Bounded Power-Law Characterizations of Solar Flares

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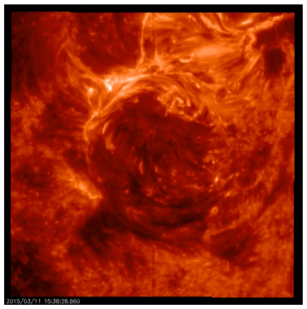
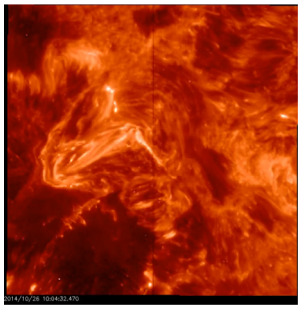
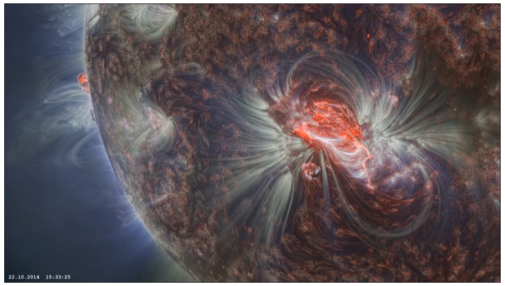
Solar Flares



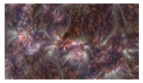


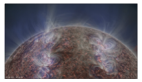
- What are they?
  - Bursts of light and radiation caused by an impulsive release of stored magnetic energy from the sun
  - Observable at all wavelengths between radio and γ-ray
- What do they look like?
- Where do they occur?
  - Active regions in the corona



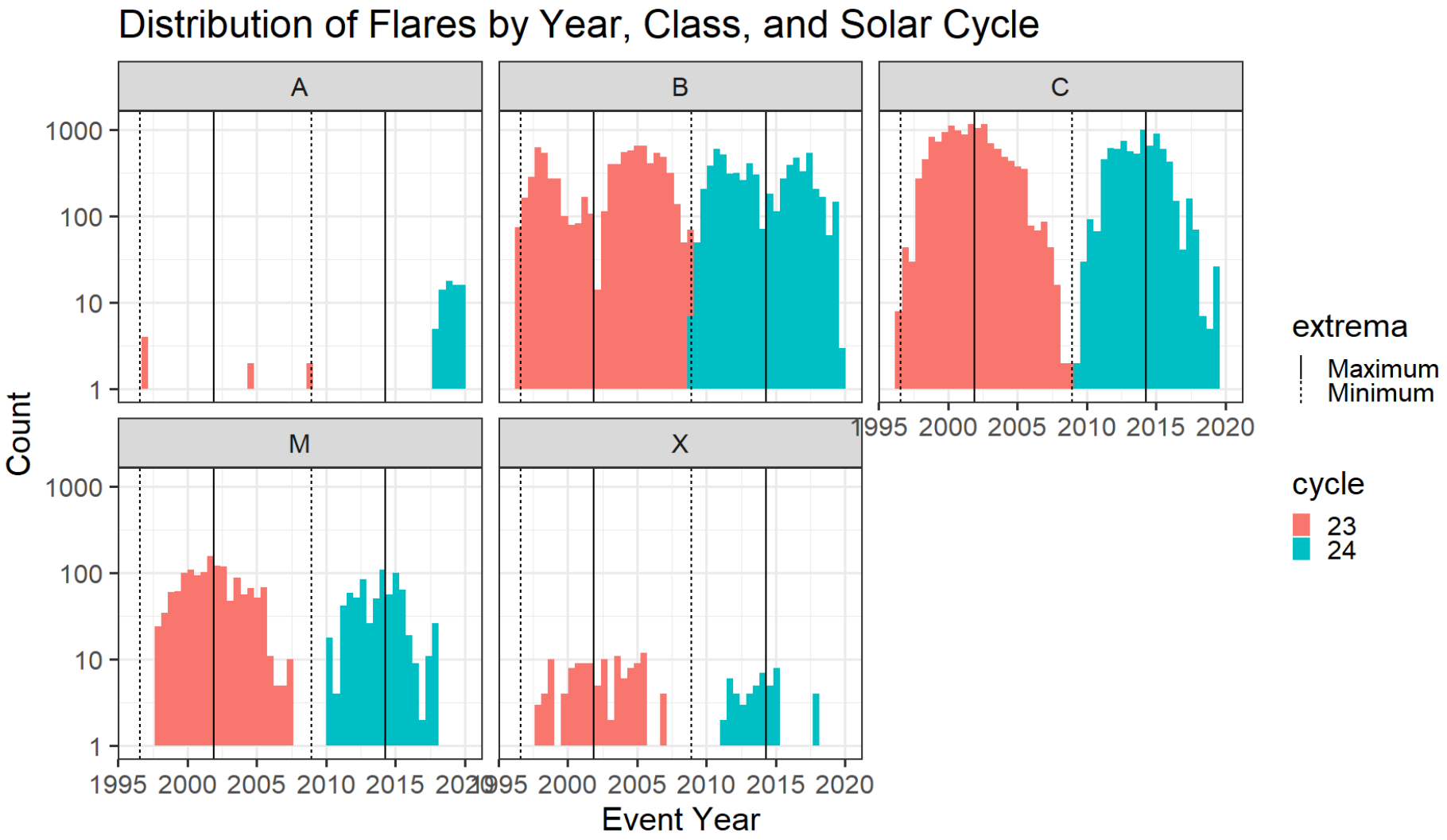
Solar Flares

- Solar Cycle
  - 11-year cycle where the sun goes through different levels of solar activity
- Flare classes
  - A-class: 1e-5 to 1e-4 erg/s/cm^2 at Earth
  - B-class: 1e-4 to 1e-3 erg/s/cm^2 at Earth
  - C-class: 1e-3 to 0.01 erg/s/cm^2 at Earth
  - M-class: 0.01 to 0.1 erg/s/cm^2 at Earth
  - X class: > 0.1 erg/s/cm^2 at Earth



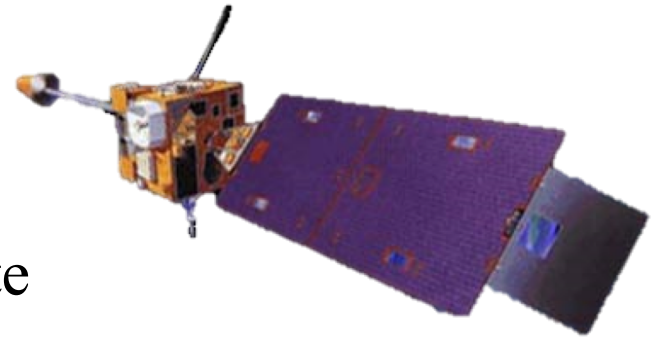


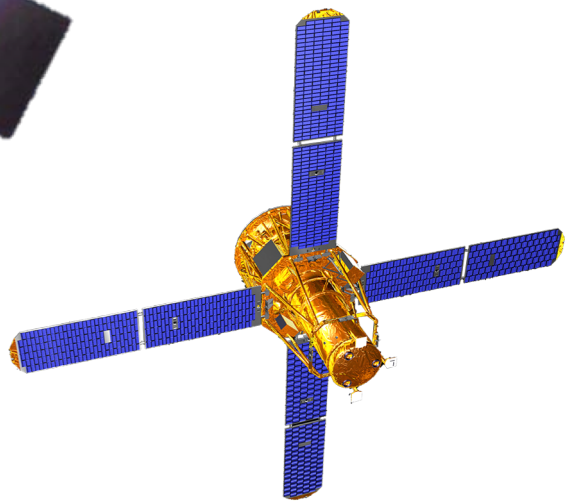
- Flare Frequency
  - Flare frequencies change throughout the solar cycle
  - Flares increase in frequency and intensity near the solar maximum

