Long memory and fractality among global equity markets: A multivariate wavelet approach

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Abstract

This paper seeks to understand the long memory behaviour of global equity returns using novel methods from wavelet analysis. We implement the wavelet based multivariate long memory approach, which possibly is the first application of wavelet based multivariate long memory technique in finance and economics. In doing so, long-run correlation structures among global equity returns are captured within the framework of wavelet-multivariate long memory methods, enabling one to analyze the long-run correlation among several markets exhibiting both similar and dissimilar fractal structures.

Keywords: Long memory, Fractal connectivity, Wavelets, Hurst exponent

JEL Classification: C13, C14, C22, C32, G15.

1.1 Introduction

The estimation and the analysis of long memory parameters have mainly focused on the analysis of long-range dependence in stock return volatility using traditional time and spectral domain estimators of long memory. The definitive ubiquity and existence of long memory in the volatility, estimated or generated using various methods, of stock returns is an established stylized fact. The presence of long memory requires major revisions in the standard estimation procedures without which the estimated results can be seriously biased. In this paper on long memory among global equity markets, several wavelet based estimators are applied to test for the presence of long memory in the global equity returns and returns volatility. The presence of long memory in the volatility of the stock returns as well as some returns themselves is demonstrated from the empirical evidences. Furthermore, phases of efficiency and inefficiency of markets, as adjudicated by presence of both long memory and no-memory, is evidenced when the analysis is performed using rolling windows. The existence or absence of long memory in

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stock returns can be used to determine the stage of market development in terms of efficiency and inefficiency. According to the weak-form version of the *Efficient Market Hypothesis* (EMH), equity prices contain all available information about the equity price, acquired from past trading. This suggests that prediction of prices, when the EMH hold, is not possible. On the other hand, the presence of long-memory in equity returns and volatility implies that distant observations in the equity returns and volatility series are related to each other. This implication leads to the rejection of efficient markets as the presence of long range dependence is incompatible with the basic tenets of efficient market hypothesis (EMH).

The analysis of long memory is further extended to estimate long-run correlation matrix of global equity returns using wavelet based multivariate long memory estimator. The estimates generated from multivariate long memory model allow one to detect mechanisms that give rise to long memory. Long memory among several groups of equity markets can either be the result of some same underlying process generating the data or it can be a product of multiple mechanisms (Wendt et al., 2009). The long-run correlation matrix, also known as the fractal connectivity matrix, generated from the multivariate long-memory model helps in determining the convergence of wavelet correlations of long-range dependent processes. The convergence to an asymptotic value over a range of low-frequency wavelet scales helps one in determining regimes of fractal connectivity (Achard et al., 2008; Achard and Gannaz, 2016). In doing so, associations and similarities between the processes that generate equity market returns of various markets can be highlighted. Furthermore, a hierarchical clustering algorithm is implemented on the elements of the generated fractal connectivity matrix to group markets having similar long-run correlation behavior. Significant rise in long-run correlations is evidenced during the subprime crisis period. However, long-run correlations among all equity markets are very low[2]. Nonetheless, comparisons can be drawn with regard to the long-memory behavior of global equity markets during both normal and crisis-hit periods. In this paper, the issue of multifractality of equity returns is also highlighted via the implementation of a rolling window long memory procedure. The resulting estimates of long memory parameters, with varying degrees of fractal structures, are not always stable and fluctuate between regimes of efficiency and inefficiency. This implies

[2] This is not to be confused with the regular wavelet correlation where correlations tend to be strong in the long-run. Correlations based on fractal connectivity are used to determine the similarity in mechanisms that generate the underlying long memory behaviour among markets.

that markets are not always efficient in the weak sense and arbitrage opportunities exist. The pattern of evolution of long memory parameter, as verified from the time-series of Hurst exponents, is in agreement with the adaptive markets hypothesis which allow interplay of "*[...] complex market dynamics, with cycles as well as trends, and panics, manias, bubbles, crashes, and other phenomena that are routinely witnessed in natural market ecologies.*" (Lo, 2004, p. 24). In the next section some relevant works related to long memory behaviour of global equity markets are reviewed.

1.2 Literature Review

Since the groundbreaking work of Hurst (1951), where he investigated the flow of river Nile and found evidence of long range dependence, there has been significant interest, spanning researchers across disciplines, in the phenomena of long memory. Mandelbrot and Van Ness (1968), using the idea of Hurst exponent, employed the idea of long-memory processes in conjunction with fractional Brownian motion and related stochastic processes. However, in the field of time series analysis, Granger and Joyeux (1980) and Hosking (1981) were among the first to integrate long memory processes with time series methods. Since then, a plethora of time-series based models of long memory has been developed to analyze long-range dependence in stochastic processes. However, a majority of research articles that focuses on the estimation of long memory parameters and detection of the same relies on the traditional rescaled range (R/S) approaches of Mandelbrot (1965) and its modified version developed by Lo (1991). The spectral domain approach proposed by Geweke and Porter-Hudak (1983) to estimate the long memory parameter has been used by many researchers too. This section reviews some important works on long memory concerning the analysis of global equity markets. Numerous studies have been carried out to test the presence of long run dependence in stock markets. Works related to the estimation and analysis of long memory parameters have mainly focused on the analysis of long-range dependence in stock return volatility. The concurrent use of squared returns and absolute returns as a measure of volatility is very evident from the literature that focuses on the analysis of stock returns volatility (see for eg. Ding et al., 1993; Granger and Ding, 1995; Lobato and Velasco, 2000). Studies which analyzes the long memory parameters and confirms the existence of long memory in stock returns volatility are abundant. However, since the prime focus of this paper will be in investigating long memory in global equity returns, importance will be given to studies analyzing long memory in equity returns instead of returns volatility.

