Nine Lepton Flavor Changing Decays

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Introduction

From the experimental observation of neutrino oscillations, lepton flavor violation (LFV) in the neutrino sector has been observed. However, that violation has not yet been observed in the charged-lepton sector and it is not quite certain where it is most likely to be observed first. After discovery of Higgs boson it is pertinent to ask if there could be a connection or mixing between the Higgs sector and the mechanism responsible for the nonconservation of lepton number, and to find out whether some remnant effect could show up in lepton decays, which may be detectable at present or future colliders.

In this investigation we numerically compute the processes $\ell_1^\pm \to \ell_2^\pm \gamma$, $Z \to \ell_1^\pm \ell_2^\mp$, and $h \to \ell_1^\pm \ell_2^\mp$, where ℓ_1 and ℓ_2 are charged leptons with different flavours, in a simple extension of the Standard Model (SM). That extension is a particular case of the scheme proposed in ref. [1]: a multi-Higgs-doublet model where all the Yukawa-coupling matrices are diagonal in lepton-flavour space (because of the invariance of the dimension-four terms in the Lagrangian under the lepton-flavour symmetries) and the violation of those symmetries arises only *softly* through the dimension-three Majorana mass terms of three right-handed neutrinos. In ref. [1] the above-mentioned processes have been computed analytically within that general scheme. We check that analytical computation but express the amplitudes through Passarino–Veltman (PV) functions. That allows us to use the formulas in high-precision numerical computations and to establish the impact of the separate amplitudes on the branching ratios (BRs) of the LFV decays. Although the analytical results allow one to study the LFV processes in a model with an arbitrary number of scalar doublets, in this analysis we only perform the numerical computation in the context of a simple version of the two Higgs doublet model (2HDM).

Scalar sector

In our numerical computations, we work in the context of a 2HDM without scalar $SU(2)$ singlets. We use, without loss of generality, the 'Higgs basis', wherein only the first scalar doublet has a vacuum expectation value:

$$\Phi_1 = \begin{pmatrix} G^\pm \\ (v + X_1 + iG^0)/\sqrt{2} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^\pm \\ (X_2 + iX_3)/\sqrt{2} \end{pmatrix}. \tag{1}$$

In Φ_1, $v \approx 246$ GeV is real, G^0 is the neutral Goldstone boson, and $G^\pm$ is the charged Goldstone boson. In Φ_2, $H^\pm$ is a physical charged scalar with mass $m_{H^\pm}$. The fields $X_{1,2,3}$ are real. We write

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} \begin{pmatrix} S_2^0 \\ S_3^0 \\ S_4^0 \end{pmatrix}, \tag{2}$$

where the matrix T is orthogonal and $S_{2,3,4}^0$ are physical (*i.e.* eigenstate of mass) neutral scalars. In our notation, $S_1^0 \equiv G^0$.

Let $H \equiv h$ be the neutral scalar with mass $m_h \approx 125$ GeV observed at the LHC then, for definiteness, we take $h \equiv S_2^0$.

Feynman diagrams in which the neutral scalar h couples to a pair of either charged scalars or charged gauge bosons do arise in the computation of the decay $h \to \ell_1^\pm \ell_2^\mp$. The relevant vertices are given by ref. [1, 2].

Leptonic sector

Our model has three right-handed neutrinos ν_{Re}, $\nu_{R\mu}$, and $\nu_{R\tau}$. Their Majorana mass terms are given by the 3×3 symmetric matrix M_R as

$$\mathcal{L}_{\nu_R\,\text{mass}} = -\frac{1}{2} \sum_{\ell,\ell'=e,\mu,\tau} (M_R)_{\ell\ell'}\, \bar{\nu}_{R\ell}\, C\, \bar{\nu}_{R\ell'}^T + \text{H.c.}, \tag{3}$$

where C is the charge-conjugation matrix in Dirac space. The flavour-space matrix M_R is non-diagonal. Indeed, $\mathcal{L}_{\nu_R\,\text{mass}}$ is the *only* source of breaking of the family lepton numbers in our model; that breaking is thus *soft*. Because of our assumption that the family lepton numbers are symmetries of the dimension-four part of the Lagrangian, the lepton Yukawa Lagrangian conserves flavour:

