

# MODE CLASSIFICATION AND NATURAL UNITS IN PLAINCHANT

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## ABSTRACT

Many musics across the world are structured around multiple *modes*, which hold a middle ground between scales and melodies. We study whether we can classify mode in a corpus of 20,865 medieval plainchant melodies from the Cantus database. We revisit the traditional ‘textbook’ classification approach (using the final, the range and initial note) as well as the only prior computational study we are aware of, which uses pitch profiles. Both approaches work well, but largely reduce modes to scales and ignore their melodic character. Our main contribution is a model that reaches 93–95%  $F_1$  score on mode classification, compared to 86–90% using traditional pitch-based musicological methods. Importantly, it reaches 81–83% even when we discard all absolute pitch information and reduce a melody to its contour. The model uses tf-idf vectors and strongly depends on the choice of units: i.e., how the melody is segmented. If we borrow the syllable or word structure from the lyrics, the model outperforms all of our baselines. This suggests that, like language, music is made up of ‘natural’ units, in our case between the level of notes and complete phrases, a finding that may well be useful in other musics.

## 1. INTRODUCTION

In his seminal Grove entry, Harold Powers [1] points out a remarkable cross-cultural generalisation: many musics are structured around multiple *modes*. Modes are often associated with the major–minor distinction in Western music, but there are much richer systems of modes: examples include Indian *raga*, Arabic *makam*, Persian *dastgah*, *pathet* in Javanese gamelan music and the *modes* of Gregorian chant. The specifics obviously vary, but all these phenomena share properties with both scales and melodies, and are perhaps best thought of as occupying the continuum in between [1]. On the one hand, a mode is more than a scale: it might imply a hierarchy of pitch relations or favour the use of characteristic motifs. On the other hand, it is not as specific as a particular tune: a mode rather describes a melody *type*. Modes are of central importance to their musical tradition, both as means to classify the repertoire, and

as practical guides for composition and improvisation [1]. Characterising modes computationally is therefore an important problem for *computational ethnomusicology*.

Several MIR studies have investigated automatic mode classification in Indian *raga* [2, 3], Turkish *makam* [4, 5] and Persian *dastgah* [6, 7]. These studies can roughly be divided in two groups. First, studies emphasising the scalar aspect of mode usually look at pitch distributions [2, 5, 7], similar to key detection in Western music. Second, studies emphasising the melodic aspect often use sequential models or melodic motifs [3, 4]. For example, [4] trains  $n$ -gram models for 13 Turkish makams, and then classifies melodies by their perplexity under these models. Going beyond  $n$ -grams, [3] uses motifs, characteristic phrases, extracted from raga recordings to represent every recording as a vector of motif-frequencies. They weigh counts amongst others by the *inverse document frequency* (see section 3.4), which balances highly frequent motifs, and favours specific ones.

In this paper, we focus on automatic mode classification in Medieval plainchant. This has only rarely been studied computationally, even though the term (if not the phenomenon) ‘mode’ originates there. At first glance, mode in plainchant is relatively clear, though certainly not entirely unambiguous. With a second glance, it has a musicological and historical depth that inspired a vast body of scholarship going back over one thousand years. The music is indeed sufficiently distant in time from most other musics, including Western classical and pop music, to provide an interesting cross-cultural comparison. And for once, data is abundant, thanks to the immense efforts of chant scholars.

Chant has mostly figured in MIR studies in optical music recognition of medieval manuscripts: the SIMSSA project, for example, has used such systems to transcribe plainchant from the Cantus database [8]. Recent ISMIR conferences have also included analyses of Byzantine plainchant [9] and Jewish Torah tropes [10], and a comparison of five Christian chant traditions using interval  $n$ -grams [11]. But, to the best of our knowledge, Huron and Veltman’s study [12] is the only computational study addressing mode classification in chant. They took a scalar perspective on mode by using pitch class profiles, an approach which was later criticised, partly for ignoring mode’s melodic character [13].

We aim to revisit this work on a larger dataset, and also to model the melodic aspect of mode. Concretely, we compare three approaches to mode classification:

1. **Classical approach:** based the range, final, and initial note of a chant.