There is no clear consensus, among studies that attempt to detect long memory in financial data, on the existence of the phenomenon of long-range dependence. A plethora of studies that support the presence of long memory in financial time series is documented in the literature with commensurate number of articles rejecting the presence of long memory. The presence of long memory in squared daily returns of S&P 500 index is evident in the works of Ding et al. (1993) where significant autocorrelation for lags up to ten years were present. Similarly, Lobato and Savin (1998) also demonstrated the presence of long memory in the squared returns of the S&P 500 dataset spanning three decades.

Ray and Tsay (2000) unearthed the presence of strong *long-range dependence* in the volatilities of selected companies of the S&P 500 index. Granger and Ding (1995) also detected the presence of long memory in the absolute value of stock returns. Furthermore, Lobato and Velasco (2000) using a frequency domain tapering procedure in a multi stage semi-parametric method unearthed the presence of long memory in stock returns and volatility of returns. The presence of long memory in the returns of Brazilian equity market is documented in Assaf and Cavalcante (2005).

Barkoulas et al. (2000), while investigating the long memory properties of the Athens stock exchange, find evidence of long-range persistence in the returns of the Athens stock market. Moreover, the forecast performance of a long memory incorporated model significantly outdid forecasts generated from a regular random walk model. Similarly, Panas (2001), using a spectral measure of fractality along with the Levy index, found nonlinearities in Greek equity returns and unearthed the existence of long memory, thereby rejecting the weak-form efficiency of the Greek equity market. Henry (2002), using a mixture of semi-parametric and spectral estimators, found evidence of long memory in the returns of South Korean stock market. Moreover, some evidence of weak long memory was unearthed in the markets of Germany and Taiwan.

In their analysis of the EMH, Jagric et al. (2005) employed a wavelet method to test for long memory in the returns of some select European markets. The empirical investigations documented the presence of long-range dependence in four emerging eastern European markets, thereby rejecting evidence in favour of the efficient market hypothesis. Similar analysis using wavelet based methods to detect long memory in the returns of the Dow Jones Industrial average (DJI) were employed by Elder and Serletis (2007) where no evidence of long memory was detected, thereby supporting results from a vast number of studies that reject the presence of long memory in the developed markets of the U.S. However, the presence of long memory in the equity returns of some developed markets of Europe, the U.S., and Japan is documented in Ozdemir (2007). Furthermore, Ozun and Cifter (2007), also using a wavelet based estimator of long memory, found some evidence of long-range dependence in the returns of the Istanbul Stock Index, thereby rejecting the weak form efficiency of Istanbul share prices. Similarly, evidence of long memory in the equity markets of G7 countries is documented in Bilal and Nadhem (2009). On the other hand, Mariani et al (2010), using detrended fluctuation analysis and truncated Levy flight method, found evidence of long memory in several eastern European markets. However, among the countries that are part of the Organisation for Economic Co-operation and Development (OECD), long-memory, as investigated by Tolvi (2003), was only evidenced in the smaller equity markets of Denmark and Finland.

Jefferis and Thupayagale (2008), using a long-memory variant of the GARCH model, investigated long memory behaviour of some select African equity markets and found evidence supporting the presence of long-memory in the developing markets of Botswana and Zimbabwe. The presence of long memory in the developing markets of Central and Eastern European countries (CEE) is documented in the studies of Jagric et al. (2006) and Kasman et al. (2009), where the presence of long memory in equity returns is specifically limited to the developing markets of Hungary, Czech, Slovenia and Croatia.

Kristoufek and Vosvrda (2012) constructed a measure of efficiency by measuring the distance between an efficient case and a vector containing long memory and other measures of fractality. Long memory is evidenced in many developing and emerging markets whereas all developed markets show signs of efficiency, with the Japanese NIKKIEI leading all other developed markets in terms of efficiency.

Cont (2005) attempted to identify economic intuition and mechanisms behind the existence of fractality and long memory in returns and returns volatility. The possible economic factors underlying the existence of long memory in volatility are, i) heterogeneous investment horizons of market agents, ii) evolutionary trading models that employ genetic algorithms, iii) market fluctuations arising out of investors' sudden switch between several trading strategies, and iv) the inactivity of investors, operating at certain time periods and market regimes, based on trading strategies or behavioral aspects.

The presence of heterogeneous investment horizons can be one of the most important factors that generate long memory behavior in equity markets. Investment horizons, which can be successfully disaggregated into several microunits using wavelet methods to delineate price behaviors from varying time-horizons, contain varying returns and volatility structures. The aggregation of all microunits, ranging from very short run to long run, is said to produce long memory properties in the aggregate series (see Granger, 1980; Davidson and Sibbertsen, 2005). However, contemporaneous aggregation of microunits having both short-memory and long-memory can lead to spurious long-memory in the aggregate, thereby biasing the results in favour of long-range dependence. Granger and Ding (1996) attempt to theoretically explain this bias arising out of aggregation but however fail to empirically demonstrate that long memory in returns volatility of stock indices is due to aggregating volatility of individual stocks containing short-memory. Furthermore, Andersen and Bollerslev (1997) theoretically demonstrated volatility to be an assortment of various heterogeneous information structures in the short-run and concluded that the underlying volatility processes contain long memory. Nonetheless, in some major studies, estimates of long-memory are found to be uncontaminated by aggregation effects thereby supporting evidence in favour of fractality in equity returns (see Han, 2005; Souza, 2007; Kang et al., 2010), thereby rejecting any indication of spurious long memory.

