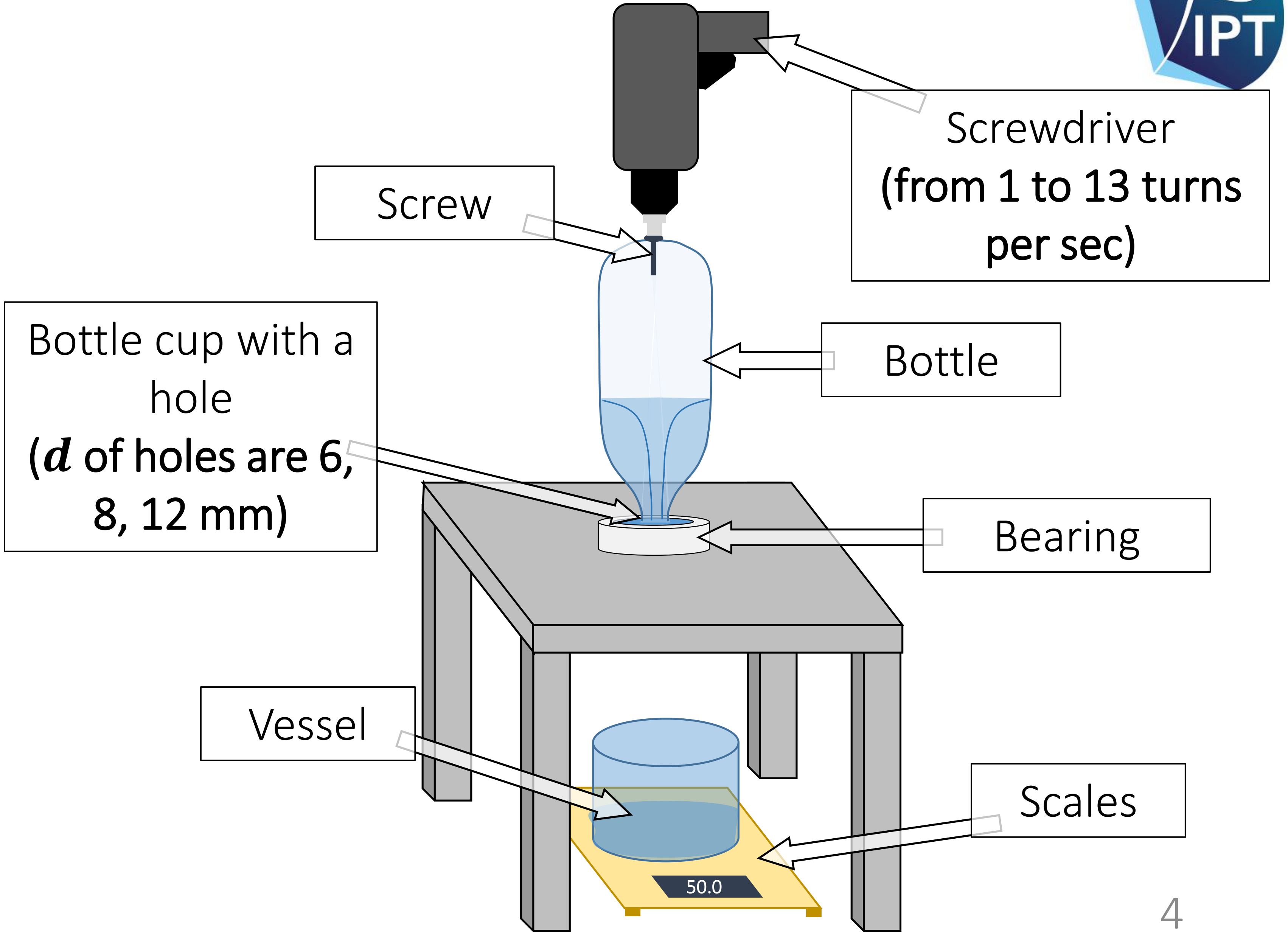
When an open bottle of water is turned upside down and slightly rotated, a whirlpool is formed. What are its characteristics? How fast can the bottle be emptied that way? What will change if the bottle is filled with sand instead?

Observation



Experimental setup



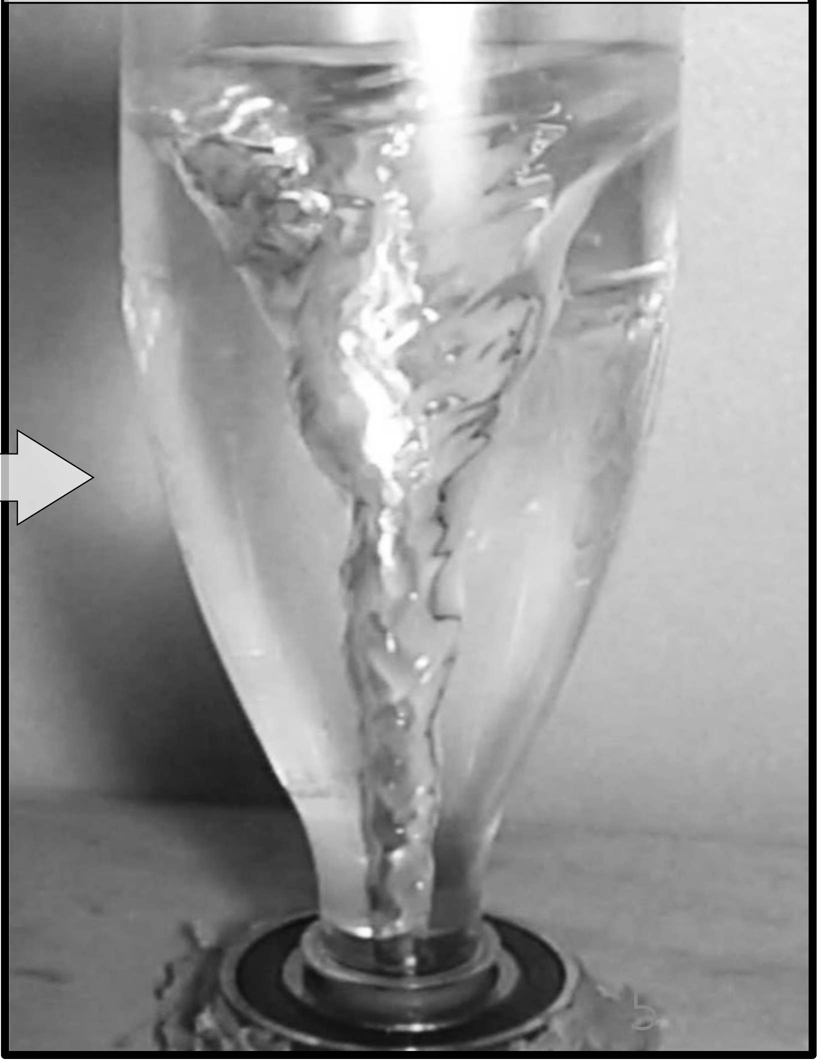


Stages of the process

1. Twist of the water in a bottle

2. Bubbles' formation

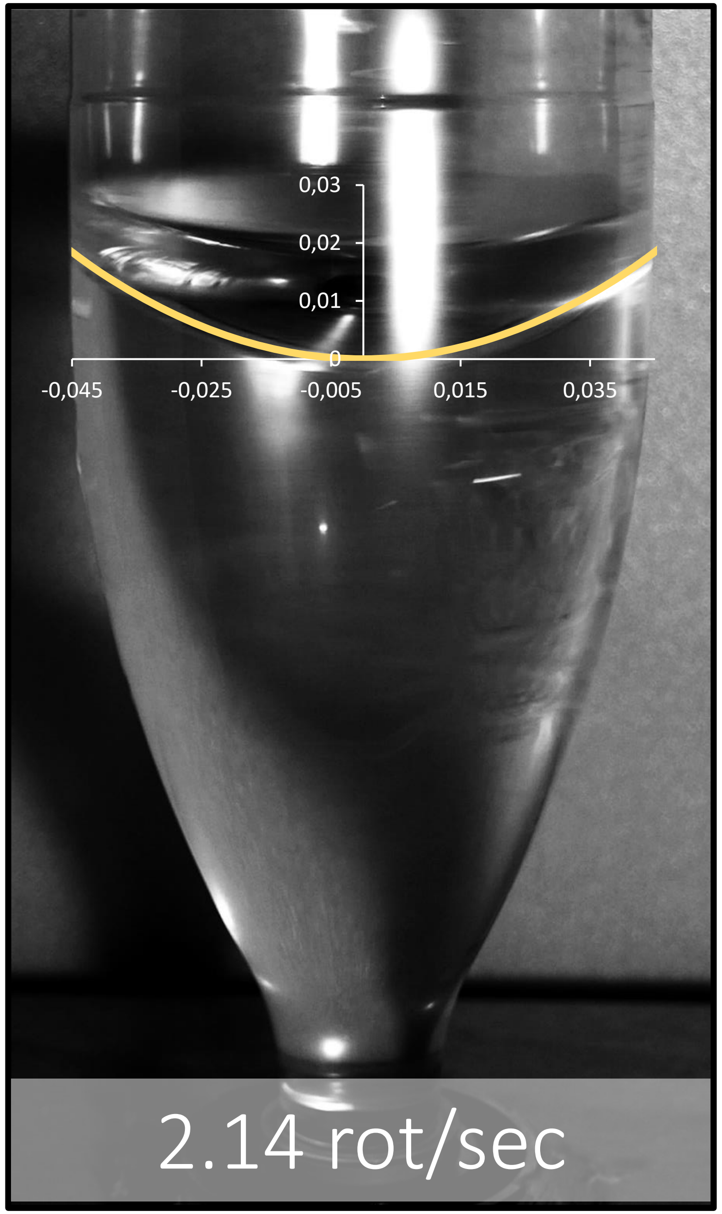
3. Formation of stable funnel

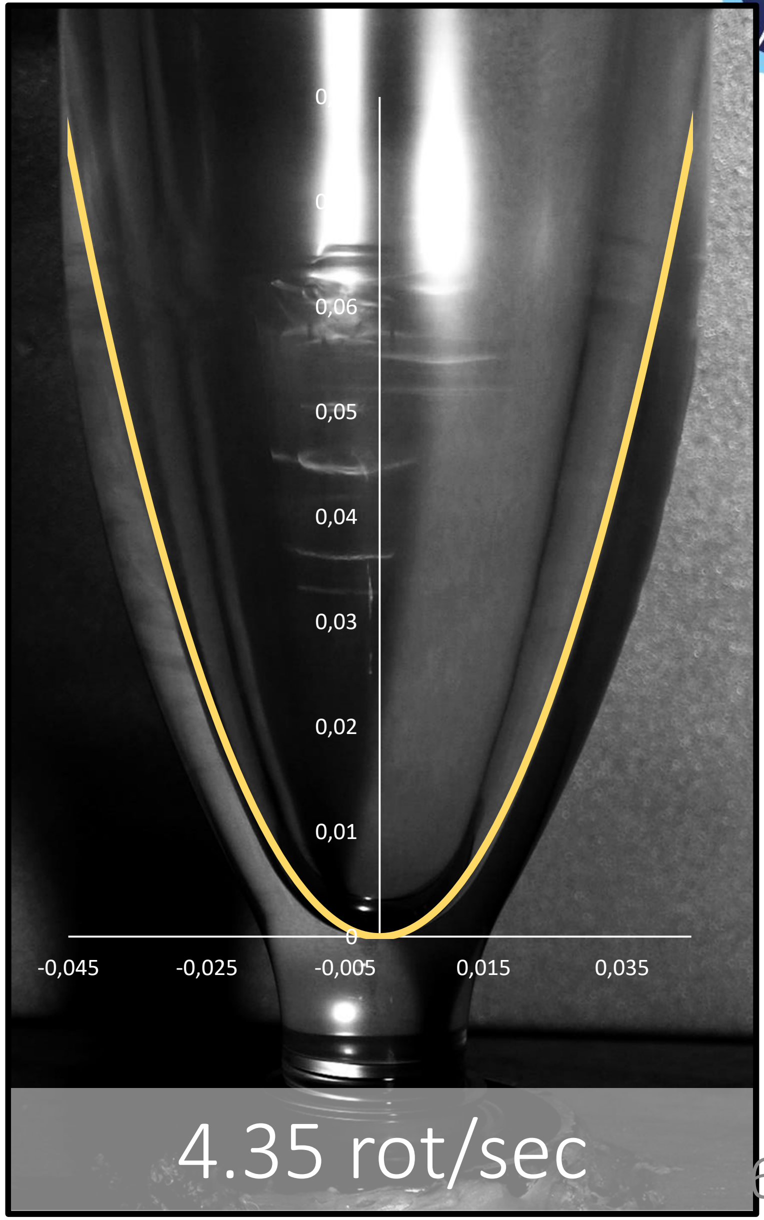


During the twist

Shape of the free surface:

