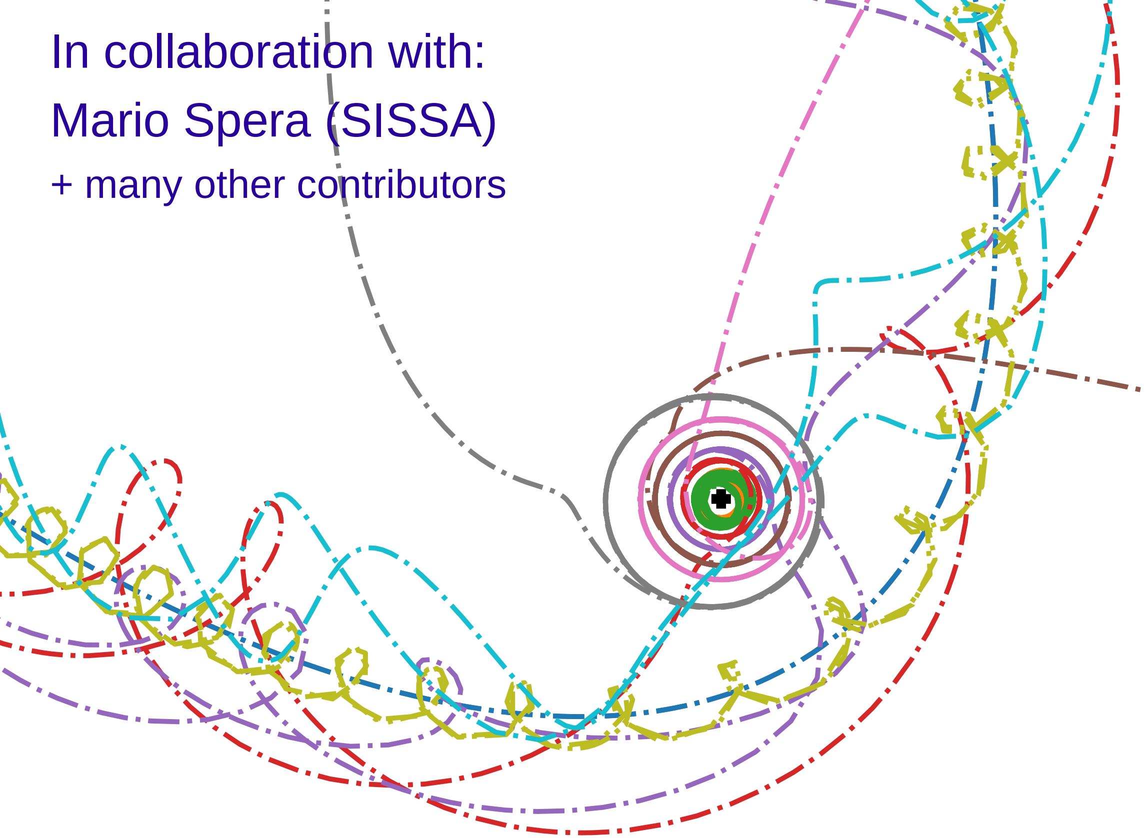
In collaboration with:
Mario Spera (SISSA)
+ many other contributors



TSUNAMI

A modern regularized code for planetary and black hole dynamics

With applications to:

- planetary and stellar scatterings
- hierarchical and multiple systems
- exomoons and black holes

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Alberto
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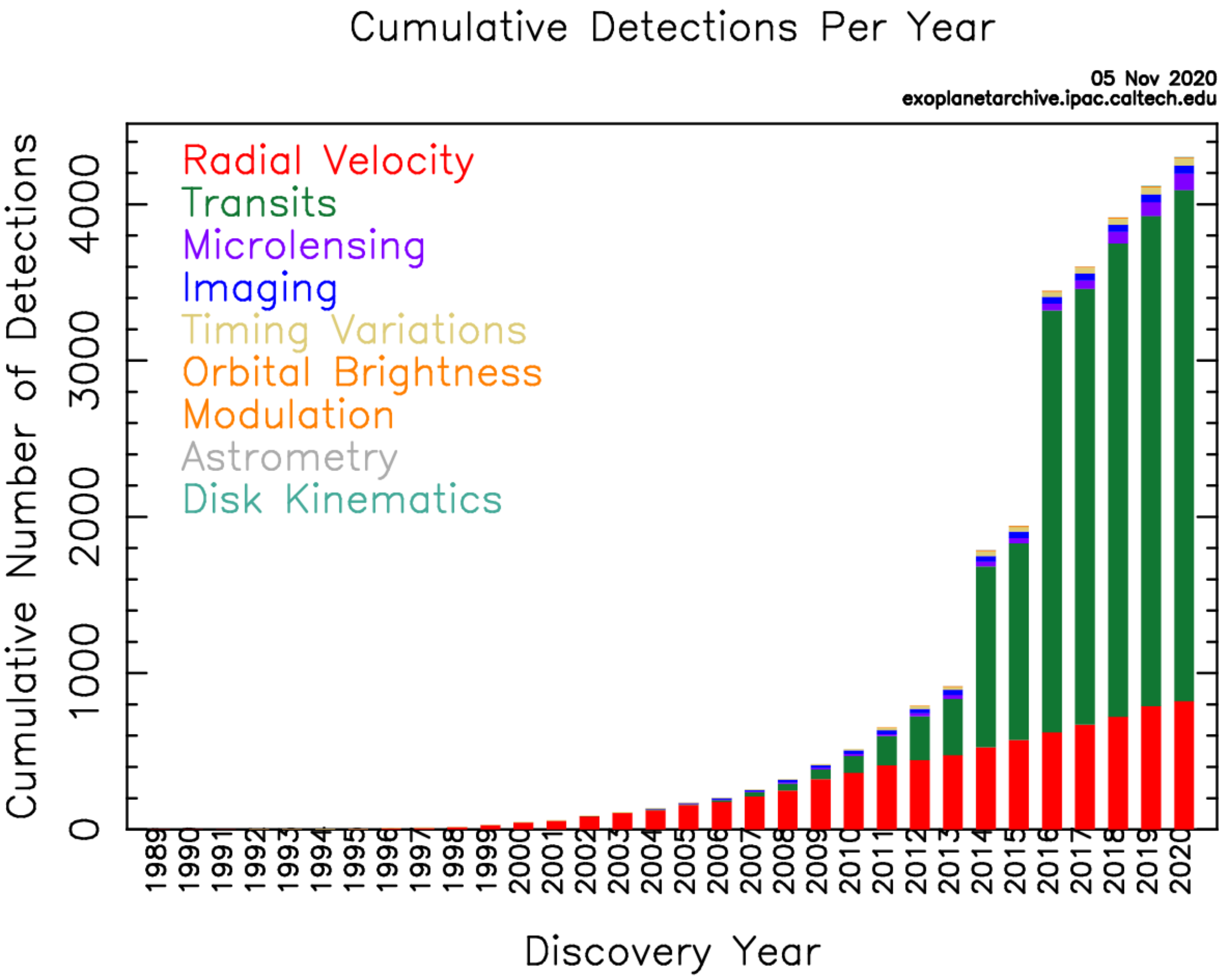
東京大学
THE UNIVERSITY OF TOKYO

We are living a “*Renaissance*” of celestial mechanics



- **Exponential growth of planet detections**

many exotic system discovered (hot jupiters, ultra-short period planets, etc)