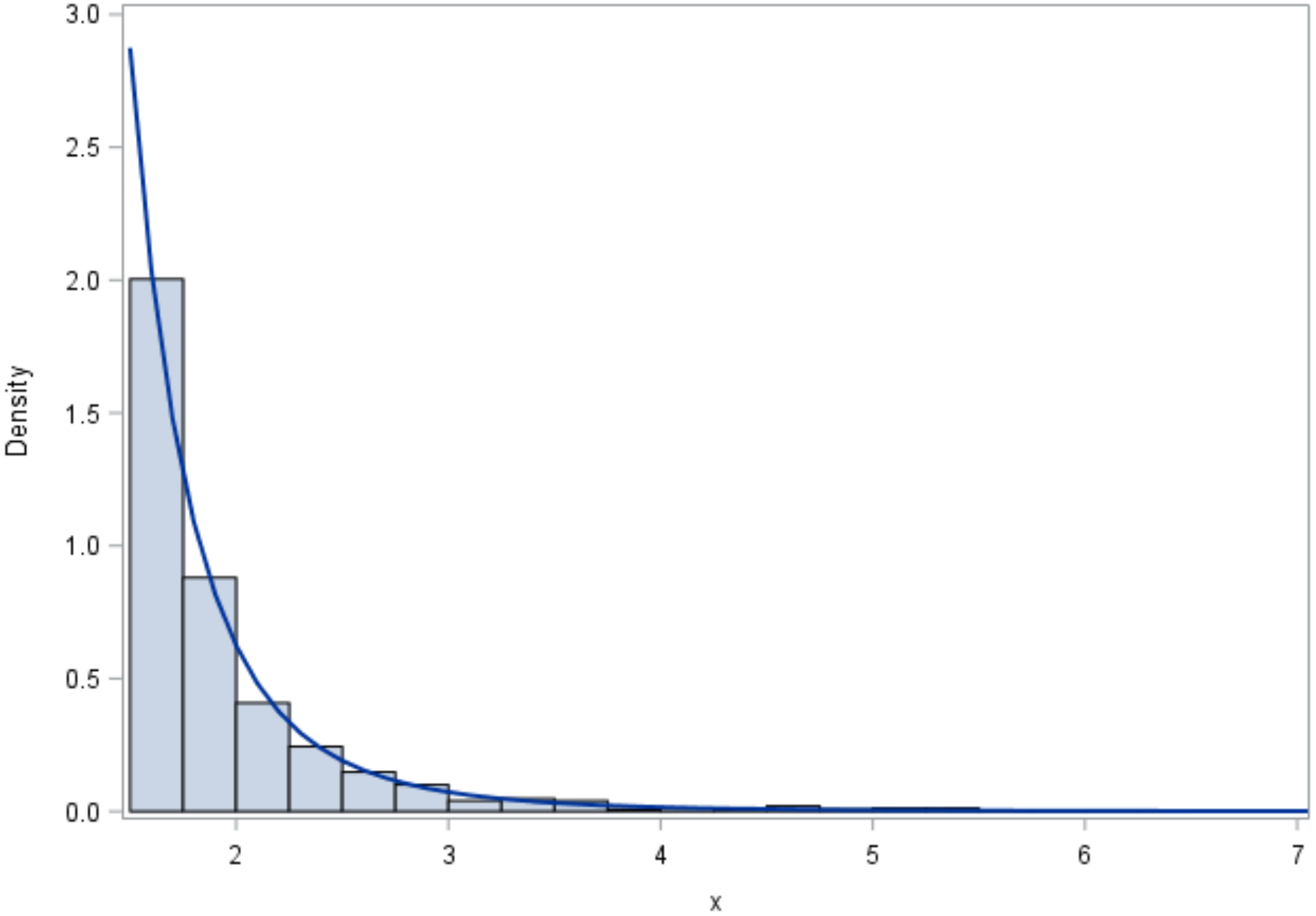

Distribution of Flares by Year, Class, and Solar Cycle
A
B
C
M
X
Count
1000
100
10
1
1995
2000
2005
2010
2015
2020
Event Year
extrema
Maximum
Minimum
cycle
23
24

Solar Flare Observations

- Satellites
  - GOES Satellites
  - RHESSI Satellite
- Properties of Interest
  - Total Energy - $ergs$
  - Peak Flux - $ergs/s/cm^2$
  - Duration - $seconds$
  - Quiet Time - $seconds$ between end and start of consecutive flares
  - Wait Time - $seconds$ between flare peaks



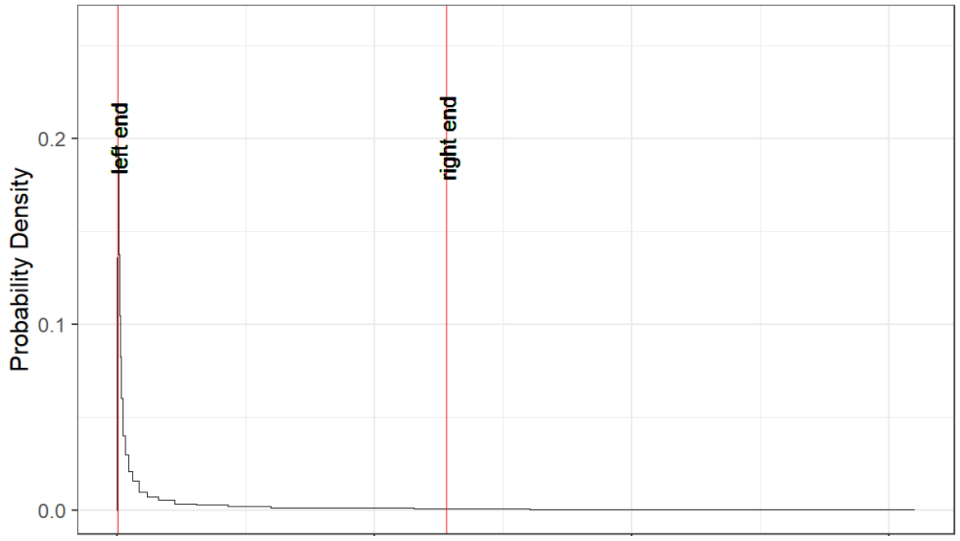


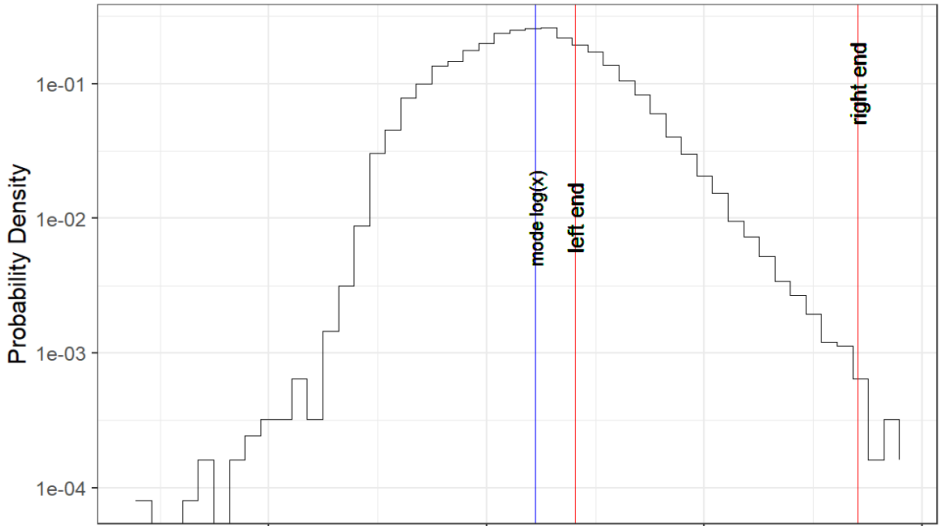

3.0
2.5
2.0
1.5
1.0
0.5
0.0
Density
2
3
4
5
6
7
x

Why is this research important?

- To understand the physical processes that cause flares to occur in the corona
- Self-Organized Criticality – occur as cascades
- Frequency of super flares
- Trends by cycle, active region, and sequences
- Solar flares are dangerous!
  - Harmful to astronauts if unprotected
  - Can affect satellites and electronics on earth
- Extrapolate results to stars similar to the sun

Estimating Power-Law Distributions





Normal scale:

$$f(x) = kx^{-\alpha}$$

Log-log scale:

$$\ln(f(x)) = \ln(k) - \alpha \ln(x)$$

- Commonly utilized statistical estimation methods
  - Simple Linear Regression in log-log scale
  - Maximum Likelihood Estimation (MLE)