Studies examining long memory in financial time series are relatively few. Jensen (1999) estimated the "long memory parameter of a fractionally integrated process" using a wavelet based OLS method. By projecting volatility in the time-frequency domain, Jensen and Whitcher (2000) demonstrated the effectiveness of the wavelet based estimator in capturing nonstationary long-memory behavior. Vuorenmaa (2005) investigated the time-varying long memory of Nokia Oyj returns using the wavelet OLS method and found significantly strong long memory during the dot-com bubble period. Ozun and Ciftr (2007), demonstrating the superiority of wavelet OLS method as compared to the spectral long memory estimator of Geweke and Porter-Hudak (1983), found significant long memory in the returns of Istanbul stock exchange. Similarly, DiSario et al. (2008), on investigating the volatility structure of S&P 500 returns using the wavelet OLS method, found evidence of long-memory in the S&P 500 returns volatility.

In the same vein as the aforementioned studies, Tan et al. (2012) while examining the fractal structure of emerging economies using wavelet OLS method demonstrated significant long memory in the returns of larger firms as compared to smaller firms. Likewise, Tan et al. (2014), using the wavelet estimator of Jensen (1999) and detrended fluctuation analysis, examined long memory behavior of equity returns and volatility of ten markets from both developing and developed economies. On the other hand, Power and Turvey (2010) investigated long memory structure of fourteen commodity futures using the Hurst estimator of Veitch and Abry (1999) and demonstrated long-range dependence in all commodities. The presence of long-memory in the equity markets of both developing and developed economies was demonstrated. Boubaker and Peguin-Feissolle (2013) proposed semiparametric wavelet base long memory estimators and demonstrated its superiority, with respect to several non-wavelet estimators, using simulation experiments. More recently, Pascoal and Monteiro (2014), investigating the predictability of the Portuguese stock returns using wavelet estimators of long memory, fractal dimension and the Holder exponent, found no evidence of long memory in the PSI20 returns, thereby confirming the efficiency of the Portuguese equity market.

This paper investigates long memory among global equity markets using estimators from the wavelet domain. Studies investigating long memory in global financial markets based on wavelet based long memory methods are relatively fewer as compared to traditional time and spectral domain estimators of long memory. Furthermore, empirical studies based on wavelet domain estimators of long-range dependence are practically nonexistent in the case of Indian equity markets.

This paper, however, implements the wavelet based approaches of Abry and Veitch (1998) and Abry et al. (2003) to examine the Hurst exponents, and its time-varying structure, of global equity markets. Moreover, an analysis of multivariate long memory of global equity markets using the recent method of Achard and Gannaz (2016) is carried out, which possibly is the first application of wavelet domain multivariate long memory technique in finance and economics.

The aforementioned multivariate method allows one to analyze the long-run correlation among several markets exhibiting fractal structures.

1.2 Data

The empirical data consists of twenty four major stock indices comprising both developed and emerging markets. The stock indices included are BSE 30 (India), Nasdaq (U.S.), S&P 500 (U.S.), DJIA (U.S.), FTSE 100 (Great Britain), CAC40 (France), DAX 30 (Germany), NIKKEI 225 (Japan), KOSPI (Korea), KLSE (Malaysia), JKSE (Indonesia), TAIEX (Taiwan), SSE (China), STI (Singapore), HSI (Hong Kong), BEL20 (Belgium), ATX (Austria), AEX (Netherlands), IBEX 35 (Spain), SMI (Switzerland), STOXX50 (Eurozone), ASX 200 (Australia), KSE100 (Pakistan), and IBOV (Brazil). The period of study ranges from 01-07-1997 to 20-01-2014 consisting of 4096 dyadic length observations making it suitable for various wavelet methods. Returns of all the stock indices are calculated by taking first order logarithmic differences.

1.3 Methodology

In this paper, wavelet based measures of long memory parameters are applied to analyze long memory behavior of global equity returns. There are several classes of wavelet based long memory estimators that can measure long-term correlations present in a time series. The wavelet based Hurst estimator of Abry and Veitch (1998) is used in a rolling window algorithm to analyze the time varying structure of the Hurst parameter and the evolution of Hurst parameter and long range dependence over time.

The long range dependence phenomenon is associated with a slow power law decay of the autocorrelation function of a stationary process *x*. The covariance function $\gamma_x(k)$ of the long memory process *x* takes the following form,

$$\gamma_x(k) \sim \mathrm{c}_\gamma \, k^{-(2-2H)}, \quad k \to +\infty \tag{1.1}$$

where c_γ is a positive constant and $H \in (0, 0.5)$. The Hurst parameter H is used to measure the presence of long memory. The spectrum $\Gamma_x(\nu)$ of the long memory process x is given by,

$$\Gamma_x(\nu) \sim c_f \,|\, \nu \,|^{1-2H}, \quad \nu \to 0 \tag{1.2}$$

where ν is the frequency, $c_f = \pi^{-1} c_\gamma \Lambda(2H-1)\sin(\pi - \pi H)$, and the Gamma function is given by Λ. This mathematical structure of long memory processes is the reason for its inclusion in a class of stochastic processes which have the $1/|\nu|^\alpha$ form. The property of long memory also finds some close association with the phenomenon of scale invariance, self-similarity and fractals. Hence, statistically self-similar processes like fractional Brownian motion (FBM) are closely related to long memory phenomenon.