TESS mission + radial velocity follow-ups

More satellites: CHEOPS, PLATO, Jasmine, JWST
Ground based: ELT, TMT, etc

We are living a “*Renaissance*” of celestial mechanics

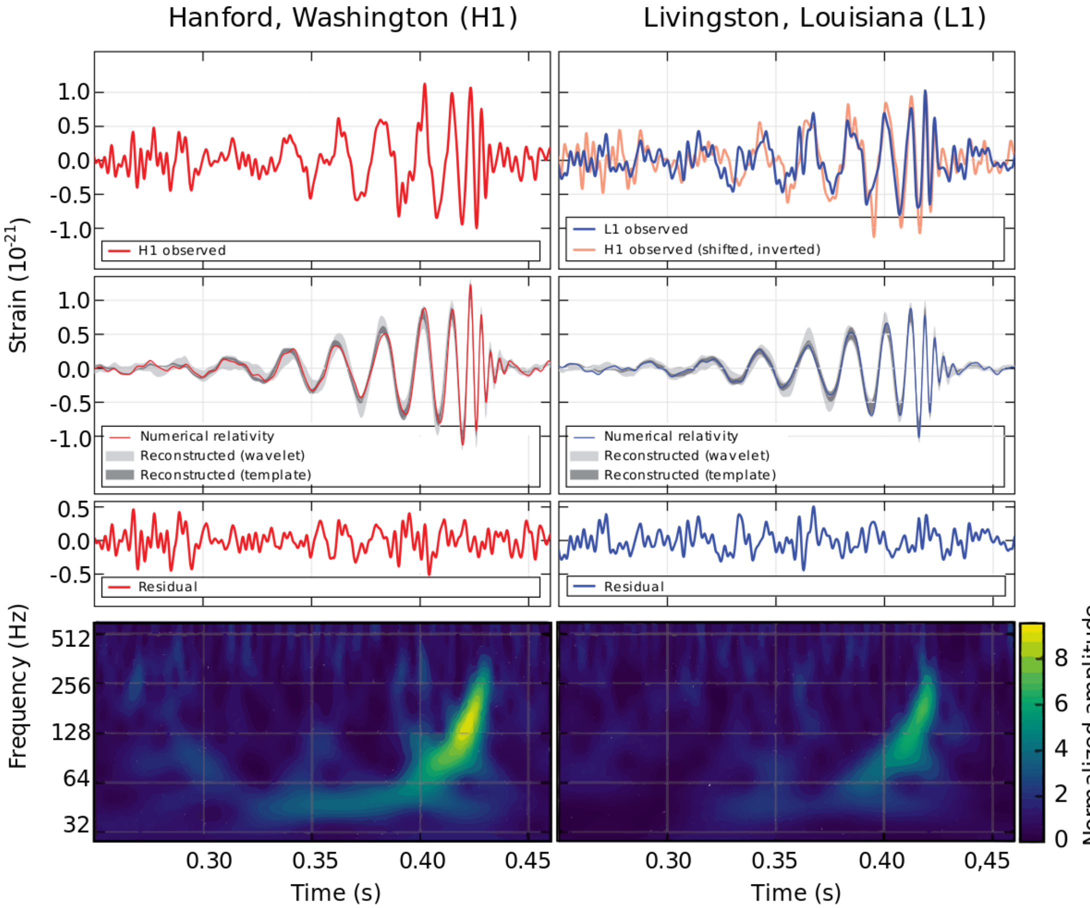
- **Birth of gravitational wave astronomy:**

gravitational waves from coalescing binary black holes and neutron stars

many formation pathways invoke few-body interactions (“scatterings”)

LIGO, VIRGO, KAGRA observatories running, LIGO-India planned

+ 3rd *generation detectors being considered (Einstein Telescope, Cosmic explorer)*

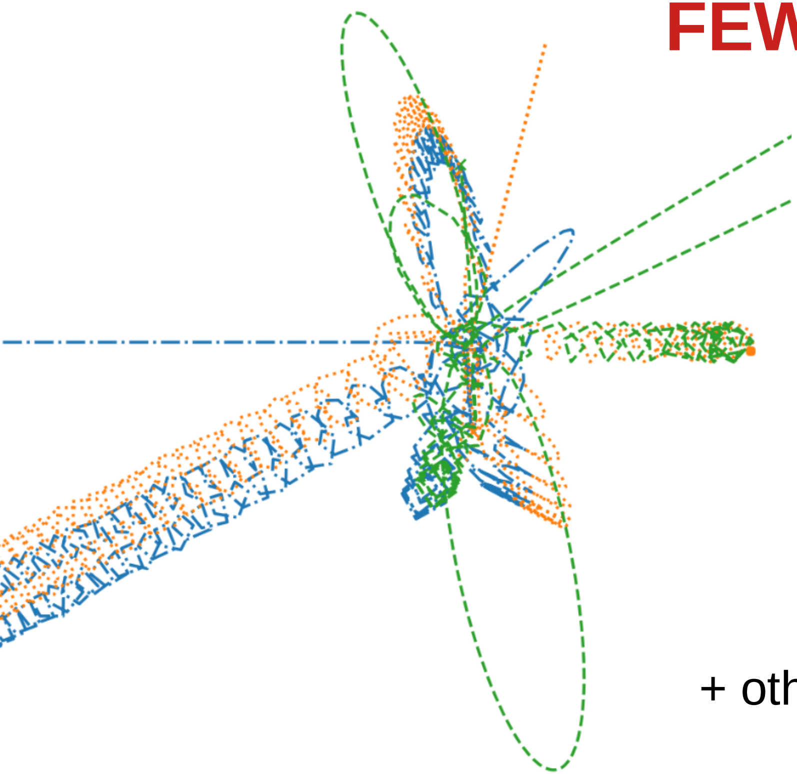


+ space-based detectors

- **Exponential growth of planet detections**

- **Birth of gravitational wave astronomy:**

RENEWED INTEREST IN THE GRAVITATIONAL FEW-BODY PROBLEM



We need to solve the Newtonian equations of motion:

$$\ddot{\mathbf{p}}_j = \sum_{i=0}^{N} \frac{G\, m_i}{|\mathbf{p}_i - \mathbf{p}_j|^3}(\mathbf{p}_i - \mathbf{p}_j)$$

(no analytic solution for N >2)

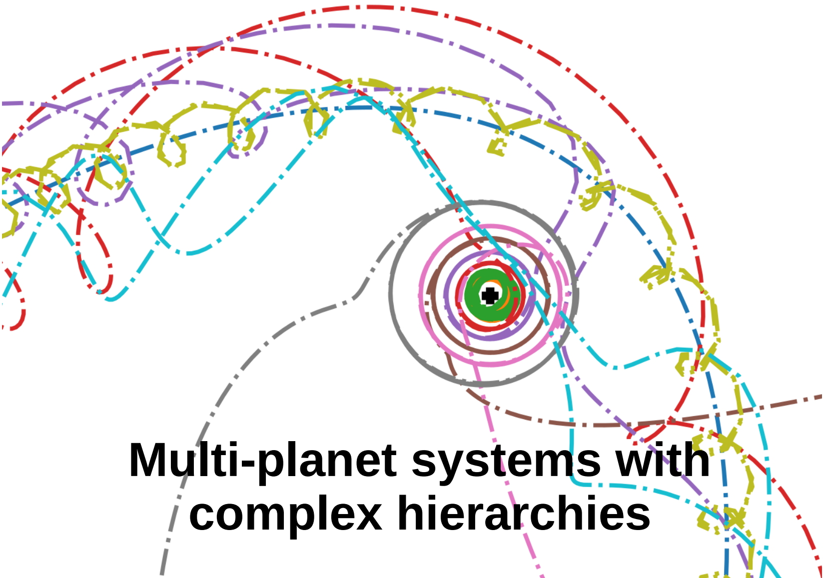
+ other forces (general relativity corrections, tidal forces, etc)

Plenty of numerical integration techniques:

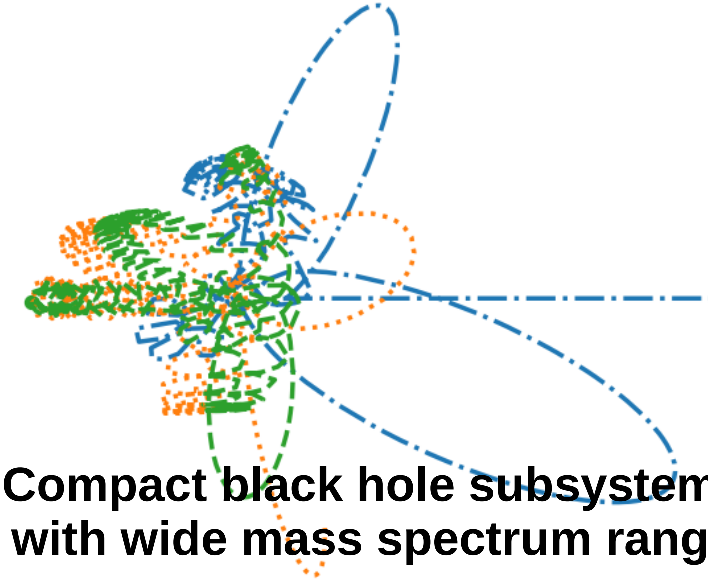
- Hermite intepolation (e.g. Makino & Aarseth 1992)
- Runge-Kutta

✗ not good for long term integration of planetary systems

- Kepler splitting (e.g. Wisdom & Holman 1991) ✗ works only in specific configurations
- Higher order symplectic (e.g. Kinoshita, Yoshida & Nakai 1991) ✗ no forces corrections



Multi-planet systems with complex hierarchies



Compact black hole subsystems with wide mass spectrum range

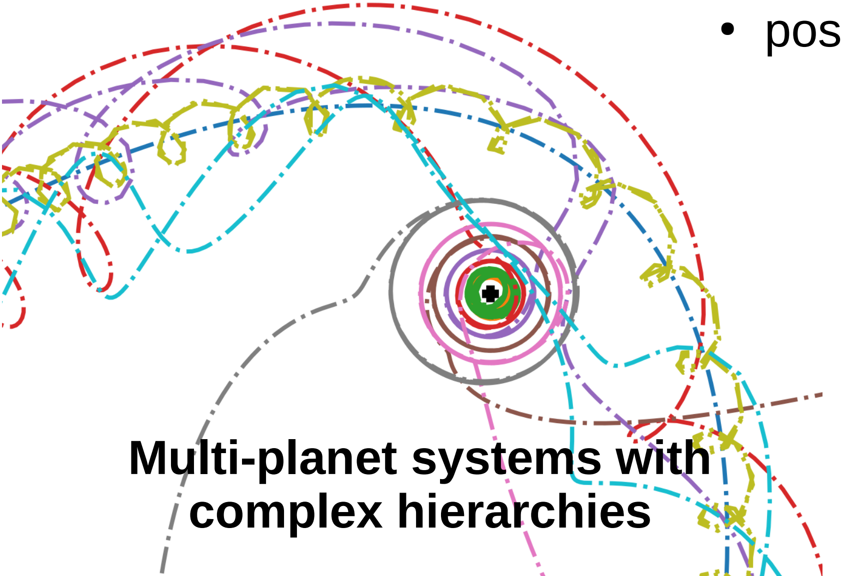
Main issue of gravity: “ultraviolet divergence”

$$\lim_{r \to 0} F = \lim_{r \to 0} \frac{G\,Mm}{r^2} \to \infty$$

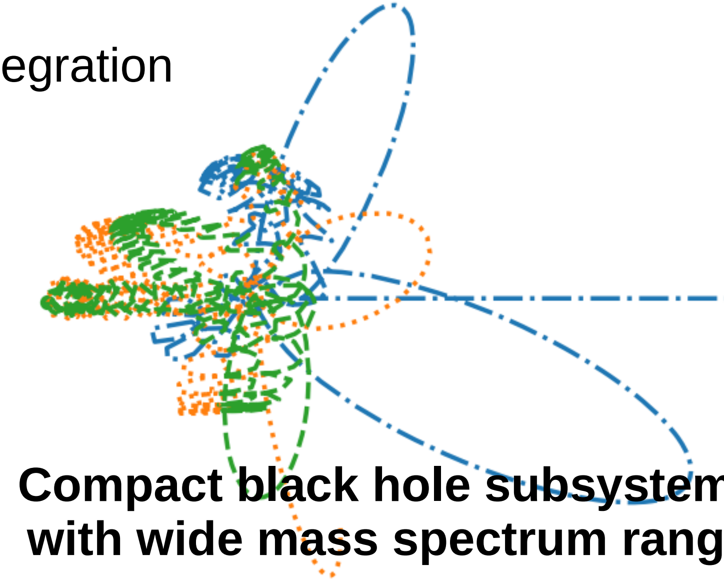
Force explodes when two particles are very close

Consequences:

- large round-off errors
- slow integration
- possible halt of the integration



Multi-planet systems with complex hierarchies



Compact black hole subsystems with wide mass spectrum range

TSUNAMI code

TSUNAMI code

Project ID: 2294947 |

- Mikkola&Tanikawa 1999 regularized Leapfrog

No $\Delta t \longrightarrow 0$ for $a \to \infty$

Solve equation of motions in a time-transformed Hamiltonian

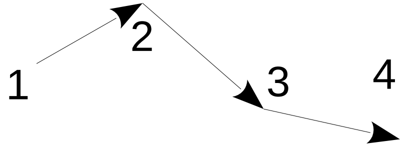
$$dt = g(\mathbf{q}, \mathbf{p}, t)\, ds \qquad g(\mathbf{q}, \mathbf{p}, t) = \frac{1}{U(r_{ij})}$$

- Chain coordinate systems

No round-off errors when

$$\delta r \ll r_{\mathrm{COM}}$$

Chain of interparticle vectors, relative coordinates

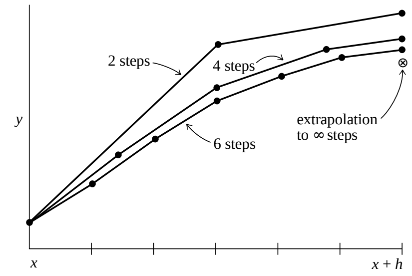


$$\mathbf{R}_1 = 0$$
$$\mathbf{R}_2 = \mathbf{r}_2 - \mathbf{r}_1$$
$$\mathbf{R}_3 = \mathbf{r}_3 - \mathbf{r}_2$$
$$\mathbf{R}_i = \mathbf{r}_i - \mathbf{r}_{i-1}$$