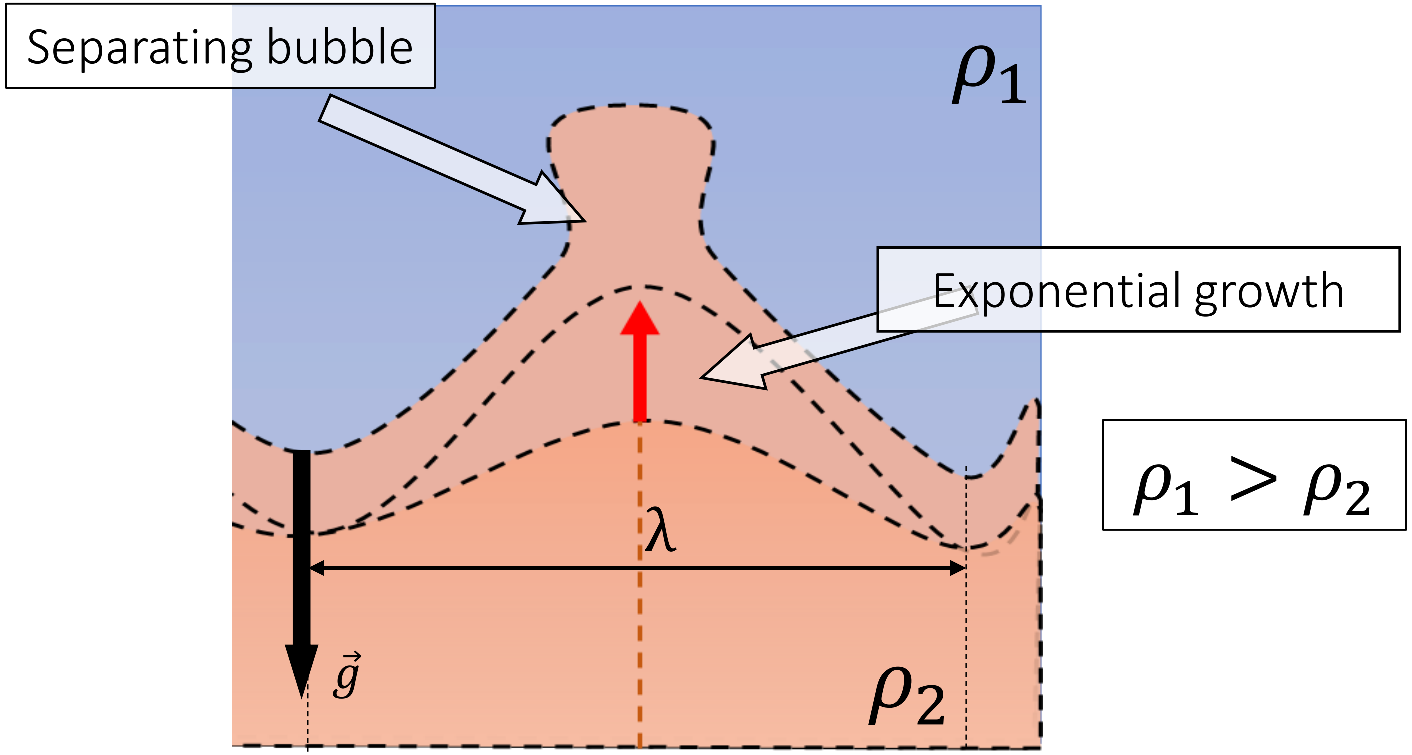
$$z(r) = \frac{\omega^2 r^2}{2g}$$





Bubbles’ formation => water runoff

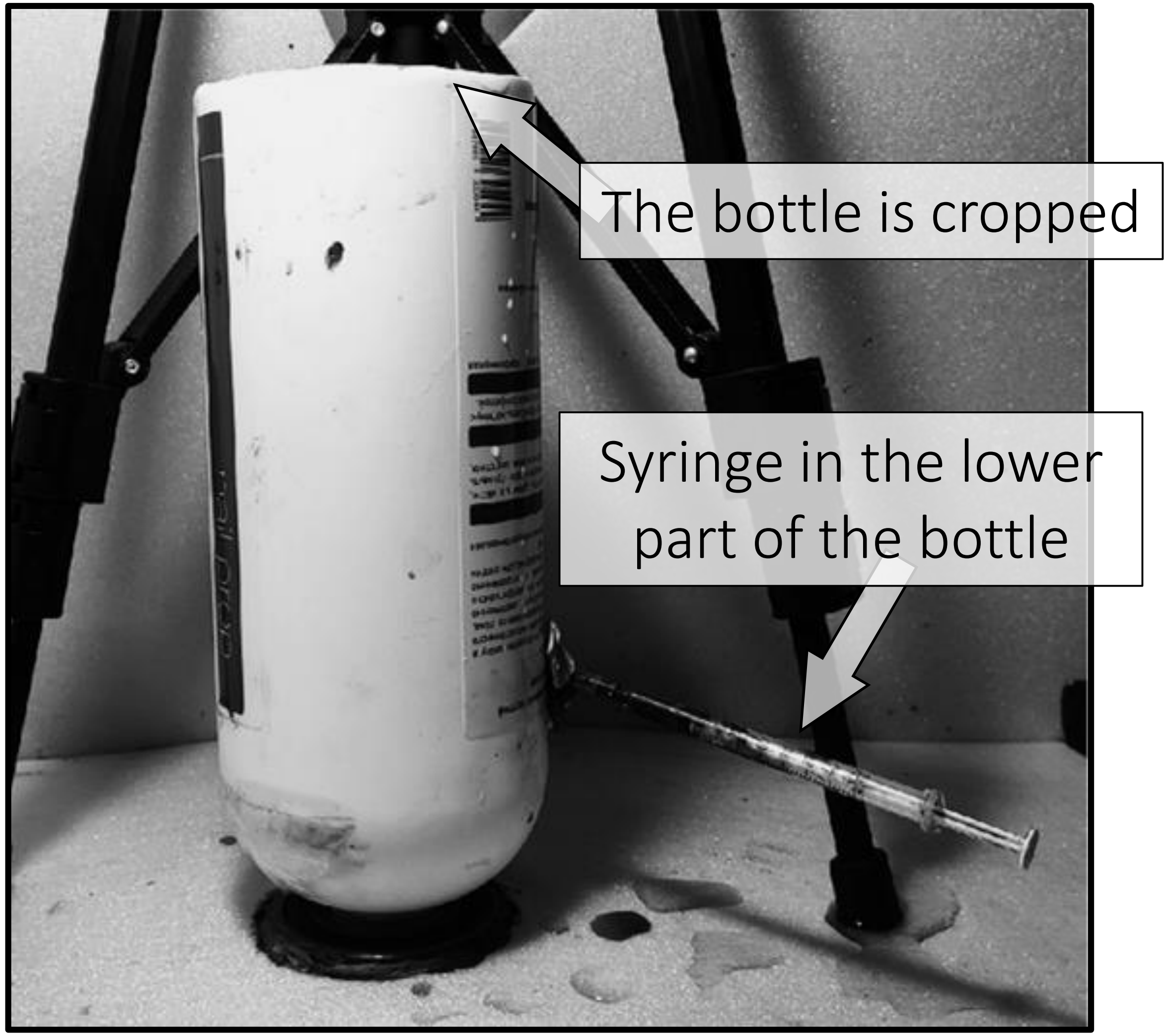
Rayleigh–Taylor instability

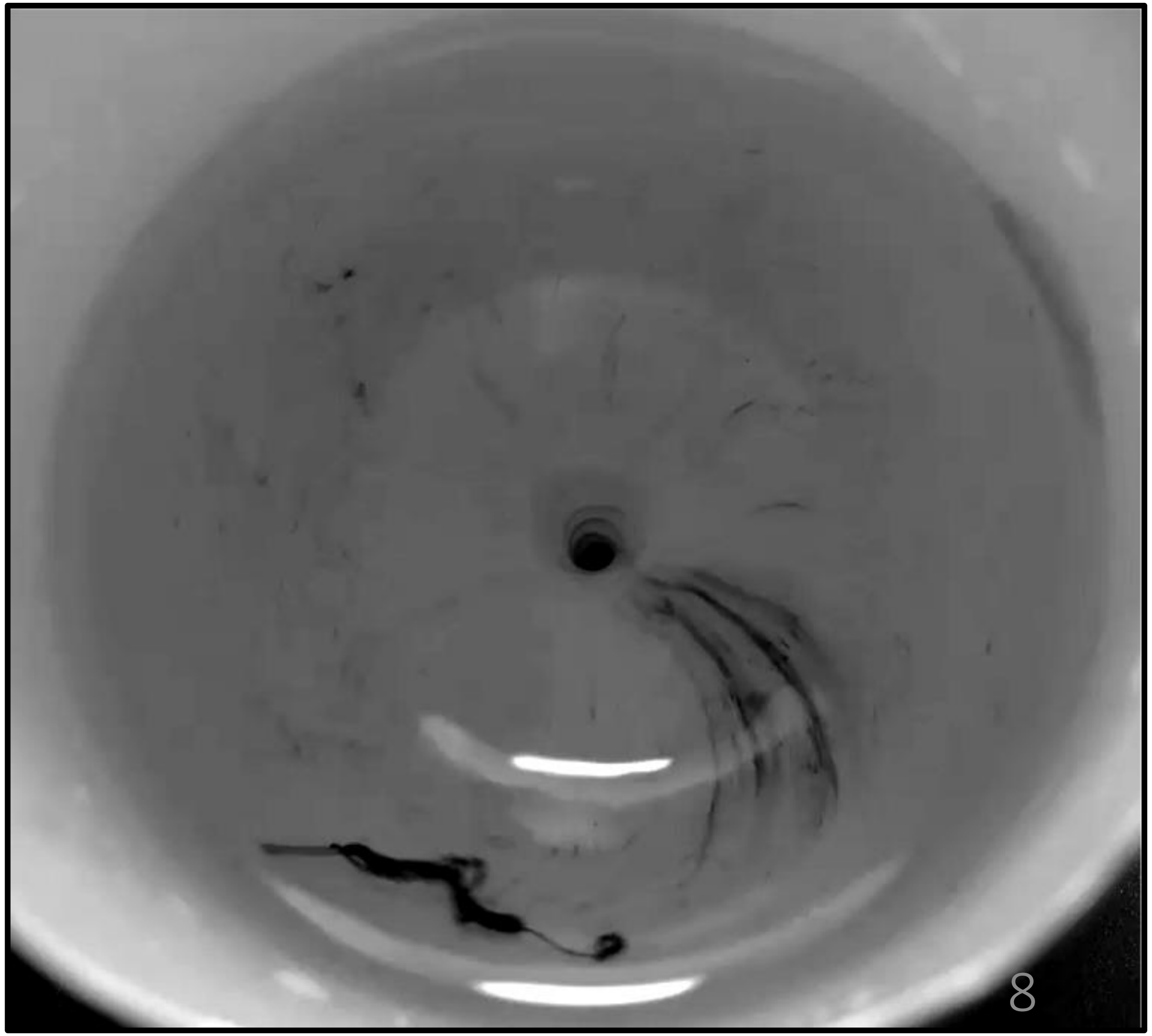


In the bottle closed on top (no air source on top) water starts to flow out due to the bubbles’ formation