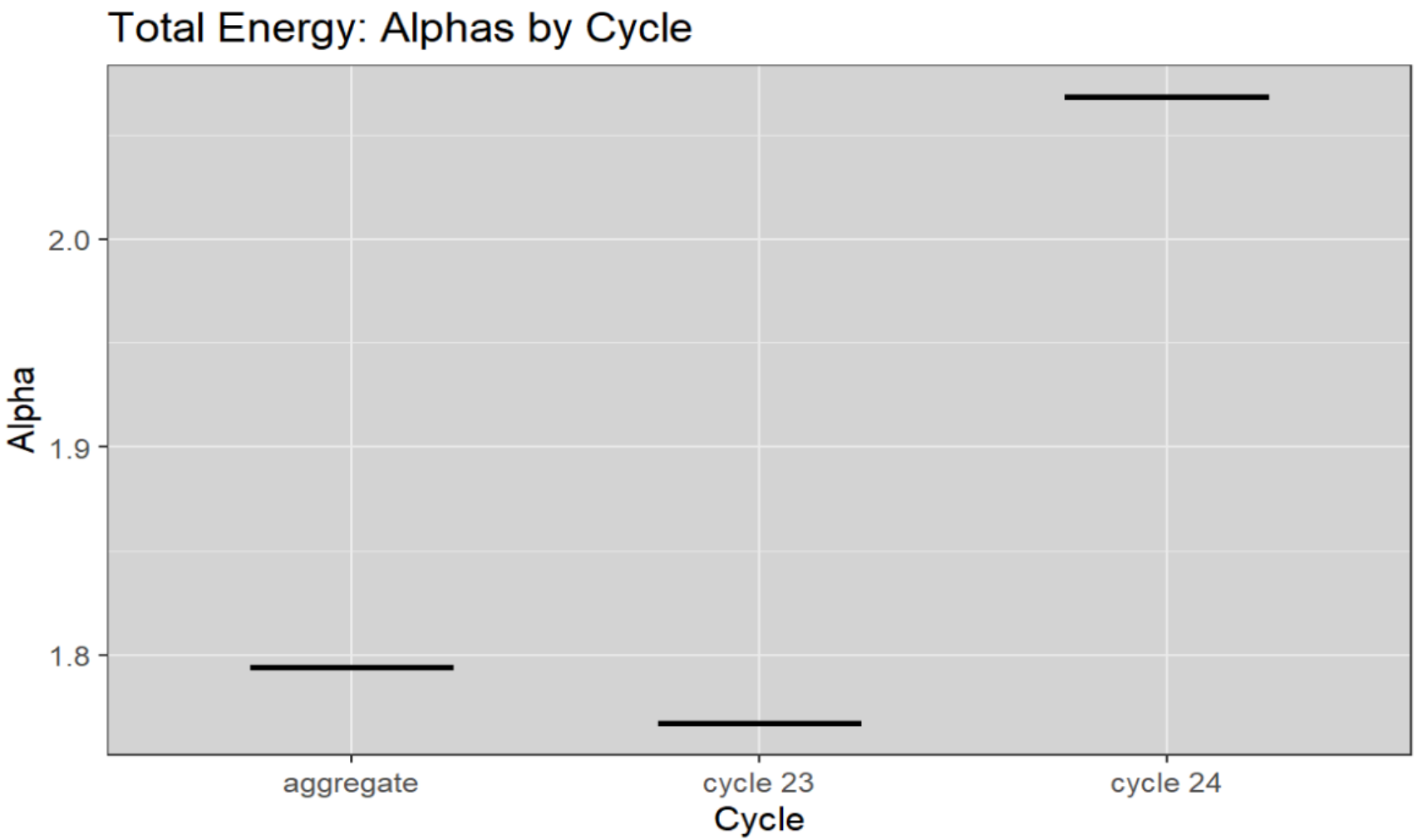
What statistical problem do we face?

- A **truly** semi-parametric problem
  - We want to find the power-law that best fits our data
  - **BUT**
  - There is no defined model that can be used to fit the trend outside the bounds
- The entire distribution does **not** follow a power-law
  - Distribution turns over due to sensitivity limitations at low flare energies and small samples at high energies
- We utilize the Maximum Product of Spacings method and Monte Carlo Analysis

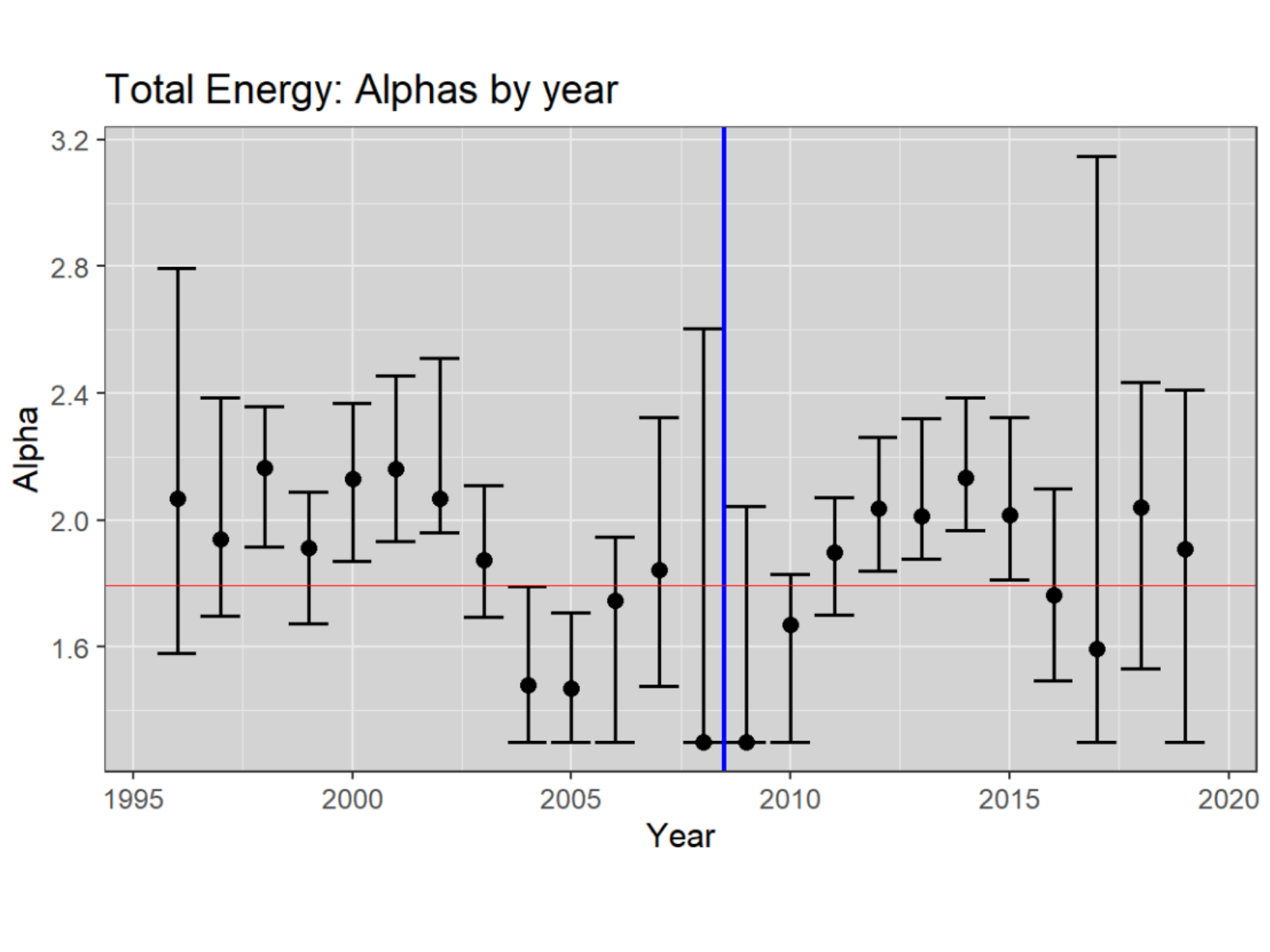
Maximum Product of Spacings (MPS)

- MPS fits a CDF (F) that maximizes the product of spacings
  - CDF – Cumulative Distribution Function
  - MLE fits to the Probability Density Function (PDF)
- $max \prod_{i=0}^{n} \{F(X_{i+1}) - F(X_i)\}$
- Xufei Wang's MPS algorithm
  - Mode ln(x)
  - Left and right ends of the region
  - Exponent
  - Normalization constant