- Bulirsch–Stoer extrapolation

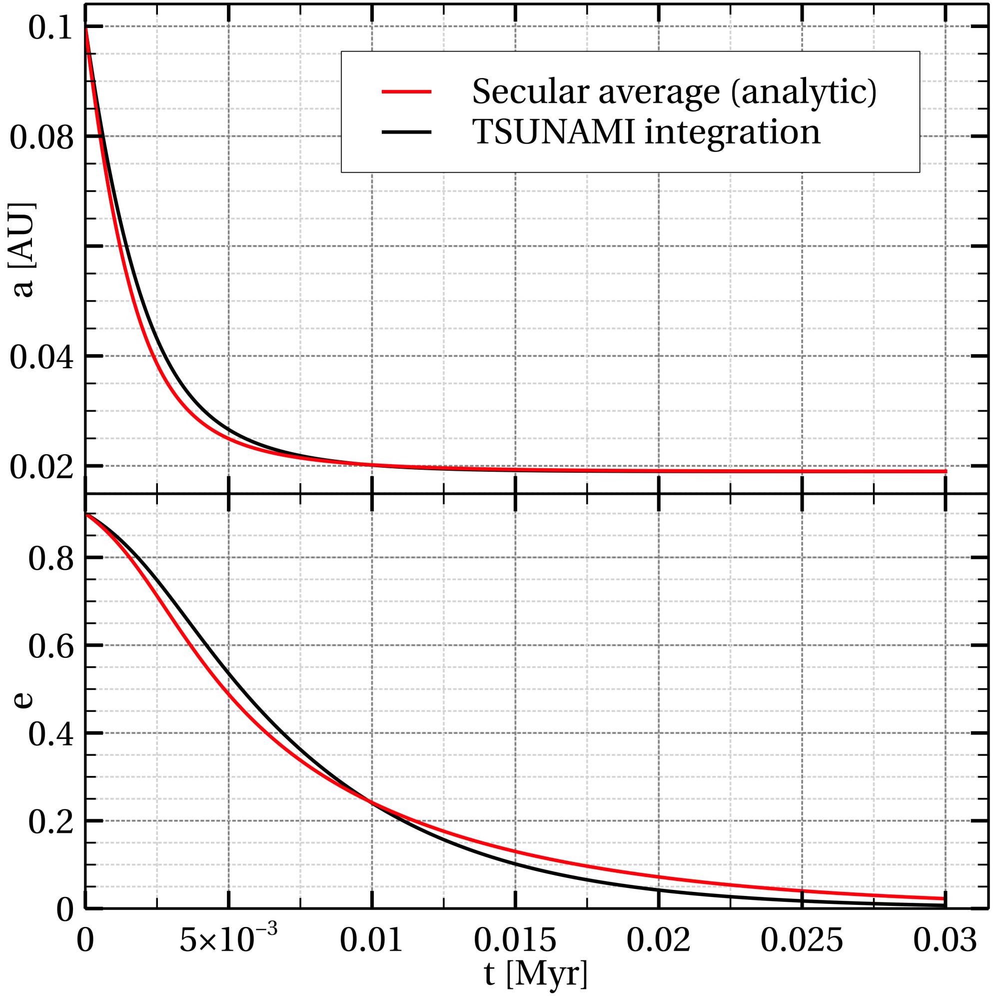
Excellent accuracy over broad dynamical range

Rational functions extrapolation as a function of timestep



TSUNAMI code +TIDAL MODELS for stars & planets

- Hut 1981 equilibrium tide model



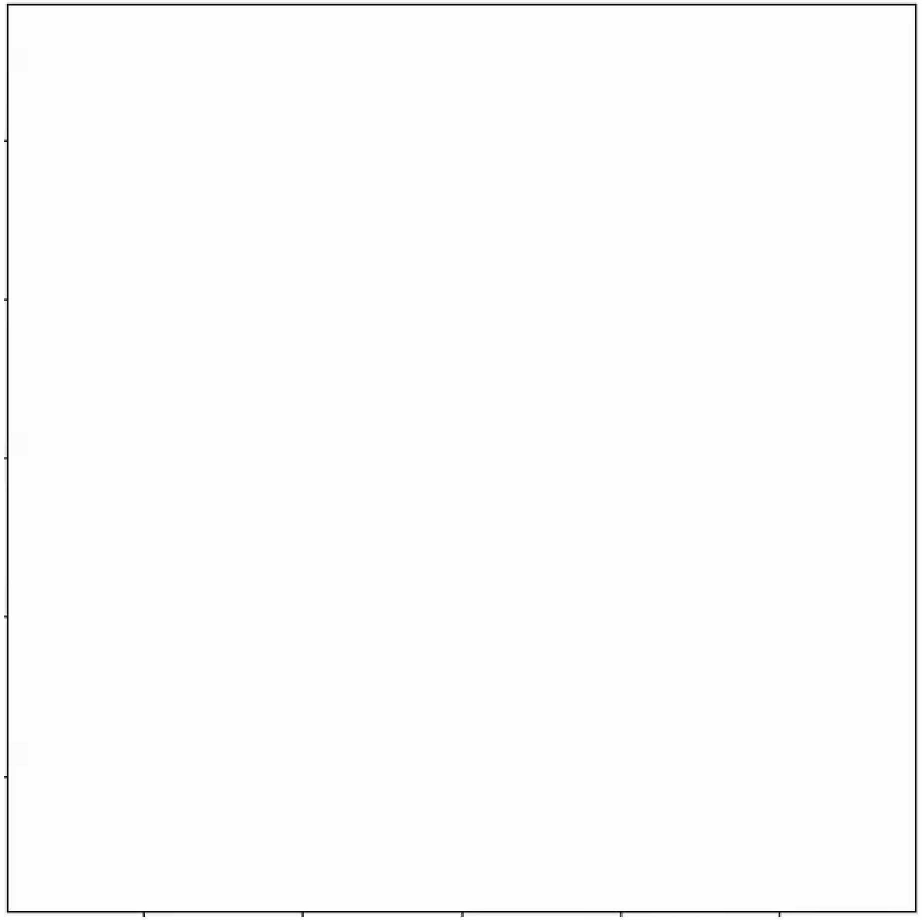
For more complicated tidal models:

TIDYMESS by Tjarda Boekholt

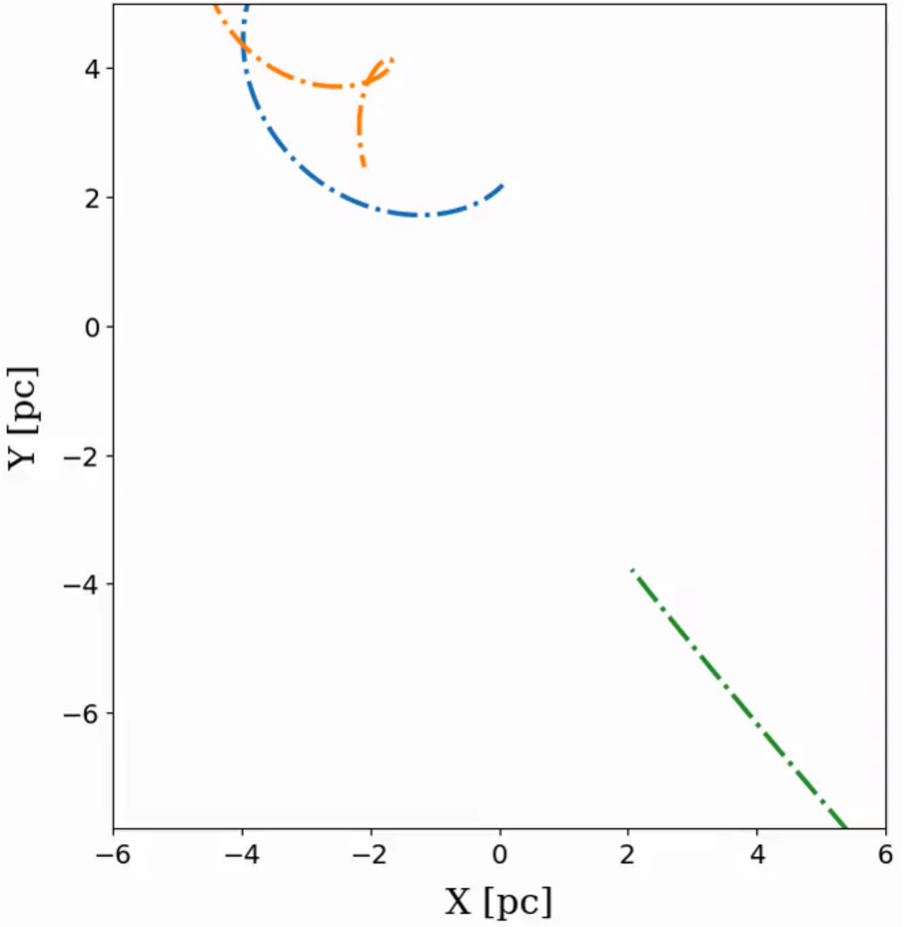
Maxwell viscoelastic rheology

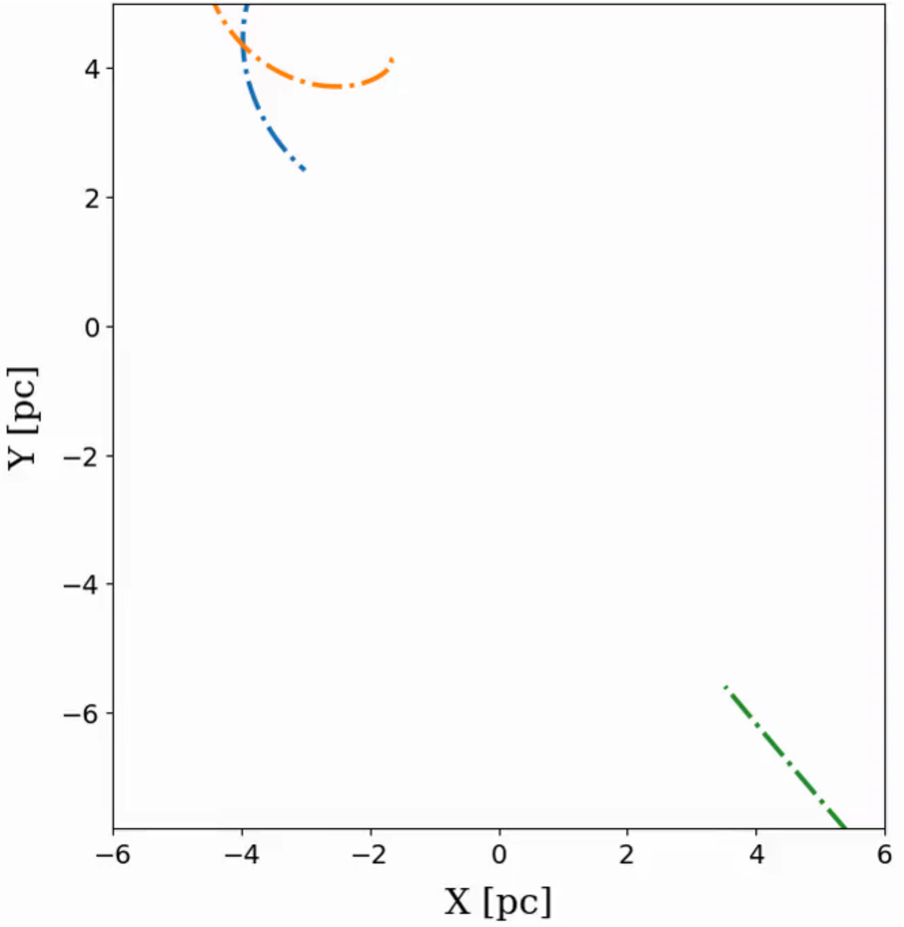
TSUNAMI code +TIDAL MODELS for stars & planets

- Dynamical tide drag model (Samsing, Leigh & AAT 2018, Press & Teukolsky 1977)



TSUNAMI code **+TIDAL MODELS for stars & planets**





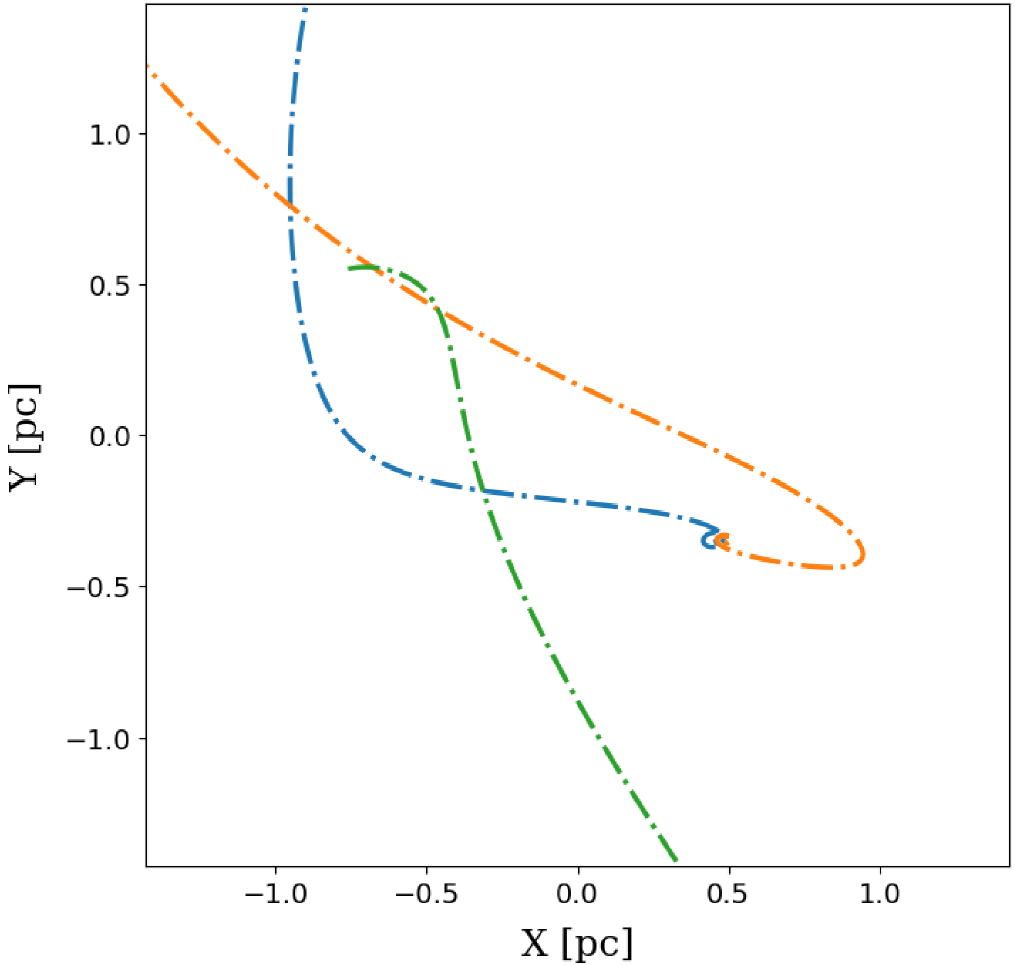
TSUNAMI code +TIDAL MODELS for stars & planets

Equlibrium tide:

Evolution of binary stars
Eccentricity migration
(Kozai-Lidov, scatterings, etc)

Dynamical tide:

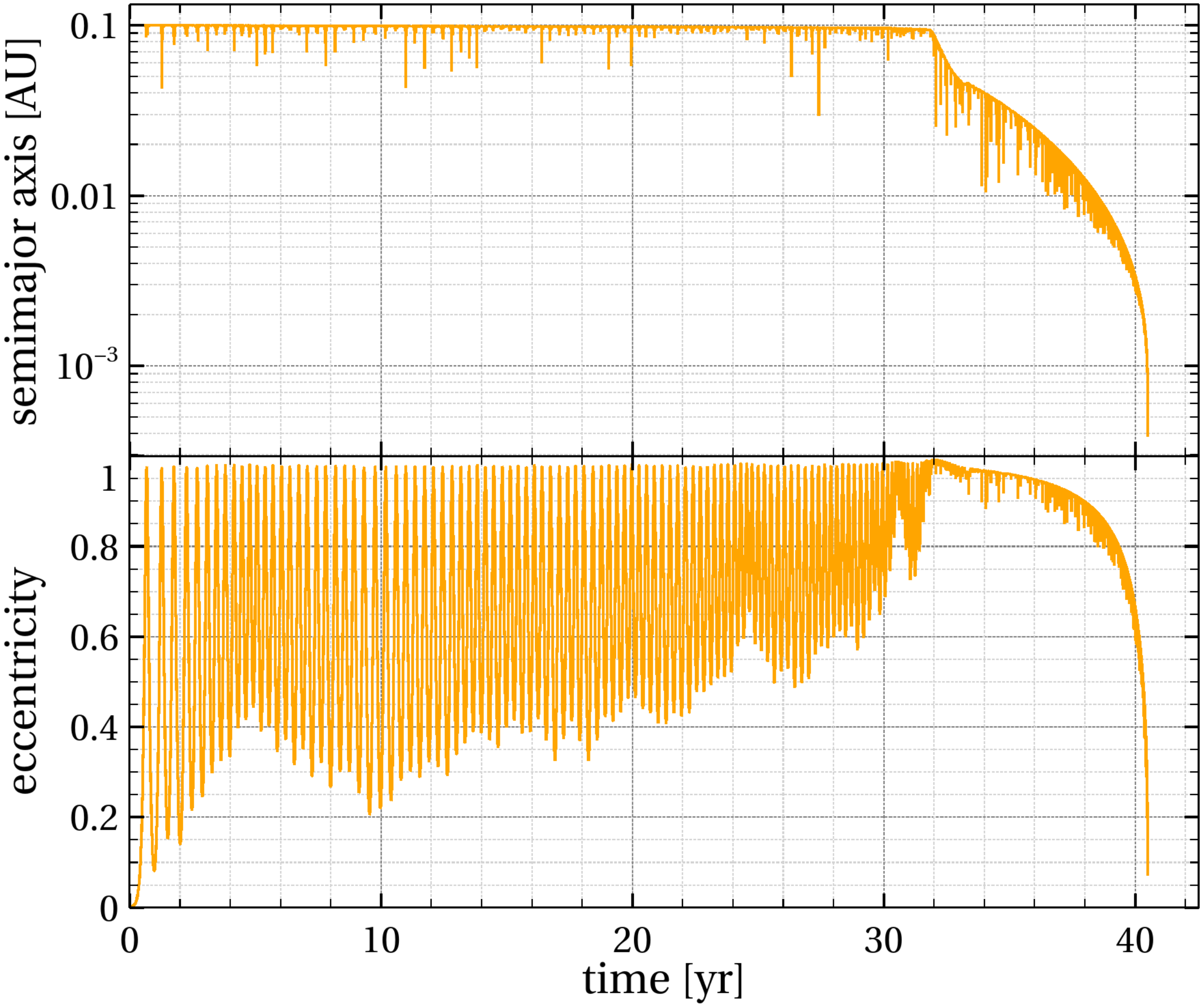
Ultra-short period planets
Tidal capture of moons
Formation of binary planets



TSUNAMI code +POST NEWTONIANS CORRECTIONS

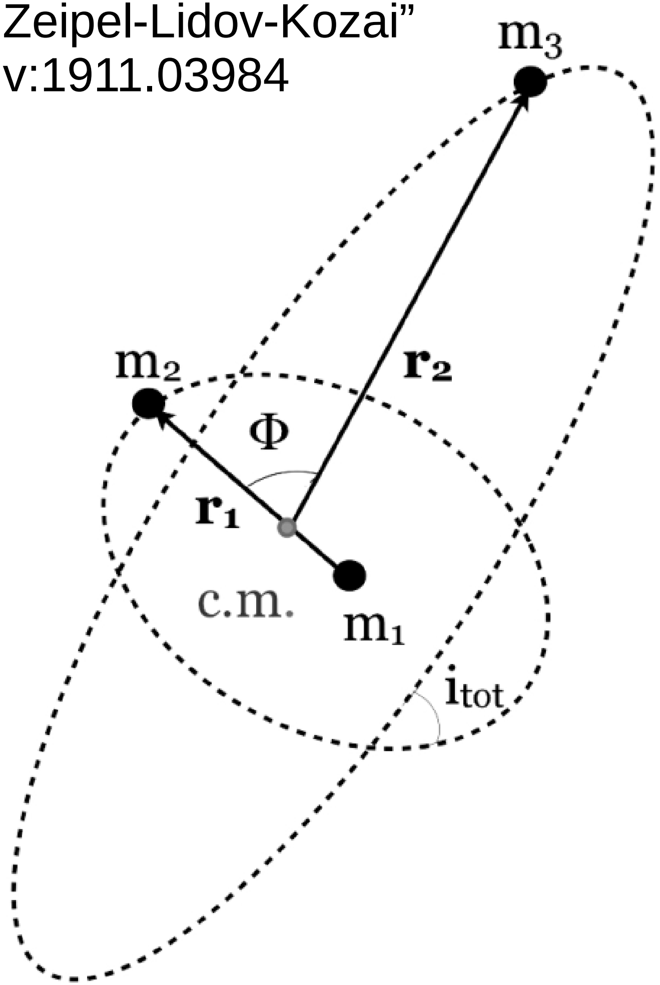
- 1PN (precession)
- 2PN (precession)
- 2.5PN (gravitational radiation)

- WIP spin-orbit coupling terms



OKINAMI module for secular Kozai-Lidov evolution

or rather “von Zeipel-Lidov-Kozai”
see arXiv:1911.03984



Long-term integration of stable triples such as

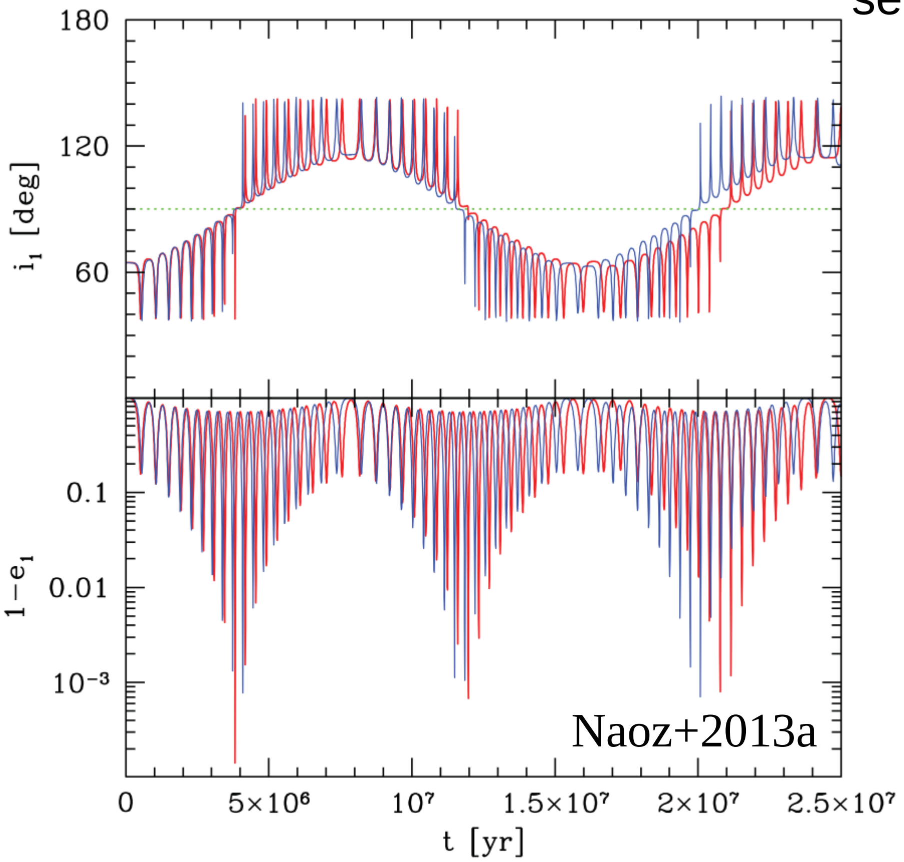
- 2-planet solar systems
- stellar triples
- planet in binary system
- binaries orbiting supermassive black holes
- moon-planet systems