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\sum_{k=1}^{2} \sum_{\ell=e,\mu,\tau} \Big[(\Gamma_k)_\ell\, \bar{\ell}_R \left(\Phi_k^- \nu_{L\ell} + \Phi_k^{0*} \ell_L \right) \\ &\quad + (\Delta_k)_\ell\, \bar{\nu}_{R\ell} \left(\Phi_k^0 \nu_{L\ell} - \Phi_k^+ \ell_L \right) \Big] + \text{H.c.},\end{aligned} \tag{4}$$

where the diagonal matrices $(\Gamma_{1,2})_\ell$ and $(\Delta_{1,2})_\ell$ are the Yukawa coupling constants. Note that $(\Gamma_1)_\ell = \left(\sqrt{2}/v\right) m_\ell$, where the m_ℓ are the charged-lepton masses. For the other Yukawa couplings appearing in equation (4) we employ the notation

$$(\Gamma_2)_\ell = \gamma_\ell, \quad (\Delta_1)_\ell = d_\ell, \quad (\Delta_2)_\ell = \delta_\ell. \tag{5}$$

These coupling constants in general are complex and dimensionless.

The neutrino Dirac mass matrix is diagonal, with $(M_D)_{\ell\ell} = \left(v/\sqrt{2}\right) d_\ell$. The 6×6 neutrino Majorana mass matrix

$$\mathcal{M} = \begin{pmatrix} 0_{3\times3} & M_D \\ M_D & M_R \end{pmatrix} \tag{6}$$

is symmetric; it is diagonalized by the 6×6 unitary matrix

$$\mathcal{U} = \begin{pmatrix} U_L \\ U_R^* \end{pmatrix} \tag{7}$$

as $\mathcal{U}^T \mathcal{M} \mathcal{U} = \hat{m} \equiv \text{diag}\,(m_1, \ldots, m_6)$. The m_i $(i = 1, \ldots, 6)$ are the neutrino masses; they are real and positive. In equation (7), the submatrices U_L and U_R are both 3×6. The leptonic charged-current, neutral-current Lagrangians and the interactions of the charged and neutral scalars with the charged leptons and with the neutrinos are given in ref. [1].

We connect the lepton mixing matrix U_L with the usual PMNS matrix through the seesaw approximation. The 3×3 symmetric matrix

$$M = -M_D M_R^{-1} M_D \tag{8}$$

is diagonalized by an unitary matrix V as

$$V^T M V = \hat{n} = \text{diag}\,(n_1, n_2, n_3), \tag{9}$$

where the n_p $(p = 1, 2, 3)$ are real and positive. In the seesaw approximation, V is the PMNS matrix. Now, it follows from equations (8) and (9) that

$$M_R = -M_D V \hat{n}^{-1} V^T M_D. \tag{10}$$

We use as input the matrix V, the matrix M_D in notations of parameters used for calculations, and the matrix $\hat{n}$. We determine M_R through equation (10). We then use M_R, together with the M_D, to construct the 6×6 matrix $\mathcal{M}$ of equation (6).

Finally, we diagonalize $\mathcal{M}$ through the unitary matrix $\mathcal{U}$ of equation (7). We thus find both U_L and the neutrino masses m_i. Because the seesaw approximation is very good, for $p = 1, 2, 3$ the m_p turn out approximately equal to the n_p, and the left 3×3 submatrix of U_L turn out approximately equal to the matrix V. For the $n_{1,2,3}$ we use the light-neutrino masses. The squared-mass differences $\Delta_{\text{solar}} = n_2^2 - n_1^2$ and $\Delta_{\text{atmospheric}} = \left|n_3^2 - n_1^2\right|$ are taken from the phenomenological fit [4]. The lightest-neutrino mass is kept free; we let it vary in between 10^{-5} eV and ~ 0.03 eV for normal ordering $(n_1 < n_3)$, and in between 10^{-5} eV and ~ 0.015 eV for inverted ordering $(n_3 < n_1)$. The smallest n_p cannot be allowed to be zero because $\hat{n}^{-1}$ appears in equation (10). The upper bound on the mass of the lightest neutrino arises from the cosmological bound of *Planck* 2018 results. Since the n_p are many orders of magnitude below the Fermi scale, the matrix elements of M_R are much above the Fermi scale unless the d_ℓ are extremely small. Thus, when we want to lower the seesaw scale, we lower the d_ℓ.

