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Article

A Study of Neutrosophic Differential Equation by Using a Neutrosophic Thick Function

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Abstract: This paper, is an extension of another paper that was previously published in IJNS entitled "A study of the integration of neutrosophic thick function". In this paper, the concept of neutrosophic thick function has been introduced in the definition of other types of differential equations, which are Bernoulli's equation, Exact differentiale quation, Non- Exact differentiale quation, And integrating factors, Addition to the Ricati. Finally, solutions to this equation will be found.

Keywords: The neutrosophic thick function, the integration of neutrosophic thick function, the neutrosophic homogeneous and non-homogeneous differential equation.

1. Introduction

Neutrosophic created by F. Smarandache, is a new logic in the mathematical world, which relies on the principle of indeterminacy, and this logic is considered as a generalization of fuzzy logic [1], which differs from classical logic. Also, in 2015, F. Smarandache, has defined the concept of continuation of a neutrosophic function in [1], and neutrosophic mereo-limit [1], mereo-continuity. Moreover, in 2014, he has defined the concept of a neutro-oscillator differential in [3], and mereo-derivative. Finally in 2013 he introduced neutrosophic integration in [2], and mereo-integral, besides the classical defintions of limit, continuity, deverative, and integral respectively.

Recently, the neutrosophic crisp set theory may have application in image processing [4], [5] ,the neutrosophic sets [6] have application in the medical field [7], [8], [9], [10] , the field of geographic information systems [11] and possible applications to database [12]. Also, neutrosophic triplet group application to physics [13] . Morever Several researches have made multiple contributions to neutrosophic topological[14],[15], [16], [17], [18], [19], [20], Also More researches have made multiple contributions to neutrosophic analysis[21]. Finally the neutrosophic integration may have application in calculus the areas between tow neutrosophic functions.

2. Preliminaries

In this paper $f(x) = [f_1(x), f_2(x)]$ is called a Neutrosophic thick function. Now, we recall some definitions which are useful in this paper.

Defintion 2.1. [22] . Let $f(x) = [f_1(x), f_2(x)]$ be a neutrosophic thick function. Then we define the integration of this function such as:

$$\int f(x)dx = \int [f_1(x), f_2(x)]\, dx = \left[\int f_1(x)\, dx + c_1, \int f_2(x)\, dx + c_2\right] = [A, B]$$

Where $c_1 = a_1 + b_1 I_1 , c_2 = a_2 + b_2 I_2$.

3. Neutrosophic Bernoulli's equation.

In this section we defined the Neutrosophic bernoulli equation based on the thick function and find solutions to this equation.

Defintion 3.1 .

This equation takes the following form:

$$\acute{y} + [p_1(x), p_2(x)]y = q(x)y^n \dots\dots (1)$$

$$\acute{y} + p(x)y = [q_1(x), q_2(x)]y^n \dots\dots (2)$$

$$\acute{y} + [p_1(x), p_2(x)]y = [q_1(x), q_2(x)]y^n \dots\dots (3)$$

Now I will rely on the first model and the rest of the models in the same way:

$$\acute{y} + [p_1(x), p_2(x)]y = q(x)y^n$$

Method of solution.

1- We dived the ends of the equation (1) by y^n:

$$\acute{y}y^{-n} + [p_1(x), p_2(x)]y^{-n+1} = q(x) \dots\dots (4)$$

2- Now let:

$$z = y^{-n+1}$$

Then:

$$\acute{z} = (-n+1)y^{-n}\acute{y}$$

$$\acute{y} = \frac{y^n \acute{z}}{-n+1}$$

3- We substitute into equation (1):

$$\acute{z}y^{-n} + (-n+1)[p_1(x), p_2(x)]z = (-n+1)q(x) \dots\dots (5)$$

It's a Neutrosophic non- homogeneous linear differential equation, we studied this equation in [22], we obtain the solution of this equation:

$$z = \frac{1}{\mu(x)}\left(a + bI + \int \mu(x)q(x)dx\right)$$

Where $\mu(x)$ is the complement to the equation (5).