Let γ_0 be an arbitrary reference frequency selected by the choice of ψ_0, the mother wavelet. The amount of energy in the signal during scaled time $2^j k$ and scaled frequency $2^{-j}\nu_0$ is measured by the squared absolute value of the detail wavelet coefficient $|d_x(j,k)|^2$. A wavelet based spectral estimator of Abry et al. (1998) is constructed by taking a time average of $|d_x(j,k)|^2$ at a given scale, and is given by,

$$\hat{\Gamma}_x(2^{-j}\nu_0) = \frac{1}{n_j}\sum_k |\, d_x(j,k)\,|^2 \tag{1.3}$$

where n_j is the "number of wavelet coefficients" at level j, and $n_j = 2^{-j} n$, where n is the data length. Therefore , $\hat{\Gamma}_x(\nu)$ captures the amount of energy that lies within a given bandwidth and around some frequency ν. Hence, $\hat{\Gamma}_x(\nu)$ can be regarded as an estimator for the spectrum $\Gamma_x(\nu)$ of x. The wavelet based estimator of the Hurst exponent $\hat{H}$ is designed by performing a simple linear regression of $\log_2(\hat{\Gamma}_x(2^{-j}\nu_0))$ on j, i.e.,

$$\log_2(\hat{\Gamma}_x(2^{-j}\nu_0)) = \log_2\left(\frac{1}{n_j}\sum_k |\, d_x(j,k)\,|^2\right) = (2\hat{H}-1)\,j + \hat{c} \tag{1.4}$$

where $\hat{c}$ estimates $\log_2(c_f \int |\nu|^{(1-2H)} |\Psi_0(\nu)|^2 d\nu)$, where Ψ_0 is the Fourier transform of the mother wavelet ψ_0. A weighted least square estimator is constructed by performing a WLS fit between the wavelet scales j_1 and j_2 which gives the estimator of the "*Hurst exponent*", H.

$$\hat{H}(j_1, j_2) \equiv \frac{1}{2}\left[\frac{\sum_{j=j_1}^{j_2} S_j j n_j - \sum_{j=j_1}^{j_2} S_j j \sum_{j=j_1}^{j_2} S_j \eta_j}{\sum_{j=j_1}^{j_2} S_j \sum_{j=j_1}^{j_2} S_j j^2 - \left(\sum_{j=j_1}^{j_2} S_j j\right)^2} + 1\right] \tag{1.5}$$

where $\eta_j = \log_2\left(\frac{1}{n_j}\sum_k |d_x(j,k)|^2\right)$ and the weight $S_j = (n \ln^2 2)/2^{j+1}$ is the inverse of the theoretical asymptotic variance of η_j. The estimators of multivariate long memory and the related "*fractal connectivity matrix*", based on the above univariate estimator is given in Achard et al. (2008) and Achard and Gannaz (2016).

1.4 Empirical Results

The empirical analysis first proceeds with visually analyzing the dynamic nature of long memory of select equity returns, as given by the time-series plot of long memory estimates, which is obtained by applying the wavelet based estimator of the Hurst exponent developed by Abry and Veitch (1998) and Abry et al. (2003) in a rolling windows framework.

1.4.1 Rolling windows Hurst analysis of time varying market efficiency

Long memory in equity returns can vary with due to the change in the efficiency of equity markets over time. The advancement of equity markets, coupled with varying phases of market development, policy decisions and financial turbulence, can significantly alter long memory structure of financial markets. Therefore, the estimates of long memory parameters are not always stable for all markets. In view of the changing structures and efficiency of equity markets, the analysis of long memory behavior of equity returns constitutes an analysis of time-varying long memory behavior of equity returns. Consequently, the Hurst exponents of select equity returns are estimated in a rolling window framework. The length of the window contains 260 observations which approximately equals one year. The window is moved forward by an increment of twenty four day i.e. a one month increment. Finally, the estimation of wavelet based Hurst exponent in rolling window framework generates a time-series of Hurst exponent.

The plots of the time-series of Hurst exponents of select equity returns can be visually inspected to determine the phases of efficiency and inefficiency, as measured by the drift of the Hurst exponent from the threshold value of 0.5, given by the horizontal line around the Hurst value of 0.5 in Figure 1.1-1.2 which plots the generated Hurst series against time given in the horizontal axis in years. The vertical axis in Figures 1.1-1.2 shows the Hurst values.