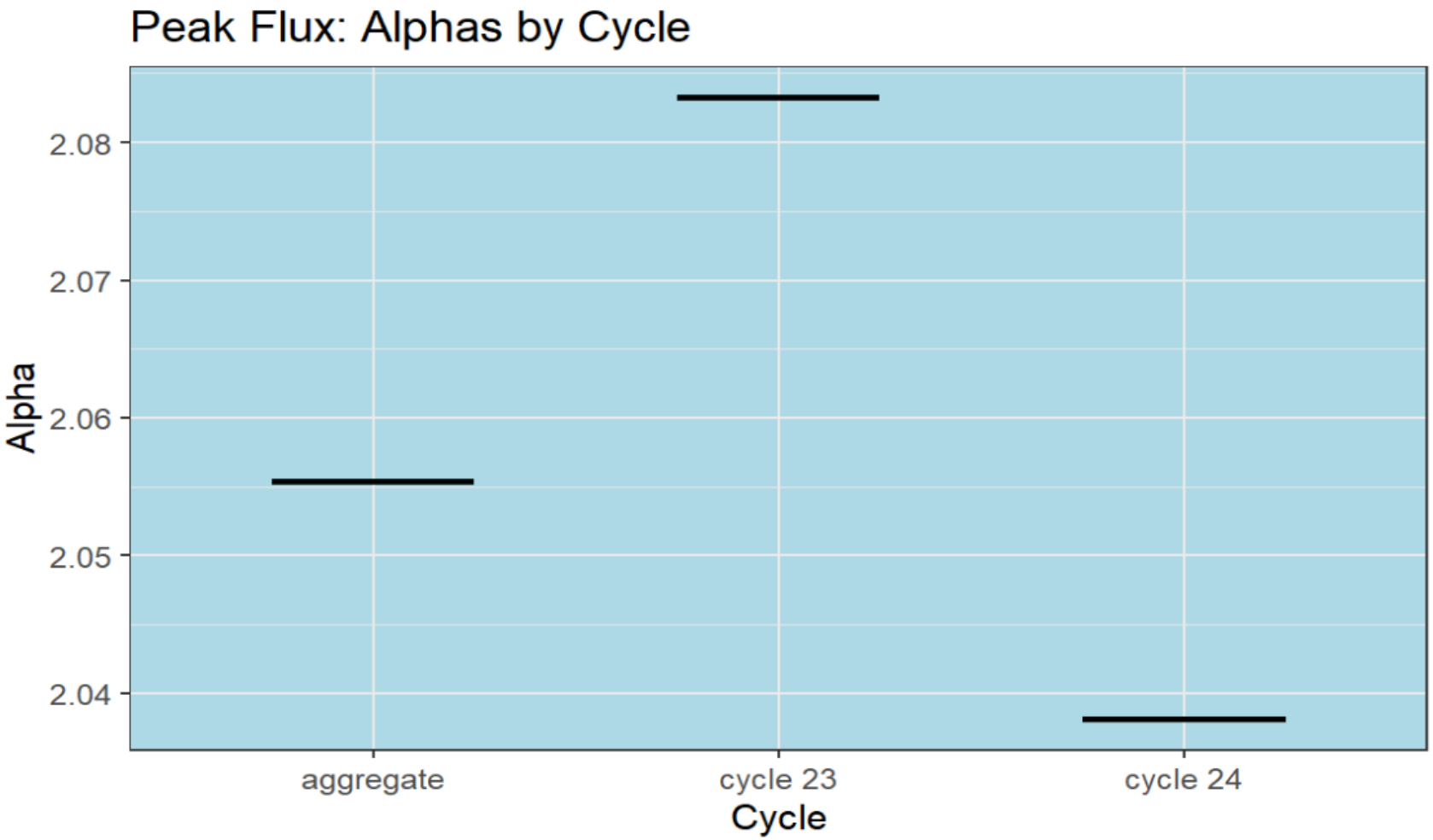
Results – Total Energy



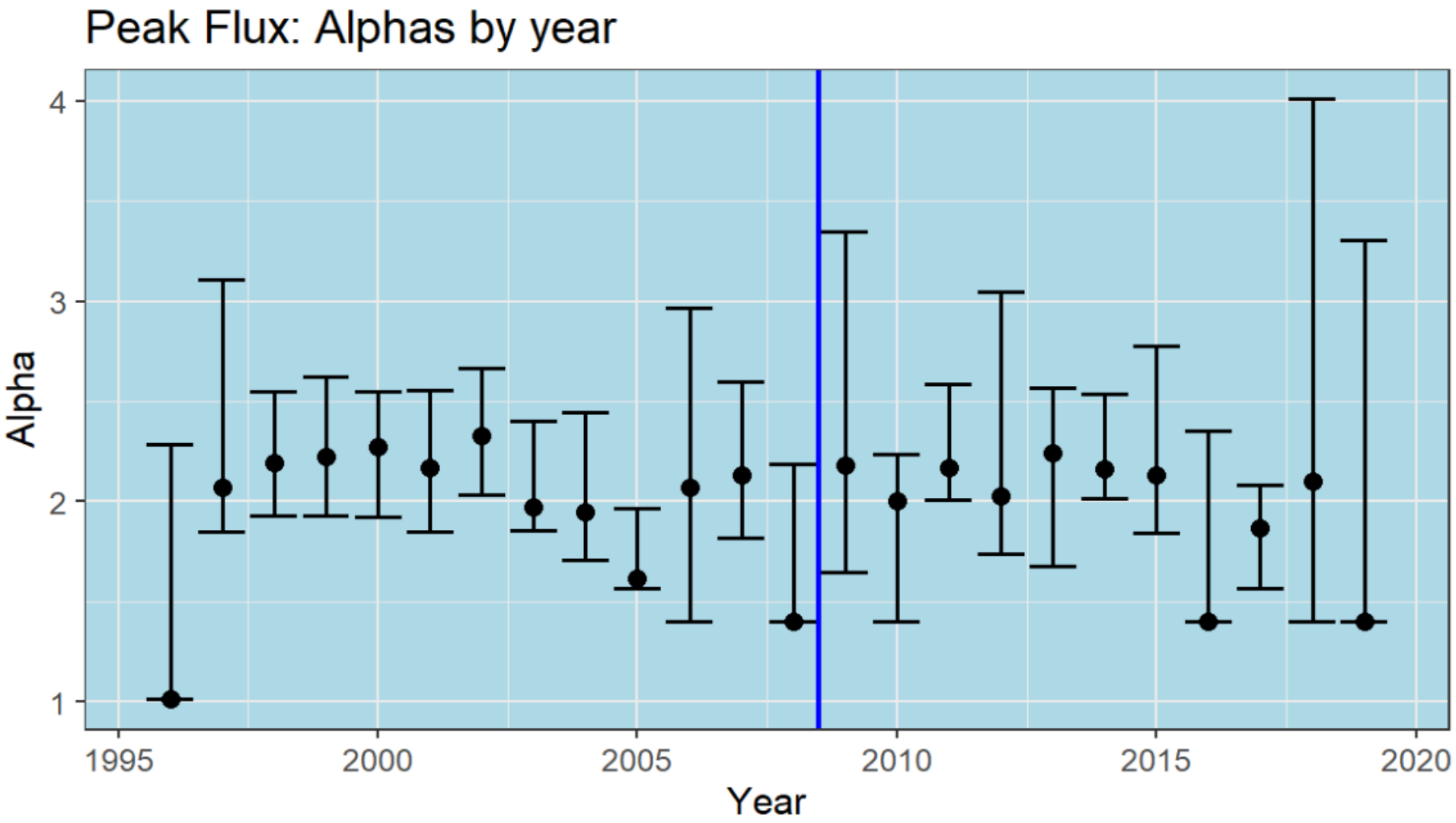
Cycle	Alpha	Mode of ln(x)	ln(left)	ln(right)
Aggregate	1.794005	63.22	63.47	68.07
23	1.767075	63.22	63.47	67.81
24	2.068505	63.20	65.51	6.78


Total Energy: Alphas by year
3.2
2.8
2.4
2.0
1.6
Alpha
1995
2000
2005
2010
2015
2020
Year

