The General Exponential form of a Neutrosophic Complex Number

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Abstract

In this paper, the general exponential form of a neutrosophic complex number is defined by virtue of the formula for indeterminacy in the angle $(\theta + \vartheta I)$, where $(\theta + \vartheta I)$ is the indeterminate angle between two indeterminate parts of the coordinate axes $(x - axis \ and \ y - axis)$, and the general trigonometric form of a neutrosophic complex number is defined. In addition, we also provide theorems with proofs for how to find the conjugate of neutrosophic complex numbers by using the general exponential form, division of neutrosophic complex numbers by the general exponential form, multiplying two neutrosophic complex numbers by the general exponential form, and the inverted neutrosophic complex number by the general exponential form.

Keywords: classical neutrosophic numbers, neutrosophic complex numbers, indeterminacy, conjugate, the general exponential form

1. Introduction

As an alternative to the existing logic, Smarandache proposed the neutrosophic logic for representing a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy and contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [4][8]. Moreover, he presented the definition of the standard form of neutrosophic real numbers and conditions for the division of two neutrosophic real numbers, and he further defined the standard form of neutrosophic complex numbers, and found root index $n \geq 2$ of a neutrosophic real and complex number [3][5], studied the concept of the neutrosophic probability [4][6] and the neutrosophic statistics [5][7]. Then, Smarandache initiated the concept of preliminary calculus of the differential and integral calculus, where he first introduced the notion of neutrosophic mereo-limit, mereo-continuity, mereoderivative and mereo-integral [1][8]. Y. Alhasan presented the concept of neutrosophic complex numbers and its properties that include the conjugate of neutrosophic complex numbers, division of neutrosophic complex numbers, the inverted neutrosophic complex numbers and the absolute value of a neutrosophic complex numbers along with theories related to the conjugate of neutrosophic complex numbers, the product of a neutrosophic complex number by its conjugate equals the absolute value of numbers [2]. Madeleine Al-Taha presented some results on single valued neutrosophic (weak) polygroups [10]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and the right-hand side represented triangular neutrosophic numbers [11]. Chakraborty used pentagonal neutrosophic number in networking problems and shortest path problems [12][13]. Neutrophology logic provided brilliant mathematical theories as a generalization of fuzzy and crisp logic, such as the neutrosophic set theory, it followed the introduction of the neutrosophic concepts in Literature [14] and Literature [15]. Wadei Al-Omeri introduced the

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concept of neutrosophic crisp sets, investigated the properties of continuous, open and closed maps in neutrosophic crisp topological spaces [16].

Complex numbers are of great importance in our daily life, as they greatly help us in making mathematical operations, and they also provide a system for finding solutions to mathematical equations that have no solutions in the real number group, and one of the most prominent tools in the field of electrical engineering, calculating electric voltage and measuring electric current.

This paper aims to study the neutrosophic logic in the complex numbers by defining the generalized exponential form of a neutrosophic complex number. We find the conjugate, inverted of neutrosophic complex number by the general exponential form, division, and multiplying of neutrosophic complex numbers by the general exponential form. Moreover, we also provide some examples that reinforce a easy understanding of the current paper.

2. Preliminaries

2.1 Neutrosophic Real Numbers [5]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$, where a , b are real coefficients, and I represents the indeterminacy, such that $0.\mathrm{I} = 0$ and $\mathrm{I}^n = \mathrm{I}$ for all positive integers n.

2.2 Neutrosophic Complex Numbers [5]

Suppose that z is a neutrosophic complex number, then it takes the following standard form: $z = a + cI + bi + diI$, where a , b, c, d are real coefficients, and I represents the indeterminacy, such that $i^2 = -1 \Rightarrow i = \sqrt{-1}$.

Note: we can say that any real number can be considered as a neutrosophic number.

For example: $2 = 2 + 0.I$, $or: 2 = 2 + 0.I + 0.i + 0.i.I$

2.3 Multiplying two neutrosophic complex numbers [3]

Let z_1 , z_2 be two neutrosophic complex numbers, where

$\mathrm{z}_1 = \mathrm{a}_1 + \mathrm{c}_1\mathrm{I} + \mathrm{b}_1\mathrm{i} + \mathrm{d}_1\mathrm{iI}$, $\mathrm{z}_2 = \mathrm{a}_2 + \mathrm{c}_2\mathrm{I} + \mathrm{b}_2\mathrm{i} + \mathrm{d}_2\mathrm{iI}$

Then:

$$z_1 . z_2 = (a_1 + c_1I + b_1i + d_1iI)(a_2 + c_2I + b_2i + d_2iI)$$

$$= (a_1a_2 - b_1b_2) + (a_1c_2 + a_2c_1 + c_1c_2 - b_1d_2 - d_1b_2 - d_1d_2)I$$

$$+(a_1b_2 + a_2b_1)i + (a_1d_2 + c_1b_2 + c_1d_2 + b_1c_2 + a_2d_1 + d_1c_2)i.I$$