Extremely fast: orbit averaged approach, evolves osculating orbital elements

OKINAMI module for secular Kozai-Lidov evolution

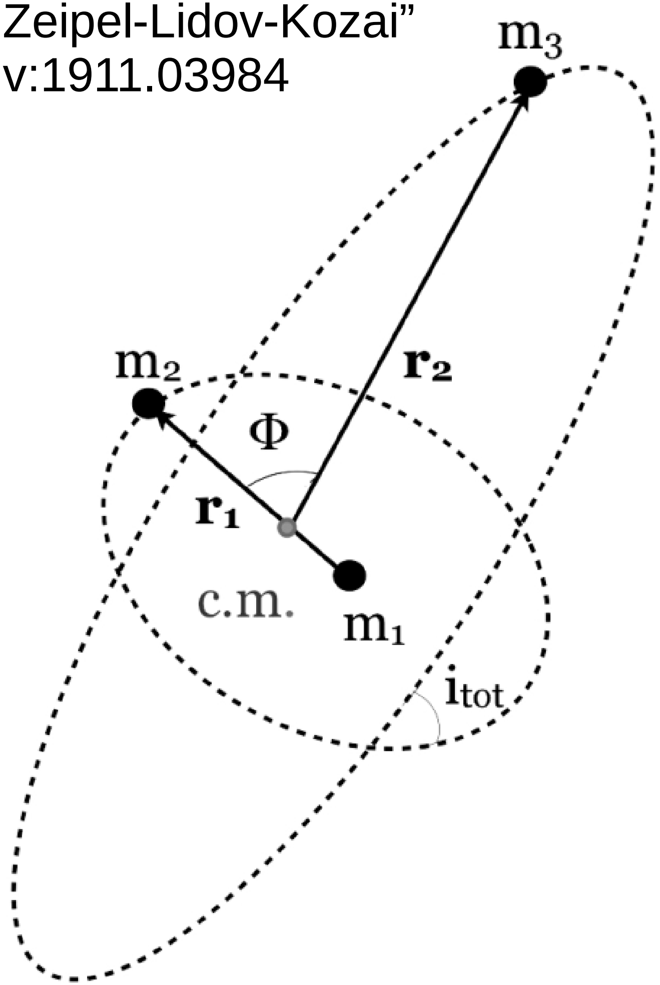
or rather “von Zeipel-Lidov-Kozai”
see arXiv:1911.03984

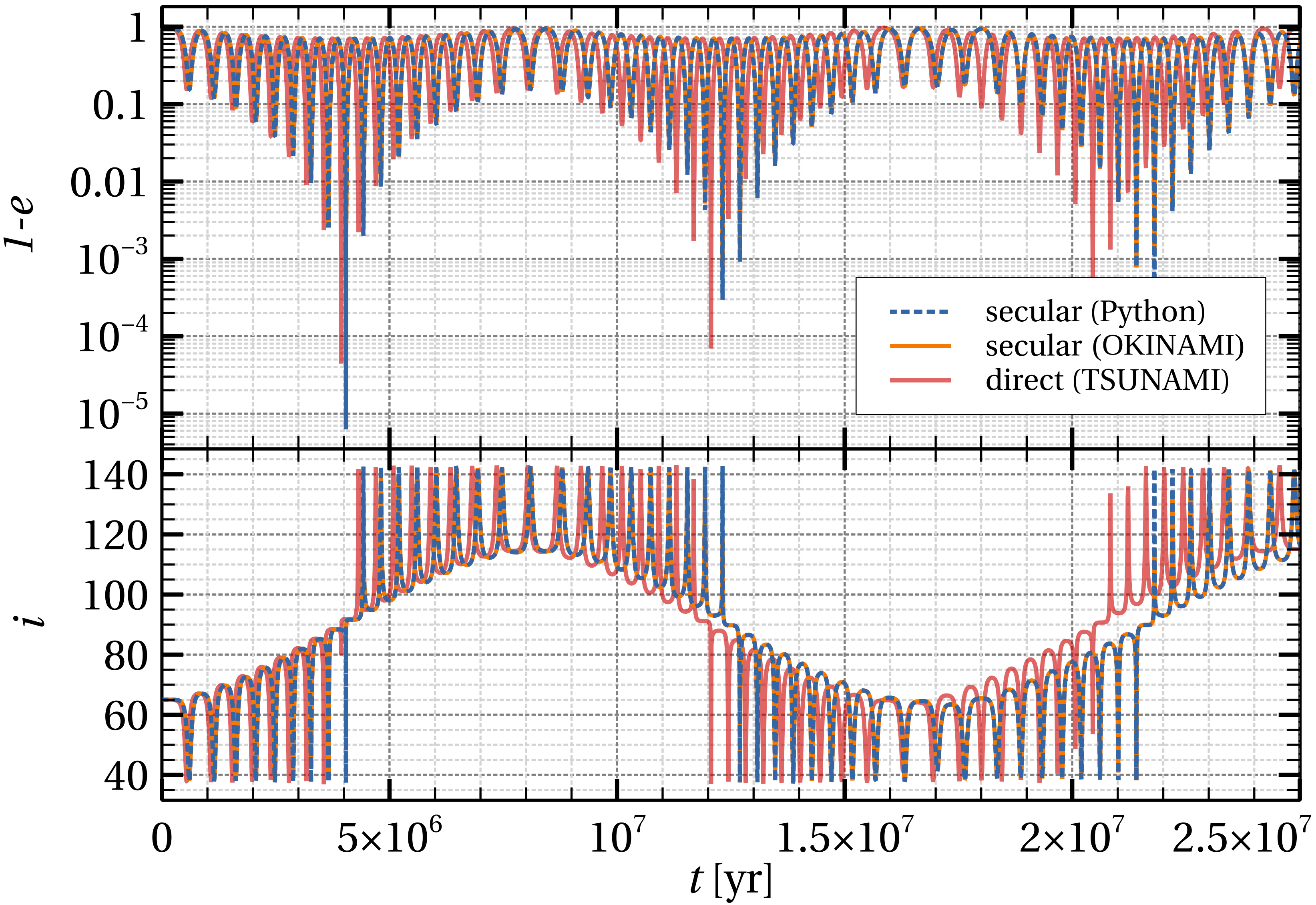
oscillations in eccentricity- inclination of planet perturbed by outer brown dwarf



Naoz+2013a

Figure 3. Comparison between a direct integration (using a B–S integrator)





oscillations in eccentricity- inclination of planet perturbed by outer brown dwarf