We use usual PMNS matrix parametrization of ref. [3] including Majorana phases α_{21} and α_{31}. The ranges for the mixing angles are taken from the phenomenological fit of ref. [4].

Details of the computations

We generated the complete set of diagrams with the Feynman gauge using FeynMaster package, with a modified version of the FeynRules SM file to account neutrinos, lepton flavour mixing and additional Higgs doublets. The automatically generated amplitudes of FeynMaster were expressed by Passarino-Veltman functions using package FeynCalc and specific functions of FeynMaster. Finally all the amplitudes was independently checked by performing computations manually. The detail expressions off the all amplitudes and formulas of decay rates are given in the ref. [2].

For numerical calculations we made two separate programs with Mathematica and with Fortran. The numerical instabilities or cancellations in the calculations have been solved with the high precision numbers for what Mathematica and lets do. The program written with Fortran allows to apply the minimization procedure to find the BRs within the ranges available in the experiment. The data is fitted by minimizing χ^2 with respect to the model parameters like the Yukawa couplings d_ℓ, δ_ℓ and γ_ℓ, the PMNS matrix parameters, mass of additional charged scalar $m_{H^\pm}$ and the parameter T_{11}.

We include various constraints in the computations which could potentially restrict decay rates of the processes. For the limitation of neutrino masses we take into account the latest experimental limits of total mass of the light neutrinos, mass relevant for neutrinoless double-beta decay and the mass relevant for standard β decay. We check compatibility of our parameters with the invisible decay of the Z boson and constraints on mass of the charged scalar $m_{H^\pm}$. Moduli of coupling constants are restricted from bellow with bound $Y_{\min} = 10^{-6}$ and they should not be larger than perturbativity bound $Y_{\max} = \sqrt{4\pi}$.

Numerical analysis

To estimate the influence of Yukawa coupling constants to branching ratios in the general case we fix couplings as ratio $d_\mu/d_e = d_\tau/d_\mu = 1.1$, $\delta_\mu/\delta_e = \delta_\tau/\delta_\mu = 1.1$ and $\gamma_\mu/\gamma_e = \gamma_\tau/\gamma_\mu = 1.1$ therefore, only one coupling can be varied while the others remain fixed. The mass of the lightest neutrino is fixed to $m_1 = 0.01$ eV while the other neutrino masses and the oscillation parameters are fixed to the central values of ref. [4] for normal ordering of the light neutrinos. The mass of the charged scalar is fixed to $m_{H^\pm} = 750$ GeV. The 2HDM parameters are fixed following $\lambda_3 = \lambda_7 = 1$ and $T_{11} = 0.99999$.