4- We obtain the general solution of the equation (1):

$$y = \{z\}^{\frac{1}{-n+1}}$$

Example 3.2. Find the general solution for the following neutrosophic bernoulli equation:

$$\acute{y} - \left[x, \frac{1}{x}\right]y = xy^3$$

Solution. The equation given from the first form:

$$\acute{y} + [p_1(x), p_2(x)]y = q(x)y^n$$

We dived the ends of the equation by y^3:

$$\acute{y}y^{-3} - \left[x, \frac{1}{x}\right]y^{-2} = x \dots\dots (6)$$

Now let:

$$z = y^{-2}$$

Then:

$$\acute{z} = -2y^{-3}\acute{y}$$
$$\acute{y} = \frac{-y^3\acute{z}}{2}$$

Then:

$$\acute{z} + \left[2x, \frac{2}{x}\right] z = -2x$$

It's a Neutrosophic non- homogeneous linear differential equation, the complement of this equation as follow:

$$\mu(x) = \left[e^{x^2}, x^2\right]$$

We find that the general solution is written as:

$$z = \frac{1}{\mu(x)}\left(a + bI + \int \mu(x)q(x)dx\right)$$
$$z = \frac{1}{\left[e^{x^2}, x^2\right]}\left(a + bI + \left[\int -2xe^{x^2}dx\,, \int -2x^3dx\,\right]\right)$$
$$z = \frac{1}{\left[e^{x^2}, x^2\right]}\left(a + bI + \left[\int -e^{x^2}dx\,, \int \frac{-x^4}{2}dx\,\right]\right)$$

We obtain the general solution of the equation:

$$y = \left\{\frac{1}{\left[e^{x^2}, x^2\right]}\left(a + bI + \left[-e^{x^2}, \frac{-x^4}{2}\right]\right)\right\}^{\frac{-1}{2}}$$

Example 3.3. Find the general solution for the following neutrosophic bernoulli equation:

$$\acute{y} + \frac{1}{x}\,y = \left[-\ln(x)\,, \frac{1}{x}\right] y^2$$

Solution. The equation given from the second form:

$$\acute{y} + p(x)y = [q_1(x), q_2(x)]y^n$$

We dived the ends of the equation by y^2:

$$\acute{y}y^{-2} + \frac{1}{x}y^{-1} = \left[-\ln(x)\,, \frac{1}{x}\right] \quad \ldots\ldots\ (7)$$

Now let:

$$z = y^{-1}$$

Then:

$$\acute{z} = -y^{-2}\acute{y}$$
$$\acute{y} = -y^2\acute{z}$$

Then:

$$\acute{z} - \frac{1}{x}z = \left[\ln(x)\,, \frac{-1}{x}\right]$$

It's a Neutrosophic non- homogeneous linear differential equation, the complement of this equation as follow:

$$\mu(x) = \frac{1}{x}$$

We find that the general solution is written as:

$$z = \frac{1}{\mu(x)}\left(a + bI + \int \mu(x)q(x)dx\right)$$

$$z = \frac{1}{\frac{1}{x}}\left(a + bI + \left[\int \frac{\ln(x)}{x}dx\,, \int \frac{-1}{x^2}dx\right]\right)$$

$$z = x\left(a + bI + \left[\ln(\ln x)\,, \frac{1}{x}\right]\right)$$

We obtain the general solution of the equation:

$$y = \left\{x\left(a + bI + \left[\ln(\ln x)\,, \frac{1}{x}\right]\right)\right\}^{-1}$$

Example 3.4. Find the general solution for the following neutrosophic bernoulli equation:

$$\acute{y} + [tanx, cotx]y = [sinx, cosx]y^2$$

Solution. The equation given from the second form:

$$\acute{y} + [p_1(x), p_2(x)]y = [q_1(x), q_2(x)]y^n$$

We dived the ends of the equation by y^2:

$$\acute{y}y^{-2} + [tanx, cotx]y^{-1} = [sinx, cosx] \;\ldots\ldots\; (8)$$

Now let:

$$z = y^{-1}$$

Then:

$$\acute{z} = -y^{-2}\acute{y}$$

$$\acute{y} = -y^2\acute{z}$$

Then:

$$\acute{z} + [-tanx, -cotx]z = [-sinx, -cosx]$$

It's a Neutrosophic non- homogeneous linear differential equation, the complement of this equation as follow:

$$\mu(x) = \left[cosx, \frac{1}{sinx}\right]$$

We find that the general solution is written as:

$$z = \frac{1}{\mu(x)}\left(a + bI + \int \mu(x)q(x)dx\right)$$

$$z = \frac{1}{\left[cosx, \frac{1}{sinx}\right]}\left(a + bI + \left[\int -sinx\; cosxdx\,, \int \frac{-cosx}{sinx}dx\right]\right)$$

$$z = \frac{1}{\left[cosx, \frac{1}{sinx}\right]}\left(a + bI + \left[\frac{1}{4}cos2x\,, -\ln(sinx)\right]\right)$$

We obtain the general solution of the equation:

$$y=\left\{\frac{1}{\left[cosx,\frac{1}{sinx}\right]}\left(a+bI+\left[\frac{1}{4}cos2x\,,-\ln(sinx)\right]\right)\right\}^{-1}$$

4. Neutrosophic Exact differential equation.

In this section we defined the Neutrosophic Exact differential equation based on the thick function and find solutions to this equation.