2.4 Division of neutrosophic real numbers [5]

Suppose that w_1, w_2 are two neutrosophic numbers, where

$$w_1 = a_1 + b_1I \ , \qquad w_2 = a_2 + b_2I$$

To find $(a_1 + b_1I) \div (a_2 + b_2I)$, we can write:

$$\frac{a_1 + b_1I}{a_2 + b_2I} \equiv x + yI$$

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where x and y are real unknowns.

$$a_1 + b_1 I \equiv (a_2 + b_2 I)(x + yI)$$

$$a_1 + b_1 I \equiv a_2 x + (b_2 x + a_2 y + b_2 y)I$$

by identifying the coefficients, we get

$$a_1 = a_2 x$$

$$b_1 = b_2 x + (a_2 + b_{2)} y$$

We only obtain unique one solution, which is shown as follows:

$$\begin{vmatrix} a_2 & 0 \\ b_2 & a_2 + b_2 \end{vmatrix} \neq 0 \quad \Rightarrow \quad a_2(a_2 + b_2) \neq 0$$

Hence: $a_2 \neq 0 \quad and \quad a_2 \neq -b_2$ are the conditions for the division of two neutrosophic real numbers to exist.

Then we have

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} . I$$

2.5 The absolute value of a neutrosophic complex number[2]

Suppose that $z = a + cI + bi + d.iI$ is a neutrosophic complex number, the absolute value of a neutrosophic complex number defined by the following form:

$$|Z| = \sqrt{(a + cI)^2 + (b + dI)^2}$$

2.6 Conjugate of a neutrosophic complex number[2]

Suppose that z is a neutrosophic complex number, where $z = a + cI + bi + d.iI$. We denote the conjugate of a neutrosophic complex number by $\bar{z}$ and define it by the following form:

$$\bar{z} = a + cI - bi - d.iI$$

3. The general exponential form of a neutrosophic complex number

Theorem 3.1

The general exponential form of the neutrosophic complex number is given by the formula:

$$z = |z| e^{i(\theta + \vartheta I)} = r e^{i(\theta + \vartheta I)}$$

where $r = |z|$ is the absolute value of a neutrosophic complex number.

Proof:

We know that:

$$z = a + cI + bi + diI$$

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$$z = \mathrm{r}\left(\frac{a+cI}{r} + \frac{b+dI}{r}\, i\right)$$

$$z = \mathrm{r}\left(\cos(\theta + \vartheta I) + sin\,(\theta + \vartheta I)\, i\right)$$

$$\cos(\theta + \vartheta I) = \frac{x}{r} = \frac{a\ +\ bI}{r} \quad , \quad sin\,(\theta + \vartheta I) = \frac{y}{r} = \frac{c\ +\ dI}{r}$$

Where $(\theta + \vartheta I)$ is the indeterminate angle between two indeterminate parts of the coordinate axes ($x - axis\ and\ y - axis$).

Hece we get:

$$z = re^{i(\theta+\vartheta I)}$$

3.1 The general Trigonometric form of a neutrosophic complex number

Definition 3.1:

The following formula:

$$z = \mathrm{r}\left(\cos(\theta + \vartheta I) + sin\,(\theta + \vartheta I)\, i\right)$$

is called the general Trigonometric form of a neutrosophic complex number.

3.2 Multiplying two neutrosophic complex numbers by the general exponential form

Theorem 3.2

Let $z_1 = r_1 e^{i(\theta_1+\vartheta_1 I)}$ and $z_2 = r_2 e^{i(\theta_2+\vartheta_2 I)}$

Then:

$$z_1 z_2 = r_1 r_2\ e^{i(\theta_1+\theta_2+(\vartheta_1+\vartheta_2)I)}$$

Proof:

$$z_1 z_2 = r_1 e^{i(\theta_1+\vartheta_1 I)} . r_2 e^{i(\theta_2+\vartheta_2 I)}$$

$$z_1 z_2 = r_1 r_2\big((cos(\theta_1 + \vartheta_1 I) + i \sin(\theta_1 + \vartheta_1 I))(cos(\theta_2 + \vartheta_2 I) + i \sin(\theta_2 + \vartheta_2 I))\big)$$

$$z_1 z_2 = r_1 r_2(\ [cos(\theta_1 + \vartheta_1 I)\ cos(\theta_2 + \vartheta_2 I) - \sin(\theta_1 + \vartheta_1 I) \sin(\theta_2 + \vartheta_2 I)] + i[\sin(\theta_1 + \vartheta_1 I) cos(\theta_2 + \vartheta_2 I) + cos(\theta_1 + \vartheta_1 I) \sin(\theta_2 + \vartheta_2 I)])$$

$$z_1 z_2 = r_1 r_2(\ \cos(\theta_1 + \theta_2 + (\vartheta_1 + \vartheta_2)I) + i \sin(\theta_1 + \theta_2 + (\vartheta_1 + \vartheta_2)I))$$

Hence:

$$z_1 z_2 = r_1 r_2\ e^{i(\theta_1+\theta_2+(\vartheta_1+\vartheta_2)I)}$$

Example 3.1

Let $z_1 = 2e^{i(\frac{\pi}{6}+\frac{\pi}{3}I)}$ and $z_2 = 4e^{i(\frac{\pi}{4}+\frac{\pi}{2}I)}$