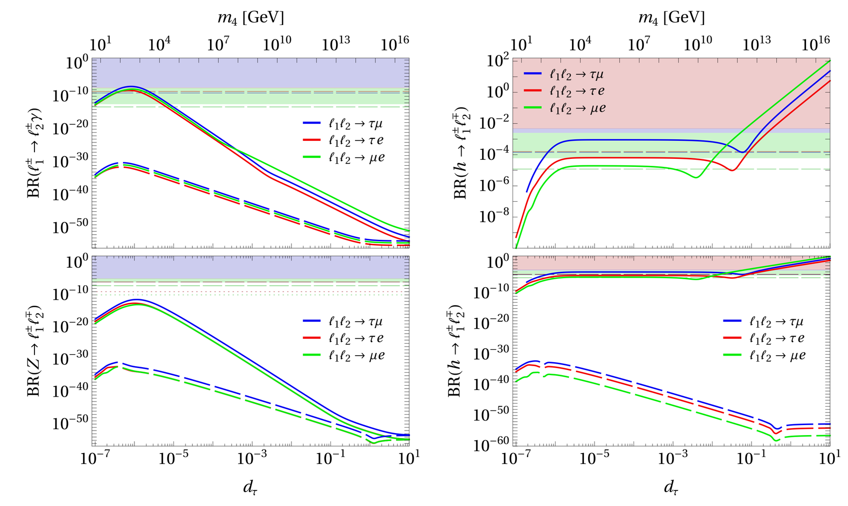

Figure 1: The branching ratios as a functions of the Yukawa coupling d_τ. The couplings $\delta_\tau = \gamma_\tau = Y_{\max}$ and the other couplings are fixed through the rates described above. Full lines represent the contribution of all amplitudes while dashed lines represent the contribution of amplitudes without additional charged scalar $H^\pm$. The shadowed bands in the plots are excluded by present experimental data but dashed and dotted lines shows future experimental sensitivities.

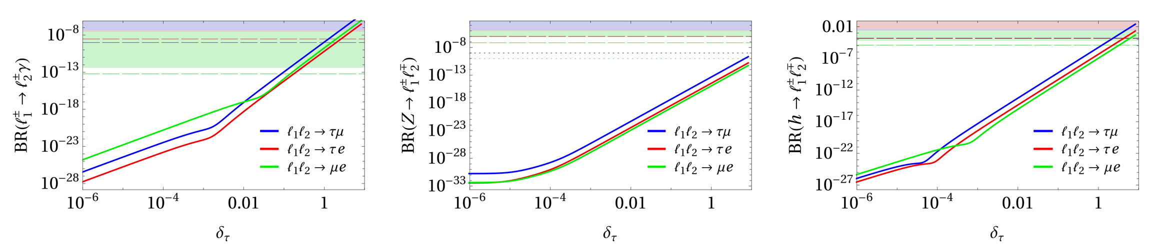

Figure 2: The branching ratios as a functions of the Yukawa coupling δ_τ. The couplings $d_\tau = 10^{-6}$, $\gamma_\tau = Y_{\max}$ and the other couplings are fixed through the rates described above.

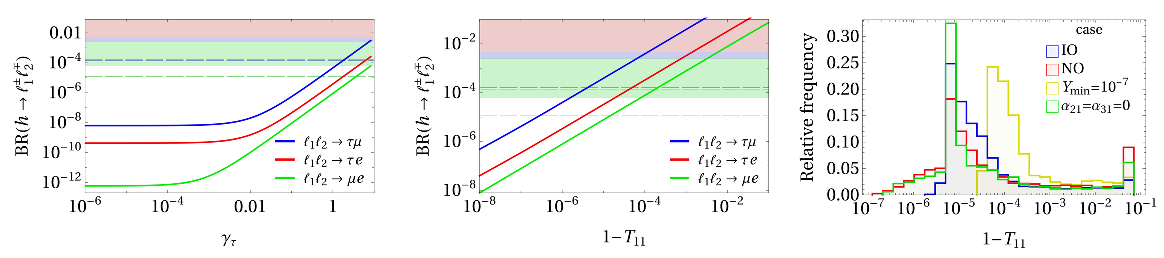

Figure 3: The left plot shows the dependence of Higgs decay rates from Yukawa coupling γ_τ. The right plot shows the dependence of the Higgs decay rates from the parameter T_{11}. In these two plots, the other parameters are fixed as described above. The right plot shows the distribution of the parameter T_{11} for four cases described bellow.

In order to find adequate numerical values for the parameters we perform global minimization procedure. We compute all nine decays and collect only the points that are within the current experimental bounds and future sensitivities for all the lepton and the Higgs decays simultaneously. These statistical data allow to elucidate which are the relevant and the less relevant parameters of that model for the LFV decays.

In the figures bellow we compare four different cases of computations. In the first case (labelled by 'IO') is assumed inverted ordering of the light neutrinos while in the second case (labelled by 'NO') is assumed normal ordering of the light neutrinos. In the third cases the lower bound of the Yukawa couplings is expanded to $Y_{\min} = 10^{-7}$ and in the fourth case is assumed that Majorana phases $\alpha_{21} = \alpha_{31} = 0$. In all cases the oscillation parameters are varied in the 3σ experimental ranges and the mass of $H^\pm$ is fixed at $m_{H^\pm} = 750$ GeV. There are assumed the real Yukawa couplings.