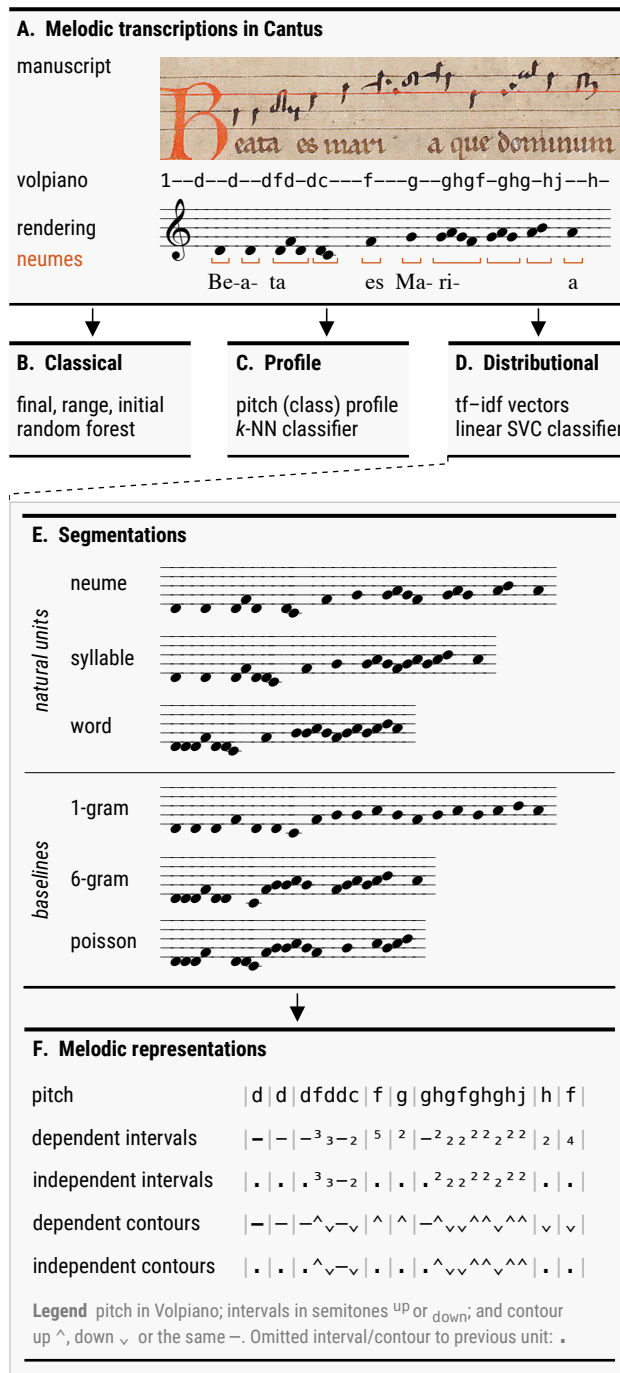
2. **Profile approach:** uses pitch, pitch class and repetition profiles (cf. [12]).
3. **Distributional approach:** uses tf-idf vectors based on various segmentations and representation of the melody.

## 2. GREGORIAN CHANT

Gregorian chant is the monophonic, Latin chant sung during services in the Roman church. It started out as an oral tradition, coexisting with several others in late Antiquity. Although the specifics are debated [14, ch. 2], from the 9th century onwards it gradually turned into a (partly) written tradition, displacing other chant traditions. Initially, only the texts of the chants were written down, as singers would know the melodies by heart. Chant is rooted in recitation, and the music and text are intimately related: “the basic unit of music-writing [was] not the note, but the syllable” [15], the smallest singable unit of text. Accordingly, the earliest notation lived between the lines of text: signs, called *neumes*, reminding the singer of the contour of the melody: perhaps how many notes and their direction, but not *which* exact pitches. The earliest melodies are therefore unknown, but later manuscripts use a pitch-specific notation by placing neumes on staff lines, preserving those melodies to the present day (see Figure 1A).

There are different chant genres for different parts of the liturgy, each with own musical characteristics [16]. Some genres consist of recitations of a sacred text mostly on a fixed pitch, with common starting and ending formulae, while others use elaborate melodies and few repeated notes. Genres also differ in their *melismaticness*: the number of notes per syllable (see Figure S5). In *syllabic* genres like *antiphons*, every syllable of text aligns with roughly one note. More melismatic genres like *responsories* align single syllables to long melismas of ten notes or more. In this paper, we focus on antiphons and responsories, two melodic and common genres.

Gregorian chant uses a distinct tonal system of eight modes, usually numbered 1–8, but sometimes named like church scales. Modes come in pairs that share the same scale (*Dorian, Phrygian, Lydian* or *Mixolydian*), but have a different range or *ambitus*: *authentic* modes moves mostly above the tonal center or the *final*, *plagal* ones mostly around it. Mode 3 is for example also called *Phrygian authentic*, and melodies in this mode rarely go below the final note E. The standard way of determining the mode is to first determine the final, and then the range [16]. For the majority of the chants this will be sufficient, but one might further consider the initial note, characteristic phrases or circumstantial evidence (e.g. psalm tones). Nevertheless, the mode of some chants will remain ambiguous: the *theory* of eight modes was borrowed from Byzantine theory in the 8th century, and applied to an already existing chant repertoire (with its own modalities [13]). The fit between theory and practice was reasonable, but not perfect [1]. This also suggests that perfect classification accuracy is likely out of reach.