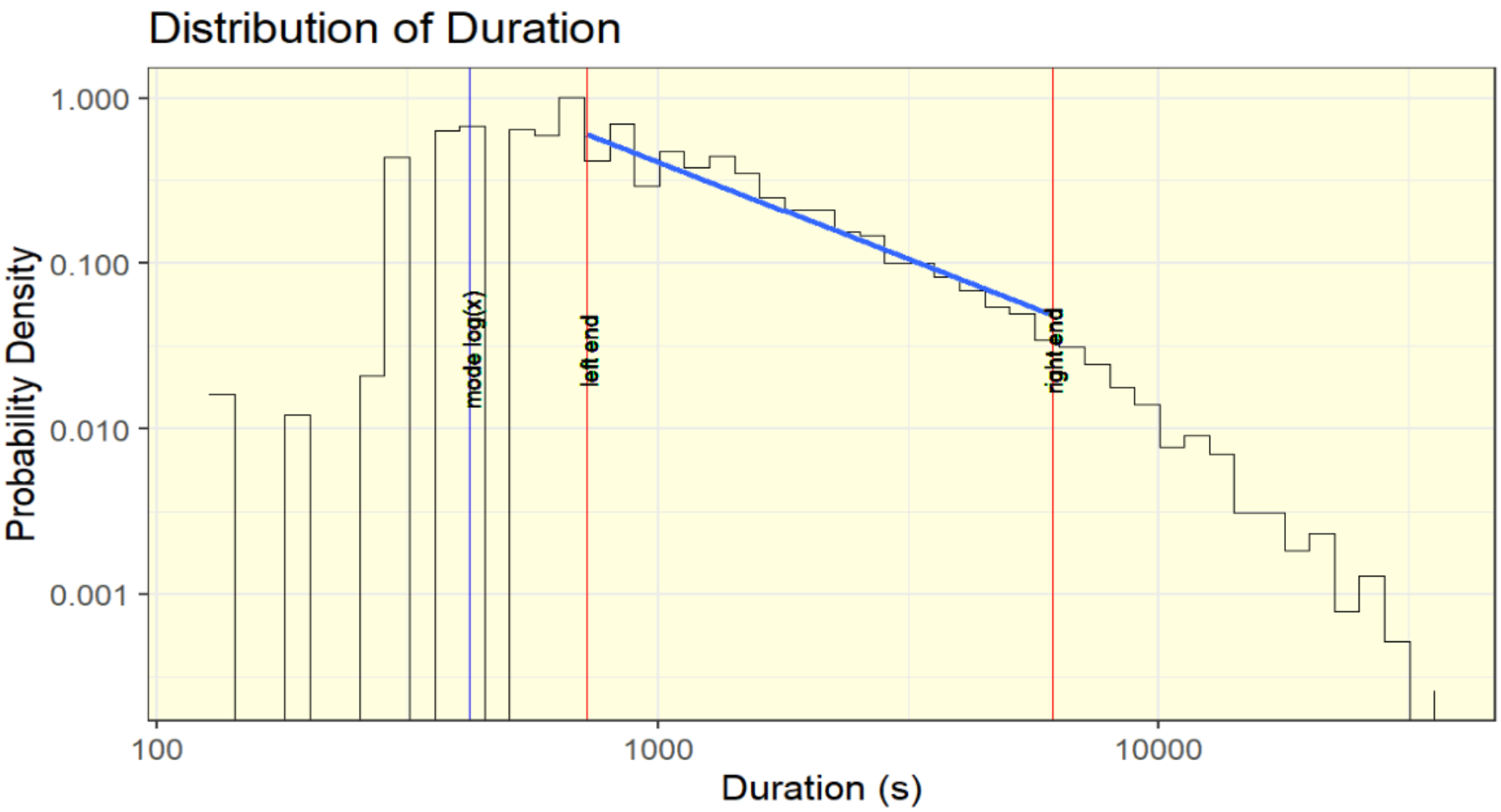
Results – Peak Flux



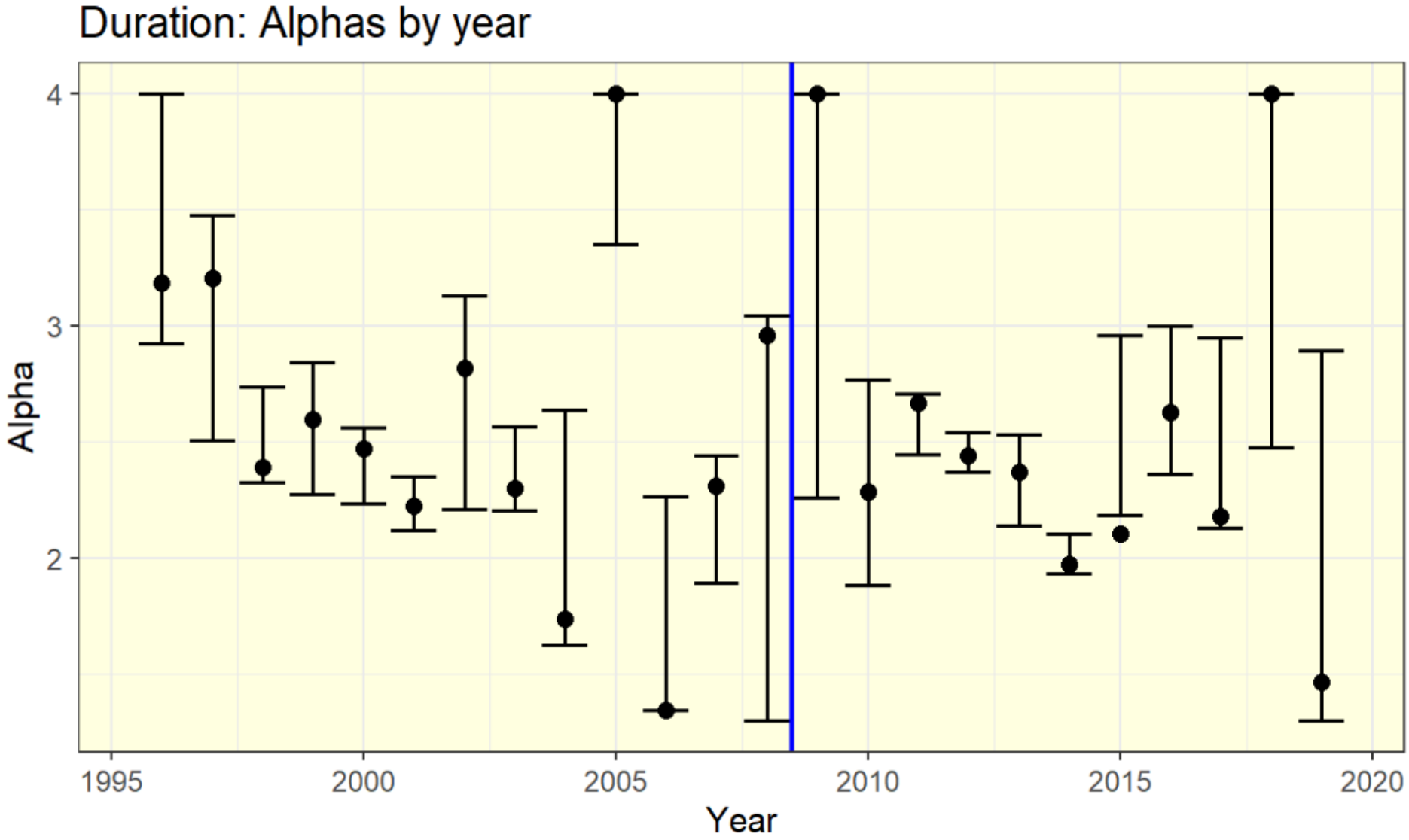
Cycle	Alpha	Mode of ln(x)	ln(left)	ln(right)
Aggregate	2.055357	-6.91	-6.27	-2.12
23	2.08328	-6.91	-6.21	-2.32
24	2.038141	-6.93	-4.61	-2.38


Peak Flux: Alphas by year
Alpha
4
3
2
1
1995
2000
2005
2010
2015
2020
Year

Results – Duration



Alpha	Mode of ln(x)	ln(left)	ln(right)
2.180884	6.043652	6.582649	8.719979


Duration: Alphas by year
Alpha
4
3
2
1995
2000
2005
2010
2015
2020
Year

Summary of Results

- We utilize the MPS method to get estimates of the power-law bounds and exponents and a Monte Carlo analysis for error bars
- Total energy alphas differ by cycle and there may be a trend in alphas by year
- Peak flux alphas are consistent across cycle and year
- Duration follows a power-law, but there is high uncertainty

Moving Forward

- Quiet and Wait time distributions
- Map flares to active regions
- Estimate distributions by order in a sequence
- Continue to test robustness of MPS through simulations
- Confidence intervals

Acknowledgements

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- Drs. Katherine Reeves and Chad Madsen from the CfA
  - Putting on the Solar REU for 2020!
- NSF-REU solar physics program at SAO, grant number AGS-1850750

References

- IRIS videos
  - https://iris.lmsal.com/mod?cmd=view-pods
- By-class flare videos - Miloslav Druckmuller
  - http://www.zam.fme.vutbr.cz/~druck/Sdo/Pm-nafe/Archive.htm