To see the behaviour of the separate parameters we choose benchmark point with $m_1 = 16.5$ meV, $m_{H^\pm} = 750$ GeV, $T_{11} = 0.9999$, $d_e = d_\tau = 10^{-6}$, $d_\mu = 4 \times 10^{-6}$, $\delta_e = \delta_\mu = \delta_\tau = 3.5$, $\gamma_e = 0$, $\gamma_\mu = 1$, $\gamma_\tau = 3$, $\alpha_{21} = 3.515$ and $\alpha_{31} = 1.06$. The other oscillation parameters are fixed to the central values of ref. [4] for NO.

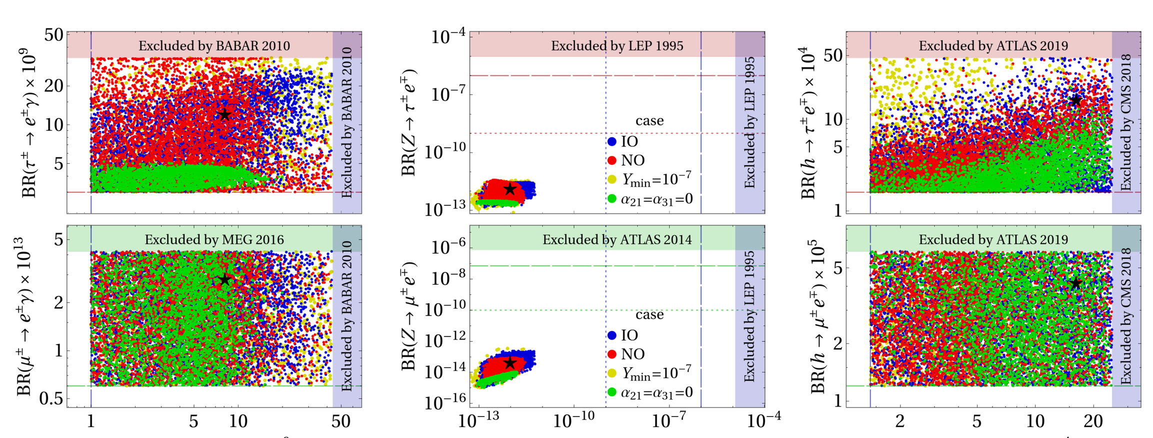

Figure 4: The scatter plots of the branching ratios of LFV decays for four different cases. The black star indicate the benchmark point. The shadowed bands in the plots are excluded by present experimental data but dashed and dotted lines shows future experimental sensitivities.

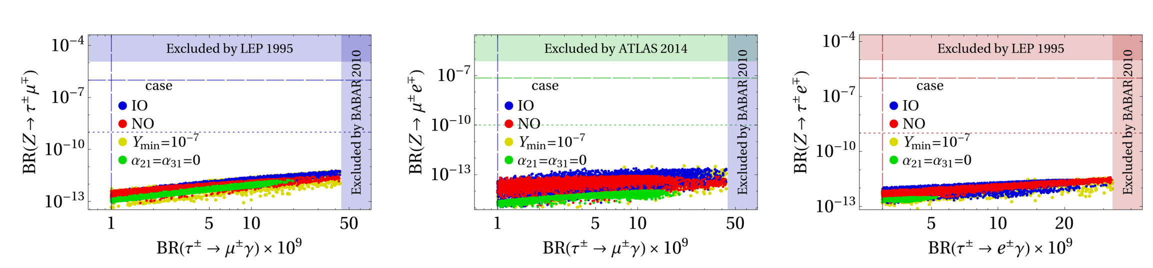

Figure 5: Correlation plots for the charged lepton and the Z boson decays for four different cases. The shadowed bands in the plots are excluded by present experimental data but dashed and dotted lines shows future experimental sensitivities.