**Figure 1. Overview of this study** which compares three approaches to mode classification in a corpus of Gregorian chant. Cantus contributors have transcribed a vast number of melodies from medieval manuscripts (A). We classify mode based on the final, range and initial in the *classical approach* (B), and based on pitch (class) and repetition profiles in the *profile approach* (C). Finally in the *distributional approach* (D), we use tf-idf vectors where we tweak two parameters: the *segmentation*, or which melodic units we use (E), and the *representation* (F), where we gradually discard information about the scale when we move from pitches to contours. In this way we aim to capture the melodic, rather than scalar, aspect of mode.

### 3. METHODS

The design of this study is visualized in Figure 1.

#### 3.1 Data: the Cantus Database

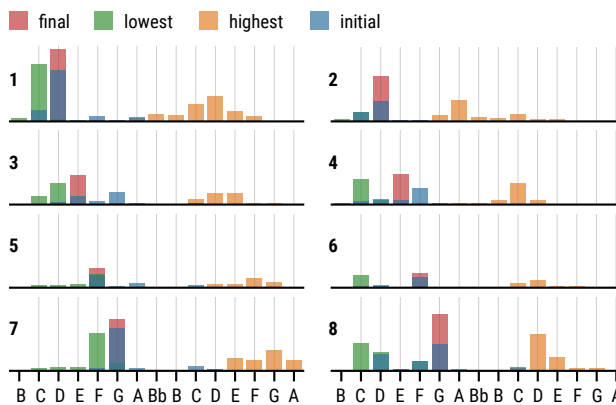
We use chant transcriptions from the Cantus database [17]. This is primarily a digital index of medieval chant manuscripts, recording the chant location in the manuscript, its full text, and properties like the mode, the liturgical feast, but also links to manuscript images. Cantus currently consists of almost 150 manuscripts, containing over 450,000 chants, contributed by chant scholars from all over the world. Over 60,000 chants also contain melodic transcriptions written in Volpiano.<sup>1</sup> It sets plain text as musical notes on a five-line staff, as illustrated in Figure 1a. Volpiano also supports some accidentals, clefs, liquescents, bar-lines and strokes. All submissions to Cantus are subject to strict guidelines and manually checked by the Cantus editors (see also [18]). This ensures the quality and consistency of database, making it a valuable resource for computational research.

We scraped the entire database of 497,071 chants via its REST API and we have released this as the CantusCorpus.<sup>2</sup> We here only consider chants that have a Volpiano transcription (63,628 chants) and further filter out chants with incomplete or non-standard transcriptions, without a complete melody, without ‘simple’ mode annotation, and exact duplicates (see section S1). This resulted in 7031 responsories (966,871 notes, avg. length 138 notes) and 13,865 antiphons (825,143 notes, avg. length 60 notes). We fixed a 70/30 train/test split for all datasets and only used training data in exploratory analyses. Cantus often contains multiple variants of any particular melody, transcribed from different manuscripts (see Figure S11). One may wonder whether the simple train/test split is sufficient, or whether even more care is needed to avoid overlap between such melodic variants in the train and test sets. This is a difficult issue that also applies to other musical corpora (e.g., the Essen folk-song corpus), and for which there is no perfect solution. We tried repeating our experiments on a subset without variants and return to this issue in section 4.4.

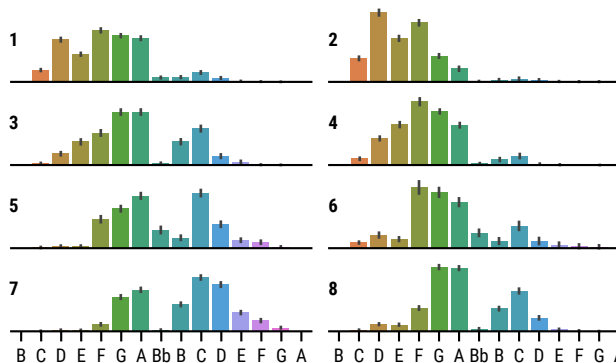
According to the transcription guidelines, flat symbols are transcribed only once, directly before the first flattened note. We replace the first and later flattened notes by the corresponding accidental, a Volpiano character that sits at a specific staff line. In this way, flat notes are also encoded by a single Volpiano character. We discard characters like clefs and pausas, and only retain the notes, accidentals and boundaries (hyphens). The resulting string is used in our three classification experiments, which we now discuss.