Defintion 4.1 . Let the differential equation:

$$[p_1(x,y),p_2(x,y)]dx+[q_1(x,y),q_2(x,y)]dy=0 \quad \ldots\ldots (9)$$

We say the Neutrosophic differential equation (9) , is a Exact differential equation if …. The conditions:

$$\frac{\partial p_1}{\partial y}=\frac{\partial q_1}{\partial x}$$

$$\frac{\partial p_2}{\partial y}=\frac{\partial p_2}{\partial x}$$

We obtain the general solution of this equation:

$$\left[\int_{x_0}^{x}\boldsymbol{p_1(x,y)dx}\,,\int_{x_0}^{x}\boldsymbol{p_2(x,y)dx}\right]+\left[\int_{y_0}^{y}\boldsymbol{q_1(x_0,y)dy}\,,\int_{y_0}^{y}\boldsymbol{q_2(x_0,y)dy}\right]=\boldsymbol{a+bI}\ \ldots\ldots\ \mathbf{(10)}$$

Which $\boldsymbol{x_0},\boldsymbol{y_0}$ is arbitrary constants.

Example 4.2. Prove the equation is a exact differential equation, and Find the general solution for this equation:

$$\boldsymbol{[3x^2+6xy^2,y-2x^3]dx+[6xy^2+4y^3,x]dy=0}$$

Solution. We note:

$$\frac{\partial p_1}{\partial y}=12xy\,,\frac{\partial q_1}{\partial x}=12xy\ \Rightarrow\frac{\partial p_1}{\partial y}=\frac{\partial q_1}{\partial x}$$

$$\frac{\partial p_2}{\partial y}=1\,,\frac{\partial q_2}{\partial x}=1\ \Rightarrow\frac{\partial p_2}{\partial y}=\frac{\partial q_2}{\partial x}$$

Then the equation is exact and this solution is:

$$\left[\int_{x_0}^{x}\boldsymbol{p_1(x,y)dx}\,,\int_{x_0}^{x}\boldsymbol{p_2(x,y)dx}\right]+\left[\int_{y_0}^{y}\boldsymbol{q_1(x_0,y)dy}\,,\int_{y_0}^{y}\boldsymbol{q_2(x_0,y)dy}\right]=\boldsymbol{a+bI}$$

Now let $\boldsymbol{x_0=0},\boldsymbol{y_0=0}$, then:

$$\left[\int_{0}^{x}3x^2+6xy^2dx\,,\int_{0}^{x}y-2x^3dx\right]+\left[\int_{0}^{y}4y^3dy\,,\int_{0}^{y}0dy\right]=a+bI$$

$$\left[x^3+3x^2y^2, yx-\frac{1}{2}x^4\right]+[y^4, a_1+b_1I_1] = a+bI$$

$$\left[x^3+3x^2y^2+y^4\ , yx-\frac{1}{2}x^4+a_1+b_1I_1\right] = a+bI$$

5. Neutrosophic non-Exact differential equation and complement factors.

Defintion 5.1 **.** Let the differential equation:

$$[p_1(x,y), p_2(x,y)]dx+[q_1(x,y), q_2(x,y)]dy=0 \ \dots\dots \ (11)$$

We say the Neutrosophic differential equation(11), is a non-Exact differential equation if:

$$\frac{\partial p_1}{\partial y}\neq\frac{\partial q_1}{\partial x}$$

$$\boldsymbol{\frac{\partial p_2}{\partial y}\neq\frac{\partial p_2}{\partial x}}$$

Method of solution.

1- We find the complement to the equation (11) as follows:

$$\mu(z)=[\mu_1(z), \mu_2(z)]$$

Which

$$z=z(x,y)$$

$$\frac{d\ln\mu_1(z)}{dz}=\frac{\frac{\partial p_1}{\partial y}-\frac{\partial q_1}{\partial x}}{q_1\frac{\partial z}{\partial x}-p_1\frac{\partial z}{\partial y}}$$

$$\frac{d\ln\mu_2(z)}{dz}=\frac{\frac{\partial p_2}{\partial y}-\frac{\partial q_2}{\partial x}}{q_2\frac{\partial z}{\partial x}-p_2\frac{\partial z}{\partial y}}$$

2- We multiply the ends of the equation (11) by the complement factor:

$$\boldsymbol{[\mu_1(z)p_1(x,y), \mu_2(z)p_2(x,y)]dx+[\mu_1(z)q_1(x,y), \mu_2(z)q_2(x,y)]dy=0}$$

Then the equation is a exact and obtain this solution by$(\mathbf{10})$.