Water flows' visualization

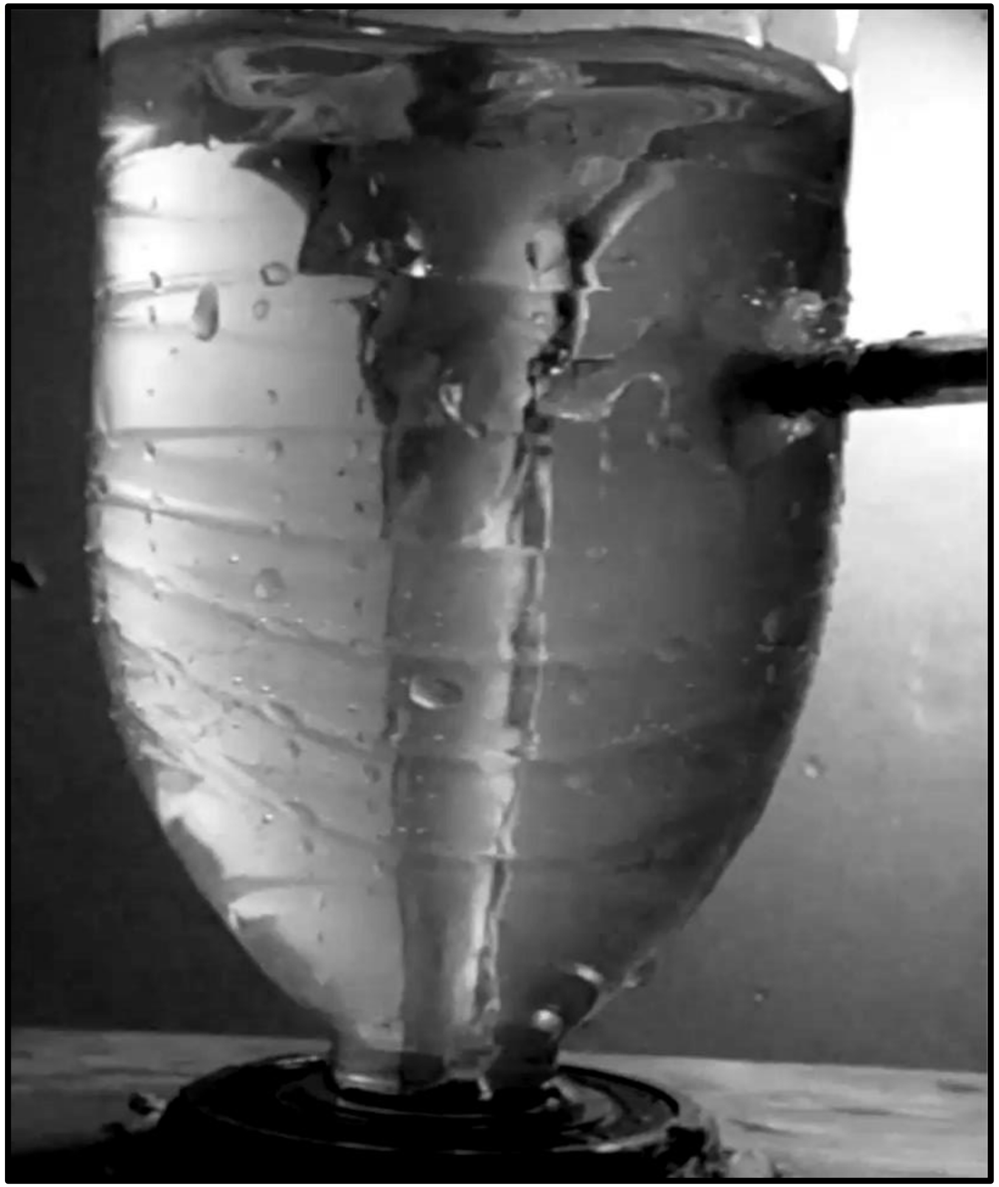
Water flows to center in the **boundary layer near the bottom**





Water flows' visualization

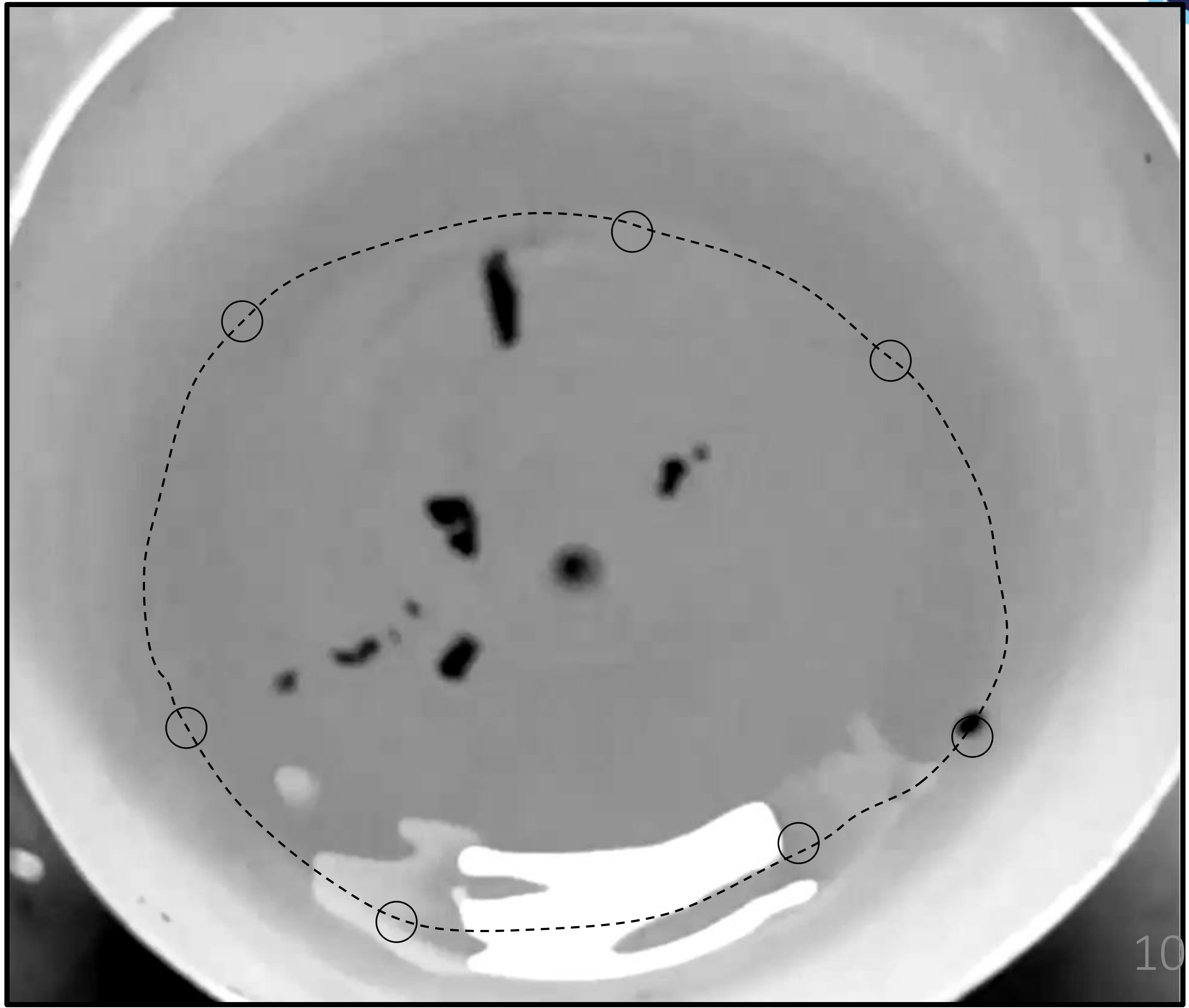
Water flows to center in the **air border**



Water flows' visualization

In the bulk water moves in a circle

Water flows out from the boundary layer and the surface layer



Stable funnel – Rankine vortex model

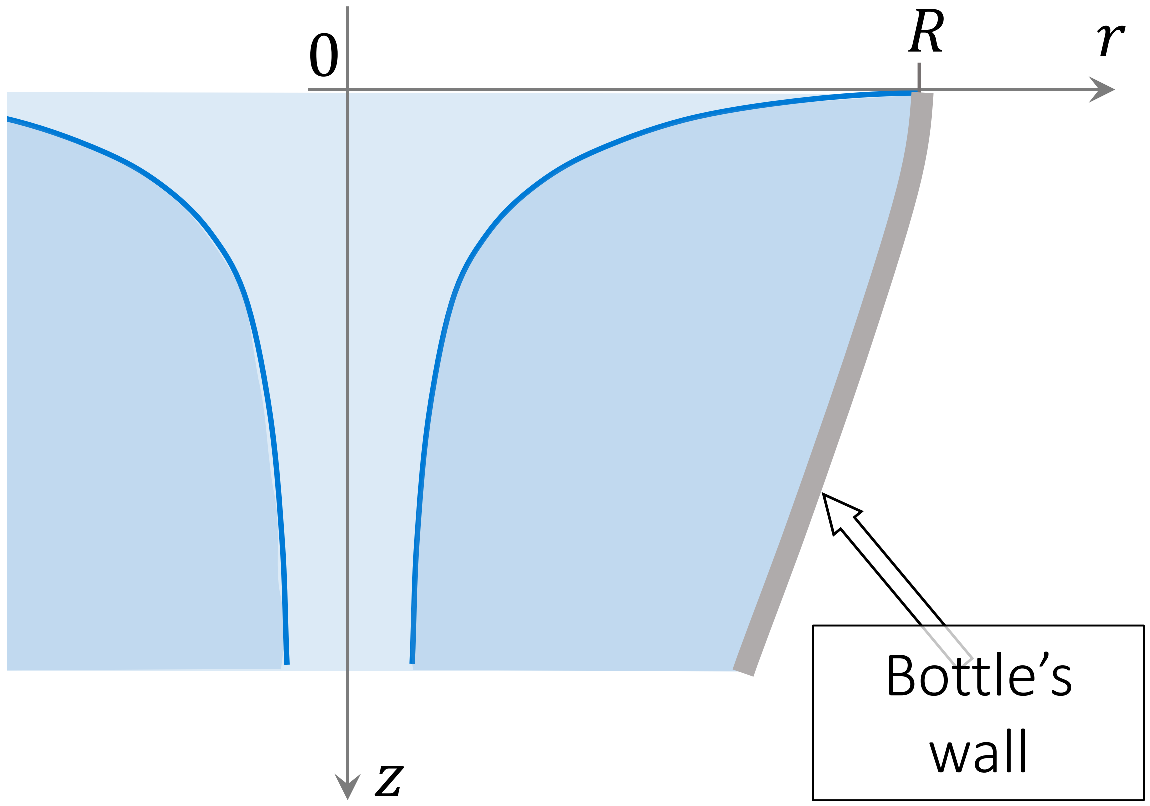
Speed of the water's rotation:

$$v_\theta = \frac{\Gamma}{2\pi r}$$

v_θ - speed of the water's in some point

Γ – circulation

r – distance between the center and that point



$$\Pi + \rho g\left(z - \frac{\Gamma^2}{8\pi^2 g R^2}\right) - \frac{\Gamma^2 \rho}{8\pi^2 r^2} = p$$

Π – external pressure

Near the surface $p = \Pi$:

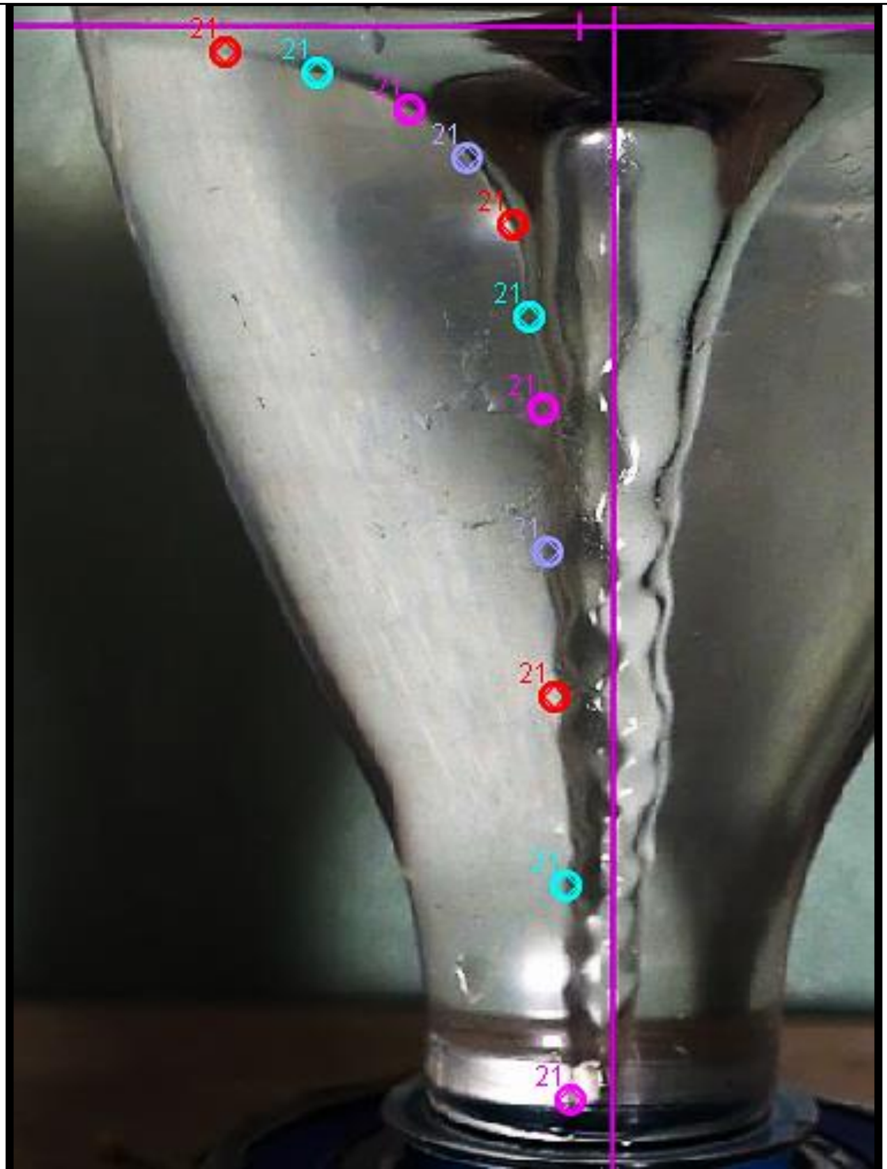
$$z = \frac{\Gamma^2}{8\pi^2 g}\left(\frac{1}{r^2} - \frac{1}{R^2}\right)$$