#### 3.2 Classical Approach: Final, Range, Initial

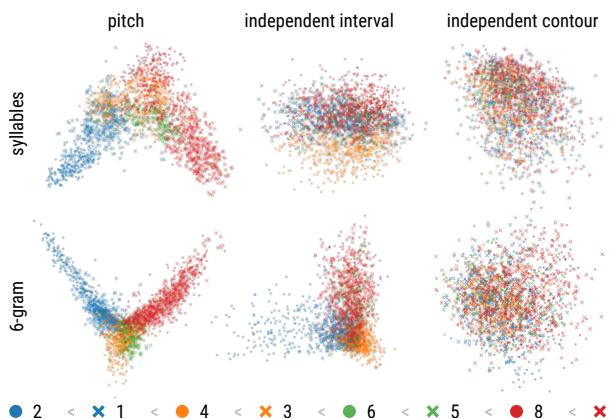
The first approach is motivated by the classical procedure for mode classification. We extract three features from every chant: the final pitch, the range (lowest and highest pitches)



**Figure 2. Classical features.** The classical approach uses the final, range and initial to determine the mode. The overall distribution for each of the modes (1–8) is clearly different, although not entirely without ambiguity.



**Figure 3. Pitch profiles** showing the relative frequency of every pitch in each of the 8 modes. Again, although the distribution of individual modes are clearly distinct, some residual ambiguity remains.



**Figure 4. PCA of tf-idf vectors.** Principal component projection of the tf-idf vectors of responsories in several conditions. The figure suggests that classification gets harder when moving from a pitch to a contour representation. The legend shows a theoretical ordering of the modes based on their range. See Figure S9 and Figure S10 for larger plots.

<sup>1</sup> Volpiano is a typeface developed by David Hiley and Fabian Weber for notating plainchant. See [fawe.de/volpiano/](http://fawe.de/volpiano/)

<sup>2</sup> See [github.com/bacor/cantuscorpus](https://github.com/bacor/cantuscorpus), here we use v0.2.

and the initial pitch. Theory suggests that the final alone should give an accuracy of roughly 50%, and adding the range should further increase that by roughly 50%, if there is no ambiguity. Figure 2 shows the feature distributions for all modes. It suggests that there is some ambiguity, and so numbers will be a little lower. For this task we use random forest classifiers [19], which aggregate multiple decision trees. Training details of all models are discussed below.

### 3.3 Profile Approach: Pitch (Class) Profiles

The second approach is inspired by Huron & Veltman [12]. Using 97 chants from the Liber Usualis, they compute average *pitch class profiles* (the relative frequency of each pitch class) for each of the modes and then classified chants to closest profile. We take a similar approach and use  $k$ -nearest neighbour classification, where  $k$  is tuned (see section 3.5). In a commentary, Wiering [13] argued for using actual pitches rather than pitch classes, as the pitches an octave above the final have a very different role than those an octave below it. We follow that suggestion by also computing *pitch profiles* (Figure 3). Finally, we propose a *repetition profile* aiming to describe which notes function like a recitation tone. For every Volpiano pitch  $q$  we compute a repetition score  $r(q)$ , which is the relative frequency of direct repetitions, and collect these to get a repetition profile. Formally, if a chant has pitches  $p_1, \dots, p_N$ , then  $r(q) = \#\{i : p_i = q \text{ and } p_{i+1} = q\} / (N - 1)$  since there are  $N - 1$  possible repetitions.

### 3.4 Distributional Approach: tf-idf Vectors

Our third approach aims to capture the melodic aspect of mode. In short, we use a bag of ‘words’ model (cf. [3]) and tweak two parameters: the segmentation (which melodic units to use as ‘words’) and the representation (pitches, intervals and contours). The idea is to discard more and more information about the scale, and see if we can nevertheless determine the mode.

First, the units. For chant, three natural segmentations suggest themselves: one can segment the melody (1) at neume boundaries, but also wherever we find (2) a syllable or (3) a word boundary in the lyrics. Given the close relation between text and music in chant, there is some reason to believe that these are meaningful units. Conveniently, all of these boundaries are explicitly encoded in Volpiano, by a single, double and triple dash respectively. Note that these natural units are nested: neumes never cross syllable boundaries. We compare the natural units to two types of baselines. The first is an  $n$ -gram baseline where we slice the melody after every  $n$  notes, for  $n = 1, \dots, 16$ . The second is a random, variable-length baseline. Here the melody is segmented randomly, but in such a way that the segment length is approximately Poisson distributed with a mean length of 3, 5, or 7. We stress that all these units are proper segmentations: units do not overlap. In particular, we choose not to use a higher-order model (using  $n$ -grams of units), because we are only interested in comparing different segmentations.