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Then:

$$z_1 z_2 = 8e^{i(\frac{5\pi}{12}+\frac{5\pi}{6}I)}$$

3.3 Conjugate of a neutrosophic complex number by the general exponential form

Theorem 3.3

Let $z = re^{i(\theta+\vartheta I)}$, then the conjugate of a neutrosophic complex number by the general exponential form is given by the formula:

$$\bar{z} = re^{-i(\theta+\vartheta I)}$$

Proof:

We know that:

$$\bar{z} = a + cI - bi - diI$$

$$\bar{z} = \mathrm{r}\left(\frac{a+cI}{r} - \frac{b+dI}{r}\ i\right)$$

$$\bar{z} = \mathrm{r}\left(\cos(\theta+\vartheta I) - sin\ (\theta+\vartheta I)\ i\right)$$

$$\bar{z} = \mathrm{r}\left(\cos(-(\theta+\vartheta I)) + sin\ (-(\theta+\vartheta I))\ i\right)$$

Hence we get:

$$\bar{z} = re^{-i(\theta+\vartheta I)}$$

3.4 Inverted Neutrosophic complex number by the general exponential form

Let $z = re^{i(\theta+\vartheta I)}$, then

$$\frac{1}{z} = \frac{1}{\mathrm{r}} e^{-i(\theta+\vartheta I)}$$

Proof:

$$\frac{1}{z} = \frac{1}{\mathrm{r}\left(\cos(\theta+\vartheta I) + sin\ (\theta+\vartheta I)\ i\right)}$$

multiply the numerator and denominator by conjugate $(\cos(\theta+\vartheta I) - sin\ (\theta+\vartheta I)\ i)$

then:

$$\frac{1}{z} = \frac{1}{\mathrm{r}} \frac{1}{(\cos(\theta+\vartheta I) + sin\ (\theta+\vartheta I)\ i)} \cdot \frac{(\cos(\theta+\vartheta I) - sin\ (\theta+\vartheta I)\ i)}{(\cos(\theta+\vartheta I) - sin\ (\theta+\vartheta I)\ i)}$$

$$\frac{1}{z} = \frac{1}{\mathrm{r}} \frac{\cos(\theta+\vartheta I) - sin\ (\theta+\vartheta I)\ i}{(\cos^2(\theta+\vartheta I) + sin^2\ (\theta+\vartheta I)\ )} \qquad (*)$$

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But:

$$\cos(\theta + \vartheta I) - sin\,(\theta + \vartheta I)i = e^{-i(\theta+\vartheta I)}$$

$$\cos^2(\theta + \vartheta I) + sin^2\,(\theta + \vartheta I)\ = 1$$

By substitution in (*), we get:

$$\frac{1}{z} = \frac{1}{\mathrm{r}} e^{-i(\theta+\vartheta I)}$$

Special case:

When $r = 1$, then:

$$\bar{z} = \frac{1}{z}$$

Example 3.2

Let $\mathrm{z} = 2e^{i(\frac{\pi}{6}+\frac{\pi}{3}I)}$

Then:

$$\frac{1}{z} = \frac{1}{2} e^{-i(\frac{\pi}{6}+\frac{\pi}{3}I)}$$

Example 3.3

Let $\mathrm{z} = e^{i(\frac{\pi}{4}+\frac{\pi}{2}I)}$

Then:

$$\frac{1}{z} = e^{-i(\frac{\pi}{4}+\frac{\pi}{2}I)}$$

3.5 Division two neutrosophic complex numbers by the general exponential form

Theorem 3.4

Let $z_1 = r_1 e^{i(\theta_1+\vartheta_1 I)}$ $\quad and \quad$ $z_2 = r_2 e^{i(\theta_2+\vartheta_2 I)}$

Then:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2+(\vartheta_1-\vartheta_2)I)}$$

Proof:

$$\frac{z_1}{z_2} = z_1.\frac{1}{z_2}$$

$$\frac{z_1}{z_2} = r_1 e^{i(\theta_1+\vartheta_1 I)}.\frac{1}{r_2} e^{-i(\theta_2+\vartheta_2 I)}$$

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$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1+\vartheta_1 I)} . e^{-i(\theta_2+\vartheta_2 I)}$$

Hence we get:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2+(\vartheta_1-\vartheta_2)I)}$$

Example 3.4

Let $z_1 = 2e^{i(\frac{\pi}{6}+\frac{\pi}{3}I)}$ *and* $z_2 = 4e^{i(\frac{\pi}{4}+\frac{\pi}{2}I)}$

Then:

$$z_1 z_2 = \frac{1}{2} e^{-i(\frac{\pi}{12}+\frac{\pi}{6}I)}$$

5. Conclusions

In this paper, we define the general exponential form of a neutrosophic complex number, and arithmetic perations on it, multiplication, division, conjugate and of a neutrosophic complex number by the general exponential form, and find inverted neutrosophic complex numbers by the general exponential form. This research is considered as one of the important researches in neutrosophic complex numbers.

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