

# 由素数的次幂和引出的欧拉乘积公式

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**摘要:** 将多个素数的次幂和与自然数的次幂和联系起来。

**关键词:** 欧拉乘积公式, 黎曼猜想。

如果我没有计算不同素数幂的乘积之和, 恐怕我永远不会和欧拉有任何关系, 因为知道魔术是如何工作的, 这就很简单了。

$$\forall m, n, k, d, n_k \in \mathbb{N}, \forall p, p_k \in \text{素数}, \forall p^n = \frac{p^n - 1}{1 - p^{-n}}, \quad \forall \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \frac{p_k^n - 1}{1 - p_k^{-n}},$$

$$\forall p^n \text{ 的约数, 例如, } p^0, p^1, p^2, p^3, p^4, \dots, p^n, \forall p^n \text{ 的约数和, } S_n = \sum_{m=0}^n p^m = \frac{p^{n+1} - p^0}{p^1 - 1},$$

$$\forall p^n \text{ 的约数的倒数, 例如, } p^0, p^{-1}, p^{-2}, p^{-3}, p^{-4}, \dots, p^{-n},$$

$$\forall p^n \text{ 的约数的倒数和, } S_{-n} = \sum_{m=0}^n p^{-m} = \frac{p^{-n-1} - p^0}{p^{-1} - 1} = \frac{p^{n+1} - p^0}{p^n(p^1 - 1)},$$

$$\Rightarrow \forall p^n = \frac{S_n}{S_{-n}} = \frac{\sum_{m=0}^n p^m}{\sum_{m=0}^n p^{-m}} = \frac{p^n - p^0}{p^1 - 1} / \frac{p^{-n-1} - p^0}{p^{-1} - 1}, \Rightarrow \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \frac{\sum_{m=0}^n p_k^m}{\sum_{m=0}^n p_k^{-m}} = \prod_{k=0}^{\infty} \left( \frac{p_k^n - p_k^0}{p_k^1 - 1} / \frac{p_k^{-n-1} - p_k^0}{p_k^{-1} - 1} \right),$$

$$n \rightarrow \infty, \Rightarrow \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \left( \frac{p_k^n - p_k^0}{p_k^1 - 1} / \frac{p_k^{-n-1} - p_k^0}{p_k^{-1} - 1} \right) = \frac{\sum_{d=1}^{\infty} d^{+1}}{\sum_{d=1}^{\infty} d^{-1}}, \Rightarrow \sum_{d=1}^{\infty} d^{\pm 1} = \prod_{k=0}^{\infty} \frac{p_k^{\pm(n+1)} - p_k^0}{p_k^{\pm 1} - 1},$$

$$n \rightarrow \infty, n_k \rightarrow \infty, \Rightarrow \prod_{k=0}^{\infty} p_k^{s \cdot n_k} = \frac{\sum_{d=1}^{\infty} d^{+s}}{\sum_{d=1}^{\infty} d^{-s}}, \Rightarrow \sum_{d=1}^{\infty} d^{\pm s} = \prod_{k=0}^{\infty} \frac{p_k^{\pm s(n_k+1)} - p_k^0}{p_k^{\pm s} - 1}.$$

$$\Rightarrow \sum_{d=1}^{\infty} d^{-s} = \prod_{k=0}^{\infty} \frac{p_k^{-s(n_k+1)} - p_k^0}{p_k^{-s} - 1} = \left( \sum_{d=1}^{\infty} d^{+s} \right) \prod_{k=0}^{\infty} \frac{1 - p_k^{-s \cdot n_k}}{p_k^{s \cdot n_k} - 1}.$$

因此, 欧拉乘积公式是不准确的, 即,  $\sum_{d=1}^{\infty} d^{-s} \neq \prod_{k=0}^{\infty} \frac{1}{1 - p_k^{-s}}$ 。

所以, 所有引用欧拉乘积公式的文献都是有问题的, 与欧拉乘积公式相关的黎曼猜想是错误的。

**参考文献:** 无。

# The Euler Product Formula derived from the Sum of the Power of Primes

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**Abstract:** Relate the sum of powers of multiple primes to the sum of powers of natural numbers.