Figure 1.1

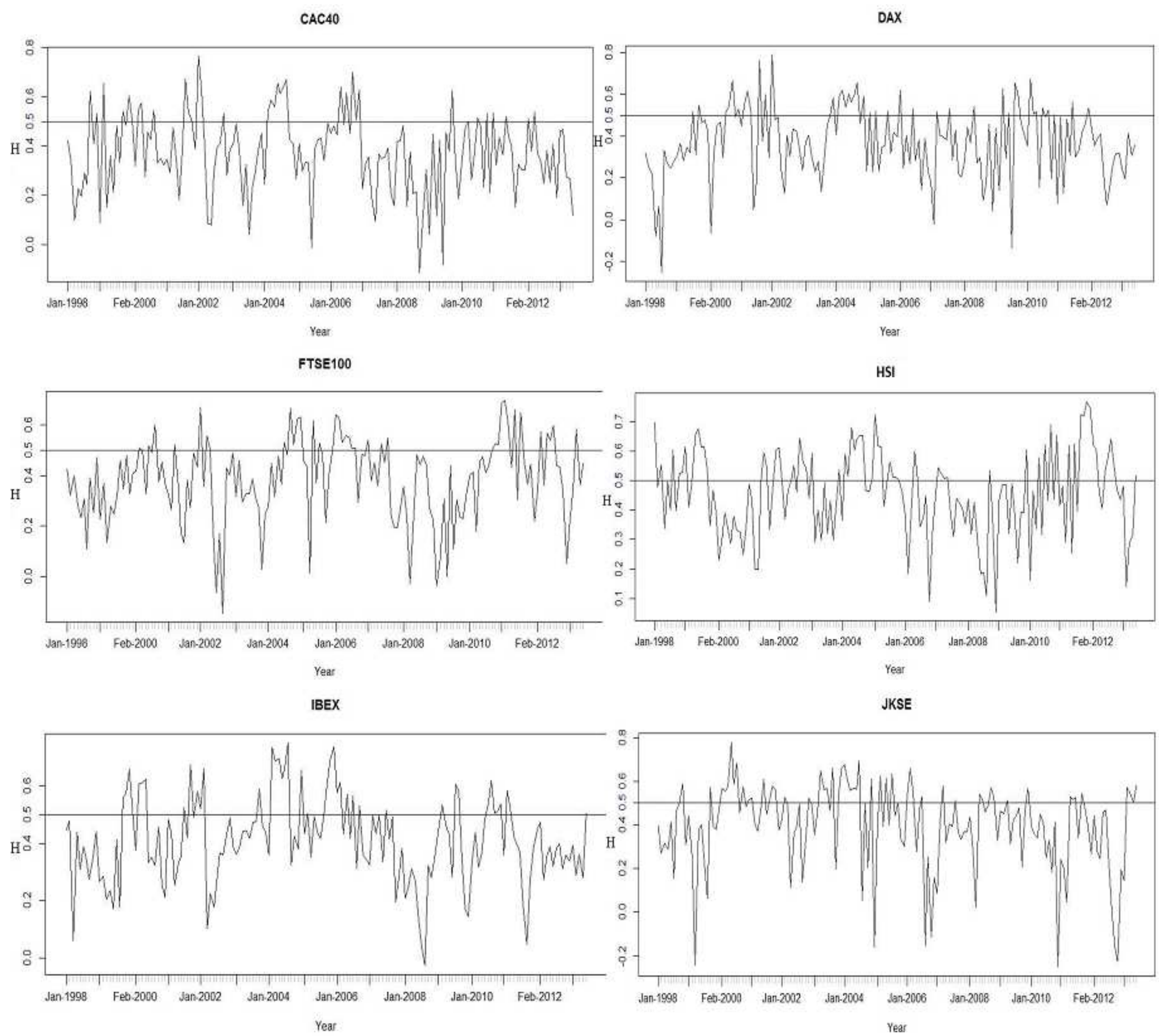
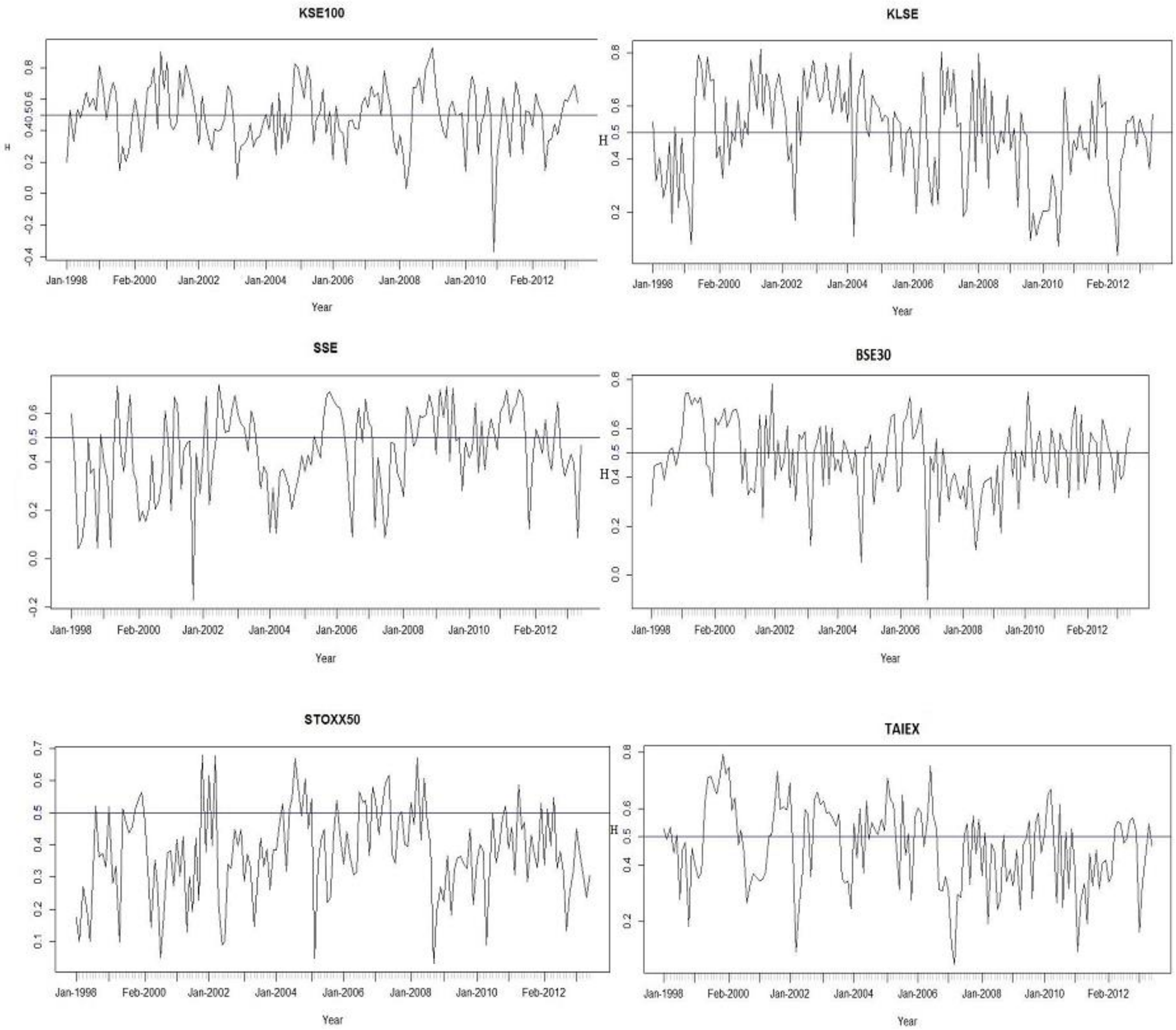