Numerical analysis

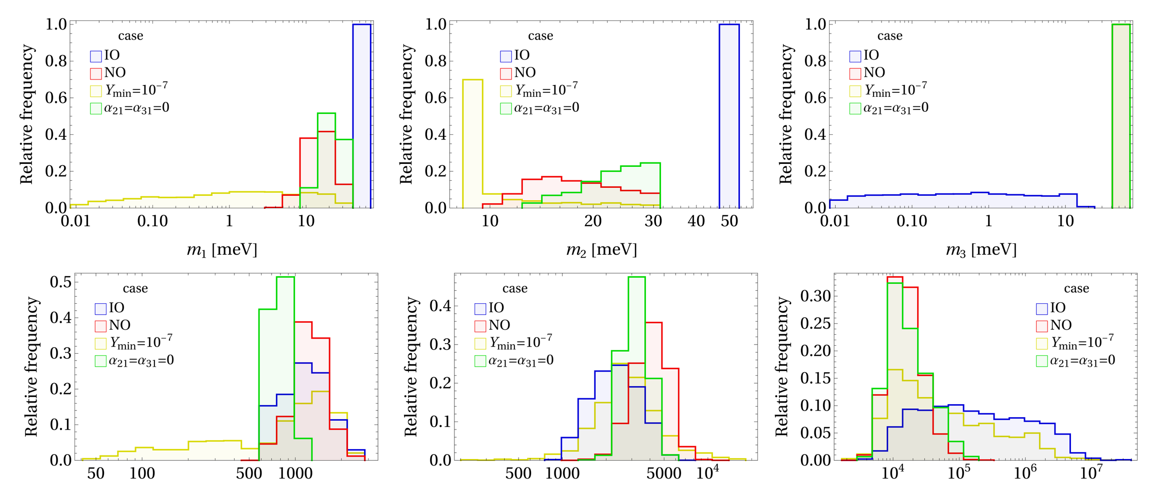

Figure 6: The distributions of the neutrino masses for four different cases.