**Key words:** Euler product formula, Riemann conjecture.

If I hadn't calculated the sum of the products of different prime powers, I'm afraid I would never have anything to do with Euler, because knowing how magic works, it would be very simple.

$$\forall m, n, k, d, n_k \in \mathbb{N}, \forall p, p_k, \in \text{prime numbers}, \forall p^n = \frac{p^n - 1}{1 - p^{-n}}, \forall \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \frac{p_k^{n+1} - 1}{1 - p_k^{-n}},$$

$$\text{Divisor of } \forall p^n, \text{ Such as, } p^0, p^1, p^2, p^3, p^4, \dots, p^n, \text{ Sum of divisors of } \forall p^n, S_n = \sum_{m=0}^n p^m = \frac{p^{n+1} - p^0}{p^1 - 1},$$

The reciprocal of the divisor of  $\forall p^n$ , Such as,  $p^0, p^{-1}, p^{-2}, p^{-3}, p^{-4}, \dots, p^{-n}$ ,

$$\text{The reciprocal sum of the divisor of } \forall p^n, S_{-n} = \sum_{m=0}^n p^{-m} = \frac{p^{-n-1} - p^0}{p^{-1} - 1} = \frac{p^{n+1} - p^0}{p^n(p^1 - 1)},$$

$$\Rightarrow \forall p^n = \frac{S_n}{S_{-n}} = \frac{\sum_{m=0}^n p^m}{\sum_{m=0}^n p^{-m}} = \frac{p^{n+1} - p^0}{p^1 - 1} / \frac{p^{-n-1} - p^0}{p^{-1} - 1}, \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \frac{\sum_{m=0}^n p_k^m}{\sum_{m=0}^n p_k^{-m}} = \prod_{k=0}^{\infty} \left( \frac{p_k^{n+1} - p_k^0}{p_k^1 - 1} / \frac{p_k^{-n-1} - p_k^0}{p_k^{-1} - 1} \right),$$

$$n \rightarrow \infty, \Rightarrow \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \left( \frac{p_k^{n+1} - p_k^0}{p_k^1 - 1} / \frac{p_k^{-n-1} - p_k^0}{p_k^{-1} - 1} \right) = \frac{\sum_{d=1}^{\infty} d^{n+1}}{\sum_{d=1}^{\infty} d^{-n-1}}, \sum_{d=1}^{\infty} d^{\pm 1} = \prod_{k=0}^{\infty} \frac{p_k^{\pm(n+1)} - p_k^0}{p_k^{\pm 1} - 1},$$

$$n \rightarrow \infty, n_k \rightarrow \infty, \Rightarrow \prod_{k=0}^{\infty} p_k^{s+n_k} = \frac{\sum_{d=1}^{\infty} d^{s+1}}{\sum_{d=1}^{\infty} d^{-s-1}}, \sum_{d=1}^{\infty} d^{\pm s} = \prod_{k=0}^{\infty} \frac{p_k^{\pm s(n_k+1)} - p_k^0}{p_k^{\pm s} - 1}.$$

$$\Rightarrow \sum_{d=1}^{\infty} d^{-s} = \prod_{k=0}^{\infty} \frac{p_k^{-s(n_k+1)} - p_k^0}{p_k^{-s} - 1} = \left( \sum_{d=1}^{\infty} d^{+s} \right) \prod_{k=0}^{\infty} \frac{1 - p_k^{-s+n_k}}{p_k^{-s+n_k} - 1},$$

So, the Euler product formula is not accurate, that is,  $\sum_{d=1}^{\infty} d^{-s} \neq \prod_{k=0}^{\infty} \frac{1}{1 - p_k^{-s}}$ .

Therefore, all papers quoting the Euler product formula are problematic, and the Riemann conjecture related to the Euler product formula is wrong.

**Reference:** none.