Figure 1.2



The plots of Hurst series given in Figures 1.1-1.2 reveal the time-varying nature of Hurst exponents. The developed equity markets of Europe show relatively less degree of persistence in returns with Hurst exponent below 0.5 for most of the time period. However, equity returns of France (CAC 40) and Germany (DAX) exhibited long-range dependence during the first three quarters of 2004, thereby allowing some possibility for returns predictability during that period. Nevertheless, equity returns of France and Germany has been relatively unpredictable throughout the studied time period. The same holds true for the Eurozone (STOXX 50) equity returns. On the other hand, the emerging markets equity returns seems to exhibit varying phases of return predictability with high values of Hurst exponent during some time intervals. For example, some indication of persistence in the returns of the Indian equity market (BSE 30) can be observed during the one year period of January 1999-January 2000 which is then followed by a sharp drop in Hurst exponent around February 2000, which can be attributed to market fluctuations arising out of the dot-com bubble. However, long-memory rises again after March 2000 extending up to January 2001 indicating some evidence of returns predictability during this period. Some evidence of returns predictability, as indicated by the presence of long-memory with Hurst value above 0.5, is also observed during Feb 2006-November 2006 and the last half of 2012. Moreover, the Asian markets of Hong Kong (HSI), China (SSE), Indonesia (KLSE) and Taiwan (TAIEX) exhibit evidence of returns predictability. Persistence in equity returns can be evidenced for, i) HSI during mid 2011-mid 2012, ii) SSE during mid 2005- early 2006, mid 2008-January 2009 and late 2010-late 2011, iii) KLSE during mid 1999-February 2000 and 2001-2002 and, iv) TAIEX during January 1999-mid 2000.

Interestingly, with the exception of equity returns of Pakistan (KSE 100) and China (SSE), returns markets from both developed and emerging economies exhibit anti-persistence (short-memory) during the financial crisis period of 2008, thereby eliminating any scope for returns predictability during this period. Moreover, barring periods of abrupt changes in Hurst parameter beyond and within the threshold range of 0.5, the phases of market efficiency are more pronounced for the developed equity markets where Hurst exponents of these markets' equity returns tend to lie below the threshold range of 0.5. However, efficiency of both developed and emerging equity markets is not stable throughout the studied time-period, allowing investors some arbitrage opportunities. Nonetheless, investors operating in emerging equity markets have more scope for arbitrage as these markets exhibit relatively more phases of long-range dependence.

1.4.2 Long-range correlation among global equity returns

The long memory structure, of both developed and emerging markets, and its dynamic evolution can vary with stock returns of different markets and their underlying structure as evidenced from the previous section. However, there can be similarities in the fractal structure of the stock returns of some markets. Therefore, in this section, an attempt is made to investigate the long-range correlation among global equity returns using the newly developed multivariate long-memory estimators of Achard and Gannaz (2016) which offer an efficient way to estimate long-memory and analyze similarities in fractal structure among equity markets. The resulting long-run correlation matrix, estimated using the aforementioned method, aids in scrutinizing the correlation structure among equity returns operating at long-range frequencies. The long run correlation matrix, also known as the fractal connectivity matrix, furthermore assists in analyzing similarity of fractal structures among equity markets. The elements of long-range correlation matrix, of equity returns exhibiting LRD, is clustered using the hierarchical clustering algorithm to analyze the structure of equity returns correlations during both stable and turbulent financial phases, thereby assisting in identifying *fractally similar* market groups.