Second, the representation. We represent melodies in three ways: as a sequence of *pitches*, *intervals* (the number of semitones between successive notes) and *contours* (the contour between successive notes: up, down or level). There is one complication when segmenting sequences of intervals or contours: we introduce dependencies between the units. All units would, for example, start with the interval from the previous unit. We call this a *dependent* segmentation. Alternatively, you could discard the intervals between units to obtain an *independent* version. This effectively makes every unit one interval shorter. We analyse both independent and dependent versions, but in the independent one we found it convenient to start all units (including the first) with a dot to keep the segmentation identical across representations. You can think of the dot as marking the omitted interval to the previous unit.

Third, the model. Given a segmentation, we represent every chant by a vector of unit frequencies, but weighted to favour frequent, yet *specific* units: units that do not occur in too many chants. A standard way of doing this in textual information retrieval is using *term-frequency inverse-document-frequency* (tf-idf) scores, which multiply the frequency of a term in a document (tf) by the inverse document frequency (idf): the inverse of the number of documents containing the term. We use +1 smoothing for the idf, at most 5000 features, and found it was important *not* to set a minimum or maximum document frequency. We train a linear support vector machine to classify mode using the resulting tf-idf vectors.

In sum, we analyse 22 segmentations (3 natural ones, 16  $n$ -grams, 3 random) and 5 representations (pitch and dependent/independent interval/contour), giving a total of 110 conditions.

### 3.5 Training

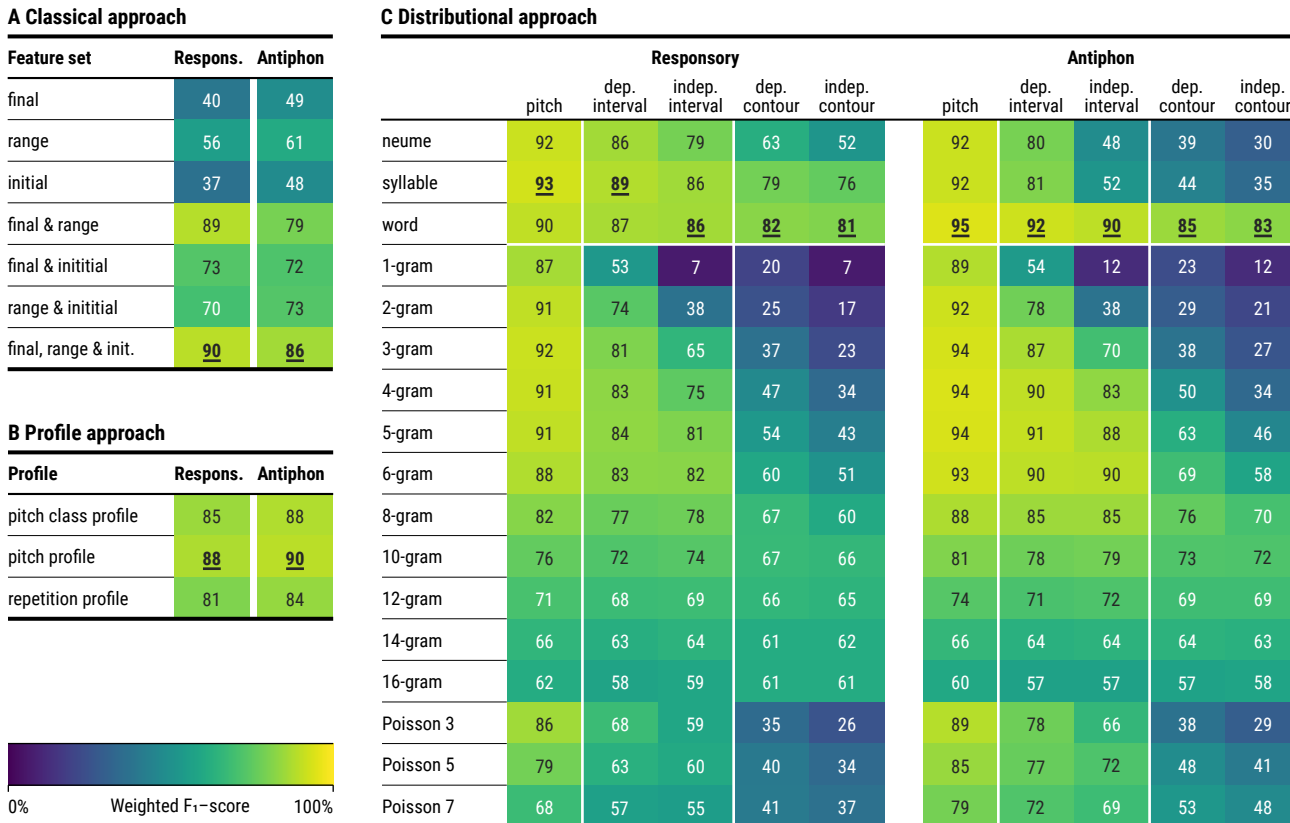
We tune every model using a randomised hyperparameter search with 5-fold stratified cross-validation. That is to say that we randomly sample hyperparameters from a suitable grid (determined by extensive manual analyses) and determine their performance using 5-fold cross-validation on the training set, where we ensure the class frequencies are similar in all folds. We use the hyperparameters yielding the highest cross-validation test accuracy to train the final model. All models were implemented in Python using scikit-learn [20] and data and code are available online.<sup>3</sup>