Sir Horace Lamb, "Hydrodynamics"

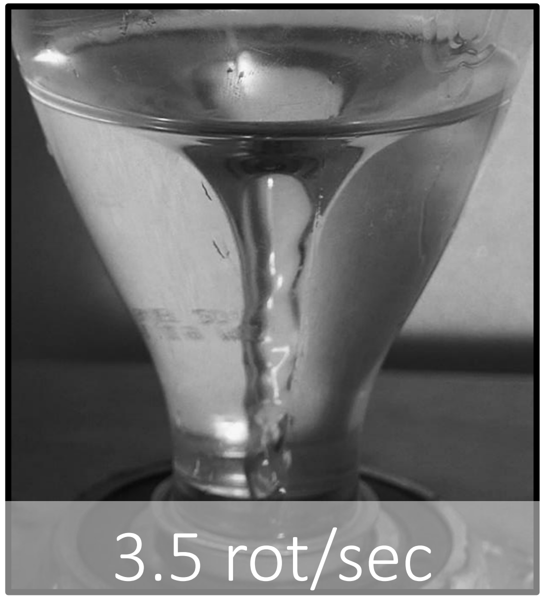
Experiment processing technique

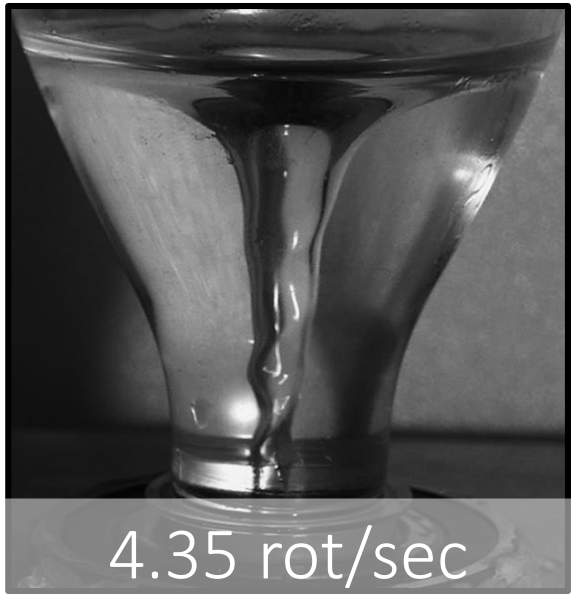
1) In “Tracker” we marked the points on the funnel’s surface in some frame
2) We entered the coordinates of those points into Excel
3) We plotted fit curve
4) We determined the circulation in the certain moment of time by the curve’s coeffitient

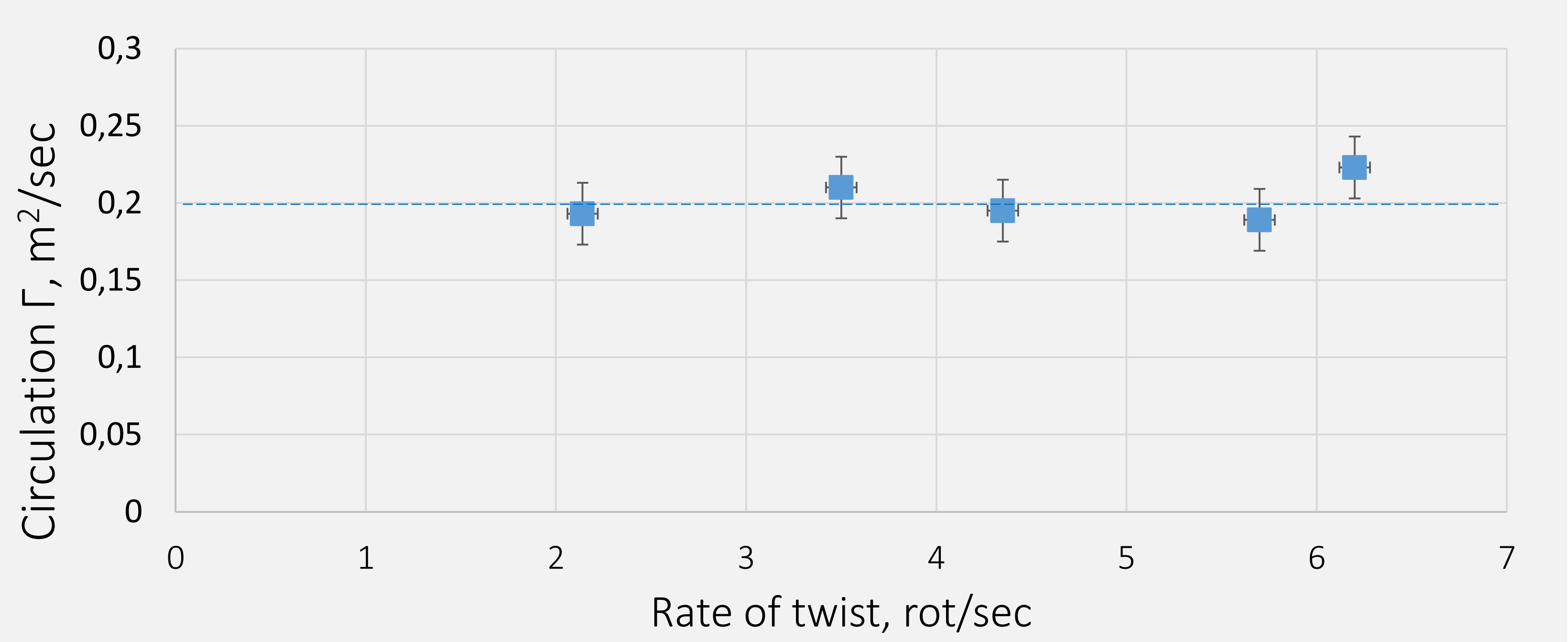
Processing using “Tracker”



Dependence on the initial rate of twist

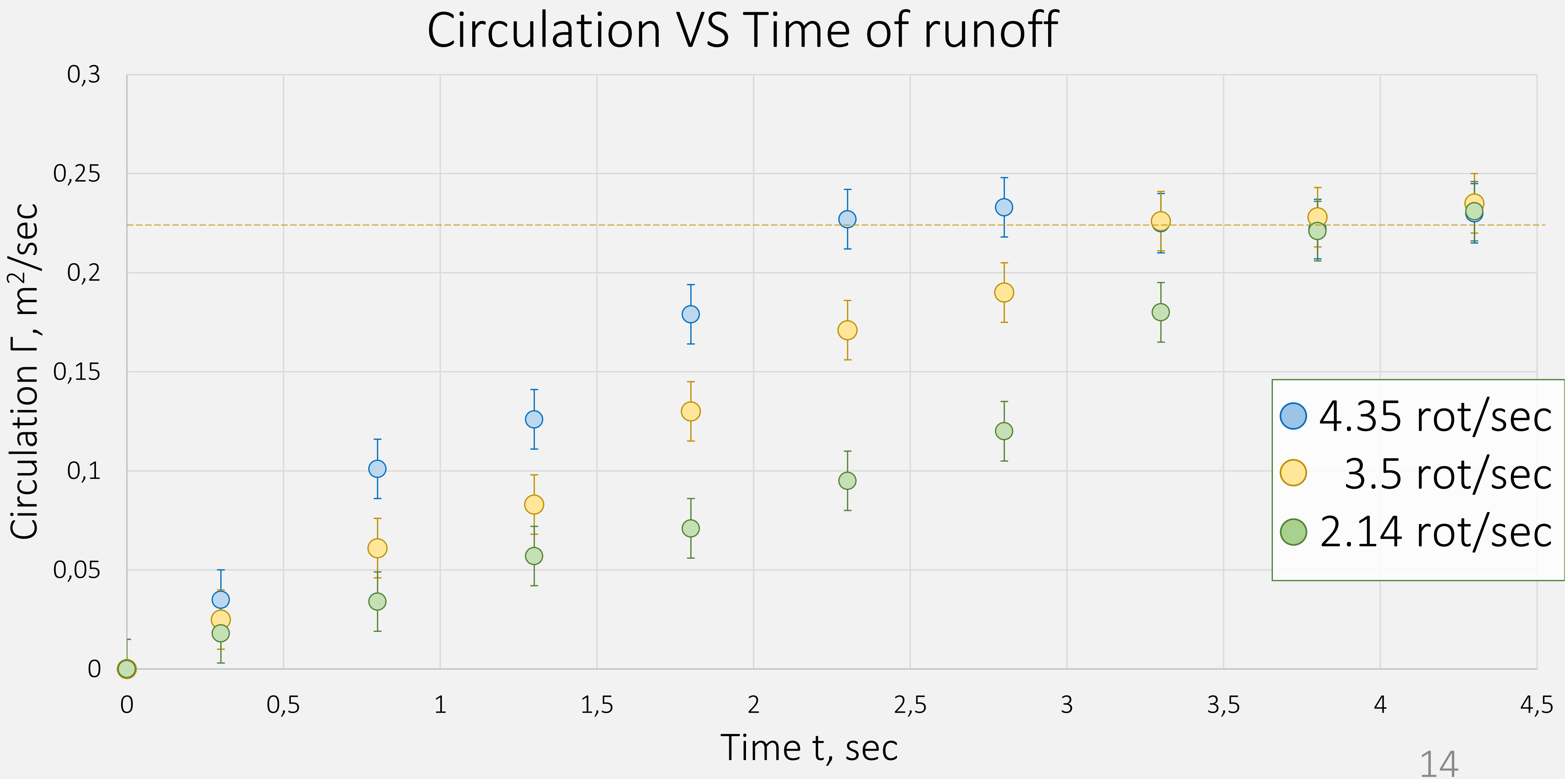






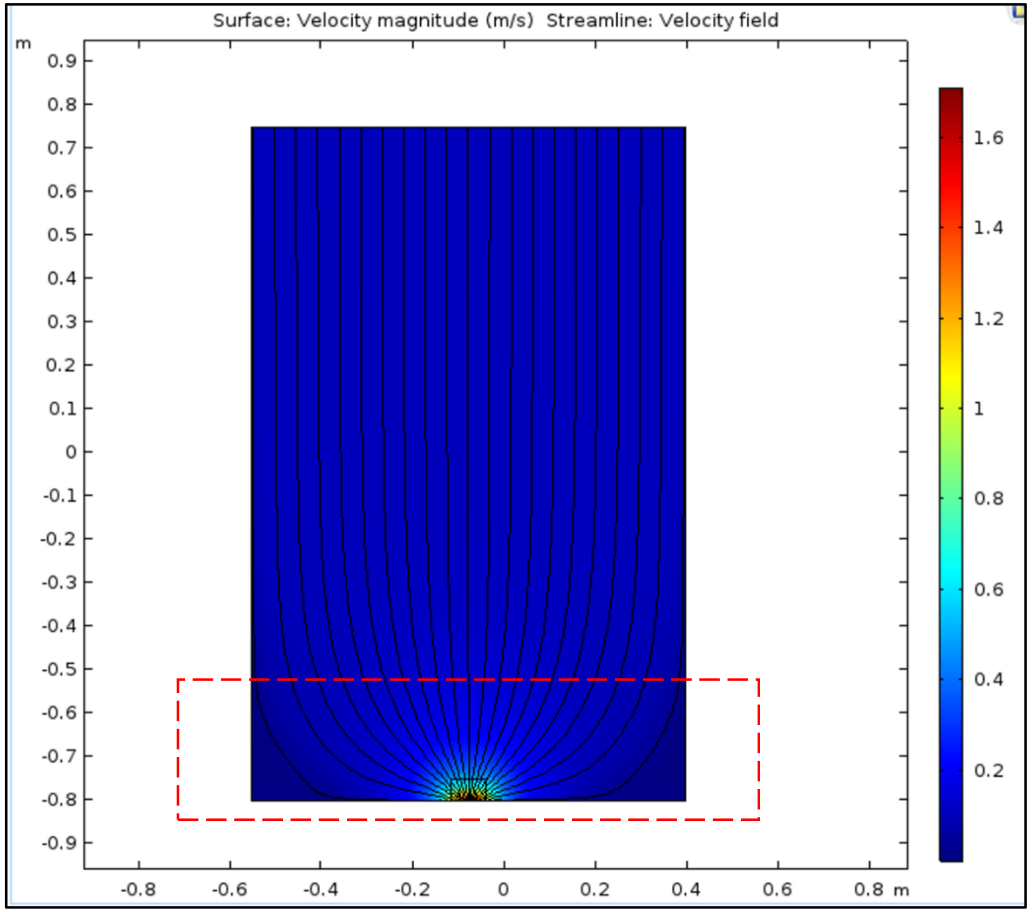
Circulation **doesn't depend on** the rate of twist