Figure 1.3

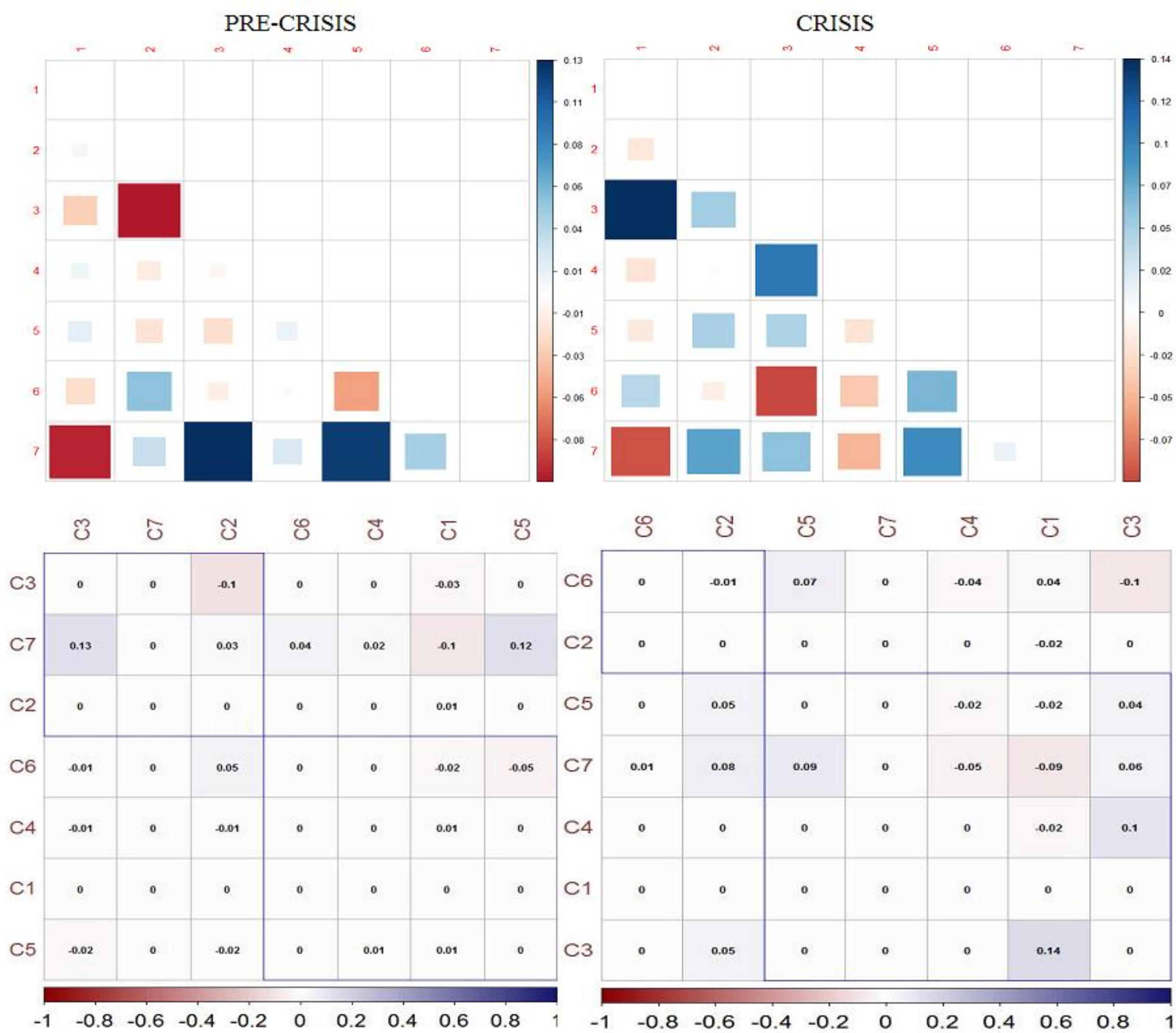


Figure 1.3 gives the long-run correlation matrix, displaying long-run correlation among seven equity markets of the U.S. (SP500), France (CAC40), Germany (DAX), Japan (NIKKEI), South Korea (KOSPI), Indonesia (JKSE) and India (BSE30). The upper panel of Figure 1.3 shows the fractal connectivity matrix whereas the lower panel gives the clustered version, using hierarchical clustering algorithm, of the long-run matrix of correlations. The left panel shows the correlation matrix of equity returns during the pre-subprime crisis period whereas the right panel gives the matrix of equity returns during the crisis period. The color coded legend, on the right side of fractal connectivity matrix and towards the bottom of the clustered matrix, helps in identifying the strength of long-range correlations. The strength of correlation rises as we move from red (low) to blue (high). The returns of seven aforementioned equity markets are labeled numerically from 1 to 7 in the upper panel and alphanumerically from C1 to C7 in the lower panel, where “C1, C2, C3, C4, C5, C6 and C7” correspond to SP500, CAC40, DAX, NIKKEI, KOSPI, JKSE and BSE30, respectively. It is evident from the long-run correlation matrix (upper panel) that long-range correlations significantly rises during the subprime crisis period, as indicated by larger number of elements in blue depicting positive correlations. The clustering of markets according to similar fractal structures is different during pre-crisis and crisis periods. Moreover, five markets (C1, C3, C4, C5 and C7) are clustered together during the subprime crisis period reflecting similar long memory behavior among these markets during crisis period. This is in line with the results from previous section where fractal structure of returns from SP500 (C1), DAX (C2), NIKKEI (C3), KOSPI (C4) and BSE30 (C7) behave similarly during the subprime period.