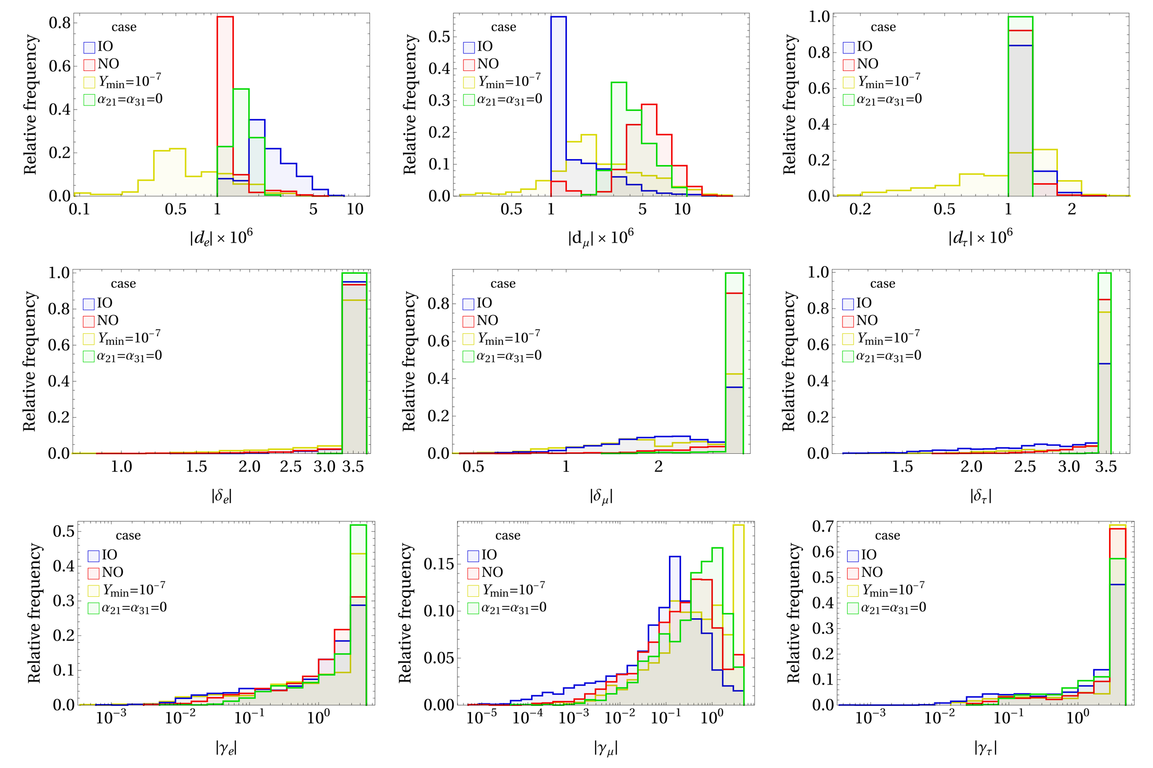

Figure 7: The distributions of the Yukawa couplings for four different cases.

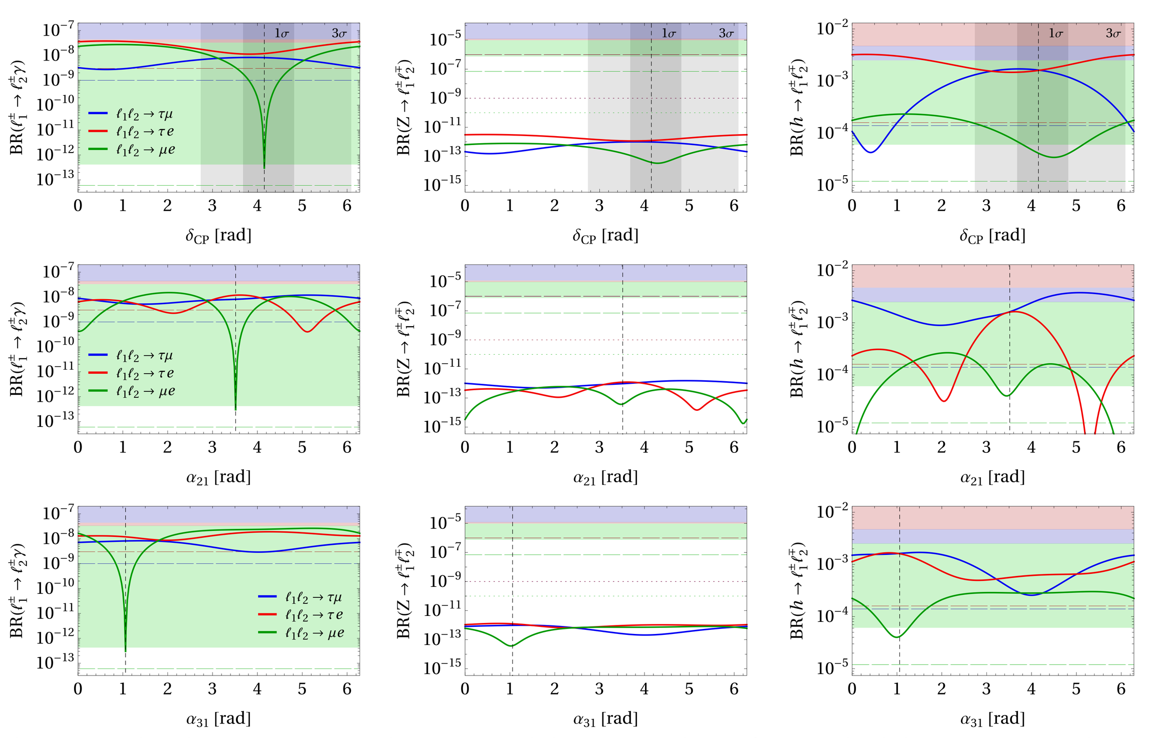

Figure 8: The decay rates as a functions of the Dirac phase δ_{CP} and of the Majorana phases α_{21} and α_{31}. The other parameters are taken from the benchmark point. Vertical dashed line indicate fixed parameters of the benchmark point. Notice that the similar fine-tuning for the other oscillation parameters is needed for the decays with $\ell_1 = \mu$ and $\ell_2 = e$.