## 4. RESULTS

Figure 5 gives support-weighted<sup>4</sup> averages of  $F_1$ -scores obtained on the full test sets for all three approaches. The scores are averages of five independent runs of the experiment, using different train/test-splits. Standard deviations were small and are included in figure S12. We now compare the three approaches and then discuss the effect of representation and segmentation on the distributional approach.

<sup>3</sup> See [github.com/bacor/ismir2020](https://github.com/bacor/ismir2020)

<sup>4</sup> The retrieval scores for all classes (modes) are averaged, weighted by the number of instances in each class.



**Figure 5. Classification results.** Weighted  $F_1$ -score for three approaches to mode classification, using two chant genres: responsories and antiphons. Scores are averages of five independent runs of the experiment. The classical approach (A) using the final, range and initial reaches  $F_1$ -scores of 90% and 86%. The profile approach (B) works better for antiphons (90% vs. 86%) and somewhat worse for responsories (88% vs. 90%). As [13] suspected, pitch profiles outperform pitch class profiles by a small margin. The distributional approach (C) reaches the highest  $F_1$  scores of 95% on both responsories and antiphons. The choice of segmentation (vertically) is crucial: classification is improved by using ‘natural’ units, word-based units in particular, rather than  $n$ -grams. As the representation (horizontally) becomes cruder, from pitches to intervals and finally to contours, the task becomes much harder. But, when using word-based segmentation, performance remains high.

#### 4.1 Approaches: Distributional Approach Works Best

First of all, we report the highest classification scores with our distributional approach using pitch representations: an  $F_1$ -score of 93% for responsories and 95% for antiphons. This corresponds of an error reduction of 30–60% compared to the classical approach (90% and 86%). The classical approach confirms the rule of thumb: the range and final are very informative features. Using only these, we obtain  $F_1$ -scores of 89% and 79%, which are further increased by also adding the initial. The profile approach outperforms the classical approach for antiphons (90% vs. 86%), but is outperformed for responsories (88% vs. 90%). Our results support Wiering’s [13] intuition that pitch profiles more accurately describe mode than pitch class profiles, but the effect is small: it increases  $F_1$  scores by 2–3%. Repetition profiles appear to be less useful for both genres.

In broad strokes, our results validate the classical and profile approach, both of which peak around a 90%  $F_1$ -score, using simple features. The distributional approach improves this, up to 95% using complex features. Importantly, we now show that the distributional approach maintains high performance when using interval or contour representations.

#### 4.2 Representations: Contours are Sufficient

We find that the classification task gets harder when the representation gets cruder, from those based on pitch, to intervals and finally to contours (figure 5C, horizontally). This was anticipated: cruder representations are obtained by discarding information from every unit. Shorter units are impacted more by this information loss. For example, the performance with 1-grams drops by over 75% when moving from pitch to independent contour representation. At that point it performs at majority baseline (a 7%  $F_1$ -score for responsories and 12% for antiphons).<sup>5</sup> For longer units such as 10-grams, the drop is not as dramatic (around 10%). However, this comes at the cost of a comparatively low performance using the pitch-representation, presumably because of increasing sparsity.

Natural units, however, escape this trade-off. Word-based segmentations perform consistently well, dropping

<sup>5</sup> For 1-grams in independent interval and contour representation, every unit is identical: a dot representing the omitted contour to the previous note. The majority class for both responsories and antiphons is mode 8, taking up 21% and 28% of the test data respectively (see table S3). This is precisely the accuracy of the model in those conditions.

only 3% below the classical baseline using the highly impoverished independent contour representation. In contrast to the other representations, the contours do not carry any information about the scale: the same contour can be reproduced in any scale. Apparently, we can discard the scalar aspect of mode, and still classify it: contours alone contain sufficient information for mode classification. The success of pitch-based methods might obscure the fact that mode is as much a melodic phenomenon as a scalar one.

It is interesting to note that the earliest chant notation used *unpitched* neumes that mainly described the contour of the melody—not the exact pitches. Our results reinforce the idea that contour is highly informative—so informative that given a mode, text and contour, an experienced singer could reconstruct the chant melody.