Figure 1.4

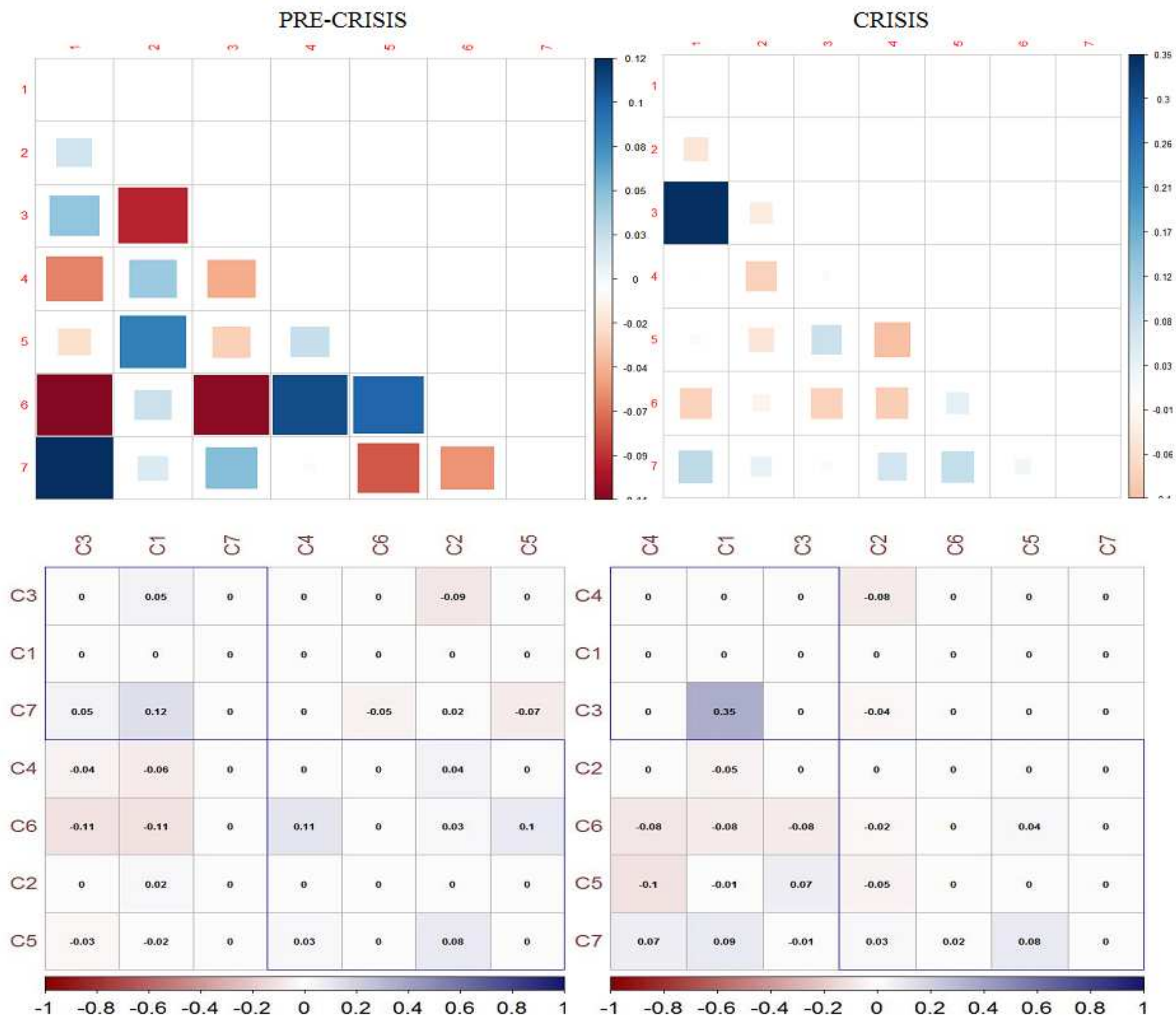


Figure 1.4 gives the long-run correlation matrix among seven Asian equity markets of South Korea (KOSPI), Malaysia (KLSE), Taiwan (TAIEX), China (SSE), Singapore (STI), Hong Kong (HSI) and India (BSE30) .The fractal structure of Asian equity returns given in Figure 1.4 evidences the rise in long-range correlation between South Korean and Taiwanese equity returns during the subprime crisis. The long-range correlation among other Asian markets are very low during both crisis and non-crisis periods indicating dissimilar fractal structures. This is also evidenced from clustering of equity returns from these markets where markets forming clusters are almost similar during both crisis and non-crisis periods. This is in contrast with the developed

western markets where fractal structures, based on long-range correlation coefficients and clustering of the same, are not similar during crisis and non-crisis periods.

1.5 Conclusion

This paper investigated the phenomenon of long memory among global equity returns using methods from both univariate and multivariate class of wavelet based long memory estimators. Since dependence structure of equity returns over time can be time-varying, the analysis of long memory is first carried out in a time varying framework. This helps in examining the evolution of long memory parameter over time, thereby allowing one to detect phases of market efficiency and inefficiency. Therefore, analysis of the evolutionary nature of long memory is captured using rolling windows estimation method where long memory of equity returns from both emerging and developed markets are investigated. The results indicate that the developed equity markets of Europe and the U.S. show relatively less degree of persistence. However, phases of long-memory, though smaller, are detected for some developed markets. In contrast, markets from emerging economies are found to have relatively more phases of inefficiency, indicating presence of arbitrage opportunities. Moreover, emerging markets' equity returns are found to move between phases of long memory and short memory. Furthermore, long memory is not evidenced during the subprime crisis period of 2008 for majority of markets which is in line with the wavelet based study of Tan et al. (2014), where faster information disseminated among investors during financial crises is said to curtail speculative behavior, thereby affecting predictability of markets. Likewise, the time-varying nature of long memory and varying phases and stages of market efficiency is consistent with the conception of adaptive markets, where market efficiency should be viewed from an evolutionary framework.

The results obtained from time varying long memory analysis reinforces the notion that markets are not always efficient. On the other hand, markets tend to traverse through different dynamics and are subjected to evolutionary patterns, where stages of both efficiency and inefficiency come into play. Since, evidence of heterogeneity in market efficiency during both stable and turbulent periods is demonstrated via several measure of market fractality, investors should take cognizance of the dynamic and evolutionary nature of market efficiency; and tactically formulate investment strategies based on the fractal structures of equity markets which are evolutionary and time varying in nature. The structure of fractality in equity returns based on fractal connectivity matrix, which aids one in investigating the long memory properties of equity markets in greater detail, is used to analyze similarities in long memory behavior among equity markets. The analysis revealed the existence of similarities of fractal structures among returns from developed markets during periods of financial turbulence. Therefore, multivariate wavelet estimator of long-range correlation and fractal similarities provide an efficient way of analyzing equity markets' correlation structure. However, analyses of global equity returns using the aforementioned method demands a more thorough investigation as there are no studies in literature analyzing fractal connectivity of financial markets.

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