Summary

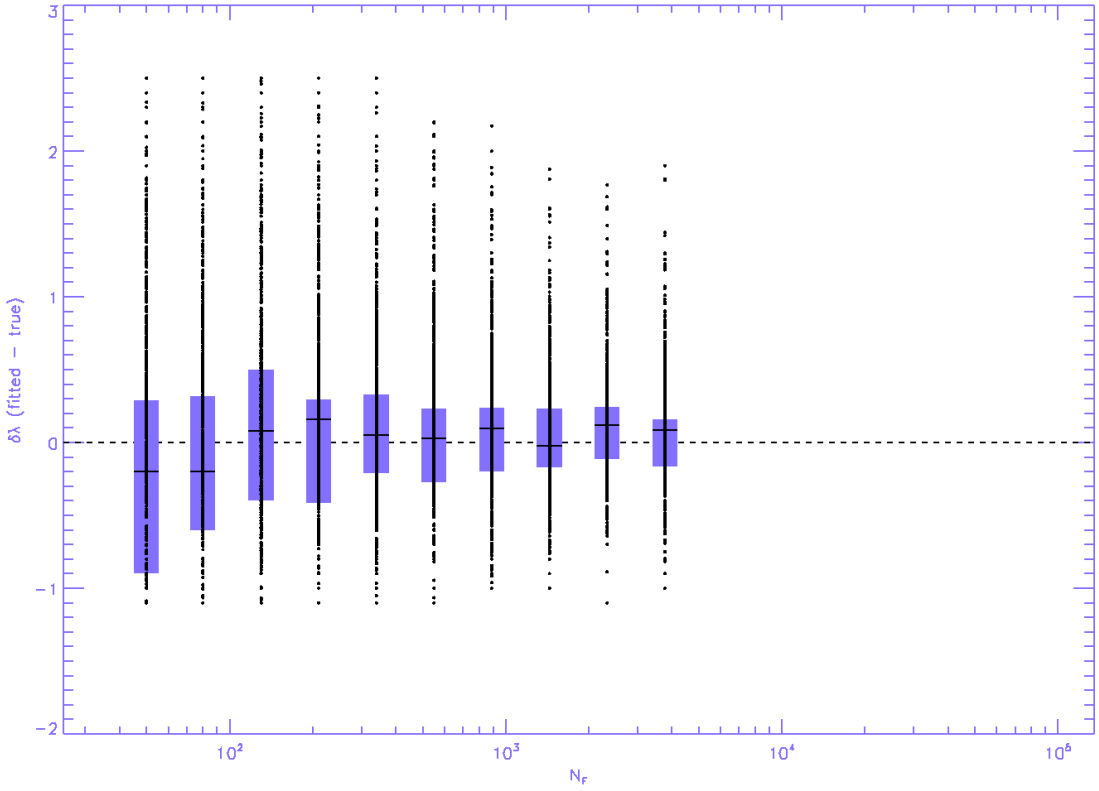
- Results
  - We utilize the MPS method to get estimates of the power-law bounds and exponents and a Monte Carlo analysis for error bars
  - Total energy alphas differ by cycle and there may be a trend in alphas by year
  - Peak flux alphas are consistent across cycle and year
  - Duration follows a power-law, but there is high uncertainty
- Future work
  - Map flares to active regions
  - Estimate distributions by order in a sequence
  - Continue to test robustness of MPS through simulations
  - Confidence intervals

Bonus Slides

Simulations

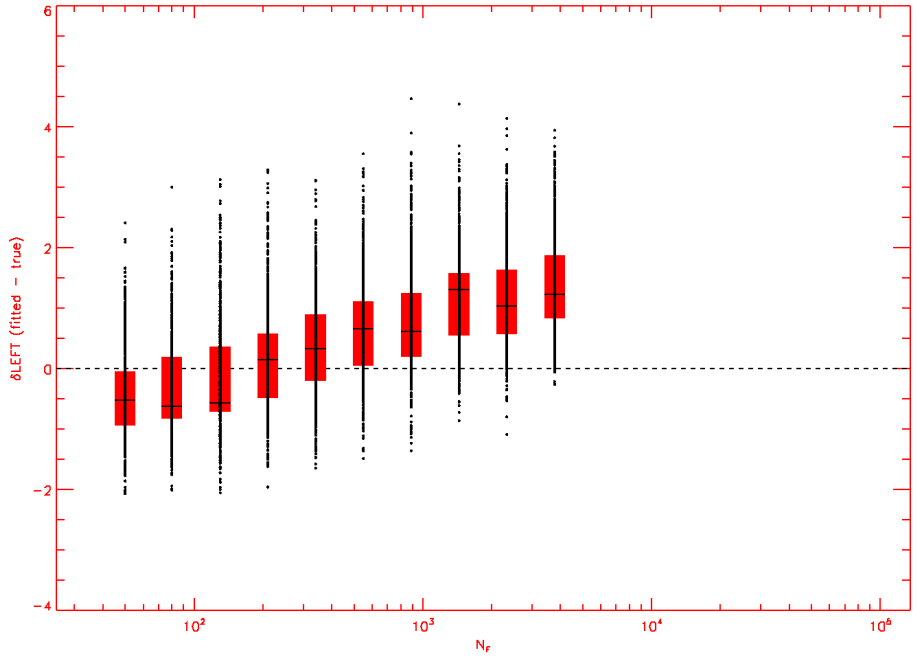
MPS Simulation Performance

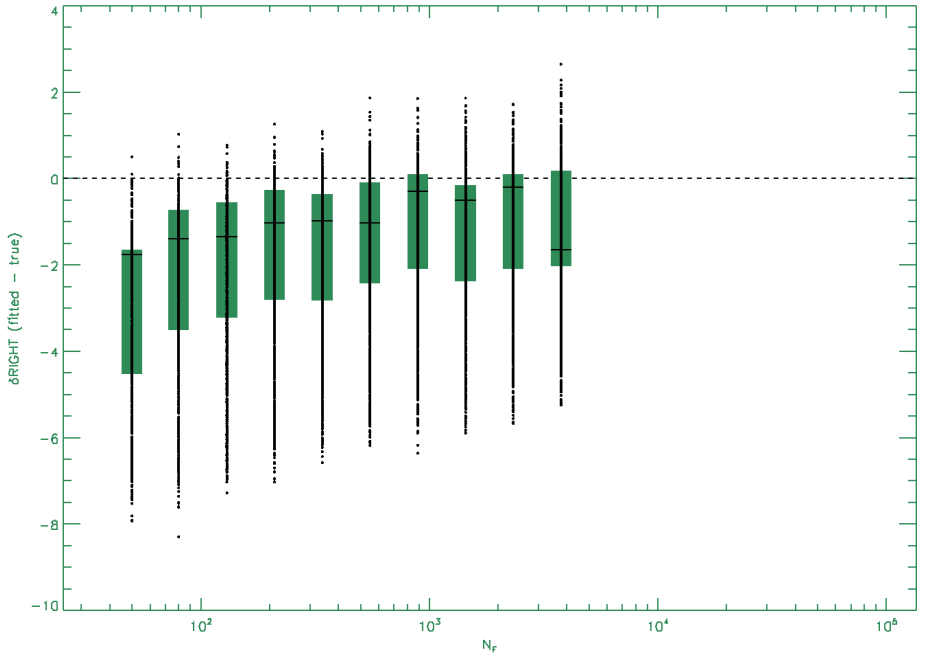
- MPS tends to perform well (in most cases) and converges to the true value as the sample size increases



MPS Simulation Performance

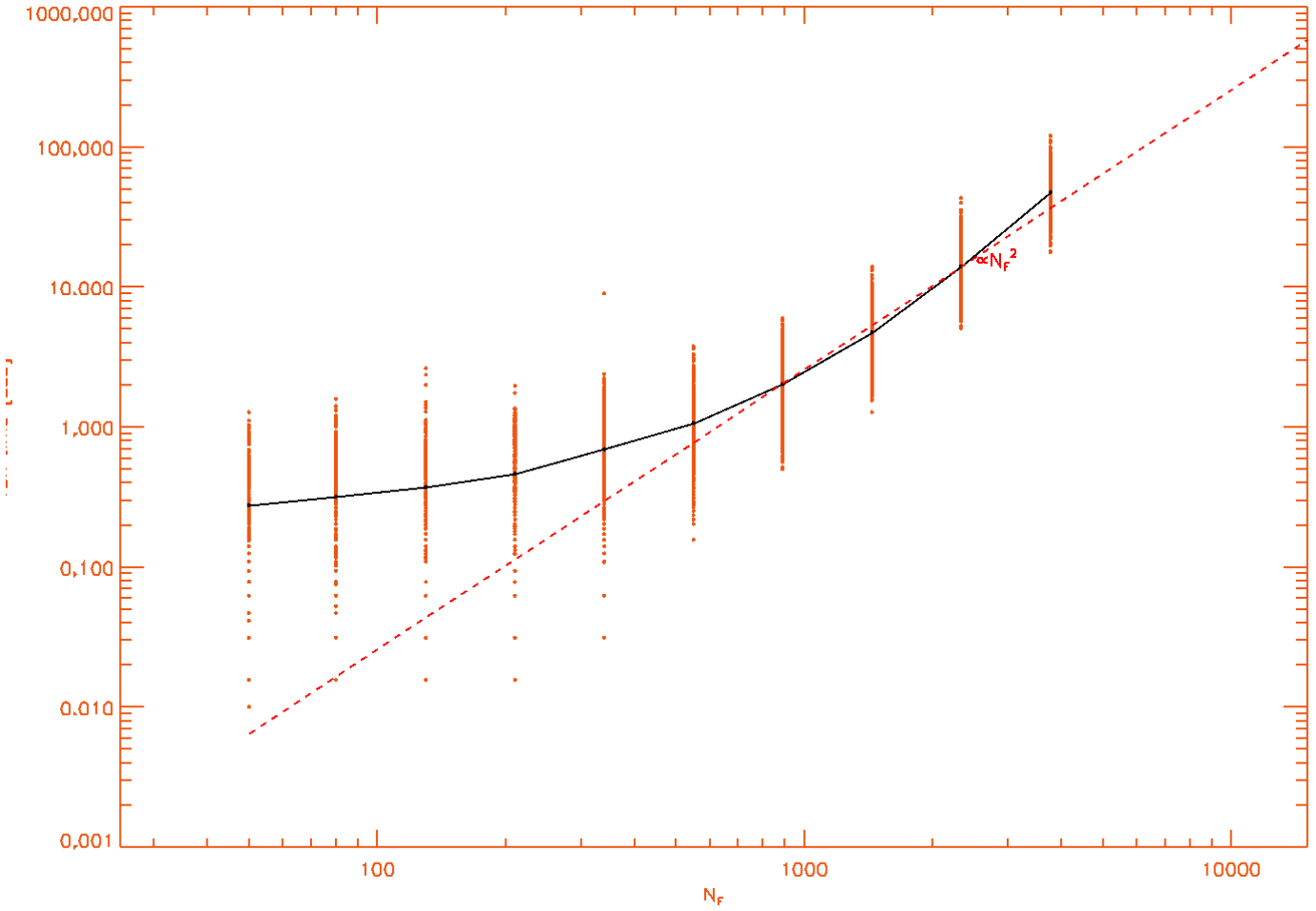
- The region estimates tend to be more "conservative" (underestimate)