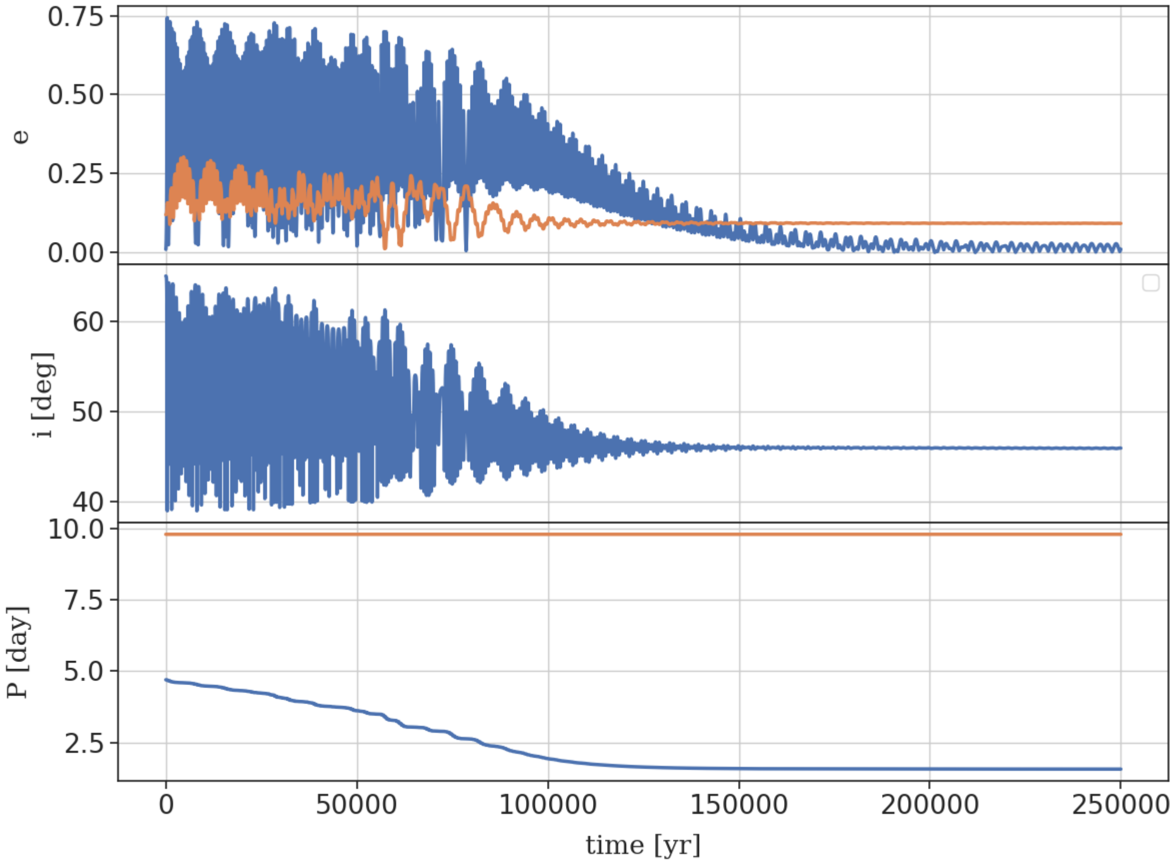
OKINAMI module for secular Kozai-Lidov

+ Post-Newtonian precession and gravitational wave decay

+Inject external derivatives: coupled evolution with binary and single stellar evolution

e.g. BSE with Kozai-Lidov oscillations via Python interface / AMUSE

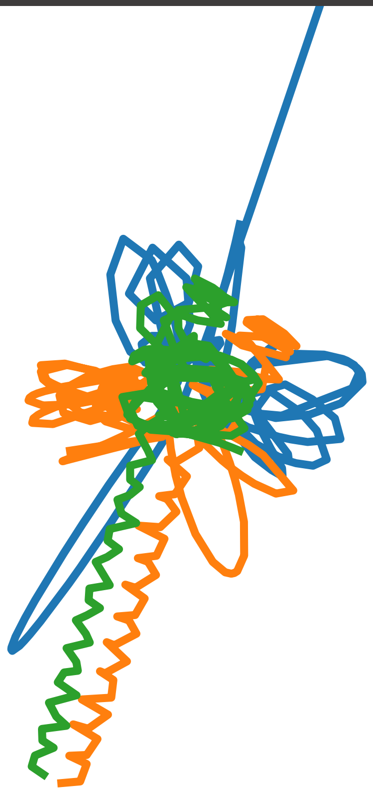
+ Tidal decay for inner and outer orbit



TSUNAMI Python interface

```
import tsunami
import numpy as np
import matplotlib.pyplot as plt

# Initialize units
code = tsunami.Tsunami()

# Initial conditions - pythagorean problem
mass = np.array([3., 4., 5.])
print("m", mass)

pos = np.array([[1.,3.,0.], [-2.,-1.,0.], [1.,-1.,0.]])
print("p", pos)

vel = np.array([[0.,0.,0.], [0.,0.,0.], [0.,0.,0.]])
print("v", vel)

radius = np.array([0., 0., 0.])
ptype = np.array([-1, -1, -1])
code.add_particle_set(pos, vel, mass, radius, ptype)

code.sync_position_and_velocities(pos, vel)
totpos = pos

dt = 0.1
ftime = 70
ctime = 0
while(ctime < ft):
        ctime += dt

        code.evolve_system(ctime)
        ctime = code.ctime
        code.sync_position_and_velocities(pos, vel)

        totp = np.vstack((totpos, pos))
```

```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.set_aspect('equal')
ax.plot(totp[::3,0], totp[::3,1], lw=2.5)
ax.plot(totp[1::3,0], totp[1::3,1], lw=2.5)
ax.plot(totp[2::3,0], totp[2::3,1], lw=2.5)
plt.show()
```



PYTHON INTERFACE (both native and through AMUSE):

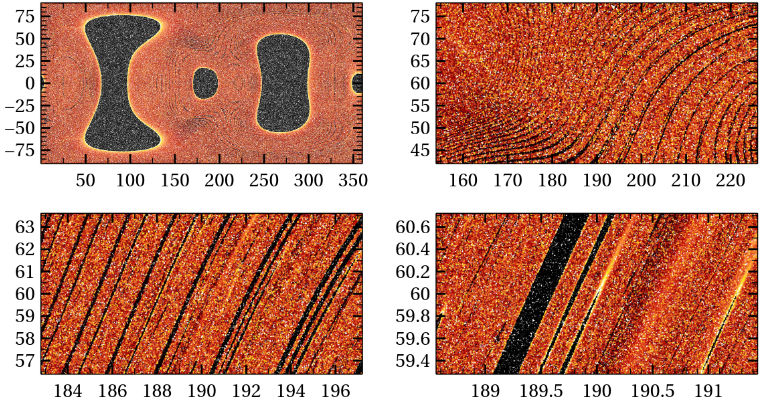
Can be used as a “building block”, embedded in more complicated simulation framework

1. PROMENADE: Monte-Carlo code for the evolution of compact binaries around supermassive black holes (**AAT**, in prep)

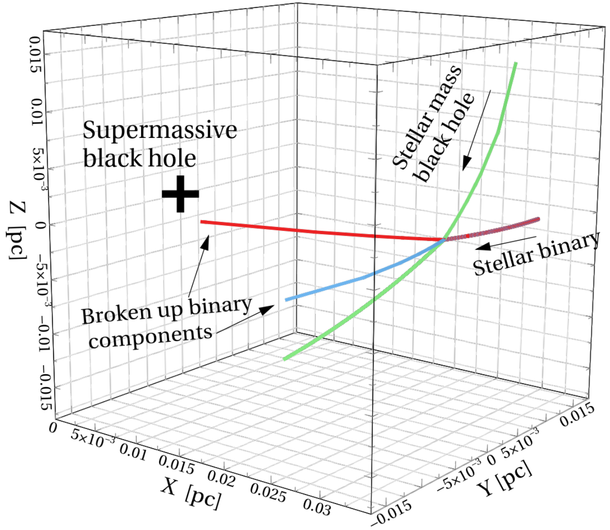
2. CuspBuild: Monte-Carlo code for the repeated black hole scatterings in stellar clusters (Atallah, in prep.)

Possible applications:

- *Exomoon Fates around Migrating Hot Jupiters* (**AAT**, Hamers, Geller & Spera, 2020)
- *Understanding Chaos in the Three-Body Problem* (Manwadkar, **AAT** & Leigh, 2020)



- *Black Hole Dynamics close to Supermassive Black Holes* (**AAT**, et al. 2019)
- *Explaining the Origin of S-Stars in the Galactic Center* (**AAT**, Fujii & Spera 2019)



\+ many ongoing works from me and collaborators

TSUNAMI is a modern, user friendly, regularized code for few-body dynamics

+OKINAMI
its secular counterpart for hierarchical triples

Ideally suited for

- Scatterings simulations
- Unstable planetary systems
- Complex hierarchical systems with high mass ratios
- Tidally evolving stellar/planetary systems
- Black hole dynamics

TSUNAMI code

Project ID: 2294947

“*There is no ‘best’ integrator: only the most suitable for your physical problem*” – generic quote

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