Example 5.2 Find the general solution for the following equation:

$$\left[2xy+x^2y+\frac{1}{3}y^3, \frac{y}{x^2}-2x\right]dx+\left[x^2+y^2, \frac{1}{x}\right]dy=0$$

Which $\mu(x)=[\mu_1(x), \mu_2(x)]$.

solution.

$$\mu(x)=[\mu_1(x), \mu_2(x)]$$

$$\frac{d\ln\mu_1(x)}{dx}=\frac{\frac{\partial p_1}{\partial y}-\frac{\partial q_1}{\partial x}}{q_1\frac{\partial z}{\partial x}-p_1\frac{\partial z}{\partial y}}=1\Longrightarrow\mu_1(x)=e^x$$

$$\frac{d\ln\mu_2(z)}{dz}=\frac{\frac{\partial p_2}{\partial y}-\frac{\partial q_2}{\partial x}}{q_2\frac{\partial z}{\partial x}-p_2\frac{\partial z}{\partial y}}=\frac{1}{x}\Longrightarrow\mu_2(x)=x^2$$

Then:

$$\mu(x)=[e^x,x^2]$$

We multiply the ends of the equation by the complement factor

$$\left[e^x\left(2xy+x^2y+\frac{1}{3}y^3\right),y-2x^3\right]dx+[e^x(x^2+y^2),x]dy=0$$

The last equation is a exact, and this siolution obtain as follow:

$$\left[\int_{x_0}^{x}e^x\left(2xy+x^2y+\frac{1}{3}y^3\right)dx\,,\int_{x_0}^{x}(y-2x^3)dx\right]+\left[\int_{y_0}^{y}e^{x_0}({x_0}^2+y^2)dy\,,\int_{y_0}^{y}x_0dy\right]=a+bI$$

Now let $\boldsymbol{x_0}=\mathbf{0},\boldsymbol{y_0}=\mathbf{0}$, then:

$$\left[\int_{0}^{x}e^x\left(2xy+x^2y+\frac{1}{3}y^3\right)dx\,,\int_{0}^{x}(y-2x^3)dx\right]+\left[\int_{0}^{y}y^2dy\,,\int_{y_0}^{y}(0)\,dy\right]=a+bI$$

$$\left[yx^2e^x+\frac{1}{3}y^3e^x,yx-\frac{x^4}{2}\right]+\left[\frac{1}{3}y^3,a_1+b_1I_1\right]=a+bI$$

$$\left[yx^2e^x+\frac{1}{3}y^3e^x+\frac{1}{3}y^3,yx-\frac{x^2}{4}+a_1+b_1I_1\right]=a+bI$$

Example 5.3 Find the general solution for the following equation:

$$[5x^2+2xy+3y^3,x^2-y^2+2x]dx+[3x^2+3xy^2+6y^3,x^2-y^2-2y]dy=0$$

Which $\mu(x)=[\mu_1(x),\mu_2(x)]$

solution.

$$\mu(x)=[\mu_1(x),\mu_2(x)]$$

$$\frac{d\ln\mu_1(x)}{dx}=\frac{\frac{\partial p_1}{\partial y}-\frac{\partial q_1}{\partial x}}{q_1\frac{\partial z}{\partial x}-p_1\frac{\partial z}{\partial y}}=\frac{2}{x+y}\Longrightarrow\mu_1(x)=(x+y)^2$$

$$\frac{d\ln\mu_2(z)}{dz}=\frac{\frac{\partial p_2}{\partial y}-\frac{\partial q_2}{\partial x}}{q_2\frac{\partial z}{\partial x}-p_2\frac{\partial z}{\partial y}}=1\Longrightarrow\mu_2(x)=e^{x+y}$$

Then:

$$\mu(x)=[(x+y)^2,e^{x+y}]$$

We multiply the ends of the equation by the complement factor

$$[(x+y)^2(5x^2+2xy+3y^3),(x^2-y^2+2x)e^{x+y}]dx$$
$$+[(x+y)^2(3x^2+3xy^2+6y^3),(x^2-y^2-2y)e^{x+y}]dy=0$$

The last equation is a exact, and this solution obtain as follow:

$$\left[\int_{x_0}^{x}(x+y)^2(5x^2+2xy+3y^3)dx\,,\int_{x_0}^{x}(x^2-y^2+2x)e^{x+y}dx\right]$$

$$+\left[\int_{y_0}^{y}(x_0+y)^2(3{x_0}^2+3x_0y^2+6y^3)dy\,,\int_{y_0}^{y}({x_0}^2-y^2-2y)e^{x_0+y}dy\right]=a+bI$$