Circulation VS time

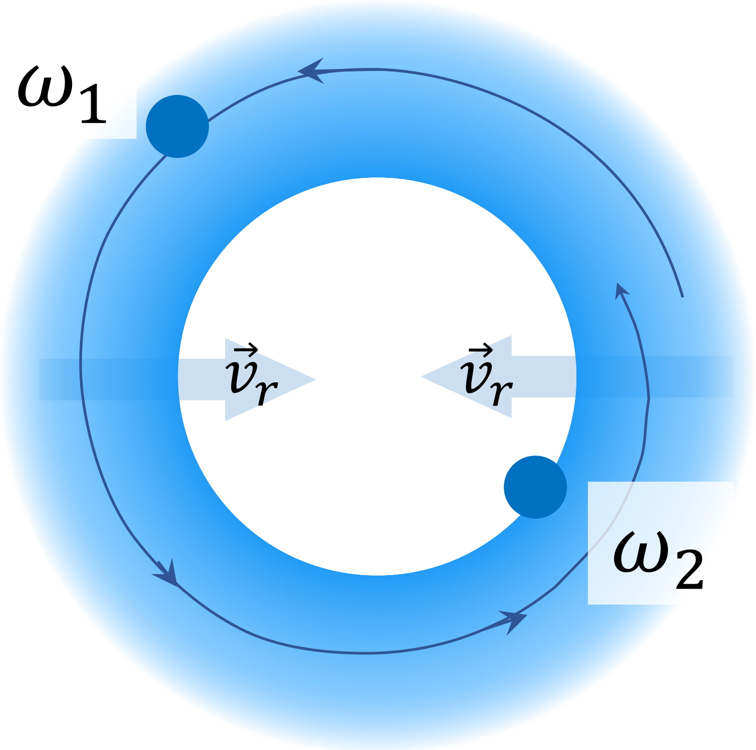


Initial growth of circulation

Water's runoff (COMSOL Multiphysics):



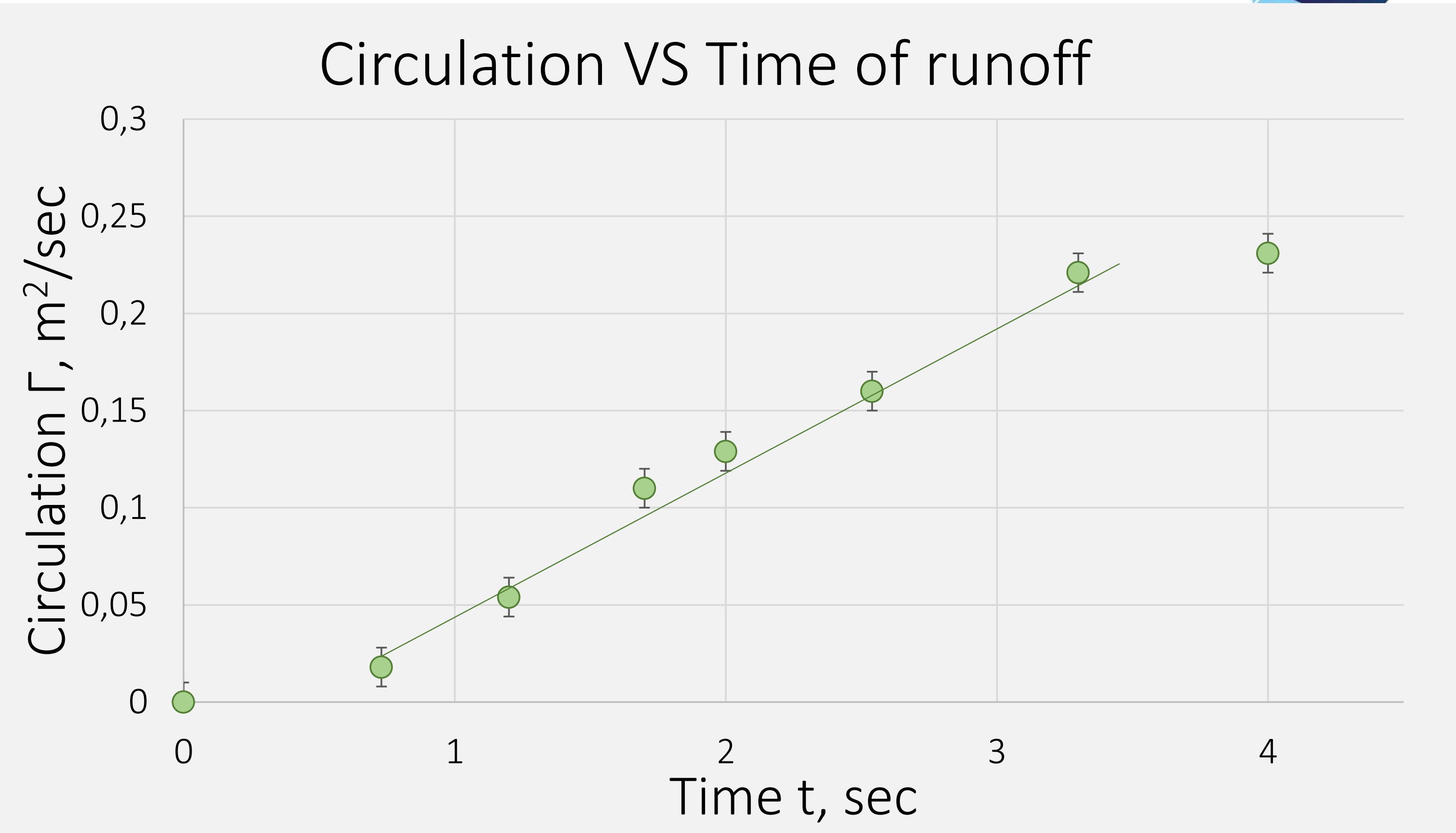
Water flows to the center:



$$\omega_1 < \omega_2$$

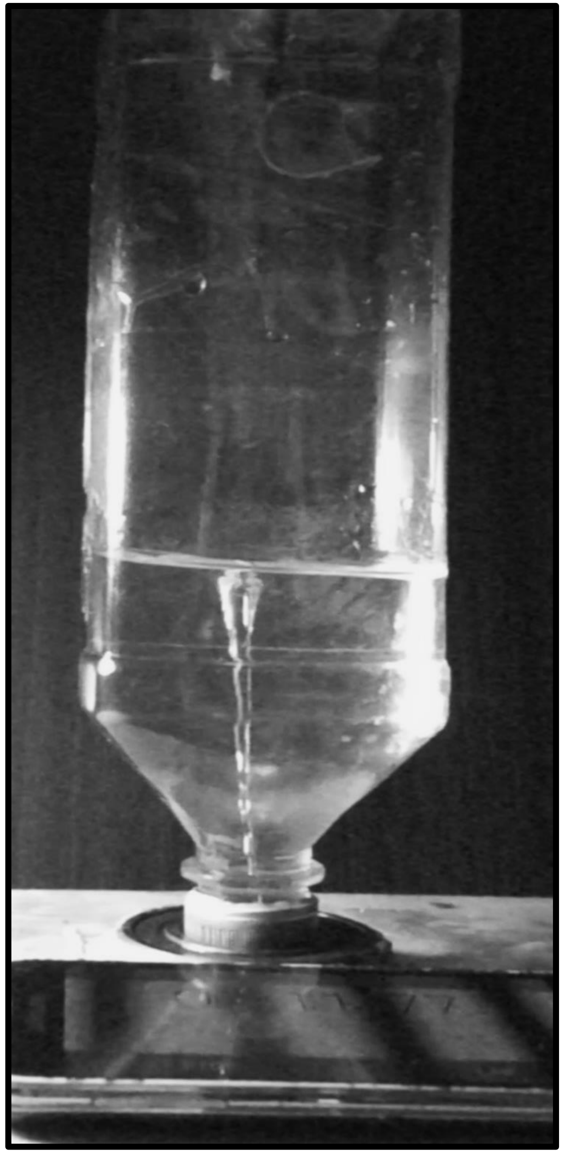
Runoff induces the flow to the center,
ω grows, Γ increases

Dynamics of circulation growth



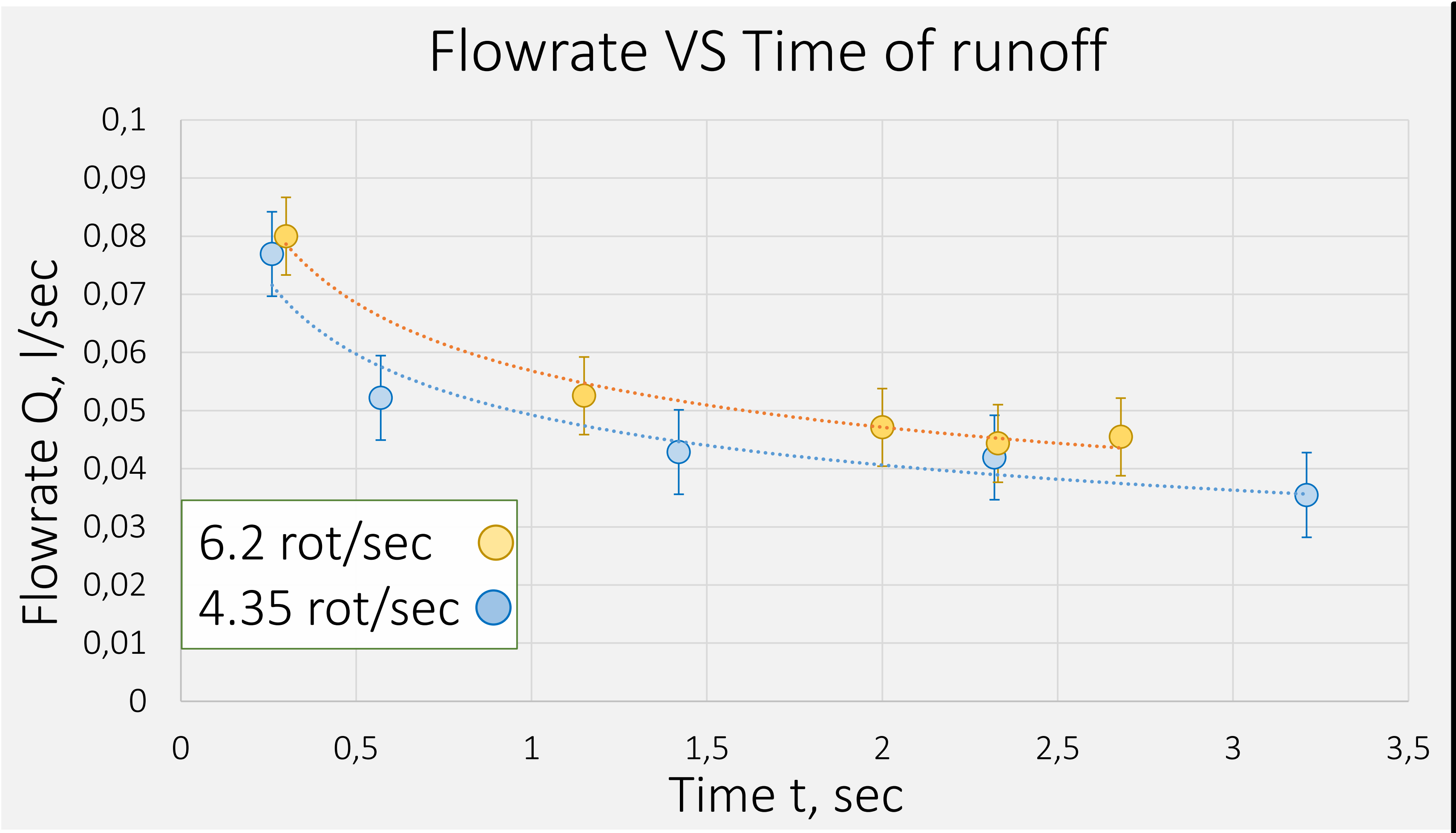
At the initial stage circulation grows linearly