### 4.3 Segmentations: Natural Units Work Best.

Our most important result is that among all the representations we considered, natural units (neume, syllables, and words) yield the highest classification performance. The 4- and 6-gram baselines also reach top  $F_1$ -scores in antiphons, but only when we use representations that include information about pitch. Furthermore, the success of natural units cannot be explained solely by their length. In responsories, neumes, syllables and words are on average 2.3, 3.0 and 7.1 notes long, respectively (see table S6), and yet the performance of these natural units is consistently higher than  $n$ -grams of comparable length. The performance of the natural units is also consistently higher than that of the variable-length Poisson baselines, which are intended to mimic the overall distribution of natural lengths but ignore musical and textual semantics.

A few other observations merit discussion. Firstly, although neume and syllable segmentations behave differently for responsories, they behave similarly to each other for antiphons. The reason may be that in antiphons, neumes and syllables more often coincide. Antiphons are less *melismatic* than responsories (i.e., they use fewer notes per syllable, 1.5 to be precise). Secondly, both the  $n$ -grams and the Poisson baseline perform better on antiphons than on responsories, possibly because the  $n$ -grams are more likely to end up being coincidentally aligned with the natural units the less melismatic the genre.

### 4.4 Controlling for Melodic Variants

We repeated all experiments on a subset of the data from which we removed melody variants (see supplement S13 for details). In terms of the number of notes, this meant a 75% and 66% reduction in data size for responsories and antiphons respectively. The performance of all models decreased on this subset, and for responsories more than for antiphons. Our main findings that contours are sufficient and that natural units work best across representations stand. We do observe some reorderings: some already high-performing  $n$ -grams in antiphons now for example slightly overtake word segmentations, although only for pitch and dependent interval representations. The distributional approach works best for antiphons regardless of including or

excluding chant variants, but for responsories, the distributional approach drops slightly below the classical approach on the subset (where the profile approach is worst). These findings might be explained by increased sparsity in the smaller dataset: natural units in responsories are, after all, longer. Exploring these issues further is left for future work.

## 5. DISCUSSION AND CONCLUSION

In this paper, we analyzed three approaches to mode classification in a large corpus of plainchant: (1) the classical approach using the final, range and initial; (2) the profile approach using pitch (class) profiles and (3) the distributional approach using a tf-idf vector model and various segmentations and representations. We found that the distributional approach performs best, and that it can maintain high performance on contour representations if using the right segmentation: at word boundaries, in this case. The main findings were largely upheld when we removed melody variants, but the handling of variants is an issue that deserves further investigation and that has implications beyond this study.

Although our results are specific to one corpus of medieval music and one classification task, we believe our conclusions are of wider relevance. We often fall back on  $n$ -grams because they are well understood and easy to use. A more natural segmentation may be harder to obtain, but if finding them can have such a large effect on a relatively simple task like mode classification, their advantages may be even stronger for more complex tasks.

A first next step could be to explore whether lyrics yield equally useful units in other vocal musics. As noted, the link between text and music in plainchant is particularly tight. This at least suggests that the text may be useful in other types of chant, like Byzantine chant or Torah trope. For folk melodies designed to standard poetic meters, it is not as obvious whether lyrics would help or hinder the identification of useful units. This is worth investigating, as characteristic motifs and repeated pattern are commonly used in computational folk-song studies, in particular for tune family identification [21, 22].

Our results raise another question: is chant indeed composed by stringing together certain melodic units, much like a sentence is composed of words? It has been suggested (and disputed) that Gregorian chant is composed in a process of *centonization*, and that a chant is a patchwork of existing melodic chunks called *centos*. A recent study used the tf-idf weighting to discover centos in Arab-Andalusian music [23]. This raises the possibility that classification using natural units may have been successful because they indeed are the building blocks, the centos.

Chant is not yet commonly studied in the MIR community, but we hope that this study shows that chant is an interesting repertoire that can yield insights of broader relevance. The immense efforts of chant scholars mean that data are abundant. In short, we think chant can aid the development of models that apply beyond Western classical and pop music, and embrace the true diversity of musics around the world.