Now let $\boldsymbol{x_0=0, y_0=0}$, then:

$$\left[\int_{0}^{x}(x+y)^2(5x^2+2xy+3y^3)dx\,,\int_{0}^{x}(x^2-y^2+2x)e^{x+y}dx\right]+\left[\int_{0}^{y}6y^5dy\,,\int_{0}^{y}(-y^2-2y)e^{y}dy\right]=a+bI$$

$$[x^5+3yx^4+(y^3+2xy+3y^3)x^3+(3y^4+y^3)x^2+3xy^5,(x^2-y^2)e^{x+y}]+[y^6,-y^2e^y]=a+bI$$

$$[(y^3+2xy+3y^3)x^3+(3y^4+y^3)x^2+3xy^5+y^6,(x^2-y^2)e^{x+y}-y^2e^y]=a+bI$$

6. Neutrosophic Recati equation.

In this section we defined the Neutrosophic recati equation based on the thick function and find solutions to this equation.

Defintion 6.1 . We define the Neutrosophic bernoulli equation by a neutrosophic thick function form:

$$\acute{y}+[p_1(x),p_2(x)]y^2+[q_1(x),q_2(x)]y+[r_1(x),r_2(x)]=0\;\ldots\ldots\;(12)$$

And takes a particular solution:

$$y_1=[f_1(x),f_2(x)]$$

Example 6.2. Find the general solution for the following neutrosophic ricati equation:

$$\acute{y}+\left[\frac{cosx}{1-sinx\,cosx},\frac{1}{1-x^3}\right]y^2+\left[\frac{-1}{1-sinx\,cosx},\frac{-x^2}{1-x^3}\right]y+\left[\frac{sinx}{1-sinx\,cosx},\frac{-2x}{1-x^3}\right]=0\;\ldots\ldots\;(13)$$

If a particular solution is:

$$y_1=[cosx,-x^2]$$

Solution. We let:

$$y=[cosx+z_1,-x^2+z_2]$$

$$\acute{y}=[-sinx+\acute{z}_1\,,-2x+\acute{z}_2]$$

We substitute into equation (13):

$$[-sinx+\acute{z}_1\,,-2x+\acute{z}_2]\left[\frac{cosx}{1-sinx\,cosx},\frac{1}{1-x^3}\right][{z_1}^2+2cosxz_1+cos^2x\,,{z_2}^2-2xz_1+4x^4]$$

$$+\left[\frac{-1}{1-sinx\,cosx},\frac{-x^2}{1-x^3}\right][cosx+z_1,-x^2+z_2]+\left[\frac{sinx}{1-sinx\,cosx},\frac{-2x}{1-x^3}\right]=0$$

$$\left[\acute{z}_1-\frac{2cos^2x-1}{1-sinx\,cosx}z_1-\frac{cosx}{1-sinx\,cosx}{z_1}^2,\acute{z}_2+\frac{3x^2}{1-x^3}z_2-\frac{1}{1-x^3}{z_1}^2\right]=0$$

Then:

$$\acute{z}_1-\frac{2cos^2x-1}{1-sinx\,cosx}z_1-\frac{cosx}{1-sinx\,cosx}{z_1}^2=0$$

$$\acute{z}_2+\frac{3x^2}{1-x^3}z_2-\frac{1}{1-x^3}{z_1}^2=0$$

We obtain:

$$z_1 = \left\{\frac{1}{1 - sinx\ cosx}(a_1 + b_1 I_1 + sinx)\right\}^{-1}$$

$$z_2 = \left\{\frac{1}{1 - x^3}(a_2 + b_2 I_2 - x)\right\}^{-1}$$

Then the general solution for (13) is:

$$y = \left[cosx + \left\{\frac{1}{1 - sinx\ cosx}(a_1 + b_1 I_1 + sinx)\right\}^{-1}, -x^2 + \left\{\frac{1}{1 - x^3}(a_2 + b_2 I_2 - x)\right\}^{-1}\right]$$

7. Conclusion

In this paper, a new type of neutrosophic integration has been defined by using the thick function, Moreover, we studied a bernoulli's differential equation, exact differentiale quation, non-exact differentiale quation, Addition to the ricati based on the thick function, and found solutions to this equation. Also solutions of other types of neutrosophic differential equations can be found depending on the thick function such as lagrange equation, claurout equation, darbowx equation. We will work on this in the future.

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