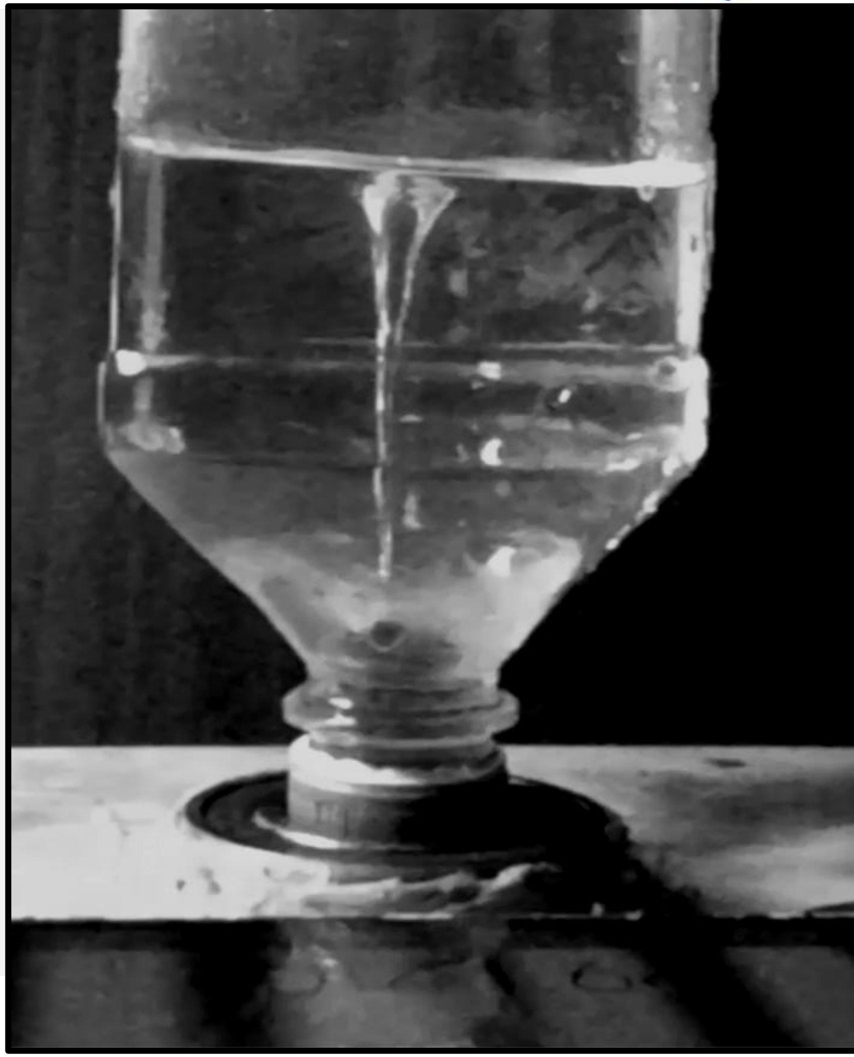
Technique of measuring the volumetric rate



1) Water flew out into the vessel placed on the scales
2) Stopwatch was mounted to be in frame
3) We entered the time and corresponding balance reading into Excel
4) Volumetric flow rate was calculated by the changing of mass in time

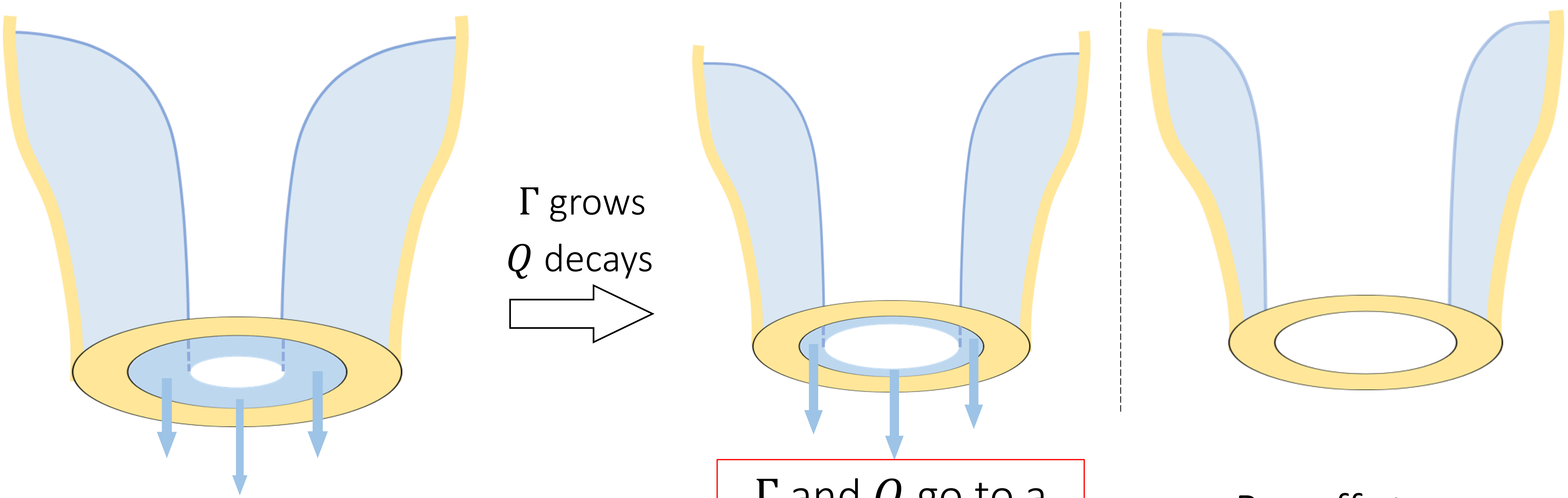
Dynamics of the flowrate at the initial stage





WHY circulation stabilizes during the runoff

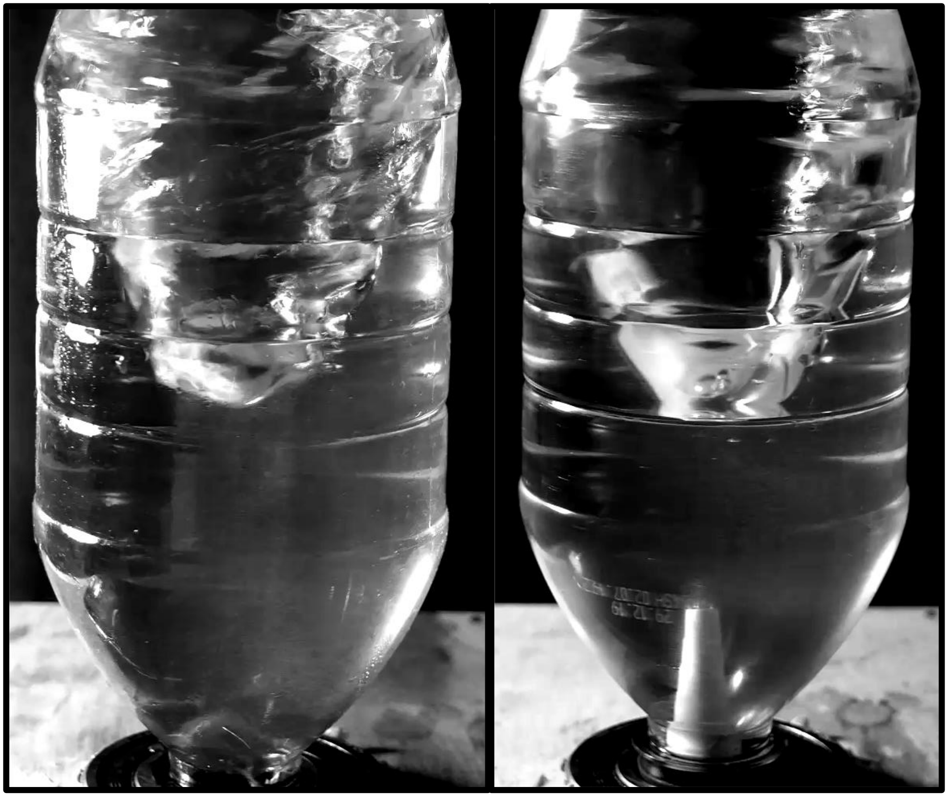
With viscosity: | No viscosity:



Γ and Q go to a constant

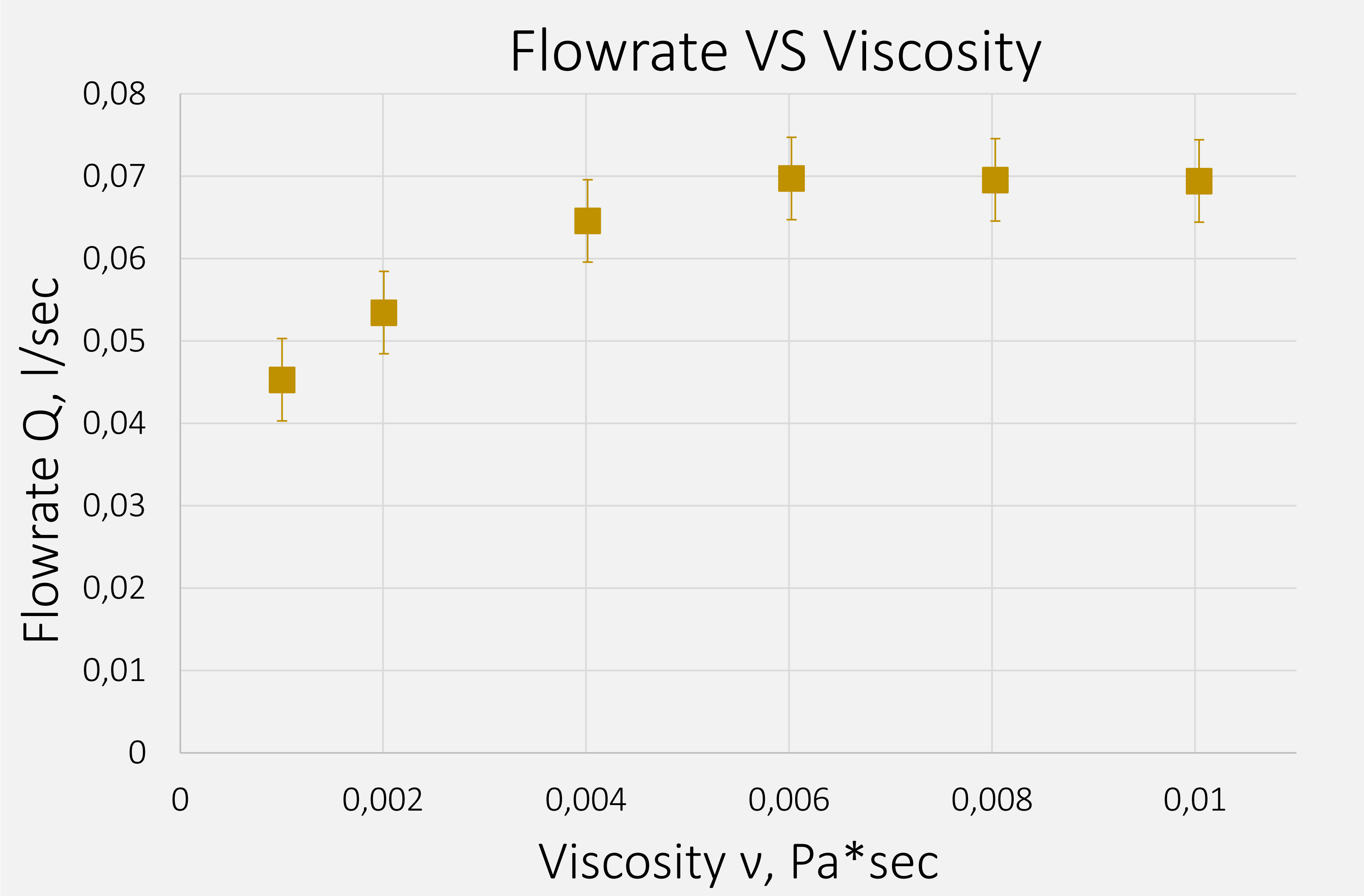
Runoff stops

Flowrate VS Viscosity

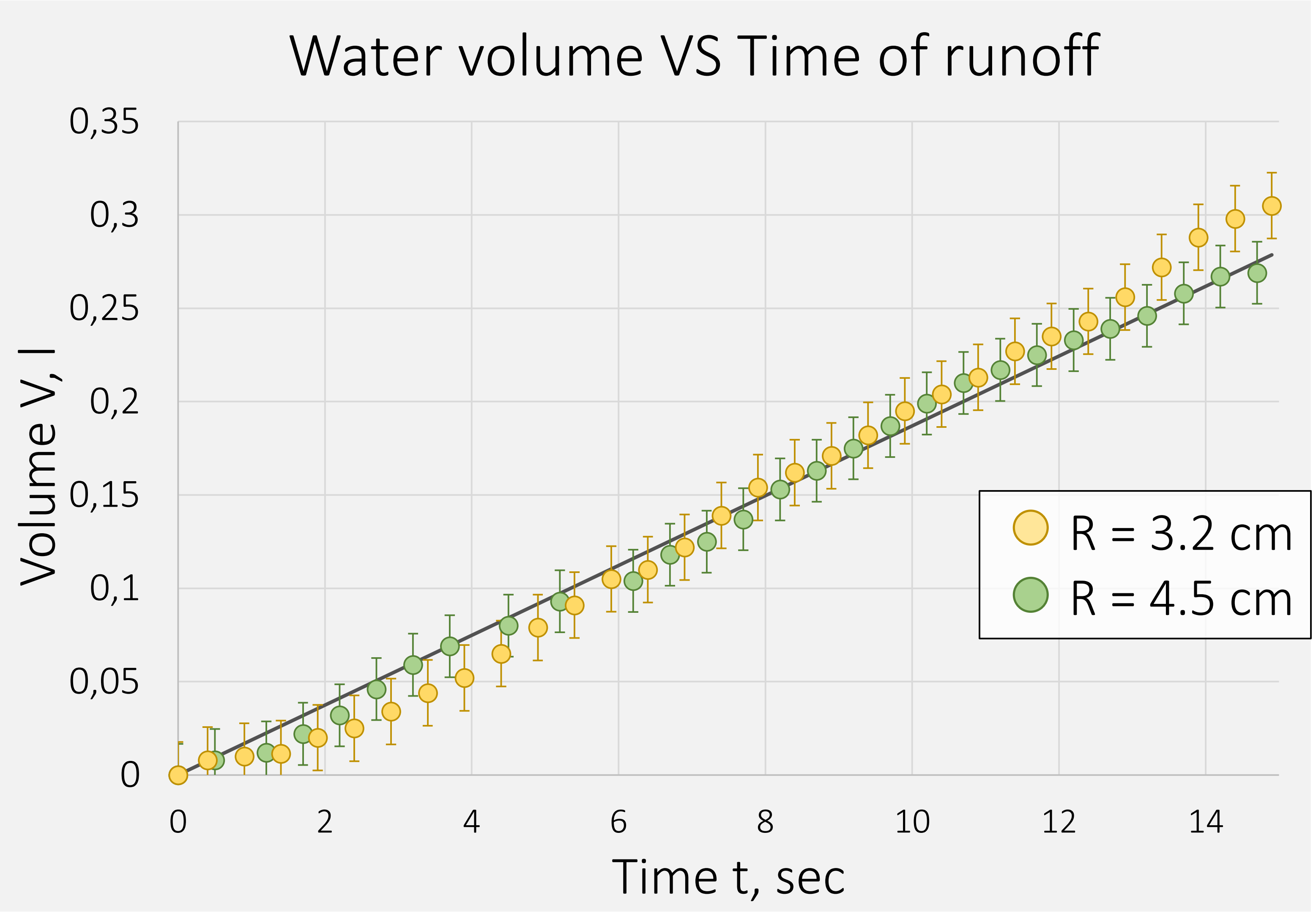


×2 of water viscosity

×10 of water viscosity



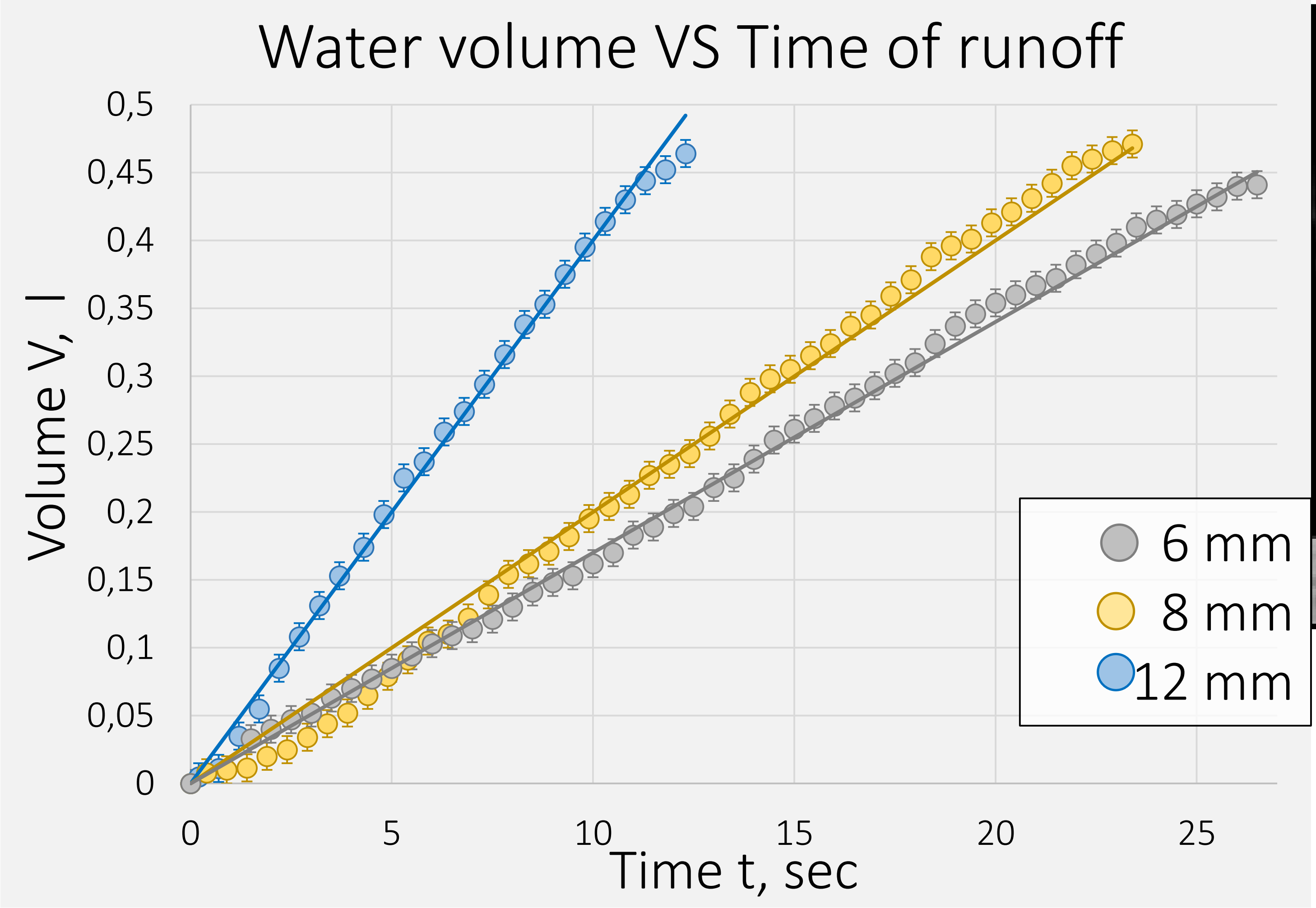
Flowrate in the different bottles

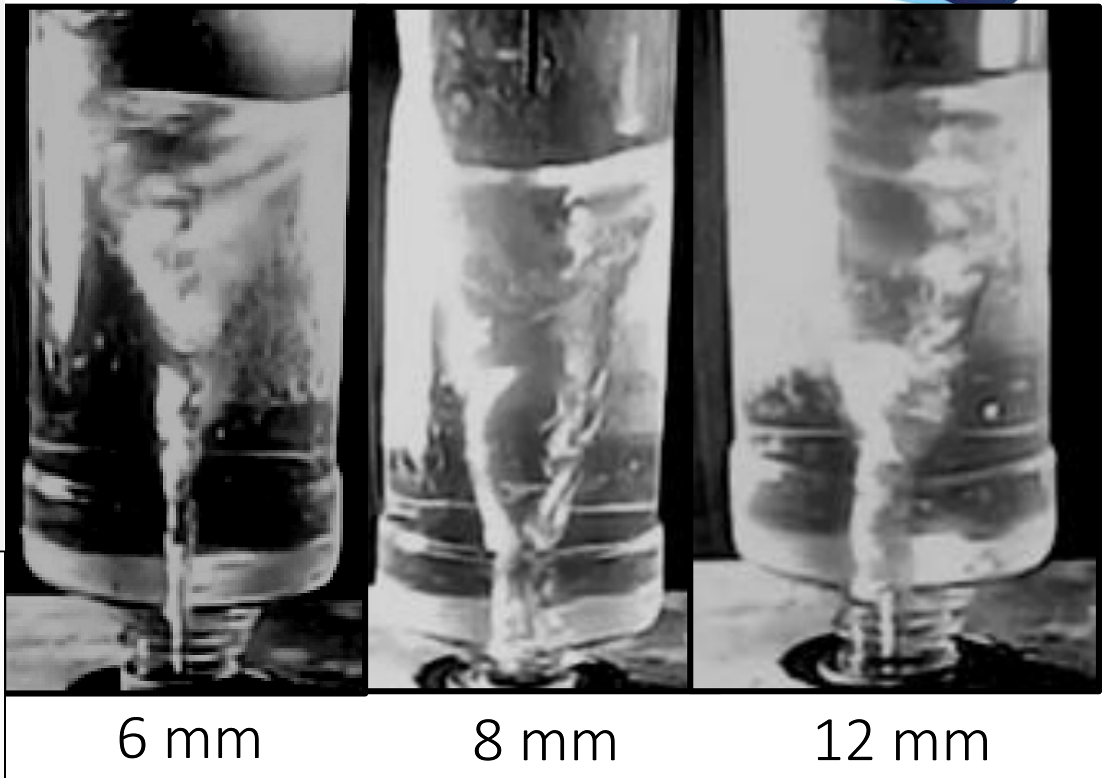


R = 4.2 cm

R = 2.8 cm

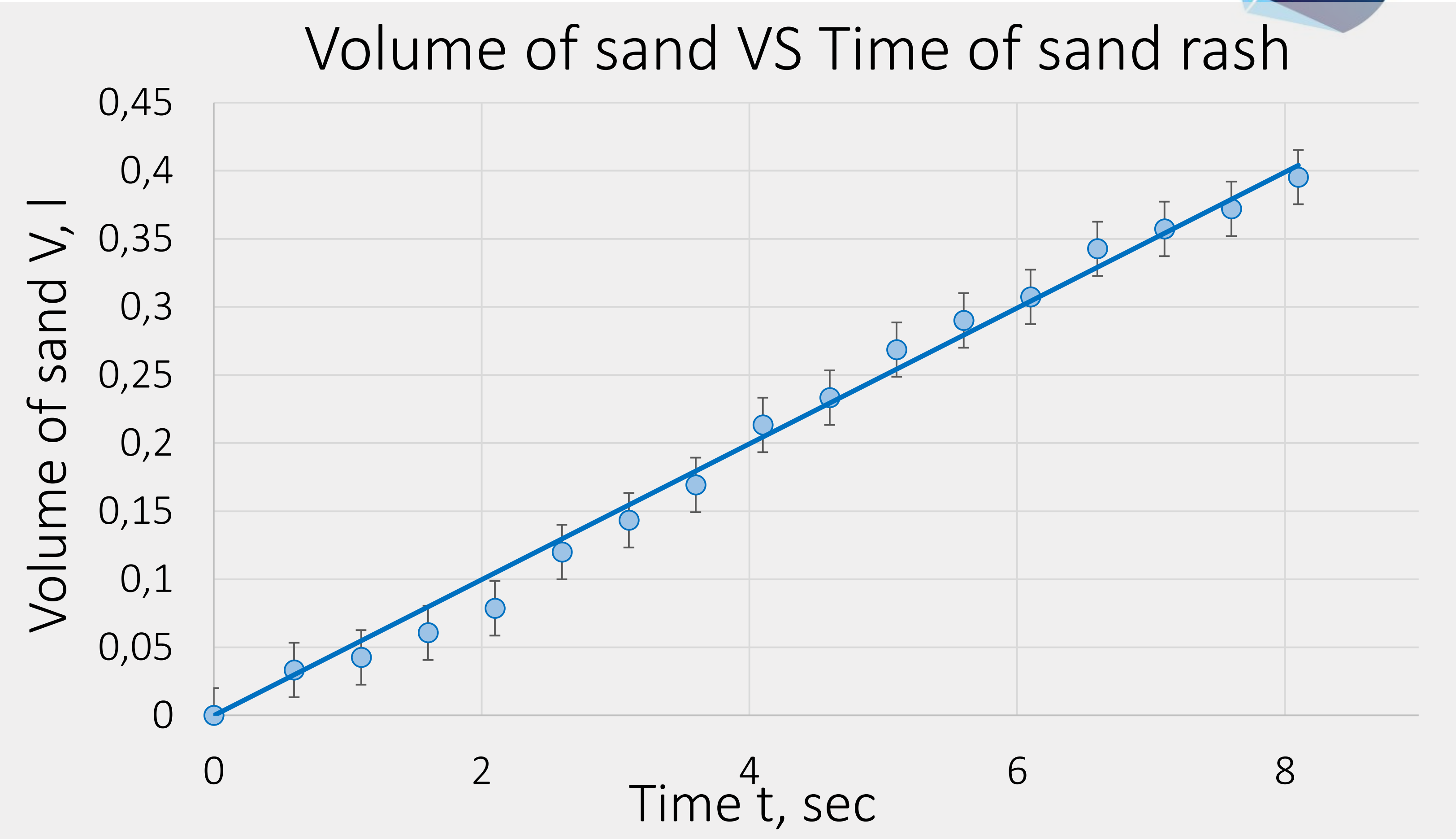
Different diameters of the hole





Q doesn't depend on the water column height

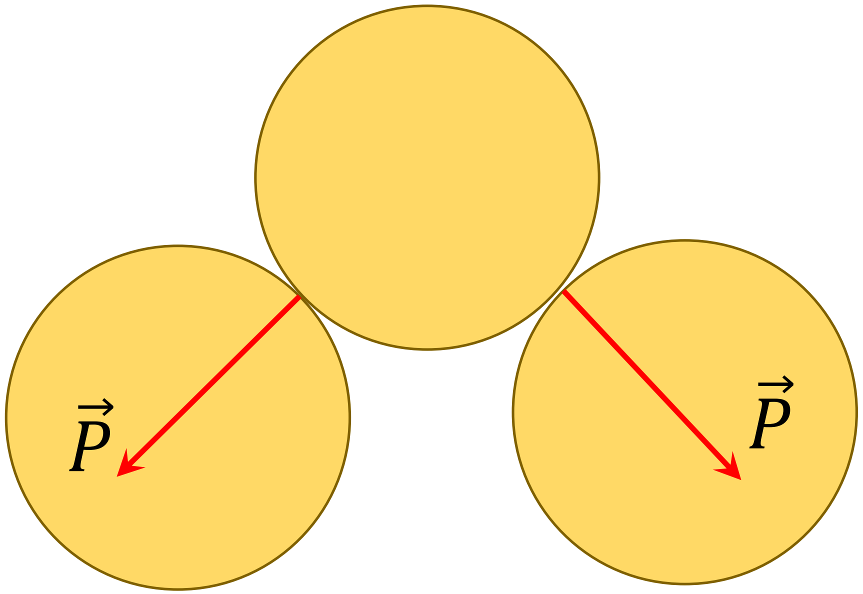
Experiment with a sand



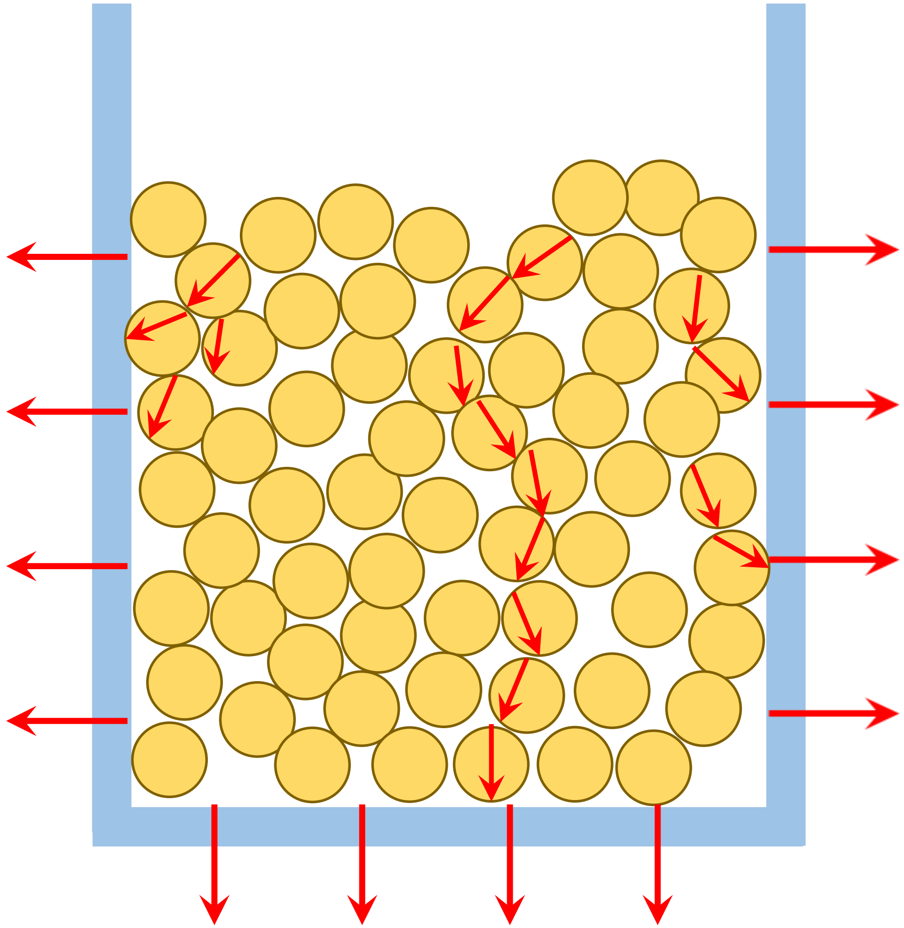
Q doesn't depend on the sand column height

Distribution of pressure in a vessel with sand

How upper particle press on the lower particles:



Pressure is distributed along the walls of vessel:



Theoretical model

(wasn’t in the report)

Model two-dimensional flow

Euler equation for the axisymmetric flow:

$$\left(\frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r}\right)(r v_\theta) = 0$$

Due to the runoff we have:

$$v_r = -\frac{Q}{r}$$

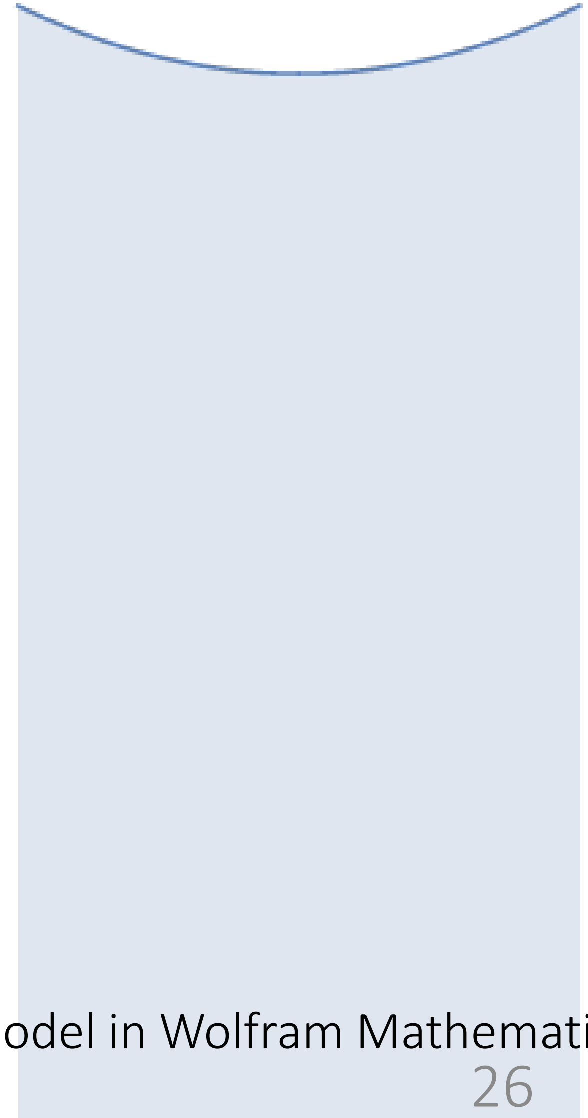
Let's consider the initial condition:

$$v_\theta = \omega r$$

Solving the equation we get:

$$\boxed{v_\theta = \omega r - \frac{\omega Q t}{r}}$$

At the initial time circulation grows linearly



Model in Wolfram Mathematica

Flow rate VS viscosity