## 6. REFERENCES

- [1] H. S. Powers, F. Wiering, J. Porter, J. Cowdery, R. Widdess, R. Davis, M. Perlman, S. Jones, and A. Marett, "Mode," in *Grove Music Online*. Oxford University Press, 2001. [Online]. Available: <https://doi.org/10.1093/gmo/9781561592630.article.43718>
- [2] P. Chordia and A. Rae, "Raag recognition using pitch-class and pitch-class dyad distributions," in *Proceedings of the 8th International Society for Music Information Retrieval Conference*, 2007, pp. 431–436.
- [3] S. Gulati, J. Serra, V. Ishwar, S. Senturk, and X. Serra, "Phrase-based rāga recognition using vector space modeling," in *2016 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Shanghai, 2016, pp. 66–70.
- [4] E. Ünal, B. Bozkurt, and M. K. Karaosmanoğlu, "N-gram based statistical makam detection on makam music in Turkey using symbolic data," in *Proceedings of the 13th International Conference on Music Information Retrieval*, 2012, p. 43–48.
- [5] N. B. Atalay and S. Yöre, "Pitch distribution, melodic contour or both? modeling makam schema with multidimensional scaling and self-organizing maps," *New Ideas in Psychology*, vol. 56, p. 100746, Jan. 2020.
- [6] S. Abdoli, "Iranian traditional music dastgah classification," in *Proceedings of the 12th International Conference on Music Information Retrieval*, 2011, pp. 275–280.
- [7] P. Heydarian and D. Bainbridge, "Dastgāh recognition in iranian music: Different features and optimized parameters," in *Proceedings of the 6th International Conference on Digital Libraries for Musicology*. The Hague, the Netherlands: ACM Press, 2019, pp. 53–57.
- [8] K. Helsen, J. Bain, I. Fujinaga, A. Hankinson, and D. Lacoste, "Optical music recognition and manuscript chant sources," *Early Music*, vol. 42, no. 4, pp. 555–558, 2014.
- [9] M. Panteli and H. Purwins, "A computational comparison of theory and practice of scale intonation in Byzantine chant," in *Proceedings of the 14th International Conference on Music Information Retrieval*, 2013, pp. 169–174.
- [10] P. van Kranenburg, D. P. Biro, S. Ness, and G. Tzanetakis, "A computational investigation of melodic contour stability in Jewish Torah trope performance traditions," in *Proceedings of the 12th International Conference on Music Information Retrieval*, 2011, pp. 163–168.
- [11] P. van Kranenburg and G. Maessen, "Comparing offertory melodies of five medieval Christian chant traditions," in *Proceedings of the 18th International Conference on Music Information Retrieval*, 2017, pp. 163–168.
- [12] D. Huron and J. Veltman, "A cognitive approach to medieval mode: Evidence for an historical antecedent to the major/minor system," *Empirical Musicology Review*, vol. 1, no. 1, pp. 33–55, 2006.
- [13] F. Wiering, "Comment on Huron and Veltman: Does a cognitive approach to medieval mode make sense?" *Empirical Musicology Review*, vol. 1, no. 1, pp. 56–60, 2006.
- [14] P. Jeffery, *Re-Envisioning Past Musical Cultures: Ethnomusicology in the Study of Gregorian Chant*, ser. Chicago Studies in Ethnomusicology. University of Chicago Press, 1992.
- [15] T. F. Kelly, "Notation I," in *The Cambridge History of Medieval Music*. Cambridge University Press, 2018, vol. 1, pp. 236–262.
- [16] D. Hiley, *Gregorian Chant*. Cambridge University Press, 2009.
- [17] D. Lacoste, T. Bailey, R. Steiner, and J. Koláček, "Cantus: A database for Latin ecclesiastical chant," <http://cantus.uwaterloo.ca/>, 1987-2019, directed by Debra Lacoste (2011-), Terence Bailey (1997-2010), and Ruth Steiner (1987-1996). Web developer, Jan Koláček (2011-).
- [18] K. Helsen and D. Lacoste, "A report on the encoding of melodic incipits in the Cantus database with the music font 'Volpiano'," *Plainsong and Medieval Music*, vol. 20, no. 01, pp. 51–65, Apr. 2011.
- [19] L. Breiman, "Random forests," *Machine Learning*, vol. 45, no. 1, pp. 5–32, 2001.
- [20] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay, "Scikit-learn: Machine learning in Python," *Journal of Machine Learning Research*, vol. 12, pp. 2825–2830, 2011.
- [21] A. Volk and P. van Kranenburg, "Melodic similarity among folk songs: An annotation study on similarity-based categorization in music," *Musicae Scientiæ*, vol. 16, no. 3, pp. 317–339, 2012.
- [22] B. Janssen, P. van Kranenburg, and A. Volk, "Finding occurrences of melodic segments in folk songs employing symbolic similarity measures," *Journal of New Music Research*, vol. 46, no. 2, pp. 118–134, 2017.
- [23] T. Nuttall, M. G. Casado, V. N. Tarifa, R. C. Repetto, and X. Serra, "Contributing to new musicological theories with computational methods: The case of centonization in Arab-Andalusian music," in *Proceedings of the 20th International Conference on Music Information Retrieval*, 2019, pp. 223–228.