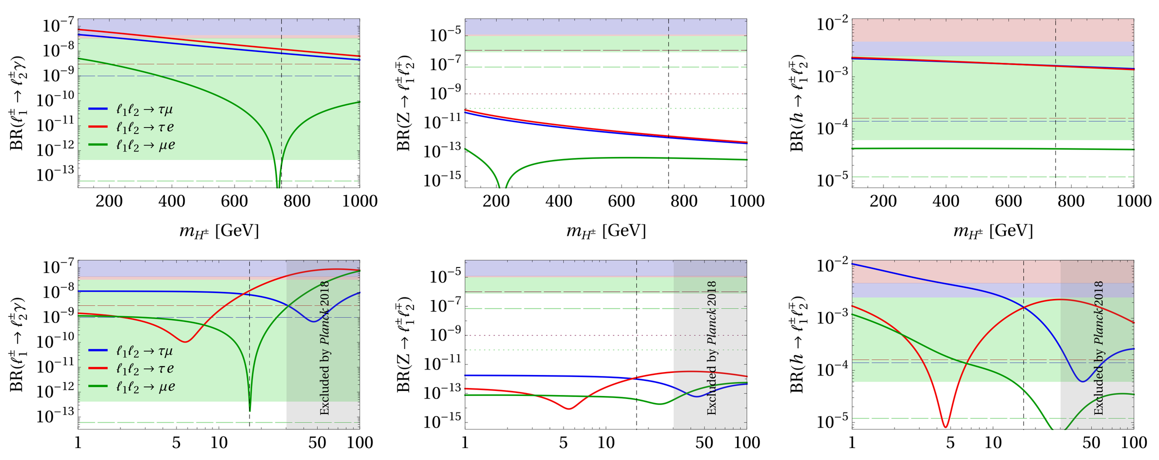

Figure 9: The decay rates as a functions of the mass of the charged scalar $m_{H^\pm}$ and of the mass of the lightest neutrino m_1. The other parameters are taken from the benchmark point. Vertical dashed line indicate fixed parameters of the benchmark point.

Conclusions

- The amplitudes with the additional charged scalar gives the main impact and allows to get decay rates for the lepton and Higgs processes close to the experimental bounds as illustrated in fig. 1.
- The decays $\ell_1^\pm \to \ell_2^\pm \gamma$ require small values of Yukawa couplings d_ℓ (with very light right-handed neutrinos) and large values of couplings δ_ℓ in order to be visible.
- The decays $Z \to \ell_1^\pm \ell_2^\mp$ correlate with the decays $\ell_1^\pm \to \ell_2^\pm \gamma$ and have similar behaviour, unfortunately due to the large total decay width of Z boson, these decays are invisible in all the planned experiments.
- The decays $h \to \ell_1^\pm \ell_2^\mp$ have different behaviour than the other decays and prefers larger values of d_ℓ but thanks to the freedom to adjust γ_ℓ and T_{11} they are visible for future experiments.
- The Majorana phases have significant impact to the branching ratios for all processes.
- The case of the complex Yukawa couplings *vs.* the real Yukawa couplings or the case of the 3σ *vs.* 1σ of experimental ranges of the oscillation parameters do not have significant influence to the decay rates.
- The case of normal ordering of the light neutrinos and the case of inverted ordering of the light neutrinos gives the similar decay rates.
- As the mass of the charged scalar increases, the values of BRs decrease, but for $m_{H^\pm} < 1000$ GeV the lepton and Higgs decays are visible for future experiments.
- Fine-tuning is needed in order that $\text{BR}(\mu^\pm \to e^\pm \gamma)$ and $\text{BR}(h \to \mu^\pm e^\mp)$ does not become too large when all other four branching ratios are simultaneously close to their experimental limits. The minimization procedure helps to achieve this perfectly as shown in fig. 8 and fig. 9.

References

[1] W. Grimus and L. Lavoura, Phys. Rev. D **66** (2002) 014016, hep-ph/0204070.

[2] D. Jurčiukonis and L. Lavoura, *in preparation* (2020).

[3] **Particle Data Group** Collaboration, M. Tanabashi et al., *Review of Particle Physics*, Phys. Rev. **D98** (2018), no. 3 030001.

[4] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tórtola, and J. W. F. Valle, Phys. Lett. **B782** (2018), arXiv:1708.01186.

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