Lets consider, how changed the hydrodynamic impulse of part of liquid, falling out from a bottle

$$\frac{dp}{dt} = mg - \frac{C\rho v^2 S}{2} \qquad (1)$$

From the other hand

$$p \sim \Gamma \rho \delta^2 \qquad (2)$$

where δ is the thickness of boundary layer (the flowing down happens mostly in the boundary layer.
Putting (1) into (2)

$$\rho\delta^2 \frac{d\Gamma}{dt} = \rho\delta^2 \pi R g - \frac{C\rho v^2 \pi R \delta}{2}$$

$$\delta \frac{d\Gamma}{dt} = \delta \pi R g - \frac{\rho v^2 \pi R}{2}$$

We know that circulation Γ doesn't change, so

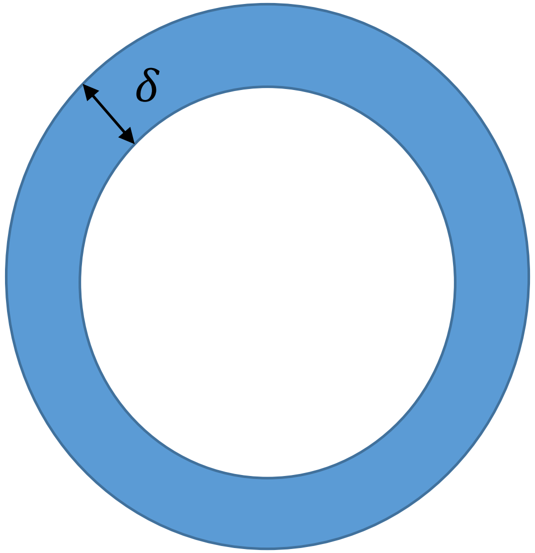
$$\delta \pi R g = \frac{\rho v^2 \pi R}{2}$$

Such a way we can estimate v

$$v = \sqrt{2g\delta}$$

Finally we obtain volume discharge. Knowing $\delta \sim \sqrt{\nu}$

$$Q = 2\pi R v \delta \sim \pi R \sqrt{2g} \nu^{3/4}$$



Conclusion

1) After the start of runoff a funnel is formed. Shape of its free surface can be described by the Rankine vortex model. Shape of its free surface and velocity field are determined by the circulation.
2) At the initial stage circulation grows linearly, then it stabilizes.
3) Water flowrate depends on:
   - viscosity; it grows with viscosity growth;
   - diameter of the hole; it grows with diameter growth.
4) Water flowrate doesn’t depend on the water column height. Similarly, sand flowrate doesn’t depend on the sand column height.

Thank you for attention!