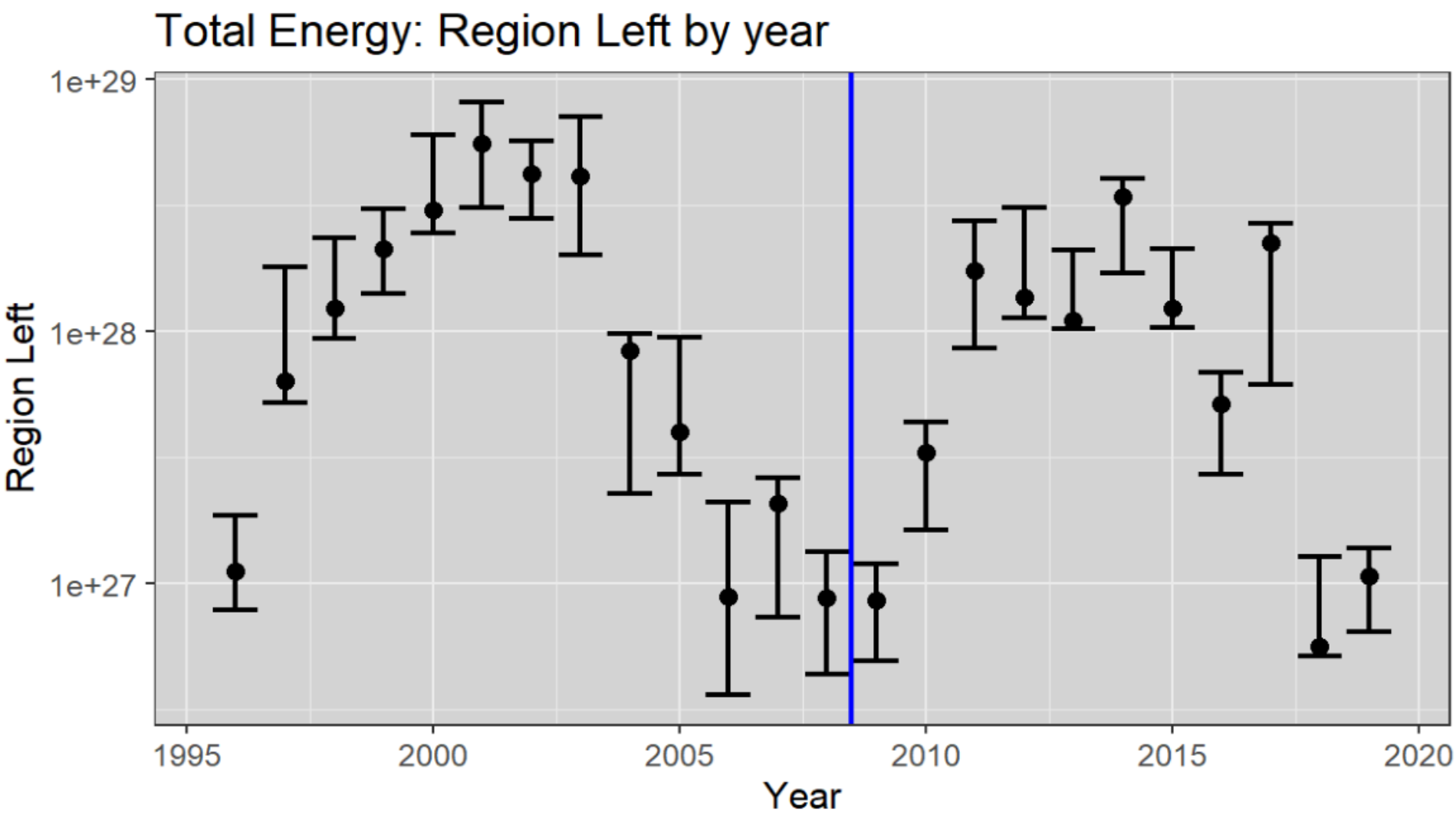


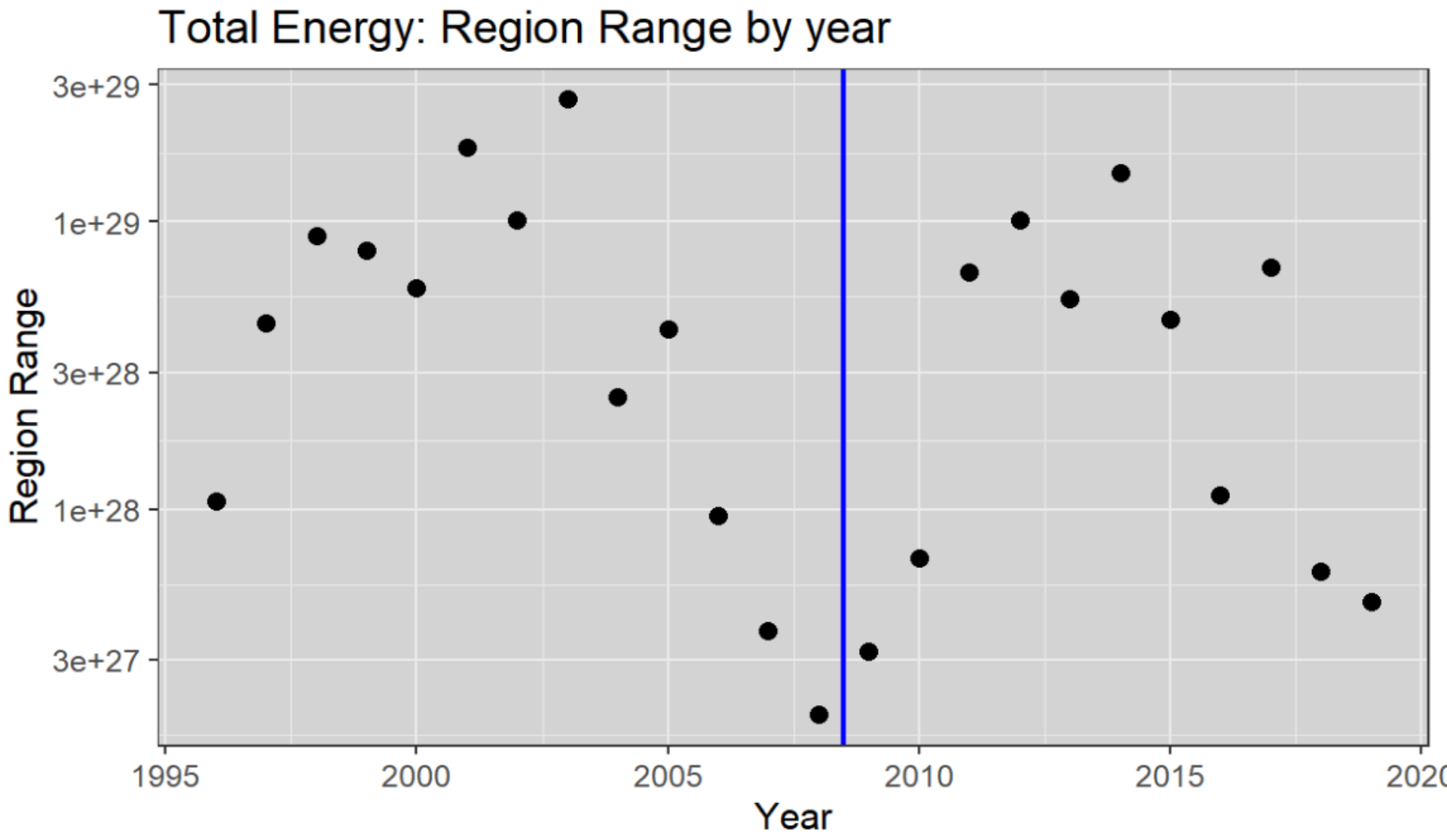
MPS Simulation Performance

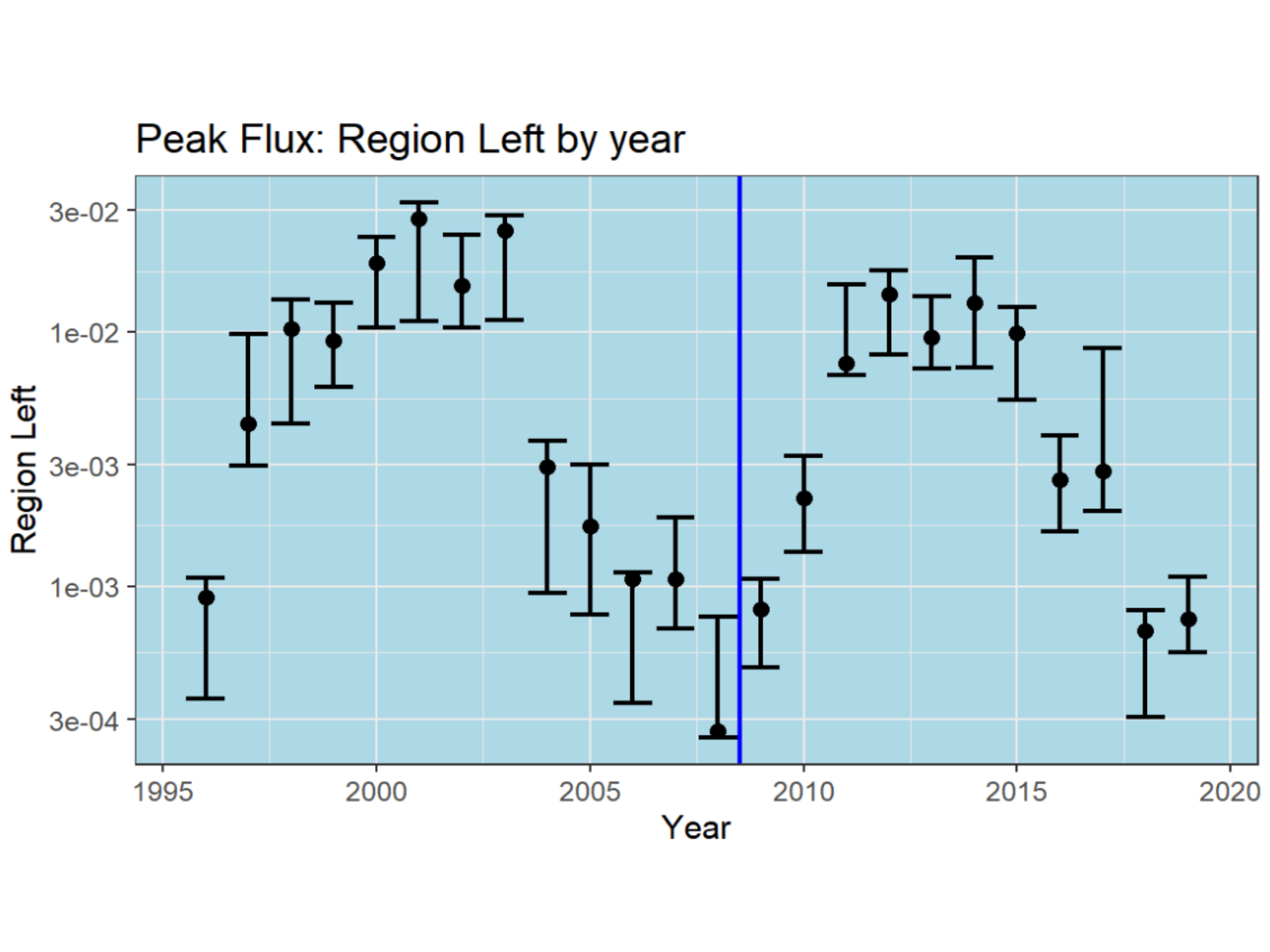
- Algorithm's run time increases as N^2

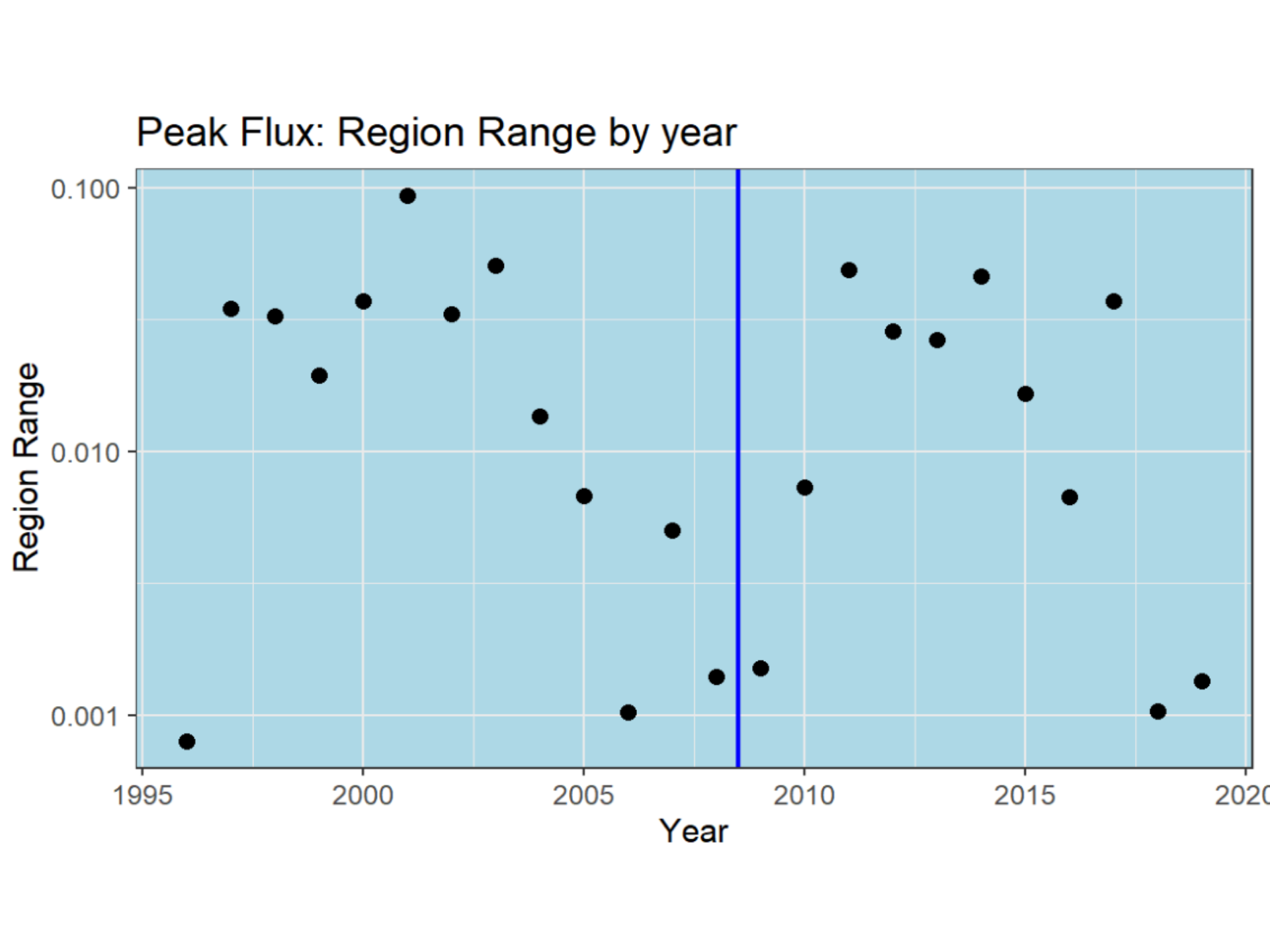


Results


Total Energy: Region Left by year
1e+29
1e+28
1e+27
Region Left
1995
2000
2005
2010
2015
2020
Year


Total Energy: Region Range by year
Region Range
3e+29
1e+29
3e+28
1e+28
3e+27
1995
2000
2005
2010
2015
2020
Year


Peak Flux: Region Left by year
Region Left
3e-02
1e-02
3e-03
1e-03
3e-04
1995
2000
2005
2010
2015
2020
Year


Peak Flux: Region Range by year
0.100
0.010
0.001
Region Range
1995
2000
2005
2010
2015
2020
Year